# Computer Algebra
# Independent Integration Tests

Summer 2023 edition with Rubi V 4.17.3

## 3-Logarithms/60-3.2.2-f+g-x-^m-h+i-x-^q-A+B-log-e-a+b-x-over-c+d-x-^n-^p

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# Contents

# CHAPTER 1

## Introduction

This report gives the result of running the computer algebra independent integration test.The download section in on the main webpage contains links to download the problems in plain text format used for all CAS systems. The number of integrals in this report is [ 263 ]. This is test number [ 60 ].

# 1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.3.1 (August 16, 2023) on windows 10.
2. Rubi 4.17.3 (Sept 25, 2023) on Mathematica 13.3.1 on windows 10
3. Maple 2023.1 (July, 12, 2023) on windows 10.
4. Maxima 5.47 (June 1, 2023) using Lisp SBCL 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
5. FriCAS 1.3.9 (July 8, 2023) based on sbcl 2.3.0 on Linux via sagemath 10.1 (Aug 20, 2023).
6. Giac/Xcas 1.9.0-57 (June 26, 2023) on Linux via sagemath 10.1 (Aug 20, 2023).
7. Sympy 1.12 (May 10, 2023) Using Python 3.11.3 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was run directly in Python not via sagemath.

## 1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed. If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

| System | % solved | % Failed |
|---|---|---|
| Rubi | 100.00 ( 263 ) | 0.00 ( 0 ) |
| Mathematica | 94.68 ( 249 ) | 5.32 ( 14 ) |
| Maxima | 68.44 ( 180 ) | 31.56 ( 83 ) |
| Maple | 61.22 ( 161 ) | 38.78 ( 102 ) |
| Fricas | 59.32 ( 156 ) | 40.68 ( 107 ) |
| Mupad | 48.29 ( 127 ) | 51.71 ( 136 ) |
| Giac | 47.53 ( 125 ) | 52.47 ( 138 ) |
| Sympy | 20.53 ( 54 ) | 79.47 ( 209 ) |

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

| grade | description |
|---|---|
| A | Integral was solved and antiderivative is optimal in quality and leaf size. |
| B | Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size. |
| C | Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons<br>1. antiderivative contains a hypergeometric function and the optimal antiderivative does not.<br>2. antiderivative contains a special function and the optimal antiderivative does not.<br>3. antiderivative contains the imaginary unit and the optimal antiderivative does not. |
| F | Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised. |

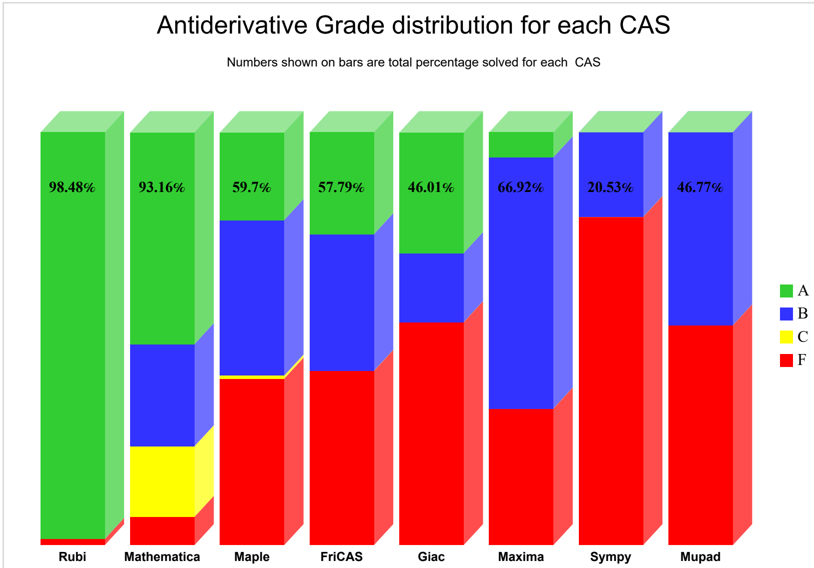
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

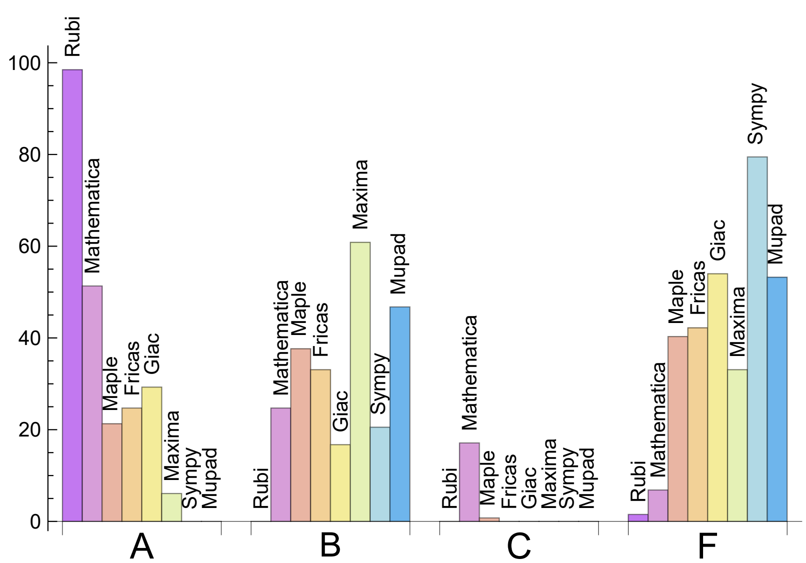
| System | % A grade | % B grade | % C grade | % F grade |
|---|---|---|---|---|
| Rubi | 98.479 | 0.000 | 0.000 | 1.521 |
| Mathematica | 51.331 | 24.715 | 17.110 | 6.844 |
| Giac | 29.278 | 16.730 | 0.000 | 53.992 |
| Fricas | 24.715 | 33.080 | 0.000 | 42.205 |
| Maple | 21.293 | 37.643 | 0.760 | 40.304 |
| Maxima | 6.084 | 60.837 | 0.000 | 33.080 |
| Mupad | 0.000 | 46.768 | 0.000 | 53.232 |
| Sympy | 0.000 | 20.532 | 0.000 | 79.468 |

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.




The figure below compares the grades of the CAS systems.




The following table shows the distribution of the different types of failures for each CAS. There are 3 types failures. The first is when CAS returns the input within the time limit, which means it could not solve it. This is the typical failure and given as **F**.

The second failure is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned. This is assigned **F(-1)**.

The third is due to an exception generated, indicated as **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in the CAS itself. This type of error requires more investigation to determine the cause.

| System | Number failed | Percentage normal failure | Percentage timeout failure | Percentage exception failure |
|---|---|---|---|---|
| Rubi | 0 | 0.00 | 0.00 | 0.00 |
| Mathematica | 14 | 100.00 | 0.00 | 0.00 |
| Maxima | 83 | 100.00 | 0.00 | 0.00 |
| Maple | 102 | 100.00 | 0.00 | 0.00 |
| Fricas | 107 | 100.00 | 0.00 | 0.00 |
| Mupad | 136 | 0.00 | 100.00 | 0.00 |
| Giac | 138 | 92.75 | 5.80 | 1.45 |
| Sympy | 209 | 22.01 | 73.21 | 4.78 |

Table 1.4: Failure statistics for each CAS

## 1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

Similarly the **Normalized median** is relative to the median leaf size of the optimal.

For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

| System | Mean time (sec) |
|---|---|
| Fricas | 0.36 |
| Maxima | 0.39 |
| Rubi | 0.68 |
| Mathematica | 0.85 |
| Mupad | 3.87 |
| Maple | 15.62 |
| Sympy | 21.50 |
| Giac | 27.40 |

Table 1.5: Time performance for each CAS

| System | Mean size | Normalized mean | Median size | Normalized median |
|---|---|---|---|---|
| Rubi | 281.34 | 0.90 | 227.00 | 0.88 |
| Fricas | 660.65 | 2.43 | 438.00 | 2.09 |
| Maple | 718.80 | 2.66 | 509.00 | 2.19 |
| Mathematica | 878.90 | 2.60 | 374.00 | 1.26 |
| Giac | 942.44 | 4.02 | 394.00 | 1.58 |
| Sympy | 948.35 | 4.84 | 869.50 | 4.35 |
| Mupad | 1048.30 | 3.36 | 638.00 | 2.70 |
| Maxima | 2290.62 | 7.49 | 1392.00 | 4.91 |

Table 1.6: Leaf size performance for each CAS

## 1.4 Performance based on number of rules Rubi used

This section shows how each CAS performed based on the number of rules Rubi needed to solve the same integral. One diagram is given for each CAS.

On the $y$ axis is the percentage solved which Rubi itself needed the number of rules given the $x$ axis. These plots show that as more rules are needed then most CAS system percentage of solving decreases which indicates the integral is becoming more complicated to solve.

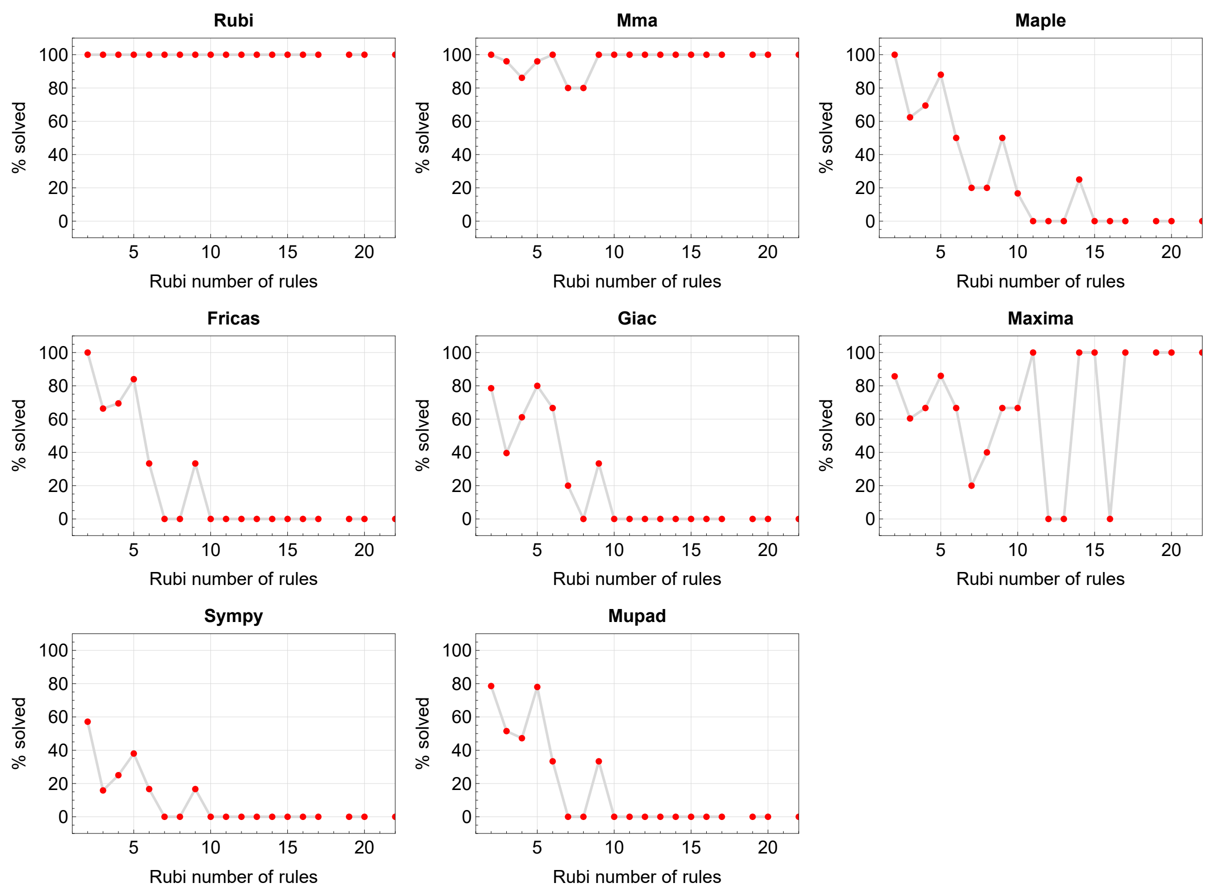



Figure 1.1: Solving statistics per number of Rubi rules used

## 1.5 Performance based on number of steps Rubi used

This section shows how each CAS performed based on the number of steps Rubi needed to solve the same integral. Note that the number of steps Rubi needed can be much higher than the number of rules, as the same rule could be used more than once.

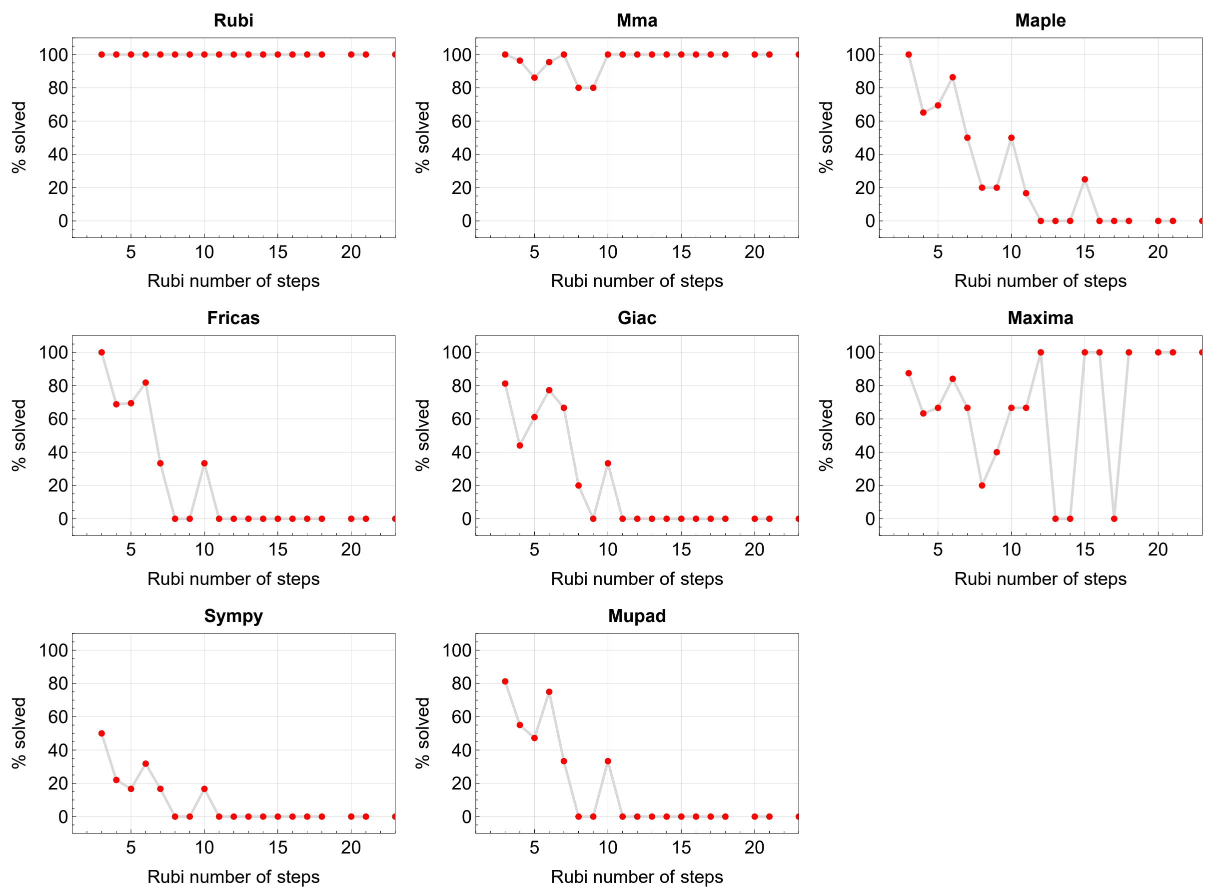



Figure 1.2: Solving statistics per number of Rubi steps used

The above diagram show that the precentage of solved intergals decreases for most CAS systems as the number of steps increases. As expected, for integrals that required less steps by Rubi, CAS systems had more success which indicates the integral was not as hard to solve. As Rubi needed more steps to solve the integral, the solved percentage decreased for most CAS systems which indicates the integral is becoming harder to solve.

## 1.6 Solved integrals histogram based on leaf size of result

The following shows the distribution of solved integrals for each CAS system based on leaf size of the antiderivatives produced by each CAS. It shows that most integrals solved produced leaf size less than about 100 to 150. The bin size used is 40.

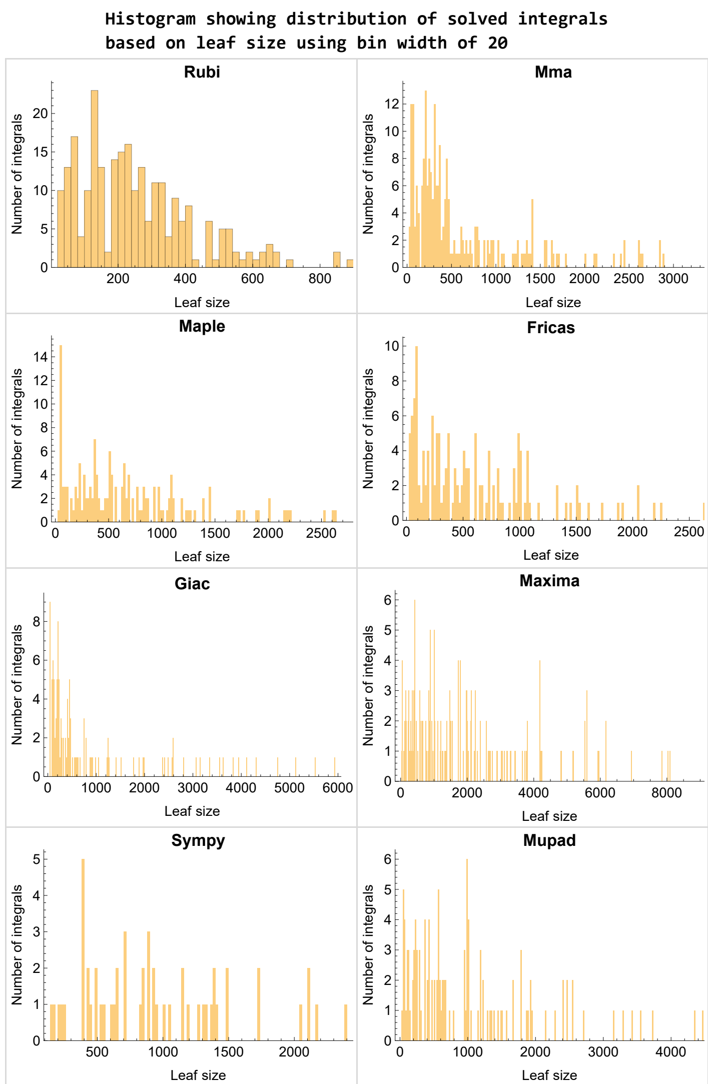



Figure 1.3: Solved integrals based on leaf size distribution

## 1.7 Solved integrals histogram based on CPU time used

The following shows the distribution of solved integrals for each CAS system based on CPU time used in seconds. The bin size used is 0.1 second.

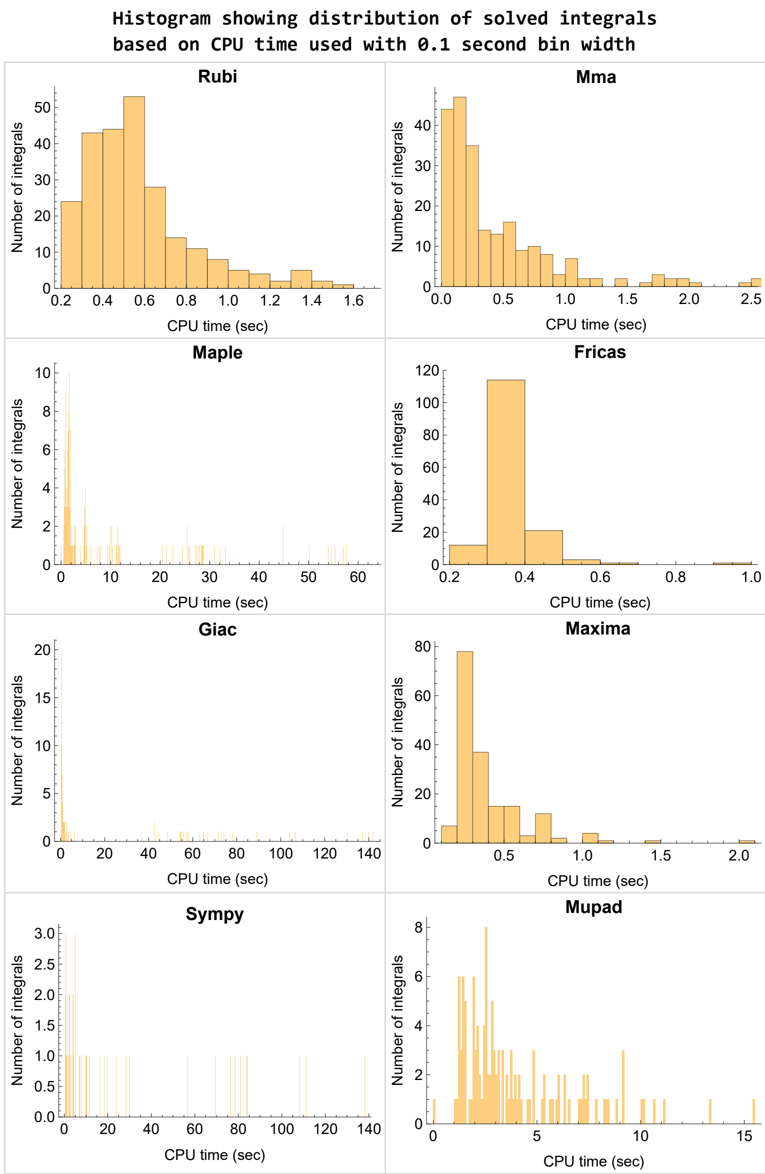



Figure 1.4: Solved integrals histogram based on CPU time used

## 1.8 Leaf size vs. CPU time used

The following gives the relation between the CPU time used to solve an integral and the leaf size of the antiderivative.

The result for Fricas, Maxima and Giac is shifted more to the right than the other CAS system due to the use of sagemath to call them, which causes an initial slight delay in the timing to start the integration due to overhead of starting a new process each time.

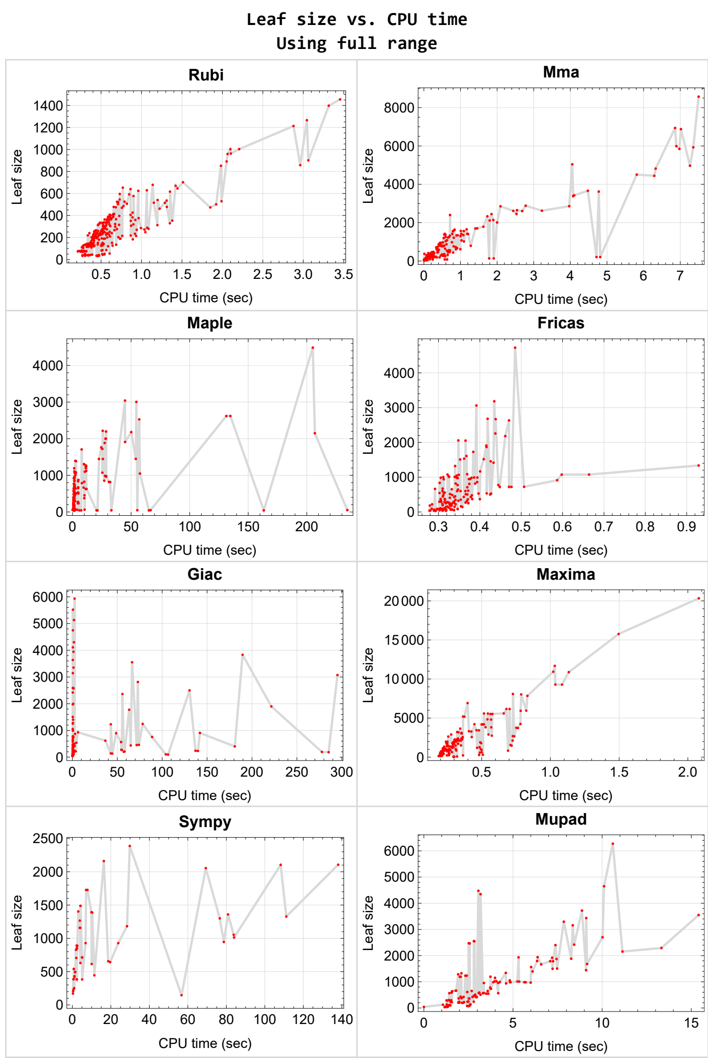



Figure 1.5: Leaf size vs. CPU time. Full range

## 1.9 list of integrals with no known antiderivative

{253, 254, 262, 263}

## 1.10 List of integrals solved by CAS but has no known antiderivative

**Rubi** {}

**Mathematica** {}

**Maple** {}

**Maxima** {}

**Fricas** {}

**Sympy** {}

**Giac** {}

**Mupad** {}

## 1.11 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not necessarily mean that the anti-derivative is wrong as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it possible to do further investigation to determine why the result could not be verified.

**Rubi** {88, 96, 104, 190, 198, 206, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 249, 250, 251, 252, 259, 260, 261}

**Mathematica** {}

**Maple** {252, 261}

**Maxima** Verification phase not currently implemented.

**Fricas** Verification phase not currently implemented.

**Sympy** Verification phase not currently implemented.

**Giac** Verification phase not currently implemented.

**Mupad** Verification phase not currently implemented.

## 1.12 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

## 1.13 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

# 1.14 Important notes about some of the results

## 1.14.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See `https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/` for reference.

### 1.14.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

### 1.14.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at `https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/`

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

### 1.14.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

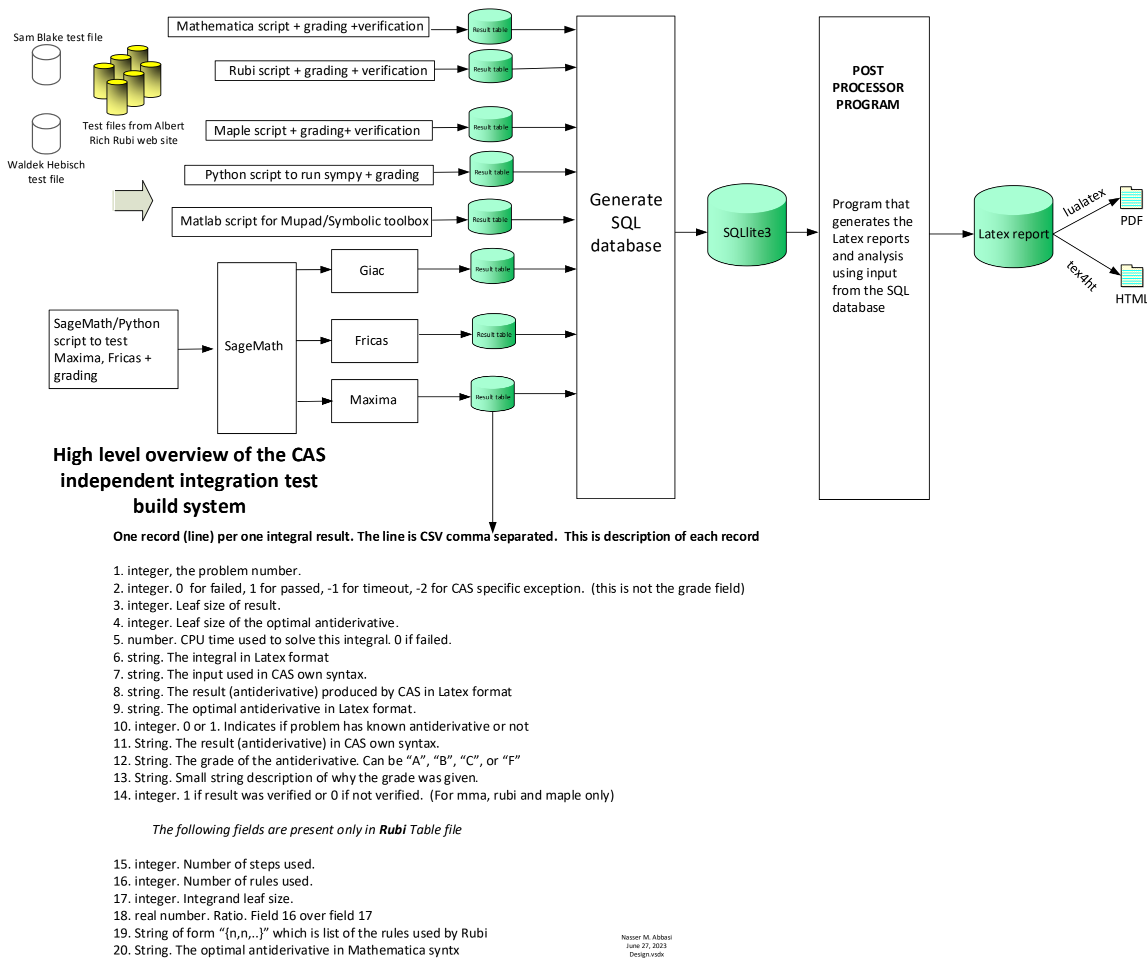
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

# 1.15 Design of the test system

The following diagram gives a high level view of the current test build system.



# CHAPTER 2

## DETAILED SUMMARY TABLES OF RESULTS

## 2.1 List of integrals sorted by grade for each CAS



### 2.1.1 Rubi

**A grade** { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 255, 256, 257, 258, 259, 260, 261 }

**B grade** { }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### 2.1.2 Mma

**A grade** { 1, 2, 3, 4, 5, 6, 8, 9, 10, 11, 12, 13, 14, 15, 16, 19, 20, 21, 22, 23, 24, 25, 26, 27, 31, 32, 33, 34, 39, 40, 41, 42, 47, 48, 50, 55, 56, 58, 65, 66, 67, 75, 76, 77, 84, 85, 87, 88, 89, 90, 91, 96, 97, 98, 99, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 115, 116, 117, 118, 119, 120, 121, 122, 123, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 146, 151, 152, 154, 159, 160, 162, 169, 170, 171, 179, 180, 181, 186, 187, 189, 190, 212, 213, 214, 218, 219, 220, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248 }

**B grade** { 7, 17, 18, 28, 29, 30, 49, 57, 59, 60, 64, 68, 69, 70, 74, 78, 79, 80, 86, 92, 93, 94, 100, 101, 114, 124, 125, 153, 161, 163, 164, 168, 172, 173, 174, 178, 182, 183, 184, 185, 188, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 203, 206, 207, 208, 209, 251, 252, 255, 256, 257, 258, 260, 261 }

**C grade** { 35, 36, 37, 38, 43, 44, 45, 46, 51, 52, 53, 54, 61, 62, 63, 71, 72, 73, 81, 82, 83, 95, 102, 103, 139, 140, 141, 142, 147, 148, 149, 150, 155, 156, 157, 158, 165, 166, 167, 175, 176, 177, 197, 204, 205 }

**F normal fail** { 210, 211, 215, 216, 217, 221, 222, 223, 226, 227, 240, 249, 250, 259 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### 2.1.3 Maple

**A grade** { 3, 4, 7, 8, 9, 11, 12, 13, 18, 19, 30, 34, 35, 36, 37, 38, 41, 42, 43, 44, 45, 46, 47, 48, 50, 51, 52, 53, 54, 95, 96, 103, 139, 140, 146, 147, 154, 197, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 245, 255, 256, 257, 258 }

**B grade** { 1, 2, 5, 6, 10, 14, 15, 16, 17, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 31, 32, 33, 39, 40, 49, 61, 62, 63, 71, 72, 73, 81, 82, 83, 87, 88, 89, 90, 91, 97, 98, 99, 102, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 129, 130, 141, 142, 148, 149, 150, 153, 155, 156, 157, 158, 165, 166, 167, 175, 176, 177, 190, 191, 192, 193, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 213, 214, 219, 220 }

**C grade** { 252, 261 }

**F normal fail** { 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 92, 93, 94, 100, 101, 112, 113, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 151, 152, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 194, 195, 196, 202, 203, 210, 211, 212, 215, 216, 217, 218, 221, 222, 223, 226, 227, 236, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 259, 260 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### 2.1.4 Fricas

**A grade** { 3, 4, 10, 11, 12, 22, 35, 36, 37, 38, 42, 43, 44, 45, 50, 51, 52, 89, 90, 91, 95, 96, 97, 98, 103, 104, 105, 139, 140, 141, 146, 147, 148, 154, 155, 197, 198, 215, 216, 220, 221, 222, 224, 225, 230, 231, 232, 234, 235, 236, 237, 238, 239, 241, 242, 243, 244, 245, 246, 247, 248, 255, 256, 257, 258 }

**B grade** { 1, 2, 7, 8, 9, 13, 17, 18, 19, 20, 21, 23, 28, 29, 30, 46, 49, 53, 54, 61, 62, 63, 71, 72, 73, 81, 82, 83, 88, 99, 102, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 129, 130, 142, 149, 150, 153, 156, 157, 158, 165, 166, 167, 175, 176, 177, 190, 191, 192, 193, 199, 200, 201, 204, 205, 206, 207, 208, 209, 212, 213, 214, 217, 218, 219, 223, 228, 229, 233 }

**C grade** { }

**F normal fail** { 5, 6, 14, 15, 16, 24, 25, 26, 27, 31, 32, 33, 34, 39, 40, 41, 47, 48, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 92, 93, 94, 100, 101, 112, 113, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 151, 152, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 194, 195, 196, 202, 203, 210, 211, 226, 227, 240, 249, 250, 251, 252, 259, 260, 261 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### 2.1.5 Maxima

**A grade** { 4, 5, 14, 33, 42, 50, 111, 112, 146, 154, 231, 232, 237, 238, 239, 245 }

**B grade** { 1, 2, 3, 7, 8, 9, 10, 11, 12, 13, 15, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 28, 29, 30, 31, 32, 35, 36, 37, 38, 39, 40, 43, 44, 45, 46, 47, 49, 51, 52, 53, 54, 55, 56, 57, 58, 61, 62, 63, 64, 65, 66, 67, 71, 72, 73, 74, 75, 76, 77, 81, 82, 83, 88, 89, 90, 91, 95, 96, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 114, 115, 116, 117, 118, 119, 120, 121, 122, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 135, 136, 137, 139, 140, 141, 142, 143, 144, 147, 148, 149, 150, 151, 153, 155, 156, 157, 158, 159, 160, 161, 162, 165, 166, 167, 168, 169, 170, 171, 175, 176, 177, 178, 179, 180, 181, 190, 191, 192, 193, 197, 198, 199, 200, 201, 204, 205, 206, 207, 208, 209, 228, 229, 230, 233, 255, 256, 257, 258 }

**C grade** { }

**F normal fail** { 6, 16, 27, 34, 41, 48, 59, 60, 68, 69, 70, 78, 79, 80, 84, 85, 86, 87, 92, 93, 94, 100, 101, 113, 123, 134, 138, 145, 152, 163, 164, 172, 173, 174, 182, 183, 184, 185, 186, 187, 188, 189, 194, 195, 196, 202, 203, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 259, 260, 261 }

**F(-1) timedout fail** { }

**F(-2) exception fail** { }

### 2.1.6 Giac

**A grade** { 7, 8, 9, 17, 18, 19, 28, 29, 30, 37, 38, 42, 43, 44, 45, 46, 49, 50, 51, 61, 62, 63, 71, 72, 73, 81, 82, 83, 90, 91, 95, 96, 97, 98, 99, 102, 103, 104, 114, 115, 116, 124, 125, 126, 139, 141, 142, 146, 147, 148, 149, 150, 153, 154, 155, 165, 166, 167, 175, 176, 177, 192, 197, 198, 199, 204, 205, 206, 224, 225, 231, 234, 235, 236, 237, 238, 239 }

**B grade** { 1, 2, 3, 4, 10, 11, 12, 13, 20, 21, 22, 23, 31, 32, 33, 34, 35, 39, 40, 41, 88, 108, 109, 110, 111, 117, 118, 119, 120, 127, 128, 129, 130, 135, 136, 137, 138, 143, 144, 145, 190, 232, 233, 245 }

**C grade** { }

**F normal fail** { 5, 6, 14, 15, 16, 24, 25, 26, 27, 36, 47, 48, 52, 53, 54, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 89, 92, 93, 94, 100, 101, 105, 106, 107, 112, 113, 121, 122, 123, 131, 132, 133, 134, 140, 151, 152, 156, 157, 158, 159, 160, 161, 162, 163, 164, 170, 171, 172, 173, 174, 180, 181, 182, 183, 184, 186, 187, 188, 189, 191, 194, 195, 196, 202, 203, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 228, 229, 230, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 255, 256, 257, 258, 259, 260, 261 }

**F(-1) timedout fail** { 168, 169, 178, 179, 185, 193, 200, 201 }

**F(-2) exception fail** { 226, 227 }

### 2.1.7 Mupad

**A grade** { }

**B grade** { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 28, 29, 30, 35, 36, 37, 38, 42, 43, 44, 45, 46, 49, 50, 51, 52, 53, 54, 61, 62, 63, 71, 72, 73, 81, 82, 83, 88, 89, 90, 91, 95, 96, 97, 98, 99, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 114, 115, 116, 117, 118, 119, 120, 124, 125, 126, 127, 128, 129, 130, 139, 140, 141, 142, 146, 147, 148, 149, 150, 153, 154, 155, 156, 157, 158, 165, 166, 167, 175, 176, 177, 190, 191, 192, 193, 197, 198, 200, 201, 204, 205, 206, 207, 208, 209, 228, 229, 230, 231, 232, 233, 237, 238, 239, 245 }

**C grade** { }

**F normal fail** { }

**F(-1) timedout fail** { 5, 6, 14, 15, 16, 24, 25, 26, 27, 31, 32, 33, 34, 39, 40, 41, 47, 48, 55, 56, 57, 58, 59, 60, 64, 65, 66, 67, 68, 69, 70, 74, 75, 76, 77, 78, 79, 80, 84, 85, 86, 87, 92, 93, 94, 100, 101, 112, 113, 121, 122, 123, 131, 132, 133, 134, 135, 136, 137, 138, 143, 144, 145, 151, 152, 159, 160, 161, 162, 163, 164, 168, 169, 170, 171, 172, 173, 174, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 194, 195, 196, 199, 202, 203, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 234, 235, 236, 240, 241, 242, 243, 244, 246, 247, 248, 249, 250, 251, 252, 255, 256, 257, 258, 259, 260, 261 }

**F(-2) exception fail** { }

### 2.1.8 Sympy

**A grade** { }

**B grade** { 1, 2, 3, 4, 7, 8, 9, 10, 11, 12, 13, 17, 18, 19, 20, 21, 22, 23, 35, 36, 37, 38, 42, 43, 44, 49, 50, 51, 53, 61, 62, 71, 72, 88, 89, 90, 91, 95, 96, 97, 102, 103, 104, 109, 110, 111, 114, 119, 120, 130, 146, 153, 154, 245 }

**C grade** { }

**F normal fail** { 5, 14, 16, 24, 26, 27, 31, 32, 33, 34, 47, 48, 59, 68, 70, 78, 80, 84, 85, 86, 87, 100, 101, 112, 121, 123, 131, 134, 135, 136, 137, 138, 139, 152, 163, 165, 174, 175, 188, 189, 190, 197, 203, 204, 205, 246 }

**F(-1) timedout fail** { 6, 15, 25, 28, 29, 30, 39, 40, 41, 45, 46, 52, 54, 55, 56, 57, 58, 60, 63, 64, 65, 66, 67, 69, 73, 74, 75, 76, 77, 79, 81, 82, 83, 92, 93, 94, 98, 99, 105, 106, 107, 108, 113, 115, 116, 117, 118, 122, 124, 125, 126, 127, 128, 129, 132, 133, 140, 141, 142, 143, 144, 145, 147, 148, 149, 150, 151, 155, 156, 157, 158, 159, 160, 161, 162, 164, 166, 167, 168, 169, 170, 171, 172, 173, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 191, 192, 193, 194, 195, 196, 198, 199, 200, 201, 202, 206, 207, 208, 209, 210, 211, 217, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263 }

**F(-2) exception fail** { 212, 213, 214, 215, 216, 218, 219, 220, 221, 222 }

## 2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by the table below. The elapsed time is in seconds. For failed result it is given as **F(-1)** if the failure was due to timeout. It is given as **F(-2)** if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given as **F**.

In this table, the column `N.S.` means **normalized size** and is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To make the table fit the page, the name `Mathematica` was abbreviated to `MMA`.

| Problem 1 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 212 | 189 | 261 | 574 | 1022 | 504 | 1158 | 3637 | 1195 |
| N.S. | 1 | 0.89 | 1.23 | 2.71 | 4.82 | 2.38 | 5.46 | 17.16 | 5.64 |
| time (sec) | N/A | 0.395 | 0.155 | 0.957 | 0.241 | 0.370 | 4.095 | 0.506 | 2.014 |

| Problem 2 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 180 | 165 | 217 | 396 | 671 | 370 | 850 | 2418 | 638 |
| N.S. | 1 | 0.92 | 1.21 | 2.20 | 3.73 | 2.06 | 4.72 | 13.43 | 3.54 |
| time (sec) | N/A | 0.375 | 0.111 | 0.800 | 0.216 | 0.382 | 2.478 | 0.457 | 1.589 |

| Problem 3 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 140 | 137 | 181 | 206 | 361 | 226 | 498 | 1254 | 282 |
| N.S. | 1 | 0.98 | 1.29 | 1.47 | 2.58 | 1.61 | 3.56 | 8.96 | 2.01 |
| time (sec) | N/A | 0.332 | 0.164 | 0.634 | 0.206 | 0.327 | 1.444 | 0.497 | 1.312 |

| Problem 4 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 81 | 77 | 70 | 108 | 144 | 127 | 253 | 627 | 126 |
| N.S. | 1 | 0.95 | 0.86 | 1.33 | 1.78 | 1.57 | 3.12 | 7.74 | 1.56 |
| time (sec) | N/A | 0.236 | 0.025 | 0.559 | 0.207 | 0.325 | 0.943 | 0.407 | 1.068 |

| Problem 5 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 133 | 126 | 164 | 417 | 241 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.95 | 1.23 | 3.14 | 1.81 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.608 | 0.078 | 1.491 | 0.262 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 6 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 142 | 134 | 175 | 370 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.94 | 1.23 | 2.61 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.484 | 0.096 | 1.378 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 7 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 72 | 208 | 153 | 570 | 177 | 384 | 134 | 197 |
| N.S. | 1 | 0.85 | 2.45 | 1.80 | 6.71 | 2.08 | 4.52 | 1.58 | 2.32 |
| time (sec) | N/A | 0.242 | 0.101 | 0.810 | 0.219 | 0.330 | 2.273 | 0.427 | 2.139 |

| Problem 8 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 136 | 187 | 279 | 933 | 363 | 629 | 270 | 361 |
| N.S. | 1 | 0.79 | 1.08 | 1.61 | 5.39 | 2.10 | 3.64 | 1.56 | 2.09 |
| time (sec) | N/A | 0.311 | 0.237 | 0.953 | 0.240 | 0.371 | 4.169 | 0.531 | 2.523 |

| Problem 9 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 269 | 201 | 210 | 390 | 1386 | 602 | 928 | 426 | 590 |
| N.S. | 1 | 0.75 | 0.78 | 1.45 | 5.15 | 2.24 | 3.45 | 1.58 | 2.19 |
| time (sec) | N/A | 0.369 | 0.277 | 1.506 | 0.261 | 0.365 | 6.868 | 0.532 | 3.114 |

| Problem 10 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 423 | 375 | 429 | 925 | 1789 | 724 | 1727 | 4746 | 2473 |
| N.S. | 1 | 0.89 | 1.01 | 2.19 | 4.23 | 1.71 | 4.08 | 11.22 | 5.85 |
| time (sec) | N/A | 0.600 | 0.229 | 1.185 | 0.244 | 0.507 | 7.850 | 0.597 | 2.519 |

| Problem 11 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 337 | 301 | 362 | 636 | 1200 | 534 | 1266 | 3144 | 1287 |
| N.S. | 1 | 0.89 | 1.07 | 1.89 | 3.56 | 1.58 | 3.76 | 9.33 | 3.82 |
| time (sec) | N/A | 0.493 | 0.164 | 1.003 | 0.228 | 0.417 | 3.866 | 0.517 | 1.925 |

| Problem 12 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 239 | 227 | 216 | 396 | 671 | 367 | 850 | 1975 | 636 |
| N.S. | 1 | 0.95 | 0.90 | 1.66 | 2.81 | 1.54 | 3.56 | 8.26 | 2.66 |
| time (sec) | N/A | 0.385 | 0.133 | 0.812 | 0.216 | 0.400 | 2.347 | 0.483 | 1.588 |

| Problem 13 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 118 | 105 | 97 | 206 | 280 | 223 | 491 | 1056 | 290 |
| N.S. | 1 | 0.89 | 0.82 | 1.75 | 2.37 | 1.89 | 4.16 | 8.95 | 2.46 |
| time (sec) | N/A | 0.257 | 0.029 | 0.672 | 0.204 | 0.341 | 1.434 | 0.436 | 1.272 |

| Problem 14 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 276 | 307 | 252 | 698 | 518 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.11 | 0.91 | 2.53 | 1.88 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.827 | 0.132 | 1.570 | 0.261 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 15 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 247 | 223 | 221 | 568 | 992 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 0.89 | 2.30 | 4.02 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.462 | 0.140 | 1.556 | 0.266 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 16 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 230 | 203 | 244 | 522 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 1.06 | 2.27 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.688 | 0.187 | 1.489 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 17 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 74 | 315 | 185 | 1515 | 271 | 614 | 140 | 423 |
| N.S. | 1 | 0.83 | 3.54 | 2.08 | 17.02 | 3.04 | 6.90 | 1.57 | 4.75 |
| time (sec) | N/A | 0.268 | 0.204 | 1.013 | 0.252 | 0.335 | 10.028 | 0.503 | 2.731 |

| Problem 18 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 138 | 454 | 319 | 2218 | 510 | 928 | 282 | 647 |
| N.S. | 1 | 0.76 | 2.51 | 1.76 | 12.25 | 2.82 | 5.13 | 1.56 | 3.57 |
| time (sec) | N/A | 0.337 | 0.240 | 1.591 | 0.304 | 0.366 | 23.848 | 0.549 | 3.578 |

| Problem 19 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 205 | 344 | 430 | 3029 | 807 | 1300 | 444 | 941 |
| N.S. | 1 | 0.73 | 1.22 | 1.53 | 10.78 | 2.87 | 4.63 | 1.58 | 3.35 |
| time (sec) | N/A | 0.388 | 0.499 | 1.701 | 0.355 | 0.374 | 76.593 | 0.548 | 4.614 |

| Problem 20 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 457 | 401 | 586 | 1194 | 2637 | 912 | 2161 | 5520 | 4347 |
| N.S. | 1 | 0.88 | 1.28 | 2.61 | 5.77 | 2.00 | 4.73 | 12.08 | 9.51 |
| time (sec) | N/A | 0.613 | 0.408 | 1.438 | 0.257 | 0.587 | 16.359 | 0.669 | 3.180 |

| Problem 21 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 371 | 327 | 429 | 925 | 1789 | 722 | 1727 | 4110 | 2465 |
| N.S. | 1 | 0.88 | 1.16 | 2.49 | 4.82 | 1.95 | 4.65 | 11.08 | 6.64 |
| time (sec) | N/A | 0.510 | 0.216 | 1.157 | 0.250 | 0.470 | 7.138 | 0.551 | 2.573 |

| Problem 22 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 271 | 253 | 261 | 568 | 1022 | 502 | 1158 | 2589 | 1192 |
| N.S. | 1 | 0.93 | 0.96 | 2.10 | 3.77 | 1.85 | 4.27 | 9.55 | 4.40 |
| time (sec) | N/A | 0.402 | 0.135 | 0.886 | 0.311 | 0.425 | 3.835 | 0.477 | 2.063 |

| Problem 23 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 129 | 120 | 314 | 439 | 322 | 706 | 1506 | 566 |
| N.S. | 1 | 0.87 | 0.81 | 2.11 | 2.95 | 2.16 | 4.74 | 10.11 | 3.80 |
| time (sec) | N/A | 0.279 | 0.040 | 0.738 | 0.265 | 0.348 | 2.024 | 0.473 | 1.429 |

| Problem 24 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 356 | 462 | 352 | 1086 | 850 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.30 | 0.99 | 3.05 | 2.39 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.217 | 0.194 | 1.806 | 0.315 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 25 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 373 | 326 | 374 | 863 | 1501 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.87 | 1.00 | 2.31 | 4.02 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.576 | 0.244 | 1.658 | 0.349 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 26 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 345 | 293 | 314 | 732 | 2302 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.85 | 0.91 | 2.12 | 6.67 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.550 | 0.231 | 1.608 | 0.359 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 27 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 310 | 270 | 308 | 685 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.87 | 0.99 | 2.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.949 | 0.261 | 1.609 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 28 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 74 | 427 | 185 | 3107 | 355 | 0 | 140 | 780 |
| N.S. | 1 | 0.83 | 4.80 | 2.08 | 34.91 | 3.99 | 0.00 | 1.57 | 8.76 |
| time (sec) | N/A | 0.280 | 0.278 | 1.575 | 0.353 | 0.354 | 0.000 | 0.547 | 3.723 |

| Problem 29 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 138 | 608 | 357 | 4218 | 644 | 0 | 282 | 1053 |
| N.S. | 1 | 0.76 | 3.36 | 1.97 | 23.30 | 3.56 | 0.00 | 1.56 | 5.82 |
| time (sec) | N/A | 0.355 | 0.354 | 1.602 | 0.447 | 0.347 | 0.000 | 0.546 | 4.818 |

| Problem 30 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 205 | 642 | 481 | 5524 | 991 | 0 | 444 | 1396 |
| N.S. | 1 | 0.73 | 2.28 | 1.71 | 19.66 | 3.53 | 0.00 | 1.58 | 4.97 |
| time (sec) | N/A | 0.419 | 0.581 | 2.259 | 0.564 | 0.394 | 0.000 | 0.612 | 6.099 |

| Problem 31 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | F | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 252 | 298 | 354 | 1105 | 790 | 0 | 0 | 3552 | 0 |
| N.S. | 1 | 1.18 | 1.40 | 4.38 | 3.13 | 0.00 | 0.00 | 14.10 | 0.00 |
| time (sec) | N/A | 0.712 | 0.184 | 1.646 | 0.260 | 0.000 | 0.000 | 66.529 | 0.000 |

| Problem 32 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | F | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 198 | 225 | 254 | 657 | 477 | 0 | 0 | 2364 | 0 |
| N.S. | 1 | 1.14 | 1.28 | 3.32 | 2.41 | 0.00 | 0.00 | 11.94 | 0.00 |
| time (sec) | N/A | 0.579 | 0.123 | 1.773 | 0.263 | 0.000 | 0.000 | 55.791 | 0.000 |

| Problem 33 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | F | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 145 | 162 | 332 | 221 | 0 | 0 | 1230 | 0 |
| N.S. | 1 | 1.16 | 1.30 | 2.66 | 1.77 | 0.00 | 0.00 | 9.84 | 0.00 |
| time (sec) | N/A | 0.397 | 0.065 | 1.343 | 0.247 | 0.000 | 0.000 | 42.824 | 0.000 |

| Problem 34 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 76 | 81 | 95 | 146 | 0 | 0 | 0 | 617 | 0 |
| N.S. | 1 | 1.07 | 1.25 | 1.92 | 0.00 | 0.00 | 0.00 | 8.12 | 0.00 |
| time (sec) | N/A | 0.533 | 0.024 | 1.370 | 0.000 | 0.000 | 0.000 | 36.747 | 0.000 |

| Problem 35 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 44 | 207 | 72 | 172 | 60 | 170 | 104 | 69 |
| N.S. | 1 | 1.00 | 4.70 | 1.64 | 3.91 | 1.36 | 3.86 | 2.36 | 1.57 |
| time (sec) | N/A | 0.292 | 0.078 | 0.622 | 0.210 | 0.319 | 0.310 | 0.324 | 2.516 |

| Problem 36 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 173 | 127 | 292 | 230 | 424 | 144 | 386 | 0 | 241 |
| N.S. | 1 | 0.73 | 1.69 | 1.33 | 2.45 | 0.83 | 2.23 | 0.00 | 1.39 |
| time (sec) | N/A | 0.399 | 0.176 | 0.977 | 0.217 | 0.358 | 0.618 | 0.000 | 2.446 |

| Problem 37 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 255 | 194 | 418 | 365 | 885 | 349 | 889 | 139 | 545 |
| N.S. | 1 | 0.76 | 1.64 | 1.43 | 3.47 | 1.37 | 3.49 | 0.55 | 2.14 |
| time (sec) | N/A | 0.426 | 0.223 | 1.257 | 0.243 | 0.341 | 2.588 | 42.817 | 3.504 |

| Problem 38 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 373 | 262 | 492 | 500 | 1469 | 611 | 1392 | 270 | 970 |
| N.S. | 1 | 0.70 | 1.32 | 1.34 | 3.94 | 1.64 | 3.73 | 0.72 | 2.60 |
| time (sec) | N/A | 0.505 | 0.386 | 1.634 | 0.310 | 0.372 | 9.881 | 55.044 | 5.995 |

| Problem 39 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | F | F(-1) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 341 | 304 | 359 | 846 | 1341 | 0 | 0 | 2814 | 0 |
| N.S. | 1 | 0.89 | 1.05 | 2.48 | 3.93 | 0.00 | 0.00 | 8.25 | 0.00 |
| time (sec) | N/A | 0.605 | 0.257 | 1.674 | 0.275 | 0.000 | 0.000 | 72.934 | 0.000 |

| Problem 40 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | F | F(-1) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 260 | 227 | 239 | 532 | 886 | 0 | 0 | 1774 | 0 |
| N.S. | 1 | 0.87 | 0.92 | 2.05 | 3.41 | 0.00 | 0.00 | 6.82 | 0.00 |
| time (sec) | N/A | 0.479 | 0.145 | 1.488 | 0.273 | 0.000 | 0.000 | 63.252 | 0.000 |

| Problem 41 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F(-1) | B | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 160 | 146 | 175 | 295 | 0 | 0 | 0 | 895 | 0 |
| N.S. | 1 | 0.91 | 1.09 | 1.84 | 0.00 | 0.00 | 0.00 | 5.59 | 0.00 |
| time (sec) | N/A | 0.338 | 0.102 | 1.475 | 0.000 | 0.000 | 0.000 | 48.777 | 0.000 |

| Problem 42 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 98 | 73 | 104 | 82 | 134 | 88 | 231 | 115 | 106 |
| N.S. | 1 | 0.74 | 1.06 | 0.84 | 1.37 | 0.90 | 2.36 | 1.17 | 1.08 |
| time (sec) | N/A | 0.203 | 0.031 | 0.595 | 0.200 | 0.346 | 0.613 | 0.398 | 1.440 |

| Problem 43 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 109 | 292 | 231 | 421 | 151 | 386 | 233 | 247 |
| N.S. | 1 | 0.70 | 1.87 | 1.48 | 2.70 | 0.97 | 2.47 | 1.49 | 1.58 |
| time (sec) | N/A | 0.364 | 0.164 | 0.901 | 0.223 | 0.339 | 0.601 | 0.434 | 2.503 |

| Problem 44 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 261 | 182 | 324 | 366 | 859 | 334 | 828 | 133 | 415 |
| N.S. | 1 | 0.70 | 1.24 | 1.40 | 3.29 | 1.28 | 3.17 | 0.51 | 1.59 |
| time (sec) | N/A | 0.378 | 0.245 | 1.136 | 0.238 | 0.306 | 1.900 | 44.472 | 2.927 |

| Problem 45 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 364 | 254 | 453 | 506 | 1721 | 664 | 0 | 276 | 984 |
| N.S. | 1 | 0.70 | 1.24 | 1.39 | 4.73 | 1.82 | 0.00 | 0.76 | 2.70 |
| time (sec) | N/A | 0.483 | 0.373 | 2.875 | 0.330 | 0.292 | 0.000 | 54.532 | 5.636 |

| Problem 46 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 457 | 313 | 520 | 640 | 2560 | 1019 | 0 | 436 | 1679 |
| N.S. | 1 | 0.68 | 1.14 | 1.40 | 5.60 | 2.23 | 0.00 | 0.95 | 3.67 |
| time (sec) | N/A | 0.489 | 0.705 | 2.645 | 0.428 | 0.325 | 0.000 | 64.990 | 9.157 |

| Problem 47 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 361 | 300 | 317 | 681 | 2037 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.83 | 0.88 | 1.89 | 5.64 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.561 | 0.254 | 1.598 | 0.316 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 48 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 251 | 217 | 245 | 463 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.86 | 0.98 | 1.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.442 | 0.196 | 1.561 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 49 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 85 | 72 | 207 | 174 | 567 | 185 | 382 | 132 | 198 |
| N.S. | 1 | 0.85 | 2.44 | 2.05 | 6.67 | 2.18 | 4.49 | 1.55 | 2.33 |
| time (sec) | N/A | 0.252 | 0.102 | 0.806 | 0.217 | 0.279 | 2.324 | 0.405 | 2.197 |

| Problem 50 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 144 | 138 | 111 | 221 | 255 | 221 | 422 | 236 | 208 |
| N.S. | 1 | 0.96 | 0.77 | 1.53 | 1.77 | 1.53 | 2.93 | 1.64 | 1.44 |
| time (sec) | N/A | 0.312 | 0.072 | 0.771 | 0.209 | 0.285 | 1.084 | 0.394 | 1.982 |

| Problem 51 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 243 | 182 | 418 | 367 | 885 | 355 | 889 | 442 | 545 |
| N.S. | 1 | 0.75 | 1.72 | 1.51 | 3.64 | 1.46 | 3.66 | 1.82 | 2.24 |
| time (sec) | N/A | 0.409 | 0.267 | 1.217 | 0.255 | 0.316 | 2.505 | 0.406 | 3.614 |

| Problem 52 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 365 | 253 | 452 | 506 | 1721 | 672 | 0 | 0 | 983 |
| N.S. | 1 | 0.69 | 1.24 | 1.39 | 4.72 | 1.84 | 0.00 | 0.00 | 2.69 |
| time (sec) | N/A | 0.429 | 0.378 | 2.830 | 0.327 | 0.310 | 0.000 | 0.000 | 5.707 |

| Problem 53 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | B | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 463 | 319 | 533 | 649 | 2380 | 1011 | 2106 | 0 | 1443 |
| N.S. | 1 | 0.69 | 1.15 | 1.40 | 5.14 | 2.18 | 4.55 | 0.00 | 3.12 |
| time (sec) | N/A | 0.478 | 0.631 | 2.085 | 0.352 | 0.326 | 138.117 | 0.000 | 9.101 |

| Problem 54 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 563 | 388 | 637 | 787 | 3816 | 1509 | 0 | 0 | 2291 |
| N.S. | 1 | 0.69 | 1.13 | 1.40 | 6.78 | 2.68 | 0.00 | 0.00 | 4.07 |
| time (sec) | N/A | 0.586 | 0.887 | 7.993 | 0.570 | 0.352 | 0.000 | 0.000 | 13.330 |

| Problem 55 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 539 | 617 | 905 | 0 | 3186 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.14 | 1.68 | 0.00 | 5.91 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.352 | 0.449 | 0.000 | 0.331 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 56 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 450 | 516 | 680 | 0 | 2243 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.15 | 1.51 | 0.00 | 4.98 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.148 | 0.342 | 0.000 | 0.307 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 57 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 343 | 405 | 869 | 0 | 1252 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.18 | 2.53 | 0.00 | 3.65 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.821 | 0.405 | 0.000 | 0.295 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 58 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 227 | 205 | 0 | 633 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.12 | 1.01 | 0.00 | 3.12 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.568 | 0.119 | 0.000 | 0.284 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 59 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 286 | 272 | 1214 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.95 | 4.24 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.992 | 0.818 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 60 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 241 | 218 | 1407 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 5.84 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.865 | 1.207 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 61 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 114 | 765 | 330 | 1987 | 289 | 714 | 206 | 469 |
| N.S. | 1 | 0.81 | 5.43 | 2.34 | 14.09 | 2.05 | 5.06 | 1.46 | 3.33 |
| time (sec) | N/A | 0.319 | 0.527 | 0.797 | 0.304 | 0.315 | 5.020 | 0.452 | 2.812 |

| Problem 62 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 287 | 218 | 1032 | 643 | 3282 | 601 | 1387 | 455 | 955 |
| N.S. | 1 | 0.76 | 3.60 | 2.24 | 11.44 | 2.09 | 4.83 | 1.59 | 3.33 |
| time (sec) | N/A | 0.416 | 0.596 | 0.988 | 0.424 | 0.310 | 10.342 | 0.511 | 4.173 |

| Problem 63 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 445 | 334 | 1255 | 926 | 4808 | 985 | 0 | 744 | 1870 |
| N.S. | 1 | 0.75 | 2.82 | 2.08 | 10.80 | 2.21 | 0.00 | 1.67 | 4.20 |
| time (sec) | N/A | 0.500 | 0.687 | 1.609 | 0.568 | 0.306 | 0.000 | 0.557 | 7.441 |

| Problem 64 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 711 | 963 | 1559 | 0 | 5178 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.35 | 2.19 | 0.00 | 7.28 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.051 | 0.794 | 0.000 | 0.374 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 65 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 761 | 852 | 1194 | 0 | 3656 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.12 | 1.57 | 0.00 | 4.80 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.943 | 0.541 | 0.000 | 0.344 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 66 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 589 | 670 | 677 | 0 | 2259 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.14 | 1.15 | 0.00 | 3.84 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.444 | 0.356 | 0.000 | 0.318 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 67 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 334 | 360 | 287 | 0 | 1202 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.08 | 0.86 | 0.00 | 3.60 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.934 | 0.129 | 0.000 | 0.285 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 68 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 535 | 501 | 2615 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.94 | 4.89 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.967 | 2.442 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 69 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 442 | 366 | 2649 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.83 | 5.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.713 | 2.542 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 70 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 387 | 329 | 3426 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.85 | 8.85 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.370 | 4.105 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 71 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 119 | 1352 | 367 | 5532 | 444 | 1182 | 219 | 1153 |
| N.S. | 1 | 0.81 | 9.20 | 2.50 | 37.63 | 3.02 | 8.04 | 1.49 | 7.84 |
| time (sec) | N/A | 0.384 | 1.080 | 1.049 | 0.578 | 0.310 | 28.408 | 0.520 | 3.993 |

| Problem 72 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 220 | 1703 | 685 | 8031 | 837 | 2055 | 479 | 1940 |
| N.S. | 1 | 0.74 | 5.70 | 2.29 | 26.86 | 2.80 | 6.87 | 1.60 | 6.49 |
| time (sec) | N/A | 0.490 | 1.456 | 1.638 | 0.788 | 0.303 | 69.337 | 0.577 | 6.382 |

| Problem 73 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 463 | 336 | 2010 | 968 | 10880 | 1323 | 0 | 780 | 3434 |
| N.S. | 1 | 0.73 | 4.34 | 2.09 | 23.50 | 2.86 | 0.00 | 1.68 | 7.42 |
| time (sec) | N/A | 0.576 | 1.999 | 1.812 | 1.134 | 0.340 | 0.000 | 0.566 | 9.112 |

| Problem 74 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 1089 | 1398 | 2330 | 0 | 6921 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.28 | 2.14 | 0.00 | 6.36 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.478 | 1.735 | 0.000 | 0.397 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 75 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 908 | 1213 | 1555 | 0 | 5196 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.34 | 1.71 | 0.00 | 5.72 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.998 | 0.798 | 0.000 | 0.364 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 76 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 730 | 959 | 901 | 0 | 3218 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.31 | 1.23 | 0.00 | 4.41 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.074 | 0.444 | 0.000 | 0.322 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 77 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 420 | 515 | 389 | 0 | 1789 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.23 | 0.93 | 0.00 | 4.26 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.314 | 0.191 | 0.000 | 0.287 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 78 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 712 | 857 | 5044 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.20 | 7.08 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.032 | 4.053 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 79 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 692 | 577 | 4506 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.83 | 6.51 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.917 | 5.813 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 80 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 604 | 486 | 5989 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.80 | 9.92 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.786 | 6.894 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 81 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 147 | 119 | 2401 | 367 | 11688 | 559 | 0 | 218 | 1565 |
| N.S. | 1 | 0.81 | 16.33 | 2.50 | 79.51 | 3.80 | 0.00 | 1.48 | 10.65 |
| time (sec) | N/A | 0.371 | 0.715 | 1.646 | 1.032 | 0.315 | 0.000 | 0.644 | 6.018 |

| Problem 82 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 299 | 220 | 2456 | 722 | 15765 | 1045 | 0 | 479 | 3720 |
| N.S. | 1 | 0.74 | 8.21 | 2.41 | 52.73 | 3.49 | 0.00 | 1.60 | 12.44 |
| time (sec) | N/A | 0.486 | 2.529 | 1.857 | 1.496 | 0.344 | 0.000 | 0.641 | 8.871 |

| Problem 83 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 463 | 336 | 2606 | 1019 | 20330 | 1610 | 0 | 780 | 6275 |
| N.S. | 1 | 0.73 | 5.63 | 2.20 | 43.91 | 3.48 | 0.00 | 1.68 | 13.55 |
| time (sec) | N/A | 0.566 | 2.686 | 2.524 | 2.080 | 0.368 | 0.000 | 0.665 | 10.601 |

| Problem 84 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 718 | 629 | 1020 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 1.42 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.093 | 1.093 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 85 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 536 | 470 | 727 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 1.36 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.792 | 0.869 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 86 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 283 | 260 | 1227 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 4.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.513 | 0.662 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 87 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 127 | 120 | 252 | 338 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.94 | 1.98 | 2.66 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.405 | 0.580 | 1.407 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 88 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 37 | 79 | 106 | 397 | 87 | 206 | 132 | 96 |
| N.S. | 1 | 0.84 | 1.80 | 2.41 | 9.02 | 1.98 | 4.68 | 3.00 | 2.18 |
| time (sec) | N/A | 0.332 | 0.242 | 0.664 | 0.222 | 0.310 | 0.376 | 0.467 | 2.579 |

| Problem 89 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | B | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 183 | 136 | 186 | 400 | 1008 | 231 | 541 | 0 | 419 |
| N.S. | 1 | 0.74 | 1.02 | 2.19 | 5.51 | 1.26 | 2.96 | 0.00 | 2.29 |
| time (sec) | N/A | 0.442 | 0.430 | 0.984 | 0.262 | 0.310 | 0.743 | 0.000 | 2.971 |

| Problem 90 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 343 | 248 | 318 | 767 | 2115 | 540 | 1488 | 211 | 981 |
| N.S. | 1 | 0.72 | 0.93 | 2.24 | 6.17 | 1.57 | 4.34 | 0.62 | 2.86 |
| time (sec) | N/A | 0.544 | 0.650 | 1.307 | 0.350 | 0.313 | 4.236 | 58.157 | 4.841 |

| Problem 91 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 507 | 364 | 442 | 1076 | 3434 | 940 | 2388 | 451 | 1882 |
| N.S. | 1 | 0.72 | 0.87 | 2.12 | 6.77 | 1.85 | 4.71 | 0.89 | 3.71 |
| time (sec) | N/A | 0.618 | 0.800 | 1.839 | 0.480 | 0.313 | 29.821 | 71.588 | 8.266 |

| Problem 92 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 722 | 592 | 4443 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.82 | 6.15 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.835 | 6.293 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 93 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 469 | 380 | 2622 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.81 | 5.59 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.610 | 3.223 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 94 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 261 | 239 | 1412 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 5.41 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.450 | 1.022 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 95 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 152 | 111 | 315 | 182 | 416 | 155 | 432 | 173 | 222 |
| N.S. | 1 | 0.73 | 2.07 | 1.20 | 2.74 | 1.02 | 2.84 | 1.14 | 1.46 |
| time (sec) | N/A | 0.258 | 0.240 | 0.677 | 0.216 | 0.321 | 1.140 | 0.400 | 2.256 |

| Problem 96 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | B | A | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 214 | 140 | 187 | 398 | 1004 | 236 | 539 | 339 | 423 |
| N.S. | 1 | 0.65 | 0.87 | 1.86 | 4.69 | 1.10 | 2.52 | 1.58 | 1.98 |
| time (sec) | N/A | 0.511 | 0.352 | 0.982 | 0.261 | 0.332 | 0.725 | 0.430 | 2.923 |

| Problem 97 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 365 | 254 | 307 | 766 | 1995 | 515 | 1404 | 203 | 731 |
| N.S. | 1 | 0.70 | 0.84 | 2.10 | 5.47 | 1.41 | 3.85 | 0.56 | 2.00 |
| time (sec) | N/A | 0.544 | 0.518 | 1.220 | 0.315 | 0.328 | 3.080 | 57.239 | 3.762 |

| Problem 98 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 523 | 364 | 466 | 1080 | 4187 | 1005 | 0 | 460 | 1497 |
| N.S. | 1 | 0.70 | 0.89 | 2.07 | 8.01 | 1.92 | 0.00 | 0.88 | 2.86 |
| time (sec) | N/A | 0.603 | 0.703 | 2.713 | 0.511 | 0.312 | 0.000 | 74.128 | 7.222 |

| Problem 99 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 682 | 475 | 613 | 1386 | 6160 | 1534 | 0 | 754 | 2701 |
| N.S. | 1 | 0.70 | 0.90 | 2.03 | 9.03 | 2.25 | 0.00 | 1.11 | 3.96 |
| time (sec) | N/A | 0.675 | 0.997 | 2.935 | 0.703 | 0.362 | 0.000 | 88.805 | 10.025 |

| Problem 100 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 635 | 504 | 5929 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.79 | 9.34 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.697 | 7.357 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 101 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 410 | 358 | 3384 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.87 | 8.25 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.578 | 4.077 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 102 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 115 | 767 | 344 | 1966 | 295 | 712 | 198 | 474 |
| N.S. | 1 | 0.82 | 5.44 | 2.44 | 13.94 | 2.09 | 5.05 | 1.40 | 3.36 |
| time (sec) | N/A | 0.316 | 0.497 | 0.902 | 0.295 | 0.302 | 5.086 | 0.456 | 2.856 |

| Problem 103 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 296 | 219 | 444 | 483 | 848 | 373 | 892 | 371 | 507 |
| N.S. | 1 | 0.74 | 1.50 | 1.63 | 2.86 | 1.26 | 3.01 | 1.25 | 1.71 |
| time (sec) | N/A | 0.327 | 0.227 | 0.973 | 0.228 | 0.305 | 2.210 | 0.438 | 2.881 |

| Problem 104 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | B | A | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 375 | 351 | 290 | 769 | 2116 | 545 | 1488 | 672 | 984 |
| N.S. | 1 | 0.94 | 0.77 | 2.05 | 5.64 | 1.45 | 3.97 | 1.79 | 2.62 |
| time (sec) | N/A | 0.832 | 0.466 | 1.368 | 0.337 | 0.302 | 4.201 | 0.470 | 4.861 |

| Problem 105 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | A | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 525 | 366 | 453 | 1080 | 4188 | 1008 | 0 | 0 | 1505 |
| N.S. | 1 | 0.70 | 0.86 | 2.06 | 7.98 | 1.92 | 0.00 | 0.00 | 2.87 |
| time (sec) | N/A | 0.591 | 0.665 | 2.782 | 0.499 | 0.339 | 0.000 | 0.000 | 7.474 |

| Problem 106 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 685 | 478 | 611 | 1396 | 5583 | 1517 | 0 | 0 | 2155 |
| N.S. | 1 | 0.70 | 0.89 | 2.04 | 8.15 | 2.21 | 0.00 | 0.00 | 3.15 |
| time (sec) | N/A | 0.663 | 0.877 | 2.337 | 0.545 | 0.409 | 0.000 | 0.000 | 11.142 |

| Problem 107 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 851 | 596 | 793 | 1708 | 9282 | 2257 | 0 | 0 | 3550 |
| N.S. | 1 | 0.70 | 0.93 | 2.01 | 10.91 | 2.65 | 0.00 | 0.00 | 4.17 |
| time (sec) | N/A | 0.719 | 1.281 | 7.752 | 1.084 | 0.437 | 0.000 | 0.000 | 15.415 |

| Problem 108 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 197 | 269 | 1195 | 1118 | 720 | 0 | 3945 | 1237 |
| N.S. | 1 | 0.88 | 1.21 | 5.36 | 5.01 | 3.23 | 0.00 | 17.69 | 5.55 |
| time (sec) | N/A | 0.440 | 0.161 | 11.437 | 0.217 | 0.476 | 0.000 | 1.234 | 2.406 |

| Problem 109 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 190 | 173 | 225 | 876 | 740 | 529 | 1013 | 2571 | 663 |
| N.S. | 1 | 0.91 | 1.18 | 4.61 | 3.89 | 2.78 | 5.33 | 13.53 | 3.49 |
| time (sec) | N/A | 0.408 | 0.121 | 5.002 | 0.207 | 0.427 | 84.025 | 1.317 | 1.813 |

| Problem 110 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 149 | 145 | 189 | 507 | 393 | 309 | 643 | 1276 | 295 |
| N.S. | 1 | 0.97 | 1.27 | 3.40 | 2.64 | 2.07 | 4.32 | 8.56 | 1.98 |
| time (sec) | N/A | 0.365 | 0.182 | 1.716 | 0.197 | 0.368 | 19.632 | 0.659 | 1.533 |

| Problem 111 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | A | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 86 | 81 | 74 | 250 | 156 | 162 | 382 | 580 | 134 |
| N.S. | 1 | 0.94 | 0.86 | 2.91 | 1.81 | 1.88 | 4.44 | 6.74 | 1.56 |
| time (sec) | N/A | 0.252 | 0.014 | 0.685 | 0.195 | 0.354 | 5.086 | 0.462 | 1.290 |

| Problem 112 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | A | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 141 | 134 | 172 | 0 | 276 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.95 | 1.22 | 0.00 | 1.96 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.662 | 0.084 | 0.000 | 0.482 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 113 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 150 | 142 | 189 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.95 | 1.26 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.510 | 0.102 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 114 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 76 | 216 | 237 | 582 | 250 | 1360 | 100 | 204 |
| N.S. | 1 | 0.85 | 2.43 | 2.66 | 6.54 | 2.81 | 15.28 | 1.12 | 2.29 |
| time (sec) | N/A | 0.252 | 0.103 | 5.162 | 0.204 | 0.372 | 80.892 | 0.784 | 1.984 |

| Problem 115 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 143 | 196 | 455 | 945 | 478 | 0 | 234 | 374 |
| N.S. | 1 | 0.79 | 1.08 | 2.51 | 5.22 | 2.64 | 0.00 | 1.29 | 2.07 |
| time (sec) | N/A | 0.326 | 0.278 | 10.203 | 0.217 | 0.381 | 0.000 | 1.077 | 2.117 |

| Problem 116 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 281 | 211 | 220 | 986 | 1398 | 773 | 0 | 394 | 610 |
| N.S. | 1 | 0.75 | 0.78 | 3.51 | 4.98 | 2.75 | 0.00 | 1.40 | 2.17 |
| time (sec) | N/A | 0.397 | 0.322 | 27.713 | 0.247 | 0.346 | 0.000 | 1.363 | 2.435 |

| Problem 117 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 442 | 388 | 441 | 1761 | 1978 | 1074 | 0 | 5133 | 2555 |
| N.S. | 1 | 0.88 | 1.00 | 3.98 | 4.48 | 2.43 | 0.00 | 11.61 | 5.78 |
| time (sec) | N/A | 0.626 | 0.256 | 24.523 | 0.240 | 0.598 | 0.000 | 1.803 | 2.807 |

| Problem 118 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 352 | 311 | 374 | 1274 | 1336 | 774 | 0 | 3352 | 1328 |
| N.S. | 1 | 0.88 | 1.06 | 3.62 | 3.80 | 2.20 | 0.00 | 9.52 | 3.77 |
| time (sec) | N/A | 0.520 | 0.180 | 11.129 | 0.227 | 0.445 | 0.000 | 1.271 | 2.106 |

| Problem 119 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 250 | 234 | 224 | 837 | 740 | 530 | 1054 | 1997 | 661 |
| N.S. | 1 | 0.94 | 0.90 | 3.35 | 2.96 | 2.12 | 4.22 | 7.99 | 2.64 |
| time (sec) | N/A | 0.404 | 0.129 | 4.756 | 0.204 | 0.397 | 83.821 | 1.003 | 1.784 |

| Problem 120 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 124 | 109 | 101 | 463 | 309 | 297 | 656 | 990 | 303 |
| N.S. | 1 | 0.88 | 0.81 | 3.73 | 2.49 | 2.40 | 5.29 | 7.98 | 2.44 |
| time (sec) | N/A | 0.275 | 0.019 | 1.958 | 0.196 | 0.338 | 18.637 | 0.647 | 1.401 |

| Problem 121 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 289 | 319 | 264 | 0 | 580 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.10 | 0.91 | 0.00 | 2.01 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.886 | 0.130 | 0.000 | 0.487 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 122 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 259 | 235 | 233 | 0 | 1190 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.91 | 0.90 | 0.00 | 4.59 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.515 | 0.141 | 0.000 | 0.467 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 123 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 242 | 215 | 258 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.89 | 1.07 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.761 | 0.198 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 124 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 93 | 78 | 329 | 376 | 1544 | 409 | 0 | 108 | 421 |
| N.S. | 1 | 0.84 | 3.54 | 4.04 | 16.60 | 4.40 | 0.00 | 1.16 | 4.53 |
| time (sec) | N/A | 0.282 | 0.192 | 9.999 | 0.250 | 0.327 | 0.000 | 1.554 | 2.226 |

| Problem 125 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 145 | 474 | 855 | 2247 | 710 | 0 | 250 | 652 |
| N.S. | 1 | 0.77 | 2.51 | 4.52 | 11.89 | 3.76 | 0.00 | 1.32 | 3.45 |
| time (sec) | N/A | 0.358 | 0.258 | 27.074 | 0.302 | 0.370 | 0.000 | 1.978 | 2.683 |

| Problem 126 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 293 | 215 | 357 | 1049 | 3058 | 1087 | 0 | 418 | 954 |
| N.S. | 1 | 0.73 | 1.22 | 3.58 | 10.44 | 3.71 | 0.00 | 1.43 | 3.26 |
| time (sec) | N/A | 0.430 | 0.560 | 57.556 | 0.351 | 0.369 | 0.000 | 2.578 | 3.365 |

| Problem 127 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 477 | 414 | 631 | 2179 | 2901 | 1336 | 0 | 5934 | 4476 |
| N.S. | 1 | 0.87 | 1.32 | 4.57 | 6.08 | 2.80 | 0.00 | 12.44 | 9.38 |
| time (sec) | N/A | 0.652 | 0.369 | 50.102 | 0.278 | 0.929 | 0.000 | 2.503 | 3.060 |

| Problem 128 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 387 | 337 | 441 | 1721 | 1978 | 1075 | 0 | 4300 | 2547 |
| N.S. | 1 | 0.87 | 1.14 | 4.45 | 5.11 | 2.78 | 0.00 | 11.11 | 6.58 |
| time (sec) | N/A | 0.537 | 0.223 | 25.411 | 0.251 | 0.664 | 0.000 | 1.781 | 2.835 |

| Problem 129 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F(-1) | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 283 | 260 | 269 | 1119 | 1118 | 721 | 0 | 2584 | 1234 |
| N.S. | 1 | 0.92 | 0.95 | 3.95 | 3.95 | 2.55 | 0.00 | 9.13 | 4.36 |
| time (sec) | N/A | 0.425 | 0.142 | 11.618 | 0.227 | 0.448 | 0.000 | 1.179 | 2.328 |

| Problem 130 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 156 | 133 | 124 | 652 | 479 | 429 | 945 | 1402 | 588 |
| N.S. | 1 | 0.85 | 0.79 | 4.18 | 3.07 | 2.75 | 6.06 | 8.99 | 3.77 |
| time (sec) | N/A | 0.301 | 0.029 | 4.810 | 0.207 | 0.378 | 78.682 | 0.801 | 1.552 |

| Problem 131 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 373 | 478 | 368 | 0 | 935 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.28 | 0.99 | 0.00 | 2.51 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.339 | 0.182 | 0.000 | 0.499 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 132 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 390 | 343 | 394 | 0 | 1785 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 1.01 | 0.00 | 4.58 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.596 | 0.240 | 0.000 | 0.496 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 133 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 361 | 309 | 331 | 0 | 2746 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.86 | 0.92 | 0.00 | 7.61 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.558 | 0.230 | 0.000 | 0.573 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 134 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 326 | 286 | 326 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.986 | 0.254 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 135 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 269 | 315 | 370 | 0 | 1003 | 0 | 0 | 3832 | 0 |
| N.S. | 1 | 1.17 | 1.38 | 0.00 | 3.73 | 0.00 | 0.00 | 14.25 | 0.00 |
| time (sec) | N/A | 0.717 | 0.182 | 0.000 | 0.494 | 0.000 | 0.000 | 189.494 | 0.000 |

| Problem 136 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 211 | 238 | 266 | 0 | 627 | 0 | 0 | 2499 | 0 |
| N.S. | 1 | 1.13 | 1.26 | 0.00 | 2.97 | 0.00 | 0.00 | 11.84 | 0.00 |
| time (sec) | N/A | 0.567 | 0.121 | 0.000 | 0.500 | 0.000 | 0.000 | 130.459 | 0.000 |

| Problem 137 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 134 | 154 | 170 | 0 | 306 | 0 | 0 | 1242 | 0 |
| N.S. | 1 | 1.15 | 1.27 | 0.00 | 2.28 | 0.00 | 0.00 | 9.27 | 0.00 |
| time (sec) | N/A | 0.395 | 0.079 | 0.000 | 0.509 | 0.000 | 0.000 | 78.291 | 0.000 |

| Problem 138 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 80 | 85 | 101 | 0 | 0 | 0 | 0 | 566 | 0 |
| N.S. | 1 | 1.06 | 1.26 | 0.00 | 0.00 | 0.00 | 0.00 | 7.08 | 0.00 |
| time (sec) | N/A | 0.516 | 0.025 | 0.000 | 0.000 | 0.000 | 0.000 | 54.287 | 0.000 |

| Problem 139 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | F | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 50 | 219 | 84 | 175 | 74 | 0 | 92 | 76 |
| N.S. | 1 | 1.00 | 4.38 | 1.68 | 3.50 | 1.48 | 0.00 | 1.84 | 1.52 |
| time (sec) | N/A | 0.297 | 0.074 | 1.456 | 0.203 | 0.317 | 0.000 | 0.426 | 2.428 |

| Problem 140 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 181 | 134 | 304 | 269 | 427 | 194 | 0 | 0 | 239 |
| N.S. | 1 | 0.74 | 1.68 | 1.49 | 2.36 | 1.07 | 0.00 | 0.00 | 1.32 |
| time (sec) | N/A | 0.397 | 0.169 | 4.793 | 0.216 | 0.310 | 0.000 | 0.000 | 2.798 |

| Problem 141 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 266 | 204 | 434 | 628 | 888 | 483 | 0 | 104 | 573 |
| N.S. | 1 | 0.77 | 1.63 | 2.36 | 3.34 | 1.82 | 0.00 | 0.39 | 2.15 |
| time (sec) | N/A | 0.451 | 0.216 | 11.918 | 0.252 | 0.312 | 0.000 | 104.068 | 3.080 |

| Problem 142 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 389 | 275 | 518 | 1072 | 1472 | 859 | 0 | 232 | 986 |
| N.S. | 1 | 0.71 | 1.33 | 2.76 | 3.78 | 2.21 | 0.00 | 0.60 | 2.53 |
| time (sec) | N/A | 0.509 | 0.383 | 25.467 | 0.290 | 0.393 | 0.000 | 139.847 | 3.893 |

| Problem 143 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 359 | 322 | 375 | 0 | 1892 | 0 | 0 | 3072 | 0 |
| N.S. | 1 | 0.90 | 1.04 | 0.00 | 5.27 | 0.00 | 0.00 | 8.56 | 0.00 |
| time (sec) | N/A | 0.666 | 0.249 | 0.000 | 0.490 | 0.000 | 0.000 | 295.165 | 0.000 |

| Problem 144 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 275 | 242 | 252 | 0 | 1273 | 0 | 0 | 1898 | 0 |
| N.S. | 1 | 0.88 | 0.92 | 0.00 | 4.63 | 0.00 | 0.00 | 6.90 | 0.00 |
| time (sec) | N/A | 0.515 | 0.144 | 0.000 | 0.485 | 0.000 | 0.000 | 221.565 | 0.000 |

| Problem 145 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | B | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 168 | 154 | 183 | 0 | 0 | 0 | 0 | 906 | 0 |
| N.S. | 1 | 0.92 | 1.09 | 0.00 | 0.00 | 0.00 | 0.00 | 5.39 | 0.00 |
| time (sec) | N/A | 0.360 | 0.113 | 0.000 | 0.000 | 0.000 | 0.000 | 142.033 | 0.000 |

| Problem 146 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 102 | 77 | 114 | 81 | 136 | 105 | 444 | 91 | 113 |
| N.S. | 1 | 0.75 | 1.12 | 0.79 | 1.33 | 1.03 | 4.35 | 0.89 | 1.11 |
| time (sec) | N/A | 0.217 | 0.031 | 2.079 | 0.187 | 0.305 | 11.438 | 0.557 | 1.437 |

| Problem 147 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 166 | 119 | 304 | 269 | 424 | 196 | 0 | 208 | 241 |
| N.S. | 1 | 0.72 | 1.83 | 1.62 | 2.55 | 1.18 | 0.00 | 1.25 | 1.45 |
| time (sec) | N/A | 0.383 | 0.159 | 4.760 | 0.207 | 0.328 | 0.000 | 1.046 | 1.491 |

| Problem 148 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 273 | 192 | 342 | 666 | 862 | 450 | 0 | 96 | 432 |
| N.S. | 1 | 0.70 | 1.25 | 2.44 | 3.16 | 1.65 | 0.00 | 0.35 | 1.58 |
| time (sec) | N/A | 0.403 | 0.252 | 9.417 | 0.219 | 0.340 | 0.000 | 106.593 | 2.063 |

| Problem 149 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 380 | 267 | 478 | 977 | 1724 | 946 | 0 | 238 | 1016 |
| N.S. | 1 | 0.70 | 1.26 | 2.57 | 4.54 | 2.49 | 0.00 | 0.63 | 2.67 |
| time (sec) | N/A | 0.508 | 0.385 | 28.773 | 0.290 | 0.374 | 0.000 | 137.129 | 3.973 |

| Problem 150 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 477 | 329 | 549 | 1910 | 2563 | 1458 | 0 | 401 | 1665 |
| N.S. | 1 | 0.69 | 1.15 | 4.00 | 5.37 | 3.06 | 0.00 | 0.84 | 3.49 |
| time (sec) | N/A | 0.471 | 0.717 | 44.855 | 0.362 | 0.426 | 0.000 | 180.849 | 6.581 |

| Problem 151 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 382 | 321 | 334 | 0 | 2894 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.84 | 0.87 | 0.00 | 7.58 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.602 | 0.246 | 0.000 | 0.550 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 152 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 263 | 229 | 259 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.87 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.429 | 0.202 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 153 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 89 | 76 | 215 | 237 | 578 | 250 | 1328 | 101 | 205 |
| N.S. | 1 | 0.85 | 2.42 | 2.66 | 6.49 | 2.81 | 14.92 | 1.13 | 2.30 |
| time (sec) | N/A | 0.254 | 0.102 | 4.635 | 0.198 | 0.318 | 111.169 | 0.804 | 1.930 |

| Problem 154 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 142 | 115 | 272 | 259 | 266 | 2103 | 207 | 221 |
| N.S. | 1 | 0.94 | 0.76 | 1.80 | 1.72 | 1.76 | 13.93 | 1.37 | 1.46 |
| time (sec) | N/A | 0.323 | 0.075 | 4.592 | 0.196 | 0.338 | 108.219 | 0.770 | 1.570 |

| Problem 155 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | A | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 254 | 192 | 434 | 628 | 888 | 487 | 0 | 418 | 573 |
| N.S. | 1 | 0.76 | 1.71 | 2.47 | 3.50 | 1.92 | 0.00 | 1.65 | 2.26 |
| time (sec) | N/A | 0.411 | 0.211 | 11.281 | 0.232 | 0.333 | 0.000 | 1.516 | 3.368 |

| Problem 156 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 381 | 266 | 477 | 962 | 1724 | 949 | 0 | 0 | 1018 |
| N.S. | 1 | 0.70 | 1.25 | 2.52 | 4.52 | 2.49 | 0.00 | 0.00 | 2.67 |
| time (sec) | N/A | 0.441 | 0.386 | 28.546 | 0.300 | 0.377 | 0.000 | 0.000 | 4.070 |

| Problem 157 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 483 | 335 | 561 | 1450 | 2383 | 1416 | 0 | 0 | 1341 |
| N.S. | 1 | 0.69 | 1.16 | 3.00 | 4.93 | 2.93 | 0.00 | 0.00 | 2.78 |
| time (sec) | N/A | 0.481 | 0.683 | 53.995 | 0.328 | 0.433 | 0.000 | 0.000 | 4.597 |

| Problem 158 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 587 | 407 | 671 | 2149 | 3819 | 2181 | 0 | 0 | 2400 |
| N.S. | 1 | 0.69 | 1.14 | 3.66 | 6.51 | 3.72 | 0.00 | 0.00 | 4.09 |
| time (sec) | N/A | 0.599 | 0.909 | 206.922 | 0.528 | 0.461 | 0.000 | 0.000 | 7.382 |

| Problem 159 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 584 | 646 | 949 | 0 | 3764 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.11 | 1.62 | 0.00 | 6.45 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.429 | 0.470 | 0.000 | 0.752 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 160 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 487 | 541 | 716 | 0 | 2691 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.11 | 1.47 | 0.00 | 5.53 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.191 | 0.339 | 0.000 | 0.726 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 161 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 372 | 426 | 937 | 0 | 1542 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.15 | 2.52 | 0.00 | 4.15 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.880 | 0.440 | 0.000 | 0.710 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 162 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 220 | 239 | 216 | 0 | 825 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 0.98 | 0.00 | 3.75 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.619 | 0.075 | 0.000 | 0.692 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 163 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 306 | 288 | 1372 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.94 | 4.48 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.047 | 0.588 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 164 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 261 | 234 | 1564 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 5.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.910 | 1.092 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 165 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 122 | 801 | 530 | 2017 | 600 | 0 | 195 | 561 |
| N.S. | 1 | 0.81 | 5.30 | 3.51 | 13.36 | 3.97 | 0.00 | 1.29 | 3.72 |
| time (sec) | N/A | 0.335 | 0.502 | 4.830 | 0.315 | 0.345 | 0.000 | 1.705 | 3.313 |

| Problem 166 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 307 | 238 | 1082 | 1099 | 3312 | 1167 | 0 | 501 | 993 |
| N.S. | 1 | 0.78 | 3.52 | 3.58 | 10.79 | 3.80 | 0.00 | 1.63 | 3.23 |
| time (sec) | N/A | 0.450 | 0.631 | 10.491 | 0.407 | 0.401 | 0.000 | 2.306 | 4.264 |

| Problem 167 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 475 | 364 | 1319 | 2197 | 4838 | 1868 | 0 | 871 | 1794 |
| N.S. | 1 | 0.77 | 2.78 | 4.63 | 10.19 | 3.93 | 0.00 | 1.83 | 3.78 |
| time (sec) | N/A | 0.545 | 0.709 | 28.621 | 0.542 | 0.416 | 0.000 | 3.065 | 6.355 |

| Problem 168 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | B | F | F(-1) | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 766 | 1003 | 1634 | 0 | 5952 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.31 | 2.13 | 0.00 | 7.77 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.193 | 0.837 | 0.000 | 0.824 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 169 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 819 | 891 | 1254 | 0 | 4247 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.09 | 1.53 | 0.00 | 5.19 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.066 | 0.595 | 0.000 | 0.782 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 170 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 635 | 702 | 713 | 0 | 2662 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.11 | 1.12 | 0.00 | 4.19 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.491 | 0.365 | 0.000 | 0.737 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 171 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 361 | 376 | 303 | 0 | 1473 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.04 | 0.84 | 0.00 | 4.08 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.936 | 0.073 | 0.000 | 0.718 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 172 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 572 | 529 | 2852 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 4.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.983 | 2.092 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 173 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 472 | 396 | 2885 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.84 | 6.11 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.720 | 2.779 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 174 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 417 | 353 | 3662 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.85 | 8.78 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.386 | 4.478 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 175 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 157 | 127 | 1418 | 825 | 5588 | 975 | 0 | 216 | 1195 |
| N.S. | 1 | 0.81 | 9.03 | 5.25 | 35.59 | 6.21 | 0.00 | 1.38 | 7.61 |
| time (sec) | N/A | 0.374 | 1.184 | 9.980 | 0.517 | 0.346 | 0.000 | 3.773 | 3.734 |

| Problem 176 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 319 | 240 | 1787 | 1884 | 8087 | 1729 | 0 | 541 | 1934 |
| N.S. | 1 | 0.75 | 5.60 | 5.91 | 25.35 | 5.42 | 0.00 | 1.70 | 6.06 |
| time (sec) | N/A | 0.495 | 1.625 | 27.470 | 0.727 | 0.384 | 0.000 | 4.546 | 5.310 |

| Problem 177 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 493 | 366 | 2112 | 2528 | 10936 | 2633 | 0 | 931 | 3296 |
| N.S. | 1 | 0.74 | 4.28 | 5.13 | 22.18 | 5.34 | 0.00 | 1.89 | 6.69 |
| time (sec) | N/A | 0.584 | 1.798 | 56.947 | 1.021 | 0.470 | 0.000 | 6.304 | 7.852 |

| Problem 178 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | B | F | F(-1) | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 1172 | 1455 | 2448 | 0 | 7845 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.24 | 2.09 | 0.00 | 6.69 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.516 | 1.847 | 0.000 | 0.833 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 179 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 976 | 1266 | 1627 | 0 | 5931 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.30 | 1.67 | 0.00 | 6.08 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.021 | 0.837 | 0.000 | 0.784 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 180 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 786 | 1002 | 945 | 0 | 3724 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.27 | 1.20 | 0.00 | 4.74 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 2.154 | 0.441 | 0.000 | 0.764 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 181 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | B | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 454 | 535 | 409 | 0 | 2129 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.18 | 0.90 | 0.00 | 4.69 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.340 | 0.121 | 0.000 | 0.725 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 182 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 762 | 901 | 4969 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.18 | 6.52 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 3.083 | 7.267 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 183 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 739 | 624 | 4818 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.84 | 6.52 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.987 | 6.329 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 184 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 644 | 526 | 6938 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.82 | 10.77 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.816 | 6.860 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 185 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F(-1) | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 561 | 473 | 8570 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.84 | 15.28 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.929 | 7.503 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 186 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 768 | 679 | 1072 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 1.40 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.104 | 1.026 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 187 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 573 | 507 | 763 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 1.33 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.811 | 0.813 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 188 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 303 | 280 | 1385 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 4.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.523 | 0.556 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 189 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 128 | 268 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.93 | 1.96 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.422 | 0.522 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 190 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | B | B | F | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 50 | 40 | 90 | 118 | 407 | 149 | 0 | 167 | 122 |
| N.S. | 1 | 0.80 | 1.80 | 2.36 | 8.14 | 2.98 | 0.00 | 3.34 | 2.44 |
| time (sec) | N/A | 0.339 | 0.166 | 1.438 | 0.219 | 0.319 | 0.000 | 0.552 | 2.600 |

| Problem 191 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 199 | 152 | 793 | 509 | 1018 | 428 | 0 | 0 | 361 |
| N.S. | 1 | 0.76 | 3.98 | 2.56 | 5.12 | 2.15 | 0.00 | 0.00 | 1.81 |
| time (sec) | N/A | 0.460 | 0.449 | 4.862 | 0.266 | 0.338 | 0.000 | 0.000 | 2.554 |

| Problem 192 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 369 | 274 | 975 | 1259 | 2126 | 1068 | 0 | 196 | 1011 |
| N.S. | 1 | 0.74 | 2.64 | 3.41 | 5.76 | 2.89 | 0.00 | 0.53 | 2.74 |
| time (sec) | N/A | 0.580 | 0.767 | 11.703 | 0.337 | 0.349 | 0.000 | 277.816 | 5.248 |

| Problem 193 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | F(-1) | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 543 | 400 | 1295 | 2216 | 3445 | 1910 | 0 | 0 | 1921 |
| N.S. | 1 | 0.74 | 2.38 | 4.08 | 6.34 | 3.52 | 0.00 | 0.00 | 3.54 |
| time (sec) | N/A | 0.653 | 1.001 | 25.753 | 0.462 | 0.416 | 0.000 | 0.000 | 7.198 |

| Problem 194 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 770 | 640 | 5850 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.83 | 7.60 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.872 | 6.978 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 195 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 500 | 411 | 2859 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.82 | 5.72 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.671 | 3.969 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 196 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 282 | 260 | 1570 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.92 | 5.57 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.481 | 1.010 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 197 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | A | B | A | F | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 163 | 119 | 331 | 294 | 428 | 263 | 0 | 175 | 237 |
| N.S. | 1 | 0.73 | 2.03 | 1.80 | 2.63 | 1.61 | 0.00 | 1.07 | 1.45 |
| time (sec) | N/A | 0.278 | 0.235 | 2.163 | 0.205 | 0.326 | 0.000 | 0.994 | 1.988 |

| Problem 198 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | A | F(-1) | A | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 231 | 151 | 789 | 509 | 1014 | 432 | 0 | 368 | 365 |
| N.S. | 1 | 0.65 | 3.42 | 2.20 | 4.39 | 1.87 | 0.00 | 1.59 | 1.58 |
| time (sec) | N/A | 0.531 | 0.403 | 5.056 | 0.265 | 0.361 | 0.000 | 0.900 | 1.956 |

| Problem 199 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 392 | 281 | 870 | 1307 | 2006 | 983 | 0 | 185 | 0 |
| N.S. | 1 | 0.72 | 2.22 | 3.33 | 5.12 | 2.51 | 0.00 | 0.47 | 0.00 |
| time (sec) | N/A | 0.550 | 0.617 | 9.718 | 0.309 | 0.380 | 0.000 | 285.296 | 0.000 |

| Problem 200 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | F(-1) | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 560 | 401 | 1340 | 2001 | 4198 | 2052 | 0 | 0 | 1784 |
| N.S. | 1 | 0.72 | 2.39 | 3.57 | 7.50 | 3.66 | 0.00 | 0.00 | 3.19 |
| time (sec) | N/A | 0.620 | 0.925 | 28.097 | 0.508 | 0.365 | 0.000 | 0.000 | 7.250 |

| Problem 201 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | F(-1) | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 729 | 522 | 1695 | 3041 | 6171 | 3183 | 0 | 0 | 3157 |
| N.S. | 1 | 0.72 | 2.33 | 4.17 | 8.47 | 4.37 | 0.00 | 0.00 | 4.33 |
| time (sec) | N/A | 0.699 | 1.405 | 44.816 | 0.680 | 0.435 | 0.000 | 0.000 | 8.352 |

| Problem 202 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 676 | 545 | 6878 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.81 | 10.17 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.750 | 7.019 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 203 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 441 | 389 | 3622 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 8.21 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.611 | 4.776 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 204 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 151 | 123 | 803 | 530 | 1995 | 600 | 0 | 195 | 565 |
| N.S. | 1 | 0.81 | 5.32 | 3.51 | 13.21 | 3.97 | 0.00 | 1.29 | 3.74 |
| time (sec) | N/A | 0.328 | 0.558 | 4.680 | 0.306 | 0.353 | 0.000 | 1.913 | 4.172 |

| Problem 205 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | C | B | B | B | F | A | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 317 | 240 | 464 | 672 | 861 | 654 | 0 | 407 | 505 |
| N.S. | 1 | 0.76 | 1.46 | 2.12 | 2.72 | 2.06 | 0.00 | 1.28 | 1.59 |
| time (sec) | N/A | 0.353 | 0.222 | 4.859 | 0.239 | 0.332 | 0.000 | 1.088 | 3.175 |

| Problem 206 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | A | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 402 | 383 | 971 | 1236 | 2126 | 1076 | 0 | 747 | 1007 |
| N.S. | 1 | 0.95 | 2.42 | 3.07 | 5.29 | 2.68 | 0.00 | 1.86 | 2.50 |
| time (sec) | N/A | 0.895 | 0.519 | 11.421 | 0.331 | 0.322 | 0.000 | 1.195 | 5.346 |

| Problem 207 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 562 | 403 | 1334 | 2000 | 4199 | 2057 | 0 | 0 | 1785 |
| N.S. | 1 | 0.72 | 2.37 | 3.56 | 7.47 | 3.66 | 0.00 | 0.00 | 3.18 |
| time (sec) | N/A | 0.664 | 0.892 | 28.472 | 0.516 | 0.348 | 0.000 | 0.000 | 7.089 |

| Problem 208 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 732 | 525 | 1653 | 3006 | 5594 | 3062 | 0 | 0 | 2419 |
| N.S. | 1 | 0.72 | 2.26 | 4.11 | 7.64 | 4.18 | 0.00 | 0.00 | 3.30 |
| time (sec) | N/A | 0.695 | 1.168 | 54.526 | 0.661 | 0.392 | 0.000 | 0.000 | 8.431 |

| Problem 209 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | B | B | B | F(-1) | F | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 908 | 653 | 2138 | 4486 | 9293 | 4725 | 0 | 0 | 4649 |
| N.S. | 1 | 0.72 | 2.35 | 4.94 | 10.23 | 5.20 | 0.00 | 0.00 | 5.12 |
| time (sec) | N/A | 0.771 | 1.896 | 205.155 | 1.036 | 0.485 | 0.000 | 0.000 | 10.108 |

| Problem 210 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 189 | 189 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.526 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 211 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 190 | 190 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.545 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 212 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | B | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 292 | 237 | 206 | 0 | 0 | 2680 | 0 | 0 | 0 |
| N.S. | 1 | 0.81 | 0.71 | 0.00 | 0.00 | 9.18 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.582 | 4.714 | 0.000 | 0.000 | 0.419 | 0.000 | 0.000 | 0.000 |

| Problem 213 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 210 | 180 | 134 | 2620 | 0 | 991 | 0 | 0 | 0 |
| N.S. | 1 | 0.86 | 0.64 | 12.48 | 0.00 | 4.72 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.480 | 1.908 | 134.763 | 0.000 | 0.386 | 0.000 | 0.000 | 0.000 |

| Problem 214 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 123 | 78 | 817 | 0 | 274 | 0 | 0 | 0 |
| N.S. | 1 | 0.96 | 0.61 | 6.38 | 0.00 | 2.14 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.364 | 0.634 | 32.119 | 0.000 | 0.324 | 0.000 | 0.000 | 0.000 |

| Problem 215 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 125 | 125 | 0 | 0 | 0 | 98 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.78 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.505 | 0.000 | 0.000 | 0.000 | 0.306 | 0.000 | 0.000 | 0.000 |

| Problem 216 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 206 | 201 | 0 | 0 | 0 | 293 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 0.00 | 0.00 | 0.00 | 1.42 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.594 | 0.000 | 0.000 | 0.000 | 0.307 | 0.000 | 0.000 | 0.000 |

| Problem 217 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 295 | 266 | 0 | 0 | 0 | 818 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 0.00 | 0.00 | 0.00 | 2.77 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.686 | 0.000 | 0.000 | 0.000 | 0.341 | 0.000 | 0.000 | 0.000 |

| Problem 218 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | B | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 309 | 252 | 206 | 0 | 0 | 2669 | 0 | 0 | 0 |
| N.S. | 1 | 0.82 | 0.67 | 0.00 | 0.00 | 8.64 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.591 | 4.815 | 0.000 | 0.000 | 0.438 | 0.000 | 0.000 | 0.000 |

| Problem 219 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | B | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 223 | 192 | 134 | 2619 | 0 | 983 | 0 | 0 | 0 |
| N.S. | 1 | 0.86 | 0.60 | 11.74 | 0.00 | 4.41 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.502 | 1.784 | 131.303 | 0.000 | 0.344 | 0.000 | 0.000 | 0.000 |

| Problem 220 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | B | F | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 137 | 132 | 78 | 821 | 0 | 269 | 0 | 0 | 0 |
| N.S. | 1 | 0.96 | 0.57 | 5.99 | 0.00 | 1.96 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.371 | 0.571 | 31.007 | 0.000 | 0.365 | 0.000 | 0.000 | 0.000 |

| Problem 221 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 128 | 128 | 0 | 0 | 0 | 93 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 0.00 | 0.00 | 0.00 | 0.73 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.515 | 0.000 | 0.000 | 0.000 | 0.284 | 0.000 | 0.000 | 0.000 |

| Problem 222 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | A | F(-2) | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 214 | 209 | 0 | 0 | 0 | 284 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 0.00 | 0.00 | 0.00 | 1.33 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.613 | 0.000 | 0.000 | 0.000 | 0.321 | 0.000 | 0.000 | 0.000 |

| Problem 223 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | B | F(-1) | F | F(-1) |
| verified | N/A | Yes | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 306 | 276 | 0 | 0 | 0 | 815 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 0.00 | 0.00 | 0.00 | 2.66 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.707 | 0.000 | 0.000 | 0.000 | 0.389 | 0.000 | 0.000 | 0.000 |

| Problem 224 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 40 | 43 | 0 | 65 | 0 | 41 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 1.05 | 0.00 | 1.59 | 0.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.301 | 0.022 | 7.372 | 0.000 | 0.333 | 0.000 | 0.293 | 0.000 |

| Problem 225 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | A | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 41 | 40 | 63 | 0 | 65 | 0 | 41 | 0 |
| N.S. | 1 | 1.00 | 0.98 | 1.54 | 0.00 | 1.59 | 0.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.364 | 0.002 | 10.279 | 0.000 | 0.304 | 0.000 | 0.299 | 0.000 |

| Problem 226 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | No | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 193 | 189 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.737 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 227 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F(-1) | F(-2) | F(-1) |
| verified | N/A | No | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 194 | 190 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.761 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 228 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F(-1) | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 34 | 43 | 44 | 766 | 375 | 0 | 0 | 141 |
| N.S. | 1 | 0.76 | 0.96 | 0.98 | 17.02 | 8.33 | 0.00 | 0.00 | 3.13 |
| time (sec) | N/A | 0.453 | 0.016 | 33.157 | 0.247 | 0.315 | 0.000 | 0.000 | 2.613 |

| Problem 229 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F(-1) | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 34 | 43 | 44 | 387 | 188 | 0 | 0 | 100 |
| N.S. | 1 | 0.76 | 0.96 | 0.98 | 8.60 | 4.18 | 0.00 | 0.00 | 2.22 |
| time (sec) | N/A | 0.444 | 0.014 | 5.816 | 0.219 | 0.332 | 0.000 | 0.000 | 1.487 |

| Problem 230 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | A | F(-1) | F | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 44 | 43 | 62 | 151 | 72 | 0 | 0 | 71 |
| N.S. | 1 | 0.98 | 0.96 | 1.38 | 3.36 | 1.60 | 0.00 | 0.00 | 1.58 |
| time (sec) | N/A | 0.391 | 0.015 | 3.843 | 0.210 | 0.310 | 0.000 | 0.000 | 1.363 |

| Problem 231 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 40 | 39 | 43 | 49 | 45 | 0 | 41 | 40 |
| N.S. | 1 | 0.98 | 0.95 | 1.05 | 1.20 | 1.10 | 0.00 | 1.00 | 0.98 |
| time (sec) | N/A | 0.488 | 0.061 | 21.342 | 0.295 | 0.303 | 0.000 | 0.302 | 1.293 |

| Problem 232 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 30 | 41 | 43 | 81 | 86 | 0 | 99 | 42 |
| N.S. | 1 | 0.70 | 0.95 | 1.00 | 1.88 | 2.00 | 0.00 | 2.30 | 0.98 |
| time (sec) | N/A | 0.469 | 0.015 | 65.195 | 0.321 | 0.329 | 0.000 | 0.286 | 1.251 |

| Problem 233 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | B | B | F(-1) | B | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 45 | 34 | 43 | 44 | 220 | 238 | 0 | 321 | 72 |
| N.S. | 1 | 0.76 | 0.96 | 0.98 | 4.89 | 5.29 | 0.00 | 7.13 | 1.60 |
| time (sec) | N/A | 0.470 | 0.017 | 163.308 | 0.364 | 0.327 | 0.000 | 0.292 | 1.284 |

| Problem 234 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | A | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 49 | 48 | 47 | 51 | 0 | 81 | 0 | 49 | 0 |
| N.S. | 1 | 0.98 | 0.96 | 1.04 | 0.00 | 1.65 | 0.00 | 1.00 | 0.00 |
| time (sec) | N/A | 0.516 | 0.022 | 234.622 | 0.000 | 0.346 | 0.000 | 0.292 | 0.000 |

| Problem 235 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | F | A | F(-1) | A | F(-1) |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 55 | 54 | 51 | 57 | 0 | 85 | 0 | 53 | 0 |
| N.S. | 1 | 0.98 | 0.93 | 1.04 | 0.00 | 1.55 | 0.00 | 0.96 | 0.00 |
| time (sec) | N/A | 0.601 | 0.034 | 0.023 | 0.000 | 0.301 | 0.000 | 0.299 | 0.000 |

| Problem 236 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-1) | A | F(-1) |
| verified | N/A | No | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 52 | 51 | 50 | 0 | 0 | 83 | 0 | 51 | 0 |
| N.S. | 1 | 0.98 | 0.96 | 0.00 | 0.00 | 1.60 | 0.00 | 0.98 | 0.00 |
| time (sec) | N/A | 0.551 | 0.003 | 0.000 | 0.000 | 0.306 | 0.000 | 0.283 | 0.000 |

| Problem 237 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 41 | 40 | 39 | 43 | 49 | 45 | 0 | 41 | 40 |
| N.S. | 1 | 0.98 | 0.95 | 1.05 | 1.20 | 1.10 | 0.00 | 1.00 | 0.98 |
| time (sec) | N/A | 0.479 | 0.007 | 20.474 | 0.295 | 0.294 | 0.000 | 0.274 | 0.002 |

| Problem 238 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 47 | 46 | 43 | 49 | 53 | 51 | 0 | 45 | 44 |
| N.S. | 1 | 0.98 | 0.91 | 1.04 | 1.13 | 1.09 | 0.00 | 0.96 | 0.94 |
| time (sec) | N/A | 0.519 | 0.100 | 66.617 | 0.300 | 0.338 | 0.000 | 0.275 | 1.341 |

| Problem 239 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | F(-1) | A | B |
| verified | N/A | No | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 44 | 43 | 42 | 46 | 51 | 48 | 0 | 43 | 42 |
| N.S. | 1 | 0.98 | 0.95 | 1.05 | 1.16 | 1.09 | 0.00 | 0.98 | 0.95 |
| time (sec) | N/A | 0.498 | 0.008 | 55.309 | 0.297 | 0.312 | 0.000 | 0.291 | 1.204 |

| Problem 240 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | No | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 88 | 86 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.538 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 241 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 75 | 0 | 0 | 110 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 1.47 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.311 | 0.023 | 0.000 | 0.000 | 0.315 | 0.000 | 0.000 | 0.000 |

| Problem 242 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 75 | 0 | 0 | 88 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 1.17 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.303 | 0.018 | 0.000 | 0.000 | 0.308 | 0.000 | 0.000 | 0.000 |

| Problem 243 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 75 | 0 | 0 | 66 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.88 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.270 | 0.016 | 0.000 | 0.000 | 0.290 | 0.000 | 0.000 | 0.000 |

| Problem 244 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 72 | 72 | 72 | 0 | 0 | 40 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.56 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.244 | 0.067 | 0.000 | 0.000 | 0.289 | 0.000 | 0.000 | 0.000 |

| Problem 245 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | A | A | A | B | B | B |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 34 | 35 | 37 | 34 | 146 | 88 | 33 |
| N.S. | 1 | 1.00 | 1.03 | 1.06 | 1.12 | 1.03 | 4.42 | 2.67 | 1.00 |
| time (sec) | N/A | 0.259 | 0.053 | 1.105 | 0.301 | 0.293 | 56.715 | 0.486 | 1.167 |

| Problem 246 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 71 | 71 | 71 | 0 | 0 | 40 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.56 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.267 | 0.058 | 0.000 | 0.000 | 0.280 | 0.000 | 0.000 | 0.000 |

| Problem 247 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 75 | 0 | 0 | 66 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 0.88 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.280 | 0.013 | 0.000 | 0.000 | 0.329 | 0.000 | 0.000 | 0.000 |

| Problem 248 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | A | F | F | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 75 | 75 | 75 | 0 | 0 | 88 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 1.00 | 0.00 | 0.00 | 1.17 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.300 | 0.015 | 0.000 | 0.000 | 0.297 | 0.000 | 0.000 | 0.000 |

| Problem 249 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | No | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 361 | 312 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.86 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.144 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 250 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | No | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 282 | 248 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.88 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.001 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 251 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | No | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 183 | 1415 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.90 | 6.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.865 | 0.776 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 252 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 120 | 304 | 1447 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.98 | 2.47 | 11.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.567 | 0.257 | 22.578 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 253 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 45 | 43 | 45 | 76 | 0 | 45 | 45 |
| N.S. | 1 | 1.00 | 1.05 | 1.00 | 1.05 | 1.77 | 0.00 | 1.05 | 1.05 |
| time (sec) | N/A | 0.431 | 0.169 | 0.228 | 0.342 | 0.304 | 0.000 | 0.315 | 1.238 |

| Problem 254 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 43 | 43 | 45 | 43 | 506 | 145 | 0 | 45 | 45 |
| N.S. | 1 | 1.00 | 1.05 | 1.00 | 11.77 | 3.37 | 0.00 | 1.05 | 1.05 |
| time (sec) | N/A | 0.432 | 0.328 | 0.026 | 0.378 | 0.306 | 0.000 | 0.342 | 1.288 |

| Problem 255 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 33 | 33 | 298 | 42 | 357 | 32 | 0 | 0 | 0 |
| N.S. | 1 | 1.00 | 9.03 | 1.27 | 10.82 | 0.97 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.292 | 0.130 | 1.342 | 0.204 | 0.318 | 0.000 | 0.000 | 0.000 |

| Problem 256 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 38 | 375 | 46 | 344 | 38 | 0 | 0 | 0 |
| N.S. | 1 | 1.41 | 13.89 | 1.70 | 12.74 | 1.41 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.257 | 0.224 | 1.898 | 0.201 | 0.310 | 0.000 | 0.000 | 0.000 |

| Problem 257 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 38 | 375 | 46 | 336 | 38 | 0 | 0 | 0 |
| N.S. | 1 | 1.41 | 13.89 | 1.70 | 12.44 | 1.41 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.319 | 0.170 | 1.823 | 0.203 | 0.278 | 0.000 | 0.000 | 0.000 |

| Problem 258 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | A | B | A | F(-1) | F | F(-1) |
| verified | N/A | Yes | Yes | Yes | TBD | TBD | TBD | TBD | TBD |
| size | 27 | 38 | 375 | 46 | 343 | 38 | 0 | 0 | 0 |
| N.S. | 1 | 1.41 | 13.89 | 1.70 | 12.70 | 1.41 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.319 | 0.013 | 1.861 | 0.201 | 0.289 | 0.000 | 0.000 | 0.000 |

| Problem 259 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | F | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | No | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 282 | 279 | 0 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 0.99 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 1.078 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 260 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | F | F | F | F(-1) | F | F(-1) |
| verified | N/A | No | Yes | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 203 | 214 | 1415 | 0 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.05 | 6.97 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.915 | 0.573 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 261 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | A | B | C | F | F | F(-1) | F | F(-1) |
| verified | N/A | No | Yes | No | TBD | TBD | TBD | TBD | TBD |
| size | 123 | 151 | 303 | 1447 | 0 | 0 | 0 | 0 | 0 |
| N.S. | 1 | 1.23 | 2.46 | 11.76 | 0.00 | 0.00 | 0.00 | 0.00 | 0.00 |
| time (sec) | N/A | 0.617 | 0.178 | 25.905 | 0.000 | 0.000 | 0.000 | 0.000 | 0.000 |

| Problem 262 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 53 | 51 | 53 | 76 | 0 | 53 | 53 |
| N.S. | 1 | 1.00 | 1.04 | 1.00 | 1.04 | 1.49 | 0.00 | 1.04 | 1.04 |
| time (sec) | N/A | 0.746 | 0.088 | 0.394 | 0.345 | 0.346 | 0.000 | 0.307 | 1.127 |

| Problem 263 | Optimal | Rubi | MMA | Maple | Maxima | Fricas | Sympy | Giac | Mupad |
|---|---|---|---|---|---|---|---|---|---|
| grade | N/A | N/A | N/A | N/A | N/A | N/A | F(-1) | N/A | N/A |
| verified | N/A | N/A | N/A | N/A | TBD | TBD | TBD | TBD | TBD |
| size | 51 | 51 | 53 | 51 | 506 | 145 | 0 | 53 | 53 |
| N.S. | 1 | 1.00 | 1.04 | 1.00 | 9.92 | 2.84 | 0.00 | 1.04 | 1.04 |
| time (sec) | N/A | 0.730 | 0.139 | 0.033 | 0.408 | 0.290 | 0.000 | 0.346 | 1.213 |

## 2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi only. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is also given. The larger this ratio is, the harder the integral is to solve. In this test file, problem number [74] had the largest ratio of [.5238099999999999998]

Table 2.1: Rubi specific breakdown of results for each integral

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 1 | A | 5 | 5 | 0.89 | 38 | 0.132 |
| 2 | A | 5 | 5 | 0.92 | 38 | 0.132 |
| 3 | A | 5 | 5 | 0.98 | 36 | 0.139 |
| 4 | A | 4 | 4 | 0.95 | 28 | 0.143 |
| 5 | A | 9 | 8 | 0.95 | 38 | 0.211 |
| 6 | A | 6 | 5 | 0.94 | 38 | 0.132 |
| 7 | A | 3 | 2 | 0.85 | 38 | 0.053 |
| 8 | A | 6 | 5 | 0.79 | 38 | 0.132 |
| 9 | A | 6 | 5 | 0.75 | 38 | 0.132 |
| 10 | A | 6 | 5 | 0.89 | 40 | 0.125 |
| 11 | A | 6 | 5 | 0.89 | 40 | 0.125 |
| 12 | A | 6 | 5 | 0.95 | 38 | 0.132 |
| 13 | A | 4 | 4 | 0.89 | 30 | 0.133 |
| 14 | A | 11 | 10 | 1.11 | 40 | 0.250 |
| 15 | A | 4 | 3 | 0.90 | 40 | 0.075 |
| 16 | A | 8 | 7 | 0.88 | 40 | 0.175 |
| 17 | A | 3 | 2 | 0.83 | 40 | 0.050 |
| 18 | A | 6 | 5 | 0.76 | 40 | 0.125 |
| 19 | A | 6 | 5 | 0.73 | 40 | 0.125 |
| 20 | A | 6 | 5 | 0.88 | 40 | 0.125 |
| 21 | A | 6 | 5 | 0.88 | 40 | 0.125 |
| 22 | A | 6 | 5 | 0.93 | 38 | 0.132 |

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| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 23 | A | 4 | 4 | 0.87 | 30 | 0.133 |
| 24 | A | 15 | 14 | 1.30 | 40 | 0.350 |
| 25 | A | 4 | 3 | 0.87 | 40 | 0.075 |
| 26 | A | 4 | 3 | 0.85 | 40 | 0.075 |
| 27 | A | 10 | 9 | 0.87 | 40 | 0.225 |
| 28 | A | 3 | 2 | 0.83 | 40 | 0.050 |
| 29 | A | 6 | 5 | 0.76 | 40 | 0.125 |
| 30 | A | 6 | 5 | 0.73 | 40 | 0.125 |
| 31 | A | 7 | 6 | 1.18 | 40 | 0.150 |
| 32 | A | 6 | 5 | 1.14 | 40 | 0.125 |
| 33 | A | 5 | 4 | 1.16 | 38 | 0.105 |
| 34 | A | 8 | 7 | 1.07 | 30 | 0.233 |
| 35 | A | 3 | 2 | 1.00 | 40 | 0.050 |
| 36 | A | 6 | 5 | 0.73 | 40 | 0.125 |
| 37 | A | 6 | 5 | 0.76 | 40 | 0.125 |
| 38 | A | 6 | 5 | 0.70 | 40 | 0.125 |
| 39 | A | 6 | 5 | 0.89 | 40 | 0.125 |
| 40 | A | 5 | 4 | 0.87 | 40 | 0.100 |
| 41 | A | 4 | 3 | 0.91 | 38 | 0.079 |
| 42 | A | 3 | 2 | 0.74 | 30 | 0.067 |
| 43 | A | 5 | 4 | 0.70 | 40 | 0.100 |
| 44 | A | 4 | 3 | 0.70 | 40 | 0.075 |
| 45 | A | 6 | 5 | 0.70 | 40 | 0.125 |
| 46 | A | 4 | 3 | 0.68 | 40 | 0.075 |
| 47 | A | 5 | 4 | 0.83 | 40 | 0.100 |
| 48 | A | 4 | 3 | 0.86 | 40 | 0.075 |
| 49 | A | 3 | 2 | 0.85 | 38 | 0.053 |
| 50 | A | 4 | 4 | 0.96 | 30 | 0.133 |
| 51 | A | 4 | 3 | 0.75 | 40 | 0.075 |
| 52 | A | 4 | 3 | 0.69 | 40 | 0.075 |
| 53 | A | 4 | 3 | 0.69 | 40 | 0.075 |
| 54 | A | 6 | 5 | 0.69 | 40 | 0.125 |
| 55 | A | 12 | 11 | 1.14 | 40 | 0.275 |
| 56 | A | 11 | 10 | 1.15 | 40 | 0.250 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 57 | A | 10 | 9 | 1.18 | 38 | 0.237 |
| 58 | A | 8 | 7 | 1.12 | 30 | 0.233 |
| 59 | A | 9 | 8 | 0.95 | 40 | 0.200 |
| 60 | A | 8 | 7 | 0.90 | 40 | 0.175 |
| 61 | A | 4 | 3 | 0.81 | 40 | 0.075 |
| 62 | A | 4 | 3 | 0.76 | 40 | 0.075 |
| 63 | A | 4 | 3 | 0.75 | 40 | 0.075 |
| 64 | A | 18 | 17 | 1.35 | 42 | 0.405 |
| 65 | A | 16 | 15 | 1.12 | 42 | 0.357 |
| 66 | A | 15 | 14 | 1.14 | 40 | 0.350 |
| 67 | A | 12 | 11 | 1.08 | 32 | 0.344 |
| 68 | A | 13 | 12 | 0.94 | 42 | 0.286 |
| 69 | A | 4 | 3 | 0.83 | 42 | 0.071 |
| 70 | A | 11 | 10 | 0.85 | 42 | 0.238 |
| 71 | A | 4 | 3 | 0.81 | 42 | 0.071 |
| 72 | A | 4 | 3 | 0.74 | 42 | 0.071 |
| 73 | A | 4 | 3 | 0.73 | 42 | 0.071 |
| 74 | A | 23 | 22 | 1.28 | 42 | 0.524 |
| 75 | A | 21 | 20 | 1.34 | 42 | 0.476 |
| 76 | A | 20 | 19 | 1.31 | 40 | 0.475 |
| 77 | A | 16 | 15 | 1.23 | 32 | 0.469 |
| 78 | A | 17 | 16 | 1.20 | 42 | 0.381 |
| 79 | A | 4 | 3 | 0.83 | 42 | 0.071 |
| 80 | A | 4 | 3 | 0.80 | 42 | 0.071 |
| 81 | A | 4 | 3 | 0.81 | 42 | 0.071 |
| 82 | A | 4 | 3 | 0.74 | 42 | 0.071 |
| 83 | A | 4 | 3 | 0.73 | 42 | 0.071 |
| 84 | A | 4 | 3 | 0.88 | 42 | 0.071 |
| 85 | A | 4 | 3 | 0.88 | 42 | 0.071 |
| 86 | A | 4 | 3 | 0.92 | 40 | 0.075 |
| 87 | A | 5 | 4 | 0.94 | 32 | 0.125 |
| 88 | A | 4 | 3 | 0.84 | 42 | 0.071 |
| 89 | A | 4 | 3 | 0.74 | 42 | 0.071 |
| 90 | A | 4 | 3 | 0.72 | 42 | 0.071 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 91 | A | 4 | 3 | 0.72 | 42 | 0.071 |
| 92 | A | 4 | 3 | 0.82 | 42 | 0.071 |
| 93 | A | 4 | 3 | 0.81 | 42 | 0.071 |
| 94 | A | 4 | 3 | 0.92 | 40 | 0.075 |
| 95 | A | 4 | 3 | 0.73 | 32 | 0.094 |
| 96 | A | 7 | 6 | 0.65 | 42 | 0.143 |
| 97 | A | 4 | 3 | 0.70 | 42 | 0.071 |
| 98 | A | 4 | 3 | 0.70 | 42 | 0.071 |
| 99 | A | 4 | 3 | 0.70 | 42 | 0.071 |
| 100 | A | 4 | 3 | 0.79 | 42 | 0.071 |
| 101 | A | 4 | 3 | 0.87 | 42 | 0.071 |
| 102 | A | 4 | 3 | 0.82 | 40 | 0.075 |
| 103 | A | 4 | 3 | 0.74 | 32 | 0.094 |
| 104 | A | 10 | 9 | 0.94 | 42 | 0.214 |
| 105 | A | 4 | 3 | 0.70 | 42 | 0.071 |
| 106 | A | 4 | 3 | 0.70 | 42 | 0.071 |
| 107 | A | 4 | 3 | 0.70 | 42 | 0.071 |
| 108 | A | 5 | 5 | 0.88 | 41 | 0.122 |
| 109 | A | 5 | 5 | 0.91 | 41 | 0.122 |
| 110 | A | 5 | 5 | 0.97 | 39 | 0.128 |
| 111 | A | 4 | 4 | 0.94 | 31 | 0.129 |
| 112 | A | 9 | 8 | 0.95 | 41 | 0.195 |
| 113 | A | 6 | 5 | 0.95 | 41 | 0.122 |
| 114 | A | 3 | 2 | 0.85 | 41 | 0.049 |
| 115 | A | 6 | 5 | 0.79 | 41 | 0.122 |
| 116 | A | 6 | 5 | 0.75 | 41 | 0.122 |
| 117 | A | 6 | 5 | 0.88 | 43 | 0.116 |
| 118 | A | 6 | 5 | 0.88 | 43 | 0.116 |
| 119 | A | 6 | 5 | 0.94 | 41 | 0.122 |
| 120 | A | 4 | 4 | 0.88 | 33 | 0.121 |
| 121 | A | 11 | 10 | 1.10 | 43 | 0.233 |
| 122 | A | 4 | 3 | 0.91 | 43 | 0.070 |
| 123 | A | 8 | 7 | 0.89 | 43 | 0.163 |
| 124 | A | 3 | 2 | 0.84 | 43 | 0.047 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 125 | A | 6 | 5 | 0.77 | 43 | 0.116 |
| 126 | A | 6 | 5 | 0.73 | 43 | 0.116 |
| 127 | A | 6 | 5 | 0.87 | 43 | 0.116 |
| 128 | A | 6 | 5 | 0.87 | 43 | 0.116 |
| 129 | A | 6 | 5 | 0.92 | 41 | 0.122 |
| 130 | A | 4 | 4 | 0.85 | 33 | 0.121 |
| 131 | A | 15 | 14 | 1.28 | 43 | 0.326 |
| 132 | A | 4 | 3 | 0.88 | 43 | 0.070 |
| 133 | A | 4 | 3 | 0.86 | 43 | 0.070 |
| 134 | A | 10 | 9 | 0.88 | 43 | 0.209 |
| 135 | A | 7 | 6 | 1.17 | 43 | 0.140 |
| 136 | A | 6 | 5 | 1.13 | 43 | 0.116 |
| 137 | A | 5 | 4 | 1.15 | 41 | 0.098 |
| 138 | A | 8 | 7 | 1.06 | 33 | 0.212 |
| 139 | A | 3 | 2 | 1.00 | 43 | 0.047 |
| 140 | A | 6 | 5 | 0.74 | 43 | 0.116 |
| 141 | A | 6 | 5 | 0.77 | 43 | 0.116 |
| 142 | A | 6 | 5 | 0.71 | 43 | 0.116 |
| 143 | A | 6 | 5 | 0.90 | 43 | 0.116 |
| 144 | A | 5 | 4 | 0.88 | 43 | 0.093 |
| 145 | A | 4 | 3 | 0.92 | 41 | 0.073 |
| 146 | A | 3 | 2 | 0.75 | 33 | 0.061 |
| 147 | A | 5 | 4 | 0.72 | 43 | 0.093 |
| 148 | A | 4 | 3 | 0.70 | 43 | 0.070 |
| 149 | A | 6 | 5 | 0.70 | 43 | 0.116 |
| 150 | A | 4 | 3 | 0.69 | 43 | 0.070 |
| 151 | A | 5 | 4 | 0.84 | 43 | 0.093 |
| 152 | A | 4 | 3 | 0.87 | 43 | 0.070 |
| 153 | A | 3 | 2 | 0.85 | 41 | 0.049 |
| 154 | A | 4 | 4 | 0.94 | 33 | 0.121 |
| 155 | A | 4 | 3 | 0.76 | 43 | 0.070 |
| 156 | A | 4 | 3 | 0.70 | 43 | 0.070 |
| 157 | A | 4 | 3 | 0.69 | 43 | 0.070 |
| 158 | A | 6 | 5 | 0.69 | 43 | 0.116 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 159 | A | 12 | 11 | 1.11 | 43 | 0.256 |
| 160 | A | 11 | 10 | 1.11 | 43 | 0.233 |
| 161 | A | 10 | 9 | 1.15 | 41 | 0.220 |
| 162 | A | 8 | 7 | 1.09 | 33 | 0.212 |
| 163 | A | 9 | 8 | 0.94 | 43 | 0.186 |
| 164 | A | 8 | 7 | 0.90 | 43 | 0.163 |
| 165 | A | 4 | 3 | 0.81 | 43 | 0.070 |
| 166 | A | 4 | 3 | 0.78 | 43 | 0.070 |
| 167 | A | 4 | 3 | 0.77 | 43 | 0.070 |
| 168 | A | 18 | 17 | 1.31 | 45 | 0.378 |
| 169 | A | 16 | 15 | 1.09 | 45 | 0.333 |
| 170 | A | 15 | 14 | 1.11 | 43 | 0.326 |
| 171 | A | 12 | 11 | 1.04 | 35 | 0.314 |
| 172 | A | 13 | 12 | 0.92 | 45 | 0.267 |
| 173 | A | 4 | 3 | 0.84 | 45 | 0.067 |
| 174 | A | 11 | 10 | 0.85 | 45 | 0.222 |
| 175 | A | 4 | 3 | 0.81 | 45 | 0.067 |
| 176 | A | 4 | 3 | 0.75 | 45 | 0.067 |
| 177 | A | 4 | 3 | 0.74 | 45 | 0.067 |
| 178 | A | 23 | 22 | 1.24 | 45 | 0.489 |
| 179 | A | 21 | 20 | 1.30 | 45 | 0.444 |
| 180 | A | 20 | 19 | 1.27 | 43 | 0.442 |
| 181 | A | 16 | 15 | 1.18 | 35 | 0.429 |
| 182 | A | 17 | 16 | 1.18 | 45 | 0.356 |
| 183 | A | 4 | 3 | 0.84 | 45 | 0.067 |
| 184 | A | 4 | 3 | 0.82 | 45 | 0.067 |
| 185 | A | 14 | 13 | 0.84 | 45 | 0.289 |
| 186 | A | 4 | 3 | 0.88 | 45 | 0.067 |
| 187 | A | 4 | 3 | 0.88 | 45 | 0.067 |
| 188 | A | 4 | 3 | 0.92 | 43 | 0.070 |
| 189 | A | 5 | 4 | 0.93 | 35 | 0.114 |
| 190 | A | 4 | 3 | 0.80 | 45 | 0.067 |
| 191 | A | 4 | 3 | 0.76 | 45 | 0.067 |
| 192 | A | 4 | 3 | 0.74 | 45 | 0.067 |

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Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 193 | A | 4 | 3 | 0.74 | 45 | 0.067 |
| 194 | A | 4 | 3 | 0.83 | 45 | 0.067 |
| 195 | A | 4 | 3 | 0.82 | 45 | 0.067 |
| 196 | A | 4 | 3 | 0.92 | 43 | 0.070 |
| 197 | A | 4 | 3 | 0.73 | 35 | 0.086 |
| 198 | A | 7 | 6 | 0.65 | 45 | 0.133 |
| 199 | A | 4 | 3 | 0.72 | 45 | 0.067 |
| 200 | A | 4 | 3 | 0.72 | 45 | 0.067 |
| 201 | A | 4 | 3 | 0.72 | 45 | 0.067 |
| 202 | A | 4 | 3 | 0.81 | 45 | 0.067 |
| 203 | A | 4 | 3 | 0.88 | 45 | 0.067 |
| 204 | A | 4 | 3 | 0.81 | 43 | 0.070 |
| 205 | A | 4 | 3 | 0.76 | 35 | 0.086 |
| 206 | A | 10 | 9 | 0.95 | 45 | 0.200 |
| 207 | A | 4 | 3 | 0.72 | 45 | 0.067 |
| 208 | A | 4 | 3 | 0.72 | 45 | 0.067 |
| 209 | A | 4 | 3 | 0.72 | 45 | 0.067 |
| 210 | A | 4 | 3 | 1.00 | 49 | 0.061 |
| 211 | A | 4 | 3 | 1.00 | 49 | 0.061 |
| 212 | A | 5 | 4 | 0.81 | 49 | 0.082 |
| 213 | A | 4 | 3 | 0.86 | 49 | 0.061 |
| 214 | A | 3 | 2 | 0.96 | 47 | 0.043 |
| 215 | A | 4 | 3 | 1.00 | 49 | 0.061 |
| 216 | A | 5 | 4 | 0.98 | 49 | 0.082 |
| 217 | A | 6 | 5 | 0.90 | 49 | 0.102 |
| 218 | A | 5 | 4 | 0.82 | 49 | 0.082 |
| 219 | A | 4 | 3 | 0.86 | 49 | 0.061 |
| 220 | A | 3 | 2 | 0.96 | 47 | 0.043 |
| 221 | A | 4 | 3 | 1.00 | 49 | 0.061 |
| 222 | A | 5 | 4 | 0.98 | 49 | 0.082 |
| 223 | A | 6 | 5 | 0.90 | 49 | 0.102 |
| 224 | A | 4 | 3 | 1.00 | 35 | 0.086 |
| 225 | A | 5 | 4 | 1.00 | 42 | 0.095 |
| 226 | A | 5 | 4 | 0.98 | 50 | 0.080 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 227 | A | 5 | 4 | 0.98 | 50 | 0.080 |
| 228 | A | 5 | 4 | 0.76 | 40 | 0.100 |
| 229 | A | 5 | 4 | 0.76 | 40 | 0.100 |
| 230 | A | 4 | 3 | 0.98 | 38 | 0.079 |
| 231 | A | 5 | 4 | 0.98 | 40 | 0.100 |
| 232 | A | 5 | 4 | 0.70 | 40 | 0.100 |
| 233 | A | 5 | 4 | 0.76 | 40 | 0.100 |
| 234 | A | 5 | 4 | 0.98 | 40 | 0.100 |
| 235 | A | 5 | 4 | 0.98 | 46 | 0.087 |
| 236 | A | 6 | 5 | 0.98 | 50 | 0.100 |
| 237 | A | 5 | 4 | 0.98 | 40 | 0.100 |
| 238 | A | 5 | 4 | 0.98 | 46 | 0.087 |
| 239 | A | 6 | 5 | 0.98 | 50 | 0.100 |
| 240 | A | 5 | 4 | 0.98 | 40 | 0.100 |
| 241 | A | 4 | 3 | 1.00 | 35 | 0.086 |
| 242 | A | 4 | 3 | 1.00 | 35 | 0.086 |
| 243 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 244 | A | 4 | 3 | 1.00 | 28 | 0.107 |
| 245 | A | 4 | 3 | 1.00 | 35 | 0.086 |
| 246 | A | 4 | 3 | 1.00 | 28 | 0.107 |
| 247 | A | 4 | 3 | 1.00 | 33 | 0.091 |
| 248 | A | 4 | 3 | 1.00 | 35 | 0.086 |
| 249 | A | 9 | 8 | 0.86 | 43 | 0.186 |
| 250 | A | 8 | 7 | 0.88 | 43 | 0.163 |
| 251 | A | 7 | 6 | 0.90 | 43 | 0.140 |
| 252 | A | 6 | 5 | 0.98 | 41 | 0.122 |
| 253 | N/A | 3 | 0 | 1.00 | 43 | 0.000 |
| 254 | N/A | 3 | 0 | 1.00 | 43 | 0.000 |
| 255 | A | 5 | 4 | 1.00 | 40 | 0.100 |
| 256 | A | 3 | 2 | 1.41 | 38 | 0.053 |
| 257 | A | 4 | 3 | 1.41 | 33 | 0.091 |
| 258 | A | 4 | 3 | 1.41 | 38 | 0.079 |
| 259 | A | 8 | 7 | 0.99 | 51 | 0.137 |

Continued on next page

Table 2.1 – continued from previous page

| # | grade | number of steps used | number of unique rules | normalized antiderivative leaf size | integrand leaf size | $\frac{\text{number of rules}}{\text{integrand leaf size}}$ |
|---|---|---|---|---|---|---|
| 260 | A | 7 | 6 | 1.05 | 51 | 0.118 |
| 261 | A | 6 | 5 | 1.23 | 49 | 0.102 |
| 262 | N/A | 5 | 0 | 1.00 | 51 | 0.000 |
| 263 | N/A | 5 | 0 | 1.00 | 51 | 0.000 |

## 2.3. Detailed conclusion table specific for Rubi results

# CHAPTER 3

## Listing of integrals

# 3.1 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$



## 3.1.1 Optimal result

Integrand size = 38, antiderivative size = 212

$$
\begin{aligned}
&\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx \\
&= -\frac{B(bc-ad)^4g^3ix}{20bd^3}+\frac{B(bc-ad)^3g^3i(a+bx)^2}{40b^2d^2} \\
&\quad -\frac{B(bc-ad)^2g^3i(a+bx)^3}{60b^2d}+\frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5b} \\
&\quad +\frac{(bc-ad)g^3i(a+bx)^4\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2}+\frac{B(bc-ad)^5g^3i\log(c+dx)}{20b^2d^4}
\end{aligned}
$$

output
```
-1/20*B*(-a*d+b*c)^4*g^3*i*x/b/d^3+1/40*B*(-a*d+b*c)^3*g^3*i*(b*x+a)^2/b^2
/d^2-1/60*B*(-a*d+b*c)^2*g^3*i*(b*x+a)^3/b^2/d+1/5*g^3*i*(b*x+a)^4*(d*x+c)
*(A+B*ln(e*(b*x+a)/(d*x+c)))/b+1/20*(-a*d+b*c)*g^3*i*(b*x+a)^4*(A-B+B*ln(e
*(b*x+a)/(d*x+c)))/b^2+1/20*B*(-a*d+b*c)^5*g^3*i*ln(d*x+c)/b^2/d^4
```

### 3.1.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 261, normalized size of antiderivative = 1.23

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{g^3i\left(30(bc-ad)(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+24d(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-\frac{5B(bc-ad)^2(6bd(bc}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)
]),x]
```

output
```
(g^3*i*(30*(b*c - a*d)*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) +
24*d*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (5*B*(b*c - a*d)^2
*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*
x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/d^4 + (2*B*(b*c - a*d)*(12*b*d*(b*c
- a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)*(a + b*x)
^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]))/d^4))/(120*b^2)
```

### 3.1.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.89, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2960, 27, 2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)\,dx$$

$$\Big\downarrow \text{2960}$$

$$\frac{i(bc-ad)\int g^3(a+bx)^3\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{5b}+\frac{g^3i(a+bx)^4(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^3 i(bc-ad)\int (a+bx)^3\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}$$

$$\Big\downarrow \ 2948$$

$$\frac{g^3 i(bc-ad)\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{4b}-\frac{B(bc-ad)\int \frac{(a+bx)^3}{c+dx}dx}{4b}\right)}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}$$

$$\Big\downarrow \ 49$$

$$\frac{g^3 i(bc-ad)\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{4b}-\frac{B(bc-ad)\int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{4b}\right)}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}$$

$$\Big\downarrow \ 2009$$

$$\frac{g^3 i(bc-ad)\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{4b}-\frac{B(bc-ad)\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{4b}\right)}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^3*i*(a + b*x)^4*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*b) +
 ((b*c - a*d)*g^3*i*(((a + b*x)^4*(A - B + B*Log[(e*(a + b*x))/(c + d*x)])
)/(4*b) - (B*(b*c - a*d)*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(a + b*x)
^2)/(2*d^2) + (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4))/(4*b)
))/(5*b)
```

### 3.1.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

rule 2960
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Sim
p[(f + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*
(m + 2))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B
*n + B*Log[e*((a + b*x)^n/(c + d*x)^n)]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, i, A, B, m, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IGtQ[m, -2]
```

### 3.1.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 573 vs. 2(200) = 400.

Time = 0.96 (sec) , antiderivative size = 574, normalized size of antiderivative = 2.71

| method | result |
|---|---|
| risch | $\frac{i\,g^3b^3Ac\,x^4}{4}+\frac{i\,g^3b^2dBa\,x^4}{20}-\frac{i\,g^3b^3Bc\,x^4}{20}+i\,g^3bdA\,a^2x^3+\frac{11i\,g^3bdB\,a^2x^3}{60}-\frac{i\,g^3b^3B\,c^2x^3}{60d}+\frac{i\,g^3dA\,a^3x^{}}{2}$ |
| parallelrisch | Expression too large to display |
| parts | Expression too large to display |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURN
VERBOSE)
```

output
```
1/4*i*g^3*b^3*A*c*x^4+1/20*i*g^3*b^2*d*B*a*x^4-1/20*i*g^3*b^3*B*c*x^4+i*g^
3*b*d*A*a^2*x^3+11/60*i*g^3*b*d*B*a^2*x^3-1/60*i*g^3*b^3/d*B*c^2*x^3+1/2*i
*g^3*d*A*a^3*x^2+9/40*i*g^3*d*B*a^3*x^2+1/40*i*g^3*b^3/d^2*B*c^3*x^2+i*g^3
*b^2*A*a*c*x^3-1/6*i*g^3*b^2*B*a*c*x^3+3/2*i*g^3*b*A*a^2*c*x^2-1/8*i*g^3*b
*B*a^2*c*x^2-1/8*i*g^3*b^2/d*B*a*c^2*x^2+i*g^3*A*a^3*c*x+1/20*i*g^3/b*d*B*
a^4*x-1/20*i*g^3*b^3/d^3*B*c^4*x-1/2*i*g^3/d*B*ln(-d*x-c)*a^3*c^2+1/4*i*g^
3*B*a^3*c*x-1/2*i*g^3*b/d*B*a^2*c^2*x+1/4*i*g^3*b^2/d^2*B*a*c^3*x+1/2*i*g^
3*b/d^2*B*ln(-d*x-c)*a^2*c^3-1/4*i*g^3*b^2/d^3*B*ln(-d*x-c)*a*c^4+1/20*i*g
^3*b^3/d^4*B*ln(-d*x-c)*c^5-1/20*i*g^3/b^2*d*B*ln(b*x+a)*a^5+1/4*i*g^3/b*B
*ln(b*x+a)*a^4*c+3/4*i*g^3*b^2*d*A*a*x^4+1/5*i*g^3*b^3*d*A*x^5+1/20*i*g^3*
B*x*(4*b^3*d*x^4+15*a*b^2*d*x^3+5*b^3*c*x^3+20*a^2*b*d*x^2+20*a*b^2*c*x^2+
10*a^3*d*x+30*a^2*b*c*x+20*a^3*c)*ln(e*(b*x+a)/(d*x+c))
```

### 3.1.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 504 vs. 2(200) = 400.

Time = 0.37 (sec) , antiderivative size = 504, normalized size of antiderivative = 2.38

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{24\,Ab^5d^5g^3ix^5+6\,((5\,A-B)b^5cd^4+(15\,A+B)ab^4d^5)g^3ix^4-2\,(Bb^5c^2d^3-10\,(6\,A-B)ab^4cd^4-(60\,A}{}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="fricas")
```

output
```
1/120*(24*A*b^5*d^5*g^3*i*x^5 + 6*((5*A - B)*b^5*c*d^4 + (15*A + B)*a*b^4*
d^5)*g^3*i*x^4 - 2*(B*b^5*c^2*d^3 - 10*(6*A - B)*a*b^4*c*d^4 - (60*A + 11*
B)*a^2*b^3*d^5)*g^3*i*x^3 + 3*(B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 + 5*(12*A
 - B)*a^2*b^3*c*d^4 + (20*A + 9*B)*a^3*b^2*d^5)*g^3*i*x^2 - 6*(B*b^5*c^4*d
 - 5*B*a*b^4*c^3*d^2 + 10*B*a^2*b^3*c^2*d^3 - 5*(4*A + B)*a^3*b^2*c*d^4 -
B*a^4*b*d^5)*g^3*i*x + 6*(5*B*a^4*b*c*d^4 - B*a^5*d^5)*g^3*i*log(b*x + a)
+ 6*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2
*d^3)*g^3*i*log(d*x + c) + 6*(4*B*b^5*d^5*g^3*i*x^5 + 20*B*a^3*b^2*c*d^4*g
^3*i*x + 5*(B*b^5*c*d^4 + 3*B*a*b^4*d^5)*g^3*i*x^4 + 20*(B*a*b^4*c*d^4 + B
*a^2*b^3*d^5)*g^3*i*x^3 + 10*(3*B*a^2*b^3*c*d^4 + B*a^3*b^2*d^5)*g^3*i*x^2
)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d^4)
```

### 3.1.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(194) = 388$.

Time = 4.09 (sec) , antiderivative size = 1158, normalized size of antiderivative = 5.46

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{Ab^3dg^3ix^5}{5}$$

$$-\frac{Ba^4g^3i(ad-5bc)\log\Big(x+\frac{Ba^5cd^4g^3i+\frac{Ba^5d^4g^3i(ad-5bc)}{b}-15Ba^4bc^2d^3g^3i-Ba^4cd^3g^3i(ad-5bc)+10Ba^3b^2c^3d^2g^3i-5Ba^2b^3c^4dg^3}{Ba^5d^5g^3i-5Ba^4bcd^4g^3i-10Ba^3b^2c^2d^3g^3i+10Ba^2b^3c^3d^2g^3i-5Bab^4c^4dg^3i+Bb^5c^5g^3i}}{20b^2}$$

$$-\frac{Bc^2g^3i(10a^3d^3-10a^2bcd^2+5ab^2c^2d-b^3c^3)\log\Big(x+\frac{Ba^5cd^4g^3i-15Ba^4bc^2d^3g^3i+10Ba^3b^2c^3d^2g^3i-5Ba^2b^3c^4dg^3i+B}{Ba^5d^5g^3i-5Ba^4bcd^4g^3i-}}{20d^4}$$

$$+x^4\cdot\left(\frac{3Aab^2dg^3i}{4}+\frac{Ab^3cg^3i}{4}+\frac{Bab^2dg^3i}{20}-\frac{Bb^3cg^3i}{20}\right)$$

$$+x^3\left(Aa^2bdg^3i+Aab^2cg^3i+\frac{11Ba^2bdg^3i}{60}-\frac{Bab^2cg^3i}{6}-\frac{Bb^3c^2g^3i}{60d}\right)$$

$$+x^2\left(\frac{Aa^3dg^3i}{2}+\frac{3Aa^2bcg^3i}{2}+\frac{9Ba^3dg^3i}{40}-\frac{Ba^2bcg^3i}{8}-\frac{Bab^2c^2g^3i}{8d}+\frac{Bb^3c^3g^3i}{40d^2}\right)$$

$$+x\left(Aa^3cg^3i+\frac{Ba^4dg^3i}{20b}+\frac{Ba^3cg^3i}{4}-\frac{Ba^2bc^2g^3i}{2d}+\frac{Bab^2c^3g^3i}{4d^2}-\frac{Bb^3c^4g^3i}{20d^3}\right)$$

$$+\left(Ba^3cg^3ix+\frac{Ba^3dg^3ix^2}{2}+\frac{3Ba^2bcg^3ix^2}{2}+Ba^2bdg^3ix^3+Bab^2cg^3ix^3+\frac{3Bab^2dg^3ix^4}{4}+\frac{Bb^3cg^3ix^4}{4}+\frac{Bb^3dg^3ix^5}{5}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b**3*d*g**3*i*x**5/5 - B*a**4*g**3*i*(a*d - 5*b*c)*log(x + (B*a**5*c*d**
4*g**3*i + B*a**5*d**4*g**3*i*(a*d - 5*b*c)/b - 15*B*a**4*b*c**2*d**3*g**3
*i - B*a**4*c*d**3*g**3*i*(a*d - 5*b*c) + 10*B*a**3*b**2*c**3*d**2*g**3*i
- 5*B*a**2*b**3*c**4*d*g**3*i + B*a*b**4*c**5*g**3*i)/(B*a**5*d**5*g**3*i
- 5*B*a**4*b*c*d**4*g**3*i - 10*B*a**3*b**2*c**2*d**3*g**3*i + 10*B*a**2*b
**3*c**3*d**2*g**3*i - 5*B*a*b**4*c**4*d*g**3*i + B*b**5*c**5*g**3*i))/(20
*b**2) - B*c**2*g**3*i*(10*a**3*d**3 - 10*a**2*b*c*d**2 + 5*a*b**2*c**2*d
- b**3*c**3)*log(x + (B*a**5*c*d**4*g**3*i - 15*B*a**4*b*c**2*d**3*g**3*i
+ 10*B*a**3*b**2*c**3*d**2*g**3*i - 5*B*a**2*b**3*c**4*d*g**3*i + B*a*b**4
*c**5*g**3*i + B*a*b*c**2*g**3*i*(10*a**3*d**3 - 10*a**2*b*c*d**2 + 5*a*b*
*2*c**2*d - b**3*c**3) - B*b**2*c**3*g**3*i*(10*a**3*d**3 - 10*a**2*b*c*d*
*2 + 5*a*b**2*c**2*d - b**3*c**3)/d)/(B*a**5*d**5*g**3*i - 5*B*a**4*b*c*d*
*4*g**3*i - 10*B*a**3*b**2*c**2*d**3*g**3*i + 10*B*a**2*b**3*c**3*d**2*g**
3*i - 5*B*a*b**4*c**4*d*g**3*i + B*b**5*c**5*g**3*i))/(20*d**4) + x**4*(3*
A*a*b**2*d*g**3*i/4 + A*b**3*c*g**3*i/4 + B*a*b**2*d*g**3*i/20 - B*b**3*c*
g**3*i/20) + x**3*(A*a**2*b*d*g**3*i + A*a*b**2*c*g**3*i + 11*B*a**2*b*d*g
**3*i/60 - B*a*b**2*c*g**3*i/6 - B*b**3*c**2*g**3*i/(60*d)) + x**2*(A*a**3
*d*g**3*i/2 + 3*A*a**2*b*c*g**3*i/2 + 9*B*a**3*d*g**3*i/40 - B*a**2*b*c*g*
*3*i/8 - B*a*b**2*c**2*g**3*i/(8*d) + B*b**3*c**3*g**3*i/(40*d**2)) + x*(A
*a**3*c*g**3*i + B*a**4*d*g**3*i/(20*b) + B*a**3*c*g**3*i/4 - B*a**2*b*...
```

### 3.1.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(200) = 400$.

Time = 0.24 (sec) , antiderivative size = 1022, normalized size of antiderivative = 4.82

$$\int (ag + bgx)^3(ci + dix)\left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right)\,dx$$
$$= \frac{1}{5}Ab^3dg^3ix^5 + \frac{1}{4}Ab^3cg^3ix^4 + \frac{3}{4}Aab^2dg^3ix^4 + Aab^2cg^3ix^3 + Aa^2bdg^3ix^3 + \frac{3}{2}Aa^2bcg^3ix^2$$
$$+ \frac{1}{2}Aa^3dg^3ix^2 + \left(x\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right)Ba^3cg^3i$$
$$+ \frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)Ba^2bcg^3i$$
$$+ \frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a}{b^2d^2}\right.$$
$$+ \frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{6a^4\log(bx+a)}{b^4} + \frac{6c^4\log(dx+c)}{d^4} - \frac{2(b^3cd^2-ab^2d^3)x^3 - 3(b^3c^2}{}\right.$$
$$+ \frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)Ba^3dg^3i$$
$$+ \frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a}{b^2d^2}\right.$$
$$+ \frac{1}{8}\left(6x^4\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{6a^4\log(bx+a)}{b^4} + \frac{6c^4\log(dx+c)}{d^4} - \frac{2(b^3cd^2-ab^2d^3)x^3 - 3(b^3c^2c}{b^3}\right.$$
$$+ \frac{1}{60}\left(12x^5\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{12a^5\log(bx+a)}{b^5} - \frac{12c^5\log(dx+c)}{d^5} - \frac{3(b^4cd^3-ab^3d^4)x^4 - 4(}{}\right.$$
$$+ Aa^3cg^3ix$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="maxima")
```

output
```
1/5*A*b^3*d*g^3*i*x^5 + 1/4*A*b^3*c*g^3*i*x^4 + 3/4*A*a*b^2*d*g^3*i*x^4 +
A*a*b^2*c*g^3*i*x^3 + A*a^2*b*d*g^3*i*x^3 + 3/2*A*a^2*b*c*g^3*i*x^2 + 1/2*
A*a^3*d*g^3*i*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x +
a)/b - c*log(d*x + c)/d)*B*a^3*c*g^3*i + 3/2*(x^2*log(b*e*x/(d*x + c) + a*
e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x
/(b*d))*B*a^2*b*c*g^3*i + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))
+ 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x
^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2*c*g^3*i + 1/24*(6*x^4*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x
 + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2
 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*c*g^3*i + 1/2*(x^2*log(b*e*x/
(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 -
 (b*c - a*d)*x/(b*d))*B*a^3*d*g^3*i + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e
/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d
- a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b*d*g^3*i + 1/8
*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*
c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2
*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a*b^2*d*g^3*i + 1/60*(
12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12
*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2...
```

### 3.1.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3637 vs. $2(200) = 400$.

Time = 0.51 (sec) , antiderivative size = 3637, normalized size of antiderivative = 17.16

$$\int (ag + bgx)^3 (ci + dix) \left(A + B \log \left(\frac{e(a + bx)}{c + dx}\right)\right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="giac")
```

output
```
-1/120*(6*(B*b^9*c^6*e^6*g^3*i - 6*B*a*b^8*c^5*d*e^6*g^3*i + 15*B*a^2*b^7*
c^4*d^2*e^6*g^3*i - 20*B*a^3*b^6*c^3*d^3*e^6*g^3*i + 15*B*a^4*b^5*c^2*d^4*
e^6*g^3*i - 6*B*a^5*b^4*c*d^5*e^6*g^3*i + B*a^6*b^3*d^6*e^6*g^3*i - 5*(b*e
*x + a*e)*B*b^8*c^6*d*e^5*g^3*i/(d*x + c) + 30*(b*e*x + a*e)*B*a*b^7*c^5*d
^2*e^5*g^3*i/(d*x + c) - 75*(b*e*x + a*e)*B*a^2*b^6*c^4*d^3*e^5*g^3*i/(d*x
 + c) + 100*(b*e*x + a*e)*B*a^3*b^5*c^3*d^4*e^5*g^3*i/(d*x + c) - 75*(b*e*
x + a*e)*B*a^4*b^4*c^2*d^5*e^5*g^3*i/(d*x + c) + 30*(b*e*x + a*e)*B*a^5*b^
3*c*d^6*e^5*g^3*i/(d*x + c) - 5*(b*e*x + a*e)*B*a^6*b^2*d^7*e^5*g^3*i/(d*x
 + c) + 10*(b*e*x + a*e)^2*B*b^7*c^6*d^2*e^4*g^3*i/(d*x + c)^2 - 60*(b*e*x
 + a*e)^2*B*a*b^6*c^5*d^3*e^4*g^3*i/(d*x + c)^2 + 150*(b*e*x + a*e)^2*B*a^
2*b^5*c^4*d^4*e^4*g^3*i/(d*x + c)^2 - 200*(b*e*x + a*e)^2*B*a^3*b^4*c^3*d^
5*e^4*g^3*i/(d*x + c)^2 + 150*(b*e*x + a*e)^2*B*a^4*b^3*c^2*d^6*e^4*g^3*i/
(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^5*b^2*c*d^7*e^4*g^3*i/(d*x + c)^2 + 1
0*(b*e*x + a*e)^2*B*a^6*b*d^8*e^4*g^3*i/(d*x + c)^2 - 10*(b*e*x + a*e)^3*B
*b^6*c^6*d^3*e^3*g^3*i/(d*x + c)^3 + 60*(b*e*x + a*e)^3*B*a*b^5*c^5*d^4*e^
3*g^3*i/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*a^2*b^4*c^4*d^5*e^3*g^3*i/(d*x
 + c)^3 + 200*(b*e*x + a*e)^3*B*a^3*b^3*c^3*d^6*e^3*g^3*i/(d*x + c)^3 - 15
0*(b*e*x + a*e)^3*B*a^4*b^2*c^2*d^7*e^3*g^3*i/(d*x + c)^3 + 60*(b*e*x + a*
e)^3*B*a^5*b*c*d^8*e^3*g^3*i/(d*x + c)^3 - 10*(b*e*x + a*e)^3*B*a^6*d^9*e^
3*g^3*i/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/(b^5*d^4*e^5 - 5*(b*e...
```

### 3.1.9 Mupad [B] (verification not implemented)

Time = 2.01 (sec) , antiderivative size = 1195, normalized size of antiderivative = 5.64

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x*((a*c*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d - B*b
*c))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d) - (b*g^3*i*(24*A*a^
2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d - 2*B*a*b*c*d
))/(4*d) + A*a*b^2*c*g^3*i))/(b*d) - ((20*a*d + 20*b*c)*(((20*a*d + 20*b*c
)*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d - B*b*c))/5
 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d) - (b*g^3*i*(24*A*a^2*d^2
+ 4*A*b^2*c^2 + 3*B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d - 2*B*a*b*c*d))/(4*
d) + A*a*b^2*c*g^3*i))/(20*b*d) - (a*c*((b^2*g^3*i*(20*A*a*d + 10*A*b*c +
B*a*d - B*b*c))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(b*d) + (a*g^3*i*
(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 12*A*a*b*c*d))/d))/(2
0*b*d) + (a^2*g^3*i*(2*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - 3*B*b^2*c^2
+ 16*A*a*b*c*d + 2*B*a*b*c*d))/(2*b*d)) + x^4*((b^2*g^3*i*(20*A*a*d + 10*A
*b*c + B*a*d - B*b*c))/20 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/80) - x^3*(((2
0*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d - B*b*c))/5 - (A*
b^2*g^3*i*(20*a*d + 20*b*c))/20))/(60*b*d) - (b*g^3*i*(24*A*a^2*d^2 + 4*A*
b^2*c^2 + 3*B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d - 2*B*a*b*c*d))/(12*d) +
(A*a*b^2*c*g^3*i)/3) + x^2*(((20*a*d + 20*b*c)*(((20*a*d + 20*b*c)*((b^2*g
^3*i*(20*A*a*d + 10*A*b*c + B*a*d - B*b*c))/5 - (A*b^2*g^3*i*(20*a*d + 20*
b*c))/20))/(20*b*d) - (b*g^3*i*(24*A*a^2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2 -
 B*b^2*c^2 + 32*A*a*b*c*d - 2*B*a*b*c*d))/(4*d) + A*a*b^2*c*g^3*i))/(40...
```

3.1. $\int (ag + bgx)^3 (ci + dix) \left( A + B \log \left( \frac{e(a+bx)}{c+dx} \right) \right) \, dx$

# 3.2 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$



## 3.2.1 Optimal result

Integrand size = 38, antiderivative size = 180

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{B(bc-ad)^3g^2ix}{12bd^2}-\frac{B(bc-ad)^2g^2i(a+bx)^2}{24b^2d} + \frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b} + \frac{(bc-ad)g^2i(a+bx)^3\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2}-\frac{B(bc-ad)^4g^2i\log(c+dx)}{12b^2d^3}$$

output
```
1/12*B*(-a*d+b*c)^3*g^2*i*x/b/d^2-1/24*B*(-a*d+b*c)^2*g^2*i*(b*x+a)^2/b^2/
d+1/4*g^2*i*(b*x+a)^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b+1/12*(-a*d+b*c
)*g^2*i*(b*x+a)^3*(A-B+B*ln(e*(b*x+a)/(d*x+c)))/b^2-1/12*B*(-a*d+b*c)^4*g^
2*i*ln(d*x+c)/b^2/d^3
```

### 3.2.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 217, normalized size of antiderivative = 1.21

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{g^2i\left(8(bc-ad)(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+6d(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{4B(bc-ad)^2(2bd(bc-a}{}\right.}{24b^2}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)
]),x]
```

output
```
(g^2*i*(8*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6
*d*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (4*B*(b*c - a*d)^2*(
2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]))/d^3
 - (B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^
2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/d^3))/(24*b^2)
```

### 3.2.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 165, normalized size of antiderivative = 0.92, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2960, 27, 2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$
$$\Big\downarrow \text{ 2960}$$
$$\frac{i(bc-ad)\int g^2(a+bx)^2\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{4b}+\frac{g^2i(a+bx)^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{g^2 i(bc-ad)\int (a+bx)^2 \left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}$$

$$\Big\downarrow \text{2948}$$

$$\frac{g^2 i(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{3b}-\frac{B(bc-ad)\int \frac{(a+bx)^2}{c+dx}dx}{3b}\right)}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}$$

$$\Big\downarrow \text{49}$$

$$\frac{g^2 i(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{3b}-\frac{B(bc-ad)\int \left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}\right)}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2 i(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{3b}-\frac{B(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}\right)}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b}$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g^2*i*(a + b*x)^3*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*b) +
 ((b*c - a*d)*g^2*i*(((a + b*x)^3*(A - B + B*Log[(e*(a + b*x))/(c + d*x)])
)/(3*b) - (B*(b*c - a*d)*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(2*d) + (
(b*c - a*d)^2*Log[c + d*x])/d^3))/(3*b)))/(4*b)
```

## 3.2. $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$

### 3.2.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

rule 2960
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Sim
p[(f + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*
(m + 2))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B
*n + B*Log[e*((a + b*x)^n/(c + d*x)^n)]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, i, A, B, m, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IGtQ[m, -2]
```

### 3.2.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 395 vs. $2(170) = 340$.

Time = 0.80 (sec) , antiderivative size = 396, normalized size of antiderivative = 2.20

| method | result |
|---|---|
| risch | $\frac{i\,g^2 B x\left(3b^2 d\,x^3+8abd\,x^2+4b^2 c\,x^2+6a^2 dx+12abcx+12a^2 c\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{12}+\frac{i\,g^2 b^2 dA\,x^4}{4}+\frac{2i\,g^2 bdAa\,x^3}{3}+\frac{i\,g^2 b^2 Ac}{3}$ |
| parallelrisch | $16A\,x^3 a\,b^3 d^4 g^2 i+8A\,x^3 b^4 c\,d^3 g^2 i-36A\,a^3 bc\,d^3 g^2 i+6B\,x^4\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4 d^4 g^2 i+16B\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^3 d^4 g^2 i+7Ba\,b^3$ |
| parts | Expression too large to display |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURN
VERBOSE)
```

output
```
1/12*i*g^2*B*x*(3*b^2*d*x^3+8*a*b*d*x^2+4*b^2*c*x^2+6*a^2*d*x+12*a*b*c*x+1
2*a^2*c)*ln(e*(b*x+a)/(d*x+c))+1/4*i*g^2*b^2*d*A*x^4+2/3*i*g^2*b*d*A*a*x^3
+1/3*i*g^2*b^2*A*c*x^3+1/12*i*g^2*b*d*B*a*x^3-1/12*i*g^2*b^2*B*c*x^3+1/2*i
*g^2*d*A*a^2*x^2+i*g^2*b*A*a*c*x^2+5/24*i*g^2*d*B*a^2*x^2-1/6*i*g^2*b*B*a*
c*x^2-1/24*i*g^2*b^2/d*B*c^2*x^2+i*g^2*A*a^2*c*x-1/12*i*g^2/b^2*d*B*ln(b*x
+a)*a^4+1/3*i*g^2/b*B*ln(b*x+a)*a^3*c-1/2*i*g^2/d*B*ln(-d*x-c)*a^2*c^2+1/3
*i*g^2*b/d^2*B*ln(-d*x-c)*a*c^3-1/12*i*g^2*b^2/d^3*B*ln(-d*x-c)*c^4+1/12*i
*g^2/b*d*B*a^3*x+1/6*i*g^2*B*a^2*c*x-1/3*i*g^2*b/d*B*a*c^2*x+1/12*i*g^2*b^
2/d^2*B*c^3*x
```

### 3.2.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(170) = 340$.

Time = 0.38 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.06

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{6\,Ab^4d^4g^2ix^4+2\,((4\,A-B)b^4cd^3+(8\,A+B)ab^3d^4)g^2ix^3-(Bb^4c^2d^2-4\,(6\,A-B)ab^3cd^3-(12\,A+5\,B}{}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="fricas")
```

output
```
1/24*(6*A*b^4*d^4*g^2*i*x^4 + 2*((4*A - B)*b^4*c*d^3 + (8*A + B)*a*b^3*d^4
)*g^2*i*x^3 - (B*b^4*c^2*d^2 - 4*(6*A - B)*a*b^3*c*d^3 - (12*A + 5*B)*a^2*
b^2*d^4)*g^2*i*x^2 + 2*(B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 2*(6*A + B)*a^2*
b^2*c*d^3 + B*a^3*b*d^4)*g^2*i*x + 2*(4*B*a^3*b*c*d^3 - B*a^4*d^4)*g^2*i*l
og(b*x + a) - 2*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2)*g^2*i*
log(d*x + c) + 2*(3*B*b^4*d^4*g^2*i*x^4 + 12*B*a^2*b^2*c*d^3*g^2*i*x + 4*(
B*b^4*c*d^3 + 2*B*a*b^3*d^4)*g^2*i*x^3 + 6*(2*B*a*b^3*c*d^3 + B*a^2*b^2*d^
4)*g^2*i*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d^3)
```

### 3.2.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(163) = 326$.

Time = 2.48 (sec) , antiderivative size = 850, normalized size of antiderivative = 4.72

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{Ab^2dg^2ix^4}{4} - \frac{Ba^3g^2i(ad-4bc)\log\left(x+\frac{Ba^4cd^3g^2i+\frac{Ba^4d^3g^2i(ad-4bc)}{b}-10Ba^3bc^2d^2g^2i-Ba^3cd^2g^2i(ad-4bc)+4Ba^2b^2c^3dg^2i-Bab^3c^4g^2i}{Ba^4d^4g^2i-4Ba^3bcd^3g^2i-6Ba^2b^2c^2d^2g^2i+4Bab^3c^3dg^2i-Bb^4c^4g^2i}\right)}{12b^2} - \frac{Bc^2g^2i(6a^2d^2-4abcd+b^2c^2)\log\left(x+\frac{Ba^4cd^3g^2i-10Ba^3bc^2d^2g^2i+4Ba^2b^2c^3dg^2i-Bab^3c^4g^2i+Babc^2g^2i(6a^2d^2-4abcd+b^2\cdots}{Ba^4d^4g^2i-4Ba^3bcd^3g^2i-6Ba^2b^2c^2d^2g^2i+4Bab^3c^3dg^2i-B\cdots}\right)}{12d^3} + x^3\cdot\left(\frac{2Aabdg^2i}{3}+\frac{Ab^2cg^2i}{3}+\frac{Babdg^2i}{12}-\frac{Bb^2cg^2i}{12}\right) + x^2\left(\frac{Aa^2dg^2i}{2}+Aabcg^2i+\frac{5Ba^2dg^2i}{24}-\frac{Babcg^2i}{6}-\frac{Bb^2c^2g^2i}{24d}\right) + x\left(Aa^2cg^2i+\frac{Ba^3dg^2i}{12b}+\frac{Ba^2cg^2i}{6}-\frac{Babc^2g^2i}{3d}+\frac{Bb^2c^3g^2i}{12d^2}\right) + \left(Ba^2cg^2ix+\frac{Ba^2dg^2ix^2}{2}+Babcg^2ix^2+\frac{2Babdg^2ix^3}{3}+\frac{Bb^2cg^2ix^3}{3}+\frac{Bb^2dg^2ix^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b**2*d*g**2*i*x**4/4 - B*a**3*g**2*i*(a*d - 4*b*c)*log(x + (B*a**4*c*d**
3*g**2*i + B*a**4*d**3*g**2*i*(a*d - 4*b*c)/b - 10*B*a**3*b*c**2*d**2*g**2
*i - B*a**3*c*d**2*g**2*i*(a*d - 4*b*c) + 4*B*a**2*b**2*c**3*d*g**2*i - B*
a*b**3*c**4*g**2*i)/(B*a**4*d**4*g**2*i - 4*B*a**3*b*c*d**3*g**2*i - 6*B*a
**2*b**2*c**2*d**2*g**2*i + 4*B*a*b**3*c**3*d*g**2*i - B*b**4*c**4*g**2*i)
)/(12*b**2) - B*c**2*g**2*i*(6*a**2*d**2 - 4*a*b*c*d + b**2*c**2)*log(x +
(B*a**4*c*d**3*g**2*i - 10*B*a**3*b*c**2*d**2*g**2*i + 4*B*a**2*b**2*c**3*
d*g**2*i - B*a*b**3*c**4*g**2*i + B*a*b*c**2*g**2*i*(6*a**2*d**2 - 4*a*b*c
*d + b**2*c**2) - B*b**2*c**3*g**2*i*(6*a**2*d**2 - 4*a*b*c*d + b**2*c**2)
/d)/(B*a**4*d**4*g**2*i - 4*B*a**3*b*c*d**3*g**2*i - 6*B*a**2*b**2*c**2*d*
*2*g**2*i + 4*B*a*b**3*c**3*d*g**2*i - B*b**4*c**4*g**2*i))/(12*d**3) + x*
*3*(2*A*a*b*d*g**2*i/3 + A*b**2*c*g**2*i/3 + B*a*b*d*g**2*i/12 - B*b**2*c*
g**2*i/12) + x**2*(A*a**2*d*g**2*i/2 + A*a*b*c*g**2*i + 5*B*a**2*d*g**2*i/
24 - B*a*b*c*g**2*i/6 - B*b**2*c**2*g**2*i/(24*d)) + x*(A*a**2*c*g**2*i +
B*a**3*d*g**2*i/(12*b) + B*a**2*c*g**2*i/6 - B*a*b*c**2*g**2*i/(3*d) + B*b
**2*c**3*g**2*i/(12*d**2)) + (B*a**2*c*g**2*i*x + B*a**2*d*g**2*i*x**2/2 +
 B*a*b*c*g**2*i*x**2 + 2*B*a*b*d*g**2*i*x**3/3 + B*b**2*c*g**2*i*x**3/3 +
B*b**2*d*g**2*i*x**4/4)*log(e*(a + b*x)/(c + d*x))
```

### 3.2.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(170) = 340$.

Time = 0.22 (sec) , antiderivative size = 671, normalized size of antiderivative = 3.73

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= \frac{1}{4}Ab^2dg^2ix^4 + \frac{1}{3}Ab^2cg^2ix^3 + \frac{2}{3}Aabdg^2ix^3 + Aabcg^2ix^2 + \frac{1}{2}Aa^2dg^2ix^2$$
$$+ \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) + \frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right)Ba^2cg^2i$$
$$+ \left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)Babcg^2i$$
$$+\frac{1}{6}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a}{b^2d^2}\right.$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)Ba^2dg^2i$$
$$+\frac{1}{3}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a}{b^2d^2}\right.$$
$$+\frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) - \frac{6a^4\log(bx+a)}{b^4} + \frac{6c^4\log(dx+c)}{d^4} - \frac{2(b^3cd^2-ab^2d^3)x^3 - 3(b^3c}{}\right.$$
$$+ Aa^2cg^2ix$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="maxima")
```

output
```
1/4*A*b^2*d*g^2*i*x^4 + 1/3*A*b^2*c*g^2*i*x^3 + 2/3*A*a*b*d*g^2*i*x^3 + A*
a*b*c*g^2*i*x^2 + 1/2*A*a^2*d*g^2*i*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a^2*c*g^2*i + (x^2*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d
^2 - (b*c - a*d)*x/(b*d))*B*a*b*c*g^2*i + 1/6*(2*x^3*log(b*e*x/(d*x + c) +
 a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*
c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*c*g^2*i + 1
/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*
log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^2*d*g^2*i + 1/3*(2*x^3*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c
)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a
*b*d*g^2*i + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(
b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^2*d
*g^2*i + A*a^2*c*g^2*i*x
```

### 3.2.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2418 vs. $2(170) = 340$.

Time = 0.46 (sec) , antiderivative size = 2418, normalized size of antiderivative = 13.43

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="giac")
```

output
```
1/24*(2*(B*b^7*c^5*e^5*g^2*i - 5*B*a*b^6*c^4*d*e^5*g^2*i + 10*B*a^2*b^5*c^
3*d^2*e^5*g^2*i - 10*B*a^3*b^4*c^2*d^3*e^5*g^2*i + 5*B*a^4*b^3*c*d^4*e^5*g
^2*i - B*a^5*b^2*d^5*e^5*g^2*i - 4*(b*e*x + a*e)*B*b^6*c^5*d*e^4*g^2*i/(d*
x + c) + 20*(b*e*x + a*e)*B*a*b^5*c^4*d^2*e^4*g^2*i/(d*x + c) - 40*(b*e*x
+ a*e)*B*a^2*b^4*c^3*d^3*e^4*g^2*i/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^3*
c^2*d^4*e^4*g^2*i/(d*x + c) - 20*(b*e*x + a*e)*B*a^4*b^2*c*d^5*e^4*g^2*i/(
d*x + c) + 4*(b*e*x + a*e)*B*a^5*b*d^6*e^4*g^2*i/(d*x + c) + 6*(b*e*x + a*
e)^2*B*b^5*c^5*d^2*e^3*g^2*i/(d*x + c)^2 - 30*(b*e*x + a*e)^2*B*a*b^4*c^4*
d^3*e^3*g^2*i/(d*x + c)^2 + 60*(b*e*x + a*e)^2*B*a^2*b^3*c^3*d^4*e^3*g^2*i
/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^3*b^2*c^2*d^5*e^3*g^2*i/(d*x + c)^2
+ 30*(b*e*x + a*e)^2*B*a^4*b*c*d^6*e^3*g^2*i/(d*x + c)^2 - 6*(b*e*x + a*e)
^2*B*a^5*d^7*e^3*g^2*i/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/(b^4*d^3*
e^4 - 4*(b*e*x + a*e)*b^3*d^4*e^3/(d*x + c) + 6*(b*e*x + a*e)^2*b^2*d^5*e^
2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^6*e/(d*x + c)^3 + (b*e*x + a*e)^4*d^
7/(d*x + c)^4) + (2*A*b^8*c^5*e^5*g^2*i + B*b^8*c^5*e^5*g^2*i - 10*A*a*b^7
*c^4*d*e^5*g^2*i - 5*B*a*b^7*c^4*d*e^5*g^2*i + 20*A*a^2*b^6*c^3*d^2*e^5*g^
2*i + 10*B*a^2*b^6*c^3*d^2*e^5*g^2*i - 20*A*a^3*b^5*c^2*d^3*e^5*g^2*i - 10
*B*a^3*b^5*c^2*d^3*e^5*g^2*i + 10*A*a^4*b^4*c*d^4*e^5*g^2*i + 5*B*a^4*b^4*
c*d^4*e^5*g^2*i - 2*A*a^5*b^3*d^5*e^5*g^2*i - B*a^5*b^3*d^5*e^5*g^2*i - 8*
(b*e*x + a*e)*A*b^7*c^5*d*e^4*g^2*i/(d*x + c) - 2*(b*e*x + a*e)*B*b^7*c...
```

## 3.2. $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

### 3.2.9 Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 638, normalized size of antiderivative = 3.54

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= x^3\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d-B\,b\,c)}{12}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{36}\right)$$

$$-x^2\left(\frac{\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)(12\,a\,d+12\,b\,c)}{24\,b\,d}\right.$$

$$\left.-\frac{g^2\,i\,(9\,A\,a^2\,d^2+3\,A\,b^2\,c^2+2\,B\,a^2\,d^2-B\,b^2\,c^2+18\,A\,a\,b\,c\,d-B\,a\,b\,c\,d)}{6\,d}+\frac{A\,a\,b\,c\,g^2\,i}{2}\right)$$

$$+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(B\,a^2\,c\,g^2\,i\,x+\frac{B\,a\,g^2\,i\,x^2\,(a\,d+2\,b\,c)}{2}+\frac{B\,b\,g^2\,i\,x^3\,(2\,a\,d+b\,c)}{3}\right.$$

$$\left.+\frac{B\,b^2\,d\,g^2\,i\,x^4}{4}\right)$$

$$+x\left(\frac{(12\,a\,d+12\,b\,c)\left(\frac{\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)(12\,a\,d+12\,b\,c)}{12\,b\,d}-\frac{g^2\,i\,(9\,A\,a^2\,d^2+3\,A\,b^2\,c^2+2\,B\,a^2 \cdots}{3 \cdots}\right.}{12\,b\,d}\right.$$

$$-\frac{a\,c\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)}{b\,d}$$

$$\left.+\frac{a\,g^2\,i\,(2\,A\,a^2\,d^2+6\,A\,b^2\,c^2+B\,a^2\,d^2-2\,B\,b^2\,c^2+12\,A\,a\,b\,c\,d+B\,a\,b\,c\,d)}{2\,b\,d}\right)$$

$$-\frac{\ln(c+d\,x)\,(6\,B\,i\,a^2\,c^2\,d^2\,g^2-4\,B\,i\,a\,b\,c^3\,d\,g^2+B\,i\,b^2\,c^4\,g^2)}{12\,d^3}$$

$$-\frac{\ln(a+b\,x)\,(B\,a^4\,d\,g^2\,i-4\,B\,a^3\,b\,c\,g^2\,i)}{12\,b^2}+\frac{A\,b^2\,d\,g^2\,i\,x^4}{4}$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x^3*((b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d - B*b*c))/12 - (A*b*g^2*i*(12*a*
d + 12*b*c))/36) - x^2*((((b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d - B*b*c))/4
 - (A*b*g^2*i*(12*a*d + 12*b*c))/12)*(12*a*d + 12*b*c))/(24*b*d) - (g^2*i*
(9*A*a^2*d^2 + 3*A*b^2*c^2 + 2*B*a^2*d^2 - B*b^2*c^2 + 18*A*a*b*c*d - B*a*
b*c*d))/(6*d) + (A*a*b*c*g^2*i)/2) + log((e*(a + b*x))/(c + d*x))*(B*a^2*c
*g^2*i*x + (B*a*g^2*i*x^2*(a*d + 2*b*c))/2 + (B*b*g^2*i*x^3*(2*a*d + b*c))
/3 + (B*b^2*d*g^2*i*x^4)/4) + x*(((12*a*d + 12*b*c)*((((b*g^2*i*(12*A*a*d
+ 8*A*b*c + B*a*d - B*b*c))/4 - (A*b*g^2*i*(12*a*d + 12*b*c))/12)*(12*a*d
+ 12*b*c))/(12*b*d) - (g^2*i*(9*A*a^2*d^2 + 3*A*b^2*c^2 + 2*B*a^2*d^2 - B*
b^2*c^2 + 18*A*a*b*c*d - B*a*b*c*d))/(3*d) + A*a*b*c*g^2*i))/(12*b*d) - (a
*c*((b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d - B*b*c))/4 - (A*b*g^2*i*(12*a*d
+ 12*b*c))/12))/(b*d) + (a*g^2*i*(2*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2 -
2*B*b^2*c^2 + 12*A*a*b*c*d + B*a*b*c*d))/(2*b*d)) - (log(c + d*x)*(B*b^2*c
^4*g^2*i + 6*B*a^2*c^2*d^2*g^2*i - 4*B*a*b*c^3*d*g^2*i))/(12*d^3) - (log(a
 + b*x)*(B*a^4*d*g^2*i - 4*B*a^3*b*c*g^2*i))/(12*b^2) + (A*b^2*d*g^2*i*x^4
)/4
```

## 3.2. $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

# 3.3 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$



## 3.3.1 Optimal result

Integrand size = 36, antiderivative size = 140

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=-\frac{B(bc-ad)^2gix}{6bd}+\frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b}$$
$$+\frac{(bc-ad)gi(a+bx)^2\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2}+\frac{B(bc-ad)^3gi\log(c+dx)}{6b^2d^2}$$

output
```
-1/6*B*(-a*d+b*c)^2*g*i*x/b/d+1/3*g*i*(b*x+a)^2*(d*x+c)*(A+B*ln(e*(b*x+a)/
(d*x+c)))/b+1/6*(-a*d+b*c)*g*i*(b*x+a)^2*(A-B+B*ln(e*(b*x+a)/(d*x+c)))/b^2
+1/6*B*(-a*d+b*c)^3*g*i*ln(d*x+c)/b^2/d^2
```

## 3.3.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 181, normalized size of antiderivative = 1.29

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{gi\Big(-a^2Bd^2(3bc+ad)\log(a+bx)+b\Big(dx(a^2Bd^2-b^2Bc(c+dx)+Ab^2dx(3c+2dx)+abd(6Ac+3Adx}{}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
,x]
```

output
```
(g*i*(-(a^2*B*d^2*(3*b*c + a*d)*Log[a + b*x]) + b*(d*x*(a^2*B*d^2 - b^2*B*
c*(c + d*x) + A*b^2*d*x*(3*c + 2*d*x) + a*b*d*(6*A*c + 3*A*d*x + B*d*x)) +
 B*d^2*(6*a^2*c + 3*a*b*x*(2*c + d*x) + b^2*x^2*(3*c + 2*d*x))*Log[(e*(a +
 b*x))/(c + d*x)] + B*c*(b^2*c^2 - 3*a*b*c*d + 6*a^2*d^2)*Log[c + d*x])))/
(6*b^2*d^2)
```

### 3.3.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 137, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.139$, Rules used = {2960, 27, 2948, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{ 2960}$$

$$\frac{i(bc-ad)\int g(a+bx)\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{gi(bc-ad)\int (a+bx)\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{gi(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{2b}-\frac{B(bc-ad)\int \frac{a+bx}{c+dx}dx}{2b}\right)}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{gi(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{2b}-\frac{B(bc-ad)\int\left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{2b}\right)}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}$$

$$\Big\downarrow \; 2009$$

$$\frac{gi(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{2b}-\frac{B(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{2b}\right)}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b}$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(g*i*(a + b*x)^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b) + (
(b*c - a*d)*g*i*(((a + b*x)^2*(A - B + B*Log[(e*(a + b*x))/(c + d*x)]))/(2
*b) - (B*(b*c - a*d)*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/(2*b)))/(
3*b)
```

### 3.3.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

rule 2960
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Sim
p[(f + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*
(m + 2))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B
*n + B*Log[e*((a + b*x)^n/(c + d*x)^n)]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, i, A, B, m, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IGtQ[m, -2]
```

### 3.3.4 Maple [A] (verified)

Time = 0.63 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.47

| method | result |
|---|---|
| risch | $\frac{giBx(2bd\,x^2+3xad+3bcx+6ca)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{6}+\frac{igbdA\,x^3}{3}+\frac{igdAa\,x^2}{2}+\frac{igbAc\,x^2}{2}+\frac{igdBa\,x^2}{6}-\frac{igbBc\,x^2}{6}+i$ |
| parallelrisch | $\frac{-B\ln(bx+a)a^3d^3gi+B\ln(bx+a)b^3c^3gi+2A\,x^3b^3d^3gi-B\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3c^3gi+3B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3c\,d^2gi+6Axa\,b^2c\,d^2}{}$ |
| parts | $Agi\left(xca+\frac{(ad+cb)x^2}{2}+\frac{bd\,x^3}{3}\right)-\frac{Bgi(ad-cb)^2e^2\left(d^2(ad-cb)\left(-\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{2e^2b^2d}-\frac{1}{2ebd\left(\left(\frac{be}{d}+\frac{(ad-}{d(dx}\right.\right.}\right.\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(A\,d^2egi(a^2d^2-2abcd+b^2c^2)\left(-\frac{1}{2d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}+\frac{be}{3d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}\right)+B\,d^2egi(a^2\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(A\,d^2egi(a^2d^2-2abcd+b^2c^2)\left(-\frac{1}{2d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}+\frac{be}{3d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}\right)+B\,d^2egi(a^2\right.}{}$ |

input
```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVE
RBOSE)
```

output
```
1/6*g*i*B*x*(2*b*d*x^2+3*a*d*x+3*b*c*x+6*a*c)*ln(e*(b*x+a)/(d*x+c))+1/3*i*
g*b*d*A*x^3+1/2*i*g*d*A*a*x^2+1/2*i*g*b*A*c*x^2+1/6*i*g*d*B*a*x^2-1/6*i*g*
b*B*c*x^2+i*g*A*a*c*x-1/6*i*g/b^2*d*B*ln(b*x+a)*a^3+1/2*i*g/b*B*ln(b*x+a)*
a^2*c-1/2*i*g/d*B*ln(-d*x-c)*a*c^2+1/6*i*g*b/d^2*B*ln(-d*x-c)*c^3+1/6*i*g/
b*d*B*a^2*x-1/6*i*g*b/d*B*c^2*x
```

### 3.3.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 226, normalized size of antiderivative = 1.61

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{2\,Ab^3d^3gix^3 + ((3\,A-B)b^3cd^2 + (3\,A+B)ab^2d^3)gix^2 - (Bb^3c^2d - 6\,Aab^2cd^2 - Ba^2bd^3)gix + (3\,Ba^2bcd}{}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorith
m="fricas")
```

output
```
1/6*(2*A*b^3*d^3*g*i*x^3 + ((3*A - B)*b^3*c*d^2 + (3*A + B)*a*b^2*d^3)*g*i
*x^2 - (B*b^3*c^2*d - 6*A*a*b^2*c*d^2 - B*a^2*b*d^3)*g*i*x + (3*B*a^2*b*c*
d^2 - B*a^3*d^3)*g*i*log(b*x + a) + (B*b^3*c^3 - 3*B*a*b^2*c^2*d)*g*i*log(
d*x + c) + (2*B*b^3*d^3*g*i*x^3 + 6*B*a*b^2*c*d^2*g*i*x + 3*(B*b^3*c*d^2 +
 B*a*b^2*d^3)*g*i*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d^2)
```

### 3.3.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(126) = 252$.

Time = 1.44 (sec) , antiderivative size = 498, normalized size of antiderivative = 3.56

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{Abdgix^3}{3} - \frac{Ba^2gi(ad-3bc)\log\left(x+\frac{Ba^3cd^2gi+\frac{Ba^3d^2gi(ad-3bc)}{b}-6Ba^2bc^2dgi-Ba^2cdgi(ad-3bc)+Bab^2c^3gi}{Ba^3d^3gi-3Ba^2bcd^2gi-3Bab^2c^2dgi+Bb^3c^3gi}\right)}{6b^2} - \frac{Bc^2gi(3ad-bc)\log\left(x+\frac{Ba^3cd^2gi-6Ba^2bc^2dgi+Bab^2c^3gi+Babc^2gi(3ad-bc)-\frac{Bb^2c^3gi(3ad-bc)}{d}}{Ba^3d^3gi-3Ba^2bcd^2gi-3Bab^2c^2dgi+Bb^3c^3gi}\right)}{6d^2} + x^2\left(\frac{Aadgi}{2}+\frac{Abcgi}{2}+\frac{Badgi}{6}-\frac{Bbcgi}{6}\right) + x\left(Aacgi+\frac{Ba^2dgi}{6b}-\frac{Bbc^2gi}{6d}\right) + \left(Bacgix+\frac{Badgix^2}{2}+\frac{Bbcgix^2}{2}+\frac{Bbdgix^3}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b*d*g*i*x**3/3 - B*a**2*g*i*(a*d - 3*b*c)*log(x + (B*a**3*c*d**2*g*i + B
*a**3*d**2*g*i*(a*d - 3*b*c)/b - 6*B*a**2*b*c**2*d*g*i - B*a**2*c*d*g*i*(a
*d - 3*b*c) + B*a*b**2*c**3*g*i)/(B*a**3*d**3*g*i - 3*B*a**2*b*c*d**2*g*i
- 3*B*a*b**2*c**2*d*g*i + B*b**3*c**3*g*i))/(6*b**2) - B*c**2*g*i*(3*a*d -
 b*c)*log(x + (B*a**3*c*d**2*g*i - 6*B*a**2*b*c**2*d*g*i + B*a*b**2*c**3*g
*i + B*a*b*c**2*g*i*(3*a*d - b*c) - B*b**2*c**3*g*i*(3*a*d - b*c)/d)/(B*a*
*3*d**3*g*i - 3*B*a**2*b*c*d**2*g*i - 3*B*a*b**2*c**2*d*g*i + B*b**3*c**3*
g*i))/(6*d**2) + x**2*(A*a*d*g*i/2 + A*b*c*g*i/2 + B*a*d*g*i/6 - B*b*c*g*i
/6) + x*(A*a*c*g*i + B*a**2*d*g*i/(6*b) - B*b*c**2*g*i/(6*d)) + (B*a*c*g*i
*x + B*a*d*g*i*x**2/2 + B*b*c*g*i*x**2/2 + B*b*d*g*i*x**3/3)*log(e*(a + b*
x)/(c + d*x))
```

### 3.3.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 361 vs. $2(132) = 264$.

Time = 0.21 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.58

$$\int (ag + bgx)(ci + dix)\left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right) dx = \frac{1}{3}\,Abdgix^3 + \frac{1}{2}\,Abcgix^2 + \frac{1}{2}\,Aadgix^2 + \left(x\log\left(\frac{bex}{dx + c} + \frac{ae}{dx + c}\right) + \frac{a\log\left(bx + a\right)}{b} - \frac{c\log\left(dx + c\right)}{d}\right)Bacgi + \frac{1}{2}\left(x^2\log\left(\frac{bex}{dx + c} + \frac{ae}{dx + c}\right) - \frac{a^2\log\left(bx + a\right)}{b^2} + \frac{c^2\log\left(dx + c\right)}{d^2} - \frac{(bc - ad)x}{bd}\right)Bbcgi + \frac{1}{2}\left(x^2\log\left(\frac{bex}{dx + c} + \frac{ae}{dx + c}\right) - \frac{a^2\log\left(bx + a\right)}{b^2} + \frac{c^2\log\left(dx + c\right)}{d^2} - \frac{(bc - ad)x}{bd}\right)Badgi + \frac{1}{6}\left(2\,x^3\log\left(\frac{bex}{dx + c} + \frac{ae}{dx + c}\right) + \frac{2\,a^3\log\left(bx + a\right)}{b^3} - \frac{2\,c^3\log\left(dx + c\right)}{d^3} - \frac{(b^2cd - abd^2)x^2 - 2\,(b^2c^2 - a}{b^2d^2} + Aacgix$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorith
m="maxima")
```

output
```
1/3*A*b*d*g*i*x^3 + 1/2*A*b*c*g*i*x^2 + 1/2*A*a*d*g*i*x^2 + (x*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a*c*g*i
 + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 +
c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*b*c*g*i + 1/2*(x^2*log(b*e*x
/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
- (b*c - a*d)*x/(b*d))*B*a*d*g*i + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a
*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b*d*g*i + A*a*c*g*i*x
```

### 3.3.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1254 vs. $2(132) = 264$.

Time = 0.50 (sec) , antiderivative size = 1254, normalized size of antiderivative = 8.96

$$\int (ag + bgx)(ci + dix)\left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorith
m="giac")
```

output
```
-1/6*((B*b^5*c^4*e^4*g*i - 4*B*a*b^4*c^3*d*e^4*g*i + 6*B*a^2*b^3*c^2*d^2*e
^4*g*i - 4*B*a^3*b^2*c*d^3*e^4*g*i + B*a^4*b*d^4*e^4*g*i - 3*(b*e*x + a*e)
*B*b^4*c^4*d*e^3*g*i/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^3*c^3*d^2*e^3*g*i/
(d*x + c) - 18*(b*e*x + a*e)*B*a^2*b^2*c^2*d^3*e^3*g*i/(d*x + c) + 12*(b*e
*x + a*e)*B*a^3*b*c*d^4*e^3*g*i/(d*x + c) - 3*(b*e*x + a*e)*B*a^4*d^5*e^3*
g*i/(d*x + c))*log((b*e*x + a*e)/(d*x + c))/(b^3*d^2*e^3 - 3*(b*e*x + a*e)
*b^2*d^3*e^2/(d*x + c) + 3*(b*e*x + a*e)^2*b*d^4*e/(d*x + c)^2 - (b*e*x +
a*e)^3*d^5/(d*x + c)^3) + (A*b^6*c^4*e^4*g*i - 4*A*a*b^5*c^3*d*e^4*g*i + 6
*A*a^2*b^4*c^2*d^2*e^4*g*i - 4*A*a^3*b^3*c*d^3*e^4*g*i + A*a^4*b^2*d^4*e^4
*g*i - 3*(b*e*x + a*e)*A*b^5*c^4*d*e^3*g*i/(d*x + c) + (b*e*x + a*e)*B*b^5
*c^4*d*e^3*g*i/(d*x + c) + 12*(b*e*x + a*e)*A*a*b^4*c^3*d^2*e^3*g*i/(d*x +
 c) - 4*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e^3*g*i/(d*x + c) - 18*(b*e*x + a*e)
*A*a^2*b^3*c^2*d^3*e^3*g*i/(d*x + c) + 6*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e
^3*g*i/(d*x + c) + 12*(b*e*x + a*e)*A*a^3*b^2*c*d^4*e^3*g*i/(d*x + c) - 4*
(b*e*x + a*e)*B*a^3*b^2*c*d^4*e^3*g*i/(d*x + c) - 3*(b*e*x + a*e)*A*a^4*b*
d^5*e^3*g*i/(d*x + c) + (b*e*x + a*e)*B*a^4*b*d^5*e^3*g*i/(d*x + c) - (b*e
*x + a*e)^2*B*b^4*c^4*d^2*e^2*g*i/(d*x + c)^2 + 4*(b*e*x + a*e)^2*B*a*b^3*
c^3*d^3*e^2*g*i/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^2*b^2*c^2*d^4*e^2*g*i/
(d*x + c)^2 + 4*(b*e*x + a*e)^2*B*a^3*b*c*d^5*e^2*g*i/(d*x + c)^2 - (b*e*x
 + a*e)^2*B*a^4*d^6*e^2*g*i/(d*x + c)^2)/(b^4*d^2*e^3 - 3*(b*e*x + a*e)...
```

### 3.3.9 Mupad [B] (verification not implemented)

Time = 1.31 (sec) , antiderivative size = 282, normalized size of antiderivative = 2.01

$$
\begin{aligned}
&\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx \\
&= x^2\left(\frac{g\,i\,(6\,A\,a\,d+6\,A\,b\,c+B\,a\,d-B\,b\,c)}{6}-\frac{A\,g\,i\,(6\,a\,d+6\,b\,c)}{12}\right) \\
&\quad - x\left(\frac{\left(\frac{g\,i\,(6\,A\,a\,d+6\,A\,b\,c+B\,a\,d-B\,b\,c)}{3}-\frac{A\,g\,i\,(6\,a\,d+6\,b\,c)}{6}\right)(6\,a\,d+6\,b\,c)}{6\,b\,d}+A\,a\,c\,g\,i\right. \\
&\qquad \left.-\frac{g\,i\,(2\,A\,a^2\,d^2+2\,A\,b^2\,c^2+B\,a^2\,d^2-B\,b^2\,c^2+8\,A\,a\,b\,c\,d)}{2\,b\,d}\right) \\
&\quad + \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,b\,d\,g\,i\,x^3}{3}+\frac{B\,g\,i\,(a\,d+b\,c)\,x^2}{2}+B\,a\,c\,g\,i\,x\right) \\
&\quad - \frac{\ln(a+b\,x)\,\left(B\,a^3\,d\,g\,i-3\,B\,a^2\,b\,c\,g\,i\right)}{6\,b^2} \\
&\quad + \frac{\ln(c+d\,x)\,\left(B\,b\,c^3\,g\,i-3\,B\,a\,c^2\,d\,g\,i\right)}{6\,d^2}+\frac{A\,b\,d\,g\,i\,x^3}{3}
\end{aligned}
$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x^2*((g*i*(6*A*a*d + 6*A*b*c + B*a*d - B*b*c))/6 - (A*g*i*(6*a*d + 6*b*c))
/12) - x*((((g*i*(6*A*a*d + 6*A*b*c + B*a*d - B*b*c))/3 - (A*g*i*(6*a*d +
6*b*c))/6)*(6*a*d + 6*b*c))/(6*b*d) + A*a*c*g*i - (g*i*(2*A*a^2*d^2 + 2*A*
b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 8*A*a*b*c*d))/(2*b*d)) + log((e*(a + b*x
))/(c + d*x))*((B*g*i*x^2*(a*d + b*c))/2 + (B*b*d*g*i*x^3)/3 + B*a*c*g*i*x
) - (log(a + b*x)*(B*a^3*d*g*i - 3*B*a^2*b*c*g*i))/(6*b^2) + (log(c + d*x)
*(B*b*c^3*g*i - 3*B*a*c^2*d*g*i))/(6*d^2) + (A*b*d*g*i*x^3)/3
```

## 3.3. $\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

# 3.4 $\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$



## 3.4.1 Optimal result

Integrand size = 28, antiderivative size = 81

$$\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = -\frac{B(bc-ad)ix}{2b} - \frac{B(bc-ad)^2 i\log(a+bx)}{2b^2d} + \frac{i(c+dx)^2\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d}$$

output
```
-1/2*B*(-a*d+b*c)*i*x/b-1/2*B*(-a*d+b*c)^2*i*ln(b*x+a)/b^2/d+1/2*i*(d*x+c)
^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d
```

## 3.4.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 70, normalized size of antiderivative = 0.86

$$\int (ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{i\left(-\frac{B(bc-ad)(bdx+(bc-ad)\log(a+bx))}{b^2} + (c+dx)^2\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\right)}{2d}$$

input
```
Integrate[(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(i*(-((B*(b*c - a*d)*(b*d*x + (b*c - a*d)*Log[a + b*x]))/b^2) + (c + d*x)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(2*d)
```

### 3.4.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.95, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d}-\frac{B(bc-ad)\int \frac{i^2(c+dx)}{a+bx}dx}{2di}$$

$$\Big\downarrow \text{27}$$

$$\frac{i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d}-\frac{Bi(bc-ad)\int \frac{c+dx}{a+bx}dx}{2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d}-\frac{Bi(bc-ad)\int \left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d}-\frac{Bi(bc-ad)\left(\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}\right)}{2d}$$

input
```
Int[(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
-1/2*(B*(b*c - a*d)*i*((d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2))/d + (i*(
c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d)
```

### 3.4.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

## 3.4.4 Maple [A] (verified)

Time = 0.56 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| risch | $\frac{iBx(dx+2c)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2}+\frac{idA\,x^2}{2}+iAcx-\frac{B\,c^2 i\ln(-dx-c)}{2d}-\frac{idB\ln(bx+a)a^2}{2b^2}+\frac{iB\ln(bx+a)ac}{b}+\frac{idBa}{2b}\cdots$ |
| parallelrisch | $\frac{B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^2i+A\,x^2b^2d^2i+2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2cdi+2Ax\,b^2cdi-B\ln(bx+a)a^2d^2i+2B\ln(bx+a)abcdi-B\ln(b\cdots}{2b^2d}$ |
| parts | $Ai\left(\frac{1}{2}d\,x^2+xc\right)-Bi(ad-cb)^2e^2\Bigg(-\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{2e^2b^2d}-\frac{1}{2ebd\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}+\cdots$ |
| derivativedivides | $-\frac{e(ad-cb)\Bigg(-\frac{Adei(ad-cb)}{2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}-B\,d^2ei(ad-cb)\Bigg(\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{2b^2e^2d}-\frac{1}{2bed\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\cdots}{d^2}$ |
| default | $-\frac{e(ad-cb)\Bigg(-\frac{Adei(ad-cb)}{2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}-B\,d^2ei(ad-cb)\Bigg(\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{2b^2e^2d}-\frac{1}{2bed\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\cdots}{d^2}$ |

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/2*i*B*x*(d*x+2*c)*ln(e*(b*x+a)/(d*x+c))+1/2*i*d*A*x^2+i*A*c*x-1/2*B*c^2*
i/d*ln(-d*x-c)-1/2*i/b^2*d*B*ln(b*x+a)*a^2+i/b*B*ln(b*x+a)*a*c+1/2*i/b*d*B
*a*x-1/2*i*B*c*x
```

### 3.4.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 127, normalized size of antiderivative = 1.57

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{Ab^2d^2ix^2-Bb^2c^2i\log(dx+c)+((2A-B)b^2cd+Babd^2)ix+(2Babcd-Ba^2d^2)i\log(bx+a)+(Bb^2d}{2b^2d}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas")
```

output
```
1/2*(A*b^2*d^2*i*x^2 - B*b^2*c^2*i*log(d*x + c) + ((2*A - B)*b^2*c*d + B*a
*b*d^2)*i*x + (2*B*a*b*c*d - B*a^2*d^2)*i*log(b*x + a) + (B*b^2*d^2*i*x^2
+ 2*B*b^2*c*d*i*x)*log((b*e*x + a*e)/(d*x + c)))/(b^2*d)
```

### 3.4.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 253 vs. 2(68) = 136.

Time = 0.94 (sec) , antiderivative size = 253, normalized size of antiderivative = 3.12

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{Adix^2}{2}-\frac{Bai(ad-2bc)\log\left(x+\frac{Ba^2cdi+\frac{Ba^2di(ad-2bc)}{b}-3Babc^2i-Baci(ad-2bc)}{Ba^2d^2i-2Babcdi-Bb^2c^2i}\right)}{2b^2}$$
$$-\frac{Bc^2i\log\left(x+\frac{Ba^2cdi-2Babc^2i-\frac{Bb^2c^3i}{d}}{Ba^2d^2i-2Babcdi-Bb^2c^2i}\right)}{2d}+x\left(Aci+\frac{Badi}{2b}-\frac{Bci}{2}\right)$$
$$+\left(Bcix+\frac{Bdix^2}{2}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*d*i*x**2/2 - B*a*i*(a*d - 2*b*c)*log(x + (B*a**2*c*d*i + B*a**2*d*i*(a*d
 - 2*b*c)/b - 3*B*a*b*c**2*i - B*a*c*i*(a*d - 2*b*c))/(B*a**2*d**2*i - 2*B
*a*b*c*d*i - B*b**2*c**2*i))/(2*b**2) - B*c**2*i*log(x + (B*a**2*c*d*i - 2
*B*a*b*c**2*i - B*b**2*c**3*i/d)/(B*a**2*d**2*i - 2*B*a*b*c*d*i - B*b**2*c
**2*i))/(2*d) + x*(A*c*i + B*a*d*i/(2*b) - B*c*i/2) + (B*c*i*x + B*d*i*x**
2/2)*log(e*(a + b*x)/(c + d*x))
```

### 3.4.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 144, normalized size of antiderivative = 1.78

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{1}{2}\,Adix^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bci$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bdi$$
$$+Acix$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima")
```

output
```
1/2*A*d*i*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b
 - c*log(d*x + c)/d)*B*c*i + 1/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))
 - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*d*
i + A*c*i*x
```

### 3.4.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(75) = 150$.

Time = 0.41 (sec) , antiderivative size = 627, normalized size of antiderivative = 7.74

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{1}{2}\left(\frac{(Bb^3c^3e^3i-3\,Bab^2c^2de^3i+3\,Ba^2bcd^2e^3i-Ba^3d^3e^3i)\log\left(\frac{bex+ae}{dx+c}\right)}{b^2de^2-\frac{2\,(bex+ae)bd^2e}{dx+c}+\frac{(bex+ae)^2d^3}{(dx+c)^2}}+\frac{Ab^4c^3e^3i-Bb^4c^3e^3i-3\,Aab^3c^2}{}\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/2*((B*b^3*c^3*e^3*i - 3*B*a*b^2*c^2*d*e^3*i + 3*B*a^2*b*c*d^2*e^3*i - B*
a^3*d^3*e^3*i)*log((b*e*x + a*e)/(d*x + c))/(b^2*d*e^2 - 2*(b*e*x + a*e)*b
*d^2*e/(d*x + c) + (b*e*x + a*e)^2*d^3/(d*x + c)^2) + (A*b^4*c^3*e^3*i - B
*b^4*c^3*e^3*i - 3*A*a*b^3*c^2*d*e^3*i + 3*B*a*b^3*c^2*d*e^3*i + 3*A*a^2*b
^2*c*d^2*e^3*i - 3*B*a^2*b^2*c*d^2*e^3*i - A*a^3*b*d^3*e^3*i + B*a^3*b*d^3
*e^3*i + (b*e*x + a*e)*B*b^3*c^3*d*e^2*i/(d*x + c) - 3*(b*e*x + a*e)*B*a*b
^2*c^2*d^2*e^2*i/(d*x + c) + 3*(b*e*x + a*e)*B*a^2*b*c*d^3*e^2*i/(d*x + c)
 - (b*e*x + a*e)*B*a^3*d^4*e^2*i/(d*x + c))/(b^3*d*e^2 - 2*(b*e*x + a*e)*b
^2*d^2*e/(d*x + c) + (b*e*x + a*e)^2*b*d^3/(d*x + c)^2) + (B*b^3*c^3*e*i -
 3*B*a*b^2*c^2*d*e*i + 3*B*a^2*b*c*d^2*e*i - B*a^3*d^3*e*i)*log(-b*e + (b*
e*x + a*e)*d/(d*x + c))/(b^2*d) - (B*b^3*c^3*e*i - 3*B*a*b^2*c^2*d*e*i + 3
*B*a^2*b*c*d^2*e*i - B*a^3*d^3*e*i)*log((b*e*x + a*e)/(d*x + c))/(b^2*d))*
(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.4.9 Mupad [B] (verification not implemented)

Time = 1.07 (sec) , antiderivative size = 126, normalized size of antiderivative = 1.56

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = x\left(\frac{i\,(2\,A\,a\,d+4\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,b} - \frac{A\,i\,(2\,a\,d+2\,b\,c)}{2\,b}\right) + \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,d\,i\,x^2}{2}+B\,c\,i\,x\right) - \frac{\ln(a+b\,x)\,\left(B\,a^2\,d\,i-2\,B\,a\,b\,c\,i\right)}{2\,b^2} + \frac{A\,d\,i\,x^2}{2} - \frac{B\,c^2\,i\,\ln(c+d\,x)}{2\,d}$$

input
```
int((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x*((i*(2*A*a*d + 4*A*b*c + B*a*d - B*b*c))/(2*b) - (A*i*(2*a*d + 2*b*c))/(
2*b)) + log((e*(a + b*x))/(c + d*x))*((B*d*i*x^2)/2 + B*c*i*x) - (log(a +
b*x)*(B*a^2*d*i - 2*B*a*b*c*i))/(2*b^2) + (A*d*i*x^2)/2 - (B*c^2*i*log(c +
 d*x))/(2*d)
```

# 3.5 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$



## 3.5.1 Optimal result

Integrand size = 38, antiderivative size = 133

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg} - \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g} + \frac{B(bc-ad)i\,\mathrm{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{b^2g}$$

output
```
i*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g-(-a*d+b*c)*i*ln((a*d-b*c)/d/(b*x
+a))*(A-B+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g+B*(-a*d+b*c)*i*polylog(2,1+(-a*d+
b*c)/d/(b*x+a))/b^2/g
```

## 3.5.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 164, normalized size of antiderivative = 1.23

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \frac{i\left((-bBc+aBd)\log^2(a+bx)+2\left(Abdx+Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+(-bBc+aBd)\log(c+dx)\right)+2(b\cdots}{2b^2g}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*
x),x]
```

output
```
(i*((-(b*B*c) + a*B*d)*Log[a + b*x]^2 + 2*(A*b*d*x + B*d*(a + b*x)*Log[(e*
(a + b*x))/(c + d*x)] + (-(b*B*c) + a*B*d)*Log[c + d*x]) + 2*(b*c - a*d)*L
og[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)] + B*Log[(b*(c + d*x))/(b*c
 - a*d)]) + 2*B*(b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/(2*
b^2*g)
```

### 3.5.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 126, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.211$, Rules used = {2960, 27, 2942, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{ag+bgx}dx$$

$$\Big\downarrow \text{2960}$$

$$\frac{i(bc-ad)\int \frac{A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{g(a+bx)}dx}{b}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{27}$$

$$\frac{i(bc-ad)\int \frac{A-B+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a+bx}dx}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2942}$$

$$\frac{i(bc-ad)\left(\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \text{2858}$$

$$\frac{i(bc-ad)\left(\frac{B(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \; 27$$

$$\frac{i(bc-ad)\left(\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \; 2778$$

$$\frac{i(bc-ad)\left(-\frac{B(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \; 2005$$

$$\frac{i(bc-ad)\left(-\frac{B(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

$$\Big\downarrow \; 2752$$

$$\frac{i(bc-ad)\left(\frac{B\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A-B\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{bg}$$

---

3.5. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x),x]
```

output
```
(i*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*g) + ((b*c - a*d)*i*
(-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A - B + B*Log[(e*(a + b*x))/(c + d*
x)]))/b) + (B*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/b))/(b*g)
```

### 3.5.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2942
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a
+ b*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
 - a*d)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x],
 x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*
c - a*d, 0] && EqQ[b*f - a*g, 0]
```

rule 2960
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Sim
p[(f + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*
(m + 2))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B
*n + B*Log[e*((a + b*x)^n/(c + d*x)^n)]), x], x] /; FreeQ[{a, b, c, d, e, f
, g, h, i, A, B, m, n}, x] && EqQ[n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d
, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IGtQ[m, -2]
```

### 3.5.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(132) = 264$.

Time = 1.49 (sec) , antiderivative size = 417, normalized size of antiderivative = 3.14

| method | result |
|---|---|
| parts | $\frac{iA\left(\frac{xd}{b}+\frac{(-ad+cb)\ln(bx+a)}{b^2}\right)}{g} - \frac{iB(ad-cb)^2e^2\left(\frac{d^3\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)b^2e^2} - \frac{d^4\left(\frac{\mathrm{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}+\frac{\ln\left(\frac{be}{d}+\frac{(a}{d}\right.}{}\right.}{(ad-cb)b^2}\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{i\,d^2eA\left(\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^2e^2}+\frac{1}{be\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^2e^2}\right)}{g}+\frac{i\,d^2eB\left(-\frac{d\left(\frac{\mathrm{dilog}\left(-\right.}{}\right.}{}\right.}{}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(\frac{i\,d^2eA\left(\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^2e^2}+\frac{1}{be\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^2e^2}\right)}{g}+\frac{i\,d^2eB\left(-\frac{d\left(\frac{\mathrm{dilog}\left(-\right.}{}\right.}{}\right.}{}\right.}{}$ |
| risch | Expression too large to display |

3.5. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x,method=_RETURNVE
RBOSE)
```

output
```
i*A/g*(x*d/b+(-a*d+b*c)/b^2*ln(b*x+a))-i*B/g/d^3*(a*d-b*c)^2*e^2*(1/2/(a*d
-b*c)*d^3/b^2/e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/(a*d-b*c)*d^4/b^2/e^
2*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)+1/(a*d-b*c)
*d^4/b/e*(1/b/e/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d
*x+c))*d-b*e)))
```

### 3.5.5 Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorith
m="fricas")
```

output
```
integral((A*d*i*x + A*c*i + (B*d*i*x + B*c*i)*log((b*e*x + a*e)/(d*x + c))
)/(b*g*x + a*g), x)
```

### 3.5.6 Sympy [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$=\frac{i\left(\int \frac{Ac}{a+bx}\,dx+\int \frac{Adx}{a+bx}\,dx+\int \frac{Bc\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx+\int \frac{Bdx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx\right)}{g}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

output
```
i*(Integral(A*c/(a + b*x), x) + Integral(A*d*x/(a + b*x), x) + Integral(B*
c*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(B*d*x*log(
a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g
```

### 3.5.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.81

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = Adi\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)+\frac{Aci\log(bgx+ag)}{bg} - \frac{Bci\log(dx+c)}{bg}+\frac{(bci-adi)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^2g} + \frac{2\,Bbdix\log(e)+(bci-adi)B\log(bx+a)^2+2\,(Bbdix+(bci\log(e)-(i\log(e)-i)ad)B)\log(bx+a}{2\,b^2g}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorith
m="maxima")
```

output
```
A*d*i*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + A*c*i*log(b*g*x + a*g)/(b*g) -
B*c*i*log(d*x + c)/(b*g) + (b*c*i - a*d*i)*(log(b*x + a)*log((b*d*x + a*d)
/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^2*g) + 1/2*(2*
B*b*d*i*x*log(e) + (b*c*i - a*d*i)*B*log(b*x + a)^2 + 2*(B*b*d*i*x + (b*c*
i*log(e) - (i*log(e) - i)*a*d)*B)*log(b*x + a) - 2*(B*b*d*i*x + (b*c*i - a
*d*i)*B*log(b*x + a))*log(d*x + c))/(b^2*g)
```

### 3.5.8 Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algorith
m="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g),
x)
```

### 3.5.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx=\int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x),x)
```

output
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x), x)
```

# 3.6 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$



## 3.6.1 Optimal result

Integrand size = 38, antiderivative size = 142

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = -\frac{Bi(c+dx)}{bg^2(a+bx)} - \frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{Bdi\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$

output

```
-B*i*(d*x+c)/b/g^2/(b*x+a)-i*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g^2/(b*
x+a)-d*i*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/g^2+B*d
*i*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g^2
```

## 3.6.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.23

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \frac{i\left(-2(A+B)(bc-ad) - Bd(a+bx)\log^2(a+bx) + 2(-bBc+aBd)\log\left(\frac{e(a+bx)}{c+dx}\right) + 2Bd(a+bx)\log(c+\right.}{2b}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*
x)^2,x]
```

output
```
(i*(-2*(A + B)*(b*c - a*d) - B*d*(a + b*x)*Log[a + b*x]^2 + 2*(-(b*B*c) +
a*B*d)*Log[(e*(a + b*x))/(c + d*x)] + 2*B*d*(a + b*x)*Log[c + d*x] + 2*d*(
a + b*x)*Log[a + b*x]*(A - B + B*Log[(e*(a + b*x))/(c + d*x)] + B*Log[(b*(
c + d*x))/(b*c - a*d)]) + 2*B*d*(a + b*x)*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)]))/(2*b^2*g^2*(a + b*x))
```

### 3.6.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2962, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{2780}$$

$$\frac{i\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^2}$$

$$\Big\downarrow \text{2741}$$

$$\frac{i\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{b}\right)}{g^2}$$

$$\Big\downarrow \text{2779}$$

$$\frac{i\left(\frac{d\left(\frac{B\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{b}\right)}{g^2}$$

$$\Big\downarrow \text{2838}$$

$$\frac{i\left(\frac{d\left(\frac{B\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{b}\right)}{g^2}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^2,x
]
```

output
```
(i*((-((B*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(a + b*x))/(c +
 d*x)]))/(a + b*x))/b + (d*(-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1
- (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(a + b
*x))])/b))/b))/g^2
```

### 3.6.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))    Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

---

3.6. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.6.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(142) = 284$.

Time = 1.38 (sec) , antiderivative size = 370, normalized size of antiderivative = 2.61

| method | result |
|---|---|
| parts | $\frac{iA\left(\frac{d\ln(bx+a)}{b^2}-\frac{-ad+cb}{b^2(bx+a)}\right)}{g^2}-\frac{iB(ad-cb)^2e^2\left(-\frac{d^3\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2be}-\frac{d^4\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^2b^2e^2}+\frac{d^5\cdots}{\cdots}\cdots\right)}{g^2d^3\cdots}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{i\,d^2eA\left(-\frac{d\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^2e^2}-\frac{1}{be\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^2e^2}\right)}{(ad-cb)g^2}-\frac{i\,d^2eB\left(\frac{d\ln\left(\frac{be}{d}+\frac{(ad-\cdots}{d(dx\cdots}\right)}{2b^2e^2}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{i\,d^2eA\left(-\frac{d\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^2e^2}-\frac{1}{be\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^2e^2}\right)}{(ad-cb)g^2}-\frac{i\,d^2eB\left(\frac{d\ln\left(\frac{be}{d}+\frac{(ad-\cdots}{d(dx\cdots}\right)}{2b^2e^2}\cdots\right)}{\cdots}\cdots\right)}{\cdots}$ |
| risch | Expression too large to display |

## 3.6. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x,method=_RETURN
VERBOSE)
```

output
```
i*A/g^2*(d/b^2*ln(b*x+a)-(-a*d+b*c)/b^2/(b*x+a))-i*B/g^2/d^3*(a*d-b*c)^2*e
^2*(-1/(a*d-b*c)^2*d^3/b/e*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d
-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-1/2/(a*d-b*c)^2*d^4/b^
2/e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+1/(a*d-b*c)^2*d^5/b^2/e^2*(dilog(-
((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d))
```

### 3.6.5 Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algori
thm="fricas")
```

output
```
integral((A*d*i*x + A*c*i + (B*d*i*x + B*c*i)*log((b*e*x + a*e)/(d*x + c))
)/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

### 3.6.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

### 3.6.7 Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algori
thm="maxima")
```

output
```
-B*d*i*(((b*x + a)*log(b*x + a) + a)*log(d*x + c)/(b^3*g^2*x + a*b^2*g^2)
- integrate((b^2*d*x^2*log(e) + a^2*d + (b^2*c*log(e) + a*b*d)*x + (2*b^2*
d*x^2 + a^2*d + (b^2*c + 2*a*b*d)*x)*log(b*x + a))/(b^4*d*g^2*x^3 + a^2*b^
2*c*g^2 + (b^4*c*g^2 + 2*a*b^3*d*g^2)*x^2 + (2*a*b^3*c*g^2 + a^2*b^2*d*g^2
)*x), x)) + A*d*i*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - B
*c*i*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*
g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/(
(b^2*c - a*b*d)*g^2)) - A*c*i/(b^2*g^2*x + a*b*g^2)
```

### 3.6.8 Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algori
thm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)^2
, x)
```

### 3.6.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{{(a\,g+b\,g\,x)}^2}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2,x
)
```

output
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2,
x)
```

# 3.7 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$



## 3.7.1 Optimal result

Integrand size = 38, antiderivative size = 85

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = -\frac{Bi(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)g^3(a+bx)^2}$$

output
```
-1/4*B*i*(d*x+c)^2/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*i*(d*x+c)^2*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*d+b*c)/g^3/(b*x+a)^2
```

## 3.7.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 208 vs. 2(85) = 170.

Time = 0.10 (sec) , antiderivative size = 208, normalized size of antiderivative = 2.45

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \frac{i\left(-\frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2(a+bx)^2} - \frac{d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2(a+bx)} - \frac{Bd\left(\frac{1}{a+bx}+\frac{d\log(a+bx)}{bc-ad}-\frac{d\log(c+dx)}{bc-ad}\right)}{b^2} - \frac{B\left(\frac{bc-ad}{(a+bx)^2}-\frac{2d}{a+bx}-\frac{2d^2\log(a+bx)}{bc-ad}\right.}{4b^2}\right.}{g^3}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*
x)^3,x]
```

output
```
(i*(-1/2*((b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*(a + b*x)
^2) - (d*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*(a + b*x)) - (B*d*((a
+ b*x)^(-1) + (d*Log[a + b*x])/(b*c - a*d) - (d*Log[c + d*x])/(b*c - a*d))
)/b^2 - (B*((b*c - a*d)/(a + b*x)^2 - (2*d)/(a + b*x) - (2*d^2*Log[a + b*x
])/(b*c - a*d) + (2*d^2*Log[c + d*x])/(b*c - a*d)))/(4*b^2)))/g^3
```

### 3.7.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2962, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^3}dx$$

$$\Big\downarrow \ 2962$$

$$\frac{i\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)}$$

$$\Big\downarrow \ 2741$$

$$\frac{i\left(-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}\right)}{g^3(bc-ad)}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^3,x
]
```

output
```
(i*(-1/4*(B*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(2*(a + b*x)^2)))/((b*c - a*d)*g^3)
```

### 3.7.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.7.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 153, normalized size of antiderivative = 1.80

| method | result |
|---|---|
| parts | $\frac{iA\left(-\frac{-ad+cb}{2b^2(bx+a)^2}-\frac{d}{b^2(bx+a)}\right)}{g^3}-\frac{iBe^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{g^3(ad-cb)}$ |
| norman | $\frac{\frac{Bcdix\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}-\frac{2Aadi+2Abci+Badi+Bbci}{4g\,b^2}-\frac{(2Adi+Bdi)x}{2gb}+\frac{Bi\,c^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2g(ad-cb)}+\frac{B\,d^2i\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2(ad-cb)g}}{(bx+a)^2g^2}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{i\,d^2eA}{2(ad-cb)^2g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{i\,d^2eB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{(ad-cb)^2g^3}\right)}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{i\,d^2eA}{2(ad-cb)^2g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{i\,d^2eB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{(ad-cb)^2g^3}\right)}{d^2}$ |
| parallelrisch | $-\frac{-2A\,c^2i\,b^4d+2A\,a^2b^2d^3i+B\,a^2b^2d^3i-B\,b^4c^2di-4Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4c\,d^2i-2B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4d^3i+4Axa\,b^3d^3i-4A}{4g^3(bx+a)^2(ad-cb)b^4d}$ |
| risch | $-\frac{Bi(2bdx+ad+cb)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2g^3(bx+a)^2b^2}-\frac{i(2B\ln(dx+c)b^2d^2x^2-2B\ln(-bx-a)b^2d^2x^2+4B\ln(dx+c)ab\,d^2x-4B\ln(-bx-a}{}$ |

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x,method=_RETURN
VERBOSE)
```

output
```
i*A/g^3*(-1/2*(-a*d+b*c)/b^2/(b*x+a)^2-d/b^2/(b*x+a))-i*B/g^3/(a*d-b*c)*e^
2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/
4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)
```

### 3.7.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 177 vs. $2(81) = 162$.

Time = 0.33 (sec) , antiderivative size = 177, normalized size of antiderivative = 2.08

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = -\frac{2\left((2A+B)b^2cd-(2A+B)abd^2\right)ix+\left((2A+B)b^2c^2-(2A+B)a^2d^2\right)i+2\left(Bb^2d^2ix^2+2Bb^2cdix\right.}{4\left((b^5c-ab^4d)g^3x^2+2\left(ab^4c-a^2b^3d\right)g^3x+\left(a^2b^3c-a^3b^2d\right)g^3\right)}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algori
thm="fricas")
```

output
```
-1/4*(2*((2*A + B)*b^2*c*d - (2*A + B)*a*b*d^2)*i*x + ((2*A + B)*b^2*c^2 -
 (2*A + B)*a^2*d^2)*i + 2*(B*b^2*d^2*i*x^2 + 2*B*b^2*c*d*i*x + B*b^2*c^2*i
)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c -
a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3)
```

### 3.7.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 384 vs. $2(73) = 146$.

Time = 2.27 (sec) , antiderivative size = 384, normalized size of antiderivative = 4.52

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$
$$= -\frac{Bd^2i\log\left(x+\frac{-\frac{Ba^2d^4i}{ad-bc}+\frac{2Babcd^3i}{ad-bc}+Bad^3i-\frac{Bb^2c^2d^2i}{ad-bc}+Bbcd^2i}{2Bbd^3i}\right)}{2b^2g^3(ad-bc)}$$
$$+\frac{Bd^2i\log\left(x+\frac{\frac{Ba^2d^4i}{ad-bc}-\frac{2Babcd^3i}{ad-bc}+Bad^3i+\frac{Bb^2c^2d^2i}{ad-bc}+Bbcd^2i}{2Bbd^3i}\right)}{2b^2g^3(ad-bc)}$$
$$+\frac{-2Aadi-2Abci-Badi-Bbci+x(-4Abdi-2Bbdi)}{4a^2b^2g^3+8ab^3g^3x+4b^4g^3x^2}$$
$$+\frac{(-Badi-Bbci-2Bbdix)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2b^2g^3+4ab^3g^3x+2b^4g^3x^2}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

output
```
-B*d**2*i*log(x + (-B*a**2*d**4*i/(a*d - b*c) + 2*B*a*b*c*d**3*i/(a*d - b*
c) + B*a*d**3*i - B*b**2*c**2*d**2*i/(a*d - b*c) + B*b*c*d**2*i)/(2*B*b*d*
*3*i))/(2*b**2*g**3*(a*d - b*c)) + B*d**2*i*log(x + (B*a**2*d**4*i/(a*d -
b*c) - 2*B*a*b*c*d**3*i/(a*d - b*c) + B*a*d**3*i + B*b**2*c**2*d**2*i/(a*d
 - b*c) + B*b*c*d**2*i)/(2*B*b*d**3*i))/(2*b**2*g**3*(a*d - b*c)) + (-2*A*
a*d*i - 2*A*b*c*i - B*a*d*i - B*b*c*i + x*(-4*A*b*d*i - 2*B*b*d*i))/(4*a**
2*b**2*g**3 + 8*a*b**3*g**3*x + 4*b**4*g**3*x**2) + (-B*a*d*i - B*b*c*i -
2*B*b*d*i*x)*log(e*(a + b*x)/(c + d*x))/(2*a**2*b**2*g**3 + 4*a*b**3*g**3*
x + 2*b**4*g**3*x**2)
```

### 3.7.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 570 vs. $2(81) = 162$.

Time = 0.22 (sec) , antiderivative size = 570, normalized size of antiderivative = 6.71

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = -\frac{1}{4}\,Bdi\left(\frac{2\,(2\,bx+a)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^4g^3x^2+2\,ab^3g^3x+a^2b^2g^3}+\frac{3\,abc-a^2d+2\,(2\,b^2c-abd)x}{(b^5c-ab^4d)g^3x^2+2\,(ab^4c-a^2b^3d)g^3x+(a^2b^3c-a^3b^2d)g^3}+\right. + \frac{1}{4}\,Bci\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}-\frac{2\,\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^3g^3x^2+2\,ab^2g^3x+a^2bg^3}+\frac{}{(b^3c}\right. - \frac{(2\,bx+a)Adi}{2\,(b^4g^3x^2+2\,ab^3g^3x+a^2b^2g^3)}-\frac{Aci}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algori
thm="maxima")
```

output
```
-1/4*B*d*i*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^3*x^
2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*
x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c
 - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*
c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a
*b^3*c*d + a^2*b^2*d^2)*g^3)) + 1/4*B*c*i*((2*b*d*x - b*c + 3*a*d)/((b^4*c
 - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d
)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3
*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*
g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2
*(2*b*x + a)*A*d*i/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*A*c*i
/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

### 3.7.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.58

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(\frac{2\,(dx+c)^2Be^3i\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^2g^3}+\frac{(2\,Ae^3i+Be^3i)(dx+c)^2}{(bex+ae)^2g^3}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc}\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algori
thm="giac")
```

output
```
-1/4*(2*(d*x + c)^2*B*e^3*i*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^2*
g^3) + (2*A*e^3*i + B*e^3*i)*(d*x + c)^2/((b*e*x + a*e)^2*g^3))*(b*c/((b*c
*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.7.9 Mupad [B] (verification not implemented)

Time = 2.14 (sec) , antiderivative size = 197, normalized size of antiderivative = 2.32

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$
$$= -\frac{x\,(2\,A\,b\,d\,i+B\,b\,d\,i)+A\,a\,d\,i+A\,b\,c\,i+\frac{B\,a\,d\,i}{2}+\frac{B\,b\,c\,i}{2}}{2\,a^2\,b^2\,g^3+4\,a\,b^3\,g^3\,x+2\,b^4\,g^3\,x^2} - \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,c\,i}{2\,b^2\,g^3}+\frac{B\,a\,d\,i}{2\,b^3\,g^3}+\frac{B\,d\,i\,x}{b^2\,g^3}\right)}{2\,a\,x+b\,x^2+\frac{a^2}{b}} - \frac{B\,d^2\,i\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,1\mathrm{i}}{b^2\,g^3\,\left(a\,d-b\,c\right)}$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3,x
)
```

output
```
- (x*(2*A*b*d*i + B*b*d*i) + A*a*d*i + A*b*c*i + (B*a*d*i)/2 + (B*b*c*i)/2
)/(2*a^2*b^2*g^3 + 2*b^4*g^3*x^2 + 4*a*b^3*g^3*x) - (log((e*(a + b*x))/(c
+ d*x))*((B*c*i)/(2*b^2*g^3) + (B*a*d*i)/(2*b^3*g^3) + (B*d*i*x)/(b^2*g^3)
))/(2*a*x + b*x^2 + a^2/b) - (B*d^2*i*atan((b*c*2i + b*d*x*2i)/(a*d - b*c)
 + 1i)*1i)/(b^2*g^3*(a*d - b*c))
```

# 3.8 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$



## 3.8.1 Optimal result

Integrand size = 38, antiderivative size = 173

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = \frac{Bdi(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2} - \frac{bBi(c+dx)^3}{9(bc-ad)^2g^4(a+bx)^3} + \frac{di(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2g^4(a+bx)^3}$$

output
```
1/4*B*d*i*(d*x+c)^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-1/9*b*B*i*(d*x+c)^3/(-a*d+b
*c)^2/g^4/(b*x+a)^3+1/2*d*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*
c)^2/g^4/(b*x+a)^2-1/3*b*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c
)^2/g^4/(b*x+a)^3
```

### 3.8.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 187, normalized size of antiderivative = 1.08

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{i\left(\frac{12Abc}{(a+bx)^3}+\frac{4bBc}{(a+bx)^3}-\frac{12aAd}{(a+bx)^3}-\frac{4aBd}{(a+bx)^3}+\frac{18Ad}{(a+bx)^2}+\frac{3Bd}{(a+bx)^2}-\frac{6Bd^2}{(bc-ad)(a+bx)}-\frac{6Bd^3\log(a+bx)}{(bc-ad)^2}+\frac{6B(2bc+ad+3bdx}{(a+bx}\right.}{36b^2g^4}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*
x)^4,x]
```

output
```
-1/36*(i*((12*A*b*c)/(a + b*x)^3 + (4*b*B*c)/(a + b*x)^3 - (12*a*A*d)/(a +
 b*x)^3 - (4*a*B*d)/(a + b*x)^3 + (18*A*d)/(a + b*x)^2 + (3*B*d)/(a + b*x)
^2 - (6*B*d^2)/((b*c - a*d)*(a + b*x)) - (6*B*d^3*Log[a + b*x])/(b*c - a*d
)^2 + (6*B*(2*b*c + a*d + 3*b*d*x)*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)
^3 + (6*B*d^3*Log[c + d*x])/(b*c - a*d)^2))/(b^2*g^4)
```

### 3.8.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2962, 2772, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^4}\,dx$$

$$\Big\downarrow \; 2962$$

$$\frac{i\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)^2}$$

$$\Big\downarrow \; 2772$$

$$\frac{i\left(-B\int -\frac{(c+dx)^4\left(2b-\frac{3d(a+bx)}{c+dx}\right)}{6(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}\right)}{g^4(bc-ad)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{i\left(\frac{1}{6}B\int \frac{(c+dx)^4\left(2b-\frac{3d(a+bx)}{c+dx}\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}\right)}{g^4(bc-ad)^2}$$

$$\Big\downarrow \text{53}$$

$$\frac{i\left(\frac{1}{6}B\int \left(\frac{2b(c+dx)^4}{(a+bx)^4}-\frac{3d(c+dx)^3}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}\right)}{g^4(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{b(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{1}{6}B\left(\frac{3d(c+dx)^2}{2(a+bx)^2}-\frac{2b(c+dx)^3}{3(a+bx)^3}\right)\right)}{g^4(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^4,x
]
```

output
```
(i*((B*((3*d*(c + d*x)^2)/(2*(a + b*x)^2) - (2*b*(c + d*x)^3)/(3*(a + b*x)
^3)))/6 + (d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x
)^2) - (b*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(a + b*x)^3
)))/((b*c - a*d)^2*g^4)
```

### 3.8.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

---

3.8. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.8.4 Maple [A] (verified)

Time = 0.95 (sec) , antiderivative size = 279, normalized size of antiderivative = 1.61

| method | result |
|---|---|
| parts | $\frac{iA\left(-\frac{-ad+cb}{3b^2(bx+a)^3}-\frac{d}{2b^2(bx+a)^2}\right)}{g^4} - \frac{iB(ad-cb)^2e^2\left(\frac{d^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{(ad-cb)^4} - \frac{d^3be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(a}{d(}\right.}{3\left(\frac{be}{d}+\frac{(ad}{d(d}\right.}\cdots\right.}{\cdots}\cdots\right.}{g^4d^3}\cdots$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{i\,d^2e^2Ab}{3(ad-cb)^3g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i\,d^3eA}{2(ad-cb)^3g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{i\,d^2e^2Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-}{d(dx}\right.}\cdots\right.}{(ad-cb)^3g^4}\cdots\right.}{d^2}\cdots$ |
| default | $-\frac{e(ad-cb)\left(\frac{i\,d^2e^2Ab}{3(ad-cb)^3g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i\,d^3eA}{2(ad-cb)^3g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{i\,d^2e^2Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-}{d(dx}\right.}\cdots\right.}{(ad-cb)^3g^4}\cdots\right.}{d^2}\cdots$ |
| norman | $\frac{-\frac{6A\,a^2d^2i+6Aabcdi-12A\,b^2c^2i+3B\,a^2d^2i+5Babcdi-4B\,b^2c^2i}{36b^2g(ad-cb)}-\frac{\left(6Aa\,d^2i-6Abcdi+3Ba\,d^2i-Bbcdi\right)x}{12g(ad-cb)b}+\frac{B\,d^2ib\,x^3}{18a(ad-cb)g}+\frac{Bi\,c^2(3ac\cdots}{6\left(a^2d\cdots\right.}\cdots}{(bx+a)^3g^3}\cdots$ |
| risch | $-\frac{Bi(3bdx+ad+2cb)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{6(bx+a)^3b^2g^4} - \frac{(-6B\ln(-bx-a)b^3d^3x^3+6B\ln(dx+c)b^3d^3x^3-18B\ln(-bx-a)a\,b^2d^3x^2+18B\ln\cdots}{\cdots}$ |
| parallelrisch | $-\frac{-36Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^5c\,d^3i-6B\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)b^6d^4i+6B\,x^2a\,b^5d^4i-6B\,x^2b^6c\,d^3i+18Ax\,a^2b^4d^4i+18Ax\,b^6c^2d^2i+\cdots}{\cdots}$ |

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x,method=_RETURN
VERBOSE)
```

output
```
i*A/g^4*(-1/3*(-a*d+b*c)/b^2/(b*x+a)^3-1/2*d/b^2/(b*x+a)^2)-i*B/g^4/d^3*(a
*d-b*c)^2*e^2*(d^4/(a*d-b*c)^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-d^3/(a*d-b
*c)^4*b*e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

## 3.8. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$

### 3.8.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 363 vs. $2(165) = 330$.

Time = 0.37 (sec) , antiderivative size = 363, normalized size of antiderivative = 2.10

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag + bgx)^4}\,dx$$
$$= \frac{6\,(Bb^3cd^2 - Bab^2d^3)ix^2 - 3\,((6\,A + B)b^3c^2d - 6\,(2\,A + B)ab^2cd^2 + (6\,A + 5\,B)a^2bd^3)ix - (4\,(3\,A + B)b^3}{36\,((b^7c^2 - 2\,ab^6cd + a^2b^5d^2)g^4x^3 + 3\,(ab^6c^2 - 2\,a^2b^5}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algori
thm="fricas")
```

output
```
1/36*(6*(B*b^3*c*d^2 - B*a*b^2*d^3)*i*x^2 - 3*((6*A + B)*b^3*c^2*d - 6*(2*
A + B)*a*b^2*c*d^2 + (6*A + 5*B)*a^2*b*d^3)*i*x - (4*(3*A + B)*b^3*c^3 - 9
*(2*A + B)*a*b^2*c^2*d + (6*A + 5*B)*a^3*d^3)*i + 6*(B*b^3*d^3*i*x^3 + 3*B
*a*b^2*d^3*i*x^2 - 3*(B*b^3*c^2*d - 2*B*a*b^2*c*d^2)*i*x - (2*B*b^3*c^3 -
3*B*a*b^2*c^2*d)*i)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c^2 - 2*a*b^6*c*d
+ a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x
^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 -
2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4)
```

### 3.8.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 629 vs. $2(158) = 316$.

Time = 4.17 (sec) , antiderivative size = 629, normalized size of antiderivative = 3.64

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$
$$=-\frac{Bd^3i\log\left(x+\frac{-\frac{Ba^3d^6i}{(ad-bc)^2}+\frac{3Ba^2bcd^5i}{(ad-bc)^2}-\frac{3Bab^2c^2d^4i}{(ad-bc)^2}+Bad^4i+\frac{Bb^3c^3d^3i}{(ad-bc)^2}+Bbcd^3i}{2Bbd^4i}\right)}{6b^2g^4(ad-bc)^2}$$
$$+\frac{Bd^3i\log\left(x+\frac{\frac{Ba^3d^6i}{(ad-bc)^2}-\frac{3Ba^2bcd^5i}{(ad-bc)^2}+\frac{3Bab^2c^2d^4i}{(ad-bc)^2}+Bad^4i-\frac{Bb^3c^3d^3i}{(ad-bc)^2}+Bbcd^3i}{2Bbd^4i}\right)}{6b^2g^4(ad-bc)^2}$$
$$+\frac{(-Badi-2Bbci-3Bbdix)\log\left(\frac{e(a+bx)}{c+dx}\right)}{6a^3b^2g^4+18a^2b^3g^4x+18ab^4g^4x^2+6b^5g^4x^3}$$
$$+\frac{-6Aa^2d^2i-6Aabcdi+12Ab^2c^2i-5Ba^2d^2i-5Babcdi+4Bb^2c^2i-6Bb^2d^2ix^2+x(-18Aabd^2i+18A}{36a^4b^2dg^4-36a^3b^3cg^4+x^3\cdot(36ab^5dg^4-36b^6cg^4)+x^2\cdot(108a^2b^4dg^4-108ab^5cg^4)+x\,(108a}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

output
```
-B*d**3*i*log(x + (-B*a**3*d**6*i/(a*d - b*c)**2 + 3*B*a**2*b*c*d**5*i/(a*
d - b*c)**2 - 3*B*a*b**2*c**2*d**4*i/(a*d - b*c)**2 + B*a*d**4*i + B*b**3*
c**3*d**3*i/(a*d - b*c)**2 + B*b*c*d**3*i)/(2*B*b*d**4*i))/(6*b**2*g**4*(a
*d - b*c)**2) + B*d**3*i*log(x + (B*a**3*d**6*i/(a*d - b*c)**2 - 3*B*a**2*
b*c*d**5*i/(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**4*i/(a*d - b*c)**2 + B*a*d*
*4*i - B*b**3*c**3*d**3*i/(a*d - b*c)**2 + B*b*c*d**3*i)/(2*B*b*d**4*i))/(
6*b**2*g**4*(a*d - b*c)**2) + (-B*a*d*i - 2*B*b*c*i - 3*B*b*d*i*x)*log(e*(
a + b*x)/(c + d*x))/(6*a**3*b**2*g**4 + 18*a**2*b**3*g**4*x + 18*a*b**4*g*
*4*x**2 + 6*b**5*g**4*x**3) + (-6*A*a**2*d**2*i - 6*A*a*b*c*d*i + 12*A*b**
2*c**2*i - 5*B*a**2*d**2*i - 5*B*a*b*c*d*i + 4*B*b**2*c**2*i - 6*B*b**2*d*
*2*i*x**2 + x*(-18*A*a*b*d**2*i + 18*A*b**2*c*d*i - 15*B*a*b*d**2*i + 3*B*
b**2*c*d*i))/(36*a**4*b**2*d*g**4 - 36*a**3*b**3*c*g**4 + x**3*(36*a*b**5*
d*g**4 - 36*b**6*c*g**4) + x**2*(108*a**2*b**4*d*g**4 - 108*a*b**5*c*g**4)
 + x*(108*a**3*b**3*d*g**4 - 108*a**2*b**4*c*g**4))
```

3.8. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$

### 3.8.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 933 vs. $2(165) = 330$.

Time = 0.24 (sec) , antiderivative size = 933, normalized size of antiderivative = 5.39

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{1}{36}Bdi\left(\frac{6\,(3\,bx+a)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^5g^4x^3+3\,ab^4g^4x^2+3\,a^2b^3g^4x+a^3b^2g^4}+\frac{5\,ab^2c^2-22\,a^2bcd+5\,a^3}{(b^7c^2-2\,ab^6cd+a^2b^5d^2)g^4x^3+3\,(ab^6c^2-2\,a^2b^5c}\right.$$

$$-\frac{1}{18}Bci\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\,(b^2cd-5\,abd^2)}{(b^6c^2-2\,ab^5cd+a^2b^4d^2)g^4x^3+3\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)g^4x^2+3\,(a^2b^4c^2-2\,a^3b^3cd+a^4b}\right.$$

$$-\frac{(3\,bx+a)Adi}{6\,(b^5g^4x^3+3\,ab^4g^4x^2+3\,a^2b^3g^4x+a^3b^2g^4)}$$

$$-\frac{Aci}{3\,(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4)}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algori
thm="maxima")
```

output
```
-1/36*B*d*i*(6*(3*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^4*x
^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) + (5*a*b^2*c^2 - 22*
a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*
a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3
 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 -
2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^
2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d
 + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c
)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/18*
B*c*i*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^
2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d
+ a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*
log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^
2*b^2*g^4*x + a^3*b*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d +
3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3
*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/6*(3*b*x + a)*A*d*i/(b^5*g
^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*A*c*i/(b^4
*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3*b*g^4)
```

### 3.8.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 270, normalized size of antiderivative = 1.56

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{1}{36}\left(\frac{6\left(2\,Bbe^4i-\frac{3\,(bex+ae)Bde^3i}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^3bcg^4}{(dx+c)^3}-\frac{(bex+ae)^3adg^4}{(dx+c)^3}}+\frac{12\,Abe^4i+4\,Bbe^4i-\frac{18\,(bex+ae)Ade^3i}{dx+c}-\frac{9\,(bex+ae)Bde^3i}{dx+c}}{\frac{(bex+ae)^3bcg^4}{(dx+c)^3}-\frac{(bex+ae)^3adg^4}{(dx+c)^3}}\right)($$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algori
thm="giac")
```

output
```
-1/36*(6*(2*B*b*e^4*i - 3*(b*e*x + a*e)*B*d*e^3*i/(d*x + c))*log((b*e*x +
a*e)/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4/(d*x + c)^3 - (b*e*x + a*e)^3*a*d
*g^4/(d*x + c)^3) + (12*A*b*e^4*i + 4*B*b*e^4*i - 18*(b*e*x + a*e)*A*d*e^3
*i/(d*x + c) - 9*(b*e*x + a*e)*B*d*e^3*i/(d*x + c))/((b*e*x + a*e)^3*b*c*g
^4/(d*x + c)^3 - (b*e*x + a*e)^3*a*d*g^4/(d*x + c)^3))*(b*c/((b*c*e - a*d*
e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.8.9 Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 361, normalized size of antiderivative = 2.09

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{\frac{6\,A\,a^2\,d^2\,i-12\,A\,b^2\,c^2\,i+5\,B\,a^2\,d^2\,i-4\,B\,b^2\,c^2\,i+6\,A\,a\,b\,c\,d\,i+5\,B\,a\,b\,c\,d\,i}{6\,(a\,d-b\,c)}+\frac{x\,(6\,A\,a\,b\,d^2\,i+5\,B\,a\,b\,d^2\,i-6\,A\,b^2\,c\,d\,i-B\,b^2\,c\,d\,i)}{2\,(a\,d-b\,c)}+\frac{B\,b^2\,d^2\,i\,x^2}{a\,d-b\,c}}{6\,a^3\,b^2\,g^4+18\,a^2\,b^3\,g^4\,x+18\,a\,b^4\,g^4\,x^2+6\,b^5\,g^4\,x^3} - \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,c\,i}{3\,b^2\,g^4}+\frac{B\,a\,d\,i}{6\,b^3\,g^4}+\frac{B\,d\,i\,x}{2\,b^2\,g^4}\right)}{3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2} - \frac{B\,d^3\,i\,\mathrm{atanh}\left(\frac{6\,b^4\,c^2\,g^4-6\,a^2\,b^2\,d^2\,g^4}{6\,b^2\,g^4\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{3\,b^2\,g^4\,(a\,d-b\,c)^2}$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^4,x
)
```

output
```
- ((6*A*a^2*d^2*i - 12*A*b^2*c^2*i + 5*B*a^2*d^2*i - 4*B*b^2*c^2*i + 6*A*a
*b*c*d*i + 5*B*a*b*c*d*i)/(6*(a*d - b*c)) + (x*(6*A*a*b*d^2*i + 5*B*a*b*d^
2*i - 6*A*b^2*c*d*i - B*b^2*c*d*i))/(2*(a*d - b*c)) + (B*b^2*d^2*i*x^2)/(a
*d - b*c))/(6*a^3*b^2*g^4 + 6*b^5*g^4*x^3 + 18*a^2*b^3*g^4*x + 18*a*b^4*g^
4*x^2) - (log((e*(a + b*x))/(c + d*x))*((B*c*i)/(3*b^2*g^4) + (B*a*d*i)/(6
*b^3*g^4) + (B*d*i*x)/(2*b^2*g^4)))/(3*a^2*x + a^3/b + b^2*x^3 + 3*a*b*x^2
) - (B*d^3*i*atanh((6*b^4*c^2*g^4 - 6*a^2*b^2*d^2*g^4)/(6*b^2*g^4*(a*d - b
*c)^2) - (2*b*d*x)/(a*d - b*c)))/(3*b^2*g^4*(a*d - b*c)^2)
```

3.8. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$

# 3.9 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$



## 3.9.1 Optimal result

Integrand size = 38, antiderivative size = 269

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{Bd^2i(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2}+\frac{2bBdi(c+dx)^3}{9(bc-ad)^3g^5(a+bx)^3} - \frac{b^2Bi(c+dx)^4}{16(bc-ad)^3g^5(a+bx)^4} - \frac{d^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^5(a+bx)^4}$$

output
```
-1/4*B*d^2*i*(d*x+c)^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+2/9*b*B*d*i*(d*x+c)^3/(-
a*d+b*c)^3/g^5/(b*x+a)^3-1/16*b^2*B*i*(d*x+c)^4/(-a*d+b*c)^3/g^5/(b*x+a)^4
-1/2*d^2*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^5/(b*x+a)^
2+2/3*b*d*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^5/(b*x+a)
^3-1/4*b^2*i*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^5/(b*x+a
)^4
```

### 3.9.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 210, normalized size of antiderivative = 0.78

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{i\left(\frac{36Abc}{(a+bx)^4}+\frac{9bBc}{(a+bx)^4}-\frac{36aAd}{(a+bx)^4}-\frac{9aBd}{(a+bx)^4}+\frac{48Ad}{(a+bx)^3}+\frac{4Bd}{(a+bx)^3}-\frac{6Bd^2}{(bc-ad)(a+bx)^2}+\frac{12Bd^3}{(bc-ad)^2(a+bx)}+\frac{12Bd^4\log(a+bx)}{(bc-ad)^3}\right.}{144b^2g^5}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*
x)^5,x]
```

output
```
-1/144*(i*((36*A*b*c)/(a + b*x)^4 + (9*b*B*c)/(a + b*x)^4 - (36*a*A*d)/(a
+ b*x)^4 - (9*a*B*d)/(a + b*x)^4 + (48*A*d)/(a + b*x)^3 + (4*B*d)/(a + b*x
)^3 - (6*B*d^2)/((b*c - a*d)*(a + b*x)^2) + (12*B*d^3)/((b*c - a*d)^2*(a +
 b*x)) + (12*B*d^4*Log[a + b*x])/(b*c - a*d)^3 + (12*B*(3*b*c + a*d + 4*b*
d*x)*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^4 - (12*B*d^4*Log[c + d*x])/(
b*c - a*d)^3))/(b^2*g^5)
```

### 3.9.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2962, 2772, 27, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^3}$$

$$\Big\downarrow \text{2772}$$

$$\frac{i\left(-B\int -\frac{(c+dx)^5\left(3b^2-\frac{8d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{12(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{2bd(\cdots}{\cdots}\right.}{g^5(bc-ad)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{i\left(\frac{1}{12}B\int \frac{(c+dx)^5\left(3b^2-\frac{8d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{2bd(c+\cdots}{\cdots}\right.}{g^5(bc-ad)^3}$$

$$\Big\downarrow \; 1140$$

$$\frac{i\left(\frac{1}{12}B\int \left(\frac{3b^2(c+dx)^5}{(a+bx)^5}-\frac{8bd(c+dx)^4}{(a+bx)^4}+\frac{6d^2(c+dx)^3}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\cdots\right.}{g^5(bc-ad)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{i\left(-\frac{b^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{2bd(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{1}{12}B\left(-\frac{3b^2(c+dx)^4}{4(a+bx)^4}-\cdots\right.\right.}{g^5(bc-ad)^3}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^5,x
]
```

output
```
(i*((B*((-3*d^2*(c + d*x)^2)/(a + b*x)^2 + (8*b*d*(c + d*x)^3)/(3*(a + b*x
)^3) - (3*b^2*(c + d*x)^4)/(4*(a + b*x)^4)))/12 - (d^2*(c + d*x)^2*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) + (2*b*d*(c + d*x)^3*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(3*(a + b*x)^3) - (b^2*(c + d*x)^4*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(4*(a + b*x)^4)))/((b*c - a*d)^3*g^5)
```

## 3.9. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$

#### 3.9.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.9.4 Maple [A] (verified)

Time = 1.51 (sec) , antiderivative size = 390, normalized size of antiderivative = 1.45

| method | result |
|---|---|
| parts | $\frac{iA\left(-\frac{d}{3b^2(bx+a)^3}-\frac{-ad+cb}{4b^2(bx+a)^4}\right)}{g^5}-\frac{iB(ad-cb)^2e^2\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{(ad-cb)^5}-\frac{2d^4be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(}{}\right.}{3\left(\frac{be}{d}+\frac{(a}{d(}\right.}\right.}{}\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{i\,d^2e^3A\,b^2}{4(ad-cb)^4g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{2i\,d^3e^2Ab}{3(ad-cb)^4g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i\,d^4eA}{2(ad-cb)^4g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{i\,d^2e^3B\,b^2}{}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{i\,d^2e^3A\,b^2}{4(ad-cb)^4g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{2i\,d^3e^2Ab}{3(ad-cb)^4g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i\,d^4eA}{2(ad-cb)^4g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{i\,d^2e^3B\,b^2}{}\right.}{}$ |
| risch | $-\frac{iB(4bdx+ad+3cb)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{12(bx+a)^4b^2g^5}-\frac{(-48Ba\,b^3c\,d^3x^2-72B\,a^2b^2c\,d^3x+24Ba\,b^3c^2d^2x-72A\,a^2b^2c^2d^2+96Aa\,b^3c^3d-}{}$ |
| norman | $\frac{\left(12A\,a^2c\,d^2i-24Aab\,c^2di+12A\,b^2c^3i+6B\,a^2c\,d^2i-8Bab\,c^2di+3B\,b^2c^3i\right)x}{12ga\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(12A\,a^3d^3i+12A\,a^2bc\,d^2i-60Aa\,b^2c^2di+36A\,b^3c^3i+6B\right.}{24g\,a^2\left(a^2d^2-2a\right.}$ |
| parallelrisch | $\frac{-144Ax\,a^5b^3c^5i+72Bx\,a^8c^2d^3i-36Bx\,a^5b^3c^5i+72B\ln\left(\frac{e(bx+a)}{dx+c}\right)a^8c^3d^2i+36B\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6b^2c^5i+12B\,x^4\ln\left(\frac{e(bx+}{dx+}\right.}{}$ |

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x,method=_RETURN
VERBOSE)
```

output
```
i*A/g^5*(-1/3*d/b^2/(b*x+a)^3-1/4*(-a*d+b*c)/b^2/(b*x+a)^4)-i*B/g^5/d^3*(a
*d-b*c)^2*e^2*(d^5/(a*d-b*c)^5*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*d^4/(a*d
-b*c)^5*b*e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)+d^3/(a*d-b*c)^5*e^2*b^2*(-1/4
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

## 3.9. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$

### 3.9.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 602 vs. $2(257) = 514$.

Time = 0.36 (sec) , antiderivative size = 602, normalized size of antiderivative = 2.24

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{12\,(Bb^4cd^3-Bab^3d^4)ix^3-6\,(Bb^4c^2d^2-8\,Bab^3cd^3+7\,Ba^2b^2d^4)ix^2+4\,((12\,A+B)b^4c^3d-6\,(6\,A+B}{144\,((b^9c^3-3\,ab^8c^2d+3\,a^2b}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algori
thm="fricas")
```

output
```
-1/144*(12*(B*b^4*c*d^3 - B*a*b^3*d^4)*i*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*b^
3*c*d^3 + 7*B*a^2*b^2*d^4)*i*x^2 + 4*((12*A + B)*b^4*c^3*d - 6*(6*A + B)*a
*b^3*c^2*d^2 + 18*(2*A + B)*a^2*b^2*c*d^3 - (12*A + 13*B)*a^3*b*d^4)*i*x +
 (9*(4*A + B)*b^4*c^4 - 32*(3*A + B)*a*b^3*c^3*d + 36*(2*A + B)*a^2*b^2*c^
2*d^2 - (12*A + 13*B)*a^4*d^4)*i + 12*(B*b^4*d^4*i*x^4 + 4*B*a*b^3*d^4*i*x
^3 + 6*B*a^2*b^2*d^4*i*x^2 + 4*(B*b^4*c^3*d - 3*B*a*b^3*c^2*d^2 + 3*B*a^2*
b^2*c*d^3)*i*x + (3*B*b^4*c^4 - 8*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2)*i)*
log((b*e*x + a*e)/(d*x + c)))/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2
- a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2
- a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^
2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*
d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^
2 - a^7*b^2*d^3)*g^5)
```

### 3.9.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(252) = 504$.

Time = 6.87 (sec) , antiderivative size = 928, normalized size of antiderivative = 3.45

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag + bgx)^5}\,dx$$

$$= -\frac{Bd^4 i \log\left(x + \frac{-\frac{Ba^4d^8i}{(ad-bc)^3} + \frac{4Ba^3bcd^7i}{(ad-bc)^3} - \frac{6Ba^2b^2c^2d^6i}{(ad-bc)^3} + \frac{4Bab^3c^3d^5i}{(ad-bc)^3} + Bad^5i - \frac{Bb^4c^4d^4i}{(ad-bc)^3} + Bbcd^4i}{2Bbd^5i}\right)}{12b^2g^5(ad-bc)^3}$$

$$+ \frac{Bd^4 i \log\left(x + \frac{\frac{Ba^4d^8i}{(ad-bc)^3} - \frac{4Ba^3bcd^7i}{(ad-bc)^3} + \frac{6Ba^2b^2c^2d^6i}{(ad-bc)^3} - \frac{4Bab^3c^3d^5i}{(ad-bc)^3} + Bad^5i + \frac{Bb^4c^4d^4i}{(ad-bc)^3} + Bbcd^4i}{2Bbd^5i}\right)}{12b^2g^5(ad-bc)^3}$$

$$+ \frac{(-Badi - 3Bbci - 4Bbdix)\log\left(\frac{e(a+bx)}{c+dx}\right)}{12a^4b^2g^5 + 48a^3b^3g^5x + 72a^2b^4g^5x^2 + 48ab^5g^5x^3 + 12b^6g^5x^4}$$

$$+ \frac{-12Aa^3d^3i - 12Aa^2bcd^2i + 60Aab^2c^2di - 36Ab^3c^3i - 13Ba^3d^3i - 13Ba^2bcd^2i + 23Bab^2c^2di - 9Bb}{144a^6b^2d^2g^5 - 288a^5b^3cdg^5 + 144a^4b^4c^2g^5 + x^4\cdot(144a^2b^6d^2g^5 - 288ab^7cdg^5 + 144b^8c^2g^5) + x^3\cdot(576a^3}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

output
```
-B*d**4*i*log(x + (-B*a**4*d**8*i/(a*d - b*c)**3 + 4*B*a**3*b*c*d**7*i/(a*
d - b*c)**3 - 6*B*a**2*b**2*c**2*d**6*i/(a*d - b*c)**3 + 4*B*a*b**3*c**3*d
**5*i/(a*d - b*c)**3 + B*a*d**5*i - B*b**4*c**4*d**4*i/(a*d - b*c)**3 + B*
b*c*d**4*i)/(2*B*b*d**5*i))/(12*b**2*g**5*(a*d - b*c)**3) + B*d**4*i*log(x
 + (B*a**4*d**8*i/(a*d - b*c)**3 - 4*B*a**3*b*c*d**7*i/(a*d - b*c)**3 + 6*
B*a**2*b**2*c**2*d**6*i/(a*d - b*c)**3 - 4*B*a*b**3*c**3*d**5*i/(a*d - b*c
)**3 + B*a*d**5*i + B*b**4*c**4*d**4*i/(a*d - b*c)**3 + B*b*c*d**4*i)/(2*B
*b*d**5*i))/(12*b**2*g**5*(a*d - b*c)**3) + (-B*a*d*i - 3*B*b*c*i - 4*B*b*
d*i*x)*log(e*(a + b*x)/(c + d*x))/(12*a**4*b**2*g**5 + 48*a**3*b**3*g**5*x
 + 72*a**2*b**4*g**5*x**2 + 48*a*b**5*g**5*x**3 + 12*b**6*g**5*x**4) + (-1
2*A*a**3*d**3*i - 12*A*a**2*b*c*d**2*i + 60*A*a*b**2*c**2*d*i - 36*A*b**3*
c**3*i - 13*B*a**3*d**3*i - 13*B*a**2*b*c*d**2*i + 23*B*a*b**2*c**2*d*i -
9*B*b**3*c**3*i - 12*B*b**3*d**3*i*x**3 + x**2*(-42*B*a*b**2*d**3*i + 6*B*
b**3*c*d**2*i) + x*(-48*A*a**2*b*d**3*i + 96*A*a*b**2*c*d**2*i - 48*A*b**3
*c**2*d*i - 52*B*a**2*b*d**3*i + 20*B*a*b**2*c*d**2*i - 4*B*b**3*c**2*d*i)
)/(144*a**6*b**2*d**2*g**5 - 288*a**5*b**3*c*d*g**5 + 144*a**4*b**4*c**2*g
**5 + x**4*(144*a**2*b**6*d**2*g**5 - 288*a*b**7*c*d*g**5 + 144*b**8*c**2*
g**5) + x**3*(576*a**3*b**5*d**2*g**5 - 1152*a**2*b**6*c*d*g**5 + 576*a*b*
*7*c**2*g**5) + x**2*(864*a**4*b**4*d**2*g**5 - 1728*a**3*b**5*c*d*g**5 +
864*a**2*b**6*c**2*g**5) + x*(576*a**5*b**3*d**2*g**5 - 1152*a**4*b**4*...
```

### 3.9.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1386 vs. $2(257) = 514$.

Time = 0.26 (sec) , antiderivative size = 1386, normalized size of antiderivative = 5.15

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag + bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algori
thm="maxima")
```

output
```
-1/144*B*d*i*(12*(4*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^6*g^5
*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5
) + (7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*
b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d
^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)
*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4
*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6
*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 +
 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x +
 (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5) + 12
*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^
2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*c*d^3 - a*d^4)*log(d
*x + c)/((b^6*c^4 - 4*a*b^5*c^3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 +
a^4*b^2*d^4)*g^5)) + 1/48*B*c*i*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^
2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b
^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*
a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*
a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d +
3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d
+ 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d...
```

### 3.9.8 Giac [A] (verification not implemented)

Time = 0.53 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.58

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{144}\left(\frac{12\left(3\,Bb^2e^5i-\frac{8\,(bex+ae)Bbde^4i}{dx+c}+\frac{6\,(bex+ae)^2Bd^2e^3i}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^4b^2c^2g^5}{(dx+c)^4}-\frac{2\,(bex+ae)^4abcdg^5}{(dx+c)^4}+\frac{(bex+ae)^4a^2d^2g^5}{(dx+c)^4}}+\frac{36\,Ab^2e^5i+9\,Bb^2e^5i-\frac{96\,(bex+ae)}{dx+c}}{\frac{(bex+ae)^4b^2}{(dx+c)^4}}\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algori
thm="giac")
```

output
```
-1/144*(12*(3*B*b^2*e^5*i - 8*(b*e*x + a*e)*B*b*d*e^4*i/(d*x + c) + 6*(b*e
*x + a*e)^2*B*d^2*e^3*i/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x
+ a*e)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*e*x + a*e)^4*a*b*c*d*g^5/(d*x + c)
^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d*x + c)^4) + (36*A*b^2*e^5*i + 9*B*b^2*
e^5*i - 96*(b*e*x + a*e)*A*b*d*e^4*i/(d*x + c) - 32*(b*e*x + a*e)*B*b*d*e^
4*i/(d*x + c) + 72*(b*e*x + a*e)^2*A*d^2*e^3*i/(d*x + c)^2 + 36*(b*e*x + a
*e)^2*B*d^2*e^3*i/(d*x + c)^2)/((b*e*x + a*e)^4*b^2*c^2*g^5/(d*x + c)^4 -
2*(b*e*x + a*e)^4*a*b*c*d*g^5/(d*x + c)^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d
*x + c)^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
 - a*d)))
```

## 3.9.9 Mupad [B] (verification not implemented)

Time = 3.11 (sec) , antiderivative size = 590, normalized size of antiderivative = 2.19

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{B\,d^4\,i\,\mathrm{atanh}\left(\frac{12\,a^3\,b^2\,d^3\,g^5-12\,a^2\,b^3\,c\,d^2\,g^5-12\,a\,b^4\,c^2\,d\,g^5+12\,b^5\,c^3\,g^5}{12\,b^2\,g^5\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{6\,b^2\,g^5\,(a\,d-b\,c)^3}$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,c\,i}{4\,b^2\,g^5}+\frac{B\,a\,d\,i}{12\,b^3\,g^5}+\frac{B\,d\,i\,x}{3\,b^2\,g^5}\right)}{4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3}$$
$$-\frac{\frac{12\,A\,a^3\,d^3\,i+36\,A\,b^3\,c^3\,i+13\,B\,a^3\,d^3\,i+9\,B\,b^3\,c^3\,i-60\,A\,a\,b^2\,c^2\,d\,i+12\,A\,a^2\,b\,c\,d^2\,i-23\,B\,a\,b^2\,c^2\,d\,i+13\,B\,a^2\,b\,c\,d^2\,i}{12\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{x\,(12\,A\,a^2\,b\,d^3\,i+13\,B\,a^2}{}}{12\,a^4\,b^2\,g^5+48\,a^3\,b^3\,g^5\,x+72\,a^2\,b^4\,g^5\,x^2}$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^5,x
)
```

output
```
(B*d^4*i*atanh((12*b^5*c^3*g^5 + 12*a^3*b^2*d^3*g^5 - 12*a*b^4*c^2*d*g^5 -
 12*a^2*b^3*c*d^2*g^5)/(12*b^2*g^5*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(6*b^2*g^5*(a*d - b*c)^3) - (log((e*(a
 + b*x))/(c + d*x))*((B*c*i)/(4*b^2*g^5) + (B*a*d*i)/(12*b^3*g^5) + (B*d*i
*x)/(3*b^2*g^5)))/(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3)
- ((12*A*a^3*d^3*i + 36*A*b^3*c^3*i + 13*B*a^3*d^3*i + 9*B*b^3*c^3*i - 60*
A*a*b^2*c^2*d*i + 12*A*a^2*b*c*d^2*i - 23*B*a*b^2*c^2*d*i + 13*B*a^2*b*c*d
^2*i)/(12*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(12*A*a^2*b*d^3*i + 13*B*a
^2*b*d^3*i + 12*A*b^3*c^2*d*i + B*b^3*c^2*d*i - 24*A*a*b^2*c*d^2*i - 5*B*a
*b^2*c*d^2*i))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (d*x^2*(B*b^3*c*d*i -
 7*B*a*b^2*d^2*i))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*b^3*d^3*i*x^3)
/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(12*a^4*b^2*g^5 + 12*b^6*g^5*x^4 + 48*a^
3*b^3*g^5*x + 48*a*b^5*g^5*x^3 + 72*a^2*b^4*g^5*x^2)
```

3.9. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$

# 3.10 $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$



## 3.10.1 Optimal result

Integrand size = 40, antiderivative size = 423

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx \\
&= \frac{B(bc-ad)^5g^3i^2x}{60b^2d^3}+\frac{B(bc-ad)^4g^3i^2(c+dx)^2}{120bd^4}-\frac{19B(bc-ad)^3g^3i^2(c+dx)^3}{180d^4} \\
&\quad+\frac{13bB(bc-ad)^2g^3i^2(c+dx)^4}{120d^4}-\frac{b^2B(bc-ad)g^3i^2(c+dx)^5}{30d^4} \\
&\quad+\frac{B(bc-ad)^6g^3i^2\log\left(\frac{a+bx}{c+dx}\right)}{60b^3d^4}-\frac{(bc-ad)^3g^3i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^4} \\
&\quad+\frac{3b(bc-ad)^2g^3i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4} \\
&\quad-\frac{3b^2(bc-ad)g^3i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4} \\
&\quad+\frac{b^3g^3i^2(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^4}+\frac{B(bc-ad)^6g^3i^2\log(c+dx)}{60b^3d^4}
\end{aligned}$$

output
```
1/60*B*(-a*d+b*c)^5*g^3*i^2*x/b^2/d^3+1/120*B*(-a*d+b*c)^4*g^3*i^2*(d*x+c)
^2/b/d^4-19/180*B*(-a*d+b*c)^3*g^3*i^2*(d*x+c)^3/d^4+13/120*b*B*(-a*d+b*c)
^2*g^3*i^2*(d*x+c)^4/d^4-1/30*b^2*B*(-a*d+b*c)*g^3*i^2*(d*x+c)^5/d^4+1/60*
B*(-a*d+b*c)^6*g^3*i^2*ln((b*x+a)/(d*x+c))/b^3/d^4-1/3*(-a*d+b*c)^3*g^3*i^
2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4+3/4*b*(-a*d+b*c)^2*g^3*i^2*(d*
x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4-3/5*b^2*(-a*d+b*c)*g^3*i^2*(d*x+c)^
5*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4+1/6*b^3*g^3*i^2*(d*x+c)^6*(A+B*ln(e*(b*x
+a)/(d*x+c)))/d^4+1/60*B*(-a*d+b*c)^6*g^3*i^2*ln(d*x+c)/b^3/d^4
```

### 3.10.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.01

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{g^3i^2\left(90d^4(bc-ad)^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+144d^5(bc-ad)(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)]),x]
```

output
```
(g^3*i^2*(90*d^4*(b*c - a*d)^2*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d
*x)]) + 144*d^5*(b*c - a*d)*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)
]) + 60*d^6*(a + b*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 15*B*(b*c -
 a*d)^3*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*
(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^2*(12*b*d*(
b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)*(a +
b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]) - B*(b*c - a*d
)*(60*b*d*(b*c - a*d)^4*x + 30*d^2*(-(b*c) + a*d)^3*(a + b*x)^2 + 20*d^3*(
b*c - a*d)^2*(a + b*x)^3 + 15*d^4*(-(b*c) + a*d)*(a + b*x)^4 + 12*d^5*(a +
 b*x)^5 - 60*(b*c - a*d)^5*Log[c + d*x]))/(360*b^3*d^4)
```

### 3.10.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 375, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2962, 2782, 27, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2962}$$

$$g^3i^2(bc-ad)^6\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2782}$$

$$g^3i^2(bc-ad)^6\left(-B\int -\frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{60d^4(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}+\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3b^2(\ldots}{\ldots}\right.$$

$$\Big\downarrow \text{27}$$

$$g^3i^2(bc-ad)^6\left(\frac{B\int \frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{60d^4}+\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\ldots\right.}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)\ldots}\right.$$

$$\Big\downarrow \text{2123}$$

$$g^3i^2(bc-ad)^6\left(\frac{B\int\left(-\frac{10db^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}+\frac{26db}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{19d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3b}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2b^2}+\frac{c+dx}{(a+bx)b^3}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)\ldots}\right.}{60d^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$g^3 i^2 (bc - ad)^6 \left( \frac{b^3 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{6d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{3b^2 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{5d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} + \frac{3b \left(B \log \left(\frac{e(a+bx)}{c+dx}\right) + A\right)}{4d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^4} - \frac{B \log \left(\frac{e(a+bx)}{c+dx}\right) + A}{3d^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^3}\right.$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x
]
```

output
```
(b*c - a*d)^6*g^3*i^2*((b^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*d^4*(
b - (d*(a + b*x))/(c + d*x))^6) - (3*b^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(5*d^4*(b - (d*(a + b*x))/(c + d*x))^5) + (3*b*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(4*d^4*(b - (d*(a + b*x))/(c + d*x))^4) - (A + B*Log[(e*(a
 + b*x))/(c + d*x)])/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) + (B*((-2*b^2
)/(b - (d*(a + b*x))/(c + d*x))^5 + (13*b)/(2*(b - (d*(a + b*x))/(c + d*x)
)^4) - 19/(3*(b - (d*(a + b*x))/(c + d*x))^3) + 1/(2*b*(b - (d*(a + b*x))/
(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c +
 d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(60*d^4))
```

### 3.10.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.10.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(401) = 802$.

Time = 1.18 (sec) , antiderivative size = 925, normalized size of antiderivative = 2.19

| method | result |
|---|---|
| risch | $\frac{i^2g^3B\,a^3c^2x}{12}-\frac{i^2g^3bB\,a^2c^3x}{4d}+\frac{i^2g^3b^2B\,a\,c^4x}{10d^2}+\frac{i^2g^3bB\ln(dx+c)a^2c^4}{4d^2}+\frac{7i^2g^3bdB\,a^2c\,x^3}{60}-\frac{13i^2g^3b^2B\,a\,c^2x^3}{60}$ |
| parallelrisch | Expression too large to display |
| parts | Expression too large to display |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETU
RNVERBOSE)
```

output
```
1/12*i^2*g^3*B*a^3*c^2*x-1/4*i^2*g^3*b/d*B*a^2*c^3*x+1/10*i^2*g^3*b^2/d^2*
B*a*c^4*x+1/4*i^2*g^3*b/d^2*B*ln(d*x+c)*a^2*c^4+7/60*i^2*g^3*b*d*B*a^2*c*x
^3-13/60*i^2*g^3*b^2*B*a*c^2*x^3+i^2*g^3*d*A*a^3*c*x^2+3/2*i^2*g^3*b*A*a^2
*c^2*x^2+17/60*i^2*g^3*d*B*a^3*c*x^2-1/4*i^2*g^3*b*B*a^2*c^2*x^2-1/20*i^2*
g^3*b^2/d*B*a*c^3*x^2+i^2*g^3*A*a^3*c^2*x+1/10*i^2*g^3/b*d*B*a^4*c*x-1/60*
i^2*g^3*b^3/d^3*B*c^5*x-1/3*i^2*g^3/d*B*ln(d*x+c)*a^3*c^3+1/4*i^2*g^3/b*B*
ln(-b*x-a)*a^4*c^2+1/60*i^2*g^3*b^3/d^4*B*ln(d*x+c)*c^6+1/60*i^2*g^3/b^3*d
^2*B*ln(-b*x-a)*a^6+3/5*i^2*g^3*b^2*d^2*A*a*x^5+2/5*i^2*g^3*b^3*d*A*c*x^5+
1/30*i^2*g^3*b^2*d^2*B*a*x^5-1/30*i^2*g^3*b^3*d*B*c*x^5+3/4*i^2*g^3*b*d^2*
A*a^2*x^4+1/4*i^2*g^3*b^3*A*c^2*x^4+13/120*i^2*g^3*b*d^2*B*a^2*x^4-7/120*i
^2*g^3*b^3*B*c^2*x^4+1/3*i^2*g^3*d^2*A*a^3*x^3+19/180*i^2*g^3*d^2*B*a^3*x^
3-1/180*i^2*g^3*b^3/d*B*c^3*x^3+1/120*i^2*g^3/b*d^2*B*a^4*x^2+1/120*i^2*g^
3*b^3/d^2*B*c^4*x^2-1/60*i^2*g^3/b^2*d^2*B*a^5*x-1/10*i^2*g^3*b^2/d^3*B*ln
(d*x+c)*a*c^5-1/10*i^2*g^3/b^2*d*B*ln(-b*x-a)*a^5*c+1/6*i^2*g^3*b^3*d^2*A*
x^6+3/2*i^2*g^3*b^2*d*A*a*c*x^4-1/20*i^2*g^3*b^2*d*B*a*c*x^4+2*i^2*g^3*b*d
*A*a^2*c*x^3+i^2*g^3*b^2*A*a*c^2*x^3+1/60*i^2*g^3*B*x*(10*b^3*d^2*x^5+36*a
*b^2*d^2*x^4+24*b^3*c*d*x^4+45*a^2*b*d^2*x^3+90*a*b^2*c*d*x^3+15*b^3*c^2*x
^3+20*a^3*d^2*x^2+120*a^2*b*c*d*x^2+60*a*b^2*c^2*x^2+60*a^3*c*d*x+90*a^2*b
*c^2*x+60*a^3*c^2)*ln(e*(b*x+a)/(d*x+c))
```

### 3.10.5 Fricas [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 724, normalized size of antiderivative = 1.71

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{60\,Ab^6d^6g^3i^2x^6+12\,((12\,A-B)b^6cd^5+(18\,A+B)ab^5d^6)g^3i^2x^5+3\,((30\,A-7\,B)b^6c^2d^4+6\,(30\,A-B)a}{}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="fricas")
```

output
```
1/360*(60*A*b^6*d^6*g^3*i^2*x^6 + 12*((12*A - B)*b^6*c*d^5 + (18*A + B)*a*
b^5*d^6)*g^3*i^2*x^5 + 3*((30*A - 7*B)*b^6*c^2*d^4 + 6*(30*A - B)*a*b^5*c*
d^5 + (90*A + 13*B)*a^2*b^4*d^6)*g^3*i^2*x^4 - 2*(B*b^6*c^3*d^3 - 3*(60*A
- 13*B)*a*b^5*c^2*d^4 - 3*(120*A + 7*B)*a^2*b^4*c*d^5 - (60*A + 19*B)*a^3*
b^3*d^6)*g^3*i^2*x^3 + 3*(B*b^6*c^4*d^2 - 6*B*a*b^5*c^3*d^3 + 30*(6*A - B)
*a^2*b^4*c^2*d^4 + 2*(60*A + 17*B)*a^3*b^3*c*d^5 + B*a^4*b^2*d^6)*g^3*i^2*
x^2 - 6*(B*b^6*c^5*d - 6*B*a*b^5*c^4*d^2 + 15*B*a^2*b^4*c^3*d^3 - 5*(12*A
+ B)*a^3*b^3*c^2*d^4 - 6*B*a^4*b^2*c*d^5 + B*a^5*b*d^6)*g^3*i^2*x + 6*(15*
B*a^4*b^2*c^2*d^4 - 6*B*a^5*b*c*d^5 + B*a^6*d^6)*g^3*i^2*log(b*x + a) + 6*
(B*b^6*c^6 - 6*B*a*b^5*c^5*d + 15*B*a^2*b^4*c^4*d^2 - 20*B*a^3*b^3*c^3*d^3
)*g^3*i^2*log(d*x + c) + 6*(10*B*b^6*d^6*g^3*i^2*x^6 + 60*B*a^3*b^3*c^2*d^
4*g^3*i^2*x + 12*(2*B*b^6*c*d^5 + 3*B*a*b^5*d^6)*g^3*i^2*x^5 + 15*(B*b^6*c
^2*d^4 + 6*B*a*b^5*c*d^5 + 3*B*a^2*b^4*d^6)*g^3*i^2*x^4 + 20*(3*B*a*b^5*c^
2*d^4 + 6*B*a^2*b^4*c*d^5 + B*a^3*b^3*d^6)*g^3*i^2*x^3 + 30*(3*B*a^2*b^4*c
^2*d^4 + 2*B*a^3*b^3*c*d^5)*g^3*i^2*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^
3*d^4)
```

### 3.10.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1727 vs. $2(398) = 796$.

Time = 7.85 (sec) , antiderivative size = 1727, normalized size of antiderivative = 4.08

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b**3*d**2*g**3*i**2*x**6/6 + B*a**4*g**3*i**2*(a**2*d**2 - 6*a*b*c*d + 1
5*b**2*c**2)*log(x + (B*a**6*c*d**5*g**3*i**2 - 6*B*a**5*b*c**2*d**4*g**3*
i**2 + B*a**5*d**4*g**3*i**2*(a**2*d**2 - 6*a*b*c*d + 15*b**2*c**2)/b + 35
*B*a**4*b**2*c**3*d**3*g**3*i**2 - B*a**4*c*d**3*g**3*i**2*(a**2*d**2 - 6*
a*b*c*d + 15*b**2*c**2) - 15*B*a**3*b**3*c**4*d**2*g**3*i**2 + 6*B*a**2*b*
*4*c**5*d*g**3*i**2 - B*a*b**5*c**6*g**3*i**2)/(B*a**6*d**6*g**3*i**2 - 6*
B*a**5*b*c*d**5*g**3*i**2 + 15*B*a**4*b**2*c**2*d**4*g**3*i**2 + 20*B*a**3
*b**3*c**3*d**3*g**3*i**2 - 15*B*a**2*b**4*c**4*d**2*g**3*i**2 + 6*B*a*b**
5*c**5*d*g**3*i**2 - B*b**6*c**6*g**3*i**2))/(60*b**3) - B*c**3*g**3*i**2*
(20*a**3*d**3 - 15*a**2*b*c*d**2 + 6*a*b**2*c**2*d - b**3*c**3)*log(x + (B
*a**6*c*d**5*g**3*i**2 - 6*B*a**5*b*c**2*d**4*g**3*i**2 + 35*B*a**4*b**2*c
**3*d**3*g**3*i**2 - 15*B*a**3*b**3*c**4*d**2*g**3*i**2 + 6*B*a**2*b**4*c*
*5*d*g**3*i**2 - B*a*b**5*c**6*g**3*i**2 - B*a*b**2*c**3*g**3*i**2*(20*a**
3*d**3 - 15*a**2*b*c*d**2 + 6*a*b**2*c**2*d - b**3*c**3) + B*b**3*c**4*g**
3*i**2*(20*a**3*d**3 - 15*a**2*b*c*d**2 + 6*a*b**2*c**2*d - b**3*c**3)/d)/
(B*a**6*d**6*g**3*i**2 - 6*B*a**5*b*c*d**5*g**3*i**2 + 15*B*a**4*b**2*c**2
*d**4*g**3*i**2 + 20*B*a**3*b**3*c**3*d**3*g**3*i**2 - 15*B*a**2*b**4*c**4
*d**2*g**3*i**2 + 6*B*a*b**5*c**5*d*g**3*i**2 - B*b**6*c**6*g**3*i**2))/(6
0*d**4) + x**5*(3*A*a*b**2*d**2*g**3*i**2/5 + 2*A*b**3*c*d*g**3*i**2/5 + B
*a*b**2*d**2*g**3*i**2/30 - B*b**3*c*d*g**3*i**2/30) + x**4*(3*A*a**2*b...
```

### 3.10.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. $2(401) = 802$.

Time = 0.24 (sec) , antiderivative size = 1789, normalized size of antiderivative = 4.23

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="maxima")
```

output
```
1/6*A*b^3*d^2*g^3*i^2*x^6 + 2/5*A*b^3*c*d*g^3*i^2*x^5 + 3/5*A*a*b^2*d^2*g^
3*i^2*x^5 + 1/4*A*b^3*c^2*g^3*i^2*x^4 + 3/2*A*a*b^2*c*d*g^3*i^2*x^4 + 3/4*
A*a^2*b*d^2*g^3*i^2*x^4 + A*a*b^2*c^2*g^3*i^2*x^3 + 2*A*a^2*b*c*d*g^3*i^2*
x^3 + 1/3*A*a^3*d^2*g^3*i^2*x^3 + 3/2*A*a^2*b*c^2*g^3*i^2*x^2 + A*a^3*c*d*
g^3*i^2*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b -
 c*log(d*x + c)/d)*B*a^3*c^2*g^3*i^2 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/
(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(
b*d))*B*a^2*b*c^2*g^3*i^2 + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2*c^2*g^3*i^2 + 1/24*(6*x
^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*l
og(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^
3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*c^2*g^3*i^2 + (x^2*log(
b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)
/d^2 - (b*c - a*d)*x/(b*d))*B*a^3*c*d*g^3*i^2 + (2*x^3*log(b*e*x/(d*x + c)
 + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^
2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b*c*d*g^3
*i^2 + 1/4*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a
)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c
^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a*b^2*c*d...
```

### 3.10.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4746 vs. $2(401) = 802$.

Time = 0.60 (sec) , antiderivative size = 4746, normalized size of antiderivative = 11.22

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="giac")
```

output
```
-1/360*(6*(B*b^10*c^7*e^7*g^3*i^2 - 7*B*a*b^9*c^6*d*e^7*g^3*i^2 + 21*B*a^2
*b^8*c^5*d^2*e^7*g^3*i^2 - 35*B*a^3*b^7*c^4*d^3*e^7*g^3*i^2 + 35*B*a^4*b^6
*c^3*d^4*e^7*g^3*i^2 - 21*B*a^5*b^5*c^2*d^5*e^7*g^3*i^2 + 7*B*a^6*b^4*c*d^
6*e^7*g^3*i^2 - B*a^7*b^3*d^7*e^7*g^3*i^2 - 6*(b*e*x + a*e)*B*b^9*c^7*d*e^
6*g^3*i^2/(d*x + c) + 42*(b*e*x + a*e)*B*a*b^8*c^6*d^2*e^6*g^3*i^2/(d*x +
c) - 126*(b*e*x + a*e)*B*a^2*b^7*c^5*d^3*e^6*g^3*i^2/(d*x + c) + 210*(b*e*
x + a*e)*B*a^3*b^6*c^4*d^4*e^6*g^3*i^2/(d*x + c) - 210*(b*e*x + a*e)*B*a^4
*b^5*c^3*d^5*e^6*g^3*i^2/(d*x + c) + 126*(b*e*x + a*e)*B*a^5*b^4*c^2*d^6*e
^6*g^3*i^2/(d*x + c) - 42*(b*e*x + a*e)*B*a^6*b^3*c*d^7*e^6*g^3*i^2/(d*x +
 c) + 6*(b*e*x + a*e)*B*a^7*b^2*d^8*e^6*g^3*i^2/(d*x + c) + 15*(b*e*x + a*
e)^2*B*b^8*c^7*d^2*e^5*g^3*i^2/(d*x + c)^2 - 105*(b*e*x + a*e)^2*B*a*b^7*c
^6*d^3*e^5*g^3*i^2/(d*x + c)^2 + 315*(b*e*x + a*e)^2*B*a^2*b^6*c^5*d^4*e^5
*g^3*i^2/(d*x + c)^2 - 525*(b*e*x + a*e)^2*B*a^3*b^5*c^4*d^5*e^5*g^3*i^2/(
d*x + c)^2 + 525*(b*e*x + a*e)^2*B*a^4*b^4*c^3*d^6*e^5*g^3*i^2/(d*x + c)^2
 - 315*(b*e*x + a*e)^2*B*a^5*b^3*c^2*d^7*e^5*g^3*i^2/(d*x + c)^2 + 105*(b*
e*x + a*e)^2*B*a^6*b^2*c*d^8*e^5*g^3*i^2/(d*x + c)^2 - 15*(b*e*x + a*e)^2*
B*a^7*b*d^9*e^5*g^3*i^2/(d*x + c)^2 - 20*(b*e*x + a*e)^3*B*b^7*c^7*d^3*e^4
*g^3*i^2/(d*x + c)^3 + 140*(b*e*x + a*e)^3*B*a*b^6*c^6*d^4*e^4*g^3*i^2/(d*
x + c)^3 - 420*(b*e*x + a*e)^3*B*a^2*b^5*c^5*d^5*e^4*g^3*i^2/(d*x + c)^3 +
 700*(b*e*x + a*e)^3*B*a^3*b^4*c^4*d^6*e^4*g^3*i^2/(d*x + c)^3 - 700*(b...
```

### 3.10.9 Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 2473, normalized size of antiderivative = 5.85

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x
)
```

output

```
x^3*((g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3 - B*b^3*c^3 + 48*A
*a*b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2))/(12*
d) + ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B
*b*c))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*
b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2 - 2*B*b^2*c^2
 + 60*A*a*b*c*d - B*a*b*c*d))/5 + A*a*b^2*c*d*g^3*i^2))/(180*b*d) - (a*c*(
(b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B*b*c))/6 - (A*b^2*d*g^3*i^2
*(60*a*d + 60*b*c))/60))/(3*b*d)) - x^4*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A
*b*c + B*a*d - B*b*c))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d
 + 60*b*c))/(240*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*
d^2 - 2*B*b^2*c^2 + 60*A*a*b*c*d - B*a*b*c*d))/20 + (A*a*b^2*c*d*g^3*i^2)/
4) + x^2*((a*c*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B*b*c))/6
- (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (b
*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2 - 2*B*b^2*c^2 + 60*A*a
*b*c*d - B*a*b*c*d))/5 + A*a*b^2*c*d*g^3*i^2))/(2*b*d) - ((60*a*d + 60*b*c
)*((g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3 - B*b^3*c^3 + 48*A*a
*b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2))/(4*d)
+ ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d - B*b*
c))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d
) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2 - 2*B*b^2*c^2...
```

## 3.10. $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

# 3.11 $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$



## 3.11.1 Optimal result

Integrand size = 40, antiderivative size = 337

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\
&= -\frac{B(bc-ad)^4g^2i^2x}{30b^2d^2} - \frac{B(bc-ad)^3g^2i^2(c+dx)^2}{60bd^3} + \frac{B(bc-ad)^2g^2i^2(c+dx)^3}{10d^3} \\
&\quad - \frac{bB(bc-ad)g^2i^2(c+dx)^4}{20d^3} - \frac{B(bc-ad)^5g^2i^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} \\
&\quad + \frac{(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^3} \\
&\quad - \frac{b(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^3} \\
&\quad + \frac{b^2g^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3} - \frac{B(bc-ad)^5g^2i^2\log(c+dx)}{30b^3d^3}
\end{aligned}$$

output
```
-1/30*B*(-a*d+b*c)^4*g^2*i^2*x/b^2/d^2-1/60*B*(-a*d+b*c)^3*g^2*i^2*(d*x+c)
^2/b/d^3+1/10*B*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3/d^3-1/20*b*B*(-a*d+b*c)*g^2
*i^2*(d*x+c)^4/d^3-1/30*B*(-a*d+b*c)^5*g^2*i^2*ln((b*x+a)/(d*x+c))/b^3/d^3
+1/3*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3-1/2*b*
(-a*d+b*c)*g^2*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3+1/5*b^2*g^2*i
^2*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3-1/30*B*(-a*d+b*c)^5*g^2*i^2*l
n(d*x+c)/b^3/d^3
```

### 3.11.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 362, normalized size of antiderivative = 1.07

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{g^2i^2\left(20d^3(bc-ad)^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+30d^4(bc-ad)(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\cdots\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)]),x]
```

output
```
(g^2*i^2*(20*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d
*x)]) + 30*d^4*(b*c - a*d)*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]
) + 12*d^5*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 10*B*(b*c -
a*d)^3*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*
x]) - 5*B*(b*c - a*d)^2*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a +
 b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + B*(b*c - a*d
)*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c -
 a*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x])))/(
60*b^3*d^3)
```

### 3.11.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 301, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2962, 2782, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2962}$$

$$g^2i^2(bc-ad)^5\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2782}$$

$$g^2i^2(bc-ad)^5\left(-B\int \frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{30d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right.$$

$$\Big\downarrow \; 27$$

$$g^2i^2(bc-ad)^5\left(-\frac{B\int \frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{30d^3}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{B\log}{3d^3}\right.$$

$$\Big\downarrow \; 1195$$

$$g^2i^2(bc-ad)^5\left(-\frac{B\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{9d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{6bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{30d^3}+\frac{b^2}{}\right.$$

$$\Big\downarrow \; 2009$$

$$g^2i^2(bc-ad)^5\left(\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right.}{b^3}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x
]
```

output
```
(b*c - a*d)^5*g^2*i^2*((b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*d^3*(
b - (d*(a + b*x))/(c + d*x))^5) - (b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^4) + (A + B*Log[(e*(a + b*x))/(c + d
*x)])/(3*d^3*(b - (d*(a + b*x))/(c + d*x))^3) - (B*((3*b)/(2*(b - (d*(a +
b*x))/(c + d*x))^4) - 3/(b - (d*(a + b*x))/(c + d*x))^3 + 1/(2*b*(b - (d*(
a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a +
 b*x)/(c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(30*d^3))
```

## 3.11. $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

### 3.11.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.11.4 Maple [A] (verified)

Time = 1.00 (sec) , antiderivative size = 636, normalized size of antiderivative = 1.89

| method | result |
|---|---|
| risch | $\frac{i^2g^2b^2dAc\,x^4}{2}+\frac{i^2g^2b\,d^2Ba\,x^4}{20}-\frac{i^2g^2b^2dBc\,x^4}{20}+\frac{i^2g^2d^2A\,a^2x^3}{3}+\frac{i^2g^2b^2A\,c^2x^3}{3}+\frac{i^2g^2d^2B\,a^2x^3}{10}-\frac{i^2g^2b^2B\,c}{10}$ |
| parallelrisch | Expression too large to display |
| parts | Expression too large to display |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETU
RNVERBOSE)
```

output
```
1/2*i^2*g^2*b^2*d*A*c*x^4+1/20*i^2*g^2*b*d^2*B*a*x^4-1/20*i^2*g^2*b^2*d*B*
c*x^4+1/3*i^2*g^2*d^2*A*a^2*x^3+1/3*i^2*g^2*b^2*A*c^2*x^3+1/10*i^2*g^2*d^2
*B*a^2*x^3-1/10*i^2*g^2*b^2*B*c^2*x^3+1/60*i^2*g^2/b*d^2*B*a^3*x^2-1/60*i^
2*g^2*b^2/d*B*c^3*x^2-1/30*i^2*g^2/b^2*d^2*B*a^4*x+1/30*i^2*g^2*b^2/d^2*B*
c^4*x+1/30*i^2*g^2/b^3*d^2*B*ln(-b*x-a)*a^5-1/30*i^2*g^2*b^2/d^3*B*ln(d*x+
c)*c^5+1/3*i^2*g^2/b*B*ln(-b*x-a)*a^3*c^2+1/2*i^2*g^2*b*d^2*A*a*x^4-1/3*i^
2*g^2/d*B*ln(d*x+c)*a^2*c^3+1/30*i^2*g^2*B*x*(6*b^2*d^2*x^4+15*a*b*d^2*x^3
+15*b^2*c*d*x^3+10*a^2*d^2*x^2+40*a*b*c*d*x^2+10*b^2*c^2*x^2+30*a^2*c*d*x+
30*a*b*c^2*x+30*a^2*c^2)*ln(e*(b*x+a)/(d*x+c))+4/3*i^2*g^2*b*d*A*a*c*x^3+i
^2*g^2*d*A*a^2*c*x^2+i^2*g^2*b*A*a*c^2*x^2+1/4*i^2*g^2*d*B*a^2*c*x^2-1/4*i
^2*g^2*b*B*a*c^2*x^2+i^2*g^2*A*a^2*c^2*x+1/6*i^2*g^2/b*d*B*a^3*c*x-1/6*i^2
*g^2*b/d*B*a*c^3*x-1/6*i^2*g^2/b^2*d*B*ln(-b*x-a)*a^4*c+1/6*i^2*g^2*b/d^2*
B*ln(d*x+c)*a*c^4+1/5*i^2*g^2*b^2*d^2*A*x^5
```

### 3.11.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 534, normalized size of antiderivative = 1.58

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{12\,Ab^5d^5g^2i^2x^5+3\left((10\,A-B)b^5cd^4+(10\,A+B)ab^4d^5\right)g^2i^2x^4+2\left((10\,A-3\,B)b^5c^2d^3+40\,Aab^4cd^4+(\right.}{}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="fricas")
```

output
```
1/60*(12*A*b^5*d^5*g^2*i^2*x^5 + 3*((10*A - B)*b^5*c*d^4 + (10*A + B)*a*b^
4*d^5)*g^2*i^2*x^4 + 2*((10*A - 3*B)*b^5*c^2*d^3 + 40*A*a*b^4*c*d^4 + (10*
A + 3*B)*a^2*b^3*d^5)*g^2*i^2*x^3 - (B*b^5*c^3*d^2 - 15*(4*A - B)*a*b^4*c^
2*d^3 - 15*(4*A + B)*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*g^2*i^2*x^2 + 2*(B*b^5
*c^4*d - 5*B*a*b^4*c^3*d^2 + 30*A*a^2*b^3*c^2*d^3 + 5*B*a^3*b^2*c*d^4 - B*
a^4*b*d^5)*g^2*i^2*x + 2*(10*B*a^3*b^2*c^2*d^3 - 5*B*a^4*b*c*d^4 + B*a^5*d
^5)*g^2*i^2*log(b*x + a) - 2*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c
^3*d^2)*g^2*i^2*log(d*x + c) + 2*(6*B*b^5*d^5*g^2*i^2*x^5 + 30*B*a^2*b^3*c
^2*d^3*g^2*i^2*x + 15*(B*b^5*c*d^4 + B*a*b^4*d^5)*g^2*i^2*x^4 + 10*(B*b^5*
c^2*d^3 + 4*B*a*b^4*c*d^4 + B*a^2*b^3*d^5)*g^2*i^2*x^3 + 30*(B*a*b^4*c^2*d
^3 + B*a^2*b^3*c*d^4)*g^2*i^2*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d^3)
```

### 3.11.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1266 vs. $2(311) = 622$.

Time = 3.87 (sec) , antiderivative size = 1266, normalized size of antiderivative = 3.76

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{Ab^2d^2g^2i^2x^5}{5}$$

$$+\frac{Ba^3g^2i^2(a^2d^2-5abcd+10b^2c^2)\log\left(x+\frac{Ba^5cd^4g^2i^2-5Ba^4bc^2d^3g^2i^2+\frac{Ba^4d^3g^2i^2\left(a^2d^2-5abcd+10b^2c^2\right)}{b}+20Ba^3b^2c^3d^2g^2i^2-}{Ba^5d^5g^2i^2-5Ba^4bcd^4g^2i^2+10Ba^3b^2c^2d^3g^2i^2+10Ba^2b}\right.}{30b^3}$$

$$-\frac{Bc^3g^2i^2\cdot(10a^2d^2-5abcd+b^2c^2)\log\left(x+\frac{Ba^5cd^4g^2i^2-5Ba^4bc^2d^3g^2i^2+20Ba^3b^2c^3d^2g^2i^2-5Ba^2b^3c^4dg^2i^2+Bab^4c^5g^2i^2-}{Ba^5d^5g^2i^2-5Ba^4bcd^4g^2i^2+10Ba^3b^2c^2d^3g^2i^2+10Bc}\right.}{30d^3}$$

$$+x^4\left(\frac{Aabd^2g^2i^2}{2}+\frac{Ab^2cdg^2i^2}{2}+\frac{Babd^2g^2i^2}{20}-\frac{Bb^2cdg^2i^2}{20}\right)$$

$$+x^3\left(\frac{Aa^2d^2g^2i^2}{3}+\frac{4Aabcdg^2i^2}{3}+\frac{Ab^2c^2g^2i^2}{3}+\frac{Ba^2d^2g^2i^2}{10}-\frac{Bb^2c^2g^2i^2}{10}\right)$$

$$+x^2\left(Aa^2cdg^2i^2+Aabc^2g^2i^2+\frac{Ba^3d^2g^2i^2}{60b}+\frac{Ba^2cdg^2i^2}{4}-\frac{Babc^2g^2i^2}{4}-\frac{Bb^2c^3g^2i^2}{60d}\right)$$

$$+x\left(Aa^2c^2g^2i^2-\frac{Ba^4d^2g^2i^2}{30b^2}+\frac{Ba^3cdg^2i^2}{6b}-\frac{Babc^3g^2i^2}{6d}+\frac{Bb^2c^4g^2i^2}{30d^2}\right)$$

$$+\left(Ba^2c^2g^2i^2x+Ba^2cdg^2i^2x^2+\frac{Ba^2d^2g^2i^2x^3}{3}+Babc^2g^2i^2x^2+\frac{4Babcdg^2i^2x^3}{3}\right.$$

$$\left.+\frac{Babd^2g^2i^2x^4}{2}+\frac{Bb^2c^2g^2i^2x^3}{3}+\frac{Bb^2cdg^2i^2x^4}{2}+\frac{Bb^2d^2g^2i^2x^5}{5}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b**2*d**2*g**2*i**2*x**5/5 + B*a**3*g**2*i**2*(a**2*d**2 - 5*a*b*c*d + 1
0*b**2*c**2)*log(x + (B*a**5*c*d**4*g**2*i**2 - 5*B*a**4*b*c**2*d**3*g**2*
i**2 + B*a**4*d**3*g**2*i**2*(a**2*d**2 - 5*a*b*c*d + 10*b**2*c**2)/b + 20
*B*a**3*b**2*c**3*d**2*g**2*i**2 - B*a**3*c*d**2*g**2*i**2*(a**2*d**2 - 5*
a*b*c*d + 10*b**2*c**2) - 5*B*a**2*b**3*c**4*d*g**2*i**2 + B*a*b**4*c**5*g
**2*i**2)/(B*a**5*d**5*g**2*i**2 - 5*B*a**4*b*c*d**4*g**2*i**2 + 10*B*a**3
*b**2*c**2*d**3*g**2*i**2 + 10*B*a**2*b**3*c**3*d**2*g**2*i**2 - 5*B*a*b**
4*c**4*d*g**2*i**2 + B*b**5*c**5*g**2*i**2))/(30*b**3) - B*c**3*g**2*i**2*
(10*a**2*d**2 - 5*a*b*c*d + b**2*c**2)*log(x + (B*a**5*c*d**4*g**2*i**2 -
5*B*a**4*b*c**2*d**3*g**2*i**2 + 20*B*a**3*b**2*c**3*d**2*g**2*i**2 - 5*B*
a**2*b**3*c**4*d*g**2*i**2 + B*a*b**4*c**5*g**2*i**2 - B*a*b**2*c**3*g**2*
i**2*(10*a**2*d**2 - 5*a*b*c*d + b**2*c**2) + B*b**3*c**4*g**2*i**2*(10*a*
*2*d**2 - 5*a*b*c*d + b**2*c**2)/d)/(B*a**5*d**5*g**2*i**2 - 5*B*a**4*b*c*
d**4*g**2*i**2 + 10*B*a**3*b**2*c**2*d**3*g**2*i**2 + 10*B*a**2*b**3*c**3*
d**2*g**2*i**2 - 5*B*a*b**4*c**4*d*g**2*i**2 + B*b**5*c**5*g**2*i**2))/(30
*d**3) + x**4*(A*a*b*d**2*g**2*i**2/2 + A*b**2*c*d*g**2*i**2/2 + B*a*b*d**
2*g**2*i**2/20 - B*b**2*c*d*g**2*i**2/20) + x**3*(A*a**2*d**2*g**2*i**2/3
+ 4*A*a*b*c*d*g**2*i**2/3 + A*b**2*c**2*g**2*i**2/3 + B*a**2*d**2*g**2*i**
2/10 - B*b**2*c**2*g**2*i**2/10) + x**2*(A*a**2*c*d*g**2*i**2 + A*a*b*c**2
*g**2*i**2 + B*a**3*d**2*g**2*i**2/(60*b) + B*a**2*c*d*g**2*i**2/4 - B*...
```

### 3.11.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1200 vs. 2(319) = 638.

Time = 0.23 (sec) , antiderivative size = 1200, normalized size of antiderivative = 3.56

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="maxima")
```

output
```
1/5*A*b^2*d^2*g^2*i^2*x^5 + 1/2*A*b^2*c*d*g^2*i^2*x^4 + 1/2*A*a*b*d^2*g^2*
i^2*x^4 + 1/3*A*b^2*c^2*g^2*i^2*x^3 + 4/3*A*a*b*c*d*g^2*i^2*x^3 + 1/3*A*a^
2*d^2*g^2*i^2*x^3 + A*a*b*c^2*g^2*i^2*x^2 + A*a^2*c*d*g^2*i^2*x^2 + (x*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B
*a^2*c^2*g^2*i^2 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x
 + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a*b*c^2*g^2*i^2
+ 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3
 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^
2)*x)/(b^2*d^2))*B*b^2*c^2*g^2*i^2 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x +
 c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*
B*a^2*c*d*g^2*i^2 + 2/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^
3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2
*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b*c*d*g^2*i^2 + 1/12*(6*x^4*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c
)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6
*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^2*c*d*g^2*i^2 + 1/6*(2*x^3*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^
2*d^2*g^2*i^2 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*l
og(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x...
```

### 3.11.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3144 vs. $2(319) = 638$.

Time = 0.52 (sec) , antiderivative size = 3144, normalized size of antiderivative = 9.33

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="giac")
```

output
```
1/60*(2*(B*b^8*c^6*e^6*g^2*i^2 - 6*B*a*b^7*c^5*d*e^6*g^2*i^2 + 15*B*a^2*b^
6*c^4*d^2*e^6*g^2*i^2 - 20*B*a^3*b^5*c^3*d^3*e^6*g^2*i^2 + 15*B*a^4*b^4*c^
2*d^4*e^6*g^2*i^2 - 6*B*a^5*b^3*c*d^5*e^6*g^2*i^2 + B*a^6*b^2*d^6*e^6*g^2*
i^2 - 5*(b*e*x + a*e)*B*b^7*c^6*d*e^5*g^2*i^2/(d*x + c) + 30*(b*e*x + a*e)
*B*a*b^6*c^5*d^2*e^5*g^2*i^2/(d*x + c) - 75*(b*e*x + a*e)*B*a^2*b^5*c^4*d^
3*e^5*g^2*i^2/(d*x + c) + 100*(b*e*x + a*e)*B*a^3*b^4*c^3*d^4*e^5*g^2*i^2/
(d*x + c) - 75*(b*e*x + a*e)*B*a^4*b^3*c^2*d^5*e^5*g^2*i^2/(d*x + c) + 30*
(b*e*x + a*e)*B*a^5*b^2*c*d^6*e^5*g^2*i^2/(d*x + c) - 5*(b*e*x + a*e)*B*a^
6*b*d^7*e^5*g^2*i^2/(d*x + c) + 10*(b*e*x + a*e)^2*B*b^6*c^6*d^2*e^4*g^2*i
^2/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a*b^5*c^5*d^3*e^4*g^2*i^2/(d*x + c)^
2 + 150*(b*e*x + a*e)^2*B*a^2*b^4*c^4*d^4*e^4*g^2*i^2/(d*x + c)^2 - 200*(b
*e*x + a*e)^2*B*a^3*b^3*c^3*d^5*e^4*g^2*i^2/(d*x + c)^2 + 150*(b*e*x + a*e
)^2*B*a^4*b^2*c^2*d^6*e^4*g^2*i^2/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^5*b
*c*d^7*e^4*g^2*i^2/(d*x + c)^2 + 10*(b*e*x + a*e)^2*B*a^6*d^8*e^4*g^2*i^2/
(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/(b^5*d^3*e^5 - 5*(b*e*x + a*e)*b
^4*d^4*e^4/(d*x + c) + 10*(b*e*x + a*e)^2*b^3*d^5*e^3/(d*x + c)^2 - 10*(b*
e*x + a*e)^3*b^2*d^6*e^2/(d*x + c)^3 + 5*(b*e*x + a*e)^4*b*d^7*e/(d*x + c)
^4 - (b*e*x + a*e)^5*d^8/(d*x + c)^5) + (2*A*b^10*c^6*e^6*g^2*i^2 - 12*A*a
*b^9*c^5*d*e^6*g^2*i^2 + 30*A*a^2*b^8*c^4*d^2*e^6*g^2*i^2 - 40*A*a^3*b^7*c
^3*d^3*e^6*g^2*i^2 + 30*A*a^4*b^6*c^2*d^4*e^6*g^2*i^2 - 12*A*a^5*b^5*c*...
```

### 3.11.9 Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 1287, normalized size of antiderivative = 3.82

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x
)
```

output
```
log((e*(a + b*x))/(c + d*x))*((B*g^2*i^2*x^3*(a^2*d^2 + b^2*c^2 + 4*a*b*c*
d))/3 + B*a^2*c^2*g^2*i^2*x + (B*b^2*d^2*g^2*i^2*x^5)/5 + B*a*c*g^2*i^2*x^
2*(a*d + b*c) + (B*b*d*g^2*i^2*x^4*(a*d + b*c))/2) - x^3*(((30*a*d + 30*b*
c)*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))/5 - (A*b*d*g^2*i^2
*(30*a*d + 30*b*c))/30))/(90*b*d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 +
B*a^2*d^2 - B*b^2*c^2 + 18*A*a*b*c*d))/6 + (A*a*b*c*d*g^2*i^2)/3) + x*((a*
c*(((30*a*d + 30*b*c)*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))
/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30))/(30*b*d) - (g^2*i^2*(6*A*a^2*d
^2 + 6*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 18*A*a*b*c*d))/2 + A*a*b*c*d*g^
2*i^2))/(b*d) - ((30*a*d + 30*b*c)*(((30*a*d + 30*b*c)*(((30*a*d + 30*b*c)
*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))/5 - (A*b*d*g^2*i^2*(
30*a*d + 30*b*c))/30))/(30*b*d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*
a^2*d^2 - B*b^2*c^2 + 18*A*a*b*c*d))/2 + A*a*b*c*d*g^2*i^2))/(30*b*d) + (g
^2*i^2*(3*A*a^3*d^3 + 3*A*b^3*c^3 + B*a^3*d^3 - B*b^3*c^3 + 27*A*a*b^2*c^2
*d + 27*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2))/(3*b*d) - (a*c
*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*b*c))/5 - (A*b*d*g^2*i^2*(
30*a*d + 30*b*c))/30))/(b*d)))/(30*b*d) + (a*c*g^2*i^2*(3*A*a^2*d^2 + 3*A*
b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 9*A*a*b*c*d))/(b*d)) + x^2*(((30*a*d + 3
0*b*c)*(((30*a*d + 30*b*c)*((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d - B*
b*c))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30))/(30*b*d) - (g^2*i^2*(6...
```

3.11. $\int (ag + bgx)^2(ci + dix)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$

# 3.12 $\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$



## 3.12.1 Optimal result

Integrand size = 38, antiderivative size = 239

$$\begin{aligned}\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx &= \frac{B(bc-ad)^3gi^2x}{12b^2d} + \frac{B(bc-ad)^2gi^2(c+dx)^2}{24bd^2} - \frac{B(bc-ad)gi^2(c+dx)^3}{12d^2} \\ &\quad + \frac{B(bc-ad)^4gi^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2} - \frac{(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^2} \\ &\quad + \frac{bgi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2} + \frac{B(bc-ad)^4gi^2\log(c+dx)}{12b^3d^2}\end{aligned}$$

output
```
1/12*B*(-a*d+b*c)^3*g*i^2*x/b^2/d+1/24*B*(-a*d+b*c)^2*g*i^2*(d*x+c)^2/b/d^
2-1/12*B*(-a*d+b*c)*g*i^2*(d*x+c)^3/d^2+1/12*B*(-a*d+b*c)^4*g*i^2*ln((b*x+
a)/(d*x+c))/b^3/d^2-1/3*(-a*d+b*c)*g*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+
c)))/d^2+1/4*b*g*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2+1/12*B*(-a*
d+b*c)^4*g*i^2*ln(d*x+c)/b^3/d^2
```

### 3.12.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.90

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{gi^2\left(\frac{4B(bc-ad)^2(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx))}{b^3}-\frac{B(bc-ad)(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad}{b^3}\right.}{24d^2}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
]),x]
```

output
```
(g*i^2*((4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
 - a*d)^2*Log[a + b*x]))/b^3 - (B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*b
^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b
*x]))/b^3 - 8*(b*c - a*d)*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])
 + 6*b*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(24*d^2)
```

### 3.12.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.132, Rules used = {2962, 2782, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$
$$\Big\downarrow \text{2962}$$
$$gi^2(bc-ad)^4\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$
$$\Big\downarrow \text{2782}$$
$$gi^2(bc-ad)^4\left(-B\int -\frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{12d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)$$

$\Big\downarrow$ 27

$$gi^2(bc-ad)^4\left(\frac{B\int \frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)$$

$\Big\downarrow$ 86

$$gi^2(bc-ad)^4\left(\frac{B\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.$$

$\Big\downarrow$ 2009

$$gi^2(bc-ad)^4\left(-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^3}+\frac{1}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{}{2b\left(b-\right.}\right.}{12d^2}\right.$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(b*c - a*d)^4*g*i^2*((b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*d^2*(b -
(d*(a + b*x))/(c + d*x))^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*d^2*
(b - (d*(a + b*x))/(c + d*x))^3) + (B*(-(b - (d*(a + b*x))/(c + d*x))^(-3)
 + 1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c
+ d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]
/b^3))/(12*d^2))
```

## 3.12. $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

### 3.12.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.12.4 Maple [A] (verified)

Time = 0.81 (sec) , antiderivative size = 396, normalized size of antiderivative = 1.66

| method | result |
|---|---|
| risch | $\frac{g i^2 B x (3b d^2 x^3+4a d^2 x^2+8bcd x^2+12acdx+6b c^2 x+12a c^2) \ln\left(\frac{e(bx+a)}{dx+c}\right)}{12}+\frac{i^2 g b d^2 A x^4}{4}+\frac{i^2 g d^2 A a x^3}{3}+\frac{2 i^2 g b d A \cdots}{3}$ |
| parallelrisch | $\frac{-2B\ln\left(\frac{e(bx+a)}{dx+c}\right) b^4 c^4 g i^2-36 A a b^3 c^3 d g i^2+2B\ln(bx+a) a^4 d^4 g i^2+2B\ln(bx+a) b^4 c^4 g i^2+6A x^4 b^4 d^4 g i^2+4B x^2 a b^3 c d^3 \cdots}{\cdots}$ |
| parts | $A g i^2\left(\frac{b d^2 x^4}{4}+\frac{(a d^2+2bcd)x^3}{3}+\frac{(2acd+b c^2)x^2}{2}+x a c^2\right)-\frac{B g i^2 (ad-cb)^3 e^3 \left(b d^3 e (ad-cb)\left(\frac{\cdots}{8b^2 e^2 d\left(\left(\frac{be}{d}+\cdots\right.\right.}\cdots\right.\right.}{\cdots}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-A d^2 e^2 g i^2 (a^3 d^3-3a^2 bc d^2+3a b^2 c^2 d-b^3 c^3)\left(\frac{be}{4d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^4}-\frac{1}{3d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}\cdots\right.\right.}{\cdots}$ |
| default | $-\frac{e(ad-cb)\left(-A d^2 e^2 g i^2 (a^3 d^3-3a^2 bc d^2+3a b^2 c^2 d-b^3 c^3)\left(\frac{be}{4d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^4}-\frac{1}{3d^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^3}\cdots\right.\right.}{\cdots}$ |

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURN
VERBOSE)
```

output
```
1/12*g*i^2*B*x*(3*b*d^2*x^3+4*a*d^2*x^2+8*b*c*d*x^2+12*a*c*d*x+6*b*c^2*x+1
2*a*c^2)*ln(e*(b*x+a)/(d*x+c))+1/4*i^2*g*b*d^2*A*x^4+1/3*i^2*g*d^2*A*a*x^3
+2/3*i^2*g*b*d*A*c*x^3+1/12*i^2*g*d^2*B*a*x^3-1/12*i^2*g*b*d*B*c*x^3+i^2*g
*d*A*a*c*x^2+1/2*i^2*g*b*A*c^2*x^2+1/24*i^2*g/b*d^2*B*a^2*x^2+1/6*i^2*g*d*
B*a*c*x^2-5/24*i^2*g*b*B*c^2*x^2+i^2*g*A*a*c^2*x-1/3*i^2*g/d*B*ln(d*x+c)*a
*c^3+1/12*i^2*g*b/d^2*B*ln(d*x+c)*c^4+1/12*i^2*g/b^3*d^2*B*ln(-b*x-a)*a^4-
1/3*i^2*g/b^2*d*B*ln(-b*x-a)*a^3*c+1/2*i^2*g/b*B*ln(-b*x-a)*a^2*c^2-1/12*i
^2*g/b^2*d^2*B*a^3*x+1/3*i^2*g/b*d*B*a^2*c*x-1/6*i^2*g*B*a*c^2*x-1/12*i^2*
g*b/d*B*c^3*x
```

### 3.12.5 Fricas [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.54

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{6\,Ab^4d^4gi^2x^4+2\,((8\,A-B)b^4cd^3+(4\,A+B)ab^3d^4)gi^2x^3+((12\,A-5\,B)b^4c^2d^2+4\,(6\,A+B)ab^3cd^3+}{}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="fricas")
```

output
```
1/24*(6*A*b^4*d^4*g*i^2*x^4 + 2*((8*A - B)*b^4*c*d^3 + (4*A + B)*a*b^3*d^4
)*g*i^2*x^3 + ((12*A - 5*B)*b^4*c^2*d^2 + 4*(6*A + B)*a*b^3*c*d^3 + B*a^2*
b^2*d^4)*g*i^2*x^2 - 2*(B*b^4*c^3*d - 2*(6*A - B)*a*b^3*c^2*d^2 - 4*B*a^2*
b^2*c*d^3 + B*a^3*b*d^4)*g*i^2*x + 2*(6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^
3 + B*a^4*d^4)*g*i^2*log(b*x + a) + 2*(B*b^4*c^4 - 4*B*a*b^3*c^3*d)*g*i^2*
log(d*x + c) + 2*(3*B*b^4*d^4*g*i^2*x^4 + 12*B*a*b^3*c^2*d^2*g*i^2*x + 4*(
2*B*b^4*c*d^3 + B*a*b^3*d^4)*g*i^2*x^3 + 6*(B*b^4*c^2*d^2 + 2*B*a*b^3*c*d^
3)*g*i^2*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d^2)
```

### 3.12.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(221) = 442$.

Time = 2.35 (sec) , antiderivative size = 850, normalized size of antiderivative = 3.56

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{Abd^2gi^2x^4}{4} + \frac{Ba^2gi^2(a^2d^2-4abcd+6b^2c^2)\log\left(x+\frac{Ba^4cd^3gi^2-4Ba^3bc^2d^2gi^2+\frac{Ba^3d^2gi^2\left(a^2d^2-4abcd+6b^2c^2\right)}{b}+10Ba^2b^2c^3dgi^2-Ba^2cdgi\cdots}{Ba^4d^4gi^2-4Ba^3bcd^3gi^2+6Ba^2b^2c^2d^2gi^2+4Bab^3c^3dgi^2-\cdots}\right.}{12b^3} - \frac{Bc^3gi^2\cdot(4ad-bc)\log\left(x+\frac{Ba^4cd^3gi^2-4Ba^3bc^2d^2gi^2+10Ba^2b^2c^3dgi^2-Bab^3c^4gi^2-Bab^2c^3gi^2\cdot(4ad-bc)+\frac{Bb^3c^4gi^2\cdot(4ad-bc)}{d}}{Ba^4d^4gi^2-4Ba^3bcd^3gi^2+6Ba^2b^2c^2d^2gi^2+4Bab^3c^3dgi^2-Bb^4c^4gi^2}\right)}{12d^2} + x^3\left(\frac{Aad^2gi^2}{3}+\frac{2Abcdgi^2}{3}+\frac{Bad^2gi^2}{12}-\frac{Bbcdgi^2}{12}\right) + x^2\left(Aacdgi^2+\frac{Abc^2gi^2}{2}+\frac{Ba^2d^2gi^2}{24b}+\frac{Bacdgi^2}{6}-\frac{5Bbc^2gi^2}{24}\right) + x\left(Aac^2gi^2-\frac{Ba^3d^2gi^2}{12b^2}+\frac{Ba^2cdgi^2}{3b}-\frac{Bac^2gi^2}{6}-\frac{Bbc^3gi^2}{12d}\right) + \left(Bac^2gi^2x+Bacdgi^2x^2+\frac{Bad^2gi^2x^3}{3}+\frac{Bbc^2gi^2x^2}{2}+\frac{2Bbcdgi^2x^3}{3}+\frac{Bbd^2gi^2x^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b*d**2*g*i**2*x**4/4 + B*a**2*g*i**2*(a**2*d**2 - 4*a*b*c*d + 6*b**2*c**
2)*log(x + (B*a**4*c*d**3*g*i**2 - 4*B*a**3*b*c**2*d**2*g*i**2 + B*a**3*d*
*2*g*i**2*(a**2*d**2 - 4*a*b*c*d + 6*b**2*c**2)/b + 10*B*a**2*b**2*c**3*d*
g*i**2 - B*a**2*c*d*g*i**2*(a**2*d**2 - 4*a*b*c*d + 6*b**2*c**2) - B*a*b**
3*c**4*g*i**2)/(B*a**4*d**4*g*i**2 - 4*B*a**3*b*c*d**3*g*i**2 + 6*B*a**2*b
**2*c**2*d**2*g*i**2 + 4*B*a*b**3*c**3*d*g*i**2 - B*b**4*c**4*g*i**2))/(12
*b**3) - B*c**3*g*i**2*(4*a*d - b*c)*log(x + (B*a**4*c*d**3*g*i**2 - 4*B*a
**3*b*c**2*d**2*g*i**2 + 10*B*a**2*b**2*c**3*d*g*i**2 - B*a*b**3*c**4*g*i*
*2 - B*a*b**2*c**3*g*i**2*(4*a*d - b*c) + B*b**3*c**4*g*i**2*(4*a*d - b*c)
/d)/(B*a**4*d**4*g*i**2 - 4*B*a**3*b*c*d**3*g*i**2 + 6*B*a**2*b**2*c**2*d*
*2*g*i**2 + 4*B*a*b**3*c**3*d*g*i**2 - B*b**4*c**4*g*i**2))/(12*d**2) + x*
*3*(A*a*d**2*g*i**2/3 + 2*A*b*c*d*g*i**2/3 + B*a*d**2*g*i**2/12 - B*b*c*d*
g*i**2/12) + x**2*(A*a*c*d*g*i**2 + A*b*c**2*g*i**2/2 + B*a**2*d**2*g*i**2
/(24*b) + B*a*c*d*g*i**2/6 - 5*B*b*c**2*g*i**2/24) + x*(A*a*c**2*g*i**2 -
B*a**3*d**2*g*i**2/(12*b**2) + B*a**2*c*d*g*i**2/(3*b) - B*a*c**2*g*i**2/6
 - B*b*c**3*g*i**2/(12*d)) + (B*a*c**2*g*i**2*x + B*a*c*d*g*i**2*x**2 + B*
a*d**2*g*i**2*x**3/3 + B*b*c**2*g*i**2*x**2/2 + 2*B*b*c*d*g*i**2*x**3/3 +
B*b*d**2*g*i**2*x**4/4)*log(e*(a + b*x)/(c + d*x))
```

## 3.12. $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$

### 3.12.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(225) = 450$.

Time = 0.22 (sec) , antiderivative size = 671, normalized size of antiderivative = 2.81

$$\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$= \frac{1}{4}Abd^2gi^2x^4 + \frac{2}{3}Abcdgi^2x^3 + \frac{1}{3}Aad^2gi^2x^3 + \frac{1}{2}Abc^2gi^2x^2 + Aacdgi^2x^2$$
$$+ \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) + \frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right)Bac^2gi^2$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)Bbc^2gi^2$$
$$+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)Bacdgi^2$$
$$+\frac{1}{3}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2-2(b^2c^2-a}{b^2d^2}\right.$$
$$+\frac{1}{6}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2-2(b^2c^2-a}{b^2d^2}\right.$$
$$+\frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right) - \frac{6a^4\log(bx+a)}{b^4} + \frac{6c^4\log(dx+c)}{d^4} - \frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2}{}\right.$$
$$+ Aac^2gi^2x$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="maxima")
```

output
```
1/4*A*b*d^2*g*i^2*x^4 + 2/3*A*b*c*d*g*i^2*x^3 + 1/3*A*a*d^2*g*i^2*x^3 + 1/
2*A*b*c^2*g*i^2*x^2 + A*a*c*d*g*i^2*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*a*c^2*g*i^2 + 1/2*(x^2*lo
g(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x +
c)/d^2 - (b*c - a*d)*x/(b*d))*B*b*c^2*g*i^2 + (x^2*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*
x/(b*d))*B*a*c*d*g*i^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^
2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b*c*d*g*i^2 + 1/6*(2*x^3*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c
)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a
*d^2*g*i^2 + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(
b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b*d^2
*g*i^2 + A*a*c^2*g*i^2*x
```

### 3.12.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1975 vs. $2(225) = 450$.

Time = 0.48 (sec) , antiderivative size = 1975, normalized size of antiderivative = 8.26

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="giac")
```

output
```
-1/24*(2*(B*b^6*c^5*e^5*g*i^2 - 5*B*a*b^5*c^4*d*e^5*g*i^2 + 10*B*a^2*b^4*c
^3*d^2*e^5*g*i^2 - 10*B*a^3*b^3*c^2*d^3*e^5*g*i^2 + 5*B*a^4*b^2*c*d^4*e^5*
g*i^2 - B*a^5*b*d^5*e^5*g*i^2 - 4*(b*e*x + a*e)*B*b^5*c^5*d*e^4*g*i^2/(d*x
 + c) + 20*(b*e*x + a*e)*B*a*b^4*c^4*d^2*e^4*g*i^2/(d*x + c) - 40*(b*e*x +
 a*e)*B*a^2*b^3*c^3*d^3*e^4*g*i^2/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^2*c
^2*d^4*e^4*g*i^2/(d*x + c) - 20*(b*e*x + a*e)*B*a^4*b*c*d^5*e^4*g*i^2/(d*x
 + c) + 4*(b*e*x + a*e)*B*a^5*d^6*e^4*g*i^2/(d*x + c))*log((b*e*x + a*e)/(
d*x + c))/(b^4*d^2*e^4 - 4*(b*e*x + a*e)*b^3*d^3*e^3/(d*x + c) + 6*(b*e*x
+ a*e)^2*b^2*d^4*e^2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^5*e/(d*x + c)^3 +
 (b*e*x + a*e)^4*d^6/(d*x + c)^4) + (2*A*b^8*c^5*e^5*g*i^2 - B*b^8*c^5*e^5
*g*i^2 - 10*A*a*b^7*c^4*d*e^5*g*i^2 + 5*B*a*b^7*c^4*d*e^5*g*i^2 + 20*A*a^2
*b^6*c^3*d^2*e^5*g*i^2 - 10*B*a^2*b^6*c^3*d^2*e^5*g*i^2 - 20*A*a^3*b^5*c^2
*d^3*e^5*g*i^2 + 10*B*a^3*b^5*c^2*d^3*e^5*g*i^2 + 10*A*a^4*b^4*c*d^4*e^5*g
*i^2 - 5*B*a^4*b^4*c*d^4*e^5*g*i^2 - 2*A*a^5*b^3*d^5*e^5*g*i^2 + B*a^5*b^3
*d^5*e^5*g*i^2 - 8*(b*e*x + a*e)*A*b^7*c^5*d*e^4*g*i^2/(d*x + c) + 6*(b*e*
x + a*e)*B*b^7*c^5*d*e^4*g*i^2/(d*x + c) + 40*(b*e*x + a*e)*A*a*b^6*c^4*d^
2*e^4*g*i^2/(d*x + c) - 30*(b*e*x + a*e)*B*a*b^6*c^4*d^2*e^4*g*i^2/(d*x +
c) - 80*(b*e*x + a*e)*A*a^2*b^5*c^3*d^3*e^4*g*i^2/(d*x + c) + 60*(b*e*x +
a*e)*B*a^2*b^5*c^3*d^3*e^4*g*i^2/(d*x + c) + 80*(b*e*x + a*e)*A*a^3*b^4*c^
2*d^4*e^4*g*i^2/(d*x + c) - 60*(b*e*x + a*e)*B*a^3*b^4*c^2*d^4*e^4*g*i^...
```

### 3.12.9 Mupad [B] (verification not implemented)

Time = 1.59 (sec) , antiderivative size = 636, normalized size of antiderivative = 2.66

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= x^3\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d-B\,b\,c)}{12}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{36}\right)$$
$$-x^2\left(\frac{\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)(12\,a\,d+12\,b\,c)}{24\,b\,d}\right.$$
$$\left.-\frac{g\,i^2\,(3\,A\,a^2\,d^2+9\,A\,b^2\,c^2+B\,a^2\,d^2-2\,B\,b^2\,c^2+18\,A\,a\,b\,c\,d+B\,a\,b\,c\,d)}{6\,b}+\frac{A\,a\,c\,d\,g\,i^2}{2}\right)$$
$$+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(B\,a\,c^2\,g\,i^2\,x+\frac{B\,c\,g\,i^2\,x^2\,(2\,a\,d+b\,c)}{2}+\frac{B\,d\,g\,i^2\,x^3\,(a\,d+2\,b\,c)}{3}\right.$$
$$\left.+\frac{B\,b\,d^2\,g\,i^2\,x^4}{4}\right)$$
$$+x\left(\frac{(12\,a\,d+12\,b\,c)\left(\frac{\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)(12\,a\,d+12\,b\,c)}{12\,b\,d}-\frac{g\,i^2\,(3\,A\,a^2\,d^2+9\,A\,b^2\,c^2+B\,a^2\,d^2 \cdots}{3 \cdots}\right.}{12\,b\,d}\right.$$
$$-\frac{a\,c\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d-B\,b\,c)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)}{b\,d}$$
$$\left.+\frac{c\,g\,i^2\,(6\,A\,a^2\,d^2+2\,A\,b^2\,c^2+2\,B\,a^2\,d^2-B\,b^2\,c^2+12\,A\,a\,b\,c\,d-B\,a\,b\,c\,d)}{2\,b\,d}\right)$$
$$+\frac{\ln(a+b\,x)\,(B\,g\,a^4\,d^2\,i^2-4\,B\,g\,a^3\,b\,c\,d\,i^2+6\,B\,g\,a^2\,b^2\,c^2\,i^2)}{12\,b^3}$$
$$+\frac{\ln(c+d\,x)\,(B\,b\,c^4\,g\,i^2-4\,B\,a\,c^3\,d\,g\,i^2)}{12\,d^2}+\frac{A\,b\,d^2\,g\,i^2\,x^4}{4}$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x^3*((d*g*i^2*(8*A*a*d + 12*A*b*c + B*a*d - B*b*c))/12 - (A*d*g*i^2*(12*a*
d + 12*b*c))/36) - x^2*((((d*g*i^2*(8*A*a*d + 12*A*b*c + B*a*d - B*b*c))/4
 - (A*d*g*i^2*(12*a*d + 12*b*c))/12)*(12*a*d + 12*b*c))/(24*b*d) - (g*i^2*
(3*A*a^2*d^2 + 9*A*b^2*c^2 + B*a^2*d^2 - 2*B*b^2*c^2 + 18*A*a*b*c*d + B*a*
b*c*d))/(6*b) + (A*a*c*d*g*i^2)/2) + log((e*(a + b*x))/(c + d*x))*(B*a*c^2
*g*i^2*x + (B*c*g*i^2*x^2*(2*a*d + b*c))/2 + (B*d*g*i^2*x^3*(a*d + 2*b*c))
/3 + (B*b*d^2*g*i^2*x^4)/4) + x*(((12*a*d + 12*b*c)*((((d*g*i^2*(8*A*a*d +
 12*A*b*c + B*a*d - B*b*c))/4 - (A*d*g*i^2*(12*a*d + 12*b*c))/12)*(12*a*d
+ 12*b*c))/(12*b*d) - (g*i^2*(3*A*a^2*d^2 + 9*A*b^2*c^2 + B*a^2*d^2 - 2*B*
b^2*c^2 + 18*A*a*b*c*d + B*a*b*c*d))/(3*b) + A*a*c*d*g*i^2))/(12*b*d) - (a
*c*((d*g*i^2*(8*A*a*d + 12*A*b*c + B*a*d - B*b*c))/4 - (A*d*g*i^2*(12*a*d
+ 12*b*c))/12))/(b*d) + (c*g*i^2*(6*A*a^2*d^2 + 2*A*b^2*c^2 + 2*B*a^2*d^2
- B*b^2*c^2 + 12*A*a*b*c*d - B*a*b*c*d))/(2*b*d)) + (log(a + b*x)*(B*a^4*d
^2*g*i^2 + 6*B*a^2*b^2*c^2*g*i^2 - 4*B*a^3*b*c*d*g*i^2))/(12*b^3) + (log(c
 + d*x)*(B*b*c^4*g*i^2 - 4*B*a*c^3*d*g*i^2))/(12*d^2) + (A*b*d^2*g*i^2*x^4
)/4
```

## 3.12. $\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

# 3.13 $\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$



## 3.13.1 Optimal result

Integrand size = 30, antiderivative size = 118

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=-\frac{B(bc-ad)^2i^2x}{3b^2}-\frac{B(bc-ad)i^2(c+dx)^2}{6bd}-\frac{B(bc-ad)^3i^2\log(a+bx)}{3b^3d}+\frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d}$$

output
```
-1/3*B*(-a*d+b*c)^2*i^2*x/b^2-1/6*B*(-a*d+b*c)*i^2*(d*x+c)^2/b/d-1/3*B*(-a
*d+b*c)^3*i^2*ln(b*x+a)/b^3/d+1/3*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c))
)/d
```

## 3.13.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 97, normalized size of antiderivative = 0.82

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\frac{i^2\left(-\frac{B(bc-ad)(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx))}{2b^3}+(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\right)}{3d}$$

input
```
Integrate[(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(i^2*(-1/2*(B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
- a*d)^2*Log[a + b*x]))/b^3 + (c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)])))/(3*d)
```

### 3.13.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 105, normalized size of antiderivative = 0.89, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d}-\frac{B(bc-ad)\int \frac{i^3(c+dx)^2}{a+bx}dx}{3di}$$

$$\Big\downarrow \text{27}$$

$$\frac{i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d}-\frac{Bi^2(bc-ad)\int \frac{(c+dx)^2}{a+bx}dx}{3d}$$

$$\Big\downarrow \text{49}$$

$$\frac{i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d}-\frac{Bi^2(bc-ad)\int \left(\frac{(bc-ad)^2}{b^2(a+bx)}+\frac{d(bc-ad)}{b^2}+\frac{d(c+dx)}{b}\right)dx}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d}-\frac{Bi^2(bc-ad)\left(\frac{(bc-ad)^2\log(a+bx)}{b^3}+\frac{dx(bc-ad)}{b^2}+\frac{(c+dx)^2}{2b}\right)}{3d}$$

input
```
Int[(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
-1/3*(B*(b*c - a*d)*i^2*((d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b*c
 - a*d)^2*Log[a + b*x])/b^3))/d + (i^2*(c + d*x)^3*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(3*d)
```

#### 3.13.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

## 3.13.4 Maple [A] (verified)

Time = 0.67 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.75

| method | result |
|---|---|
| risch | $\frac{i^{2}(dx+c)^{3}B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{3d}+\frac{i^{2}d^{2}A\,x^{3}}{3}+i^{2}dAc\,x^{2}+\frac{i^{2}d^{2}Ba\,x^{2}}{6b}-\frac{i^{2}dBc\,x^{2}}{6}+i^{2}A\,c^{2}x+\frac{i^{2}d^{2}B\ln(bx+a)a}{3b^{3}}$ |
| parts | $\frac{A\,i^{2}(dx+c)^{3}}{3d}-B\,i^{2}(ad-cb)^{3}e^{3}\left(-\frac{1}{6bed\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)^{2}}+\frac{1}{3b^{2}e^{2}d\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}+\frac{\ln(}{}\right.$ |
| parallelrisch | $\frac{-5B\,a^{2}bc\,d^{2}i^{2}+2B\,a^{3}d^{3}i^{2}+4B\,b^{3}c^{3}i^{2}+6Ax\,b^{3}c^{2}d\,i^{2}-2Bx\,a^{2}b\,d^{3}i^{2}-4Bx\,b^{3}c^{2}d\,i^{2}+2B\,x^{3}\ln\left(\frac{e(bx+a)}{dx+c}\right)b^{3}d^{3}i^{2}+6A\,x^{2}b^{3}}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{Ad\,e^{2}i^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^{3}}+B\,d^{2}e^{2}i^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\left(\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{3b^{3}e^{3}d}-\frac{1}{3b^{2}e^{2}d\left(be-\left(\frac{be}{d}+\right.\right.}\right.\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(\frac{Ad\,e^{2}i^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)}{3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^{3}}+B\,d^{2}e^{2}i^{2}\left(a^{2}d^{2}-2abcd+b^{2}c^{2}\right)\left(\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{3b^{3}e^{3}d}-\frac{1}{3b^{2}e^{2}d\left(be-\left(\frac{be}{d}+\right.\right.}\right.\right.}{}$ |

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/3*i^2*(d*x+c)^3*B/d*ln(e*(b*x+a)/(d*x+c))+1/3*i^2*d^2*A*x^3+i^2*d*A*c*x^
2+1/6*i^2/b*d^2*B*a*x^2-1/6*i^2*d*B*c*x^2+i^2*A*c^2*x+1/3*i^2/b^3*d^2*B*ln
(b*x+a)*a^3-i^2/b^2*d*B*ln(b*x+a)*a^2*c+i^2/b*B*ln(b*x+a)*a*c^2-1/3*i^2/d*
B*ln(b*x+a)*c^3-1/3*i^2/b^2*d^2*B*a^2*x+i^2/b*d*B*a*c*x-2/3*i^2*B*c^2*x
```

### 3.13.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 223 vs. $2(110) = 220$.

Time = 0.34 (sec) , antiderivative size = 223, normalized size of antiderivative = 1.89

$$\int (ci + dix)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$$
$$= \frac{2\,Ab^3d^3i^2x^3 - 2\,Bb^3c^3i^2\log(dx+c) + ((6\,A-B)b^3cd^2 + Bab^2d^3)i^2x^2 + 2\,((3\,A-2\,B)b^3c^2d + 3\,Bab^2cd^2}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
1/6*(2*A*b^3*d^3*i^2*x^3 - 2*B*b^3*c^3*i^2*log(d*x + c) + ((6*A - B)*b^3*c
*d^2 + B*a*b^2*d^3)*i^2*x^2 + 2*((3*A - 2*B)*b^3*c^2*d + 3*B*a*b^2*c*d^2 -
 B*a^2*b*d^3)*i^2*x + 2*(3*B*a*b^2*c^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*i^
2*log(b*x + a) + 2*(B*b^3*d^3*i^2*x^3 + 3*B*b^3*c*d^2*i^2*x^2 + 3*B*b^3*c^
2*d*i^2*x)*log((b*e*x + a*e)/(d*x + c)))/(b^3*d)
```

### 3.13.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(100) = 200$.

Time = 1.43 (sec) , antiderivative size = 491, normalized size of antiderivative = 4.16

$$\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{Ad^2i^2x^3}{3}$$

$$+\frac{Bai^2(a^2d^2-3abcd+3b^2c^2)\log\left(x+\frac{Ba^3cd^2i^2-3Ba^2bc^2di^2+\frac{Ba^2di^2(a^2d^2-3abcd+3b^2c^2)}{b}+4Bab^2c^3i^2-Baci^2(a^2d^2-3abcd+3}{Ba^3d^3i^2-3Ba^2bcd^2i^2+3Bab^2c^2di^2+Bb^3c^3i^2}\right.}{3b^3}$$

$$-\frac{Bc^3i^2\log\left(x+\frac{Ba^3cd^2i^2-3Ba^2bc^2di^2+3Bab^2c^3i^2+\frac{Bb^3c^4i^2}{d}}{Ba^3d^3i^2-3Ba^2bcd^2i^2+3Bab^2c^2di^2+Bb^3c^3i^2}\right)}{3d}$$

$$+x^2\left(Acdi^2+\frac{Bad^2i^2}{6b}-\frac{Bcdi^2}{6}\right)+x\left(Ac^2i^2-\frac{Ba^2d^2i^2}{3b^2}+\frac{Bacdi^2}{b}-\frac{2Bc^2i^2}{3}\right)$$

$$+\left(Bc^2i^2x+Bcdi^2x^2+\frac{Bd^2i^2x^3}{3}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*d**2*i**2*x**3/3 + B*a*i**2*(a**2*d**2 - 3*a*b*c*d + 3*b**2*c**2)*log(x
+ (B*a**3*c*d**2*i**2 - 3*B*a**2*b*c**2*d*i**2 + B*a**2*d*i**2*(a**2*d**2
- 3*a*b*c*d + 3*b**2*c**2)/b + 4*B*a*b**2*c**3*i**2 - B*a*c*i**2*(a**2*d**
2 - 3*a*b*c*d + 3*b**2*c**2))/(B*a**3*d**3*i**2 - 3*B*a**2*b*c*d**2*i**2 +
 3*B*a*b**2*c**2*d*i**2 + B*b**3*c**3*i**2))/(3*b**3) - B*c**3*i**2*log(x
+ (B*a**3*c*d**2*i**2 - 3*B*a**2*b*c**2*d*i**2 + 3*B*a*b**2*c**3*i**2 + B*
b**3*c**4*i**2/d)/(B*a**3*d**3*i**2 - 3*B*a**2*b*c*d**2*i**2 + 3*B*a*b**2*
c**2*d*i**2 + B*b**3*c**3*i**2))/(3*d) + x**2*(A*c*d*i**2 + B*a*d**2*i**2/
(6*b) - B*c*d*i**2/6) + x*(A*c**2*i**2 - B*a**2*d**2*i**2/(3*b**2) + B*a*c
*d*i**2/b - 2*B*c**2*i**2/3) + (B*c**2*i**2*x + B*c*d*i**2*x**2 + B*d**2*i
**2*x**3/3)*log(e*(a + b*x)/(c + d*x))
```

### 3.13.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 280 vs. $2(110) = 220$.

Time = 0.20 (sec) , antiderivative size = 280, normalized size of antiderivative = 2.37

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{1}{3}Ad^2i^2x^3+Acdi^2x^2+\left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bc^2i^2$$
$$+\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bcdi^2$$
$$+\frac{1}{6}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a}{b^2d^2}\right.$$
$$+Ac^2i^2x$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output
```
1/3*A*d^2*i^2*x^3 + A*c*d*i^2*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*c^2*i^2 + (x^2*log(b*e*x/(d*x +
 c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c
- a*d)*x/(b*d))*B*c*d*i^2 + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)
*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*d^2*i^2 + A*c^2*i^2*x
```

### 3.13.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1056 vs. $2(110) = 220$.

Time = 0.44 (sec) , antiderivative size = 1056, normalized size of antiderivative = 8.95

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$=\frac{1}{6}\left(\frac{2\left(Bb^4c^4e^4i^2-4Bab^3c^3de^4i^2+6Ba^2b^2c^2d^2e^4i^2-4Ba^3bcd^3e^4i^2+Ba^4d^4e^4i^2\right)\log\left(\frac{bex+ae}{dx+c}\right)}{b^3de^3-\frac{3(bex+ae)b^2d^2e^2}{dx+c}+\frac{3(bex+ae)^2bd^3e}{(dx+c)^2}-\frac{(bex+ae)^3d^4}{(dx+c)^3}}+\frac{2Ab^6c^4}{\cdots}\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/6*(2*(B*b^4*c^4*e^4*i^2 - 4*B*a*b^3*c^3*d*e^4*i^2 + 6*B*a^2*b^2*c^2*d^2*
e^4*i^2 - 4*B*a^3*b*c*d^3*e^4*i^2 + B*a^4*d^4*e^4*i^2)*log((b*e*x + a*e)/(
d*x + c))/(b^3*d*e^3 - 3*(b*e*x + a*e)*b^2*d^2*e^2/(d*x + c) + 3*(b*e*x +
a*e)^2*b*d^3*e/(d*x + c)^2 - (b*e*x + a*e)^3*d^4/(d*x + c)^3) + (2*A*b^6*c
^4*e^4*i^2 - 3*B*b^6*c^4*e^4*i^2 - 8*A*a*b^5*c^3*d*e^4*i^2 + 12*B*a*b^5*c^
3*d*e^4*i^2 + 12*A*a^2*b^4*c^2*d^2*e^4*i^2 - 18*B*a^2*b^4*c^2*d^2*e^4*i^2
- 8*A*a^3*b^3*c*d^3*e^4*i^2 + 12*B*a^3*b^3*c*d^3*e^4*i^2 + 2*A*a^4*b^2*d^4
*e^4*i^2 - 3*B*a^4*b^2*d^4*e^4*i^2 + 5*(b*e*x + a*e)*B*b^5*c^4*d*e^3*i^2/(
d*x + c) - 20*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e^3*i^2/(d*x + c) + 30*(b*e*x
+ a*e)*B*a^2*b^3*c^2*d^3*e^3*i^2/(d*x + c) - 20*(b*e*x + a*e)*B*a^3*b^2*c*
d^4*e^3*i^2/(d*x + c) + 5*(b*e*x + a*e)*B*a^4*b*d^5*e^3*i^2/(d*x + c) - 2*
(b*e*x + a*e)^2*B*b^4*c^4*d^2*e^2*i^2/(d*x + c)^2 + 8*(b*e*x + a*e)^2*B*a*
b^3*c^3*d^3*e^2*i^2/(d*x + c)^2 - 12*(b*e*x + a*e)^2*B*a^2*b^2*c^2*d^4*e^2
*i^2/(d*x + c)^2 + 8*(b*e*x + a*e)^2*B*a^3*b*c*d^5*e^2*i^2/(d*x + c)^2 - 2
*(b*e*x + a*e)^2*B*a^4*d^6*e^2*i^2/(d*x + c)^2)/(b^5*d*e^3 - 3*(b*e*x + a*
e)*b^4*d^2*e^2/(d*x + c) + 3*(b*e*x + a*e)^2*b^3*d^3*e/(d*x + c)^2 - (b*e*
x + a*e)^3*b^2*d^4/(d*x + c)^3) + 2*(B*b^4*c^4*e*i^2 - 4*B*a*b^3*c^3*d*e*i
^2 + 6*B*a^2*b^2*c^2*d^2*e*i^2 - 4*B*a^3*b*c*d^3*e*i^2 + B*a^4*d^4*e*i^2)*
log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b^3*d) - 2*(B*b^4*c^4*e*i^2 - 4*B*a
*b^3*c^3*d*e*i^2 + 6*B*a^2*b^2*c^2*d^2*e*i^2 - 4*B*a^3*b*c*d^3*e*i^2 + ...
```

### 3.13.9 Mupad [B] (verification not implemented)

Time = 1.27 (sec) , antiderivative size = 290, normalized size of antiderivative = 2.46

$$\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= x^2\left(\frac{d\,i^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d-B\,b\,c)}{6\,b}-\frac{A\,d\,i^2\,(3\,a\,d+3\,b\,c)}{6\,b}\right)$$
$$-x\left(\frac{(3\,a\,d+3\,b\,c)\,\left(\frac{d\,i^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d-B\,b\,c)}{3\,b}-\frac{A\,d\,i^2\,(3\,a\,d+3\,b\,c)}{3\,b}\right)}{3\,b\,d}\right.$$
$$\left.-\frac{c\,i^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d-B\,b\,c)}{b}+\frac{A\,a\,c\,d\,i^2}{b}\right)$$
$$+\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(B\,c^2\,i^2\,x+B\,c\,d\,i^2\,x^2+\frac{B\,d^2\,i^2\,x^3}{3}\right)$$
$$+\frac{\ln(a+b\,x)\,(B\,a^3\,d^2\,i^2-3\,B\,a^2\,b\,c\,d\,i^2+3\,B\,a\,b^2\,c^2\,i^2)}{3\,b^3}$$
$$+\frac{A\,d^2\,i^2\,x^3}{3}-\frac{B\,c^3\,i^2\,\ln(c+d\,x)}{3\,d}$$

input
```
int((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x^2*((d*i^2*(3*A*a*d + 9*A*b*c + B*a*d - B*b*c))/(6*b) - (A*d*i^2*(3*a*d +
 3*b*c))/(6*b)) - x*(((3*a*d + 3*b*c)*((d*i^2*(3*A*a*d + 9*A*b*c + B*a*d -
 B*b*c))/(3*b) - (A*d*i^2*(3*a*d + 3*b*c))/(3*b)))/(3*b*d) - (c*i^2*(3*A*a
*d + 3*A*b*c + B*a*d - B*b*c))/b + (A*a*c*d*i^2)/b) + log((e*(a + b*x))/(c
 + d*x))*((B*d^2*i^2*x^3)/3 + B*c^2*i^2*x + B*c*d*i^2*x^2) + (log(a + b*x)
*(B*a^3*d^2*i^2 + 3*B*a*b^2*c^2*i^2 - 3*B*a^2*b*c*d*i^2))/(3*b^3) + (A*d^2
*i^2*x^3)/3 - (B*c^3*i^2*log(c + d*x))/(3*d)
```

## 3.13. $\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

# 3.14 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$



## 3.14.1 Optimal result

Integrand size = 40, antiderivative size = 276

$$\begin{aligned}
\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx
&= -\frac{Bd(bc-ad)i^2x}{2b^2g} - \frac{B(bc-ad)^2i^2\log\left(\frac{a+bx}{c+dx}\right)}{2b^3g} \\
&\quad + \frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g} + \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg} \\
&\quad - \frac{3B(bc-ad)^2i^2\log(c+dx)}{2b^3g} - \frac{(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad + \frac{B(bc-ad)^2i^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}
\end{aligned}$$

output
```
-1/2*B*d*(-a*d+b*c)*i^2*x/b^2/g-1/2*B*(-a*d+b*c)^2*i^2*ln((b*x+a)/(d*x+c))
/b^3/g+d*(-a*d+b*c)*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g+1/2*i^2*
(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g-3/2*B*(-a*d+b*c)^2*i^2*ln(d*x+c)
/b^3/g-(-a*d+b*c)^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+
a))/b^3/g+B*(-a*d+b*c)^2*i^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g
```

### 3.14.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.91

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \frac{i^2\left(2Abd(bc-ad)x-B(bc-ad)(bdx+(bc-ad)\log(a+bx))+2Bd(bc-ad)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+b^2\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x),x]
```

output
```
(i^2*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*(b*d*x + (b*c - a*d)*Log[a + b
*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*(c +
 d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)^2*Log[g*(a +
b*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a*d)^2*Log[c + d*x
] + B*(b*c - a*d)^2*(-(Log[g*(a + b*x)]*(Log[g*(a + b*x)] - 2*Log[(b*(c +
d*x))/(b*c - a*d)])) + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(2*b^
3*g)
```

### 3.14.3 Rubi [A] (verified)

Time = 0.83 (sec) , antiderivative size = 307, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2962, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{ag+bgx}\,dx$$

$$\Big\downarrow \; 2962$$

$$\frac{i^2(bc-ad)^2\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \; 2789$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$\downarrow$ 2756

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int \frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$\downarrow$ 54

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int \left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$\downarrow$ 2009

$$\frac{i^2(bc-ad)^2\left(\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}\right)}{g}$$

$\downarrow$ 2789

---

3.14. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

$$\frac{i^2(bc-ad)^2\left(\frac{\frac{d\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}\cdots\right.}{2d}\right.}{b}\cdots\right.}{g}$$

$$\Big\downarrow \; 2751$$

$$\frac{i^2(bc-ad)^2\left(\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\cdots}{\cdots}\right.}{\cdots}\cdots\right.}{g}$$

$$\Big\downarrow \; 16$$

$$\frac{i^2(bc-ad)^2\left(\frac{\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\cdots}{\cdots}\right.}{\cdots}\right.}{\cdots}\cdots\right.}{g}$$

$$\Big\downarrow \; 2779$$

---

3.14. $\displaystyle\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

$$i^2(bc-ad)^2\left(\frac{\frac{\frac{B\int \frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right.}{b}\right.$$

$$g$$

$\downarrow$ 2838

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{\frac{B\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\right.}{b}}{b}\right.$$

$$g$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x),x
]
```

output
```
((b*c - a*d)^2*i^2*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])/(2*d*(b - (d*
(a + b*x))/(c + d*x))^2) - (B*(1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(
a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(2*d)))/b
 + ((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*(c + d*x)*(b -
 (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*(a + b*x))/(c + d*x)])/(b*d)))/
b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a +
 b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b)/b))/g
```

## 3.14. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

### 3.14.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

---

## 3.14. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.14.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 697 vs. $2(268) = 536$.

Time $= 1.57$ (sec) , antiderivative size $= 698$, normalized size of antiderivative $= 2.53$

| method | result |
|---|---|
| parts | $\frac{i^2 A\left(\frac{d\left(\frac{1}{2}bd\,x^2-xad+2bcx\right)}{b^2}+\frac{\left(a^2d^2-2abcd+b^2c^2\right)\ln(bx+a)}{b^3}\right)}{g} - \frac{i^2B(ad-cb)^3e^3\left(-\frac{d^4\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)b^3e^3}+\frac{d^5\left(\text{dilog}\left(-\right.\right.}{}\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{A\,d^2e^2i^2(ad-cb)\left(\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^3e^3}+\frac{1}{b^2e^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^3e^3}+\frac{}{2be\left(be-\left(\right.\right.}\right)}{g}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{A\,d^2e^2i^2(ad-cb)\left(\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^3e^3}+\frac{1}{b^2e^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^3e^3}+\frac{}{2be\left(be-\left(\right.\right.}\right)}{g}\right.}{}$ |
| risch | Expression too large to display |

3.14. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x,method=_RETURN
VERBOSE)
```

output
```
i^2*A/g*(d/b^2*(1/2*b*d*x^2-x*a*d+2*b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3
*ln(b*x+a))-i^2*B/g/d^4*(a*d-b*c)^3*e^3*(-1/2/(a*d-b*c)*d^4/b^3/e^3*ln(b*e
/d+(a*d-b*c)*e/d/(d*x+c))^2+1/(a*d-b*c)*d^5/b^3/e^3*(dilog(-((b*e/d+(a*d-b
*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d
+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)-1/(a*d-b*c)*d^5/b^2/e^2*(1/b/e/d*ln
((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e
/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))+1/(a*
d-b*c)*d^5/b/e*(-1/2/e^2/b^2/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-1/2
/e/b/d/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)+1/2*ln(b*e/d+(a*d-b*c)*e/d/(d
*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-2*b*
e)/e^2/b^2/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)^2))
```

### 3.14.5 Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algori
thm="fricas")
```

output
```
integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B
*c*d*i^2*x + B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)))/(b*g*x + a*g), x)
```

### 3.14.6 Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^2\left(\int \frac{Ac^2}{a+bx}\,dx + \int \frac{Ad^2x^2}{a+bx}\,dx + \int \frac{Bc^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx + \int \frac{2Acdx}{a+bx}\,dx + \int \frac{Bd^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx + \int \frac{2Bcdx\log\left(\right.}{a\ldots}\right.}{g}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

output
```
i**2*(Integral(A*c**2/(a + b*x), x) + Integral(A*d**2*x**2/(a + b*x), x) +
 Integral(B*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Inte
gral(2*A*c*d*x/(a + b*x), x) + Integral(B*d**2*x**2*log(a*e/(c + d*x) + b*
e*x/(c + d*x))/(a + b*x), x) + Integral(2*B*c*d*x*log(a*e/(c + d*x) + b*e*
x/(c + d*x))/(a + b*x), x))/g
```

### 3.14.7 Maxima [A] (verification not implemented)

Time = 0.26 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.88

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= 2\,Acdi^2\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)+\frac{1}{2}\,Ad^2i^2\left(\frac{2\,a^2\log(bx+a)}{b^3g}+\frac{bx^2-2\,ax}{b^2g}\right)$$
$$+\frac{Ac^2i^2\log(bgx+ag)}{bg}-\frac{(3\,bc^2i^2-2\,acdi^2)B\log(dx+c)}{2\,b^2g}$$
$$+\frac{(b^2c^2i^2-2\,abcdi^2+a^2d^2i^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^3g}$$
$$+\frac{Bb^2d^2i^2x^2\log(e)+(b^2c^2i^2-2\,abcdi^2+a^2d^2i^2)B\log(bx+a)^2+((4\,i^2\log(e)-i^2)b^2cd-(2\,i^2\log(e)-}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algori
thm="maxima")
```

output
```
2*A*c*d*i^2*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + 1/2*A*d^2*i^2*(2*a^2*log(
b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A*c^2*i^2*log(b*g*x + a*g)/(
b*g) - 1/2*(3*b*c^2*i^2 - 2*a*c*d*i^2)*B*log(d*x + c)/(b^2*g) + (b^2*c^2*i
^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a
*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^3*g) + 1/2*(B*b^2*d^2*i
^2*x^2*log(e) + (b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B*log(b*x + a)
^2 + ((4*i^2*log(e) - i^2)*b^2*c*d - (2*i^2*log(e) - i^2)*a*b*d^2)*B*x + (
B*b^2*d^2*i^2*x^2 + 2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B*x + (2*b^2*c^2*i^2*l
og(e) - 4*(i^2*log(e) - i^2)*a*b*c*d + (2*i^2*log(e) - 3*i^2)*a^2*d^2)*B)*
log(b*x + a) - (B*b^2*d^2*i^2*x^2 + 2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B*x +
2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B*log(b*x + a))*log(d*x + c)
)/(b^3*g)
```

### 3.14.8 Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algori
thm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
, x)
```

### 3.14.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx=\int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x),x
)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x),
x)
```

# 3.15 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$



## 3.15.1 Optimal result

Integrand size = 40, antiderivative size = 247

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx &= -\frac{B(bc-ad)i^2(c+dx)}{b^2g^2(a+bx)}+\frac{d^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2}\\ &\quad-\frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^2(a+bx)}-\frac{Bd(bc-ad)i^2\log(c+dx)}{b^3g^2}\\ &\quad-\frac{2d(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}\\ &\quad+\frac{2Bd(bc-ad)i^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}\end{aligned}$$

output
```
-B*(-a*d+b*c)*i^2*(d*x+c)/b^2/g^2/(b*x+a)+d^2*i^2*(b*x+a)*(A+B*ln(e*(b*x+a
)/(d*x+c)))/b^3/g^2-(-a*d+b*c)*i^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2
/g^2/(b*x+a)-B*d*(-a*d+b*c)*i^2*ln(d*x+c)/b^3/g^2-2*d*(-a*d+b*c)*i^2*(A+B*
ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^2+2*B*d*(-a*d+b*c)*
i^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^2
```

### 3.15.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 221, normalized size of antiderivative = 0.89

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \frac{i^2\left(Abd^2x-\frac{B(bc-ad)^2}{a+bx}+Bd(-bc+ad)\log(a+bx)+Bd^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-\frac{(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^2,x]
```

output
```
(i^2*(A*b*d^2*x - (B*(b*c - a*d)^2)/(a + b*x) + B*d*(-(b*c) + a*d)*Log[a +
 b*x] + B*d^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - ((b*c - a*d)^2*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + 2*d*(b*c - a*d)*Log[a + b*x]
*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + B*d*(-(b*c) + a*d)*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*
x))/(-(b*c) + a*d)])))/(b^3*g^2)
```

### 3.15.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 223, normalized size of antiderivative = 0.90, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2962, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^2}\,dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i^2(bc-ad)\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{2793}$$

$$\frac{i^2(bc-ad)\int\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d^2}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d}{b^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i^2(bc-ad)\left(\frac{d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2d\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^2(a+bx)}+\frac{2Bd\,\text{PolyLo}}{}\right.}{g^2}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^2
,x]
```

output
```
((b*c - a*d)*i^2*(-((B*(c + d*x))/(b^2*(a + b*x))) - ((c + d*x)*(A + B*Log
[(e*(a + b*x))/(c + d*x)]))/(b^2*(a + b*x)) + (d^2*(a + b*x)*(A + B*Log[(e
*(a + b*x))/(c + d*x)]))/(b^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (
B*d*Log[b - (d*(a + b*x))/(c + d*x)])/b^3 - (2*d*(A + B*Log[(e*(a + b*x))/
(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b^3 + (2*B*d*PolyLog[2,
(b*(c + d*x))/(d*(a + b*x))])/b^3))/g^2
```

### 3.15.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.15.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(247) = 494$.

Time = 1.56 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.30

| method | result |
|---|---|
| parts | $\frac{i^2 A\left(\frac{x\,d^2}{b^2}-\frac{2d(ad-cb)\ln(bx+a)}{b^3}-\frac{a^2d^2-2abcd+b^2c^2}{b^3(bx+a)}\right)}{g^2} - \frac{i^2 B(ad-cb)^3 e^3\left(\frac{d^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2b^2e^2}+\frac{d^5 \cdots}{\cdots} \cdots\right.}{\cdots}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{i^2d^2e^2A\left(\frac{d}{b^2e^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}-\frac{2d\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^3e^3}-\frac{1}{b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{2d\ln\left(\frac{be}{d}+\frac{(ad\cdots}{d(d\cdots}\right.}{b^3e^3}\cdots\right.}{g^2}\cdots\right.}{\cdots}$ |
| default | $-\frac{e(ad-cb)\left(\frac{i^2d^2e^2A\left(\frac{d}{b^2e^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}-\frac{2d\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^3e^3}-\frac{1}{b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{2d\ln\left(\frac{be}{d}+\frac{(ad\cdots}{d(d\cdots}\right.}{b^3e^3}\cdots\right.}{g^2}\cdots\right.}{\cdots}$ |
| risch | Expression too large to display |

3.15. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x,method=_RETU
RNVERBOSE)
```

output
```
i^2*A/g^2*(x*d^2/b^2-2/b^3*d*(a*d-b*c)*ln(b*x+a)-1/b^3*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/(b*x+a))-i^2*B/g^2/d^4*(a*d-b*c)^3*e^3*(1/(a*d-b*c)^2*d^4/b^2/e^2
*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/
d+(a*d-b*c)*e/d/(d*x+c)))+1/(a*d-b*c)^2*d^5/b^3/e^3*ln(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^2-2/(a*d-b*c)^2*d^6/b^3/e^3*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c
))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d
/(d*x+c))*d-b*e)/b/e)/d)+1/(a*d-b*c)^2*d^6/b^2/e^2*(1/b/e/d*ln((b*e/d+(a*d
-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)
*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)))
```

### 3.15.5 Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algo
rithm="fricas")
```

output
```
integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B
*c*d*i^2*x + B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)))/(b^2*g^2*x^2 + 2*a*b
*g^2*x + a^2*g^2), x)
```

### 3.15.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

### 3.15.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 992 vs. $2(246) = 492$.

Time = 0.27 (sec) , antiderivative size = 992, normalized size of antiderivative = 4.02

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$$
$$= -A\left(\frac{a^2}{b^4g^2x+ab^3g^2}-\frac{x}{b^2g^2}+\frac{2\,a\log(bx+a)}{b^3g^2}\right)d^2i^2$$
$$+2\,Acdi^2\left(\frac{a}{b^3g^2x+ab^2g^2}+\frac{\log(bx+a)}{b^2g^2}\right)$$
$$-Bc^2i^2\left(\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{b^2g^2x+abg^2}+\frac{1}{b^2g^2x+abg^2}+\frac{d\log(bx+a)}{(b^2c-abd)g^2}-\frac{d\log(dx+c)}{(b^2c-abd)g^2}\right)$$
$$-\frac{Ac^2i^2}{b^2g^2x+abg^2}-\frac{(b^2c^2di^2+abcd^2i^2-a^2d^3i^2)B\log(dx+c)}{b^4cg^2-ab^3dg^2}$$
$$+\frac{(b^3cd^2i^2\log(e)-ab^2d^3i^2\log(e))Bx^2+(ab^2cd^2i^2\log(e)-a^2bd^3i^2\log(e))Bx+((b^3c^2di^2-2\,ab^2cd^2i^2+\cdots}{}$$
$$+\frac{2\,(bcdi^2-ad^2i^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^3g^2}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algo
rithm="maxima")
```

output
```
-A*(a^2/(b^4*g^2*x + a*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2)
)*d^2*i^2 + 2*A*c*d*i^2*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2
)) - B*c^2*i^2*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2)
 + 1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(
d*x + c)/((b^2*c - a*b*d)*g^2)) - A*c^2*i^2/(b^2*g^2*x + a*b*g^2) - (b^2*c
^2*d*i^2 + a*b*c*d^2*i^2 - a^2*d^3*i^2)*B*log(d*x + c)/(b^4*c*g^2 - a*b^3*
d*g^2) + ((b^3*c*d^2*i^2*log(e) - a*b^2*d^3*i^2*log(e))*B*x^2 + (a*b^2*c*d
^2*i^2*log(e) - a^2*b*d^3*i^2*log(e))*B*x + ((b^3*c^2*d*i^2 - 2*a*b^2*c*d^
2*i^2 + a^2*b*d^3*i^2)*B*x + (a*b^2*c^2*d*i^2 - 2*a^2*b*c*d^2*i^2 + a^3*d^
3*i^2)*B)*log(b*x + a)^2 + (2*(i^2*log(e) + i^2)*a*b^2*c^2*d - 3*(i^2*log(
e) + i^2)*a^2*b*c*d^2 + (i^2*log(e) + i^2)*a^3*d^3)*B + ((b^3*c*d^2*i^2 -
a*b^2*d^3*i^2)*B*x^2 + (2*b^3*c^2*d*i^2*log(e) - 4*(i^2*log(e) - i^2)*a*b^
2*c*d^2 + (2*i^2*log(e) - 3*i^2)*a^2*b*d^3)*B*x - (4*a^2*b*c*d^2*i^2*log(e
) - 2*(i^2*log(e) + i^2)*a*b^2*c^2*d - (2*i^2*log(e) - i^2)*a^3*d^3)*B)*lo
g(b*x + a) - ((b^3*c*d^2*i^2 - a*b^2*d^3*i^2)*B*x^2 + (a*b^2*c*d^2*i^2 - a
^2*b*d^3*i^2)*B*x + (2*a*b^2*c^2*d*i^2 - 3*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*
B + 2*((b^3*c^2*d*i^2 - 2*a*b^2*c*d^2*i^2 + a^2*b*d^3*i^2)*B*x + (a*b^2*c^
2*d*i^2 - 2*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B)*log(b*x + a))*log(d*x + c))/
(a*b^4*c*g^2 - a^2*b^3*d*g^2 + (b^5*c*g^2 - a*b^4*d*g^2)*x) + 2*(b*c*d*i^2
 - a*d^2*i^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(...
```

### 3.15.8 Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algo
rithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
^2, x)
```

### 3.15.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2
,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2
, x)
```

# 3.16 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$



## 3.16.1 Optimal result

Integrand size = 40, antiderivative size = 230

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = -\frac{Bdi^2(c+dx)}{b^2g^3(a+bx)} - \frac{Bi^2(c+dx)^2}{4bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2} - \frac{d^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{Bd^2i^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}$$

output
```
-B*d*i^2*(d*x+c)/b^2/g^3/(b*x+a)-1/4*B*i^2*(d*x+c)^2/b/g^3/(b*x+a)^2-d*i^2
*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g^3/(b*x+a)-1/2*i^2*(d*x+c)^2*(A+
B*ln(e*(b*x+a)/(d*x+c)))/b/g^3/(b*x+a)^2-d^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)
))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^3+B*d^2*i^2*polylog(2,b*(d*x+c)/d/(b*x+
a))/b^3/g^3
```

## 3.16.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 244, normalized size of antiderivative = 1.06

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \frac{i^2\left(-\frac{B(bc-ad)^2}{(a+bx)^2}+\frac{6Bd(-bc+ad)}{a+bx}-6Bd^2\log(a+bx)-\frac{2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}+\frac{8d(-bc+ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^3,x]
```

output
```
(i^2*(-((B*(b*c - a*d)^2)/(a + b*x)^2) + (6*B*d*(-(b*c) + a*d))/(a + b*x)
- 6*B*d^2*Log[a + b*x] - (2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(a + b*x)^2 + (8*d*(-(b*c) + a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/(a + b*x) + 4*d^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) +
6*B*d^2*Log[c + d*x] - 2*B*d^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(4*b^
3*g^3)
```

## 3.16.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.175$, Rules used = {2962, 2780, 2741, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^3}\,dx$$

$$\Big\downarrow \ 2962$$

$$\frac{i^2\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^3}$$

$$\Big\downarrow \ 2780$$

$$\frac{i^2\left(\frac{\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^3}$$

$\Big\downarrow$ 2741

$$\frac{i^2\left(\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}}{b}\right)}{g^3}$$

$\Big\downarrow$ 2780

$$\frac{i^2\left(\frac{d\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}}{b}\right)}{g^3}$$

$\Big\downarrow$ 2741

$$\frac{i^2\left(\frac{d\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}}{b}\right)}{g^3}$$

$\Big\downarrow$ 2779

---

3.16. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$

$$\frac{i^2\left(\frac{d\left(\frac{d\left(\frac{B\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}}{b}-\frac{B(c+dx)}{a+bx}\right)}{b}+\frac{-\frac{(c+dx)^2}{\cdots}}{\cdots}\right)}{g^3}$$

$\big\downarrow$ 2838

$$\frac{i^2\left(\frac{d\left(\frac{d\left(\frac{B\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}}{b}-\frac{B(c+dx)}{a+bx}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+\cdots}{c+\cdots}\right.\right.}{2(a+bx)^2}}{\cdots}\right)}{g^3}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^3
,x]
```

output
```
(i^2*((-1/4*(B*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(2*(a + b*x)^2))/b + (d*((-((B*(c + d*x))/(a + b*x)) -
((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x))/b + (d*(-(((A
+ B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b)
 + (B*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b))/b))/b))/g^3
```

## 3.16. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$

### 3.16.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.16.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 521 vs. $2(226) = 452$.

Time = 1.49 (sec) , antiderivative size = 522, normalized size of antiderivative = 2.27

| method | result |
|---|---|
| parts | $\frac{i^2 A\left(\frac{d^2\ln(bx+a)}{b^3}-\frac{a^2d^2-2abcd+b^2c^2}{2b^3(bx+a)^2}+\frac{2d(ad-cb)}{b^3(bx+a)}\right)}{g^3} - \frac{i^2B(ad-cb)^3e^3\left(-\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^3b^2e^2}-\frac{d^6}{\cdots}\right.}{\cdots}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{i^2d^2e^2A\left(-\frac{1}{2be\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^3e^3}-\frac{d}{b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\cdots\right.}{b^3e^3}\cdots\right.}{(ad-cb)g^3}\cdots\right.}{\cdots}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{i^2d^2e^2A\left(-\frac{1}{2be\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^3e^3}-\frac{d}{b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\cdots\right.}{b^3e^3}\cdots\right.}{(ad-cb)g^3}\cdots\right.}{\cdots}$ |
| risch | Expression too large to display |

3.16. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x,method=_RETU
RNVERBOSE)
```

output
```
i^2*A/g^3*(d^2/b^3*ln(b*x+a)-1/2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^2
+2/b^3*d*(a*d-b*c)/(b*x+a))-i^2*B/g^3/d^4*(a*d-b*c)^3*e^3*(-1/(a*d-b*c)^3*
d^5/b^2/e^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-1/2/(a*d-b*c)^3*d^6/b^3/e^3*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2+1/(a*d-b*c)^3*d^7/b^3/e^3*(dilog(-((b*e/d+(a*d-b*
c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+
(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)-1/(a*d-b*c)^3*d^4/b/e*(-1/2/(b*e/d+(
a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))^2))
```

### 3.16.5 Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algo
rithm="fricas")
```

output
```
integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B
*c*d*i^2*x + B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)))/(b^3*g^3*x^3 + 3*a*b
^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

### 3.16.6 Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$
$$=\frac{i^2\left(\int \frac{Ac^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{Ad^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{Bc^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{2Acdx}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right.}{g^3}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

output
```
i**2*(Integral(A*c**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
+ Integral(A*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
 + Integral(B*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x
 + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*c*d*x/(a**3 + 3*a**2*b*x
+ 3*a*b**2*x**2 + b**3*x**3), x) + Integral(B*d**2*x**2*log(a*e/(c + d*x)
+ b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + I
ntegral(2*B*c*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x
+ 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

### 3.16.7 Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algo
rithm="maxima")
```

output
```
-1/2*B*d^2*i^2*((4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*log(b*x + a
))*log(d*x + c)/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) - 2*integrate(
1/2*(2*b^3*d*x^3*log(e) + 7*a^2*b*d*x + 3*a^3*d + 2*(b^3*c*log(e) + 2*a*b^
2*d)*x^2 + 2*(2*b^3*d*x^3 + 3*a^2*b*d*x + a^3*d + (b^3*c + 3*a*b^2*d)*x^2)
*log(b*x + a))/(b^6*d*g^3*x^4 + a^3*b^3*c*g^3 + (b^6*c*g^3 + 3*a*b^5*d*g^3
)*x^3 + 3*(a*b^5*c*g^3 + a^2*b^4*d*g^3)*x^2 + (3*a^2*b^4*c*g^3 + a^3*b^3*d
*g^3)*x), x)) - 1/2*B*c*d*i^2*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*
x + c))/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2
*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)
*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/(
(b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x +
 c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3)) + 1/2*A*d^2*i^2*((4*a*b*x
 + 3*a^2)/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^
3*g^3)) + 1/4*B*c^2*i^2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^
2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e
*x/(d*x + c) + a*e/(d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) +
2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d
*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - (2*b*x + a)*A*c*d*i^2
/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*A*c^2*i^2/(b^3*g^3*x^2
+ 2*a*b^2*g^3*x + a^2*b*g^3)
```

### 3.16.8 Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algo
rithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
^3, x)
```

### 3.16.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(a\,g+b\,g\,x)^3}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3
,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3
, x)
```

# 3.17 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$



## 3.17.1 Optimal result

Integrand size = 40, antiderivative size = 89

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{Bi^2(c+dx)^3}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)g^4(a+bx)^3}$$

output
```
-1/9*B*i^2*(d*x+c)^3/(-a*d+b*c)/g^4/(b*x+a)^3-1/3*i^2*(d*x+c)^3*(A+B*ln(e*
(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^4/(b*x+a)^3
```

## 3.17.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 315 vs. 2(89) = 178.

Time = 0.20 (sec) , antiderivative size = 315, normalized size of antiderivative = 3.54

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{i^2\left(3Ab^3c^3+b^3Bc^3-3a^3Ad^3-a^3Bd^3+9Ab^3c^2dx+3b^3Bc^2dx-9a^2Abd^3x-3a^2bBd^3x+9Ab^3cd^2x^2\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^4,x]
```

output
```
-1/9*(i^2*(3*A*b^3*c^3 + b^3*B*c^3 - 3*a^3*A*d^3 - a^3*B*d^3 + 9*A*b^3*c^2
*d*x + 3*b^3*B*c^2*d*x - 9*a^2*A*b*d^3*x - 3*a^2*b*B*d^3*x + 9*A*b^3*c*d^2
*x^2 + 3*b^3*B*c*d^2*x^2 - 9*a*A*b^2*d^3*x^2 - 3*a*b^2*B*d^3*x^2 + 3*B*d^3
*(a + b*x)^3*Log[a + b*x] + 3*B*(b*c - a*d)*(a^2*d^2 + a*b*d*(c + 3*d*x) +
 b^2*(c^2 + 3*c*d*x + 3*d^2*x^2))*Log[(e*(a + b*x))/(c + d*x)] - 3*a^3*B*d
^3*Log[c + d*x] - 9*a^2*b*B*d^3*x*Log[c + d*x] - 9*a*b^2*B*d^3*x^2*Log[c +
 d*x] - 3*b^3*B*d^3*x^3*Log[c + d*x]))/(b^3*(b*c - a*d)*g^4*(a + b*x)^3)
```

### 3.17.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2962, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^4} \, dx$$

$$\Big\downarrow \text{ 2962}$$

$$\frac{i^2 \int \frac{(c+dx)^4 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4 (bc-ad)}$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{i^2 \left(-\frac{(c+dx)^3 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3 (a+bx)^3}-\frac{B (c+dx)^3}{9 (a+bx)^3}\right)}{g^4 (bc-ad)}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^4
,x]
```

output
```
(i^2*(-1/9*(B*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Log[(e*(a + b
*x))/(c + d*x)]))/(3*(a + b*x)^3)))/((b*c - a*d)*g^4)
```

#### 3.17.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.17.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(85) = 170$.

Time = 1.01 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.08

| method | result |
|---|---|
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{i^2d^2e^2A}{3(ad-cb)^2g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{i^2d^2e^2B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\right)}{(ad-cb)^2g^4}\right)}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{i^2d^2e^2A}{3(ad-cb)^2g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{i^2d^2e^2B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\right)}{(ad-cb)^2g^4}\right)}{d^2}$ |
| parts | $\frac{i^2A\left(-\frac{a^2d^2-2abcd+b^2c^2}{3b^3(bx+a)^3}+\frac{d(ad-cb)}{b^3(bx+a)^2}-\frac{d^2}{b^3(bx+a)}\right)}{g^4}-\frac{i^2Be^3\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\right)}{g^4(ad-cb)}$ |
| norman | $\frac{\frac{Bcd^2i^2x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}+\frac{Bc^2di^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}-\frac{3Aacd\,i^2+3Abc^2i^2+Bacd\,i^2+Bbc^2i^2}{9gb^2}+\frac{\left(3i^2Ad^2+Bd^2i^2\right)x^3}{9ag}-\frac{\left(3Acd\,i^2+Bc\right.}{3gb}}{(bx+a)^3g^3}$ |
| parallelrisch | $-\frac{-9Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5cd^3i^2-9Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5c^2d^2i^2-3Bx^3\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5d^4i^2+9Ax^2ab^4d^4i^2-9Ax^2b^5cd^3i^2-}{}$ |
| risch | $-\frac{Bi^2\left(3d^2x^2b^2+3abd^2x+3b^2cdx+a^2d^2+abcd+b^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{3(bx+a)^3g^4b^3}-\frac{i^2\left(-3B\ln(-bx-a)b^3d^3x^3+3B\ln(dx+c)b^3d^3x\right.}{}$ |

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x,method=_RETU
RNVERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(-1/3*i^2*d^2*e^2/(a*d-b*c)^2/g^4*A/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^3+i^2*d^2*e^2/(a*d-b*c)^2/g^4*B*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3
))
```

### 3.17.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 271 vs. $2(85) = 170$.

Time = 0.34 (sec) , antiderivative size = 271, normalized size of antiderivative = 3.04

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = -\frac{3\left((3A+B)b^3cd^2-(3A+B)ab^2d^3\right)i^2x^2+3\left((3A+B)b^3c^2d-(3A+B)a^2bd^3\right)i^2x+\left((3A+B)b^3c^3\right.}{9\left((b^7c-ab^6d)g^4x^3+3\left(ab^6c-a^2b^5d\right)g^4x^2+3\left(a^2b\right.\right.}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algo
rithm="fricas")
```

output
```
-1/9*(3*((3*A + B)*b^3*c*d^2 - (3*A + B)*a*b^2*d^3)*i^2*x^2 + 3*((3*A + B)
*b^3*c^2*d - (3*A + B)*a^2*b*d^3)*i^2*x + ((3*A + B)*b^3*c^3 - (3*A + B)*a
^3*d^3)*i^2 + 3*(B*b^3*d^3*i^2*x^3 + 3*B*b^3*c*d^2*i^2*x^2 + 3*B*b^3*c^2*d
*i^2*x + B*b^3*c^3*i^2)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c - a*b^6*d)*g
^4*x^3 + 3*(a*b^6*c - a^2*b^5*d)*g^4*x^2 + 3*(a^2*b^5*c - a^3*b^4*d)*g^4*x
 + (a^3*b^4*c - a^4*b^3*d)*g^4)
```

## 3.17.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 614 vs. $2(76) = 152$.

Time = 10.03 (sec) , antiderivative size = 614, normalized size of antiderivative = 6.90

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$
$$= -\frac{Bd^3i^2\log\left(x+\frac{-\frac{Ba^2d^5i^2}{ad-bc}+\frac{2Babcd^4i^2}{ad-bc}+Bad^4i^2-\frac{Bb^2c^2d^3i^2}{ad-bc}+Bbcd^3i^2}{2Bbd^4i^2}\right)}{3b^3g^4\,(ad-bc)}$$
$$+\frac{Bd^3i^2\log\left(x+\frac{\frac{Ba^2d^5i^2}{ad-bc}-\frac{2Babcd^4i^2}{ad-bc}+Bad^4i^2+\frac{Bb^2c^2d^3i^2}{ad-bc}+Bbcd^3i^2}{2Bbd^4i^2}\right)}{3b^3g^4\,(ad-bc)}$$
$$+\frac{-3Aa^2d^2i^2-3Aabcdi^2-3Ab^2c^2i^2-Ba^2d^2i^2-Babcdi^2-Bb^2c^2i^2+x^2(-9Ab^2d^2i^2-3Bb^2d^2i^2)+x(-}{9a^3b^3g^4+27a^2b^4g^4x+27ab^5g^4x^2+9b^6g^4x^3}$$
$$+\frac{(-Ba^2d^2i^2-Babcdi^2-3Babd^2i^2x-Bb^2c^2i^2-3Bb^2cdi^2x-3Bb^2d^2i^2x^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{3a^3b^3g^4+9a^2b^4g^4x+9ab^5g^4x^2+3b^6g^4x^3}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

output
```
-B*d**3*i**2*log(x + (-B*a**2*d**5*i**2/(a*d - b*c) + 2*B*a*b*c*d**4*i**2/
(a*d - b*c) + B*a*d**4*i**2 - B*b**2*c**2*d**3*i**2/(a*d - b*c) + B*b*c*d*
*3*i**2)/(2*B*b*d**4*i**2))/(3*b**3*g**4*(a*d - b*c)) + B*d**3*i**2*log(x
+ (B*a**2*d**5*i**2/(a*d - b*c) - 2*B*a*b*c*d**4*i**2/(a*d - b*c) + B*a*d*
*4*i**2 + B*b**2*c**2*d**3*i**2/(a*d - b*c) + B*b*c*d**3*i**2)/(2*B*b*d**4
*i**2))/(3*b**3*g**4*(a*d - b*c)) + (-3*A*a**2*d**2*i**2 - 3*A*a*b*c*d*i**
2 - 3*A*b**2*c**2*i**2 - B*a**2*d**2*i**2 - B*a*b*c*d*i**2 - B*b**2*c**2*i
**2 + x**2*(-9*A*b**2*d**2*i**2 - 3*B*b**2*d**2*i**2) + x*(-9*A*a*b*d**2*i
**2 - 9*A*b**2*c*d*i**2 - 3*B*a*b*d**2*i**2 - 3*B*b**2*c*d*i**2))/(9*a**3*
b**3*g**4 + 27*a**2*b**4*g**4*x + 27*a*b**5*g**4*x**2 + 9*b**6*g**4*x**3)
+ (-B*a**2*d**2*i**2 - B*a*b*c*d*i**2 - 3*B*a*b*d**2*i**2*x - B*b**2*c**2*
i**2 - 3*B*b**2*c*d*i**2*x - 3*B*b**2*d**2*i**2*x**2)*log(e*(a + b*x)/(c +
 d*x))/(3*a**3*b**3*g**4 + 9*a**2*b**4*g**4*x + 9*a*b**5*g**4*x**2 + 3*b**
6*g**4*x**3)
```

### 3.17.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1515 vs. $2(85) = 170$.

Time = 0.25 (sec) , antiderivative size = 1515, normalized size of antiderivative = 17.02

$$\int \frac{(ci + dix)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag + bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algo
rithm="maxima")
```

output
```
-1/18*B*d^2*i^2*(6*(3*b^2*x^2 + 3*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e/(
d*x + c))/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4)
+ (11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c^2 - 3*a*b^3*c*d +
 a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a^3*b*d^2)*x)/((b^8
*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2 - 2*a^2*b^6*c*d +
 a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*b^4*d^2)*g^4*
x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*(3*b^2*c^2*d - 3*
a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*
d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(d*x
+ c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4)) - 1/
18*B*c*d*i^2*(6*(3*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^5*g^4*
x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) + (5*a*b^2*c^2 - 22
*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16
*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^
3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 -
 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b
^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*
d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x +
c)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/18
*B*c^2*i^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^...
```

### 3.17.8 Giac [A] (verification not implemented)

Time = 0.50 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.57

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{1}{9}\left(\frac{3\,(dx+c)^3Be^4i^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^3g^4}+\frac{(3\,Ae^4i^2+Be^4i^2)(dx+c)^3}{(bex+ae)^3g^4}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)}\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algo
rithm="giac")
```

output
```
-1/9*(3*(d*x + c)^3*B*e^4*i^2*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^
3*g^4) + (3*A*e^4*i^2 + B*e^4*i^2)*(d*x + c)^3/((b*e*x + a*e)^3*g^4))*(b*c
/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.17.9 Mupad [B] (verification not implemented)

Time = 2.73 (sec) , antiderivative size = 423, normalized size of antiderivative = 4.75

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{x^2\,(3\,A\,b^2\,d^2\,i^2+B\,b^2\,d^2\,i^2)+x\,(3\,A\,a\,b\,d^2\,i^2+B\,a\,b\,d^2\,i^2+3\,A\,b^2\,c\,d\,i^2+B\,b^2\,c\,d\,i^2)+A\,a^2\,d^2\,i^2+A\,b^2}{3\,a^3\,b^3\,g^4+9\,a^2\,b^4\,g^4\,x+9\,a\,b^5\,g^4\,x^2+3\,b^6\,g^4\,x^3}$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(a\left(\frac{B\,a\,d^2\,i^2}{3\,b^4\,g^4}+\frac{B\,c\,d\,i^2}{3\,b^3\,g^4}\right)+x\left(b\left(\frac{B\,a\,d^2\,i^2}{3\,b^4\,g^4}+\frac{B\,c\,d\,i^2}{3\,b^3\,g^4}\right)+\frac{2\,B\,a\,d^2\,i^2}{3\,b^3\,g^4}+\frac{2\,B\,c\,d\,i^2}{3\,b^2\,g^4}\right)+\frac{B\,c^2\,i^2}{3\,b^2\,g^4}+\frac{B\,d^2\,i^2\,x^2}{b^2\,g^4}\right)}{3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2}$$

$$-\frac{B\,d^3\,i^2\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{3\,b^3\,g^4\,(a\,d-b\,c)}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^4
,x)
```

output
```
- (x^2*(3*A*b^2*d^2*i^2 + B*b^2*d^2*i^2) + x*(3*A*a*b*d^2*i^2 + B*a*b*d^2*
i^2 + 3*A*b^2*c*d*i^2 + B*b^2*c*d*i^2) + A*a^2*d^2*i^2 + A*b^2*c^2*i^2 + (
B*a^2*d^2*i^2)/3 + (B*b^2*c^2*i^2)/3 + A*a*b*c*d*i^2 + (B*a*b*c*d*i^2)/3)/
(3*a^3*b^3*g^4 + 3*b^6*g^4*x^3 + 9*a^2*b^4*g^4*x + 9*a*b^5*g^4*x^2) - (log
((e*(a + b*x))/(c + d*x))*(a*((B*a*d^2*i^2)/(3*b^4*g^4) + (B*c*d*i^2)/(3*b
^3*g^4)) + x*(b*((B*a*d^2*i^2)/(3*b^4*g^4) + (B*c*d*i^2)/(3*b^3*g^4)) + (2
*B*a*d^2*i^2)/(3*b^3*g^4) + (2*B*c*d*i^2)/(3*b^2*g^4)) + (B*c^2*i^2)/(3*b^
2*g^4) + (B*d^2*i^2*x^2)/(b^2*g^4)))/(3*a^2*x + a^3/b + b^2*x^3 + 3*a*b*x^
2) - (B*d^3*i^2*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(3*b^3*g^4*
(a*d - b*c))
```

# 3.18 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$



## 3.18.1 Optimal result

Integrand size = 40, antiderivative size = 181

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \frac{Bdi^2(c+dx)^3}{9(bc-ad)^2g^5(a+bx)^3} - \frac{bBi^2(c+dx)^4}{16(bc-ad)^2g^5(a+bx)^4} + \frac{di^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^2g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2g^5(a+bx)^4}$$

output
```
1/9*B*d*i^2*(d*x+c)^3/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/16*b*B*i^2*(d*x+c)^4/(-
a*d+b*c)^2/g^5/(b*x+a)^4+1/3*d*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(
-a*d+b*c)^2/g^5/(b*x+a)^3-1/4*b*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/
(-a*d+b*c)^2/g^5/(b*x+a)^4
```

### 3.18.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 454 vs. $2(181) = 362$.

Time = 0.24 (sec) , antiderivative size = 454, normalized size of antiderivative = 2.51

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{i^2\left(36Ab^4c^4+9b^4Bc^4-48aAb^3c^3d-16ab^3Bc^3d+12a^4Ad^4+7a^4Bd^4+96Ab^4c^3dx+20b^4Bc^3dx-14\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^5,x]
```

output
```
-1/144*(i^2*(36*A*b^4*c^4 + 9*b^4*B*c^4 - 48*a*A*b^3*c^3*d - 16*a*b^3*B*c^
3*d + 12*a^4*A*d^4 + 7*a^4*B*d^4 + 96*A*b^4*c^3*d*x + 20*b^4*B*c^3*d*x - 1
44*a*A*b^3*c^2*d^2*x - 48*a*b^3*B*c^2*d^2*x + 48*a^3*A*b*d^4*x + 28*a^3*b*
B*d^4*x + 72*A*b^4*c^2*d^2*x^2 + 6*b^4*B*c^2*d^2*x^2 - 144*a*A*b^3*c*d^3*x
^2 - 48*a*b^3*B*c*d^3*x^2 + 72*a^2*A*b^2*d^4*x^2 + 42*a^2*b^2*B*d^4*x^2 -
12*b^4*B*c*d^3*x^3 + 12*a*b^3*B*d^4*x^3 - 12*B*d^4*(a + b*x)^4*Log[a + b*x
] + 12*B*(b*c - a*d)^2*(a^2*d^2 + 2*a*b*d*(c + 2*d*x) + b^2*(3*c^2 + 8*c*d
*x + 6*d^2*x^2))*Log[(e*(a + b*x))/(c + d*x)] + 12*a^4*B*d^4*Log[c + d*x]
+ 48*a^3*b*B*d^4*x*Log[c + d*x] + 72*a^2*b^2*B*d^4*x^2*Log[c + d*x] + 48*a
*b^3*B*d^4*x^3*Log[c + d*x] + 12*b^4*B*d^4*x^4*Log[c + d*x]))/(b^3*(b*c -
a*d)^2*g^5*(a + b*x)^4)
```

### 3.18.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i^2 \int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^2}$$

$$\Big\downarrow \text{2772}$$

$$\frac{i^2\left(-B\int -\frac{(c+dx)^5\left(3b-\frac{4d(a+bx)}{c+dx}\right)}{12(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}\right)}{g^5(bc-ad)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{i^2\left(\frac{1}{12}B\int \frac{(c+dx)^5\left(3b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}\right)}{g^5(bc-ad)^2}$$

$$\Big\downarrow \text{53}$$

$$\frac{i^2\left(\frac{1}{12}B\int \left(\frac{3b(c+dx)^5}{(a+bx)^5}-\frac{4d(c+dx)^4}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}\right)}{g^5(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i^2\left(-\frac{b(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{1}{12}B\left(\frac{4d(c+dx)^3}{3(a+bx)^3}-\frac{3b(c+dx)^4}{4(a+bx)^4}\right)\right)}{g^5(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^5
,x]
```

output
```
(i^2*((B*((4*d*(c + d*x)^3)/(3*(a + b*x)^3) - (3*b*(c + d*x)^4)/(4*(a + b*
x)^4)))/12 + (d*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(a +
b*x)^3) - (b*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(a + b*x
)^4)))/((b*c - a*d)^2*g^5)
```

## 3.18. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$

### 3.18.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.18. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$

### 3.18.4 Maple [A] (verified)

Time = 1.59 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.76

| method | result |
|---|---|
| parts | $\frac{i^2 A\left(\frac{2d(ad-cb)}{3b^3(bx+a)^3}-\frac{a^2d^2-2abcd+b^2c^2}{4b^3(bx+a)^4}-\frac{d^2}{2b^3(bx+a)^2}\right)}{g^5}-\frac{i^2B(ad-cb)^3e^3\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\right)}{(ad-cb)^5}\right.}{g^5d^4}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{i^2d^2e^3Ab}{4(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^2d^3e^2A}{3(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i^2d^2e^3Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(a}{d(}\right.}\right.}{(ad-cb)^3g^5}\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(\frac{i^2d^2e^3Ab}{4(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^2d^3e^2A}{3(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i^2d^2e^3Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(a}{d(}\right.}\right.}{(ad-cb)^3g^5}\right.}{d^2}$ |
| risch | $-\frac{i^2B(6d^2x^2b^2+4abd^2x+8b^2cdx+a^2d^2+2abcd+3b^2c^2)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{12(bx+a)^4g^5b^3}-\frac{(-48Bab^3cd^3x^2-48Bab^3c^2d^2x-48Aab^3c^3}{}$ |
| norman | $\frac{\frac{(12Aac^2di^2-12Abc^3i^2+4Bac^2di^2-3Bbc^3i^2)x}{12ga(ad-cb)}+\frac{(24Aa^2cd^2i^2+12Aabc^2di^2-36Ab^2c^3i^2+8Ba^2cd^2i^2+7Babc^2di^2-9Bb^2c^3i^2}{24ga^2(ad-cb)}}{}$ |
| parallelrisch | $\frac{12Ax^4a^6bcd^4i^2-48Ax^4a^3b^4c^4di^2+7Bx^4a^6bcd^4i^2-16Bx^4a^3b^4c^4di^2+48Bx^3\ln\left(\frac{e(bx+a)}{dx+c}\right)a^7cd^4i^2-192Ax^3a^4b^3c^4}{}$ |

input

```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x,method=_RETU
RNVERBOSE)
```

output

```
i^2*A/g^5*(2/3*d*(a*d-b*c)/b^3/(b*x+a)^3-1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b
^3/(b*x+a)^4-1/2*d^2/b^3/(b*x+a)^2)-i^2*B/g^5/d^4*(a*d-b*c)^3*e^3*(d^5/(a*
d-b*c)^5*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x
+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^4/(a*d-b*c)^5*b*e*(-1/4/(b*e/d
+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^4))
```

### 3.18.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 510 vs. $2(173) = 346$.

Time = 0.37 (sec) , antiderivative size = 510, normalized size of antiderivative = 2.82

$$\int \frac{(ci + dix)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag + bgx)^5}\,dx$$
$$= \frac{12\,(Bb^4cd^3 - Bab^3d^4)i^2x^3 - 6\,((12\,A + B)b^4c^2d^2 - 8\,(3\,A + B)ab^3cd^3 + (12\,A + 7\,B)a^2b^2d^4)i^2x^2 - 4\,((24}{144\,((b^9c^2}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algo
rithm="fricas")
```

output
```
1/144*(12*(B*b^4*c*d^3 - B*a*b^3*d^4)*i^2*x^3 - 6*((12*A + B)*b^4*c^2*d^2
- 8*(3*A + B)*a*b^3*c*d^3 + (12*A + 7*B)*a^2*b^2*d^4)*i^2*x^2 - 4*((24*A +
 5*B)*b^4*c^3*d - 12*(3*A + B)*a*b^3*c^2*d^2 + (12*A + 7*B)*a^3*b*d^4)*i^2
*x - (9*(4*A + B)*b^4*c^4 - 16*(3*A + B)*a*b^3*c^3*d + (12*A + 7*B)*a^4*d^
4)*i^2 + 12*(B*b^4*d^4*i^2*x^4 + 4*B*a*b^3*d^4*i^2*x^3 - 6*(B*b^4*c^2*d^2
- 2*B*a*b^3*c*d^3)*i^2*x^2 - 4*(2*B*b^4*c^3*d - 3*B*a*b^3*c^2*d^2)*i^2*x -
 (3*B*b^4*c^4 - 4*B*a*b^3*c^3*d)*i^2)*log((b*e*x + a*e)/(d*x + c)))/((b^9*
c^2 - 2*a*b^8*c*d + a^2*b^7*d^2)*g^5*x^4 + 4*(a*b^8*c^2 - 2*a^2*b^7*c*d +
a^3*b^6*d^2)*g^5*x^3 + 6*(a^2*b^7*c^2 - 2*a^3*b^6*c*d + a^4*b^5*d^2)*g^5*x
^2 + 4*(a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*g^5*x + (a^4*b^5*c^2 -
2*a^5*b^4*c*d + a^6*b^3*d^2)*g^5)
```

### 3.18.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 928 vs. $2(165) = 330$.

Time = 23.85 (sec) , antiderivative size = 928, normalized size of antiderivative = 5.13

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$$
$$= -\frac{Bd^4i^2\log\left(x+\frac{-\frac{Ba^3d^7i^2}{(ad-bc)^2}+\frac{3Ba^2bcd^6i^2}{(ad-bc)^2}-\frac{3Bab^2c^2d^5i^2}{(ad-bc)^2}+Bad^5i^2+\frac{Bb^3c^3d^4i^2}{(ad-bc)^2}+Bbcd^4i^2}{2Bbd^5i^2}\right)}{12b^3g^5\,(ad-bc)^2}$$
$$+\frac{Bd^4i^2\log\left(x+\frac{\frac{Ba^3d^7i^2}{(ad-bc)^2}-\frac{3Ba^2bcd^6i^2}{(ad-bc)^2}+\frac{3Bab^2c^2d^5i^2}{(ad-bc)^2}+Bad^5i^2-\frac{Bb^3c^3d^4i^2}{(ad-bc)^2}+Bbcd^4i^2}{2Bbd^5i^2}\right)}{12b^3g^5\,(ad-bc)^2}$$
$$+\frac{-12Aa^3d^3i^2-12Aa^2bcd^2i^2-12Aab^2c^2di^2+36Ab^3c^3i^2-7Ba^3d^3i^2-7Ba^2bcd^2i^2-7Bab^2c^2di^2+9Bb}{144a^5b^3dg^5-144a^4b^4cg^5+x^4\cdot(144ab^7dg}$$
$$+\frac{(-Ba^2d^2i^2-2Babcdi^2-4Babd^2i^2x-3Bb^2c^2i^2-8Bb^2cdi^2x-6Bb^2d^2i^2x^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{12a^4b^3g^5+48a^3b^4g^5x+72a^2b^5g^5x^2+48ab^6g^5x^3+12b^7g^5x^4}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

output
```
-B*d**4*i**2*log(x + (-B*a**3*d**7*i**2/(a*d - b*c)**2 + 3*B*a**2*b*c*d**6
*i**2/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**5*i**2/(a*d - b*c)**2 + B*a*d**5
*i**2 + B*b**3*c**3*d**4*i**2/(a*d - b*c)**2 + B*b*c*d**4*i**2)/(2*B*b*d**
5*i**2))/(12*b**3*g**5*(a*d - b*c)**2) + B*d**4*i**2*log(x + (B*a**3*d**7*
i**2/(a*d - b*c)**2 - 3*B*a**2*b*c*d**6*i**2/(a*d - b*c)**2 + 3*B*a*b**2*c
**2*d**5*i**2/(a*d - b*c)**2 + B*a*d**5*i**2 - B*b**3*c**3*d**4*i**2/(a*d
- b*c)**2 + B*b*c*d**4*i**2)/(2*B*b*d**5*i**2))/(12*b**3*g**5*(a*d - b*c)*
*2) + (-12*A*a**3*d**3*i**2 - 12*A*a**2*b*c*d**2*i**2 - 12*A*a*b**2*c**2*d
*i**2 + 36*A*b**3*c**3*i**2 - 7*B*a**3*d**3*i**2 - 7*B*a**2*b*c*d**2*i**2
- 7*B*a*b**2*c**2*d*i**2 + 9*B*b**3*c**3*i**2 - 12*B*b**3*d**3*i**2*x**3 +
 x**2*(-72*A*a*b**2*d**3*i**2 + 72*A*b**3*c*d**2*i**2 - 42*B*a*b**2*d**3*i
**2 + 6*B*b**3*c*d**2*i**2) + x*(-48*A*a**2*b*d**3*i**2 - 48*A*a*b**2*c*d*
*2*i**2 + 96*A*b**3*c**2*d*i**2 - 28*B*a**2*b*d**3*i**2 - 28*B*a*b**2*c*d*
*2*i**2 + 20*B*b**3*c**2*d*i**2))/(144*a**5*b**3*d*g**5 - 144*a**4*b**4*c*
g**5 + x**4*(144*a*b**7*d*g**5 - 144*b**8*c*g**5) + x**3*(576*a**2*b**6*d*
g**5 - 576*a*b**7*c*g**5) + x**2*(864*a**3*b**5*d*g**5 - 864*a**2*b**6*c*g
**5) + x*(576*a**4*b**4*d*g**5 - 576*a**3*b**5*c*g**5)) + (-B*a**2*d**2*i*
*2 - 2*B*a*b*c*d*i**2 - 4*B*a*b*d**2*i**2*x - 3*B*b**2*c**2*i**2 - 8*B*b**
2*c*d*i**2*x - 6*B*b**2*d**2*i**2*x**2)*log(e*(a + b*x)/(c + d*x))/(12*a**
4*b**3*g**5 + 48*a**3*b**4*g**5*x + 72*a**2*b**5*g**5*x**2 + 48*a*b**6*...
```

### 3.18.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2218 vs. $2(173) = 346$.

Time = 0.30 (sec) , antiderivative size = 2218, normalized size of antiderivative = 12.25

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algo
rithm="maxima")
```

output
```
-1/144*B*d^2*i^2*(12*(6*b^2*x^2 + 4*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e
/(d*x + c))/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3*b^4
*g^5*x + a^4*b^3*g^5) + (13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*b*c*d^
2 - 7*a^5*d^3 - 12*(6*b^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 6*(6*
b^5*c^3 - 46*a*b^4*c^2*d + 29*a^2*b^3*c*d^2 - 7*a^3*b^2*d^3)*x^2 + 4*(10*a
*b^4*c^3 - 63*a^2*b^3*c^2*d + 33*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*x)/((b^10*c^
3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*g^5*x^4 + 4*(a*b^9*c^3
- 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*g^5*x^3 + 6*(a^2*b^8*c^
3 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*g^5*x^2 + 4*(a^3*b^7*
c^3 - 3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*g^5*x + (a^4*b^6*c^
3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3)*g^5) - 12*(6*b^2*c^2*
d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(b*x + a)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^
2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5) + 12*(6*b^2*c^2*d^2 -
4*a*b*c*d^3 + a^2*d^4)*log(d*x + c)/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*
c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^5)) - 1/72*B*c*d*i^2*(12*(4*b*x
 + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^6*g^5*x^4 + 4*a*b^5*g^5*x^3
+ 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5) + (7*a*b^3*c^3 - 33*a
^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a^4*d^3 + 12*(4*b^4*c*d^2 - a*b^3*d^3)*
x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3
- 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2 - 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*...
```

### 3.18.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.56

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{144}\left(\frac{12\left(3\,Bbe^5i^2-\frac{4\,(bex+ae)Bde^4i^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^4bcg^5}{(dx+c)^4}-\frac{(bex+ae)^4adg^5}{(dx+c)^4}}+\frac{36\,Abe^5i^2+9\,Bbe^5i^2-\frac{48\,(bex+ae)Ade^4i^2}{dx+c}-\frac{16\,(bex+ae)}{dx+c}}{\frac{(bex+ae)^4bcg^5}{(dx+c)^4}-\frac{(bex+ae)^4adg^5}{(dx+c)^4}}\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algo
rithm="giac")
```

output
```
-1/144*(12*(3*B*b*e^5*i^2 - 4*(b*e*x + a*e)*B*d*e^4*i^2/(d*x + c))*log((b*
e*x + a*e)/(d*x + c))/((b*e*x + a*e)^4*b*c*g^5/(d*x + c)^4 - (b*e*x + a*e)
^4*a*d*g^5/(d*x + c)^4) + (36*A*b*e^5*i^2 + 9*B*b*e^5*i^2 - 48*(b*e*x + a*
e)*A*d*e^4*i^2/(d*x + c) - 16*(b*e*x + a*e)*B*d*e^4*i^2/(d*x + c))/((b*e*x
 + a*e)^4*b*c*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a*d*g^5/(d*x + c)^4))*(b*c
/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.18.9 Mupad [B] (verification not implemented)

Time = 3.58 (sec) , antiderivative size = 647, normalized size of antiderivative = 3.57

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{\frac{12\,A\,a^3\,d^3\,i^2-36\,A\,b^3\,c^3\,i^2+7\,B\,a^3\,d^3\,i^2-9\,B\,b^3\,c^3\,i^2+12\,A\,a\,b^2\,c^2\,d\,i^2+12\,A\,a^2\,b\,c\,d^2\,i^2+7\,B\,a\,b^2\,c^2\,d\,i^2+7\,B\,a^2\,b\,c\,d^2\,i^2}{12\,(a\,d-b\,c)}+\frac{x^2\,(12\,A\,a\,b^2\,d^3\,i^2}{}}{12\,a^4\,b^3\,g^5+48\,a^3\,b^4\,g^5\,x+}$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(a\,\left(\frac{B\,a\,d^2\,i^2}{12\,b^4\,g^5}+\frac{B\,c\,d\,i^2}{6\,b^3\,g^5}\right)+x\,\left(b\,\left(\frac{B\,a\,d^2\,i^2}{12\,b^4\,g^5}+\frac{B\,c\,d\,i^2}{6\,b^3\,g^5}\right)+\frac{B\,a\,d^2\,i^2}{4\,b^3\,g^5}+\frac{B\,c\,d\,i^2}{2\,b^2\,g^5}\right)+\frac{B\,c^2\,i^2}{4\,b^2\,g^5}+\frac{B\,d^2\,i^2\,x^2}{2\,b^2\,g^5}\right)}{4\,a^3\,x+\frac{a^4}{b}+b^3\,x^4+6\,a^2\,b\,x^2+4\,a\,b^2\,x^3}$$

$$-\frac{B\,d^4\,i^2\,\mathrm{atanh}\left(\frac{12\,b^5\,c^2\,g^5-12\,a^2\,b^3\,d^2\,g^5}{12\,b^3\,g^5\,{\left(a\,d-b\,c\right)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{6\,b^3\,g^5\,{\left(a\,d-b\,c\right)}^2}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^5
,x)
```

output
```
- ((12*A*a^3*d^3*i^2 - 36*A*b^3*c^3*i^2 + 7*B*a^3*d^3*i^2 - 9*B*b^3*c^3*i^
2 + 12*A*a*b^2*c^2*d*i^2 + 12*A*a^2*b*c*d^2*i^2 + 7*B*a*b^2*c^2*d*i^2 + 7*
B*a^2*b*c*d^2*i^2)/(12*(a*d - b*c)) + (x^2*(12*A*a*b^2*d^3*i^2 + 7*B*a*b^2
*d^3*i^2 - 12*A*b^3*c*d^2*i^2 - B*b^3*c*d^2*i^2))/(2*(a*d - b*c)) + (x*(12
*A*a^2*b*d^3*i^2 + 7*B*a^2*b*d^3*i^2 - 24*A*b^3*c^2*d*i^2 - 5*B*b^3*c^2*d*
i^2 + 12*A*a*b^2*c*d^2*i^2 + 7*B*a*b^2*c*d^2*i^2))/(3*(a*d - b*c)) + (B*b^
3*d^3*i^2*x^3)/(a*d - b*c))/(12*a^4*b^3*g^5 + 12*b^7*g^5*x^4 + 48*a^3*b^4*
g^5*x + 48*a*b^6*g^5*x^3 + 72*a^2*b^5*g^5*x^2) - (log((e*(a + b*x))/(c + d
*x))*(a*((B*a*d^2*i^2)/(12*b^4*g^5) + (B*c*d*i^2)/(6*b^3*g^5)) + x*(b*((B*
a*d^2*i^2)/(12*b^4*g^5) + (B*c*d*i^2)/(6*b^3*g^5)) + (B*a*d^2*i^2)/(4*b^3*
g^5) + (B*c*d*i^2)/(2*b^2*g^5)) + (B*c^2*i^2)/(4*b^2*g^5) + (B*d^2*i^2*x^2
)/(2*b^2*g^5)))/(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3) -
(B*d^4*i^2*atanh((12*b^5*c^2*g^5 - 12*a^2*b^3*d^2*g^5)/(12*b^3*g^5*(a*d -
b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(6*b^3*g^5*(a*d - b*c)^2)
```

# 3.19 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$



## 3.19.1 Optimal result

Integrand size = 40, antiderivative size = 281

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = -\frac{Bd^2i^2(c+dx)^3}{9(bc-ad)^3g^6(a+bx)^3} + \frac{bBdi^2(c+dx)^4}{8(bc-ad)^3g^6(a+bx)^4} - \frac{b^2Bi^2(c+dx)^5}{25(bc-ad)^3g^6(a+bx)^5} - \frac{d^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^3g^6(a+bx)^3} + \frac{bdi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^6(a+bx)^4} - \frac{b^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^6(a+bx)^5}$$

output
```
-1/9*B*d^2*i^2*(d*x+c)^3/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/8*b*B*d*i^2*(d*x+c)^
4/(-a*d+b*c)^3/g^6/(b*x+a)^4-1/25*b^2*B*i^2*(d*x+c)^5/(-a*d+b*c)^3/g^6/(b*
x+a)^5-1/3*d^2*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^6/
(b*x+a)^3+1/2*b*d*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g
^6/(b*x+a)^4-1/5*b^2*i^2*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^
3/g^6/(b*x+a)^5
```

## 3.19.2 Mathematica [A] (verified)

Time = 0.50 (sec) , antiderivative size = 344, normalized size of antiderivative = 1.22

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$$
$$= \frac{i^2\left(-\frac{360Ab^2c^2}{(a+bx)^5}-\frac{72b^2Bc^2}{(a+bx)^5}+\frac{720aAbcd}{(a+bx)^5}+\frac{144abBcd}{(a+bx)^5}-\frac{360a^2Ad^2}{(a+bx)^5}-\frac{72a^2Bd^2}{(a+bx)^5}-\frac{900Abcd}{(a+bx)^4}-\frac{135bBcd}{(a+bx)^4}+\frac{900aAd^2}{(a+bx)^4}+\frac{135aBd^2}{(a+bx)^4}-\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^6,x]
```

output
```
(i^2*((-360*A*b^2*c^2)/(a + b*x)^5 - (72*b^2*B*c^2)/(a + b*x)^5 + (720*a*A
*b*c*d)/(a + b*x)^5 + (144*a*b*B*c*d)/(a + b*x)^5 - (360*a^2*A*d^2)/(a + b
*x)^5 - (72*a^2*B*d^2)/(a + b*x)^5 - (900*A*b*c*d)/(a + b*x)^4 - (135*b*B*
c*d)/(a + b*x)^4 + (900*a*A*d^2)/(a + b*x)^4 + (135*a*B*d^2)/(a + b*x)^4 -
 (600*A*d^2)/(a + b*x)^3 - (20*B*d^2)/(a + b*x)^3 + (30*B*d^3)/((b*c - a*d
)*(a + b*x)^2) - (60*B*d^4)/((b*c - a*d)^2*(a + b*x)) - (60*B*d^5*Log[a +
b*x])/(b*c - a*d)^3 - (60*B*(a^2*d^2 + a*b*d*(3*c + 5*d*x) + b^2*(6*c^2 +
15*c*d*x + 10*d^2*x^2))*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^5 + (60*B*
d^5*Log[c + d*x])/(b*c - a*d)^3))/(1800*b^3*g^6)
```

## 3.19.3 Rubi [A] (verified)

Time = 0.39 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 27, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^6}\,dx$$
$$\Big\downarrow \; 2962$$
$$\frac{i^2\int \frac{(c+dx)^6\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^6}d\frac{a+bx}{c+dx}}{g^6(bc-ad)^3}$$

$\Big\downarrow$ 2772

$$\frac{i^2\left(-B\int -\frac{(c+dx)^6\left(6b^2-\frac{15d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{30(a+bx)^6}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{bd}{}\right.}{g^6(bc-ad)^3}$$

$\Big\downarrow$ 27

$$\frac{i^2\left(\frac{1}{30}B\int \frac{(c+dx)^6\left(6b^2-\frac{15d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^6}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{bd(}{}\right.}{g^6(bc-ad)^3}$$

$\Big\downarrow$ 1140

$$\frac{i^2\left(\frac{1}{30}B\int \left(\frac{6b^2(c+dx)^6}{(a+bx)^6}-\frac{15bd(c+dx)^5}{(a+bx)^5}+\frac{10d^2(c+dx)^4}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}\right.}{g^6(bc-ad)^3}$$

$\Big\downarrow$ 2009

$$\frac{i^2\left(-\frac{b^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{bd(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^4}+\frac{1}{30}B\left(-\frac{6b^2(c+dx)^5}{5(a+bx)^5}-\right.\right.}{g^6(bc-ad)^3}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^6
,x]
```

output
```
(i^2*((B*((-10*d^2*(c + d*x)^3)/(3*(a + b*x)^3) + (15*b*d*(c + d*x)^4)/(4*
(a + b*x)^4) - (6*b^2*(c + d*x)^5)/(5*(a + b*x)^5)))/30 - (d^2*(c + d*x)^3
*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(a + b*x)^3) + (b*d*(c + d*x)^4*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^4) - (b^2*(c + d*x)^5*(
A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*(a + b*x)^5)))/((b*c - a*d)^3*g^6)
```

## 3.19. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$

#### 3.19.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.19.4 Maple [A] (verified)

Time = 1.70 (sec) , antiderivative size = 430, normalized size of antiderivative = 1.53

| method | result |
|---|---|
| parts | $\frac{i^2 A\left(-\frac{d^2}{3b^3(bx+a)^3}+\frac{d(ad-cb)}{2b^3(bx+a)^4}-\frac{a^2d^2-2abcd+b^2c^2}{5b^3(bx+a)^5}\right)}{g^6} - \frac{i^2 B(ad-cb)^3 e^3 \left(\frac{d^6\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\right.}{(ad-cb)^6}\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{i^2d^2e^4Ab^2}{5(ad-cb)^4g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}+\frac{i^2d^3e^3Ab}{2(ad-cb)^4g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^2d^4e^2A}{3(ad-cb)^4g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{i^2d^2e^4Bb^2}{}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{i^2d^2e^4Ab^2}{5(ad-cb)^4g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}+\frac{i^2d^3e^3Ab}{2(ad-cb)^4g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^2d^4e^2A}{3(ad-cb)^4g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{i^2d^2e^4Bb^2}{}\right.}{}$ |
| risch | $-\frac{i^2B(10d^2x^2b^2+5ab\,d^2x+15b^2cdx+a^2d^2+3abcd+6b^2c^2)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{30(bx+a)^5g^6b^3} - \frac{(60B\ln(dx+c)a^5d^5-72B\,b^5c^5+60Ba\,b^4d^5}{}$ |
| parallelrisch | $-\frac{-1800A\,x^2a^2b^7c\,d^5i^2+1800A\,x^2a\,b^8c^2d^4i^2-600B\,x^2a^2b^7c\,d^5i^2-1800Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b^7c^2d^4i^2-300B\,x^4\ln\left(\frac{e(bx+}{dx+}\right.}{}$ |
| norman | Expression too large to display |

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x,method=_RETU
RNVERBOSE)
```

output
```
i^2*A/g^6*(-1/3*d^2/b^3/(b*x+a)^3+1/2*d*(a*d-b*c)/b^3/(b*x+a)^4-1/5*(a^2*d
^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^5)-i^2*B/g^6/d^4*(a*d-b*c)^3*e^3*(d^6/(a
*d-b*c)^6*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-2*d^5/(a*d-b*c)^6*b*e*(-1/4/(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(
a*d-b*c)*e/d/(d*x+c))^4)+d^4/(a*d-b*c)^6*e^2*b^2*(-1/5/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^5))
```

3.19. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$

### 3.19.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 807 vs. $2(269) = 538$.

Time = 0.37 (sec) , antiderivative size = 807, normalized size of antiderivative = 2.87

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = -\frac{60\,(Bb^5cd^4 - Bab^4d^5)i^2x^4 - 30\,(Bb^5c^2d^3 - 10\,Bab^4cd^4 + 9\,Ba^2b^3d^5)i^2x^3 + 10\,(2\,(30\,A+B)b^5c^3d^2 - 15}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algo
rithm="fricas")
```

output
```
-1/1800*(60*(B*b^5*c*d^4 - B*a*b^4*d^5)*i^2*x^4 - 30*(B*b^5*c^2*d^3 - 10*B
*a*b^4*c*d^4 + 9*B*a^2*b^3*d^5)*i^2*x^3 + 10*(2*(30*A + B)*b^5*c^3*d^2 - 1
5*(12*A + B)*a*b^4*c^2*d^3 + 60*(3*A + B)*a^2*b^3*c*d^4 - (60*A + 47*B)*a^
3*b^2*d^5)*i^2*x^2 + 5*(9*(20*A + 3*B)*b^5*c^4*d - 20*(24*A + 5*B)*a*b^4*c
^3*d^2 + 120*(3*A + B)*a^2*b^3*c^2*d^3 - (60*A + 47*B)*a^4*b*d^5)*i^2*x +
(72*(5*A + B)*b^5*c^5 - 225*(4*A + B)*a*b^4*c^4*d + 200*(3*A + B)*a^2*b^3*
c^3*d^2 - (60*A + 47*B)*a^5*d^5)*i^2 + 60*(B*b^5*d^5*i^2*x^5 + 5*B*a*b^4*d
^5*i^2*x^4 + 10*B*a^2*b^3*d^5*i^2*x^3 + 10*(B*b^5*c^3*d^2 - 3*B*a*b^4*c^2*
d^3 + 3*B*a^2*b^3*c*d^4)*i^2*x^2 + 5*(3*B*b^5*c^4*d - 8*B*a*b^4*c^3*d^2 +
6*B*a^2*b^3*c^2*d^3)*i^2*x + (6*B*b^5*c^5 - 15*B*a*b^4*c^4*d + 10*B*a^2*b^
3*c^3*d^2)*i^2)*log((b*e*x + a*e)/(d*x + c)))/((b^11*c^3 - 3*a*b^10*c^2*d
+ 3*a^2*b^9*c*d^2 - a^3*b^8*d^3)*g^6*x^5 + 5*(a*b^10*c^3 - 3*a^2*b^9*c^2*d
 + 3*a^3*b^8*c*d^2 - a^4*b^7*d^3)*g^6*x^4 + 10*(a^2*b^9*c^3 - 3*a^3*b^8*c^
2*d + 3*a^4*b^7*c*d^2 - a^5*b^6*d^3)*g^6*x^3 + 10*(a^3*b^8*c^3 - 3*a^4*b^7
*c^2*d + 3*a^5*b^6*c*d^2 - a^6*b^5*d^3)*g^6*x^2 + 5*(a^4*b^7*c^3 - 3*a^5*b
^6*c^2*d + 3*a^6*b^5*c*d^2 - a^7*b^4*d^3)*g^6*x + (a^5*b^6*c^3 - 3*a^6*b^5
*c^2*d + 3*a^7*b^4*c*d^2 - a^8*b^3*d^3)*g^6)
```

### 3.19.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1300 vs. $2(258) = 516$.

Time = 76.59 (sec) , antiderivative size = 1300, normalized size of antiderivative = 4.63

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$$
$$= -\frac{Bd^5i^2\log\left(x+\frac{-\frac{Ba^4d^9i^2}{(ad-bc)^3}+\frac{4Ba^3bcd^8i^2}{(ad-bc)^3}-\frac{6Ba^2b^2c^2d^7i^2}{(ad-bc)^3}+\frac{4Bab^3c^3d^6i^2}{(ad-bc)^3}+Bad^6i^2-\frac{Bb^4c^4d^5i^2}{(ad-bc)^3}+Bbcd^5i^2}{2Bbd^6i^2}\right)}{30b^3g^6(ad-bc)^3}$$
$$+\frac{Bd^5i^2\log\left(x+\frac{\frac{Ba^4d^9i^2}{(ad-bc)^3}-\frac{4Ba^3bcd^8i^2}{(ad-bc)^3}+\frac{6Ba^2b^2c^2d^7i^2}{(ad-bc)^3}-\frac{4Bab^3c^3d^6i^2}{(ad-bc)^3}+Bad^6i^2+\frac{Bb^4c^4d^5i^2}{(ad-bc)^3}+Bbcd^5i^2}{2Bbd^6i^2}\right)}{30b^3g^6(ad-bc)^3}$$
$$+\frac{-60Aa^4d^4i^2-60Aa^3bcd^3i^2-60Aa^2b^2c^2d^2i^2+540Aab^3c^3di^2-360Ab^4c^4i^2-47Ba^4d^4i^2-47Ba^3bcd^3i}{1800a^7b^3d^2g^6-3600a^6b^4cdg^6+1800a^5b^5}$$
$$+\frac{(-Ba^2d^2i^2-3Babcdi^2-5Babd^2i^2x-6Bb^2c^2i^2-15Bb^2cdi^2x-10Bb^2d^2i^2x^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{30a^5b^3g^6+150a^4b^4g^6x+300a^3b^5g^6x^2+300a^2b^6g^6x^3+150ab^7g^6x^4+30b^8g^6x^5}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**6,x)
```

output
```
-B*d**5*i**2*log(x + (-B*a**4*d**9*i**2/(a*d - b*c)**3 + 4*B*a**3*b*c*d**8
*i**2/(a*d - b*c)**3 - 6*B*a**2*b**2*c**2*d**7*i**2/(a*d - b*c)**3 + 4*B*a
*b**3*c**3*d**6*i**2/(a*d - b*c)**3 + B*a*d**6*i**2 - B*b**4*c**4*d**5*i**
2/(a*d - b*c)**3 + B*b*c*d**5*i**2)/(2*B*b*d**6*i**2))/(30*b**3*g**6*(a*d
 - b*c)**3) + B*d**5*i**2*log(x + (B*a**4*d**9*i**2/(a*d - b*c)**3 - 4*B*a*
*3*b*c*d**8*i**2/(a*d - b*c)**3 + 6*B*a**2*b**2*c**2*d**7*i**2/(a*d - b*c)
**3 - 4*B*a*b**3*c**3*d**6*i**2/(a*d - b*c)**3 + B*a*d**6*i**2 + B*b**4*c*
*4*d**5*i**2/(a*d - b*c)**3 + B*b*c*d**5*i**2)/(2*B*b*d**6*i**2))/(30*b**3
*g**6*(a*d - b*c)**3) + (-60*A*a**4*d**4*i**2 - 60*A*a**3*b*c*d**3*i**2 -
60*A*a**2*b**2*c**2*d**2*i**2 + 540*A*a*b**3*c**3*d*i**2 - 360*A*b**4*c**4
*i**2 - 47*B*a**4*d**4*i**2 - 47*B*a**3*b*c*d**3*i**2 - 47*B*a**2*b**2*c**
2*d**2*i**2 + 153*B*a*b**3*c**3*d*i**2 - 72*B*b**4*c**4*i**2 - 60*B*b**4*d
**4*i**2*x**4 + x**3*(-270*B*a*b**3*d**4*i**2 + 30*B*b**4*c*d**3*i**2) + x
**2*(-600*A*a**2*b**2*d**4*i**2 + 1200*A*a*b**3*c*d**3*i**2 - 600*A*b**4*c
**2*d**2*i**2 - 470*B*a**2*b**2*d**4*i**2 + 130*B*a*b**3*c*d**3*i**2 - 20*
B*b**4*c**2*d**2*i**2) + x*(-300*A*a**3*b*d**4*i**2 - 300*A*a**2*b**2*c*d*
*3*i**2 + 1500*A*a*b**3*c**2*d**2*i**2 - 900*A*b**4*c**3*d*i**2 - 235*B*a*
*3*b*d**4*i**2 - 235*B*a**2*b**2*c*d**3*i**2 + 365*B*a*b**3*c**2*d**2*i**2
 - 135*B*b**4*c**3*d*i**2))/(1800*a**7*b**3*d**2*g**6 - 3600*a**6*b**4*c*d
*g**6 + 1800*a**5*b**5*c**2*g**6 + x**5*(1800*a**2*b**8*d**2*g**6 - 360...
```

## 3.19. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$

### 3.19.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3029 vs. $2(269) = 538$.

Time = 0.35 (sec) , antiderivative size = 3029, normalized size of antiderivative = 10.78

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algo
rithm="maxima")
```

output
```
-1/1800*B*d^2*i^2*(60*(10*b^2*x^2 + 5*a*b*x + a^2)*log(b*e*x/(d*x + c) + a
*e/(d*x + c))/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 10*a^2*b^6*g^6*x^3 + 10*a^3
*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) + (47*a^2*b^4*c^4 - 278*a^3*
b^3*c^3*d + 822*a^4*b^2*c^2*d^2 - 278*a^5*b*c*d^3 + 47*a^6*d^4 + 60*(10*b^
6*c^2*d^2 - 5*a*b^5*c*d^3 + a^2*b^4*d^4)*x^4 - 30*(10*b^6*c^3*d - 95*a*b^5
*c^2*d^2 + 46*a^2*b^4*c*d^3 - 9*a^3*b^3*d^4)*x^3 + 10*(20*b^6*c^4 - 140*a*
b^5*c^3*d + 537*a^2*b^4*c^2*d^2 - 248*a^3*b^3*c*d^3 + 47*a^4*b^2*d^4)*x^2
+ 5*(35*a*b^5*c^4 - 218*a^2*b^4*c^3*d + 702*a^3*b^3*c^2*d^2 - 278*a^4*b^2*
c*d^3 + 47*a^5*b*d^4)*x)/((b^12*c^4 - 4*a*b^11*c^3*d + 6*a^2*b^10*c^2*d^2
- 4*a^3*b^9*c*d^3 + a^4*b^8*d^4)*g^6*x^5 + 5*(a*b^11*c^4 - 4*a^2*b^10*c^3*
d + 6*a^3*b^9*c^2*d^2 - 4*a^4*b^8*c*d^3 + a^5*b^7*d^4)*g^6*x^4 + 10*(a^2*b
^10*c^4 - 4*a^3*b^9*c^3*d + 6*a^4*b^8*c^2*d^2 - 4*a^5*b^7*c*d^3 + a^6*b^6*
d^4)*g^6*x^3 + 10*(a^3*b^9*c^4 - 4*a^4*b^8*c^3*d + 6*a^5*b^7*c^2*d^2 - 4*a
^6*b^6*c*d^3 + a^7*b^5*d^4)*g^6*x^2 + 5*(a^4*b^8*c^4 - 4*a^5*b^7*c^3*d + 6
*a^6*b^6*c^2*d^2 - 4*a^7*b^5*c*d^3 + a^8*b^4*d^4)*g^6*x + (a^5*b^7*c^4 - 4
*a^6*b^6*c^3*d + 6*a^7*b^5*c^2*d^2 - 4*a^8*b^4*c*d^3 + a^9*b^3*d^4)*g^6) +
 60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(b*x + a)/((b^8*c^5 - 5*a*
b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + 5*a^4*b^4*c*d^4 - a^
5*b^3*d^5)*g^6) - 60*(10*b^2*c^2*d^3 - 5*a*b*c*d^4 + a^2*d^5)*log(d*x + c)
/((b^8*c^5 - 5*a*b^7*c^4*d + 10*a^2*b^6*c^3*d^2 - 10*a^3*b^5*c^2*d^3 + ...
```

### 3.19.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.58

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = -\frac{1}{1800}\left(\frac{60\left(6\,Bb^2e^6i^2-\frac{15\,(bex+ae)Bbde^5i^2}{dx+c}+\frac{10\,(bex+ae)^2Bd^2e^4i^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^5b^2c^2g^6}{(dx+c)^5}-\frac{2\,(bex+ae)^5abcdg^6}{(dx+c)^5}+\frac{(bex+ae)^5a^2d^2g^6}{(dx+c)^5}}+\frac{360\,Ab^2e^6i^2+72\,Bb^2e^6i^2-$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algo
rithm="giac")
```

output
```
-1/1800*(60*(6*B*b^2*e^6*i^2 - 15*(b*e*x + a*e)*B*b*d*e^5*i^2/(d*x + c) +
10*(b*e*x + a*e)^2*B*d^2*e^4*i^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))
/((b*e*x + a*e)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*e*x + a*e)^5*a*b*c*d*g^6/
(d*x + c)^5 + (b*e*x + a*e)^5*a^2*d^2*g^6/(d*x + c)^5) + (360*A*b^2*e^6*i^
2 + 72*B*b^2*e^6*i^2 - 900*(b*e*x + a*e)*A*b*d*e^5*i^2/(d*x + c) - 225*(b*
e*x + a*e)*B*b*d*e^5*i^2/(d*x + c) + 600*(b*e*x + a*e)^2*A*d^2*e^4*i^2/(d*
x + c)^2 + 200*(b*e*x + a*e)^2*B*d^2*e^4*i^2/(d*x + c)^2)/((b*e*x + a*e)^5
*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*e*x + a*e)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*
e*x + a*e)^5*a^2*d^2*g^6/(d*x + c)^5))*(b*c/((b*c*e - a*d*e)*(b*c - a*d))
- a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.19.9 Mupad [B] (verification not implemented)

Time = 4.61 (sec) , antiderivative size = 941, normalized size of antiderivative = 3.35

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$$

$$= \frac{B\,d^5\,i^2\,\mathrm{atanh}\left(\frac{30\,a^3\,b^3\,d^3\,g^6-30\,a^2\,b^4\,c\,d^2\,g^6-30\,a\,b^5\,c^2\,d\,g^6+30\,b^6\,c^3\,g^6}{30\,b^3\,g^6\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{15\,b^3\,g^6\,(a\,d-b\,c)^3}$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(a\left(\frac{B\,a\,d^2\,i^2}{30\,b^4\,g^6}+\frac{B\,c\,d\,i^2}{10\,b^3\,g^6}\right)+x\left(b\left(\frac{B\,a\,d^2\,i^2}{30\,b^4\,g^6}+\frac{B\,c\,d\,i^2}{10\,b^3\,g^6}\right)+\frac{2\,B\,a\,d^2\,i^2}{15\,b^3\,g^6}+\frac{2\,B\,c\,d\,i^2}{5\,b^2\,g^6}\right)+\frac{B\,c^2\,i^2}{5\,b^2\,g^6}+\frac{B\,d^2\,i^2\,x^2}{3\,b^2\,g^6}\right)}{5\,a^4\,x+\frac{a^5}{b}+b^4\,x^5+10\,a^3\,b\,x^2+5\,a\,b^3\,x^4+10\,a^2\,b^2\,x^3}$$

$$-\frac{\frac{60\,A\,a^4\,d^4\,i^2+360\,A\,b^4\,c^4\,i^2+47\,B\,a^4\,d^4\,i^2+72\,B\,b^4\,c^4\,i^2+60\,A\,a^2\,b^2\,c^2\,d^2\,i^2+47\,B\,a^2\,b^2\,c^2\,d^2\,i^2-540\,A\,a\,b^3\,c^3\,d\,i^2+60\,A\,a^3\,b\,c\,d^3\,i^2-153\,B\,a\,b^3}{60\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}}{}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^6
,x)
```

output
```
(B*d^5*i^2*atanh((30*b^6*c^3*g^6 + 30*a^3*b^3*d^3*g^6 - 30*a*b^5*c^2*d*g^6
 - 30*a^2*b^4*c*d^2*g^6)/(30*b^3*g^6*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(15*b^3*g^6*(a*d - b*c)^3) - (log((e
*(a + b*x))/(c + d*x))*(a*((B*a*d^2*i^2)/(30*b^4*g^6) + (B*c*d*i^2)/(10*b^
3*g^6)) + x*(b*((B*a*d^2*i^2)/(30*b^4*g^6) + (B*c*d*i^2)/(10*b^3*g^6)) + (
2*B*a*d^2*i^2)/(15*b^3*g^6) + (2*B*c*d*i^2)/(5*b^2*g^6)) + (B*c^2*i^2)/(5*
b^2*g^6) + (B*d^2*i^2*x^2)/(3*b^2*g^6)))/(5*a^4*x + a^5/b + b^4*x^5 + 10*a
^3*b*x^2 + 5*a*b^3*x^4 + 10*a^2*b^2*x^3) - ((60*A*a^4*d^4*i^2 + 360*A*b^4*
c^4*i^2 + 47*B*a^4*d^4*i^2 + 72*B*b^4*c^4*i^2 + 60*A*a^2*b^2*c^2*d^2*i^2 +
 47*B*a^2*b^2*c^2*d^2*i^2 - 540*A*a*b^3*c^3*d*i^2 + 60*A*a^3*b*c*d^3*i^2 -
 153*B*a*b^3*c^3*d*i^2 + 47*B*a^3*b*c*d^3*i^2)/(60*(a^2*d^2 + b^2*c^2 - 2*
a*b*c*d)) + (x^2*(60*A*a^2*b^2*d^4*i^2 + 47*B*a^2*b^2*d^4*i^2 + 60*A*b^4*c
^2*d^2*i^2 + 2*B*b^4*c^2*d^2*i^2 - 120*A*a*b^3*c*d^3*i^2 - 13*B*a*b^3*c*d^
3*i^2))/(6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(60*A*a^3*b*d^4*i^2 + 47*
B*a^3*b*d^4*i^2 + 180*A*b^4*c^3*d*i^2 + 27*B*b^4*c^3*d*i^2 - 300*A*a*b^3*c
^2*d^2*i^2 + 60*A*a^2*b^2*c*d^3*i^2 - 73*B*a*b^3*c^2*d^2*i^2 + 47*B*a^2*b^
2*c*d^3*i^2))/(12*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^3*(9*B*a*b^3*d^3
*i^2 - B*b^4*c*d^2*i^2))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*b^4*d^4*
i^2*x^4)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(30*a^5*b^3*g^6 + 30*b^8*g^6*x^5
 + 150*a^4*b^4*g^6*x + 150*a*b^7*g^6*x^4 + 300*a^3*b^5*g^6*x^2 + 300*a^...
```

## 3.19. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$

# 3.20 $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$



## 3.20.1 Optimal result

Integrand size = 40, antiderivative size = 457

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\
&= \frac{B(bc-ad)^6g^3i^3x}{140b^3d^3}+\frac{B(bc-ad)^5g^3i^3(c+dx)^2}{280b^2d^4}+\frac{B(bc-ad)^4g^3i^3(c+dx)^3}{420bd^4} \\
&\quad -\frac{17B(bc-ad)^3g^3i^3(c+dx)^4}{280d^4}+\frac{bB(bc-ad)^2g^3i^3(c+dx)^5}{14d^4}-\frac{b^2B(bc-ad)g^3i^3(c+dx)^6}{42d^4} \\
&\quad +\frac{B(bc-ad)^7g^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{140b^4d^4}-\frac{(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^4} \\
&\quad +\frac{3b(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^4} \\
&\quad -\frac{b^2(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^4} \\
&\quad +\frac{b^3g^3i^3(c+dx)^7\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{7d^4}+\frac{B(bc-ad)^7g^3i^3\log(c+dx)}{140b^4d^4}
\end{aligned}$$

output
```
1/140*B*(-a*d+b*c)^6*g^3*i^3*x/b^3/d^3+1/280*B*(-a*d+b*c)^5*g^3*i^3*(d*x+c
)^2/b^2/d^4+1/420*B*(-a*d+b*c)^4*g^3*i^3*(d*x+c)^3/b/d^4-17/280*B*(-a*d+b*
c)^3*g^3*i^3*(d*x+c)^4/d^4+1/14*b*B*(-a*d+b*c)^2*g^3*i^3*(d*x+c)^5/d^4-1/4
2*b^2*B*(-a*d+b*c)*g^3*i^3*(d*x+c)^6/d^4+1/140*B*(-a*d+b*c)^7*g^3*i^3*ln((
b*x+a)/(d*x+c))/b^4/d^4-1/4*(-a*d+b*c)^3*g^3*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+
a)/(d*x+c)))/d^4+3/5*b*(-a*d+b*c)^2*g^3*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d
*x+c)))/d^4-1/2*b^2*(-a*d+b*c)*g^3*i^3*(d*x+c)^6*(A+B*ln(e*(b*x+a)/(d*x+c)
))/d^4+1/7*b^3*g^3*i^3*(d*x+c)^7*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4+1/140*B*(
-a*d+b*c)^7*g^3*i^3*ln(d*x+c)/b^4/d^4
```

## 3.20.2 Mathematica [A] (verified)

Time = 0.41 (sec) , antiderivative size = 586, normalized size of antiderivative = 1.28

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{g^3 i^3\left(\frac{120b^2Bc(bc-ad)^5x}{d^3}-\frac{126bB(bc-ad)^6x}{d^3}+\frac{120abB(-bc+ad)^5x}{d^2}-\frac{60bBc(bc-ad)^4(a+bx)^2}{d^2}+\frac{60aB(bc-ad)^4(a+bx)^2}{d}+\frac{63B(bc-a\cdots}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)]),x]
```

output
```
(g^3*i^3*((120*b^2*B*c*(b*c - a*d)^5*x)/d^3 - (126*b*B*(b*c - a*d)^6*x)/d^
3 + (120*a*b*B*(-(b*c) + a*d)^5*x)/d^2 - (60*b*B*c*(b*c - a*d)^4*(a + b*x)
^2)/d^2 + (60*a*B*(b*c - a*d)^4*(a + b*x)^2)/d + (63*B*(b*c - a*d)^5*(a +
b*x)^2)/d^2 + (40*b*B*c*(b*c - a*d)^3*(a + b*x)^3)/d - (42*B*(b*c - a*d)^4
*(a + b*x)^3)/d + 40*a*B*(-(b*c) + a*d)^3*(a + b*x)^3 - 30*b*B*c*(b*c - a*
d)^2*(a + b*x)^4 + 30*a*B*d*(b*c - a*d)^2*(a + b*x)^4 + 21*B*(-(b*c) + a*d
)^3*(a + b*x)^4 + 24*b*B*c*d*(b*c - a*d)*(a + b*x)^5 - 84*B*d*(b*c - a*d)^
2*(a + b*x)^5 + 24*a*B*d^2*(-(b*c) + a*d)*(a + b*x)^5 - 20*b*B*c*d^2*(a +
b*x)^6 + 20*a*B*d^3*(a + b*x)^6 + 210*(b*c - a*d)^3*(a + b*x)^4*(A + B*Log
[(e*(a + b*x))/(c + d*x)]) + 504*d*(b*c - a*d)^2*(a + b*x)^5*(A + B*Log[(e
*(a + b*x))/(c + d*x)]) + 420*d^2*(b*c - a*d)*(a + b*x)^6*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]) + 120*d^3*(a + b*x)^7*(A + B*Log[(e*(a + b*x))/(c + d
*x)]) - (120*b*B*c*(b*c - a*d)^6*Log[c + d*x])/d^4 + (120*a*B*(b*c - a*d)^
6*Log[c + d*x])/d^3 + (126*B*(b*c - a*d)^7*Log[c + d*x])/d^4))/(840*b^4)
```

### 3.20.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2962, 2782, 27, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2962}$$

$$g^3i^3(bc-ad)^7\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^8}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2782}$$

$$g^3i^3(bc-ad)^7\left(-B\int -\frac{(c+dx)\left(b^3-\frac{7d(a+bx)b^2}{c+dx}+\frac{21d^2(a+bx)^2b}{(c+dx)^2}-\frac{35d^3(a+bx)^3}{(c+dx)^3}\right)}{140d^4(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}+\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{7d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{b^2(\ldots}{2\ldots}\right.$$

$$\Big\downarrow \text{27}$$

$$g^3i^3(bc-ad)^7\left(\frac{B\int \frac{(c+dx)\left(b^3-\frac{7d(a+bx)b^2}{c+dx}+\frac{21d^2(a+bx)^2b}{(c+dx)^2}-\frac{35d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}}{140d^4}+\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{7d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\ldots\right.}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}\right.$$

$$\Big\downarrow \text{2123}$$

$$g^3i^3(bc-ad)^7\left(\frac{B\int\left(-\frac{20db^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^7}+\frac{50db}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{34d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4b}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3b^2}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2b^3}+\frac{c+d\ldots}{(a+bx)\ldots}\right.}{140d^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$g^3i^3(bc-ad)^7\left(\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{7d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}+\frac{3b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right.$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x
]
```

output
```
(b*c - a*d)^7*g^3*i^3*((b^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(7*d^4*(
b - (d*(a + b*x))/(c + d*x))^7) - (b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^6) + (3*b*(A + B*Log[(e*(a + b*x))
/(c + d*x)]))/(5*d^4*(b - (d*(a + b*x))/(c + d*x))^5) - (A + B*Log[(e*(a +
 b*x))/(c + d*x)])/(4*d^4*(b - (d*(a + b*x))/(c + d*x))^4) + (B*((-10*b^2)
/(3*(b - (d*(a + b*x))/(c + d*x))^6) + (10*b)/(b - (d*(a + b*x))/(c + d*x)
)^5 - 17/(2*(b - (d*(a + b*x))/(c + d*x))^4) + 1/(3*b*(b - (d*(a + b*x))/(
c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^3*(b - (d*
(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^4 - Log[b - (d*(a + b*
x))/(c + d*x)]/b^4))/(140*d^4))
```

### 3.20.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.20.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1193 vs. $2(433) = 866$.

Time = 1.44 (sec) , antiderivative size = 1194, normalized size of antiderivative = 2.61

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1194 |
| parallelrisch | Expression too large to display | 1958 |
| parts | Expression too large to display | 2513 |
| derivativedivides | Expression too large to display | 2536 |
| default | Expression too large to display | 2536 |

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETU
RNVERBOSE)
```

output
```
3/10*i^3*g^3*d*B*a^3*c^2*x^2-1/40*i^3*g^3/d*b^2*B*a*c^4*x^2+i^3*g^3*A*a^3*
c^3*x+i^3*g^3*d^2*A*a^3*c*x^3+3*i^3*g^3*d*b*A*a^2*c^2*x^3+1/2*i^3*g^3*d^3*
b^2*A*a*x^6+1/2*i^3*g^3*d^2*b^3*A*c*x^6+1/42*i^3*g^3*d^3*b^2*B*a*x^6-1/42*
i^3*g^3*d^2*b^3*B*c*x^6+3/5*i^3*g^3*d^3*b*A*a^2*x^5+3/5*i^3*g^3*d*b^3*A*c^
2*x^5+1/14*i^3*g^3*d^3*b*B*a^2*x^5+7/30*i^3*g^3*d^2*B*a^3*c*x^3-1/14*i^3*g
^3*d*b^3*B*c^2*x^5+1/4*i^3*g^3*d^3*A*a^3*x^4+1/4*i^3*g^3*b^3*A*c^3*x^4+17/
280*i^3*g^3*d^3*B*a^3*x^4-17/280*i^3*g^3*b^3*B*c^3*x^4+9/5*i^3*g^3*d^2*b^2
*A*a*c*x^5+9/4*i^3*g^3*d^2*b*A*a^2*c*x^4+9/4*i^3*g^3*d*b^2*A*a*c^2*x^4+7/4
0*i^3*g^3*d^2*b*B*a^2*c*x^4-7/40*i^3*g^3*d*b^2*B*a*c^2*x^4+i^3*g^3*b^2*A*a
*c^3*x^3+1/420*i^3*g^3*d^3/b*B*a^4*x^3-1/420*i^3*g^3/d*b^3*B*c^4*x^3-1/280
*i^3*g^3*d^3/b^2*B*a^5*x^2+1/280*i^3*g^3/d^2*b^3*B*c^5*x^2+1/140*i^3*g^3*d
^3/b^3*B*a^6*x-1/140*i^3*g^3/d^3*b^3*B*c^6*x+1/140*i^3*g^3/d^4*b^3*B*ln(-d
*x-c)*c^7-1/140*i^3*g^3*d^3/b^4*B*ln(b*x+a)*a^7-1/4*i^3*g^3/d*B*ln(-d*x-c)
*a^3*c^4+1/4*i^3*g^3/b*B*ln(b*x+a)*a^4*c^3-3/10*i^3*g^3*b*B*a^2*c^3*x^2-7/
30*i^3*g^3*b^2*B*a*c^3*x^3+3/2*i^3*g^3*d*A*a^3*c^2*x^2+3/2*i^3*g^3*b*A*a^2
*c^3*x^2+1/40*i^3*g^3*d^2/b*B*a^4*c*x^2-1/20*i^3*g^3*d^2/b^2*B*a^5*c*x+3/2
0*i^3*g^3*d/b*B*a^4*c^2*x-3/20*i^3*g^3/d*b*B*a^2*c^4*x+1/20*i^3*g^3/d^2*b^
2*B*a*c^5*x+3/20*i^3*g^3/d^2*b*B*ln(-d*x-c)*a^2*c^5+1/20*i^3*g^3*d^2/b^3*B
*ln(b*x+a)*a^6*c-3/20*i^3*g^3*d/b^2*B*ln(b*x+a)*a^5*c^2-1/20*i^3*g^3/d^3*b
^2*B*ln(-d*x-c)*a*c^6+1/140*i^3*g^3*B*x*(20*b^3*d^3*x^6+70*a*b^2*d^3*x^...
```

### 3.20.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 912 vs. $2(433) = 866$.

Time = 0.59 (sec) , antiderivative size = 912, normalized size of antiderivative = 2.00

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{120\,Ab^7d^7g^3i^3x^7+20\,((21\,A-B)b^7cd^6+(21\,A+B)ab^6d^7)g^3i^3x^6+12\,((42\,A-5\,B)b^7c^2d^5+126\,Aab^6cd\ldots}{}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="fricas")
```

output
```
1/840*(120*A*b^7*d^7*g^3*i^3*x^7 + 20*((21*A - B)*b^7*c*d^6 + (21*A + B)*a
*b^6*d^7)*g^3*i^3*x^6 + 12*((42*A - 5*B)*b^7*c^2*d^5 + 126*A*a*b^6*c*d^6 +
 (42*A + 5*B)*a^2*b^5*d^7)*g^3*i^3*x^5 + 3*((70*A - 17*B)*b^7*c^3*d^4 + 7*
(90*A - 7*B)*a*b^6*c^2*d^5 + 7*(90*A + 7*B)*a^2*b^5*c*d^6 + (70*A + 17*B)*
a^3*b^4*d^7)*g^3*i^3*x^4 - 2*(B*b^7*c^4*d^3 - 14*(30*A - 7*B)*a*b^6*c^3*d^
4 - 1260*A*a^2*b^5*c^2*d^5 - 14*(30*A + 7*B)*a^3*b^4*c*d^6 - B*a^4*b^3*d^7
)*g^3*i^3*x^3 + 3*(B*b^7*c^5*d^2 - 7*B*a*b^6*c^4*d^3 + 84*(5*A - B)*a^2*b^
5*c^3*d^4 + 84*(5*A + B)*a^3*b^4*c^2*d^5 + 7*B*a^4*b^3*c*d^6 - B*a^5*b^2*d
^7)*g^3*i^3*x^2 - 6*(B*b^7*c^6*d - 7*B*a*b^6*c^5*d^2 + 21*B*a^2*b^5*c^4*d^
3 - 140*A*a^3*b^4*c^3*d^4 - 21*B*a^4*b^3*c^2*d^5 + 7*B*a^5*b^2*c*d^6 - B*a
^6*b*d^7)*g^3*i^3*x + 6*(35*B*a^4*b^3*c^3*d^4 - 21*B*a^5*b^2*c^2*d^5 + 7*B
*a^6*b*c*d^6 - B*a^7*d^7)*g^3*i^3*log(b*x + a) + 6*(B*b^7*c^7 - 7*B*a*b^6*
c^6*d + 21*B*a^2*b^5*c^5*d^2 - 35*B*a^3*b^4*c^4*d^3)*g^3*i^3*log(d*x + c)
+ 6*(20*B*b^7*d^7*g^3*i^3*x^7 + 140*B*a^3*b^4*c^3*d^4*g^3*i^3*x + 70*(B*b^
7*c*d^6 + B*a*b^6*d^7)*g^3*i^3*x^6 + 84*(B*b^7*c^2*d^5 + 3*B*a*b^6*c*d^6 +
 B*a^2*b^5*d^7)*g^3*i^3*x^5 + 35*(B*b^7*c^3*d^4 + 9*B*a*b^6*c^2*d^5 + 9*B*
a^2*b^5*c*d^6 + B*a^3*b^4*d^7)*g^3*i^3*x^4 + 140*(B*a*b^6*c^3*d^4 + 3*B*a^
2*b^5*c^2*d^5 + B*a^3*b^4*c*d^6)*g^3*i^3*x^3 + 210*(B*a^2*b^5*c^3*d^4 + B*
a^3*b^4*c^2*d^5)*g^3*i^3*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d^4)
```

### 3.20.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2161 vs. $2(427) = 854$.

Time = 16.36 (sec) , antiderivative size = 2161, normalized size of antiderivative = 4.73

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b**3*d**3*g**3*i**3*x**7/7 - B*a**4*g**3*i**3*(a**3*d**3 - 7*a**2*b*c*d*
*2 + 21*a*b**2*c**2*d - 35*b**3*c**3)*log(x + (B*a**7*c*d**6*g**3*i**3 - 7
*B*a**6*b*c**2*d**5*g**3*i**3 + 21*B*a**5*b**2*c**3*d**4*g**3*i**3 + B*a**
5*d**4*g**3*i**3*(a**3*d**3 - 7*a**2*b*c*d**2 + 21*a*b**2*c**2*d - 35*b**3
*c**3)/b - 70*B*a**4*b**3*c**4*d**3*g**3*i**3 - B*a**4*c*d**3*g**3*i**3*(a
**3*d**3 - 7*a**2*b*c*d**2 + 21*a*b**2*c**2*d - 35*b**3*c**3) + 21*B*a**3*
b**4*c**5*d**2*g**3*i**3 - 7*B*a**2*b**5*c**6*d*g**3*i**3 + B*a*b**6*c**7*
g**3*i**3)/(B*a**7*d**7*g**3*i**3 - 7*B*a**6*b*c*d**6*g**3*i**3 + 21*B*a**
5*b**2*c**2*d**5*g**3*i**3 - 35*B*a**4*b**3*c**3*d**4*g**3*i**3 - 35*B*a**
3*b**4*c**4*d**3*g**3*i**3 + 21*B*a**2*b**5*c**5*d**2*g**3*i**3 - 7*B*a*b*
*6*c**6*d*g**3*i**3 + B*b**7*c**7*g**3*i**3))/(140*b**4) - B*c**4*g**3*i**
3*(35*a**3*d**3 - 21*a**2*b*c*d**2 + 7*a*b**2*c**2*d - b**3*c**3)*log(x +
(B*a**7*c*d**6*g**3*i**3 - 7*B*a**6*b*c**2*d**5*g**3*i**3 + 21*B*a**5*b**2
*c**3*d**4*g**3*i**3 - 70*B*a**4*b**3*c**4*d**3*g**3*i**3 + 21*B*a**3*b**4
*c**5*d**2*g**3*i**3 - 7*B*a**2*b**5*c**6*d*g**3*i**3 + B*a*b**6*c**7*g**3
*i**3 + B*a*b**3*c**4*g**3*i**3*(35*a**3*d**3 - 21*a**2*b*c*d**2 + 7*a*b**
2*c**2*d - b**3*c**3) - B*b**4*c**5*g**3*i**3*(35*a**3*d**3 - 21*a**2*b*c*
d**2 + 7*a*b**2*c**2*d - b**3*c**3)/d)/(B*a**7*d**7*g**3*i**3 - 7*B*a**6*b
*c*d**6*g**3*i**3 + 21*B*a**5*b**2*c**2*d**5*g**3*i**3 - 35*B*a**4*b**3*c*
*3*d**4*g**3*i**3 - 35*B*a**3*b**4*c**4*d**3*g**3*i**3 + 21*B*a**2*b**5...
```

### 3.20.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2637 vs. 2(433) = 866.

Time = 0.26 (sec) , antiderivative size = 2637, normalized size of antiderivative = 5.77

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="maxima")
```

output
```
1/7*A*b^3*d^3*g^3*i^3*x^7 + 1/2*A*b^3*c*d^2*g^3*i^3*x^6 + 1/2*A*a*b^2*d^3*
g^3*i^3*x^6 + 3/5*A*b^3*c^2*d*g^3*i^3*x^5 + 9/5*A*a*b^2*c*d^2*g^3*i^3*x^5
+ 3/5*A*a^2*b*d^3*g^3*i^3*x^5 + 1/4*A*b^3*c^3*g^3*i^3*x^4 + 9/4*A*a*b^2*c^
2*d*g^3*i^3*x^4 + 9/4*A*a^2*b*c*d^2*g^3*i^3*x^4 + 1/4*A*a^3*d^3*g^3*i^3*x^
4 + A*a*b^2*c^3*g^3*i^3*x^3 + 3*A*a^2*b*c^2*d*g^3*i^3*x^3 + A*a^3*c*d^2*g^
3*i^3*x^3 + 3/2*A*a^2*b*c^3*g^3*i^3*x^2 + 3/2*A*a^3*c^2*d*g^3*i^3*x^2 + (x
*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/
d)*B*a^3*c^3*g^3*i^3 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2
*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a^2*b*c^
3*g^3*i^3 + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*
x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^
2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b^2*c^3*g^3*i^3 + 1/24*(6*x^4*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4
- (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*
c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^3*c^3*g^3*i^3 + 3/2*(x^2*log(b*e*x/(d*x +
 c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c
- a*d)*x/(b*d))*B*a^3*c^2*d*g^3*i^3 + 3/2*(2*x^3*log(b*e*x/(d*x + c) + a*e
/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d
- a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a^2*b*c^2*d*g^3*i^3
 + 3/8*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)...
```

### 3.20.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5520 vs. $2(433) = 866$.

Time = 0.67 (sec) , antiderivative size = 5520, normalized size of antiderivative = 12.08

$$\int (ag + bgx)^3 (ci + dix)^3 \left(A + B \log\left(\frac{e(a + bx)}{c + dx}\right)\right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="giac")
```

output
```
-1/840*(6*(B*b^11*c^8*e^8*g^3*i^3 - 8*B*a*b^10*c^7*d*e^8*g^3*i^3 + 28*B*a^
2*b^9*c^6*d^2*e^8*g^3*i^3 - 56*B*a^3*b^8*c^5*d^3*e^8*g^3*i^3 + 70*B*a^4*b^
7*c^4*d^4*e^8*g^3*i^3 - 56*B*a^5*b^6*c^3*d^5*e^8*g^3*i^3 + 28*B*a^6*b^5*c^
2*d^6*e^8*g^3*i^3 - 8*B*a^7*b^4*c*d^7*e^8*g^3*i^3 + B*a^8*b^3*d^8*e^8*g^3*
i^3 - 7*(b*e*x + a*e)*B*b^10*c^8*d*e^7*g^3*i^3/(d*x + c) + 56*(b*e*x + a*e
)*B*a*b^9*c^7*d^2*e^7*g^3*i^3/(d*x + c) - 196*(b*e*x + a*e)*B*a^2*b^8*c^6*
d^3*e^7*g^3*i^3/(d*x + c) + 392*(b*e*x + a*e)*B*a^3*b^7*c^5*d^4*e^7*g^3*i^
3/(d*x + c) - 490*(b*e*x + a*e)*B*a^4*b^6*c^4*d^5*e^7*g^3*i^3/(d*x + c) +
392*(b*e*x + a*e)*B*a^5*b^5*c^3*d^6*e^7*g^3*i^3/(d*x + c) - 196*(b*e*x + a
*e)*B*a^6*b^4*c^2*d^7*e^7*g^3*i^3/(d*x + c) + 56*(b*e*x + a*e)*B*a^7*b^3*c
*d^8*e^7*g^3*i^3/(d*x + c) - 7*(b*e*x + a*e)*B*a^8*b^2*d^9*e^7*g^3*i^3/(d*
x + c) + 21*(b*e*x + a*e)^2*B*b^9*c^8*d^2*e^6*g^3*i^3/(d*x + c)^2 - 168*(b
*e*x + a*e)^2*B*a*b^8*c^7*d^3*e^6*g^3*i^3/(d*x + c)^2 + 588*(b*e*x + a*e)^
2*B*a^2*b^7*c^6*d^4*e^6*g^3*i^3/(d*x + c)^2 - 1176*(b*e*x + a*e)^2*B*a^3*b
^6*c^5*d^5*e^6*g^3*i^3/(d*x + c)^2 + 1470*(b*e*x + a*e)^2*B*a^4*b^5*c^4*d^
6*e^6*g^3*i^3/(d*x + c)^2 - 1176*(b*e*x + a*e)^2*B*a^5*b^4*c^3*d^7*e^6*g^3
*i^3/(d*x + c)^2 + 588*(b*e*x + a*e)^2*B*a^6*b^3*c^2*d^8*e^6*g^3*i^3/(d*x
+ c)^2 - 168*(b*e*x + a*e)^2*B*a^7*b^2*c*d^9*e^6*g^3*i^3/(d*x + c)^2 + 21*
(b*e*x + a*e)^2*B*a^8*b*d^10*e^6*g^3*i^3/(d*x + c)^2 - 35*(b*e*x + a*e)^3*
B*b^8*c^8*d^3*e^5*g^3*i^3/(d*x + c)^3 + 280*(b*e*x + a*e)^3*B*a*b^7*c^7...
```

### 3.20.9 Mupad [B] (verification not implemented)

Time = 3.18 (sec) , antiderivative size = 4347, normalized size of antiderivative = 9.51

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x
)
```

output
```
x*(((140*a*d + 140*b*c)*(((140*a*d + 140*b*c)*((a*c*((((b^2*d^2*g^3*i^3*(2
8*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140
*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 +
12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^
3*i^3))/(b*d) - ((140*a*d + 140*b*c)*((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^3*c^
3 + 3*B*a^3*d^3 - 3*B*b^3*c^3 + 120*A*a*b^2*c^2*d + 120*A*a^2*b*c*d^2 - 6*
B*a*b^2*c^2*d + 6*B*a^2*b*c*d^2))/5 + ((140*a*d + 140*b*c)*((((b^2*d^2*g^3
*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*
d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2
*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d))/2 + A*a*b^2*c
*d^2*g^3*i^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B
*a*d - B*b*c))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140))/(b*d)))/(
140*b*d) + (g^3*i^3*(4*A*a^4*d^4 + 4*A*b^4*c^4 + B*a^4*d^4 - B*b^4*c^4 + 1
44*A*a^2*b^2*c^2*d^2 + 64*A*a*b^3*c^3*d + 64*A*a^3*b*c*d^3 - 8*B*a*b^3*c^3
*d + 8*B*a^3*b*c*d^3))/(4*b*d)))/(140*b*d) + (a*c*((g^3*i^3*(20*A*a^3*d^3
+ 20*A*b^3*c^3 + 3*B*a^3*d^3 - 3*B*b^3*c^3 + 120*A*a*b^2*c^2*d + 120*A*a^2
*b*c*d^2 - 6*B*a*b^2*c^2*d + 6*B*a^2*b*c*d^2))/5 + ((140*a*d + 140*b*c)*((
((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d - B*b*c))/7 - (A*b^2*d^2*g^
3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*
i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2 + 32*A*a*b*c*d...
```

3.20. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

# 3.21 $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$



## 3.21.1 Optimal result

Integrand size = 40, antiderivative size = 371

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\
&= -\frac{B(bc-ad)^5g^2i^3x}{60b^3d^2} - \frac{B(bc-ad)^4g^2i^3(c+dx)^2}{120b^2d^3} - \frac{B(bc-ad)^3g^2i^3(c+dx)^3}{180bd^3} \\
&\quad + \frac{7B(bc-ad)^2g^2i^3(c+dx)^4}{120d^3} - \frac{bB(bc-ad)g^2i^3(c+dx)^5}{30d^3} \\
&\quad - \frac{B(bc-ad)^6g^2i^3\log\left(\frac{a+bx}{c+dx}\right)}{60b^4d^3} + \frac{(bc-ad)^2g^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^3} \\
&\quad - \frac{2b(bc-ad)g^2i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^3} \\
&\quad + \frac{b^2g^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3} - \frac{B(bc-ad)^6g^2i^3\log(c+dx)}{60b^4d^3}
\end{aligned}$$

output
```
-1/60*B*(-a*d+b*c)^5*g^2*i^3*x/b^3/d^2-1/120*B*(-a*d+b*c)^4*g^2*i^3*(d*x+c
)^2/b^2/d^3-1/180*B*(-a*d+b*c)^3*g^2*i^3*(d*x+c)^3/b/d^3+7/120*B*(-a*d+b*c
)^2*g^2*i^3*(d*x+c)^4/d^3-1/30*b*B*(-a*d+b*c)*g^2*i^3*(d*x+c)^5/d^3-1/60*B
*(-a*d+b*c)^6*g^2*i^3*ln((b*x+a)/(d*x+c))/b^4/d^3+1/4*(-a*d+b*c)^2*g^2*i^3
*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3-2/5*b*(-a*d+b*c)*g^2*i^3*(d*x+c
)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3+1/6*b^2*g^2*i^3*(d*x+c)^6*(A+B*ln(e*(b
*x+a)/(d*x+c)))/d^3-1/60*B*(-a*d+b*c)^6*g^2*i^3*ln(d*x+c)/b^4/d^3
```

### 3.21.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 429, normalized size of antiderivative = 1.16

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{g^2i^3\Big(-15B(bc-ad)^3\left(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx)\right)}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)]),x]
```

output
```
(g^2*i^3*(-15*B*(b*c - a*d)^3*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(
c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) + 12*B*(b*c
 - a*d)^2*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 + 4*b^
3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)^4*Log[a + b
*x]) - B*(b*c - a*d)*(60*b*d*(b*c - a*d)^4*x + 30*b^2*(b*c - a*d)^3*(c + d
*x)^2 + 20*b^3*(b*c - a*d)^2*(c + d*x)^3 + 15*b^4*(b*c - a*d)*(c + d*x)^4
+ 12*b^5*(c + d*x)^5 + 60*(b*c - a*d)^5*Log[a + b*x]) + 90*b^4*(b*c - a*d)
^2*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 144*b^5*(b*c - a*d)*
(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 60*b^6*(c + d*x)^6*(A +
 B*Log[(e*(a + b*x))/(c + d*x)])))/(360*b^4*d^3)
```

### 3.21.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 327, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2962, 2782, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$
$$\Big\downarrow \text{ 2962}$$
$$g^2i^3(bc-ad)^6\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$
$$\Big\downarrow \text{ 2782}$$

$$g^2i^3(bc-ad)^6\left(-B\int \frac{(c+dx)\left(b^2-\frac{6d(a+bx)b}{c+dx}+\frac{15d^2(a+bx)^2}{(c+dx)^2}\right)}{60d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{2b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.$$

$\downarrow$ 27

$$g^2i^3(bc-ad)^6\left(-\frac{B\int \frac{(c+dx)\left(b^2-\frac{6d(a+bx)b}{c+dx}+\frac{15d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{60d^3}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{2b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{B\,\mathrm{lo}}{4d^3}\right.$$

$\downarrow$ 1195

$$g^2i^3(bc-ad)^6\left(-\frac{B\int\left(\frac{d}{b^4\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{14d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{10bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}+\frac{c+}{b^4(a-}\right.}{60d^3}\right.$$

$\downarrow$ 2009

$$g^2i^3(bc-ad)^6\left(\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{2b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^4}-\frac{\log\left(b-\frac{d(a+b}{c+d}\right.}{b^4}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x
]
```

output
```
(b*c - a*d)^6*g^2*i^3*((b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*d^3*(
b - (d*(a + b*x))/(c + d*x))^6) - (2*b*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(5*d^3*(b - (d*(a + b*x))/(c + d*x))^5) + (A + B*Log[(e*(a + b*x))/(c +
 d*x)])/(4*d^3*(b - (d*(a + b*x))/(c + d*x))^4) - (B*((2*b)/(b - (d*(a + b
*x))/(c + d*x))^5 - 7/(2*(b - (d*(a + b*x))/(c + d*x))^4) + 1/(3*b*(b - (d
*(a + b*x))/(c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b*x))/(c + d*x))^2) + 1/
(b^3*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^4 - Log[b
 - (d*(a + b*x))/(c + d*x)]/b^4))/(60*d^3))
```

## 3.21. $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

#### 3.21.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.21.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 924 vs. $2(351) = 702$.

Time = 1.16 (sec) , antiderivative size = 925, normalized size of antiderivative = 2.49

| method | result |
|---|---|
| risch | $-\frac{i^3 g^2 b^2 d^2 B c x^5}{30}+\frac{i^3 g^2 d^3 A a^2 x^4}{4}+\frac{3 i^3 g^2 b^2 d A c^2 x^4}{4}+\frac{7 i^3 g^2 d^3 B a^2 x^4}{120}-\frac{13 i^3 g^2 b^2 d B c^2 x^4}{120}+\frac{i^3 g^2 b^2 A c^3 x^3}{3}+$ |
| parallelrisch | Expression too large to display |
| parts | Expression too large to display |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETU
RNVERBOSE)
```

output
```
-1/30*i^3*g^2*b^2*d^2*B*c*x^5+1/4*i^3*g^2*d^3*A*a^2*x^4+3/4*i^3*g^2*b^2*d*
A*c^2*x^4+7/120*i^3*g^2*d^3*B*a^2*x^4-13/120*i^3*g^2*b^2*d*B*c^2*x^4+1/3*i
^3*g^2*b^2*A*c^3*x^3+1/180*i^3*g^2/b*d^3*B*a^3*x^3-19/180*i^3*g^2*b^2*B*c^
3*x^3-1/120*i^3*g^2/b^2*d^3*B*a^4*x^2-1/120*i^3*g^2*b^2/d*B*c^4*x^2+3/2*i^
3*g^2*b*d^2*A*a*c*x^4+1/20*i^3*g^2*b*d^2*B*a*c*x^4+i^3*g^2*d^2*A*a^2*c*x^3
+2*i^3*g^2*b*d*A*a*c^2*x^3+13/60*i^3*g^2*d^2*B*a^2*c*x^3-7/60*i^3*g^2*b*d*
B*a*c^2*x^3+3/2*i^3*g^2*d*A*a^2*c^2*x^2+i^3*g^2*b*A*a*c^3*x^2+1/20*i^3*g^2
/b*d^2*B*a^3*c*x^2+1/4*i^3*g^2*d*B*a^2*c^2*x^2-17/60*i^3*g^2*b*B*a*c^3*x^2
+i^3*g^2*A*a^2*c^3*x-1/10*i^3*g^2/b^2*d^2*B*a^4*c*x+1/4*i^3*g^2/b*d*B*a^3*
c^2*x-1/12*i^3*g^2*B*a^2*c^3*x-1/10*i^3*g^2*b/d*B*a*c^4*x+1/10*i^3*g^2*b/d
^2*B*ln(-d*x-c)*a*c^5+1/10*i^3*g^2/b^3*d^2*B*ln(b*x+a)*a^5*c-1/4*i^3*g^2/b
^2*d*B*ln(b*x+a)*a^4*c^2+2/5*i^3*g^2*b*d^3*A*a*x^5+3/5*i^3*g^2*b^2*d^2*A*c
*x^5+1/30*i^3*g^2*b*d^3*B*a*x^5+1/60*i^3*g^2/b^3*d^3*B*a^5*x+1/60*i^3*g^2*
b^2/d^2*B*c^5*x-1/60*i^3*g^2*b^2/d^3*B*ln(-d*x-c)*c^6-1/60*i^3*g^2/b^4*d^3
*B*ln(b*x+a)*a^6-1/4*i^3*g^2/d*B*ln(-d*x-c)*a^2*c^4+1/3*i^3*g^2/b*B*ln(b*x
+a)*a^3*c^3+1/60*i^3*g^2*B*x*(10*b^2*d^3*x^5+24*a*b*d^3*x^4+36*b^2*c*d^2*x
^4+15*a^2*d^3*x^3+90*a*b*c*d^2*x^3+45*b^2*c^2*d*x^3+60*a^2*c*d^2*x^2+120*a
*b*c^2*d*x^2+20*b^2*c^3*x^2+90*a^2*c^2*d*x+60*a*b*c^3*x+60*a^2*c^3)*ln(e*(
b*x+a)/(d*x+c))+1/6*i^3*g^2*b^2*d^3*A*x^6
```

### 3.21.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(351) = 702$.

Time = 0.47 (sec) , antiderivative size = 722, normalized size of antiderivative = 1.95

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{60\,Ab^6d^6g^2i^3x^6+12\,((18\,A-B)b^6cd^5+(12\,A+B)ab^5d^6)g^2i^3x^5+3\,((90\,A-13\,B)b^6c^2d^4+6\,(30\,A+B)}{}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="fricas")
```

output
```
1/360*(60*A*b^6*d^6*g^2*i^3*x^6 + 12*((18*A - B)*b^6*c*d^5 + (12*A + B)*a*
b^5*d^6)*g^2*i^3*x^5 + 3*((90*A - 13*B)*b^6*c^2*d^4 + 6*(30*A + B)*a*b^5*c
*d^5 + (30*A + 7*B)*a^2*b^4*d^6)*g^2*i^3*x^4 + 2*((60*A - 19*B)*b^6*c^3*d^
3 + 3*(120*A - 7*B)*a*b^5*c^2*d^4 + 3*(60*A + 13*B)*a^2*b^4*c*d^5 + B*a^3*
b^3*d^6)*g^2*i^3*x^3 - 3*(B*b^6*c^4*d^2 - 2*(60*A - 17*B)*a*b^5*c^3*d^3 -
30*(6*A + B)*a^2*b^4*c^2*d^4 - 6*B*a^3*b^3*c*d^5 + B*a^4*b^2*d^6)*g^2*i^3*
x^2 + 6*(B*b^6*c^5*d - 6*B*a*b^5*c^4*d^2 + 5*(12*A - B)*a^2*b^4*c^3*d^3 +
15*B*a^3*b^3*c^2*d^4 - 6*B*a^4*b^2*c*d^5 + B*a^5*b*d^6)*g^2*i^3*x + 6*(20*
B*a^3*b^3*c^3*d^3 - 15*B*a^4*b^2*c^2*d^4 + 6*B*a^5*b*c*d^5 - B*a^6*d^6)*g^
2*i^3*log(b*x + a) - 6*(B*b^6*c^6 - 6*B*a*b^5*c^5*d + 15*B*a^2*b^4*c^4*d^2
)*g^2*i^3*log(d*x + c) + 6*(10*B*b^6*d^6*g^2*i^3*x^6 + 60*B*a^2*b^4*c^3*d^
3*g^2*i^3*x + 12*(3*B*b^6*c*d^5 + 2*B*a*b^5*d^6)*g^2*i^3*x^5 + 15*(3*B*b^6
*c^2*d^4 + 6*B*a*b^5*c*d^5 + B*a^2*b^4*d^6)*g^2*i^3*x^4 + 20*(B*b^6*c^3*d^
3 + 6*B*a*b^5*c^2*d^4 + 3*B*a^2*b^4*c*d^5)*g^2*i^3*x^3 + 30*(2*B*a*b^5*c^3
*d^3 + 3*B*a^2*b^4*c^2*d^4)*g^2*i^3*x^2)*log((b*e*x + a*e)/(d*x + c)))/(b^
4*d^3)
```

### 3.21.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1727 vs. 2(347) = 694.

Time = 7.14 (sec) , antiderivative size = 1727, normalized size of antiderivative = 4.65

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b**2*d**3*g**2*i**3*x**6/6 - B*a**3*g**2*i**3*(a**3*d**3 - 6*a**2*b*c*d*
*2 + 15*a*b**2*c**2*d - 20*b**3*c**3)*log(x + (B*a**6*c*d**5*g**2*i**3 - 6
*B*a**5*b*c**2*d**4*g**2*i**3 + 15*B*a**4*b**2*c**3*d**3*g**2*i**3 + B*a**
4*d**3*g**2*i**3*(a**3*d**3 - 6*a**2*b*c*d**2 + 15*a*b**2*c**2*d - 20*b**3
*c**3)/b - 35*B*a**3*b**3*c**4*d**2*g**2*i**3 - B*a**3*c*d**2*g**2*i**3*(a
**3*d**3 - 6*a**2*b*c*d**2 + 15*a*b**2*c**2*d - 20*b**3*c**3) + 6*B*a**2*b
**4*c**5*d*g**2*i**3 - B*a*b**5*c**6*g**2*i**3)/(B*a**6*d**6*g**2*i**3 - 6
*B*a**5*b*c*d**5*g**2*i**3 + 15*B*a**4*b**2*c**2*d**4*g**2*i**3 - 20*B*a**
3*b**3*c**3*d**3*g**2*i**3 - 15*B*a**2*b**4*c**4*d**2*g**2*i**3 + 6*B*a*b*
*5*c**5*d*g**2*i**3 - B*b**6*c**6*g**2*i**3))/(60*b**4) - B*c**4*g**2*i**3
*(15*a**2*d**2 - 6*a*b*c*d + b**2*c**2)*log(x + (B*a**6*c*d**5*g**2*i**3 -
 6*B*a**5*b*c**2*d**4*g**2*i**3 + 15*B*a**4*b**2*c**3*d**3*g**2*i**3 - 35*
B*a**3*b**3*c**4*d**2*g**2*i**3 + 6*B*a**2*b**4*c**5*d*g**2*i**3 - B*a*b**
5*c**6*g**2*i**3 + B*a*b**3*c**4*g**2*i**3*(15*a**2*d**2 - 6*a*b*c*d + b**
2*c**2) - B*b**4*c**5*g**2*i**3*(15*a**2*d**2 - 6*a*b*c*d + b**2*c**2)/d)/
(B*a**6*d**6*g**2*i**3 - 6*B*a**5*b*c*d**5*g**2*i**3 + 15*B*a**4*b**2*c**2
*d**4*g**2*i**3 - 20*B*a**3*b**3*c**3*d**3*g**2*i**3 - 15*B*a**2*b**4*c**4
*d**2*g**2*i**3 + 6*B*a*b**5*c**5*d*g**2*i**3 - B*b**6*c**6*g**2*i**3))/(6
0*d**3) + x**5*(2*A*a*b*d**3*g**2*i**3/5 + 3*A*b**2*c*d**2*g**2*i**3/5 + B
*a*b*d**3*g**2*i**3/30 - B*b**2*c*d**2*g**2*i**3/30) + x**4*(A*a**2*d**...
```

### 3.21.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. $2(351) = 702$.

Time = 0.25 (sec) , antiderivative size = 1789, normalized size of antiderivative = 4.82

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="maxima")
```

output
```
1/6*A*b^2*d^3*g^2*i^3*x^6 + 3/5*A*b^2*c*d^2*g^2*i^3*x^5 + 2/5*A*a*b*d^3*g^
2*i^3*x^5 + 3/4*A*b^2*c^2*d*g^2*i^3*x^4 + 3/2*A*a*b*c*d^2*g^2*i^3*x^4 + 1/
4*A*a^2*d^3*g^2*i^3*x^4 + 1/3*A*b^2*c^3*g^2*i^3*x^3 + 2*A*a*b*c^2*d*g^2*i^
3*x^3 + A*a^2*c*d^2*g^2*i^3*x^3 + A*a*b*c^3*g^2*i^3*x^2 + 3/2*A*a^2*c^2*d*
g^2*i^3*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b -
 c*log(d*x + c)/d)*B*a^2*c^3*g^2*i^3 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x
 + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d)
)*B*a*b*c^3*g^2*i^3 + 1/6*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*
a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 -
 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*b^2*c^3*g^2*i^3 + 3/2*(x^2*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
 - (b*c - a*d)*x/(b*d))*B*a^2*c^2*d*g^2*i^3 + (2*x^3*log(b*e*x/(d*x + c) +
 a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*
c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*b*c^2*d*g^2*i
^3 + 1/8*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/
b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2
*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b^2*c^2*d*g^2*
i^3 + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)
/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^
2*d^2)*x)/(b^2*d^2))*B*a^2*c*d^2*g^2*i^3 + 1/4*(6*x^4*log(b*e*x/(d*x + ...
```

### 3.21.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4110 vs. $2(351) = 702$.

Time = 0.55 (sec) , antiderivative size = 4110, normalized size of antiderivative = 11.08

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algo
rithm="giac")
```

output
```
1/360*(6*(B*b^9*c^7*e^7*g^2*i^3 - 7*B*a*b^8*c^6*d*e^7*g^2*i^3 + 21*B*a^2*b
^7*c^5*d^2*e^7*g^2*i^3 - 35*B*a^3*b^6*c^4*d^3*e^7*g^2*i^3 + 35*B*a^4*b^5*c
^3*d^4*e^7*g^2*i^3 - 21*B*a^5*b^4*c^2*d^5*e^7*g^2*i^3 + 7*B*a^6*b^3*c*d^6*
e^7*g^2*i^3 - B*a^7*b^2*d^7*e^7*g^2*i^3 - 6*(b*e*x + a*e)*B*b^8*c^7*d*e^6*
g^2*i^3/(d*x + c) + 42*(b*e*x + a*e)*B*a*b^7*c^6*d^2*e^6*g^2*i^3/(d*x + c)
 - 126*(b*e*x + a*e)*B*a^2*b^6*c^5*d^3*e^6*g^2*i^3/(d*x + c) + 210*(b*e*x
+ a*e)*B*a^3*b^5*c^4*d^4*e^6*g^2*i^3/(d*x + c) - 210*(b*e*x + a*e)*B*a^4*b
^4*c^3*d^5*e^6*g^2*i^3/(d*x + c) + 126*(b*e*x + a*e)*B*a^5*b^3*c^2*d^6*e^6
*g^2*i^3/(d*x + c) - 42*(b*e*x + a*e)*B*a^6*b^2*c*d^7*e^6*g^2*i^3/(d*x + c
) + 6*(b*e*x + a*e)*B*a^7*b*d^8*e^6*g^2*i^3/(d*x + c) + 15*(b*e*x + a*e)^2
*B*b^7*c^7*d^2*e^5*g^2*i^3/(d*x + c)^2 - 105*(b*e*x + a*e)^2*B*a*b^6*c^6*d
^3*e^5*g^2*i^3/(d*x + c)^2 + 315*(b*e*x + a*e)^2*B*a^2*b^5*c^5*d^4*e^5*g^2
*i^3/(d*x + c)^2 - 525*(b*e*x + a*e)^2*B*a^3*b^4*c^4*d^5*e^5*g^2*i^3/(d*x
+ c)^2 + 525*(b*e*x + a*e)^2*B*a^4*b^3*c^3*d^6*e^5*g^2*i^3/(d*x + c)^2 - 3
15*(b*e*x + a*e)^2*B*a^5*b^2*c^2*d^7*e^5*g^2*i^3/(d*x + c)^2 + 105*(b*e*x
+ a*e)^2*B*a^6*b*c*d^8*e^5*g^2*i^3/(d*x + c)^2 - 15*(b*e*x + a*e)^2*B*a^7*
d^9*e^5*g^2*i^3/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/(b^6*d^3*e^6 - 6
*(b*e*x + a*e)*b^5*d^4*e^5/(d*x + c) + 15*(b*e*x + a*e)^2*b^4*d^5*e^4/(d*x
 + c)^2 - 20*(b*e*x + a*e)^3*b^3*d^6*e^3/(d*x + c)^3 + 15*(b*e*x + a*e)^4*
b^2*d^7*e^2/(d*x + c)^4 - 6*(b*e*x + a*e)^5*b*d^8*e/(d*x + c)^5 + (b*e*...
```

### 3.21.9 Mupad [B] (verification not implemented)

Time = 2.57 (sec) , antiderivative size = 2465, normalized size of antiderivative = 6.64

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x
)
```

output
```
x^3*((g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3 - 3*B*b^3*c^3 + 72*A
*a*b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d + 5*B*a^2*b*c*d^2))/(12*
b) + ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B
*b*c))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*
b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2 - 3*B*b^2*c^2
 + 60*A*a*b*c*d + B*a*b*c*d))/5 + A*a*b*c*d^2*g^2*i^3))/(180*b*d) - (a*c*(
(b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B*b*c))/6 - (A*b*d^2*g^2*i^3
*(60*a*d + 60*b*c))/60))/(3*b*d)) - x^4*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A
*b*c + B*a*d - B*b*c))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d
 + 60*b*c))/(240*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*
d^2 - 3*B*b^2*c^2 + 60*A*a*b*c*d + B*a*b*c*d))/20 + (A*a*b*c*d^2*g^2*i^3)/
4) + x^2*((a*c*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B*b*c))/6
- (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (d
*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2 - 3*B*b^2*c^2 + 60*A*a
*b*c*d + B*a*b*c*d))/5 + A*a*b*c*d^2*g^2*i^3))/(2*b*d) - ((60*a*d + 60*b*c
)*((g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3 - 3*B*b^3*c^3 + 72*A*a
*b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d + 5*B*a^2*b*c*d^2))/(4*b)
+ ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d - B*b*
c))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d
) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2 - 3*B*b^2*c^2...
```

## 3.21. $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

# 3.22 $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$



## 3.22.1 Optimal result

Integrand size = 38, antiderivative size = 271

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx \\
&= \frac{B(bc-ad)^4 gi^3 x}{20b^3 d} + \frac{B(bc-ad)^3 gi^3 (c+dx)^2}{40b^2 d^2} + \frac{B(bc-ad)^2 gi^3 (c+dx)^3}{60bd^2} \\
&\quad - \frac{B(bc-ad) gi^3 (c+dx)^4}{20d^2} + \frac{B(bc-ad)^5 gi^3 \log\left(\frac{a+bx}{c+dx}\right)}{20b^4 d^2} \\
&\quad - \frac{(bc-ad) gi^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2} \\
&\quad + \frac{bgi^3 (c+dx)^5 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5d^2} + \frac{B(bc-ad)^5 gi^3 \log(c+dx)}{20b^4 d^2}
\end{aligned}$$

output

```
1/20*B*(-a*d+b*c)^4*g*i^3*x/b^3/d+1/40*B*(-a*d+b*c)^3*g*i^3*(d*x+c)^2/b^2/
d^2+1/60*B*(-a*d+b*c)^2*g*i^3*(d*x+c)^3/b/d^2-1/20*B*(-a*d+b*c)*g*i^3*(d*x
+c)^4/d^2+1/20*B*(-a*d+b*c)^5*g*i^3*ln((b*x+a)/(d*x+c))/b^4/d^2-1/4*(-a*d+
b*c)*g*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2+1/5*b*g*i^3*(d*x+c)^5
*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2+1/20*B*(-a*d+b*c)^5*g*i^3*ln(d*x+c)/b^4/d
^2
```

### 3.22.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 261, normalized size of antiderivative = 0.96

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{gi^3\left(\frac{5B(bc-ad)^2(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx))}{b^4} - \frac{2B(bc-ad)(12bd(bc-ad)^3x+6b^2(bc-ad)^2(c+d}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)
]),x]
```

output
```
(g*i^3*((5*B*(b*c - a*d)^2*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c +
 d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 - (2*B*(b
*c - a*d)*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 + 4*b^
3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)^4*Log[a + b
*x]))/b^4 - 30*(b*c - a*d)*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]
) + 24*b*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(120*d^2)
```

### 3.22.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.93, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.132$, Rules used = {2962, 2782, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{2962}$$

$$gi^3(bc-ad)^5\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2782}$$

$$gi^3(bc-ad)^5\left(-B\int -\frac{(c+dx)\left(b-\frac{5d(a+bx)}{c+dx}\right)}{20d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)$$

$\downarrow$ 27

$$gi^3(bc-ad)^5\left(\frac{B\int \frac{(c+dx)\left(b-\frac{5d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{20d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)$$

$\downarrow$ 86

$$gi^3(bc-ad)^5\left(\frac{B\int \left(\frac{d}{b^4\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{4d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{c+dx}{b^4(a+bx)}\right)d\frac{a+bx}{c+dx}}{20d^2}-\frac{B\ldots}{4\ldots}\right.$$

$\downarrow$ 2009

$$gi^3(bc-ad)^5\left(-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^4}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^4}+\frac{1}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\ldots}{2b^2\left(b-\ldots\right.}\right.}{20d^2}\right.$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(b*c - a*d)^5*g*i^3*((b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*d^2*(b -
(d*(a + b*x))/(c + d*x))^5) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(4*d^2*
(b - (d*(a + b*x))/(c + d*x))^4) + (B*(-(b - (d*(a + b*x))/(c + d*x))^(-4)
 + 1/(3*b*(b - (d*(a + b*x))/(c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b*x))/(
c + d*x))^2) + 1/(b^3*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c +
d*x)]/b^4 - Log[b - (d*(a + b*x))/(c + d*x)]/b^4))/(20*d^2))
```

## 3.22. $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

### 3.22.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.22.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(255) = 510$.

Time = 0.89 (sec) , antiderivative size = 568, normalized size of antiderivative = 2.10

| method | result |
|---|---|
| risch | $\frac{i^3 g\, d^3 B\, a^4 x}{20 b^3} - \frac{i^3 g b B\, c^4 x}{20 d} - \frac{i^3 g\, d^3 B \ln(bx+a) a^5}{20 b^4} + \frac{i^3 g b B \ln(-dx-c) c^5}{20 d^2} + \frac{i^3 g\, d^3 A a\, x^4}{4} + \frac{3 i^3 g b\, d^2 A c\, x^4}{4} + \frac{i^3 g}{}$ |
| parallelrisch | $\frac{-6 B\, a^5 d^5 g\, i^3 + 6 B\, b^5 c^5 g\, i^3 + 27 B\, a^4 b c\, d^4 g\, i^3 - 60 B \ln(bx+a) a^3 b^2 c^2 d^3 g\, i^3 + 60 B \ln(bx+a) a^2 b^3 c^3 d^2 g\, i^3 - 30 B \ln(bx+a) a b}{}$ |
| parts | Expression too large to display |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURN
VERBOSE)
```

output
```
1/20*i^3*g/b^3*d^3*B*a^4*x-1/20*i^3*g*b/d*B*c^4*x-1/20*i^3*g/b^4*d^3*B*ln(
b*x+a)*a^5+1/20*i^3*g*b/d^2*B*ln(-d*x-c)*c^5+1/4*i^3*g*d^3*A*a*x^4+3/4*i^3
*g*b*d^2*A*c*x^4+1/20*i^3*g*d^3*B*a*x^4-1/20*i^3*g*b*d^2*B*c*x^4+i^3*g*b*d
*A*c^2*x^3+1/60*i^3*g/b*d^3*B*a^2*x^3-11/60*i^3*g*b*d*B*c^2*x^3+1/2*i^3*g*
b*A*c^3*x^2-1/40*i^3*g/b^2*d^3*B*a^3*x^2-9/40*i^3*g*b*B*c^3*x^2+i^3*g*d^2*
A*a*c*x^3+1/6*i^3*g*d^2*B*a*c*x^3+3/2*i^3*g*d*A*a*c^2*x^2+1/8*i^3*g/b*d^2*
B*a^2*c*x^2+1/8*i^3*g*d*B*a*c^2*x^2+i^3*g*A*a*c^3*x-1/4*i^3*g/b^2*d^2*B*a^
3*c*x+1/2*i^3*g/b*d*B*a^2*c^2*x-1/4*i^3*g*B*a*c^3*x+1/4*i^3*g/b^3*d^2*B*ln
(b*x+a)*a^4*c-1/2*i^3*g/b^2*d*B*ln(b*x+a)*a^3*c^2+1/2*i^3*g/b*B*ln(b*x+a)*
a^2*c^3-1/4*i^3*g/d*B*ln(-d*x-c)*a*c^4+1/5*i^3*g*b*d^3*A*x^5+1/20*g*i^3*B*
x*(4*b*d^3*x^4+5*a*d^3*x^3+15*b*c*d^2*x^3+20*a*c*d^2*x^2+20*b*c^2*d*x^2+30
*a*c^2*d*x+10*b*c^3*x+20*a*c^3)*ln(e*(b*x+a)/(d*x+c))
```

### 3.22.5 Fricas [A] (verification not implemented)

Time = 0.42 (sec) , antiderivative size = 502, normalized size of antiderivative = 1.85

$$\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{24\,Ab^5d^5gi^3x^5 + 6\left((15\,A-B)b^5cd^4 + (5\,A+B)ab^4d^5\right)gi^3x^4 + 2\left((60\,A-11\,B)b^5c^2d^3 + 10\,(6\,A+B)ab^4c\right.}{}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="fricas")
```

output
```
1/120*(24*A*b^5*d^5*g*i^3*x^5 + 6*((15*A - B)*b^5*c*d^4 + (5*A + B)*a*b^4*
d^5)*g*i^3*x^4 + 2*((60*A - 11*B)*b^5*c^2*d^3 + 10*(6*A + B)*a*b^4*c*d^4 +
 B*a^2*b^3*d^5)*g*i^3*x^3 + 3*((20*A - 9*B)*b^5*c^3*d^2 + 5*(12*A + B)*a*b
^4*c^2*d^3 + 5*B*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*g*i^3*x^2 - 6*(B*b^5*c^4*d
 - 5*(4*A - B)*a*b^4*c^3*d^2 - 10*B*a^2*b^3*c^2*d^3 + 5*B*a^3*b^2*c*d^4 -
B*a^4*b*d^5)*g*i^3*x + 6*(10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2*d^3 + 5*
B*a^4*b*c*d^4 - B*a^5*d^5)*g*i^3*log(b*x + a) + 6*(B*b^5*c^5 - 5*B*a*b^4*c
^4*d)*g*i^3*log(d*x + c) + 6*(4*B*b^5*d^5*g*i^3*x^5 + 20*B*a*b^4*c^3*d^2*g
*i^3*x + 5*(3*B*b^5*c*d^4 + B*a*b^4*d^5)*g*i^3*x^4 + 20*(B*b^5*c^2*d^3 + B
*a*b^4*c*d^4)*g*i^3*x^3 + 10*(B*b^5*c^3*d^2 + 3*B*a*b^4*c^2*d^3)*g*i^3*x^2
)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d^2)
```

### 3.22.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1158 vs. $2(252) = 504$.

Time = 3.84 (sec) , antiderivative size = 1158, normalized size of antiderivative = 4.27

$$\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \frac{Abd^3gi^3x^5}{5}$$
$$-\frac{Ba^2gi^3(a^3d^3-5a^2bcd^2+10ab^2c^2d-10b^3c^3)\log\left(x+\frac{Ba^5cd^4gi^3-5Ba^4bc^2d^3gi^3+10Ba^3b^2c^3d^2gi^3+\frac{Ba^3d^2gi^3\left(a^3d^3-5a\right.}{}}{Ba^5d^5gi^3-5Ba^4bcd^4gi^3+}\right.}{20b^4}$$
$$-\frac{Bc^4gi^3\cdot(5ad-bc)\log\left(x+\frac{Ba^5cd^4gi^3-5Ba^4bc^2d^3gi^3+10Ba^3b^2c^3d^2gi^3-15Ba^2b^3c^4dgi^3+Bab^4c^5gi^3+Bab^3c^4gi^3\cdot(5ad-bc)-\frac{B}{}}{Ba^5d^5gi^3-5Ba^4bcd^4gi^3+10Ba^3b^2c^2d^3gi^3-10Ba^2b^3c^3d^2gi^3-5Bab^4c^4dgi^3+Bb^5c^5gi}\right.}{20d^2}$$
$$+x^4\left(\frac{Aad^3gi^3}{4}+\frac{3Abcd^2gi^3}{4}+\frac{Bad^3gi^3}{20}-\frac{Bbcd^2gi^3}{20}\right)$$
$$+x^3\left(Aacd^2gi^3+Abc^2dgi^3+\frac{Ba^2d^3gi^3}{60b}+\frac{Bacd^2gi^3}{6}-\frac{11Bbc^2dgi^3}{60}\right)+x^2$$
$$\cdot\left(\frac{3Aac^2dgi^3}{2}+\frac{Abc^3gi^3}{2}-\frac{Ba^3d^3gi^3}{40b^2}+\frac{Ba^2cd^2gi^3}{8b}+\frac{Bac^2dgi^3}{8}-\frac{9Bbc^3gi^3}{40}\right)$$
$$+x\left(Aac^3gi^3+\frac{Ba^4d^3gi^3}{20b^3}-\frac{Ba^3cd^2gi^3}{4b^2}+\frac{Ba^2c^2dgi^3}{2b}-\frac{Bac^3gi^3}{4}-\frac{Bbc^4gi^3}{20d}\right)$$
$$+\left(Bac^3gi^3x+\frac{3Bac^2dgi^3x^2}{2}+Bacd^2gi^3x^3+\frac{Bad^3gi^3x^4}{4}+\frac{Bbc^3gi^3x^2}{2}+Bbc^2dgi^3x^3\right.$$
$$\left.+\frac{3Bbcd^2gi^3x^4}{4}+\frac{Bbd^3gi^3x^5}{5}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*b*d**3*g*i**3*x**5/5 - B*a**2*g*i**3*(a**3*d**3 - 5*a**2*b*c*d**2 + 10*a
*b**2*c**2*d - 10*b**3*c**3)*log(x + (B*a**5*c*d**4*g*i**3 - 5*B*a**4*b*c*
*2*d**3*g*i**3 + 10*B*a**3*b**2*c**3*d**2*g*i**3 + B*a**3*d**2*g*i**3*(a**
3*d**3 - 5*a**2*b*c*d**2 + 10*a*b**2*c**2*d - 10*b**3*c**3)/b - 15*B*a**2*
b**3*c**4*d*g*i**3 - B*a**2*c*d*g*i**3*(a**3*d**3 - 5*a**2*b*c*d**2 + 10*a
*b**2*c**2*d - 10*b**3*c**3) + B*a*b**4*c**5*g*i**3)/(B*a**5*d**5*g*i**3 -
 5*B*a**4*b*c*d**4*g*i**3 + 10*B*a**3*b**2*c**2*d**3*g*i**3 - 10*B*a**2*b*
*3*c**3*d**2*g*i**3 - 5*B*a*b**4*c**4*d*g*i**3 + B*b**5*c**5*g*i**3))/(20*
b**4) - B*c**4*g*i**3*(5*a*d - b*c)*log(x + (B*a**5*c*d**4*g*i**3 - 5*B*a*
*4*b*c**2*d**3*g*i**3 + 10*B*a**3*b**2*c**3*d**2*g*i**3 - 15*B*a**2*b**3*c
**4*d*g*i**3 + B*a*b**4*c**5*g*i**3 + B*a*b**3*c**4*g*i**3*(5*a*d - b*c) -
 B*b**4*c**5*g*i**3*(5*a*d - b*c)/d)/(B*a**5*d**5*g*i**3 - 5*B*a**4*b*c*d*
*4*g*i**3 + 10*B*a**3*b**2*c**2*d**3*g*i**3 - 10*B*a**2*b**3*c**3*d**2*g*i
**3 - 5*B*a*b**4*c**4*d*g*i**3 + B*b**5*c**5*g*i**3))/(20*d**2) + x**4*(A*
a*d**3*g*i**3/4 + 3*A*b*c*d**2*g*i**3/4 + B*a*d**3*g*i**3/20 - B*b*c*d**2*
g*i**3/20) + x**3*(A*a*c*d**2*g*i**3 + A*b*c**2*d*g*i**3 + B*a**2*d**3*g*i
**3/(60*b) + B*a*c*d**2*g*i**3/6 - 11*B*b*c**2*d*g*i**3/60) + x**2*(3*A*a*
c**2*d*g*i**3/2 + A*b*c**3*g*i**3/2 - B*a**3*d**3*g*i**3/(40*b**2) + B*a**
2*c*d**2*g*i**3/(8*b) + B*a*c**2*d*g*i**3/8 - 9*B*b*c**3*g*i**3/40) + x*(A
*a*c**3*g*i**3 + B*a**4*d**3*g*i**3/(20*b**3) - B*a**3*c*d**2*g*i**3/(4...
```

### 3.22.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1022 vs. $2(255) = 510$.

Time = 0.31 (sec) , antiderivative size = 1022, normalized size of antiderivative = 3.77

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$
$$= \frac{1}{5}Abd^3gi^3x^5 + \frac{3}{4}Abcd^2gi^3x^4 + \frac{1}{4}Aad^3gi^3x^4 + Abc^2dgi^3x^3 + Aacd^2gi^3x^3 + \frac{1}{2}Abc^3gi^3x^2$$
$$+ \frac{3}{2}Aac^2dgi^3x^2 + \left(x\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)Bac^3gi^3$$
$$+\frac{1}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bbc^3gi^3$$
$$+\frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{a^2\log(bx+a)}{b^2}+\frac{c^2\log(dx+c)}{d^2}-\frac{(bc-ad)x}{bd}\right)Bac^2dgi^3$$
$$+\frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a}{b^2d^2}\right.$$
$$+\frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{2a^3\log(bx+a)}{b^3}-\frac{2c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2(b^2c^2-a}{b^2d^2}\right.$$
$$+\frac{1}{8}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2c}{b^3}\right.$$
$$+\frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{6a^4\log(bx+a)}{b^4}+\frac{6c^4\log(dx+c)}{d^4}-\frac{2(b^3cd^2-ab^2d^3)x^3-3(b^3c^2}{}\right.$$
$$+\frac{1}{60}\left(12x^5\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)+\frac{12a^5\log(bx+a)}{b^5}-\frac{12c^5\log(dx+c)}{d^5}-\frac{3(b^4cd^3-ab^3d^4)x^4-4(}{}\right.$$
$$+Aac^3gi^3x$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="maxima")
```

3.22. $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

output
```
1/5*A*b*d^3*g*i^3*x^5 + 3/4*A*b*c*d^2*g*i^3*x^4 + 1/4*A*a*d^3*g*i^3*x^4 +
A*b*c^2*d*g*i^3*x^3 + A*a*c*d^2*g*i^3*x^3 + 1/2*A*b*c^3*g*i^3*x^2 + 3/2*A*
a*c^2*d*g*i^3*x^2 + (x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x +
a)/b - c*log(d*x + c)/d)*B*a*c^3*g*i^3 + 1/2*(x^2*log(b*e*x/(d*x + c) + a*
e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x
/(b*d))*B*b*c^3*g*i^3 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^
2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*a*c^2*d
*g*i^3 + 1/2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x +
 a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 -
 a^2*d^2)*x)/(b^2*d^2))*B*b*c^2*d*g*i^3 + 1/2*(2*x^3*log(b*e*x/(d*x + c) +
 a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*
c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B*a*c*d^2*g*i^3 +
 1/8*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4
+ 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d -
 a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*b*c*d^2*g*i^3 + 1/
24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 +
6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a
^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*a*d^3*g*i^3 + 1/60*(
12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b*x + a)/b^5 - 12
*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2...
```

### 3.22.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2589 vs. $2(255) = 510$.

Time = 0.48 (sec) , antiderivative size = 2589, normalized size of antiderivative = 9.55

$$\int (ag + bgx)(ci + dix)^3 \left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algori
thm="giac")
```

output
```
-1/120*(6*(B*b^7*c^6*e^6*g*i^3 - 6*B*a*b^6*c^5*d*e^6*g*i^3 + 15*B*a^2*b^5*
c^4*d^2*e^6*g*i^3 - 20*B*a^3*b^4*c^3*d^3*e^6*g*i^3 + 15*B*a^4*b^3*c^2*d^4*
e^6*g*i^3 - 6*B*a^5*b^2*c*d^5*e^6*g*i^3 + B*a^6*b*d^6*e^6*g*i^3 - 5*(b*e*x
 + a*e)*B*b^6*c^6*d*e^5*g*i^3/(d*x + c) + 30*(b*e*x + a*e)*B*a*b^5*c^5*d^2
*e^5*g*i^3/(d*x + c) - 75*(b*e*x + a*e)*B*a^2*b^4*c^4*d^3*e^5*g*i^3/(d*x +
 c) + 100*(b*e*x + a*e)*B*a^3*b^3*c^3*d^4*e^5*g*i^3/(d*x + c) - 75*(b*e*x
+ a*e)*B*a^4*b^2*c^2*d^5*e^5*g*i^3/(d*x + c) + 30*(b*e*x + a*e)*B*a^5*b*c*
d^6*e^5*g*i^3/(d*x + c) - 5*(b*e*x + a*e)*B*a^6*d^7*e^5*g*i^3/(d*x + c))*l
og((b*e*x + a*e)/(d*x + c))/(b^5*d^2*e^5 - 5*(b*e*x + a*e)*b^4*d^3*e^4/(d*
x + c) + 10*(b*e*x + a*e)^2*b^3*d^4*e^3/(d*x + c)^2 - 10*(b*e*x + a*e)^3*b
^2*d^5*e^2/(d*x + c)^3 + 5*(b*e*x + a*e)^4*b*d^6*e/(d*x + c)^4 - (b*e*x +
a*e)^5*d^7/(d*x + c)^5) + (6*A*b^10*c^6*e^6*g*i^3 - 5*B*b^10*c^6*e^6*g*i^3
 - 36*A*a*b^9*c^5*d*e^6*g*i^3 + 30*B*a*b^9*c^5*d*e^6*g*i^3 + 90*A*a^2*b^8*
c^4*d^2*e^6*g*i^3 - 75*B*a^2*b^8*c^4*d^2*e^6*g*i^3 - 120*A*a^3*b^7*c^3*d^3
*e^6*g*i^3 + 100*B*a^3*b^7*c^3*d^3*e^6*g*i^3 + 90*A*a^4*b^6*c^2*d^4*e^6*g*
i^3 - 75*B*a^4*b^6*c^2*d^4*e^6*g*i^3 - 36*A*a^5*b^5*c*d^5*e^6*g*i^3 + 30*B
*a^5*b^5*c*d^5*e^6*g*i^3 + 6*A*a^6*b^4*d^6*e^6*g*i^3 - 5*B*a^6*b^4*d^6*e^6
*g*i^3 - 30*(b*e*x + a*e)*A*b^9*c^6*d*e^5*g*i^3/(d*x + c) + 31*(b*e*x + a*
e)*B*b^9*c^6*d*e^5*g*i^3/(d*x + c) + 180*(b*e*x + a*e)*A*a*b^8*c^5*d^2*e^5
*g*i^3/(d*x + c) - 186*(b*e*x + a*e)*B*a*b^8*c^5*d^2*e^5*g*i^3/(d*x + c...
```

### 3.22.9 Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 1192, normalized size of antiderivative = 4.40

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx=\text{Too large to display}$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x^4*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d - B*b*c))/20 - (A*d^2*g*i^3*(
20*a*d + 20*b*c))/80) + x*((a*c*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d
+ 20*A*b*c + B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*
b*d) - (d*g*i^3*(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2 - 3*B*b^2*c^2 + 32
*A*a*b*c*d + 2*B*a*b*c*d))/(4*b) + A*a*c*d^2*g*i^3))/(b*d) - ((20*a*d + 20
*b*c)*(((20*a*d + 20*b*c)*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A
*b*c + B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d) -
 (d*g*i^3*(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2 - 3*B*b^2*c^2 + 32*A*a*b
*c*d + 2*B*a*b*c*d))/(4*b) + A*a*c*d^2*g*i^3))/(20*b*d) - (a*c*((d^2*g*i^3
*(10*A*a*d + 20*A*b*c + B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c)
)/20))/(b*d) + (c*g*i^3*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2 - B*b^2*c^2
 + 12*A*a*b*c*d))/b))/(20*b*d) + (c^2*g*i^3*(12*A*a^2*d^2 + 2*A*b^2*c^2 +
3*B*a^2*d^2 - B*b^2*c^2 + 16*A*a*b*c*d - 2*B*a*b*c*d))/(2*b*d)) - x^3*(((2
0*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d - B*b*c))/5 - (A*
d^2*g*i^3*(20*a*d + 20*b*c))/20))/(60*b*d) - (d*g*i^3*(4*A*a^2*d^2 + 24*A*
b^2*c^2 + B*a^2*d^2 - 3*B*b^2*c^2 + 32*A*a*b*c*d + 2*B*a*b*c*d))/(12*b) +
(A*a*c*d^2*g*i^3)/3) + x^2*(((20*a*d + 20*b*c)*(((20*a*d + 20*b*c)*((d^2*g
*i^3*(10*A*a*d + 20*A*b*c + B*a*d - B*b*c))/5 - (A*d^2*g*i^3*(20*a*d + 20*
b*c))/20))/(20*b*d) - (d*g*i^3*(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2 - 3
*B*b^2*c^2 + 32*A*a*b*c*d + 2*B*a*b*c*d))/(4*b) + A*a*c*d^2*g*i^3))/(40...
```

## 3.22. $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

# 3.23 $\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx$



## 3.23.1 Optimal result

Integrand size = 30, antiderivative size = 149

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = -\frac{B(bc-ad)^3 i^3 x}{4b^3} - \frac{B(bc-ad)^2 i^3 (c+dx)^2}{8b^2 d} - \frac{B(bc-ad) i^3 (c+dx)^3}{12bd} - \frac{B(bc-ad)^4 i^3 \log(a+bx)}{4b^4 d} + \frac{i^3 (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d}$$

output
```
-1/4*B*(-a*d+b*c)^3*i^3*x/b^3-1/8*B*(-a*d+b*c)^2*i^3*(d*x+c)^2/b^2/d-1/12*
B*(-a*d+b*c)*i^3*(d*x+c)^3/b/d-1/4*B*(-a*d+b*c)^4*i^3*ln(b*x+a)/b^4/d+1/4*
i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d
```

## 3.23.2 Mathematica [A] (verified)

Time = 0.04 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.81

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{i^3 \left(-\frac{B(bc-ad)(6bd(bc-ad)^2 x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx))}{6b^4} + (c+dx)^4 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\right)}{4d}$$

input
```
Integrate[(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
(i^3*(-1/6*(B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c +
d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 + (c + d*x
)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])))/(4*d)
```

### 3.23.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 129, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)dx$$

$$\Big\downarrow \text{ 2948}$$

$$\frac{i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d}-\frac{B(bc-ad)\int \frac{i^4(c+dx)^3}{a+bx}dx}{4di}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d}-\frac{Bi^3(bc-ad)\int \frac{(c+dx)^3}{a+bx}dx}{4d}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d}-\frac{Bi^3(bc-ad)\int \left(\frac{(bc-ad)^3}{b^3(a+bx)}+\frac{d(bc-ad)^2}{b^3}+\frac{d(c+dx)(bc-ad)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx}{4d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i^3(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d}-\frac{Bi^3(bc-ad)\left(\frac{(bc-ad)^3\log(a+bx)}{b^4}+\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2(bc-ad)}{2b^2}+\frac{(c+dx)^3}{3b}\right)}{4d}$$

input
```
Int[(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]),x]
```

output
```
-1/4*(B*(b*c - a*d)*i^3*((d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x)^
2)/(2*b^2) + (c + d*x)^3/(3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4))/d + (i
^3*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*d)
```

#### 3.23.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

### 3.23.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 313 vs. $2(139) = 278$.

Time = 0.74 (sec) , antiderivative size = 314, normalized size of antiderivative = 2.11

| method | result |
|---|---|
| risch | $\frac{i^3(dx+c)^4 B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{4d}+\frac{i^3d^3Ax^4}{4}+i^3d^2Ac\,x^3+\frac{i^3d^3Ba\,x^3}{12b}-\frac{i^3d^2Bc\,x^3}{12}+\frac{3i^3dA\,c^2x^2}{2}-\frac{i^3d^3B\,a^2x^2}{8b^2}$ |
| parts | $\frac{A\,i^3(dx+c)^4}{4d}-B\,i^3(ad-cb)^4e^4\left(\frac{1}{8b^2e^2d\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)^2}-\frac{1}{12bed\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)^3}-\frac{\ln(}{}\right.$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{Ad\,e^3i^3\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{4\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^4}-B\,d^2e^3i^3\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\left(\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-c}{d(dx+}\right.\right.}{4b^4e^4d}\right.\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{Ad\,e^3i^3\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}{4\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^4}-B\,d^2e^3i^3\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)\left(\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-c}{d(dx+}\right.\right.}{4b^4e^4d}\right.\right.}{}$ |
| parallelrisch | $\frac{24Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4c^3d\,i^3+24B\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4c\,d^3i^3+21B\,a^3bc\,d^3i^3+36B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4c^2d^2i^3+12B\,x^2a\,b^3c\,d^3}{}$ |

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c))),x,method=_RETURNVERBOSE)
```

output
```
1/4*i^3*(d*x+c)^4*B/d*ln(e*(b*x+a)/(d*x+c))+1/4*i^3*d^3*A*x^4+i^3*d^2*A*c*
x^3+1/12*i^3/b*d^3*B*a*x^3-1/12*i^3*d^2*B*c*x^3+3/2*i^3*d*A*c^2*x^2-1/8*i^
3/b^2*d^3*B*a^2*x^2+1/2*i^3/b*d^2*B*a*c*x^2-3/8*i^3*d*B*c^2*x^2+i^3*A*c^3*
x-1/4*i^3/b^4*d^3*B*ln(b*x+a)*a^4+i^3/b^3*d^2*B*ln(b*x+a)*a^3*c-3/2*i^3/b^
2*d*B*ln(b*x+a)*a^2*c^2+i^3/b*B*ln(b*x+a)*a*c^3-1/4*i^3/d*B*ln(b*x+a)*c^4+
1/4*i^3/b^3*d^3*B*a^3*x-i^3/b^2*d^2*B*a^2*c*x+3/2*i^3/b*d*B*a*c^2*x-3/4*i^
3*B*c^3*x
```

### 3.23.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 322 vs. $2(139) = 278$.

Time = 0.35 (sec) , antiderivative size = 322, normalized size of antiderivative = 2.16

$$\int(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx$$
$$=\frac{6\,Ab^4d^4i^3x^4-6\,Bb^4c^4i^3\log(dx+c)+2\left((12\,A-B)b^4cd^3+Bab^3d^4\right)i^3x^3+3\left(3\left(4\,A-B\right)b^4c^2d^2+4\,Bab^3\right.}{}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="fricas"
)
```

output
```
1/24*(6*A*b^4*d^4*i^3*x^4 - 6*B*b^4*c^4*i^3*log(d*x + c) + 2*((12*A - B)*b
^4*c*d^3 + B*a*b^3*d^4)*i^3*x^3 + 3*(3*(4*A - B)*b^4*c^2*d^2 + 4*B*a*b^3*c
*d^3 - B*a^2*b^2*d^4)*i^3*x^2 + 6*((4*A - 3*B)*b^4*c^3*d + 6*B*a*b^3*c^2*d
^2 - 4*B*a^2*b^2*c*d^3 + B*a^3*b*d^4)*i^3*x + 6*(4*B*a*b^3*c^3*d - 6*B*a^2
*b^2*c^2*d^2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*i^3*log(b*x + a) + 6*(B*b^4*d^
4*i^3*x^4 + 4*B*b^4*c*d^3*i^3*x^3 + 6*B*b^4*c^2*d^2*i^3*x^2 + 4*B*b^4*c^3*
d*i^3*x)*log((b*e*x + a*e)/(d*x + c)))/(b^4*d)
```

### 3.23.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 706 vs. $2(128) = 256$.

Time = 2.02 (sec) , antiderivative size = 706, normalized size of antiderivative = 4.74

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)dx = \frac{Ad^3i^3x^4}{4} - \frac{Bai^3(ad-2bc)(a^2d^2-2abcd+2b^2c^2)\log\left(x+\frac{Ba^4cd^3i^3-4Ba^3bc^2d^2i^3+6Ba^2b^2c^3di^3+\frac{Ba^2di^3(ad-2bc)\left(a^2d^2-2abcd+2b^2c\cdots\right.}{b}}{Ba^4d^4i^3-4Ba^3bcd^3i^3+6Ba^2b^2c^2d^2i^3-4\cdots}\right.}{4b^4} - \frac{Bc^4i^3\log\left(x+\frac{Ba^4cd^3i^3-4Ba^3bc^2d^2i^3+6Ba^2b^2c^3di^3-4Bab^3c^4i^3-\frac{Bb^4c^5i^3}{d}}{Ba^4d^4i^3-4Ba^3bcd^3i^3+6Ba^2b^2c^2d^2i^3-4Bab^3c^3di^3-Bb^4c^4i^3}\right)}{4d} + x^3\left(Acd^2i^3+\frac{Bad^3i^3}{12b}-\frac{Bcd^2i^3}{12}\right) + x^2\cdot\left(\frac{3Ac^2di^3}{2}-\frac{Ba^2d^3i^3}{8b^2}+\frac{Bacd^2i^3}{2b}-\frac{3Bc^2di^3}{8}\right) + x\left(Ac^3i^3+\frac{Ba^3d^3i^3}{4b^3}-\frac{Ba^2cd^2i^3}{b^2}+\frac{3Bac^2di^3}{2b}-\frac{3Bc^3i^3}{4}\right) + \left(Bc^3i^3x+\frac{3Bc^2di^3x^2}{2}+Bcd^2i^3x^3+\frac{Bd^3i^3x^4}{4}\right)\log\left(\frac{e(a+bx)}{c+dx}\right)$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c))),x)
```

output
```
A*d**3*i**3*x**4/4 - B*a*i**3*(a*d - 2*b*c)*(a**2*d**2 - 2*a*b*c*d + 2*b**
2*c**2)*log(x + (B*a**4*c*d**3*i**3 - 4*B*a**3*b*c**2*d**2*i**3 + 6*B*a**2
*b**2*c**3*d*i**3 + B*a**2*d*i**3*(a*d - 2*b*c)*(a**2*d**2 - 2*a*b*c*d + 2
*b**2*c**2)/b - 5*B*a*b**3*c**4*i**3 - B*a*c*i**3*(a*d - 2*b*c)*(a**2*d**2
 - 2*a*b*c*d + 2*b**2*c**2))/(B*a**4*d**4*i**3 - 4*B*a**3*b*c*d**3*i**3 +
6*B*a**2*b**2*c**2*d**2*i**3 - 4*B*a*b**3*c**3*d*i**3 - B*b**4*c**4*i**3))
/(4*b**4) - B*c**4*i**3*log(x + (B*a**4*c*d**3*i**3 - 4*B*a**3*b*c**2*d**2
*i**3 + 6*B*a**2*b**2*c**3*d*i**3 - 4*B*a*b**3*c**4*i**3 - B*b**4*c**5*i**
3/d)/(B*a**4*d**4*i**3 - 4*B*a**3*b*c*d**3*i**3 + 6*B*a**2*b**2*c**2*d**2*
i**3 - 4*B*a*b**3*c**3*d*i**3 - B*b**4*c**4*i**3))/(4*d) + x**3*(A*c*d**2*
i**3 + B*a*d**3*i**3/(12*b) - B*c*d**2*i**3/12) + x**2*(3*A*c**2*d*i**3/2
- B*a**2*d**3*i**3/(8*b**2) + B*a*c*d**2*i**3/(2*b) - 3*B*c**2*d*i**3/8) +
 x*(A*c**3*i**3 + B*a**3*d**3*i**3/(4*b**3) - B*a**2*c*d**2*i**3/b**2 + 3*
B*a*c**2*d*i**3/(2*b) - 3*B*c**3*i**3/4) + (B*c**3*i**3*x + 3*B*c**2*d*i**
3*x**2/2 + B*c*d**2*i**3*x**3 + B*d**3*i**3*x**4/4)*log(e*(a + b*x)/(c + d
*x))
```

### 3.23.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 439 vs. $2(139) = 278$.

Time = 0.27 (sec) , antiderivative size = 439, normalized size of antiderivative = 2.95

$$\int (ci+dix)^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) dx = \frac{1}{4}Ad^3i^3x^4 + Acd^2i^3x^3 + \frac{3}{2}Ac^2di^3x^2$$
$$+ \left(x\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right)Bc^3i^3$$
$$+ \frac{3}{2}\left(x^2\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)Bc^2di^3$$
$$+ \frac{1}{2}\left(2x^3\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{2a^3\log(bx+a)}{b^3} - \frac{2c^3\log(dx+c)}{d^3} - \frac{(b^2cd-abd^2)x^2 - 2(b^2c^2-a}{b^2d^2}\right.$$
$$+ \frac{1}{24}\left(6x^4\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{6a^4\log(bx+a)}{b^4} + \frac{6c^4\log(dx+c)}{d^4} - \frac{2(b^3cd^2-ab^2d^3)x^3 - 3(b^3c^2}{}\right.$$
$$+ Ac^3i^3x$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="maxima"
)
```

output
```
1/4*A*d^3*i^3*x^4 + A*c*d^2*i^3*x^3 + 3/2*A*c^2*d*i^3*x^2 + (x*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*B*c^3*i^3
 + 3/2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 +
c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*B*c^2*d*i^3 + 1/2*(2*x^3*log(b
*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x +
 c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*B
*c*d^2*i^3 + 1/24*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(
b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3
*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*B*d^3*i
^3 + A*c^3*i^3*x
```

### 3.23.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1506 vs. $2(139) = 278$.

Time = 0.47 (sec) , antiderivative size = 1506, normalized size of antiderivative = 10.11

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c))),x, algorithm="giac")
```

output
```
1/24*(6*(B*b^5*c^5*e^5*i^3 - 5*B*a*b^4*c^4*d*e^5*i^3 + 10*B*a^2*b^3*c^3*d^
2*e^5*i^3 - 10*B*a^3*b^2*c^2*d^3*e^5*i^3 + 5*B*a^4*b*c*d^4*e^5*i^3 - B*a^5
*d^5*e^5*i^3)*log((b*e*x + a*e)/(d*x + c))/(b^4*d*e^4 - 4*(b*e*x + a*e)*b^
3*d^2*e^3/(d*x + c) + 6*(b*e*x + a*e)^2*b^2*d^3*e^2/(d*x + c)^2 - 4*(b*e*x
 + a*e)^3*b*d^4*e/(d*x + c)^3 + (b*e*x + a*e)^4*d^5/(d*x + c)^4) + (6*A*b^
8*c^5*e^5*i^3 - 11*B*b^8*c^5*e^5*i^3 - 30*A*a*b^7*c^4*d*e^5*i^3 + 55*B*a*b
^7*c^4*d*e^5*i^3 + 60*A*a^2*b^6*c^3*d^2*e^5*i^3 - 110*B*a^2*b^6*c^3*d^2*e^
5*i^3 - 60*A*a^3*b^5*c^2*d^3*e^5*i^3 + 110*B*a^3*b^5*c^2*d^3*e^5*i^3 + 30*
A*a^4*b^4*c*d^4*e^5*i^3 - 55*B*a^4*b^4*c*d^4*e^5*i^3 - 6*A*a^5*b^3*d^5*e^5
*i^3 + 11*B*a^5*b^3*d^5*e^5*i^3 + 26*(b*e*x + a*e)*B*b^7*c^5*d*e^4*i^3/(d*
x + c) - 130*(b*e*x + a*e)*B*a*b^6*c^4*d^2*e^4*i^3/(d*x + c) + 260*(b*e*x
+ a*e)*B*a^2*b^5*c^3*d^3*e^4*i^3/(d*x + c) - 260*(b*e*x + a*e)*B*a^3*b^4*c
^2*d^4*e^4*i^3/(d*x + c) + 130*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*i^3/(d*x
+ c) - 26*(b*e*x + a*e)*B*a^5*b^2*d^6*e^4*i^3/(d*x + c) - 21*(b*e*x + a*e)
^2*B*b^6*c^5*d^2*e^3*i^3/(d*x + c)^2 + 105*(b*e*x + a*e)^2*B*a*b^5*c^4*d^3
*e^3*i^3/(d*x + c)^2 - 210*(b*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*i^3/(d*x
+ c)^2 + 210*(b*e*x + a*e)^2*B*a^3*b^3*c^2*d^5*e^3*i^3/(d*x + c)^2 - 105*(
b*e*x + a*e)^2*B*a^4*b^2*c*d^6*e^3*i^3/(d*x + c)^2 + 21*(b*e*x + a*e)^2*B*
a^5*b*d^7*e^3*i^3/(d*x + c)^2 + 6*(b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*i^3/(d
*x + c)^3 - 30*(b*e*x + a*e)^3*B*a*b^4*c^4*d^4*e^2*i^3/(d*x + c)^3 + 60...
```

3.23. $\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

### 3.23.9 Mupad [B] (verification not implemented)

Time = 1.43 (sec) , antiderivative size = 566, normalized size of antiderivative = 3.80

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$$

$$= x \left( \frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{d^2\,i^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,b}-\frac{A\,d^2\,i^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{c\,d\,i^3\,(4\,A\,a\,d+6\,A\,b\,c+B\,a\,d-B\,b\,c)}{b}+\frac{A\,a\cdots}{\cdots}\right.}{4\,b\,d} \right.$$

$$\left. + \frac{c^2\,i^3\,(12\,A\,a\,d+8\,A\,b\,c+3\,B\,a\,d-3\,B\,b\,c)}{2\,b} - \frac{a\,c\left(\frac{d^2\,i^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,b}-\frac{A\,d^2\,i^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)}{b\,d} \right)$$

$$- x^2 \left( \frac{\left(\frac{d^2\,i^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d-B\,b\,c)}{4\,b}-\frac{A\,d^2\,i^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d} - \frac{c\,d\,i^3\,(4\,A\,a\,d+6\,A\,b\,c+B\,a\,d-B\,b\,c)}{2\,b} + \frac{A\,a\,c\,d^2\,i^3}{2\,b} \right)$$

$$+ \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(B\,c^3\,i^3\,x+\frac{3\,B\,c^2\,d\,i^3\,x^2}{2}+B\,c\,d^2\,i^3\,x^3+\frac{B\,d^3\,i^3\,x^4}{4}\right)$$

$$+ x^3\left(\frac{d^2\,i^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d-B\,b\,c)}{12\,b}-\frac{A\,d^2\,i^3\,(4\,a\,d+4\,b\,c)}{12\,b}\right)$$

$$- \frac{\ln(a+b\,x)\,(B\,a^4\,d^3\,i^3-4\,B\,a^3\,b\,c\,d^2\,i^3+6\,B\,a^2\,b^2\,c^2\,d\,i^3-4\,B\,a\,b^3\,c^3\,i^3)}{4\,b^4}$$

$$+ \frac{A\,d^3\,i^3\,x^4}{4} - \frac{B\,c^4\,i^3\,\ln(c+d\,x)}{4\,d}$$

input
```
int((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))),x)
```

output
```
x*(((4*a*d + 4*b*c)*((((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d - B*b*c))/(4*b
) - (A*d^2*i^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(4*b*d) - (c*d*i^3
*(4*A*a*d + 6*A*b*c + B*a*d - B*b*c))/b + (A*a*c*d^2*i^3)/b))/(4*b*d) + (c
^2*i^3*(12*A*a*d + 8*A*b*c + 3*B*a*d - 3*B*b*c))/(2*b) - (a*c*((d^2*i^3*(4
*A*a*d + 16*A*b*c + B*a*d - B*b*c))/(4*b) - (A*d^2*i^3*(4*a*d + 4*b*c))/(4
*b)))/(b*d)) - x^2*((((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d - B*b*c))/(4*b)
 - (A*d^2*i^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(8*b*d) - (c*d*i^3*
(4*A*a*d + 6*A*b*c + B*a*d - B*b*c))/(2*b) + (A*a*c*d^2*i^3)/(2*b)) + log(
(e*(a + b*x))/(c + d*x))*((B*d^3*i^3*x^4)/4 + B*c^3*i^3*x + (3*B*c^2*d*i^3
*x^2)/2 + B*c*d^2*i^3*x^3) + x^3*((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d - B
*b*c))/(12*b) - (A*d^2*i^3*(4*a*d + 4*b*c))/(12*b)) - (log(a + b*x)*(B*a^4
*d^3*i^3 - 4*B*a*b^3*c^3*i^3 + 6*B*a^2*b^2*c^2*d*i^3 - 4*B*a^3*b*c*d^2*i^3
))/(4*b^4) + (A*d^3*i^3*x^4)/4 - (B*c^4*i^3*log(c + d*x))/(4*d)
```

## 3.23. $\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\,dx$

# 3.24 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$



## 3.24.1 Optimal result

Integrand size = 40, antiderivative size = 356

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = -\frac{5Bd(bc-ad)^2i^3x}{6b^3g}-\frac{B(bc-ad)i^3(c+dx)^2}{6b^2g}-\frac{5B(bc-ad)^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{6b^4g}+\frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g}+\frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g}+\frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg}-\frac{11B(bc-ad)^3i^3\log(c+dx)}{6b^4g}-\frac{(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}+\frac{B(bc-ad)^3i^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}$$

output
```
-5/6*B*d*(-a*d+b*c)^2*i^3*x/b^3/g-1/6*B*(-a*d+b*c)*i^3*(d*x+c)^2/b^2/g-5/6
*B*(-a*d+b*c)^3*i^3*ln((b*x+a)/(d*x+c))/b^4/g+d*(-a*d+b*c)^2*i^3*(b*x+a)*(
A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g+1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b
*x+a)/(d*x+c)))/b^2/g+1/3*i^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g-11
/6*B*(-a*d+b*c)^3*i^3*ln(d*x+c)/b^4/g-(-a*d+b*c)^3*i^3*(A+B*ln(e*(b*x+a)/(
d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g+B*(-a*d+b*c)^3*i^3*polylog(2,b*(d
*x+c)/d/(b*x+a))/b^4/g
```

### 3.24.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 352, normalized size of antiderivative = 0.99

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$=\frac{i^3\left(6Abd(bc-ad)^2x-3B(bc-ad)^2(bdx+(bc-ad)\log(a+bx))-B(bc-ad)\left(2bd(bc-ad)x+b^2(c+\right.\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x),x]
```

output
```
(i^3*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*Log
[a + b*x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
 - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/
(c + d*x)] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d
*x)]) + 2*b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c -
a*d)^3*Log[g*(a + b*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c -
a*d)^3*Log[c + d*x] - 3*B*(b*c - a*d)^3*(Log[g*(a + b*x)]*(Log[g*(a + b*x)
] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)])))/(6*b^4*g)
```

### 3.24.3 Rubi [A] (verified)

Time = 1.22 (sec) , antiderivative size = 462, normalized size of antiderivative = 1.30, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.350, Rules used = {2962, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{ag+bgx}\,dx$$
$$\Big\downarrow \text{2962}$$
$$\frac{i^3(bc-ad)^3\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \text{2789}$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \text{2756}$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\int \frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \text{54}$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\int \left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\right.\right.}{(a+bx)\left(b-\frac{d(a}{c+}\right.}}{b}\right.}{g}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i^3(bc-ad)^3\left(\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^3}+\frac{1}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{1}{2b\left(b-\frac{d(a}{c-}\right.}\right.}{3d}\right.}{b}\right.}{g}$$

$$\Big\downarrow \text{2789}$$

3.24. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

$$\frac{i^3(bc-ad)^3\left(\frac{\frac{d\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^3}\cdots\right.}{\cdots}\right.}{b}\cdots\right.}{g}$$

$$\downarrow 2756$$

$$\frac{i^3(bc-ad)^3\left(\frac{\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int \frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\cdots\right.}{\cdots}\cdots\right.}{g}$$

$$\downarrow 54$$

$$\frac{i^3(bc-ad)^3\left(\frac{\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int \left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b}+\cdots\right.}{g}$$

$$\downarrow 2009$$

3.24. $\displaystyle\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

$$\frac{i^3(bc-ad)^3\left(\frac{\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}}{b}+\frac{d\left(\frac{B\log \cdots}{3d \cdots}\right.}{\cdots}\right.}{g}$$

$\Big\downarrow$ 2789

$$\frac{i^3(bc-ad)^3\left(\frac{\frac{\frac{d\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}\cdots\right.}{2d}\right.}{b}}{b}\cdots\right.}{g}$$

$\Big\downarrow$ 2751

$$i^3(bc-ad)^3\left(\frac{\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\log\cdots\right.}{\cdots}\right.}{\cdots}}{b}\cdots\right.$$

$\Big\downarrow$ 16

---

3.24. $\displaystyle\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

$$i^3(bc-ad)^3\left(\frac{\frac{\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\cdots\right.}{b\cdots}\right.}{\cdots}\right.}{\cdots}\cdots\right.$$

$\downarrow$ 2779

$$i^3(bc-ad)^3\left(\frac{\frac{\frac{\frac{B\int \frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right.}{bd}\cdots\right.}{b}}{b}\cdots}{b}\cdots\right.$$

$\downarrow$ 2838

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{\frac{\frac{B\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\cdots\right.\right.}{b}}{b}\cdots}{b}\cdots\right.$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x),x
]
```

## 3.24. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

output
```
((b*c - a*d)^3*i^3*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*d*(b - (d*
(a + b*x))/(c + d*x))^3) - (B*(1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1
/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[
b - (d*(a + b*x))/(c + d*x)]/b^3))/(3*d)))/b + ((d*((A + B*Log[(e*(a + b*x
))/(c + d*x)])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(1/(b*(b - (d*(a
 + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x)
)/(c + d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c
 + d*x)]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*(a
+ b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*L
og[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(
a + b*x))])/b)/b)/b)/b))/g
```

### 3.24.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

## 3.24. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.24.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1085 vs. $2(344) = 688$.

Time = 1.81 (sec) , antiderivative size = 1086, normalized size of antiderivative = 3.05

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1086 |
| derivativedivides | Expression too large to display | 1178 |
| default | Expression too large to display | 1178 |
| risch | Expression too large to display | 5361 |

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x,method=_RETURN
VERBOSE)
```

output
```
i^3*A/g*(d/b^3*(1/3*b^2*d^2*x^3-1/2*a*b*d^2*x^2+3/2*b^2*c*d*x^2+a^2*d^2*x-
3*a*b*c*d*x+3*b^2*c^2*x)+(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^
4*ln(b*x+a))-i^3*B/g/d^5*(a*d-b*c)^4*e^4*(1/(a*d-b*c)*d^6/b^3/e^3*(1/b/e/d
*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))-1/
(a*d-b*c)*d^6/b^2/e^2*(-1/2/e^2/b^2/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b
*e)-1/2/e/b/d/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)+1/2*ln(b*e/d+(a*d-b*c)
*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*((b*e/d+(a*d-b*c)*e/d/(d*x+c))
*d-2*b*e)/e^2/b^2/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)^2)+1/2/(a*d-b*c)*d
^5/b^4/e^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+1/(a*d-b*c)*d^6/b/e*(-1/6/b/e
/d/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)^2+1/3/b^2/e^2/d/((b*e/d+(a*d-b*c)
*e/d/(d*x+c))*d-b*e)+1/3/b^3/e^3/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)
-1/3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(3*e^2*
b^2-3*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d*b*e+d^2*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^2)/b^3/e^3/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)^3)-1/(a*d-b*c)*d^6/b^4/
e^4*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c
)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d))
```

### 3.24.5 Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algori
thm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log((b
*e*x + a*e)/(d*x + c)))/(b*g*x + a*g), x)
```

### 3.24.6 Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \frac{i^3\left(\int \frac{Ac^3}{a+bx}\,dx+\int \frac{Ad^3x^3}{a+bx}\,dx+\int \frac{Bc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx+\int \frac{3Acd^2x^2}{a+bx}\,dx+\int \frac{3Ac^2dx}{a+bx}\,dx+\int \frac{Bd^3x^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,d\right.}{g}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x)
```

output
```
i**3*(Integral(A*c**3/(a + b*x), x) + Integral(A*d**3*x**3/(a + b*x), x) +
 Integral(B*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Inte
gral(3*A*c*d**2*x**2/(a + b*x), x) + Integral(3*A*c**2*d*x/(a + b*x), x) +
 Integral(B*d**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) +
 Integral(3*B*c*d**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x),
x) + Integral(3*B*c**2*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x),
 x))/g
```

### 3.24.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 850 vs. $2(343) = 686$.

Time = 0.31 (sec) , antiderivative size = 850, normalized size of antiderivative = 2.39

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx$$
$$= 3\,Ac^2di^3\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)-\frac{1}{6}\,Ad^3i^3\left(\frac{6\,a^3\log(bx+a)}{b^4g}-\frac{2\,b^2x^3-3\,abx^2+6\,a^2x}{b^3g}\right)$$
$$+\frac{3}{2}\,Acd^2i^3\left(\frac{2\,a^2\log(bx+a)}{b^3g}+\frac{bx^2-2\,ax}{b^2g}\right)+\frac{Ac^3i^3\log(bgx+ag)}{bg}$$
$$-\frac{(11\,b^2c^3i^3-15\,abc^2di^3+6\,a^2cd^2i^3)B\log(dx+c)}{6\,b^3g}$$
$$+\frac{(b^3c^3i^3-3\,ab^2c^2di^3+3\,a^2bcd^2i^3-a^3d^3i^3)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^4g}$$
$$+\frac{2\,Bb^3d^3i^3x^3\log(e)+((9\,i^3\log(e)-i^3)b^3cd^2-(3\,i^3\log(e)-i^3)ab^2d^3)Bx^2+3\,(b^3c^3i^3-3\,ab^2c^2di^3+3\,a}{}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algori
thm="maxima")
```

output
```
3*A*c^2*d*i^3*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - 1/6*A*d^3*i^3*(6*a^3*lo
g(b*x + a)/(b^4*g) - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/(b^3*g)) + 3/2*A*c*
d^2*i^3*(2*a^2*log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A*c^3*i^3
*log(b*g*x + a*g)/(b*g) - 1/6*(11*b^2*c^3*i^3 - 15*a*b*c^2*d*i^3 + 6*a^2*c
*d^2*i^3)*B*log(d*x + c)/(b^3*g) + (b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^
2*b*c*d^2*i^3 - a^3*d^3*i^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) +
 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^4*g) + 1/6*(2*B*b^3*d^3*i^3*
x^3*log(e) + ((9*i^3*log(e) - i^3)*b^3*c*d^2 - (3*i^3*log(e) - i^3)*a*b^2*
d^3)*B*x^2 + 3*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^2*i^3 - a^3*
d^3*i^3)*B*log(b*x + a)^2 + ((18*i^3*log(e) - 7*i^3)*b^3*c^2*d - 6*(3*i^3*
log(e) - 2*i^3)*a*b^2*c*d^2 + (6*i^3*log(e) - 5*i^3)*a^2*b*d^3)*B*x + (2*B
*b^3*d^3*i^3*x^3 + 3*(3*b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B*x^2 + 6*(3*b^3*c^
2*d*i^3 - 3*a*b^2*c*d^2*i^3 + a^2*b*d^3*i^3)*B*x + (6*b^3*c^3*i^3*log(e) -
 18*(i^3*log(e) - i^3)*a*b^2*c^2*d + 9*(2*i^3*log(e) - 3*i^3)*a^2*b*c*d^2
- (6*i^3*log(e) - 11*i^3)*a^3*d^3)*B)*log(b*x + a) - (2*B*b^3*d^3*i^3*x^3
+ 3*(3*b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2
*c*d^2*i^3 + a^2*b*d^3*i^3)*B*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a
^2*b*c*d^2*i^3 - a^3*d^3*i^3)*B*log(b*x + a))*log(d*x + c))/(b^4*g)
```

### 3.24.8 Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g),x, algori
thm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
, x)
```

### 3.24.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x),x
)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x),
x)
```

# 3.25 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$



## 3.25.1 Optimal result

Integrand size = 40, antiderivative size = 373

$$\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx \\
&= -\frac{Bd^2(bc-ad)i^3x}{2b^3g^2}-\frac{B(bc-ad)^2i^3(c+dx)}{b^3g^2(a+bx)}-\frac{Bd(bc-ad)^2i^3\log\left(\frac{a+bx}{c+dx}\right)}{2b^4g^2} \\
&\quad+\frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2} \\
&\quad-\frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2(a+bx)}+\frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^2} \\
&\quad-\frac{5Bd(bc-ad)^2i^3\log(c+dx)}{2b^4g^2}-\frac{3d(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2} \\
&\quad+\frac{3Bd(bc-ad)^2i^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}
\end{aligned}$$

output
```
-1/2*B*d^2*(-a*d+b*c)*i^3*x/b^3/g^2-B*(-a*d+b*c)^2*i^3*(d*x+c)/b^3/g^2/(b*
x+a)-1/2*B*d*(-a*d+b*c)^2*i^3*ln((b*x+a)/(d*x+c))/b^4/g^2+2*d^2*(-a*d+b*c)
*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^2-(-a*d+b*c)^2*i^3*(d*x+c)*
(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g^2/(b*x+a)+1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))/b^2/g^2-5/2*B*d*(-a*d+b*c)^2*i^3*ln(d*x+c)/b^4/g^2-3*d*(
-a*d+b*c)^2*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/
g^2+3*B*d*(-a*d+b*c)^2*i^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^2
```

### 3.25.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.00

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{i^3\left(2Abd^2(3bc-2ad)x - bBd^2(bc-ad)x - \frac{2B(bc-ad)^3}{a+bx} - a^2Bd^3\log(a+bx) - 2Bd(bc-ad)^2\log(a+bx) \cdots\right.}{\cdots}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^2,x]
```

output
```
(i^3*(2*A*b*d^2*(3*b*c - 2*a*d)*x - b*B*d^2*(b*c - a*d)*x - (2*B*(b*c - a*
d)^3)/(a + b*x) - a^2*B*d^3*Log[a + b*x] - 2*B*d*(b*c - a*d)^2*Log[a + b*x
] + 2*B*d^2*(3*b*c - 2*a*d)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*d
^3*x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - (2*(b*c - a*d)^3*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(a + b*x) + 6*d*(b*c - a*d)^2*Log[a + b*x]*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]) + b^2*B*c^2*d*Log[c + d*x] + 2*B*d*(b*c -
 a*d)^2*Log[c + d*x] - 2*B*d*(-(b*c) + a*d)*(-3*b*c + 2*a*d)*Log[c + d*x]
- 3*B*d*(b*c - a*d)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b
*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(2*b^4*g^2)
```

### 3.25.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 326, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2962, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^2}\,dx$$

$$\Big\downarrow \ 2962$$

$$\frac{i^3(bc-ad)^2\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{2793}$$

$$\frac{i^3(bc-ad)^2\int\left(\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d^2}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d^2}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{b^3(a+bx)^2}}{g^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i^3(bc-ad)^2\left(\frac{2d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{3d\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3(a+bx)}+\frac{d\left(B\log\left(\right.\right.}{2b^2\left(b-\right.}}{g^2}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^2
,x]
```

output
```
((b*c - a*d)^2*i^3*(-((B*(c + d*x))/(b^3*(a + b*x))) - (B*d)/(2*b^3*(b - (
d*(a + b*x))/(c + d*x))) - (B*d*Log[(a + b*x)/(c + d*x)])/(2*b^4) - ((c +
d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^3*(a + b*x)) + (d*(A + B*Log
[(e*(a + b*x))/(c + d*x)]))/(2*b^2*(b - (d*(a + b*x))/(c + d*x))^2) + (2*d
^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^4*(c + d*x)*(b - (d*
(a + b*x))/(c + d*x))) + (5*B*d*Log[b - (d*(a + b*x))/(c + d*x)])/(2*b^4)
- (3*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/b^4 + (3*B*d*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b^4))/g^2
```

### 3.25.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.25.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. $2(365) = 730$.

Time = 1.66 (sec) , antiderivative size = 863, normalized size of antiderivative = 2.31

| method | result |
|---|---|
| parts | $\frac{i^3 A\left(\frac{d^2\left(\frac{1}{2}bd\,x^2-2xad+3bcx\right)}{b^3}+\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)\ln(bx+a)}{b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{b^4(bx+a)}\right)}{g^2}-\frac{i^3B(ad-cb)^4e^4\left(-\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{A\,d^2e^3i^3(ad-cb)\left(-\frac{3d\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^4e^4}+\frac{2d}{b^3e^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{d}{2b^2e^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.\right.}\right.}{g^2}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{A\,d^2e^3i^3(ad-cb)\left(-\frac{3d\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{b^4e^4}+\frac{2d}{b^3e^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{d}{2b^2e^2\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.\right.}\right.}{g^2}\right.}{}$ |
| risch | Expression too large to display |

3.25. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x,method=_RETU
RNVERBOSE)
```

output
```
i^3*A/g^2*(d^2/b^3*(1/2*b*d*x^2-2*x*a*d+3*b*c*x)+3/b^4*d*(a^2*d^2-2*a*b*c*
d+b^2*c^2)*ln(b*x+a)-1/b^4*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/
(b*x+a))-i^3*B/g^2/d^5*(a*d-b*c)^4*e^4*(-2/(a*d-b*c)^2*d^7/b^3/e^3*(1/b/e/
d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*
(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))+1
/(a*d-b*c)^2*d^7/b^2/e^2*(-1/2/e^2/b^2/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*
d-b*e)-1/2/e/b/d/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)+1/2*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*((b*e/d+(a*d-b*c)*e/d/(d*x+
c))*d-2*b*e)/e^2/b^2/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)^2)-1/(a*d-b*c)^
2*d^5/b^3/e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-3/2/(a*d-b*c)^2*d^6/b^4/e^4*ln(b*e/
d+(a*d-b*c)*e/d/(d*x+c))^2+3/(a*d-b*c)^2*d^7/b^4/e^4*(dilog(-((b*e/d+(a*d-
b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/
d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d))
```

### 3.25.5 Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algo
rithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log((b
*e*x + a*e)/(d*x + c)))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

### 3.25.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

### 3.25.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1501 vs. $2(364) = 728$.

Time = 0.35 (sec) , antiderivative size = 1501, normalized size of antiderivative = 4.02

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algo
rithm="maxima")
```

output
```
-3*A*(a^2/(b^4*g^2*x + a*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^
2))*c*d^2*i^3 + 1/2*(2*a^3/(b^5*g^2*x + a*b^4*g^2) + 6*a^2*log(b*x + a)/(b
^4*g^2) + (b*x^2 - 4*a*x)/(b^3*g^2))*A*d^3*i^3 + 3*A*c^2*d*i^3*(a/(b^3*g^2
*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - B*c^3*i^3*(log(b*e*x/(d*x + c)
 + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^2*x + a*b*g^2) + d*log(
b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - A
*c^3*i^3/(b^2*g^2*x + a*b*g^2) - 1/2*(5*b^3*c^3*d*i^3 - 3*a*b^2*c^2*d^2*i^
3 - 2*a^2*b*c*d^3*i^3 + 2*a^3*d^4*i^3)*B*log(d*x + c)/(b^5*c*g^2 - a*b^4*d
*g^2) + 1/2*((b^4*c*d^3*i^3*log(e) - a*b^3*d^4*i^3*log(e))*B*x^3 + ((6*i^3
*log(e) - i^3)*b^4*c^2*d^2 - (9*i^3*log(e) - 2*i^3)*a*b^3*c*d^3 + (3*i^3*l
og(e) - i^3)*a^2*b^2*d^4)*B*x^2 + ((6*i^3*log(e) - i^3)*a*b^3*c^2*d^2 - 2*
(5*i^3*log(e) - i^3)*a^2*b^2*c*d^3 + (4*i^3*log(e) - i^3)*a^3*b*d^4)*B*x +
 3*((b^4*c^3*d*i^3 - 3*a*b^3*c^2*d^2*i^3 + 3*a^2*b^2*c*d^3*i^3 - a^3*b*d^4
*i^3)*B*x + (a*b^3*c^3*d*i^3 - 3*a^2*b^2*c^2*d^2*i^3 + 3*a^3*b*c*d^3*i^3 -
 a^4*d^4*i^3)*B)*log(b*x + a)^2 + 2*(3*(i^3*log(e) + i^3)*a*b^3*c^3*d - 6*
(i^3*log(e) + i^3)*a^2*b^2*c^2*d^2 + 4*(i^3*log(e) + i^3)*a^3*b*c*d^3 - (i
^3*log(e) + i^3)*a^4*d^4)*B + ((b^4*c*d^3*i^3 - a*b^3*d^4*i^3)*B*x^3 + 3*(
2*b^4*c^2*d^2*i^3 - 3*a*b^3*c*d^3*i^3 + a^2*b^2*d^4*i^3)*B*x^2 + (6*b^4*c^
3*d*i^3*log(e) - 18*(i^3*log(e) - i^3)*a*b^3*c^2*d^2 + 9*(2*i^3*log(e) - 3
*i^3)*a^2*b^2*c*d^3 - (6*i^3*log(e) - 11*i^3)*a^3*b*d^4)*B*x - (18*a^2*...
```

## 3.25.8 Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2,x, algo
rithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
^2, x)
```

### 3.25.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{{\left(c\,i+d\,i\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}{{\left(a\,g+b\,g\,x\right)}^2}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2
,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^2
, x)
```

# 3.26 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$



## 3.26.1 Optimal result

Integrand size = 40, antiderivative size = 345

$$\begin{aligned}\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx &= -\frac{2Bd(bc-ad)i^3(c+dx)}{b^3g^3(a+bx)}-\frac{B(bc-ad)i^3(c+dx)^2}{4b^2g^3(a+bx)^2}\\
&\quad+\frac{d^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^3}-\frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^3(a+bx)}\\
&\quad-\frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^3(a+bx)^2}-\frac{Bd^2(bc-ad)i^3\log(c+dx)}{b^4g^3}\\
&\quad-\frac{3d^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}\\
&\quad+\frac{3Bd^2(bc-ad)i^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}\end{aligned}$$

output
```
-2*B*d*(-a*d+b*c)*i^3*(d*x+c)/b^3/g^3/(b*x+a)-1/4*B*(-a*d+b*c)*i^3*(d*x+c)
^2/b^2/g^3/(b*x+a)^2+d^3*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^3-2
*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g^3/(b*x+a)-1/2*
(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g^3/(b*x+a)^2-B*d
^2*(-a*d+b*c)*i^3*ln(d*x+c)/b^4/g^3-3*d^2*(-a*d+b*c)*i^3*(A+B*ln(e*(b*x+a)
/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^3+3*B*d^2*(-a*d+b*c)*i^3*polylo
g(2,b*(d*x+c)/d/(b*x+a))/b^4/g^3
```

### 3.26.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 314, normalized size of antiderivative = 0.91

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{i^3\left(4Abd^3x-\frac{B(bc-ad)^3}{(a+bx)^2}-\frac{10Bd(bc-ad)^2}{a+bx}+10Bd^2(-bc+ad)\log(a+bx)+4Bd^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-\frac{2(bc}{}\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^3,x]
```

output
```
(i^3*(4*A*b*d^3*x - (B*(b*c - a*d)^3)/(a + b*x)^2 - (10*B*d*(b*c - a*d)^2)
/(a + b*x) + 10*B*d^2*(-(b*c) + a*d)*Log[a + b*x] + 4*B*d^3*(a + b*x)*Log[
(e*(a + b*x))/(c + d*x)] - (2*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(a + b*x)^2 - (12*d*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(a + b*x) + 12*d^2*(b*c - a*d)*Log[a + b*x]*(A + B*Log[(e*(a + b*x))
/(c + d*x)]) + 6*B*d^2*(b*c - a*d)*Log[c + d*x] + 6*B*d^2*(-(b*c) + a*d)*(
Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog
[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(4*b^4*g^3)
```

### 3.26.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 293, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2962, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^3}\,dx$$
$$\Big\downarrow \; 2962$$
$$\frac{i^3(bc-ad)\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^3}$$
$$\Big\downarrow \; 2793$$

$$\frac{i^3(bc-ad)\int\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d^3}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d^2}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d}{b^3(a+bx)^2}+\frac{(c+dx)^3\left(A+B\log\left(\frac{e}{}\right.\right.}{b^2(a+bx)^3}\right.}{g^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i^3(bc-ad)\left(\frac{d^3(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{3d^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4}-\frac{2d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3(a+bx)}-\frac{(c+dx)^2}{}\right.}{g^3}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^3
,x]
```

output
```
((b*c - a*d)*i^3*((-2*B*d*(c + d*x))/(b^3*(a + b*x)) - (B*(c + d*x)^2)/(4*
b^2*(a + b*x)^2) - (2*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b
^3*(a + b*x)) - ((c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b^2*
(a + b*x)^2) + (d^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^4*(
c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*d^2*Log[b - (d*(a + b*x))/(c
+ d*x)])/b^4 - (3*d^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c +
 d*x))/(d*(a + b*x))])/b^4 + (3*B*d^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x
))])/b^4))/g^3
```

### 3.26.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.26.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 731 vs. $2(341) = 682$.

Time = 1.61 (sec) , antiderivative size = 732, normalized size of antiderivative = 2.12

| method | result |
|---|---|
| parts | $\frac{i^3 A\left(\frac{x d^3}{b^3}-\frac{3d^2(ad-cb)\ln(bx+a)}{b^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{2b^4(bx+a)^2}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{b^4(bx+a)}\right)}{g^3} - \frac{i^3B(ad-cb)^4e^4\left(\frac{d^8\left(\frac{\ln(\ldots}{\ldots}\right.}{\ldots}\ldots\right.}{\ldots}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{i^3d^2e^3A\left(-\frac{1}{2b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{3d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^4e^4}-\frac{2d}{b^3e^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^2}{b^3e^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\ldots\right.\right.}\ldots\right)}{g^3}\ldots\right.}{\ldots}$ |
| default | $-\frac{e(ad-cb)\left(\frac{i^3d^2e^3A\left(-\frac{1}{2b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{3d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^4e^4}-\frac{2d}{b^3e^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^2}{b^3e^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\ldots\right.\right.}\ldots\right)}{g^3}\ldots\right.}{\ldots}$ |
| risch | Expression too large to display |

3.26. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x,method=_RETU
RNVERBOSE)
```

output
```
i^3*A/g^3*(x*d^3/b^3-3/b^4*d^2*(a*d-b*c)*ln(b*x+a)-1/2/b^4*(-a^3*d^3+3*a^2
*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/(b*x+a)^2-3/b^4*d*(a^2*d^2-2*a*b*c*d+b^2*c
^2)/(b*x+a))-i^3*B/g^3/d^5*(a*d-b*c)^4*e^4*(1/(a*d-b*c)^3*d^8/b^3/e^3*(1/b
/e/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c
))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)
)+2/(a*d-b*c)^3*d^6/b^3/e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+3/2/(a*d-b*c)^3*d^7/b
^4/e^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-3/(a*d-b*c)^3*d^8/b^4/e^4*(dilog(
-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)+1/(a*d-b*c)^3*d^5/b^
2/e^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

### 3.26.5 Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algo
rithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log((b
*e*x + a*e)/(d*x + c)))/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a
^3*g^3), x)
```

### 3.26.6 Sympy [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{i^3 \left(\int \frac{Ac^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{Ad^3x^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{Bc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{3Acd^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right.}{}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3,x)
```

output
```
i**3*(Integral(A*c**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
+ Integral(A*d**3*x**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
 + Integral(B*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x
 + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*A*c*d**2*x**2/(a**3 + 3*a**
2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*A*c**2*d*x/(a**3 + 3*a
**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(B*d**3*x**3*log(a*e/(c
 + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3)
, x) + Integral(3*B*c*d**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3
 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*B*c**2*d*x*log
(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**
3*x**3), x))/g**3
```

### 3.26.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2302 vs. $2(340) = 680$.

Time = 0.36 (sec) , antiderivative size = 2302, normalized size of antiderivative = 6.67

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algo
rithm="maxima")
```

output
```
-3/4*B*c^2*d*i^3*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*
g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2*b^2*c - a
*b*d)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2
*b^3*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*
a*b^3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2
 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3)) - 1/2*A*d^3*i^3*((6*a^2*b*x + 5*a^3)/(
b^6*g^3*x^2 + 2*a*b^5*g^3*x + a^2*b^4*g^3) - 2*x/(b^3*g^3) + 6*a*log(b*x +
 a)/(b^4*g^3)) + 3/2*A*c*d^2*i^3*((4*a*b*x + 3*a^2)/(b^5*g^3*x^2 + 2*a*b^4
*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) + 1/4*B*c^3*i^3*((2*b*d*
x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*
x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b
^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2
*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d
+ a^2*b*d^2)*g^3)) - 3/2*(2*b*x + a)*A*c^2*d*i^3/(b^4*g^3*x^2 + 2*a*b^3*g^
3*x + a^2*b^2*g^3) - 1/2*A*c^3*i^3/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^
3) - 1/2*(2*b^3*c^3*d^2*i^3 + 8*a*b^2*c^2*d^3*i^3 - 13*a^2*b*c*d^4*i^3 + 5
*a^3*d^5*i^3)*B*log(d*x + c)/(b^6*c^2*g^3 - 2*a*b^5*c*d*g^3 + a^2*b^4*d^2*
g^3) + 1/4*(4*(b^5*c^2*d^3*i^3*log(e) - 2*a*b^4*c*d^4*i^3*log(e) + a^2*b^3
*d^5*i^3*log(e))*B*x^3 + 8*(a*b^4*c^2*d^3*i^3*log(e) - 2*a^2*b^3*c*d^4*i^3
*log(e) + a^3*b^2*d^5*i^3*log(e))*B*x^2 + 2*(12*(i^3*log(e) + i^3)*a*b^...
```

### 3.26.8 Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3,x, algo
rithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
^3, x)
```

### 3.26.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(a\,g+b\,g\,x)^3}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3
,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^3
, x)
```

# 3.27 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$



## 3.27.1 Optimal result

Integrand size = 40, antiderivative size = 310

$$\begin{aligned}\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = &-\frac{Bd^2i^3(c+dx)}{b^3g^4(a+bx)}-\frac{Bdi^3(c+dx)^2}{4b^2g^4(a+bx)^2}\\&-\frac{Bi^3(c+dx)^3}{9bg^4(a+bx)^3}\\&-\frac{d^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^4(a+bx)}\\&-\frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^4(a+bx)^2}\\&-\frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bg^4(a+bx)^3}\\&-\frac{d^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}\\&+\frac{Bd^3i^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}\end{aligned}$$

output
```
-B*d^2*i^3*(d*x+c)/b^3/g^4/(b*x+a)-1/4*B*d*i^3*(d*x+c)^2/b^2/g^4/(b*x+a)^2
-1/9*B*i^3*(d*x+c)^3/b/g^4/(b*x+a)^3-d^2*i^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*
x+c)))/b^3/g^4/(b*x+a)-1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2
/g^4/(b*x+a)^2-1/3*i^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g^4/(b*x+a)
^3-d^3*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^4+B
*d^3*i^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^4
```

## 3.27.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 308, normalized size of antiderivative = 0.99

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{i^3\left(-\frac{4B(bc-ad)^3}{(a+bx)^3}-\frac{21Bd(bc-ad)^2}{(a+bx)^2}+\frac{66Bd^2(-bc+ad)}{a+bx}-66Bd^3\log(a+bx)-\frac{12(bc-ad)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}-\frac{54d(bc-ad}{}\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^4,x]
```

output
```
(i^3*((-4*B*(b*c - a*d)^3)/(a + b*x)^3 - (21*B*d*(b*c - a*d)^2)/(a + b*x)^
2 + (66*B*d^2*(-(b*c) + a*d))/(a + b*x) - 66*B*d^3*Log[a + b*x] - (12*(b*c
 - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^3 - (54*d*(b*c -
 a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^2 + (108*d^2*(-(b*
c) + a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + 36*d^3*Log[a +
 b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 66*B*d^3*Log[c + d*x] - 18*B*
d^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(36*b^4*g^4)
```

## 3.27.3 Rubi [A] (verified)

Time = 0.95 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.87, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.225$, Rules used = {2962, 2780, 2741, 2780, 2741, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^4} \, dx$$

$$\Big\downarrow \; \color{blue}{2962}$$

$$\frac{i^3 \int \frac{(c+dx)^4 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^4}$$

$$\Big\downarrow \; \color{blue}{2780}$$

$$\frac{i^3 \left(\frac{\int \frac{(c+dx)^4 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{b}+\frac{d \int \frac{(c+dx)^3 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^4}$$

$$\Big\downarrow \; \color{blue}{2741}$$

$$\frac{i^3 \left(\frac{d \int \frac{(c+dx)^3 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)^3 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}-\frac{B(c+dx)^3}{9(a+bx)^3}}{b}\right)}{g^4}$$

$$\Big\downarrow \; \color{blue}{2780}$$

$$\frac{i^3 \left(\frac{d \left(\frac{\int \frac{(c+dx)^3 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d \int \frac{(c+dx)^2 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^3 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}-\frac{B(c+dx)^3}{9(a+bx)^3}}{b}\right)}{g^4}$$

$$\Big\downarrow \; \color{blue}{2741}$$

---

3.27. $\int \frac{(ci+dix)^3 \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4} \, dx$

$$\frac{i^3\left(\frac{d\left(\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}}{b}\right)}{b}+\frac{-\frac{(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}-\frac{B(c+dx)^3}{9(a+bx)^3}}{b}\right)}{g^4}$$

$\downarrow$ 2780

$$\frac{i^3\left(\frac{d\left(\frac{d\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}}{b}\right)}{b}+\frac{-\frac{(\ldots}{}}{}\right)}{g^4}$$

$\downarrow$ 2741

---

3.27. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$

$$\frac{i^3\left(\frac{d\left(\frac{d\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}}{b}\right)}{b}+\cdots\right)}{g^4}$$

$\downarrow$ 2779

$$\frac{i^3\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{B\int \frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2}{\cdots}}{\cdots}\cdots\right)}{b}\cdots\right)}{g^4}$$

3.27. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$

$$\Big\downarrow \textcolor{blue}{2838}$$

$$\frac{i^3\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{B\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}\right)}{b}\right)+\frac{-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}}{b}}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+\cdots}{c+\cdots}\right.\right.}{2(a+bx)^2}\cdots}{}}{b}\cdots\right.}{g^4}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^4
,x]
```

output
```
(i^3*((-1/9*(B*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(3*(a + b*x)^3))/b + (d*((-1/4*(B*(c + d*x)^2)/(a + b*x
)^2 - ((c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2))/
b + (d*((-((B*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(a + b*x))/
(c + d*x)]))/(a + b*x))/b + (d*(-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*Lo
g[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(a
 + b*x))])/b))/b))/b))/b))/g^4
```

### 3.27.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.27.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(302) = 604$.

Time = 1.61 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.21

| method | result |
|---|---|
| parts | $\frac{i^3 A\left(-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{3b^4(bx+a)^3}+\frac{d^3\ln(bx+a)}{b^4}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{2b^4(bx+a)^2}+\frac{3d^2(ad-cb)}{b^4(bx+a)}\right)}{g^4}-\frac{i^3B(ad-cb)^4e^4\left(-\frac{d^5\left(\cdots\right)}{\cdots}\cdots\right.}{\cdots}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{i^3d^2e^3A\left(-\frac{1}{3be\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{d^3\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^4e^4}-\frac{d^2}{b^3e^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d}{2b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\cdots\right)}{(ad-cb)g^4}\cdots\right.}{\cdots}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{i^3d^2e^3A\left(-\frac{1}{3be\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{d^3\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{b^4e^4}-\frac{d^2}{b^3e^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{d}{2b^2e^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\cdots\right)}{(ad-cb)g^4}\cdots\right.}{\cdots}$ |
| risch | Expression too large to display |

3.27. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x,method=_RETU
RNVERBOSE)
```

output
```
i^3*A/g^4*(-1/3*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)
^3+d^3/b^4*ln(b*x+a)-3/2*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^2+3/b^4
*d^2*(a*d-b*c)/(b*x+a))-i^3*B/g^4/d^5*(a*d-b*c)^4*e^4*(-1/(a*d-b*c)^4*d^5/
b/e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-
1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-1/(a*d-b*c)^4*d^7/b^3/e^3*(-1/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*
e/d/(d*x+c)))-1/2/(a*d-b*c)^4*d^8/b^4/e^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
2+1/(a*d-b*c)^4*d^9/b^4/e^4*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/
b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*
d-b*e)/b/e)/d)-1/(a*d-b*c)^4*d^6/b^2/e^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

### 3.27.5 Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^4}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algo
rithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log((b
*e*x + a*e)/(d*x + c)))/(b^4*g^4*x^4 + 4*a*b^3*g^4*x^3 + 6*a^2*b^2*g^4*x^2
 + 4*a^3*b*g^4*x + a^4*g^4), x)
```

### 3.27.6 Sympy [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx$$

$$=\frac{i^3\left(\int \frac{Ac^3}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{Ad^3x^3}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\int \frac{Bc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx+\right.}{}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4,x)
```

output
```
i**3*(Integral(A*c**3/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**
3 + b**4*x**4), x) + Integral(A*d**3*x**3/(a**4 + 4*a**3*b*x + 6*a**2*b**2
*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B*c**3*log(a*e/(c + d*x)
 + b*e*x/(c + d*x))/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3
+ b**4*x**4), x) + Integral(3*A*c*d**2*x**2/(a**4 + 4*a**3*b*x + 6*a**2*b*
*2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*A*c**2*d*x/(a**4 + 4
*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B
*d**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**4 + 4*a**3*b*x + 6*a**
2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*B*c*d**2*x**2*lo
g(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 +
 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*B*c**2*d*x*log(a*e/(c + d*x)
+ b*e*x/(c + d*x))/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 +
 b**4*x**4), x))/g**4
```

### 3.27.7 Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^4}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algo
rithm="maxima")
```

output
```
-1/6*B*d^3*i^3*((18*a*b^2*x^2 + 27*a^2*b*x + 11*a^3 + 6*(b^3*x^3 + 3*a*b^2
*x^2 + 3*a^2*b*x + a^3)*log(b*x + a))*log(d*x + c)/(b^7*g^4*x^3 + 3*a*b^6*
g^4*x^2 + 3*a^2*b^5*g^4*x + a^3*b^4*g^4) - 6*integrate(1/6*(6*b^4*d*x^4*lo
g(e) + 45*a^2*b^2*d*x^2 + 38*a^3*b*d*x + 11*a^4*d + 6*(b^4*c*log(e) + 3*a*
b^3*d)*x^3 + 6*(2*b^4*d*x^4 + 6*a^2*b^2*d*x^2 + 4*a^3*b*d*x + a^4*d + (b^4
*c + 4*a*b^3*d)*x^3)*log(b*x + a))/(b^8*d*g^4*x^5 + a^4*b^4*c*g^4 + (b^8*c
*g^4 + 4*a*b^7*d*g^4)*x^4 + 2*(2*a*b^7*c*g^4 + 3*a^2*b^6*d*g^4)*x^3 + 2*(3
*a^2*b^6*c*g^4 + 2*a^3*b^5*d*g^4)*x^2 + (4*a^3*b^5*c*g^4 + a^4*b^4*d*g^4)*
x), x)) - 1/6*B*c*d^2*i^3*(6*(3*b^2*x^2 + 3*a*b*x + a^2)*log(b*e*x/(d*x +
c) + a*e/(d*x + c))/(b^6*g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3
*b^3*g^4) + (11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c^2 - 3*a
*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a^3*b*d^2
)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2 - 2*a^2
*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d + a^4*b^4
*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*(3*b^2*
c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*
a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3
)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*
g^4)) - 1/12*B*c^2*d*i^3*(6*(3*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c
))/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) + (5...
```

### 3.27.8 Giac [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(bgx+ag)^4}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4,x, algo
rithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(b*g*x + a*g)
^4, x)
```

### 3.27.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^4}\,dx=\int \frac{{\left(c\,i+d\,i\,x\right)}^3\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}{{\left(a\,g+b\,g\,x\right)}^4}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^4
,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^4
, x)
```

# 3.28 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx$



## 3.28.1 Optimal result

Integrand size = 40, antiderivative size = 89

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = -\frac{Bi^3(c+dx)^4}{16(bc-ad)g^5(a+bx)^4} - \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)g^5(a+bx)^4}$$

output
```
-1/16*B*i^3*(d*x+c)^4/(-a*d+b*c)/g^5/(b*x+a)^4-1/4*i^3*(d*x+c)^4*(A+B*ln(e
*(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^5/(b*x+a)^4
```

## 3.28.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 427 vs. 2(89) = 178.

Time = 0.28 (sec) , antiderivative size = 427, normalized size of antiderivative = 4.80

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{i^3\left(4Ab^4c^4+b^4Bc^4-4a^4Ad^4-a^4Bd^4+16Ab^4c^3dx+4b^4Bc^3dx-16a^3Abd^4x-4a^3bBd^4x+24Ab^4c^2\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^5,x]
```

output
```
-1/16*(i^3*(4*A*b^4*c^4 + b^4*B*c^4 - 4*a^4*A*d^4 - a^4*B*d^4 + 16*A*b^4*c
^3*d*x + 4*b^4*B*c^3*d*x - 16*a^3*A*b*d^4*x - 4*a^3*b*B*d^4*x + 24*A*b^4*c
^2*d^2*x^2 + 6*b^4*B*c^2*d^2*x^2 - 24*a^2*A*b^2*d^4*x^2 - 6*a^2*b^2*B*d^4*
x^2 + 16*A*b^4*c*d^3*x^3 + 4*b^4*B*c*d^3*x^3 - 16*a*A*b^3*d^4*x^3 - 4*a*b^
3*B*d^4*x^3 + 4*B*d^4*(a + b*x)^4*Log[a + b*x] + 4*B*(-(a^4*d^4) - 4*a^3*b
*d^4*x - 6*a^2*b^2*d^4*x^2 - 4*a*b^3*d^4*x^3 + b^4*c*(c^3 + 4*c^2*d*x + 6*
c*d^2*x^2 + 4*d^3*x^3))*Log[(e*(a + b*x))/(c + d*x)] - 4*a^4*B*d^4*Log[c +
 d*x] - 16*a^3*b*B*d^4*x*Log[c + d*x] - 24*a^2*b^2*B*d^4*x^2*Log[c + d*x]
- 16*a*b^3*B*d^4*x^3*Log[c + d*x] - 4*b^4*B*d^4*x^4*Log[c + d*x]))/(b^4*(b
*c - a*d)*g^5*(a + b*x)^4)
```

### 3.28.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.83, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.050$, Rules used = {2962, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i^3\int \frac{(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{i^3\left(-\frac{(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}-\frac{B(c+dx)^4}{16(a+bx)^4}\right)}{g^5(bc-ad)}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^5
,x]
```

output
```
(i^3*(-1/16*(B*(c + d*x)^4)/(a + b*x)^4 - ((c + d*x)^4*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(4*(a + b*x)^4)))/((b*c - a*d)*g^5)
```

#### 3.28.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.28.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 184 vs. $2(85) = 170$.

Time = 1.58 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.08

| method | result |
|---|---|
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{i^3d^2e^3A}{4(ad-cb)^2g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{i^3d^2e^3B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}\right)}{(ad-cb)^2g^5}\right)}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{i^3d^2e^3A}{4(ad-cb)^2g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{i^3d^2e^3B\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}\right)}{(ad-cb)^2g^5}\right)}{d^2}$ |
| parts | $\frac{i^3A\left(-\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{b^4(bx+a)^3}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{4b^4(bx+a)^4}+\frac{3d^2(ad-cb)}{2b^4(bx+a)^2}-\frac{d^3}{b^4(bx+a)}\right)}{g^5}-\frac{i^3B\,e^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\cdots}\right.}{g^5(ad\cdots}$ |
| parallelrisch | $\frac{4B\,x^4\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c\,d^4i^3+16B\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c^2d^3i^3+24B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c^3d^2i^3+16Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c^4d\,i^3+\cdots}{}$ |
| norman | $\frac{Bc\,d^3i^3x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(ad-cb)g}+\frac{B\,c^3d\,i^3x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}+\frac{\left(4A\,c^3i^3+B\,c^3i^3\right)x}{4ga}+\frac{3\left(4Aa\,c^2d\,i^3+4Ab\,c^3i^3+Ba\,c^2d\,i^3+Bb\,c^3i^3\right)x^2}{8g\,a^2}+\frac{(4A\cdots}{}$ |
| risch | $-\frac{i^3B\left(4d^3x^3b^3+6a\,b^2d^3x^2+6b^3c\,d^2x^2+4a^2b\,d^3x+4a\,b^2c\,d^2x+4b^3c^2dx+a^3d^3+a^2bc\,d^2+a\,b^2c^2d+b^3c^3\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{4(bx+a)^4g^5b^4}-\cdots$ |

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x,method=_RETU
RNVERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(-1/4*i^3*d^2*e^3/(a*d-b*c)^2/g^5*A/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^4+i^3*d^2*e^3/(a*d-b*c)^2/g^5*B*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
4))
```

## 3.28.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 355 vs. $2(85) = 170$.

Time = 0.35 (sec) , antiderivative size = 355, normalized size of antiderivative = 3.99

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{4\left((4A+B)b^4cd^3-(4A+B)ab^3d^4\right)i^3x^3+6\left((4A+B)b^4c^2d^2-(4A+B)a^2b^2d^4\right)i^3x^2+4\left((4A+B)b\cdots\right.}{16\left((b^9c-ab^8d)g^5x^4+4\left(ab^8c-\cdots\right.\right.}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algo
rithm="fricas")
```

output
```
-1/16*(4*((4*A + B)*b^4*c*d^3 - (4*A + B)*a*b^3*d^4)*i^3*x^3 + 6*((4*A + B
)*b^4*c^2*d^2 - (4*A + B)*a^2*b^2*d^4)*i^3*x^2 + 4*((4*A + B)*b^4*c^3*d -
(4*A + B)*a^3*b*d^4)*i^3*x + ((4*A + B)*b^4*c^4 - (4*A + B)*a^4*d^4)*i^3 +
 4*(B*b^4*d^4*i^3*x^4 + 4*B*b^4*c*d^3*i^3*x^3 + 6*B*b^4*c^2*d^2*i^3*x^2 +
4*B*b^4*c^3*d*i^3*x + B*b^4*c^4*i^3)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c
 - a*b^8*d)*g^5*x^4 + 4*(a*b^8*c - a^2*b^7*d)*g^5*x^3 + 6*(a^2*b^7*c - a^3
*b^6*d)*g^5*x^2 + 4*(a^3*b^6*c - a^4*b^5*d)*g^5*x + (a^4*b^5*c - a^5*b^4*d
)*g^5)
```

### 3.28.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**5,x)
```

output
```
Timed out
```

### 3.28.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3107 vs. $2(85) = 170$.

Time = 0.35 (sec) , antiderivative size = 3107, normalized size of antiderivative = 34.91

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algo
rithm="maxima")
```

output
```
-1/48*B*d^3*i^3*(12*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x + a^3)*log(b*e*x/
(d*x + c) + a*e/(d*x + c))/(b^8*g^5*x^4 + 4*a*b^7*g^5*x^3 + 6*a^2*b^6*g^5*
x^2 + 4*a^3*b^5*g^5*x + a^4*b^4*g^5) + (25*a^3*b^3*c^3 - 23*a^4*b^2*c^2*d
+ 13*a^5*b*c*d^2 - 3*a^6*d^3 + 12*(4*b^6*c^3 - 6*a*b^5*c^2*d + 4*a^2*b^4*c
*d^2 - a^3*b^3*d^3)*x^3 + 6*(18*a*b^5*c^3 - 22*a^2*b^4*c^2*d + 13*a^3*b^3*
c*d^2 - 3*a^4*b^2*d^3)*x^2 + 4*(22*a^2*b^4*c^3 - 23*a^3*b^3*c^2*d + 13*a^4
*b^2*c*d^2 - 3*a^5*b*d^3)*x)/((b^11*c^3 - 3*a*b^10*c^2*d + 3*a^2*b^9*c*d^2
 - a^3*b^8*d^3)*g^5*x^4 + 4*(a*b^10*c^3 - 3*a^2*b^9*c^2*d + 3*a^3*b^8*c*d^
2 - a^4*b^7*d^3)*g^5*x^3 + 6*(a^2*b^9*c^3 - 3*a^3*b^8*c^2*d + 3*a^4*b^7*c*
d^2 - a^5*b^6*d^3)*g^5*x^2 + 4*(a^3*b^8*c^3 - 3*a^4*b^7*c^2*d + 3*a^5*b^6*
c*d^2 - a^6*b^5*d^3)*g^5*x + (a^4*b^7*c^3 - 3*a^5*b^6*c^2*d + 3*a^6*b^5*c*
d^2 - a^7*b^4*d^3)*g^5) + 12*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 + 4*a^2*b*c*d^
3 - a^3*d^4)*log(b*x + a)/((b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^6*c^2*d^2 -
4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*g^5) - 12*(4*b^3*c^3*d - 6*a*b^2*c^2*d^2 +
4*a^2*b*c*d^3 - a^3*d^4)*log(d*x + c)/((b^8*c^4 - 4*a*b^7*c^3*d + 6*a^2*b^
6*c^2*d^2 - 4*a^3*b^5*c*d^3 + a^4*b^4*d^4)*g^5)) - 1/48*B*c*d^2*i^3*(12*(6
*b^2*x^2 + 4*a*b*x + a^2)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^7*g^5*x^
4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3*b^4*g^5*x + a^4*b^3*g^5) +
 (13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*b*c*d^2 - 7*a^5*d^3 - 12*(6*b
^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 6*(6*b^5*c^3 - 46*a*b^4*c...
```

### 3.28.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 140, normalized size of antiderivative = 1.57

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$
$$-\frac{1}{16}\left(\frac{4\,(dx+c)^4Be^5i^3\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^4g^5}+\frac{(4\,Ae^5i^3+Be^5i^3)(dx+c)^4}{(bex+ae)^4g^5}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{a}{(bce-ade}\right.$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^5,x, algo
rithm="giac")
```

output
```
-1/16*(4*(d*x + c)^4*B*e^5*i^3*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)
^4*g^5) + (4*A*e^5*i^3 + B*e^5*i^3)*(d*x + c)^4/((b*e*x + a*e)^4*g^5))*(b*
c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.28.9 Mupad [B] (verification not implemented)

Time = 3.72 (sec) , antiderivative size = 780, normalized size of antiderivative = 8.76

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^5}\,dx =$$
$$-\frac{x^3 (4 A b^3 d^3 i^3 + B b^3 d^3 i^3) + x^2 \left(6 A a b^2 d^3 i^3 + \frac{3 B a b^2 d^3 i^3}{2} + 6 A b^3 c d^2 i^3 + \frac{3 B b^3 c d^2 i^3}{2}\right) + x (4 A a^2 b d^3 \cdots}{\cdots}$$
$$-\frac{\ln\left(\frac{e(a+bx)}{c+dx}\right) \left(x^2 \left(b \left(b \left(\frac{B a d^3 i^3}{4 b^5 g^5} + \frac{B c d^2 i^3}{4 b^4 g^5}\right) + \frac{B a d^3 i^3}{2 b^4 g^5} + \frac{B c d^2 i^3}{2 b^3 g^5}\right) + \frac{3 B a d^3 i^3}{4 b^3 g^5} + \frac{3 B c d^2 i^3}{4 b^2 g^5}\right) + x \left(b \left(a \left(\frac{B a d^3}{4 b^5 g^5} \cdots\right.\right.\right.\right.}{4 \cdots}$$
$$-\frac{B d^4 i^3 \operatorname{atan}\left(\frac{b c 2i + b d x 2i}{a d - b c} + 1i\right) 1i}{2 b^4 g^5 (a d - b c)}$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^5
,x)
```

output
```
- (x^3*(4*A*b^3*d^3*i^3 + B*b^3*d^3*i^3) + x^2*(6*A*a*b^2*d^3*i^3 + (3*B*a
*b^2*d^3*i^3)/2 + 6*A*b^3*c*d^2*i^3 + (3*B*b^3*c*d^2*i^3)/2) + x*(4*A*a^2*
b*d^3*i^3 + B*a^2*b*d^3*i^3 + 4*A*b^3*c^2*d*i^3 + B*b^3*c^2*d*i^3 + 4*A*a*
b^2*c*d^2*i^3 + B*a*b^2*c*d^2*i^3) + A*a^3*d^3*i^3 + A*b^3*c^3*i^3 + (B*a^
3*d^3*i^3)/4 + (B*b^3*c^3*i^3)/4 + A*a*b^2*c^2*d*i^3 + A*a^2*b*c*d^2*i^3 +
 (B*a*b^2*c^2*d*i^3)/4 + (B*a^2*b*c*d^2*i^3)/4)/(4*a^4*b^4*g^5 + 4*b^8*g^5
*x^4 + 16*a^3*b^5*g^5*x + 16*a*b^7*g^5*x^3 + 24*a^2*b^6*g^5*x^2) - (log((e
*(a + b*x))/(c + d*x))*(x^2*(b*(b*((B*a*d^3*i^3)/(4*b^5*g^5) + (B*c*d^2*i^
3)/(4*b^4*g^5)) + (B*a*d^3*i^3)/(2*b^4*g^5) + (B*c*d^2*i^3)/(2*b^3*g^5)) +
 (3*B*a*d^3*i^3)/(4*b^3*g^5) + (3*B*c*d^2*i^3)/(4*b^2*g^5)) + x*(b*(a*((B*
a*d^3*i^3)/(4*b^5*g^5) + (B*c*d^2*i^3)/(4*b^4*g^5)) + (B*c^2*d*i^3)/(4*b^3
*g^5)) + a*(b*((B*a*d^3*i^3)/(4*b^5*g^5) + (B*c*d^2*i^3)/(4*b^4*g^5)) + (B
*a*d^3*i^3)/(2*b^4*g^5) + (B*c*d^2*i^3)/(2*b^3*g^5)) + (3*B*c^2*d*i^3)/(4*
b^2*g^5)) + a*(a*((B*a*d^3*i^3)/(4*b^5*g^5) + (B*c*d^2*i^3)/(4*b^4*g^5)) +
 (B*c^2*d*i^3)/(4*b^3*g^5)) + (B*c^3*i^3)/(4*b^2*g^5) + (B*d^3*i^3*x^3)/(b
^2*g^5)))/(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3) - (B*d^4
*i^3*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*1i)/(2*b^4*g^5*(a*d - b*c)
)
```

# 3.29 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$



## 3.29.1 Optimal result

Integrand size = 40, antiderivative size = 181

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \frac{Bdi^3(c+dx)^4}{16(bc-ad)^2g^6(a+bx)^4} - \frac{bBi^3(c+dx)^5}{25(bc-ad)^2g^6(a+bx)^5} + \frac{di^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^2g^6(a+bx)^4} - \frac{bi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^2g^6(a+bx)^5}$$

output
```
1/16*B*d*i^3*(d*x+c)^4/(-a*d+b*c)^2/g^6/(b*x+a)^4-1/25*b*B*i^3*(d*x+c)^5/(
-a*d+b*c)^2/g^6/(b*x+a)^5+1/4*d*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/
(-a*d+b*c)^2/g^6/(b*x+a)^4-1/5*b*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))
/(-a*d+b*c)^2/g^6/(b*x+a)^5
```

### 3.29.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 608 vs. $2(181) = 362$.

Time = 0.35 (sec) , antiderivative size = 608, normalized size of antiderivative = 3.36

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = -\frac{i^3\left(80Ab^5c^5+16b^5Bc^5-100aAb^4c^4d-25ab^4Bc^4d+20a^5Ad^5+9a^5Bd^5+300Ab^5c^4dx+55b^5Bc^4dx-\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^6,x]
```

output
```
-1/400*(i^3*(80*A*b^5*c^5 + 16*b^5*B*c^5 - 100*a*A*b^4*c^4*d - 25*a*b^4*B*
c^4*d + 20*a^5*A*d^5 + 9*a^5*B*d^5 + 300*A*b^5*c^4*d*x + 55*b^5*B*c^4*d*x
- 400*a*A*b^4*c^3*d^2*x - 100*a*b^4*B*c^3*d^2*x + 100*a^4*A*b*d^5*x + 45*a
^4*b*B*d^5*x + 400*A*b^5*c^3*d^2*x^2 + 60*b^5*B*c^3*d^2*x^2 - 600*a*A*b^4*
c^2*d^3*x^2 - 150*a*b^4*B*c^2*d^3*x^2 + 200*a^3*A*b^2*d^5*x^2 + 90*a^3*b^2
*B*d^5*x^2 + 200*A*b^5*c^2*d^3*x^3 + 10*b^5*B*c^2*d^3*x^3 - 400*a*A*b^4*c*
d^4*x^3 - 100*a*b^4*B*c*d^4*x^3 + 200*a^2*A*b^3*d^5*x^3 + 90*a^2*b^3*B*d^5
*x^3 - 20*b^5*B*c*d^4*x^4 + 20*a*b^4*B*d^5*x^4 - 20*B*d^5*(a + b*x)^5*Log[
a + b*x] + 20*B*(b*c - a*d)^2*(a^3*d^3 + a^2*b*d^2*(2*c + 5*d*x) + a*b^2*d
*(3*c^2 + 10*c*d*x + 10*d^2*x^2) + b^3*(4*c^3 + 15*c^2*d*x + 20*c*d^2*x^2
+ 10*d^3*x^3))*Log[(e*(a + b*x))/(c + d*x)] + 20*a^5*B*d^5*Log[c + d*x] +
100*a^4*b*B*d^5*x*Log[c + d*x] + 200*a^3*b^2*B*d^5*x^2*Log[c + d*x] + 200*
a^2*b^3*B*d^5*x^3*Log[c + d*x] + 100*a*b^4*B*d^5*x^4*Log[c + d*x] + 20*b^5
*B*d^5*x^5*Log[c + d*x]))/(b^4*(b*c - a*d)^2*g^6*(a + b*x)^5)
```

### 3.29.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^6}\,dx$$

$\downarrow$ 2962

$$\frac{i^3 \int \frac{(c+dx)^6\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^6}d\frac{a+bx}{c+dx}}{g^6(bc-ad)^2}$$

$\downarrow$ 2772

$$\frac{i^3\left(-B\int -\frac{(c+dx)^6\left(4b-\frac{5d(a+bx)}{c+dx}\right)}{20(a+bx)^6}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}+\frac{d(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}\right)}{g^6(bc-ad)^2}$$

$\downarrow$ 27

$$\frac{i^3\left(\frac{1}{20}B\int \frac{(c+dx)^6\left(4b-\frac{5d(a+bx)}{c+dx}\right)}{(a+bx)^6}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}+\frac{d(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}\right)}{g^6(bc-ad)^2}$$

$\downarrow$ 53

$$\frac{i^3\left(\frac{1}{20}B\int \left(\frac{4b(c+dx)^6}{(a+bx)^6}-\frac{5d(c+dx)^5}{(a+bx)^5}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}+\frac{d(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}\right)}{g^6(bc-ad)^2}$$

$\downarrow$ 2009

$$\frac{i^3\left(-\frac{b(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}+\frac{d(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\frac{1}{20}B\left(\frac{5d(c+dx)^4}{4(a+bx)^4}-\frac{4b(c+dx)^5}{5(a+bx)^5}\right)\right)}{g^6(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^6
,x]
```

output
```
(i^3*((B*((5*d*(c + d*x)^4)/(4*(a + b*x)^4) - (4*b*(c + d*x)^5)/(5*(a + b*
x)^5)))/20 + (d*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(a +
b*x)^4) - (b*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*(a + b*x
)^5)))/((b*c - a*d)^2*g^6)
```

## 3.29. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$

#### 3.29.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.29. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$

### 3.29.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(173) = 346$.

Time = 1.60 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.97

| method | result |
|---|---|
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{i^3 d^2 e^4 Ab}{5(ad-cb)^3 g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{i^3 d^3 e^3 A}{4(ad-cb)^3 g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^3 d^2 e^4 Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{1}{25\left(\frac{be}{d}+\frac{(a}{d(}\right.}\right.}{(ad-cb)^3 g^6}\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(\frac{i^3 d^2 e^4 Ab}{5(ad-cb)^3 g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{i^3 d^3 e^3 A}{4(ad-cb)^3 g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^3 d^2 e^4 Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{1}{25\left(\frac{be}{d}+\frac{(a}{d(}\right.}\right.}{(ad-cb)^3 g^6}\right.}{d^2}$ |
| parts | $\frac{i^3 A\left(\frac{d^2(ad-cb)}{b^4(bx+a)^3}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{4b^4(bx+a)^4}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{5b^4(bx+a)^5}-\frac{d^3}{2b^4(bx+a)^2}\right)}{g^6}-\frac{i^3B(ad-cb)^4e^4\left(d^6\left(-\frac{\ln(}{4(}\right.\right.}{}$ |
| risch | $-\frac{i^3B\left(10d^3x^3b^3+10a\,b^2d^3x^2+20b^3c\,d^2x^2+5a^2b\,d^3x+10a\,b^2c\,d^2x+15b^3c^2dx+a^3d^3+2a^2bc\,d^2+3a\,b^2c^2d+4b^3c^3\right)\ln\left(\frac{e(b}{d}\right.}{20(bx+a)^5 g^6 b^4}$ |
| parallelrisch | $-\frac{-150B\,x^2a\,b^7c^2d^4i^3+300Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^8c^4d^2i^3-400Axa\,b^7c^3d^3i^3-100Bxa\,b^7c^3d^3i^3-100B\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^7c^4d}{}$ |
| norman | $\frac{\frac{\left(20Aa\,c^3d\,i^3-20Ab\,c^4i^3+5Ba\,c^3d\,i^3-4Bb\,c^4i^3\right)x}{20ga(ad-cb)}+\frac{\left(60A\,a^2c^2d^2i^3+20Aab\,c^3d\,i^3-80A\,b^2c^4i^3+15B\,a^2c^2d^2i^3+9Bab\,c^3d\,i^3-16B\,b^2\right.}{40g\,a^2(ad-cb)}}{}$ |

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x,method=_RETU
RNVERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(1/5*i^3*d^2*e^4/(a*d-b*c)^3/g^6*A*b/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^5-1/4*i^3*d^3*e^3/(a*d-b*c)^3/g^6*A/(b*e/d+(a*d-b*c)*e/d/(d*x+
c))^4-i^3*d^2*e^4/(a*d-b*c)^3/g^6*B*b*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5)+i^
3*d^3*e^3/(a*d-b*c)^3/g^6*B*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

### 3.29.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 644 vs. $2(173) = 346$.

Time = 0.35 (sec) , antiderivative size = 644, normalized size of antiderivative = 3.56

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$$
$$= \frac{20\,(Bb^5cd^4-Bab^4d^5)i^3x^4-10\,((20\,A+B)b^5c^2d^3-10\,(4\,A+B)ab^4cd^4+(20\,A+9\,B)a^2b^3d^5)i^3x^3-10\,(}{}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algo
rithm="fricas")
```

output
```
1/400*(20*(B*b^5*c*d^4 - B*a*b^4*d^5)*i^3*x^4 - 10*((20*A + B)*b^5*c^2*d^3
 - 10*(4*A + B)*a*b^4*c*d^4 + (20*A + 9*B)*a^2*b^3*d^5)*i^3*x^3 - 10*(2*(2
0*A + 3*B)*b^5*c^3*d^2 - 15*(4*A + B)*a*b^4*c^2*d^3 + (20*A + 9*B)*a^3*b^2
*d^5)*i^3*x^2 - 5*((60*A + 11*B)*b^5*c^4*d - 20*(4*A + B)*a*b^4*c^3*d^2 +
(20*A + 9*B)*a^4*b*d^5)*i^3*x - (16*(5*A + B)*b^5*c^5 - 25*(4*A + B)*a*b^4
*c^4*d + (20*A + 9*B)*a^5*d^5)*i^3 + 20*(B*b^5*d^5*i^3*x^5 + 5*B*a*b^4*d^5
*i^3*x^4 - 10*(B*b^5*c^2*d^3 - 2*B*a*b^4*c*d^4)*i^3*x^3 - 10*(2*B*b^5*c^3*
d^2 - 3*B*a*b^4*c^2*d^3)*i^3*x^2 - 5*(3*B*b^5*c^4*d - 4*B*a*b^4*c^3*d^2)*i
^3*x - (4*B*b^5*c^5 - 5*B*a*b^4*c^4*d)*i^3)*log((b*e*x + a*e)/(d*x + c)))/
((b^11*c^2 - 2*a*b^10*c*d + a^2*b^9*d^2)*g^6*x^5 + 5*(a*b^10*c^2 - 2*a^2*b
^9*c*d + a^3*b^8*d^2)*g^6*x^4 + 10*(a^2*b^9*c^2 - 2*a^3*b^8*c*d + a^4*b^7*
d^2)*g^6*x^3 + 10*(a^3*b^8*c^2 - 2*a^4*b^7*c*d + a^5*b^6*d^2)*g^6*x^2 + 5*
(a^4*b^7*c^2 - 2*a^5*b^6*c*d + a^6*b^5*d^2)*g^6*x + (a^5*b^6*c^2 - 2*a^6*b
^5*c*d + a^7*b^4*d^2)*g^6)
```

### 3.29.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**6,x)
```

output
```
Timed out
```

### 3.29.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4218 vs. $2(173) = 346$.

Time = 0.45 (sec) , antiderivative size = 4218, normalized size of antiderivative = 23.30

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algo
rithm="maxima")
```

output
```
-1/1200*B*d^3*i^3*(60*(10*b^3*x^3 + 10*a*b^2*x^2 + 5*a^2*b*x + a^3)*log(b*
e*x/(d*x + c) + a*e/(d*x + c))/(b^9*g^6*x^5 + 5*a*b^8*g^6*x^4 + 10*a^2*b^7
*g^6*x^3 + 10*a^3*b^6*g^6*x^2 + 5*a^4*b^5*g^6*x + a^5*b^4*g^6) + (77*a^3*b
^4*c^4 - 548*a^4*b^3*c^3*d + 352*a^5*b^2*c^2*d^2 - 148*a^6*b*c*d^3 + 27*a^
7*d^4 - 60*(10*b^7*c^3*d - 10*a*b^6*c^2*d^2 + 5*a^2*b^5*c*d^3 - a^3*b^4*d^
4)*x^4 + 30*(10*b^7*c^4 - 100*a*b^6*c^3*d + 95*a^2*b^5*c^2*d^2 - 46*a^3*b^
4*c*d^3 + 9*a^4*b^3*d^4)*x^3 + 10*(50*a*b^6*c^4 - 410*a^2*b^5*c^3*d + 337*
a^3*b^4*c^2*d^2 - 148*a^4*b^3*c*d^3 + 27*a^5*b^2*d^4)*x^2 + 5*(65*a^2*b^5*
c^4 - 488*a^3*b^4*c^3*d + 352*a^4*b^3*c^2*d^2 - 148*a^5*b^2*c*d^3 + 27*a^6
*b*d^4)*x)/((b^13*c^4 - 4*a*b^12*c^3*d + 6*a^2*b^11*c^2*d^2 - 4*a^3*b^10*c
*d^3 + a^4*b^9*d^4)*g^6*x^5 + 5*(a*b^12*c^4 - 4*a^2*b^11*c^3*d + 6*a^3*b^1
0*c^2*d^2 - 4*a^4*b^9*c*d^3 + a^5*b^8*d^4)*g^6*x^4 + 10*(a^2*b^11*c^4 - 4*
a^3*b^10*c^3*d + 6*a^4*b^9*c^2*d^2 - 4*a^5*b^8*c*d^3 + a^6*b^7*d^4)*g^6*x^
3 + 10*(a^3*b^10*c^4 - 4*a^4*b^9*c^3*d + 6*a^5*b^8*c^2*d^2 - 4*a^6*b^7*c*d
^3 + a^7*b^6*d^4)*g^6*x^2 + 5*(a^4*b^9*c^4 - 4*a^5*b^8*c^3*d + 6*a^6*b^7*c
^2*d^2 - 4*a^7*b^6*c*d^3 + a^8*b^5*d^4)*g^6*x + (a^5*b^8*c^4 - 4*a^6*b^7*c
^3*d + 6*a^7*b^6*c^2*d^2 - 4*a^8*b^5*c*d^3 + a^9*b^4*d^4)*g^6) - 60*(10*b^
3*c^3*d^2 - 10*a*b^2*c^2*d^3 + 5*a^2*b*c*d^4 - a^3*d^5)*log(b*x + a)/((b^9
*c^5 - 5*a*b^8*c^4*d + 10*a^2*b^7*c^3*d^2 - 10*a^3*b^6*c^2*d^3 + 5*a^4*b^5
*c*d^4 - a^5*b^4*d^5)*g^6) + 60*(10*b^3*c^3*d^2 - 10*a*b^2*c^2*d^3 + 5*...
```

### 3.29.8 Giac [A] (verification not implemented)

Time = 0.55 (sec) , antiderivative size = 282, normalized size of antiderivative = 1.56

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = -\frac{1}{400}\left(\frac{20\left(4\,Bbe^6i^3-\frac{5\,(bex+ae)Bde^5i^3}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^5bcg^6}{(dx+c)^5}-\frac{(bex+ae)^5adg^6}{(dx+c)^5}}+\frac{80\,Abe^6i^3+16\,Bbe^6i^3-\frac{100\,(bex+ae)Ade^5i^3}{dx+c}-\frac{25\,(bex+\cdots}{dx\cdots}}{\frac{(bex+ae)^5bcg^6}{(dx+c)^5}-\frac{(bex+ae)^5adg^6}{(dx+c)^5}}\right.$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^6,x, algo
rithm="giac")
```

output
```
-1/400*(20*(4*B*b*e^6*i^3 - 5*(b*e*x + a*e)*B*d*e^5*i^3/(d*x + c))*log((b*
e*x + a*e)/(d*x + c))/((b*e*x + a*e)^5*b*c*g^6/(d*x + c)^5 - (b*e*x + a*e)
^5*a*d*g^6/(d*x + c)^5) + (80*A*b*e^6*i^3 + 16*B*b*e^6*i^3 - 100*(b*e*x +
a*e)*A*d*e^5*i^3/(d*x + c) - 25*(b*e*x + a*e)*B*d*e^5*i^3/(d*x + c))/((b*e
*x + a*e)^5*b*c*g^6/(d*x + c)^5 - (b*e*x + a*e)^5*a*d*g^6/(d*x + c)^5))*(b
*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.29.9 Mupad [B] (verification not implemented)

Time = 4.82 (sec) , antiderivative size = 1053, normalized size of antiderivative = 5.82

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx = -\frac{\frac{20\,A\,a^4\,d^4\,i^3-80\,A\,b^4\,c^4\,i^3+9\,B\,a^4\,d^4\,i^3-16\,B\,b^4\,c^4\,i^3+20\,A\,a^2\,b^2\,c^2\,d^2\,i^3+9\,B\,a^2\,b^2\,c^2\,d^2\,i^3+20\,A\,a\,b^3\,c^3\,d\,i^3+20\,A\,a^3\,b\,c\,d^3\,i^3+9\,B\,a\,b^3\,c^3\,d\,i^3\cdots}{20\,(a\,d-b\,c)}}{\cdots}$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(x^2\left(b\left(b\left(\frac{B\,a\,d^3\,i^3}{20\,b^5\,g^6}+\frac{B\,c\,d^2\,i^3}{10\,b^4\,g^6}\right)+\frac{3\,B\,a\,d^3\,i^3}{20\,b^4\,g^6}+\frac{3\,B\,c\,d^2\,i^3}{10\,b^3\,g^6}\right)+\frac{3\,B\,a\,d^3\,i^3}{10\,b^3\,g^6}+\frac{3\,B\,c\,d^2\,i^3}{5\,b^2\,g^6}\right)+x\left(b\left(a\left(\frac{B\,a\cdots}{20\,\cdots}\right.\right.\right.\cdots}{5\,a^4\cdots}$$

$$-\frac{B\,d^5\,i^3\,\mathrm{atanh}\left(\frac{20\,b^6\,c^2\,g^6-20\,a^2\,b^4\,d^2\,g^6}{20\,b^4\,g^6\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{10\,b^4\,g^6\,(a\,d-b\,c)^2}$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^6
,x)
```

output
```
- ((20*A*a^4*d^4*i^3 - 80*A*b^4*c^4*i^3 + 9*B*a^4*d^4*i^3 - 16*B*b^4*c^4*i
^3 + 20*A*a^2*b^2*c^2*d^2*i^3 + 9*B*a^2*b^2*c^2*d^2*i^3 + 20*A*a*b^3*c^3*d
*i^3 + 20*A*a^3*b*c*d^3*i^3 + 9*B*a*b^3*c^3*d*i^3 + 9*B*a^3*b*c*d^3*i^3)/(
20*(a*d - b*c)) + (x^2*(20*A*a^2*b^2*d^4*i^3 + 9*B*a^2*b^2*d^4*i^3 - 40*A*
b^4*c^2*d^2*i^3 - 6*B*b^4*c^2*d^2*i^3 + 20*A*a*b^3*c*d^3*i^3 + 9*B*a*b^3*c
*d^3*i^3))/(2*(a*d - b*c)) + (x*(20*A*a^3*b*d^4*i^3 + 9*B*a^3*b*d^4*i^3 -
60*A*b^4*c^3*d*i^3 - 11*B*b^4*c^3*d*i^3 + 20*A*a*b^3*c^2*d^2*i^3 + 20*A*a^
2*b^2*c*d^3*i^3 + 9*B*a*b^3*c^2*d^2*i^3 + 9*B*a^2*b^2*c*d^3*i^3))/(4*(a*d
- b*c)) + (x^3*(20*A*a*b^3*d^4*i^3 + 9*B*a*b^3*d^4*i^3 - 20*A*b^4*c*d^3*i^
3 - B*b^4*c*d^3*i^3))/(2*(a*d - b*c)) + (B*b^4*d^4*i^3*x^4)/(a*d - b*c))/(
20*a^5*b^4*g^6 + 20*b^9*g^6*x^5 + 100*a^4*b^5*g^6*x + 100*a*b^8*g^6*x^4 +
200*a^3*b^6*g^6*x^2 + 200*a^2*b^7*g^6*x^3) - (log((e*(a + b*x))/(c + d*x))
*(x^2*(b*(b*((B*a*d^3*i^3)/(20*b^5*g^6) + (B*c*d^2*i^3)/(10*b^4*g^6)) + (3
*B*a*d^3*i^3)/(20*b^4*g^6) + (3*B*c*d^2*i^3)/(10*b^3*g^6)) + (3*B*a*d^3*i^
3)/(10*b^3*g^6) + (3*B*c*d^2*i^3)/(5*b^2*g^6)) + x*(b*(a*((B*a*d^3*i^3)/(2
0*b^5*g^6) + (B*c*d^2*i^3)/(10*b^4*g^6)) + (3*B*c^2*d*i^3)/(20*b^3*g^6)) +
 a*(b*((B*a*d^3*i^3)/(20*b^5*g^6) + (B*c*d^2*i^3)/(10*b^4*g^6)) + (3*B*a*d
^3*i^3)/(20*b^4*g^6) + (3*B*c*d^2*i^3)/(10*b^3*g^6)) + (3*B*c^2*d*i^3)/(5*
b^2*g^6)) + a*(a*((B*a*d^3*i^3)/(20*b^5*g^6) + (B*c*d^2*i^3)/(10*b^4*g^6))
 + (3*B*c^2*d*i^3)/(20*b^3*g^6)) + (B*c^3*i^3)/(5*b^2*g^6) + (B*d^3*i^3...
```

3.29. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^6}\,dx$

# 3.30 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx$



## 3.30.1 Optimal result

Integrand size = 40, antiderivative size = 281

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = -\frac{Bd^2i^3(c+dx)^4}{16(bc-ad)^3g^7(a+bx)^4} + \frac{2bBdi^3(c+dx)^5}{25(bc-ad)^3g^7(a+bx)^5} - \frac{b^2Bi^3(c+dx)^6}{36(bc-ad)^3g^7(a+bx)^6} - \frac{d^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^7(a+bx)^4} + \frac{2bdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5(bc-ad)^3g^7(a+bx)^5} - \frac{b^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6(bc-ad)^3g^7(a+bx)^6}$$

output
```
-1/16*B*d^2*i^3*(d*x+c)^4/(-a*d+b*c)^3/g^7/(b*x+a)^4+2/25*b*B*d*i^3*(d*x+c
)^5/(-a*d+b*c)^3/g^7/(b*x+a)^5-1/36*b^2*B*i^3*(d*x+c)^6/(-a*d+b*c)^3/g^7/(
b*x+a)^6-1/4*d^2*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^
7/(b*x+a)^4+2/5*b*d*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3
/g^7/(b*x+a)^5-1/6*b^2*i^3*(d*x+c)^6*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c
)^3/g^7/(b*x+a)^6
```

### 3.30.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 642 vs. 2(281) = 562.

Time = 0.58 (sec) , antiderivative size = 642, normalized size of antiderivative = 2.28

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx$$
$$= \frac{i^3\left(-100B(bc-ad)^6+432aBd(-bc+ad)^5-432bBd(bc-ad)^5x+540aBd^2(bc-ad)^4(a+bx)+120Bd\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*
g*x)^7,x]
```

output
```
(i^3*(-100*B*(b*c - a*d)^6 + 432*a*B*d*(-(b*c) + a*d)^5 - 432*b*B*d*(b*c -
 a*d)^5*x + 540*a*B*d^2*(b*c - a*d)^4*(a + b*x) + 120*B*d*(b*c - a*d)^5*(a
 + b*x) + 540*b*B*d^2*(b*c - a*d)^4*x*(a + b*x) - 825*B*d^2*(b*c - a*d)^4*
(a + b*x)^2 + 720*a*B*d^3*(-(b*c) + a*d)^3*(a + b*x)^2 - 720*b*B*d^3*(b*c
- a*d)^3*x*(a + b*x)^2 + 1080*a*B*d^4*(b*c - a*d)^2*(a + b*x)^3 + 700*B*d^
3*(b*c - a*d)^3*(a + b*x)^3 + 1080*b*B*d^4*(b*c - a*d)^2*x*(a + b*x)^3 - 1
050*B*d^4*(b*c - a*d)^2*(a + b*x)^4 + 2160*a*B*d^5*(-(b*c) + a*d)*(a + b*x
)^4 - 2160*b*B*d^5*(b*c - a*d)*x*(a + b*x)^4 + 2100*B*d^5*(b*c - a*d)*(a +
 b*x)^5 - 2160*a*B*d^6*(a + b*x)^5*Log[a + b*x] - 2160*b*B*d^6*x*(a + b*x)
^5*Log[a + b*x] + 2100*B*d^6*(a + b*x)^6*Log[a + b*x] - 600*(b*c - a*d)^6*
(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2160*d*(-(b*c) + a*d)^5*(a + b*x)*(
A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2700*d^2*(b*c - a*d)^4*(a + b*x)^2*(
A + B*Log[(e*(a + b*x))/(c + d*x)]) + 1200*d^3*(-(b*c) + a*d)^3*(a + b*x)^
3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2160*a*B*d^6*(a + b*x)^5*Log[c +
d*x] + 2160*b*B*d^6*x*(a + b*x)^5*Log[c + d*x] - 2100*B*d^6*(a + b*x)^6*Lo
g[c + d*x]))/(3600*b^4*(b*c - a*d)^3*g^7*(a + b*x)^6)
```

### 3.30.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 205, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 27, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ag+bgx)^7}dx$$

$\Big\downarrow$ 2962

$$\frac{i^3\int \frac{(c+dx)^7\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^7}d\frac{a+bx}{c+dx}}{g^7(bc-ad)^3}$$

$\Big\downarrow$ 2772

$$\frac{i^3\left(-B\int -\frac{(c+dx)^7\left(10b^2-\frac{24d(a+bx)b}{c+dx}+\frac{15d^2(a+bx)^2}{(c+dx)^2}\right)}{60(a+bx)^7}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6(a+bx)^6}-\frac{d^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\right.}{g^7(bc-ad)^3}$$

$\Big\downarrow$ 27

$$\frac{i^3\left(\frac{1}{60}B\int \frac{(c+dx)^7\left(10b^2-\frac{24d(a+bx)b}{c+dx}+\frac{15d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^7}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6(a+bx)^6}-\frac{d^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\frac{2b}{}\right.}{g^7(bc-ad)^3}$$

$\Big\downarrow$ 1140

$$\frac{i^3\left(\frac{1}{60}B\int \left(\frac{10b^2(c+dx)^7}{(a+bx)^7}-\frac{24bd(c+dx)^6}{(a+bx)^6}+\frac{15d^2(c+dx)^5}{(a+bx)^5}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6(a+bx)^6}-\frac{d^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right.}{4(a+bx)^4}\right.}{g^7(bc-ad)^3}$$

$\Big\downarrow$ 2009

$$\frac{i^3\left(-\frac{b^2(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6(a+bx)^6}-\frac{d^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}+\frac{2bd(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5(a+bx)^5}+\frac{1}{60}B\left(-\frac{5b^2(c+dx)^6}{3(a+bx)^6}-\right.\right.}{g^7(bc-ad)^3}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a*g + b*g*x)^7
,x]
```

3.30. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}dx$

output
```
(i^3*((B*((-15*d^2*(c + d*x)^4)/(4*(a + b*x)^4) + (24*b*d*(c + d*x)^5)/(5*
(a + b*x)^5) - (5*b^2*(c + d*x)^6)/(3*(a + b*x)^6)))/60 - (d^2*(c + d*x)^4
*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(a + b*x)^4) + (2*b*d*(c + d*x)^
5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*(a + b*x)^5) - (b^2*(c + d*x)^6
*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(6*(a + b*x)^6)))/((b*c - a*d)^3*g^
7)
```

### 3.30.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.30.4 Maple [A] (verified)

Time = 2.26 (sec) , antiderivative size = 481, normalized size of antiderivative = 1.71

| method | result |
|---|---|
| parts | $\frac{i^3 A\left(-\frac{d^3}{3b^4(bx+a)^3}+\frac{3d^2(ad-cb)}{4b^4(bx+a)^4}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{5b^4(bx+a)^5}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{6b^4(bx+a)^6}\right)}{g^7}-\frac{i^3B(ad-cb)^4e^4\left(\frac{d^7\left(-\right.}{}\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{i^3d^2e^5A\,b^2}{6(ad-cb)^4g^7\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^6}+\frac{2i^3d^3e^4Ab}{5(ad-cb)^4g^7\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{i^3d^4e^3A}{4(ad-cb)^4g^7\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{i^3d^2e^5B\,b^2}{}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{i^3d^2e^5A\,b^2}{6(ad-cb)^4g^7\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^6}+\frac{2i^3d^3e^4Ab}{5(ad-cb)^4g^7\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{i^3d^4e^3A}{4(ad-cb)^4g^7\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{i^3d^2e^5B\,b^2}{}\right.}{}$ |
| risch | $-\frac{i^3B\left(20d^3x^3b^3+15a\,b^2d^3x^2+45b^3c\,d^2x^2+6a^2b\,d^3x+18a\,b^2c\,d^2x+36b^3c^2dx+a^3d^3+3a^2bc\,d^2+6a\,b^2c^2d+10b^3c^3\right)\ln\left(\frac{e(}{}\right.}{60(bx+a)^6g^7b^4}$ |
| parallelrisch | Expression too large to display |
| norman | Expression too large to display |

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^7,x,method=_RETU
RNVERBOSE)
```

output
```
i^3*A/g^7*(-1/3*d^3/b^4/(b*x+a)^3+3/4*d^2*(a*d-b*c)/b^4/(b*x+a)^4-3/5*d*(a
^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^5-1/6*(-a^3*d^3+3*a^2*b*c*d^2-3*a*b^
2*c^2*d+b^3*c^3)/b^4/(b*x+a)^6)-i^3*B/g^7/d^5*(a*d-b*c)^4*e^4*(d^7/(a*d-b*
c)^7*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)-2*d^6/(a*d-b*c)^7*b*e*(-1/5/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))^5)+d^5/(a*d-b*c)^7*e^2*b^2*(-1/6/(b*e/d+(a*d-b*c)*e/d/(d
*x+c))^6*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/36/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^6))
```

### 3.30.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(269) = 538$.

Time = 0.39 (sec) , antiderivative size = 991, normalized size of antiderivative = 3.53

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = -\frac{60\,(Bb^6cd^5 - Bab^5d^6)i^3x^5 - 30\,(Bb^6c^2d^4 - 12\,Bab^5cd^5 + 11\,Ba^2b^4d^6)i^3x^4 + 20\,((60\,A+B)b^6c^3d^3 - 9\,(}{}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^7,x, algo
rithm="fricas")
```

output
```
-1/3600*(60*(B*b^6*c*d^5 - B*a*b^5*d^6)*i^3*x^5 - 30*(B*b^6*c^2*d^4 - 12*B
*a*b^5*c*d^5 + 11*B*a^2*b^4*d^6)*i^3*x^4 + 20*((60*A + B)*b^6*c^3*d^3 - 9*
(20*A + B)*a*b^5*c^2*d^4 + 45*(4*A + B)*a^2*b^4*c*d^5 - (60*A + 37*B)*a^3*
b^3*d^6)*i^3*x^3 + 15*((180*A + 19*B)*b^6*c^4*d^2 - 24*(20*A + 3*B)*a*b^5*
c^3*d^3 + 90*(4*A + B)*a^2*b^4*c^2*d^4 - (60*A + 37*B)*a^4*b^2*d^6)*i^3*x^
2 + 6*(4*(90*A + 13*B)*b^6*c^5*d - 15*(60*A + 11*B)*a*b^5*c^4*d^2 + 150*(4
*A + B)*a^2*b^4*c^3*d^3 - (60*A + 37*B)*a^5*b*d^6)*i^3*x + (100*(6*A + B)*
b^6*c^6 - 288*(5*A + B)*a*b^5*c^5*d + 225*(4*A + B)*a^2*b^4*c^4*d^2 - (60*
A + 37*B)*a^6*d^6)*i^3 + 60*(B*b^6*d^6*i^3*x^6 + 6*B*a*b^5*d^6*i^3*x^5 + 1
5*B*a^2*b^4*d^6*i^3*x^4 + 20*(B*b^6*c^3*d^3 - 3*B*a*b^5*c^2*d^4 + 3*B*a^2*
b^4*c*d^5)*i^3*x^3 + 15*(3*B*b^6*c^4*d^2 - 8*B*a*b^5*c^3*d^3 + 6*B*a^2*b^4
*c^2*d^4)*i^3*x^2 + 6*(6*B*b^6*c^5*d - 15*B*a*b^5*c^4*d^2 + 10*B*a^2*b^4*c
^3*d^3)*i^3*x + (10*B*b^6*c^6 - 24*B*a*b^5*c^5*d + 15*B*a^2*b^4*c^4*d^2)*i
^3)*log((b*e*x + a*e)/(d*x + c)))/((b^13*c^3 - 3*a*b^12*c^2*d + 3*a^2*b^11
*c*d^2 - a^3*b^10*d^3)*g^7*x^6 + 6*(a*b^12*c^3 - 3*a^2*b^11*c^2*d + 3*a^3*
b^10*c*d^2 - a^4*b^9*d^3)*g^7*x^5 + 15*(a^2*b^11*c^3 - 3*a^3*b^10*c^2*d +
3*a^4*b^9*c*d^2 - a^5*b^8*d^3)*g^7*x^4 + 20*(a^3*b^10*c^3 - 3*a^4*b^9*c^2*
d + 3*a^5*b^8*c*d^2 - a^6*b^7*d^3)*g^7*x^3 + 15*(a^4*b^9*c^3 - 3*a^5*b^8*c
^2*d + 3*a^6*b^7*c*d^2 - a^7*b^6*d^3)*g^7*x^2 + 6*(a^5*b^8*c^3 - 3*a^6*b^7
*c^2*d + 3*a^7*b^6*c*d^2 - a^8*b^5*d^3)*g^7*x + (a^6*b^7*c^3 - 3*a^7*b^...
```

### 3.30.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**7,x)
```

output
```
Timed out
```

### 3.30.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5524 vs. $2(269) = 538$.

Time = 0.56 (sec) , antiderivative size = 5524, normalized size of antiderivative = 19.66

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^7,x, algo
rithm="maxima")
```

output
```
-1/3600*B*d^3*i^3*(60*(20*b^3*x^3 + 15*a*b^2*x^2 + 6*a^2*b*x + a^3)*log(b*
e*x/(d*x + c) + a*e/(d*x + c))/(b^10*g^7*x^6 + 6*a*b^9*g^7*x^5 + 15*a^2*b^
8*g^7*x^4 + 20*a^3*b^7*g^7*x^3 + 15*a^4*b^6*g^7*x^2 + 6*a^5*b^5*g^7*x + a^
6*b^4*g^7) + (57*a^3*b^5*c^5 - 405*a^4*b^4*c^4*d + 1470*a^5*b^3*c^3*d^2 -
730*a^6*b^2*c^2*d^3 + 245*a^7*b*c*d^4 - 37*a^8*d^5 + 60*(20*b^8*c^3*d^2 -
15*a*b^7*c^2*d^3 + 6*a^2*b^6*c*d^4 - a^3*b^5*d^5)*x^5 - 30*(20*b^8*c^4*d -
 235*a*b^7*c^3*d^2 + 171*a^2*b^6*c^2*d^3 - 67*a^3*b^5*c*d^4 + 11*a^4*b^4*d
^5)*x^4 + 20*(20*b^8*c^5 - 175*a*b^7*c^4*d + 866*a^2*b^6*c^3*d^2 - 604*a^3
*b^5*c^2*d^3 + 230*a^4*b^4*c*d^4 - 37*a^5*b^3*d^5)*x^3 + 15*(35*a*b^7*c^5
- 271*a^2*b^6*c^4*d + 1128*a^3*b^5*c^3*d^2 - 700*a^4*b^4*c^2*d^3 + 245*a^5
*b^3*c*d^4 - 37*a^6*b^2*d^5)*x^2 + 6*(47*a^2*b^6*c^5 - 345*a^3*b^5*c^4*d +
 1320*a^4*b^4*c^3*d^2 - 730*a^5*b^3*c^2*d^3 + 245*a^6*b^2*c*d^4 - 37*a^7*b
*d^5)*x)/((b^15*c^5 - 5*a*b^14*c^4*d + 10*a^2*b^13*c^3*d^2 - 10*a^3*b^12*c
^2*d^3 + 5*a^4*b^11*c*d^4 - a^5*b^10*d^5)*g^7*x^6 + 6*(a*b^14*c^5 - 5*a^2*
b^13*c^4*d + 10*a^3*b^12*c^3*d^2 - 10*a^4*b^11*c^2*d^3 + 5*a^5*b^10*c*d^4
- a^6*b^9*d^5)*g^7*x^5 + 15*(a^2*b^13*c^5 - 5*a^3*b^12*c^4*d + 10*a^4*b^11
*c^3*d^2 - 10*a^5*b^10*c^2*d^3 + 5*a^6*b^9*c*d^4 - a^7*b^8*d^5)*g^7*x^4 +
20*(a^3*b^12*c^5 - 5*a^4*b^11*c^4*d + 10*a^5*b^10*c^3*d^2 - 10*a^6*b^9*c^2
*d^3 + 5*a^7*b^8*c*d^4 - a^8*b^7*d^5)*g^7*x^3 + 15*(a^4*b^11*c^5 - 5*a^5*b
^10*c^4*d + 10*a^6*b^9*c^3*d^2 - 10*a^7*b^8*c^2*d^3 + 5*a^8*b^7*c*d^4 -...
```

### 3.30.8 Giac [A] (verification not implemented)

Time = 0.61 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.58

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx =$$

$$-\frac{1}{3600}\left(\frac{60\left(10\,Bb^2e^7i^3-\frac{24\,(bex+ae)Bbde^6i^3}{dx+c}+\frac{15\,(bex+ae)^2Bd^2e^5i^3}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^6b^2c^2g^7}{(dx+c)^6}-\frac{2\,(bex+ae)^6abcdg^7}{(dx+c)^6}+\frac{(bex+ae)^6a^2d^2g^7}{(dx+c)^6}}+\frac{600\,Ab^2e^7i^3+100\,Bb^2e^7i^3}{}\right.$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^7,x, algo
rithm="giac")
```

output
```
-1/3600*(60*(10*B*b^2*e^7*i^3 - 24*(b*e*x + a*e)*B*b*d*e^6*i^3/(d*x + c) +
 15*(b*e*x + a*e)^2*B*d^2*e^5*i^3/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c)
)/((b*e*x + a*e)^6*b^2*c^2*g^7/(d*x + c)^6 - 2*(b*e*x + a*e)^6*a*b*c*d*g^7
/(d*x + c)^6 + (b*e*x + a*e)^6*a^2*d^2*g^7/(d*x + c)^6) + (600*A*b^2*e^7*i
^3 + 100*B*b^2*e^7*i^3 - 1440*(b*e*x + a*e)*A*b*d*e^6*i^3/(d*x + c) - 288*
(b*e*x + a*e)*B*b*d*e^6*i^3/(d*x + c) + 900*(b*e*x + a*e)^2*A*d^2*e^5*i^3/
(d*x + c)^2 + 225*(b*e*x + a*e)^2*B*d^2*e^5*i^3/(d*x + c)^2)/((b*e*x + a*e
)^6*b^2*c^2*g^7/(d*x + c)^6 - 2*(b*e*x + a*e)^6*a*b*c*d*g^7/(d*x + c)^6 +
(b*e*x + a*e)^6*a^2*d^2*g^7/(d*x + c)^6))*(b*c/((b*c*e - a*d*e)*(b*c - a*d
)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.30.9 Mupad [B] (verification not implemented)

Time = 6.10 (sec) , antiderivative size = 1396, normalized size of antiderivative = 4.97

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx$$

$$= \frac{B\,d^6\,i^3\,\mathrm{atanh}\left(\frac{60\,a^3\,b^4\,d^3\,g^7-60\,a^2\,b^5\,c\,d^2\,g^7-60\,a\,b^6\,c^2\,d\,g^7+60\,b^7\,c^3\,g^7}{60\,b^4\,g^7\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{30\,b^4\,g^7\,(a\,d-b\,c)^3}$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(x^2\left(b\left(b\left(\frac{B\,a\,d^3\,i^3}{60\,b^5\,g^7}+\frac{B\,c\,d^2\,i^3}{20\,b^4\,g^7}\right)+\frac{B\,a\,d^3\,i^3}{15\,b^4\,g^7}+\frac{B\,c\,d^2\,i^3}{5\,b^3\,g^7}\right)+\frac{B\,a\,d^3\,i^3}{6\,b^3\,g^7}+\frac{B\,c\,d^2\,i^3}{2\,b^2\,g^7}\right)+x\left(b\left(a\left(\frac{B\,a\,d^3\,i^3}{60\,b^5\,g^7}\right.\right.\right.\right.}{6\,a^5\,x+\frac{a^6}{b}+b^5\,}$$

$$-\frac{60\,A\,a^5\,d^5\,i^3+600\,A\,b^5\,c^5\,i^3+37\,B\,a^5\,d^5\,i^3+100\,B\,b^5\,c^5\,i^3+60\,A\,a^2\,b^3\,c^3\,d^2\,i^3+60\,A\,a^3\,b^2\,c^2\,d^3\,i^3+37\,B\,a^2\,b^3\,c^3\,d^2\,i^3+37\,B\,a^3\,b^2\,c^2\,d^3\,i^3-840\,A}{60\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(a*g + b*g*x)^7
,x)
```

output
```
(B*d^6*i^3*atanh((60*b^7*c^3*g^7 + 60*a^3*b^4*d^3*g^7 - 60*a*b^6*c^2*d*g^7
 - 60*a^2*b^5*c*d^2*g^7)/(60*b^4*g^7*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(30*b^4*g^7*(a*d - b*c)^3) - (log((e
*(a + b*x))/(c + d*x))*(x^2*(b*(b*((B*a*d^3*i^3)/(60*b^5*g^7) + (B*c*d^2*i
^3)/(20*b^4*g^7)) + (B*a*d^3*i^3)/(15*b^4*g^7) + (B*c*d^2*i^3)/(5*b^3*g^7)
) + (B*a*d^3*i^3)/(6*b^3*g^7) + (B*c*d^2*i^3)/(2*b^2*g^7)) + x*(b*(a*((B*a
*d^3*i^3)/(60*b^5*g^7) + (B*c*d^2*i^3)/(20*b^4*g^7)) + (B*c^2*d*i^3)/(10*b
^3*g^7)) + a*(b*((B*a*d^3*i^3)/(60*b^5*g^7) + (B*c*d^2*i^3)/(20*b^4*g^7))
+ (B*a*d^3*i^3)/(15*b^4*g^7) + (B*c*d^2*i^3)/(5*b^3*g^7)) + (B*c^2*d*i^3)/
(2*b^2*g^7)) + a*(a*((B*a*d^3*i^3)/(60*b^5*g^7) + (B*c*d^2*i^3)/(20*b^4*g^
7)) + (B*c^2*d*i^3)/(10*b^3*g^7)) + (B*c^3*i^3)/(6*b^2*g^7) + (B*d^3*i^3*x
^3)/(3*b^2*g^7)))/(6*a^5*x + a^6/b + b^5*x^6 + 15*a^4*b*x^2 + 6*a*b^4*x^5
+ 20*a^3*b^2*x^3 + 15*a^2*b^3*x^4) - ((60*A*a^5*d^5*i^3 + 600*A*b^5*c^5*i^
3 + 37*B*a^5*d^5*i^3 + 100*B*b^5*c^5*i^3 + 60*A*a^2*b^3*c^3*d^2*i^3 + 60*A
*a^3*b^2*c^2*d^3*i^3 + 37*B*a^2*b^3*c^3*d^2*i^3 + 37*B*a^3*b^2*c^2*d^3*i^3
 - 840*A*a*b^4*c^4*d*i^3 + 60*A*a^4*b*c*d^4*i^3 - 188*B*a*b^4*c^4*d*i^3 +
37*B*a^4*b*c*d^4*i^3)/(60*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x^2*(60*A*a^
3*b^2*d^5*i^3 + 37*B*a^3*b^2*d^5*i^3 + 180*A*b^5*c^3*d^2*i^3 + 19*B*b^5*c^
3*d^2*i^3 - 300*A*a*b^4*c^2*d^3*i^3 + 60*A*a^2*b^3*c*d^4*i^3 - 53*B*a*b^4*
c^2*d^3*i^3 + 37*B*a^2*b^3*c*d^4*i^3))/(4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*...
```

3.30. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ag+bgx)^7}\,dx$

# 3.31 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$



## 3.31.1 Optimal result

Integrand size = 40, antiderivative size = 252

$$\begin{aligned}\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx &= \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3di} - \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2i}\\ &+ \frac{(bc-ad)^2g^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^3i}\\ &+ \frac{(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^4i}\\ &+ \frac{B(bc-ad)^3g^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}\end{aligned}$$

output
```
1/3*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i-1/6*(-a*d+b*c)*g^3*(b*x+
a)^2*(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/d^2/i+1/6*(-a*d+b*c)^2*g^3*(b*x+a)*
(6*A+5*B+6*B*ln(e*(b*x+a)/(d*x+c)))/d^3/i+1/6*(-a*d+b*c)^3*g^3*ln((-a*d+b*
c)/b/(d*x+c))*(6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/d^4/i+B*(-a*d+b*c)^3*g^
3*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i
```

### 3.31.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 354, normalized size of antiderivative = 1.40

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g^3\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*
i*x),x]
```

output
```
(g^3*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a +
b*x))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) -
6*B*(b*c - a*d)^3*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*
(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (
-(b*c) + a*d)*Log[c + d*x]) - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c
+ d*x)])*Log[i*(c + d*x)] + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c
) + a*d)] - Log[i*(c + d*x)])*Log[i*(c + d*x)] + 2*PolyLog[2, (b*(c + d*x)
)/(b*c - a*d)])))/(6*d^4*i)
```

### 3.31.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 298, normalized size of antiderivative = 1.18, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.150$, Rules used = {2962, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{ci+dix}\,dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{g^3(bc-ad)^3\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \text{2784}$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int\frac{(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{i}$$

$$\Big\downarrow \; 2784$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}}{3d}\right)}{i}$$

$$\Big\downarrow \; 2784$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{6A+11B+6B\log(\cdots}{b-\frac{d(a+b\cdots}{c+dx}}\cdots}{d}}{2d}}{3d}\cdots\right.}{i}$$

$$\Big\downarrow \; 2754$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{6B\int\frac{(c+dx)\log(1-\cdots}{a+b\cdots}\cdots}{d}\cdots}{\cdots}}{3d}\cdots\right.}{i}$$

$$\Big\downarrow \; 2838$$

---

3.31. $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{\cdots}}{\cdots}}{2}}{3d}\right.}{i}$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x),x
]
```

output
```
((b*c - a*d)^3*g^3*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*
d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2*(3*A + B +
3*B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c
+ d*x))^2) - (((a + b*x)*(6*A + 5*B + 6*B*Log[(e*(a + b*x))/(c + d*x)]))/(
d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((6*A + 11*B + 6*B*Log[(e*
(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (6*B*Pol
yLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d))/(3*d)))/i
```

### 3.31.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.31.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1104 vs. $2(244) = 488$.

Time = 1.65 (sec) , antiderivative size = 1105, normalized size of antiderivative = 4.38

| method | result | size |
|---|---|---|
| derivativedivides | Expression too large to display | 1105 |
| default | Expression too large to display | 1105 |
| parts | Expression too large to display | 1116 |
| risch | Expression too large to display | 3731 |

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x,method=_RETURN
VERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(A*d^2*g^3*(a^2*d^2-2*a*b*c*d+b^2*c^2)/e/i*(-3/2*b^2/d^
4*e^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2+1/d^4*ln(b*e-(b*e/d+(a*d-b*c
)*e/d/(d*x+c))*d)+1/3*b^3*e^3/d^4/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3+
3*b*e/d^4/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d))+B*d^2*g^3*(a^2*d^2-2*a*b*
c*d+b^2*c^2)/e/i*(3*b^2/d^3*e^2*(-1/2/b^2/e^2/d*ln(b*e-(b*e/d+(a*d-b*c)*e/
d/(d*x+c))*d)+1/2/b/e/d/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-1/2*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(2*b*e-(b*e/d+(a*d-b
*c)*e/d/(d*x+c))*d)/b^2/e^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2)+1/d^3
*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)+b^3*e^3/d^3*
(1/3/b^3/e^3/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-1/3/b^2/e^2/d/(b*e-
(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-1/6/b/e/d/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+
c))*d)^2+1/3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))
*(3*e^2*b^2-3*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d*b*e+d^2*(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^2)/b^3/e^3/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^3)+3*b*e/d^3*(1
/b/e/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)+ln(b*e/d+(a*d-b*c)*e/d/(d*x
+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*
d))))
```

### 3.31.5 Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algori
thm="fricas")
```

output
```
integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
+ (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((b
*e*x + a*e)/(d*x + c)))/(d*i*x + c*i), x)
```

### 3.31.6 Sympy [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g^3\left(\int \frac{Aa^3}{c+dx}\,dx+\int \frac{Ab^3x^3}{c+dx}\,dx+\int \frac{Ba^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx+\int \frac{3Aab^2x^2}{c+dx}\,dx+\int \frac{3Aa^2bx}{c+dx}\,dx+\int \frac{Bb^3x^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\right.}{i}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

output
```
g**3*(Integral(A*a**3/(c + d*x), x) + Integral(A*b**3*x**3/(c + d*x), x) +
 Integral(B*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Inte
gral(3*A*a*b**2*x**2/(c + d*x), x) + Integral(3*A*a**2*b*x/(c + d*x), x) +
 Integral(B*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) +
 Integral(3*B*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x),
x) + Integral(3*B*a**2*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x),
 x))/i
```

### 3.31.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 790 vs. 2(243) = 486.

Time = 0.26 (sec) , antiderivative size = 790, normalized size of antiderivative = 3.13

$$\begin{aligned}\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx &= 3\,Aa^2bg^3\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right)-\frac{1}{6}\,Ab^3g^3\left(\frac{6\,c^3\log(dx+c)}{d^4i}-\frac{2\,d^2x^3-3\,cdx^2+6\,c^2x}{d^3i}\right)\\ &+\frac{3}{2}\,Aab^2g^3\left(\frac{2\,c^2\log(dx+c)}{d^3i}+\frac{dx^2-2\,cx}{d^2i}\right)+\frac{Aa^3g^3\log(dix+ci)}{di}\\ &-\frac{(b^3c^3g^3-3\,ab^2c^2dg^3+3\,a^2bcd^2g^3-a^3d^3g^3)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^4i}\\ &+\frac{(6\,a^3d^3g^3\log(e)-(6\,g^3\log(e)+11\,g^3)b^3c^3+9\,(2\,g^3\log(e)+3\,g^3)ab^2c^2d-18\,(g^3\log(e)+g^3)a^2bcd^2)B}{6\,d^4i}\\ &+\frac{2\,Bb^3d^3g^3x^3\log(e)-((3\,g^3\log(e)+g^3)b^3cd^2-(9\,g^3\log(e)+g^3)ab^2d^3)Bx^2+3\,(b^3c^3g^3-3\,ab^2c^2dg^3+}{}\end{aligned}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algori
thm="maxima")
```

output
```
3*A*a^2*b*g^3*(x/(d*i) - c*log(d*x + c)/(d^2*i)) - 1/6*A*b^3*g^3*(6*c^3*lo
g(d*x + c)/(d^4*i) - (2*d^2*x^3 - 3*c*d*x^2 + 6*c^2*x)/(d^3*i)) + 3/2*A*a*
b^2*g^3*(2*c^2*log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A*a^3*g^3
*log(d*i*x + c*i)/(d*i) - (b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2
*g^3 - a^3*d^3*g^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dil
og(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^4*i) + 1/6*(6*a^3*d^3*g^3*log(e) - (6
*g^3*log(e) + 11*g^3)*b^3*c^3 + 9*(2*g^3*log(e) + 3*g^3)*a*b^2*c^2*d - 18*
(g^3*log(e) + g^3)*a^2*b*c*d^2)*B*log(d*x + c)/(d^4*i) + 1/6*(2*B*b^3*d^3*
g^3*x^3*log(e) - ((3*g^3*log(e) + g^3)*b^3*c*d^2 - (9*g^3*log(e) + g^3)*a*
b^2*d^3)*B*x^2 + 3*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 -
a^3*d^3*g^3)*B*log(d*x + c)^2 + ((6*g^3*log(e) + 5*g^3)*b^3*c^2*d - 6*(3*g
^3*log(e) + 2*g^3)*a*b^2*c*d^2 + (18*g^3*log(e) + 7*g^3)*a^2*b*d^3)*B*x +
(2*B*b^3*d^3*g^3*x^3 - 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B*x^2 + 6*(b^3*
c^2*d*g^3 - 3*a*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B*x + (6*a*b^2*c^2*d*g^3
- 15*a^2*b*c*d^2*g^3 + 11*a^3*d^3*g^3)*B)*log(b*x + a) - (2*B*b^3*d^3*g^3*
x^3 - 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B*x^2 + 6*(b^3*c^2*d*g^3 - 3*a*b
^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B*x)*log(d*x + c))/(d^4*i)
```

### 3.31.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3552 vs. $2(243) = 486$.

Time = 66.53 (sec) , antiderivative size = 3552, normalized size of antiderivative = 14.10

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algori
thm="giac")
```

output
```
-1/120*(6*(B*b^9*c^6*e^6*g^3 - 6*B*a*b^8*c^5*d*e^6*g^3 + 15*B*a^2*b^7*c^4*
d^2*e^6*g^3 - 20*B*a^3*b^6*c^3*d^3*e^6*g^3 + 15*B*a^4*b^5*c^2*d^4*e^6*g^3
- 6*B*a^5*b^4*c*d^5*e^6*g^3 + B*a^6*b^3*d^6*e^6*g^3 - 5*(b*e*x + a*e)*B*b^
8*c^6*d*e^5*g^3/(d*x + c) + 30*(b*e*x + a*e)*B*a*b^7*c^5*d^2*e^5*g^3/(d*x
+ c) - 75*(b*e*x + a*e)*B*a^2*b^6*c^4*d^3*e^5*g^3/(d*x + c) + 100*(b*e*x +
 a*e)*B*a^3*b^5*c^3*d^4*e^5*g^3/(d*x + c) - 75*(b*e*x + a*e)*B*a^4*b^4*c^2
*d^5*e^5*g^3/(d*x + c) + 30*(b*e*x + a*e)*B*a^5*b^3*c*d^6*e^5*g^3/(d*x + c
) - 5*(b*e*x + a*e)*B*a^6*b^2*d^7*e^5*g^3/(d*x + c) + 10*(b*e*x + a*e)^2*B
*b^7*c^6*d^2*e^4*g^3/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a*b^6*c^5*d^3*e^4*
g^3/(d*x + c)^2 + 150*(b*e*x + a*e)^2*B*a^2*b^5*c^4*d^4*e^4*g^3/(d*x + c)^
2 - 200*(b*e*x + a*e)^2*B*a^3*b^4*c^3*d^5*e^4*g^3/(d*x + c)^2 + 150*(b*e*x
 + a*e)^2*B*a^4*b^3*c^2*d^6*e^4*g^3/(d*x + c)^2 - 60*(b*e*x + a*e)^2*B*a^5
*b^2*c*d^7*e^4*g^3/(d*x + c)^2 + 10*(b*e*x + a*e)^2*B*a^6*b*d^8*e^4*g^3/(d
*x + c)^2 - 10*(b*e*x + a*e)^3*B*b^6*c^6*d^3*e^3*g^3/(d*x + c)^3 + 60*(b*e
*x + a*e)^3*B*a*b^5*c^5*d^4*e^3*g^3/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*a^
2*b^4*c^4*d^5*e^3*g^3/(d*x + c)^3 + 200*(b*e*x + a*e)^3*B*a^3*b^3*c^3*d^6*
e^3*g^3/(d*x + c)^3 - 150*(b*e*x + a*e)^3*B*a^4*b^2*c^2*d^7*e^3*g^3/(d*x +
 c)^3 + 60*(b*e*x + a*e)^3*B*a^5*b*c*d^8*e^3*g^3/(d*x + c)^3 - 10*(b*e*x +
 a*e)^3*B*a^6*d^9*e^3*g^3/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/(b^5*d
^4*e^5*i - 5*(b*e*x + a*e)*b^4*d^5*e^4*i/(d*x + c) + 10*(b*e*x + a*e)^2...
```

### 3.31.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x),x
)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x),
x)
```

# 3.32 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$



## 3.32.1 Optimal result

Integrand size = 40, antiderivative size = 198

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di} - \frac{(bc-ad)g^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i} - \frac{(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^3i} - \frac{B(bc-ad)^2g^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}$$

output
```
1/2*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i-1/2*(-a*d+b*c)*g^2*(b*x+
a)*(2*A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/d^2/i-1/2*(-a*d+b*c)^2*g^2*ln((-a*d+b
*c)/b/(d*x+c))*(2*A+3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/d^3/i-B*(-a*d+b*c)^2*g^
2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i
```

### 3.32.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 254, normalized size of antiderivative = 1.28

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^2\left(-2Abd(bc-ad)x+2Bd(-bc+ad)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2B(\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*
i*x),x]
```

output
```
(g^2*(-2*A*b*d*(b*c - a*d)*x + 2*B*d*(-(b*c) + a*d)*(a + b*x)*Log[(e*(a +
b*x))/(c + d*x)] + d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) +
2*B*(b*c - a*d)^2*Log[c + d*x] - B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log
[c + d*x]) + 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[i*(c
 + d*x)] - B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[i*(
c + d*x)])*Log[i*(c + d*x)] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(
2*d^3*i)
```

### 3.32.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.14, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{ci+dix}\,dx$$

$$\Big\downarrow \; 2962$$

$$\frac{g^2(bc-ad)^2 \int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \; 2784$$

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int \frac{(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{i}$$

$\Big\downarrow$ 2784

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int \frac{2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}}{2d}\right)}{i}$$

$\Big\downarrow$ 2754

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{2B\int \frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(2\right.}{d}}{}}{2d}\right.}{i}$$

$\Big\downarrow$ 2838

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+3B\right)}{d}-\frac{2B\,\mathrm{PolyL}}{}}{d}}{2d}\right.}{i}$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x),x
]
```

## 3.32. $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$

output
```
((b*c - a*d)^2*g^2*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*
d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(2*A + B + 2*
B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)
)) - (-(((2*A + 3*B + 2*B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*
x))/(b*(c + d*x))])/d) - (2*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/
d)/(2*d)))/i
```

### 3.32.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

3.32. $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$

### 3.32.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(192) = 384$.

Time = 1.77 (sec) , antiderivative size = 657, normalized size of antiderivative = 3.32

| method | result |
|---|---|
| parts | $\frac{g^2 A\left(\frac{b\left(\frac{1}{2} b d x^2+2 x a d-b c x\right)}{d^2}+\frac{\left(a^2 d^2-2 a b c d+b^2 c^2\right) \ln(d x+c)}{d^3}\right)}{i} - \frac{g^2 B \left( \frac{e^2 b^2 \left(a^2 d^2-2 a b c d+b^2 c^2\right)\left(-\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-b\right.}{2e^2b^2d}\right.}{} \right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{A d^2 g^2 (ad-cb)\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^3}-\frac{2be}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{b^2e^2}{2d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}\right)}{ei} - \right.}{}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{A d^2 g^2 (ad-cb)\left(-\frac{\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^3}-\frac{2be}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{b^2e^2}{2d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}\right)}{ei} - \right.}{}$ |
| risch | Expression too large to display |

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x,method=_RETURN
VERBOSE)
```

output
```
g^2*A/i*(b/d^2*(1/2*b*d*x^2+2*x*a*d-b*c*x)+(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3
*ln(d*x+c))-g^2*B/i/d*(e^2*b^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d*(-1/2/e^2/b^2
/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-1/2/e/b/d/((b*e/d+(a*d-b*c)*e/d
/(d*x+c))*d-b*e)+1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/
(d*x+c))*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-2*b*e)/e^2/b^2/((b*e/d+(a*d-b*c)
*e/d/(d*x+c))*d-b*e)^2)+2*b*e*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d*(1/b/e/d*ln((b
*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))+1/d*(a^2
*d^2-2*a*b*c*d+b^2*c^2)*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)
/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*
e)/b/e)/d))
```

### 3.32.5 Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algori
thm="fricas")
```

output
```
integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B
*a*b*g^2*x + B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)))/(d*i*x + c*i), x)
```

### 3.32.6 Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$$
$$=\frac{g^2\left(\int \frac{Aa^2}{c+dx}\,dx+\int \frac{Ab^2x^2}{c+dx}\,dx+\int \frac{Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx+\int \frac{2Aabx}{c+dx}\,dx+\int \frac{Bb^2x^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx+\int \frac{2Babx\log\left(\right.}{c}\right.}{i}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

output
```
g**2*(Integral(A*a**2/(c + d*x), x) + Integral(A*b**2*x**2/(c + d*x), x) +
 Integral(B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Inte
gral(2*A*a*b*x/(c + d*x), x) + Integral(B*b**2*x**2*log(a*e/(c + d*x) + b*
e*x/(c + d*x))/(c + d*x), x) + Integral(2*B*a*b*x*log(a*e/(c + d*x) + b*e*
x/(c + d*x))/(c + d*x), x))/i
```

### 3.32.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 477 vs. $2(191) = 382$.

Time = 0.26 (sec) , antiderivative size = 477, normalized size of antiderivative = 2.41

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = 2\,Aabg^2\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right) + \frac{1}{2}\,Ab^2g^2\left(\frac{2\,c^2\log(dx+c)}{d^3i}+\frac{dx^2-2\,cx}{d^2i}\right) + \frac{Aa^2g^2\log(dix+ci)}{di} + \frac{(b^2c^2g^2-2\,abcdg^2+a^2d^2g^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^3i} + \frac{(2\,a^2d^2g^2\log(e)+(2\,g^2\log(e)+3\,g^2)b^2c^2-4\,(g^2\log(e)+g^2)abcd)B\log(dx+c)}{2\,d^3i} + \frac{Bb^2d^2g^2x^2\log(e)-(b^2c^2g^2-2\,abcdg^2+a^2d^2g^2)B\log(dx+c)^2-((2\,g^2\log(e)+g^2)b^2cd-(4\,g^2\log(e}{}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algori
thm="maxima")
```

output
```
2*A*a*b*g^2*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + 1/2*A*b^2*g^2*(2*c^2*log(
d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A*a^2*g^2*log(d*i*x + c*i)/(
d*i) + (b^2*c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*(log(b*x + a)*log((b*d*
x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^3*i) +
 1/2*(2*a^2*d^2*g^2*log(e) + (2*g^2*log(e) + 3*g^2)*b^2*c^2 - 4*(g^2*log(e
) + g^2)*a*b*c*d)*B*log(d*x + c)/(d^3*i) + 1/2*(B*b^2*d^2*g^2*x^2*log(e) -
 (b^2*c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B*log(d*x + c)^2 - ((2*g^2*lo
g(e) + g^2)*b^2*c*d - (4*g^2*log(e) + g^2)*a*b*d^2)*B*x + (B*b^2*d^2*g^2*x
^2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B*x - (2*a*b*c*d*g^2 - 3*a^2*d^2*g^2)
*B)*log(b*x + a) - (B*b^2*d^2*g^2*x^2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B*
x)*log(d*x + c))/(d^3*i)
```

### 3.32.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2364 vs. $2(191) = 382$.

Time = 55.79 (sec) , antiderivative size = 2364, normalized size of antiderivative = 11.94

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algori
thm="giac")
```

output
```
1/24*(2*(B*b^7*c^5*e^5*g^2 - 5*B*a*b^6*c^4*d*e^5*g^2 + 10*B*a^2*b^5*c^3*d^
2*e^5*g^2 - 10*B*a^3*b^4*c^2*d^3*e^5*g^2 + 5*B*a^4*b^3*c*d^4*e^5*g^2 - B*a
^5*b^2*d^5*e^5*g^2 - 4*(b*e*x + a*e)*B*b^6*c^5*d*e^4*g^2/(d*x + c) + 20*(b
*e*x + a*e)*B*a*b^5*c^4*d^2*e^4*g^2/(d*x + c) - 40*(b*e*x + a*e)*B*a^2*b^4
*c^3*d^3*e^4*g^2/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^3*c^2*d^4*e^4*g^2/(d
*x + c) - 20*(b*e*x + a*e)*B*a^4*b^2*c*d^5*e^4*g^2/(d*x + c) + 4*(b*e*x +
a*e)*B*a^5*b*d^6*e^4*g^2/(d*x + c) + 6*(b*e*x + a*e)^2*B*b^5*c^5*d^2*e^3*g
^2/(d*x + c)^2 - 30*(b*e*x + a*e)^2*B*a*b^4*c^4*d^3*e^3*g^2/(d*x + c)^2 +
60*(b*e*x + a*e)^2*B*a^2*b^3*c^3*d^4*e^3*g^2/(d*x + c)^2 - 60*(b*e*x + a*e
)^2*B*a^3*b^2*c^2*d^5*e^3*g^2/(d*x + c)^2 + 30*(b*e*x + a*e)^2*B*a^4*b*c*d
^6*e^3*g^2/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^5*d^7*e^3*g^2/(d*x + c)^2)*
log((b*e*x + a*e)/(d*x + c))/(b^4*d^3*e^4*i - 4*(b*e*x + a*e)*b^3*d^4*e^3*
i/(d*x + c) + 6*(b*e*x + a*e)^2*b^2*d^5*e^2*i/(d*x + c)^2 - 4*(b*e*x + a*e
)^3*b*d^6*e*i/(d*x + c)^3 + (b*e*x + a*e)^4*d^7*i/(d*x + c)^4) + (2*A*b^8*
c^5*e^5*g^2 + B*b^8*c^5*e^5*g^2 - 10*A*a*b^7*c^4*d*e^5*g^2 - 5*B*a*b^7*c^4
*d*e^5*g^2 + 20*A*a^2*b^6*c^3*d^2*e^5*g^2 + 10*B*a^2*b^6*c^3*d^2*e^5*g^2 -
 20*A*a^3*b^5*c^2*d^3*e^5*g^2 - 10*B*a^3*b^5*c^2*d^3*e^5*g^2 + 10*A*a^4*b^
4*c*d^4*e^5*g^2 + 5*B*a^4*b^4*c*d^4*e^5*g^2 - 2*A*a^5*b^3*d^5*e^5*g^2 - B*
a^5*b^3*d^5*e^5*g^2 - 8*(b*e*x + a*e)*A*b^7*c^5*d*e^4*g^2/(d*x + c) - 2*(b
*e*x + a*e)*B*b^7*c^5*d*e^4*g^2/(d*x + c) + 40*(b*e*x + a*e)*A*a*b^6*c^...
```

### 3.32.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx=\int \frac{{\left(a\,g+b\,g\,x\right)}^2\,\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x),x
)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x),
x)
```

# 3.33 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$



## 3.33.1 Optimal result

Integrand size = 38, antiderivative size = 125

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di}+\frac{(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i}+\frac{B(bc-ad)g\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}$$

output
```
g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i+(-a*d+b*c)*g*ln((-a*d+b*c)/b/(d*
x+c))*(A+B+B*ln(e*(b*x+a)/(d*x+c)))/d^2/i+B*(-a*d+b*c)*g*polylog(2,d*(b*x+
a)/b/(d*x+c))/d^2/i
```

## 3.33.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.30

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g\left(2Abdx+2Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-2B(bc-ad)\log(c+dx)-2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\right.}{2d^2i}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*
x),x]
```

output
```
(g*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c -
a*d)*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log
[c + d*x] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(2*d^2*i)
```

### 3.33.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 145, normalized size of antiderivative = 1.16, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.105$, Rules used = {2962, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{ci+dix}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{g(bc-ad)\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \text{2784}$$

$$\frac{g(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int \frac{A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{i}$$

$$\Big\downarrow \text{2754}$$

$$\frac{g(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{B\int \frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A+B\right)}{d}}{d}\right)}{i}$$

$$\Big\downarrow \text{2838}$$

$$\frac{g(bc-ad)\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A+B\right)}{d}-\frac{B\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d}}{d}\right)}{i}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x),x]
```

output
```
((b*c - a*d)*g*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d
*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((A + B + B*Log[(e*(a + b*x))/(c +
 d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*PolyLog[2, (d*(a + b
*x))/(b*(c + d*x))])/d)/d))/i
```

### 3.33.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

---

3.33. $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx$

### 3.33.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 331 vs. $2(125) = 250$.

Time = 1.34 (sec) , antiderivative size = 332, normalized size of antiderivative = 2.66

| method | result |
|---|---|
| parts | $\frac{gA\left(\frac{bx}{d}+\frac{(ad-cb)\ln(dx+c)}{d^2}\right)}{i}-\frac{gB\left(be(ad-cb)\left(\frac{\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}{bed}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)}\right)+(ad-\cdots\right.}{i\cdots}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{gAb}{i\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{gA\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ei}+\frac{gB\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ei}+\frac{gdB\ln\left(\frac{be}{d}+\frac{(ad-c\cdots}{d(dx+\cdots}\right.}{ei\left(be-\left(\frac{be}{d}\cdots\right.\right.}\right.}{d^2\cdots}$ |
| default | $-\frac{e(ad-cb)\left(\frac{gAb}{i\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{gA\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ei}+\frac{gB\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ei}+\frac{gdB\ln\left(\frac{be}{d}+\frac{(ad-c\cdots}{d(dx+\cdots}\right.}{ei\left(be-\left(\frac{be}{d}\cdots\right.\right.}\right.}{d^2\cdots}$ |
| risch | $\frac{gAbx}{id}+\frac{gA\ln(dx+c)a}{id}-\frac{gA\ln(dx+c)cb}{i\,d^2}-\frac{gB\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)a}{id}+\frac{gBb\ln\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be\right)c}{i\,d^2}+\cdots$ |

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x,method=_RETURNVE
RBOSE)
```

output
```
g*A/i*(b*x/d+(a*d-b*c)/d^2*ln(d*x+c))-g*B/i/d*(b*e*(a*d-b*c)*(1/b/e/d*ln((
b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d
+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e))+(a*d-b*
c)*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)
*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d))
```

### 3.33.5 Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx=\int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorith
m="fricas")
```

output
```
integral((A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b*e*x + a*e)/(d*x + c))
)/(d*i*x + c*i), x)
```

### 3.33.6 Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \frac{g\left(\int \frac{Aa}{c+dx}\,dx+\int \frac{Abx}{c+dx}\,dx+\int \frac{Ba\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx+\int \frac{Bbx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx\right)}{i}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

output
```
g*(Integral(A*a/(c + d*x), x) + Integral(A*b*x/(c + d*x), x) + Integral(B*
a*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integral(B*b*x*log(
a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i
```

### 3.33.7 Maxima [A] (verification not implemented)

Time = 0.25 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.77

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = Abg\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right) + \frac{Aag\log(dix+ci)}{di} - \frac{(bcg-adg)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^2i} + \frac{(adg\log(e)-(g\log(e)+g)bc)B\log(dx+c)}{d^2i} - \frac{2\,Bbdgx\log(dx+c)-2\,Bbdgx\log(e)-(bcg-adg)B\log(dx+c)^2-2\,(Bbdgx+Badg)\log(bx+a)}{2\,d^2i}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorith
m="maxima")
```

output
```
A*b*g*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + A*a*g*log(d*i*x + c*i)/(d*i) -
(b*c*g - a*d*g)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-
(b*d*x + a*d)/(b*c - a*d)))*B/(d^2*i) + (a*d*g*log(e) - (g*log(e) + g)*b*c
)*B*log(d*x + c)/(d^2*i) - 1/2*(2*B*b*d*g*x*log(d*x + c) - 2*B*b*d*g*x*log
(e) - (b*c*g - a*d*g)*B*log(d*x + c)^2 - 2*(B*b*d*g*x + B*a*d*g)*log(b*x +
 a))/(d^2*i)
```

### 3.33.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1230 vs. 2(124) = 248.

Time = 42.82 (sec) , antiderivative size = 1230, normalized size of antiderivative = 9.84

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorith
m="giac")
```

output
```
-1/6*((B*b^5*c^4*e^4*g - 4*B*a*b^4*c^3*d*e^4*g + 6*B*a^2*b^3*c^2*d^2*e^4*g
 - 4*B*a^3*b^2*c*d^3*e^4*g + B*a^4*b*d^4*e^4*g - 3*(b*e*x + a*e)*B*b^4*c^4
*d*e^3*g/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^3*c^3*d^2*e^3*g/(d*x + c) - 18
*(b*e*x + a*e)*B*a^2*b^2*c^2*d^3*e^3*g/(d*x + c) + 12*(b*e*x + a*e)*B*a^3*
b*c*d^4*e^3*g/(d*x + c) - 3*(b*e*x + a*e)*B*a^4*d^5*e^3*g/(d*x + c))*log((
b*e*x + a*e)/(d*x + c))/(b^3*d^2*e^3*i - 3*(b*e*x + a*e)*b^2*d^3*e^2*i/(d*
x + c) + 3*(b*e*x + a*e)^2*b*d^4*e*i/(d*x + c)^2 - (b*e*x + a*e)^3*d^5*i/(
d*x + c)^3) + (A*b^6*c^4*e^4*g - 4*A*a*b^5*c^3*d*e^4*g + 6*A*a^2*b^4*c^2*d
^2*e^4*g - 4*A*a^3*b^3*c*d^3*e^4*g + A*a^4*b^2*d^4*e^4*g - 3*(b*e*x + a*e)
*A*b^5*c^4*d*e^3*g/(d*x + c) + (b*e*x + a*e)*B*b^5*c^4*d*e^3*g/(d*x + c) +
 12*(b*e*x + a*e)*A*a*b^4*c^3*d^2*e^3*g/(d*x + c) - 4*(b*e*x + a*e)*B*a*b^
4*c^3*d^2*e^3*g/(d*x + c) - 18*(b*e*x + a*e)*A*a^2*b^3*c^2*d^3*e^3*g/(d*x
+ c) + 6*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g/(d*x + c) + 12*(b*e*x + a*e
)*A*a^3*b^2*c*d^4*e^3*g/(d*x + c) - 4*(b*e*x + a*e)*B*a^3*b^2*c*d^4*e^3*g/
(d*x + c) - 3*(b*e*x + a*e)*A*a^4*b*d^5*e^3*g/(d*x + c) + (b*e*x + a*e)*B*
a^4*b*d^5*e^3*g/(d*x + c) - (b*e*x + a*e)^2*B*b^4*c^4*d^2*e^2*g/(d*x + c)^
2 + 4*(b*e*x + a*e)^2*B*a*b^3*c^3*d^3*e^2*g/(d*x + c)^2 - 6*(b*e*x + a*e)^
2*B*a^2*b^2*c^2*d^4*e^2*g/(d*x + c)^2 + 4*(b*e*x + a*e)^2*B*a^3*b*c*d^5*e^
2*g/(d*x + c)^2 - (b*e*x + a*e)^2*B*a^4*d^6*e^2*g/(d*x + c)^2)/(b^4*d^2*e^
3*i - 3*(b*e*x + a*e)*b^3*d^3*e^2*i/(d*x + c) + 3*(b*e*x + a*e)^2*b^2*d...
```

### 3.33.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x),x)
```

output
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x), x)
```

# 3.34 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx$



## 3.34.1 Optimal result

Integrand size = 30, antiderivative size = 76

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx=-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di}-\frac{B\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

output
```
-ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i-B*polylog(2,d*(b
*x+a)/b/(d*x+c))/d/i
```

## 3.34.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 95, normalized size of antiderivative = 1.25

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx =\frac{\log(i(c+dx))\left(2A-2B\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2B\log\left(\frac{e(a+bx)}{c+dx}\right)+B\log(i(c+dx))\right)-2B\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{bc-ad}\right)}{2di}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x),x]
```

output
```
(Log[i*(c + d*x)]*(2*A - 2*B*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*B*Log[(
e*(a + b*x))/(c + d*x)] + B*Log[i*(c + d*x)]) - 2*B*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)])/(2*d*i)
```

### 3.34.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.07, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2944, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{ci+dix}dx$$

$$\Big\downarrow \text{2944}$$

$$\frac{B(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{di}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}$$

$$\Big\downarrow \text{2858}$$

$$\frac{B(bc-ad)\int \frac{d\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)\left(\left(a-\frac{bc}{d}\right)d+b(c+dx)\right)}d(c+dx)}{d^2i}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}$$

$$\Big\downarrow \text{27}$$

$$\frac{B(bc-ad)\int -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{di}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}$$

$$\Big\downarrow \text{25}$$

$$-\frac{B(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{di}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}$$

$$\Big\downarrow \text{2778}$$

$$\frac{B(bc-ad)\int \frac{(c+dx)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{di}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}$$

$$\Big\downarrow \text{2005}$$

$$\frac{B(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{di}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{di}-\frac{B\,\text{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{di}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x),x]
```

output
```
-((Log[(b*c - a*d)/(b*(c + d*x))]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d
*i)) - (B*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(d*i)
```

### 3.34.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2944
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c +
 d*x))])*((A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/g), x] + Simp[B*n*((b*c
- a*d)/g)   Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x
] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && EqQ[d*f - c*g, 0]
```

### 3.34.4 Maple [A] (verified)

Time = 1.37 (sec) , antiderivative size = 146, normalized size of antiderivative = 1.92

| method | result |
|---|---|
| parts | $\frac{A\ln(dx+c)}{id}+\frac{B\left(-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}\right)}{i}$ |
| risch | $\frac{A\ln(dx+c)}{id}-\frac{B\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{id}-\frac{B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{id}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{dA\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ie(ad-cb)}-\frac{d^2B\left(-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}+\frac{(ad-c}{d(dx+}\right.}{be}\right.}{d}\ldots\right.}{ie(ad-cb)}\ldots\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(\frac{dA\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ie(ad-cb)}-\frac{d^2B\left(-\frac{\operatorname{dilog}\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d-be}{be}\right)}{d}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(-\frac{\left(\frac{be}{d}+\frac{(ad-c}{d(dx+}\right.}{be}\right.}{d}\ldots\right.}{ie(ad-cb)}\ldots\right.}{d^2}$ |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x,method=_RETURNVERBOSE)
```

output
```
A/i*ln(d*x+c)/d+B/i*(-dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d-
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/
b/e)/d)
```

### 3.34.5 Fricas [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{dix+ci}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="fricas")
```

output
```
integral((B*log((b*e*x + a*e)/(d*x + c)) + A)/(d*i*x + c*i), x)
```

### 3.34.6 Sympy [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx=\frac{\int \frac{A}{c+dx}\,dx+\int \frac{B\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx}{i}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x)
```

output
```
(Integral(A/(c + d*x), x) + Integral(B*log(a*e/(c + d*x) + b*e*x/(c + d*x)
)/(c + d*x), x))/i
```

### 3.34.7 Maxima [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci+dix}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{dix+ci}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="maxima")
```

output
```
-1/2*B*(log(d*x + c)^2/(d*i) - 2*integrate((log(b*x + a) + log(e))/(d*i*x
+ c*i), x)) + A*log(d*i*x + c*i)/(d*i)
```

### 3.34.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 617 vs. $2(75) = 150$.

Time = 36.75 (sec) , antiderivative size = 617, normalized size of antiderivative = 8.12

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci + dix}\,dx$$
$$= \frac{1}{2}\left(\frac{(Bb^3c^3e^3 - 3\,Bab^2c^2de^3 + 3\,Ba^2bcd^2e^3 - Ba^3d^3e^3)\log\left(\frac{bex+ae}{dx+c}\right)}{b^2de^2i - \frac{2\,(bex+ae)bd^2ei}{dx+c} + \frac{(bex+ae)^2d^3i}{(dx+c)^2}} + \frac{Ab^4c^3e^3 - Bb^4c^3e^3 - 3\,Aab^3c^2de^3 -$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i),x, algorithm="giac")
```

output
```
1/2*((B*b^3*c^3*e^3 - 3*B*a*b^2*c^2*d*e^3 + 3*B*a^2*b*c*d^2*e^3 - B*a^3*d^
3*e^3)*log((b*e*x + a*e)/(d*x + c))/(b^2*d*e^2*i - 2*(b*e*x + a*e)*b*d^2*e
*i/(d*x + c) + (b*e*x + a*e)^2*d^3*i/(d*x + c)^2) + (A*b^4*c^3*e^3 - B*b^4
*c^3*e^3 - 3*A*a*b^3*c^2*d*e^3 + 3*B*a*b^3*c^2*d*e^3 + 3*A*a^2*b^2*c*d^2*e
^3 - 3*B*a^2*b^2*c*d^2*e^3 - A*a^3*b*d^3*e^3 + B*a^3*b*d^3*e^3 + (b*e*x +
a*e)*B*b^3*c^3*d*e^2/(d*x + c) - 3*(b*e*x + a*e)*B*a*b^2*c^2*d^2*e^2/(d*x
+ c) + 3*(b*e*x + a*e)*B*a^2*b*c*d^3*e^2/(d*x + c) - (b*e*x + a*e)*B*a^3*d
^4*e^2/(d*x + c))/(b^3*d*e^2*i - 2*(b*e*x + a*e)*b^2*d^2*e*i/(d*x + c) + (
b*e*x + a*e)^2*b*d^3*i/(d*x + c)^2) + (B*b^3*c^3*e - 3*B*a*b^2*c^2*d*e + 3
*B*a^2*b*c*d^2*e - B*a^3*d^3*e)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b^2
*d*i) - (B*b^3*c^3*e - 3*B*a*b^2*c^2*d*e + 3*B*a^2*b*c*d^2*e - B*a^3*d^3*e
)*log((b*e*x + a*e)/(d*x + c))/(b^2*d*i))*(b*c/((b*c*e - a*d*e)*(b*c - a*d
)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

### 3.34.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{ci + dix}\,dx = \int \frac{A + B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{c\,i + d\,i\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(c*i + d*i*x),x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(c*i + d*i*x), x)
```

# 3.35 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx$



## 3.35.1 Optimal result

Integrand size = 40, antiderivative size = 44

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx=\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2B(bc-ad)gi}$$

output
```
1/2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/B/(-a*d+b*c)/g/i
```

## 3.35.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.08 (sec) , antiderivative size = 207, normalized size of antiderivative = 4.70

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx$$
$$=\frac{2A\log(a+bx)-B\log^2(a+bx)+2B\log(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-2A\log(c+dx)+2B\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(\ldots}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x
)),x]
```

output
```
(2*A*Log[a + b*x] - B*Log[a + b*x]^2 + 2*B*Log[a + b*x]*Log[(e*(a + b*x))/
(c + d*x)] - 2*A*Log[c + d*x] + 2*B*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[
c + d*x] - 2*B*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - B*Log[c + d*x]^
2 + 2*B*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*B*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)] + 2*B*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(2*(
b*c - a*d)*g*i)
```

### 3.35.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.050, Rules used = {2962, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)(ci+dix)}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}d\frac{a+bx}{c+dx}}{gi(bc-ad)}$$

$$\Big\downarrow \text{2738}$$

$$\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2Bgi(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x)),x]
```

output
```
(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(2*B*(b*c - a*d)*g*i)
```

### 3.35.3.1 Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.35.4 Maple [A] (verified)

Time = 0.62 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.64

| method | result | size |
|---|---|---|
| norman | $-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)^{2}}{2gi(ad-cb)}-\frac{A\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi(ad-cb)}$ | 72 |
| parallelrisch | $-\frac{a^{2}c^{2}B\ln\left(\frac{e(bx+a)}{dx+c}\right)^{2}+2A\ln\left(\frac{e(bx+a)}{dx+c}\right)a^{2}c^{2}}{2a^{2}c^{2}gi(ad-cb)}$ | 75 |
| parts | $\frac{A\left(\frac{\ln(dx+c)}{ad-cb}-\frac{\ln(bx+a)}{ad-cb}\right)}{gi}-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)^{2}}{2gi(ad-cb)}$ | 82 |
| risch | $\frac{A\ln(dx+c)}{gi(ad-cb)}-\frac{A\ln(bx+a)}{gi(ad-cb)}-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)^{2}}{2gi(ad-cb)}$ | 87 |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{d^{2}A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^{2}g}+\frac{d^{2}B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}{2ei(ad-cb)^{2}g}\right)}{d^{2}}$ | 123 |
| default | $-\frac{e(ad-cb)\left(\frac{d^{2}A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^{2}g}+\frac{d^{2}B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}{2ei(ad-cb)^{2}g}\right)}{d^{2}}$ | 123 |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x,method=_RETURNVE
RBOSE)
```

output
```
-1/2*B/g/i/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))^2-A/g/i/(a*d-b*c)*ln(e*(b*x+a)/
(d*x+c))
```

### 3.35.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 60, normalized size of antiderivative = 1.36

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx=\frac{B\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,A\log\left(\frac{bex+ae}{dx+c}\right)}{2\,(bc-ad)gi}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x, algorith
m="fricas")
```

output
```
1/2*(B*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*log((b*e*x + a*e)/(d*x + c)))/
((b*c - a*d)*g*i)
```

### 3.35.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 170 vs. 2(31) = 62.

Time = 0.31 (sec) , antiderivative size = 170, normalized size of antiderivative = 3.86

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx=A\left(\frac{\log\left(x+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{gi\,(ad-bc)}-\frac{\log\left(x+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{gi\,(ad-bc)}\right)-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2adgi-2bcgi}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x)
```

output
```
A*(log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2*c*
*2/(a*d - b*c) + b*c)/(2*b*d))/(g*i*(a*d - b*c)) - log(x + (a**2*d**2/(a*d
 - b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2*b*
d))/(g*i*(a*d - b*c))) - B*log(e*(a + b*x)/(c + d*x))**2/(2*a*d*g*i - 2*b*
c*g*i)
```

### 3.35.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 172 vs. $2(42) = 84$.

Time = 0.21 (sec) , antiderivative size = 172, normalized size of antiderivative = 3.91

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)(ci + dix)}\,dx$$
$$= B\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + A\left(\frac{\log(bx+a)}{(bc-ad)gi} - \frac{\log(dx+c)}{(bc-ad)gi}\right) - \frac{\left(\log(bx+a)^2 - 2\log(bx+a)\log(dx+c) + \log(dx+c)^2\right)B}{2(bcgi - adgi)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x, algorith
m="maxima")
```

output
```
B*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) + A*(log(b*x + a)/((b*c - a*d)*g*i) - log(d
*x + c)/((b*c - a*d)*g*i)) - 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x
+ c) + log(d*x + c)^2)*B/(b*c*g*i - a*d*g*i)
```

### 3.35.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 104 vs. $2(42) = 84$.

Time = 0.32 (sec) , antiderivative size = 104, normalized size of antiderivative = 2.36

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)(ci + dix)}\,dx$$
$$= \frac{\left(Be\log\left(\frac{bex+ae}{dx+c}\right)^2 + 2\,Ae\log\left(\frac{bex+ae}{dx+c}\right)\right)\left(\frac{bc}{(bce-ade)(bc-ad)} - \frac{ad}{(bce-ade)(bc-ad)}\right)}{2\,gi}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i),x, algorith
m="giac")
```

output
```
1/2*(B*e*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*e*log((b*e*x + a*e)/(d*x + c
)))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d))
)/(g*i)
```

### 3.35.9 Mupad [B] (verification not implemented)

Time = 2.52 (sec) , antiderivative size = 69, normalized size of antiderivative = 1.57

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)}\,dx=-\frac{B\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2-A\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{2\,g\,i\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)*(c*i + d*i*x)),x)
```

output
```
-(B*log((e*(a + b*x))/(c + d*x))^2 - A*atan((b*c*2i + b*d*x*2i)/(a*d - b*c
) + 1i)*4i)/(2*g*i*(a*d - b*c))
```

# 3.36 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx$



## 3.36.1 Optimal result

Integrand size = 40, antiderivative size = 173

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{bB(c+dx)}{(bc-ad)^2g^2i(a+bx)}+\frac{Bd\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^2g^2i} - \frac{b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i}$$

output
```
-b*B*(d*x+c)/(-a*d+b*c)^2/g^2/i/(b*x+a)+1/2*B*d*ln((b*x+a)/(d*x+c))^2/(-a*
d+b*c)^2/g^2/i-b*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^2/i/(b
*x+a)-d*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^2/g^2/i
```

## 3.36.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.18 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.69

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2d(a+bx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2d(a+bx)\left(A+\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i
*x)),x]
```

output
```
-1/2*(2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d*(a + b*x)*L
og[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(a + b*x)*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Lo
g[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[
a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)]) + B*d*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - L
og[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*
c - a*d)^2*g^2*i*(a + b*x))
```

### 3.36.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^2(ci+dix)}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i(bc-ad)^2}$$

$$\Big\downarrow \text{2772}$$

$$\frac{-B\int -\frac{(c+dx)^2\left(b+\frac{d(a+bx)\log\left(\frac{a+bx}{c+dx}\right)}{c+dx}\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{b(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i(bc-ad)^2}$$

$$\Big\downarrow \text{25}$$

$$\frac{B\int \frac{(c+dx)^2\left(b+\frac{d(a+bx)\log\left(\frac{a+bx}{c+dx}\right)}{c+dx}\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{b(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i(bc-ad)^2}$$

$$\Big\downarrow \text{2010}$$

$$\frac{B\int\left(\frac{b(c+dx)^2}{(a+bx)^2}+\frac{d\log\left(\frac{a+bx}{c+dx}\right)(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{g^2i(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+B\left(\frac{1}{2}d\log^2\left(\frac{a+bx}{c+dx}\right)-\frac{b(c+dx)}{a+bx}\right)}{g^2i(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i*x)),x
]
```

output
```
(B*(-((b*(c + d*x))/(a + b*x)) + (d*Log[(a + b*x)/(c + d*x)]^2)/2) - (b*(c
 + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - d*Log[(a + b*x)/
(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^2*g^2*i)
```

### 3.36.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] && !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.36.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 230, normalized size of antiderivative = 1.33

| method | result |
|---|---|
| parts | $\frac{A\left(\frac{d\ln(dx+c)}{(ad-cb)^2}+\frac{1}{(bx+a)(ad-cb)}-\frac{d\ln(bx+a)}{(ad-cb)^2}\right)}{g^2 i} - \frac{B\left(\frac{d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^2}-\frac{dbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{(ad-cb)^2}\right)}{g^2 id}$ |
| norman | $\frac{-\frac{(Aad+Bbc)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{Bad\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{b(Ad+Bd)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{(A+B)bx}{gia(ad-cb)}-\frac{bBdx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi\left(a^2d^2-2abcd+b^2c^2\right)}}{g(bx+a)}$ |
| parallelrisch | $-\frac{2Axa^3bc^2d+2Bxa^3bc^2d+B\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a^4c^2d-2Axa^2b^2c^3+2A\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4c^2d-2Bxa^2b^2c^3+2B\ln\left(\frac{e(bx+a)}{dx+c}\right.}{2ig^2(bx+a)(a^2d^2-2abcd+b^2c^2)c}$ |
| risch | $\frac{Ad\ln(dx+c)}{g^2i(ad-cb)^2}+\frac{A}{g^2i(bx+a)(ad-cb)}-\frac{Ad\ln(bx+a)}{g^2i(ad-cb)^2}-\frac{Bd\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2g^2i(ad-cb)^2}-\frac{Bbe\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{g^2i(ad-cb)^2\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ecb}{d(dx+c)}\right.}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{d^2Ab}{i(ad-cb)^3g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^3A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^3g^2}-\frac{d^2Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{i(ad-cb)^3g^2}+\frac{d^3B\ln}{2}\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(\frac{d^2Ab}{i(ad-cb)^3g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^3A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^3g^2}-\frac{d^2Bb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{i(ad-cb)^3g^2}+\frac{d^3B\ln}{2}\right.}{d^2}$ |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i),x,method=_RETURN
VERBOSE)
```

output
```
A/g^2/i*(d/(a*d-b*c)^2*ln(d*x+c)+1/(b*x+a)/(a*d-b*c)-d/(a*d-b*c)^2*ln(b*x+
a))-B/g^2/i/d*(1/2*d^2/(a*d-b*c)^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-d/(a*
d-b*c)^2*b*e*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x
+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

### 3.36.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 144, normalized size of antiderivative = 0.83

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{2\,(A+B)bc-2\,(A+B)ad+(Bbdx+Bad)\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,((A+B)bdx+Bbc+Aad)\log\left(\frac{bex+ae}{dx+c}\right)}{2\,((b^3c^2-2\,ab^2cd+a^2bd^2)g^2ix+(ab^2c^2-2\,a^2bcd+a^3d^2)g^2i)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i),x, algori
thm="fricas")
```

output
```
-1/2*(2*(A + B)*b*c - 2*(A + B)*a*d + (B*b*d*x + B*a*d)*log((b*e*x + a*e)/
(d*x + c))^2 + 2*((A + B)*b*d*x + B*b*c + A*a*d)*log((b*e*x + a*e)/(d*x +
c)))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^2*i*x + (a*b^2*c^2 - 2*a^2*b*c
*d + a^3*d^2)*g^2*i)
```

### 3.36.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(144) = 288$.

Time = 0.62 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.23

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{Bd\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^2d^2g^2i-4abcdg^2i+2b^2c^2g^2i} + \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{a^2dg^2i-abcg^2i+abdg^2ix-b^2cg^2ix} + (A+B)\left(\frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{g^2i\,(ad-bc)^2} - \frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{g^2i\,(ad-bc)^2} + \frac{1}{a^2dg^2i-abcg^2i+x\,(abdg^2i-b^2cg^2i)}\right)$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2/(d*i*x+c*i),x)
```

output
```
-B*d*log(e*(a + b*x)/(c + d*x))**2/(2*a**2*d**2*g**2*i - 4*a*b*c*d*g**2*i
+ 2*b**2*c**2*g**2*i) + B*log(e*(a + b*x)/(c + d*x))/(a**2*d*g**2*i - a*b*
c*g**2*i + a*b*d*g**2*i*x - b**2*c*g**2*i*x) + (A + B)*(d*log(x + (-a**3*d
**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*d - b*c)**2 - 3*a*b**2*c**2*d**2/(
a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(
g**2*i*(a*d - b*c)**2) - d*log(x + (a**3*d**4/(a*d - b*c)**2 - 3*a**2*b*c*
d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 - b**3*c*
*3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))/(g**2*i*(a*d - b*c)**2) + 1/(a**2
*d*g**2*i - a*b*c*g**2*i + x*(a*b*d*g**2*i - b**2*c*g**2*i)))
```

3.36. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx$

### 3.36.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(171) = 342$.

Time = 0.22 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.45

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-B\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log\left(\frac{}{d}\right.$$
$$\left.+\frac{ae}{dx+c}\right)$$
$$-A\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)$$
$$+\frac{((bdx+ad)\log(bx+a)^2+(bdx+ad)\log(dx+c)^2-2\,bc+2\,ad-2\,(bdx+ad)\log(bx+a)+2\,(bdx}{2\,(ab^2c^2g^2i-2\,a^2bcdg^2i+a^3d^2g^2i+(b^3c^2g^2i-2\,ab^2cdg^2i+a^2bc}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i),x, algori
thm="maxima")
```

output
```
-B*(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/(
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*g^2*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - A*(1/((b^2*c
 - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*c^2 - 2*
a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2
)*g^2*i)) + 1/2*((b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)
^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*
x + a*d)*log(b*x + a))*log(d*x + c))*B/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*
i + a^3*d^2*g^2*i + (b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*
x)
```

### 3.36.8 Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{(bgx+ag)^2(dix+ci)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i),x, algori
thm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)/((b*g*x + a*g)^2*(d*i*x + c*i
)), x)
```

## 3.36.9 Mupad [B] (verification not implemented)

Time = 2.45 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.39

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)}\,dx = \frac{A+B}{(a\,d-b\,c)\,\left(a\,g^2\,i+b\,g^2\,i\,x\right)} - \frac{B\,d\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{2\,g^2\,i\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)} + \frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,(a\,d-b\,c)}{b\,d\,g^2\,i\,\left(\frac{x}{d}+\frac{a}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)} + \frac{d\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a^2\,d^2\,g^2\,i-b^2\,c^2\,g^2\,i}{g^2\,i\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A+B)\,2\mathrm{i}}{g^2\,i\,{(a\,d-b\,c)}^2}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^2*(c*i + d*i*x)),x
)
```

output
```
(A + B)/((a*d - b*c)*(a*g^2*i + b*g^2*i*x)) + (d*atan(((2*b*d*x + (a^2*d^2
*g^2*i - b^2*c^2*g^2*i)/(g^2*i*(a*d - b*c)))*1i)/(a*d - b*c))*(A + B)*2i)/
(g^2*i*(a*d - b*c)^2) - (B*d*log((e*(a + b*x))/(c + d*x))^2)/(2*g^2*i*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*log((e*(a + b*x))/(c + d*x))*(a*d - b*c)
)/(b*d*g^2*i*(x/d + a/(b*d))*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
```

# 3.37 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx$



## 3.37.1 Optimal result

Integrand size = 40, antiderivative size = 255

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx=-\frac{B(c+dx)^2\left(b-\frac{4d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3g^3i(a+bx)^2}-\frac{Bd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i}+\frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i(a+bx)}-\frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2}+\frac{d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i}$$

output
```
-1/4*B*(d*x+c)^2*(b-4*d*(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^3/i/(b*x+a)^2-1/
2*B*d^2*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^3/i+2*b*d*(d*x+c)*(A+B*ln(e*(
b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/2*b^2*(d*x+c)^2*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+d^2*ln((b*x+a)/(d*x+c))*(A+B*
ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^3/i
```

### 3.37.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.64

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \frac{-2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4d(bc-ad)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4d^2(a+bx)^2\log(a-$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i
*x)),x]
```

output
```
(-2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d*(b*c - a*d)*(
a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d^2*(a + b*x)^2*Log[a +
b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*(A + B*Log[(
e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 4*B*d*(a + b*x)*(b*c - a*d + d*(a
+ b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*((b*c - a*d)^2 + 2*d*(
-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x)
^2*Log[c + d*x]) - 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log
[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])
 + 2*B*d^2*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x
])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c - a*d)
^3*g^3*i*(a + b*x)^2)
```

### 3.37.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 194, normalized size of antiderivative = 0.76, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^3(ci+dix)}\,dx$$
$$\Big\downarrow \; 2962$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i(bc-ad)^3}$$

$\downarrow$ 2772

$$\frac{-B\int -\frac{(c+dx)^3\left(b^2-\frac{4d(a+bx)b}{c+dx}-\frac{2d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^2}\right)}{2(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx}{c+dx}\right.\right.}{g^3i(bc-ad)^3}$$

$\downarrow$ 27

$$\frac{\frac{1}{2}B\int \frac{(c+dx)^3\left(b^2-\frac{4d(a+bx)b}{c+dx}-\frac{2d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^2}\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{g^3i(bc-ad)^3}$$

$\downarrow$ 2010

$$\frac{\frac{1}{2}B\int \left(\frac{b(c+dx)^3\left(b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)^3}-\frac{2d^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a-}{c+}\right.\right.}{g^3i(bc-ad)^3}$$

$\downarrow$ 2009

$$\frac{-\frac{b^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+\frac{2bd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}+\frac{1}{2}B\left(-d^2\log^2\right.}{g^3i(bc-ad)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i*x)),x
]
```

output
```
((B*(-1/2*((c + d*x)^2*(b - (4*d*(a + b*x))/(c + d*x))^2)/(a + b*x)^2 - d^
2*Log[(a + b*x)/(c + d*x)]^2))/2 + (2*b*d*(c + d*x)*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(a + b*x) - (b^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c +
 d*x)]))/(2*(a + b*x)^2) + d^2*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]))/((b*c - a*d)^3*g^3*i)
```

## 3.37. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx$

### 3.37.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.37.4 Maple [A] (verified)

Time = 1.26 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.43

| method | result |
|---|---|
| parts | $\frac{A\left(\frac{d^2\ln(dx+c)}{(ad-cb)^3}+\frac{1}{2(ad-cb)(bx+a)^2}+\frac{d}{(ad-cb)^2(bx+a)}-\frac{d^2\ln(bx+a)}{(ad-cb)^3}\right)}{g^3 i}-\frac{B\left(\frac{d^3\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^3}-\frac{2d^2be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-}{d(dx}\right.}{\frac{be}{d}+\frac{(ad-c}{d(dx+}}\right.}{(ad-}\right.}{}$ |
| risch | $\frac{A d^2\ln(dx+c)}{g^3 i(ad-cb)^3}+\frac{A}{2g^3 i(ad-cb)(bx+a)^2}+\frac{Ad}{g^3 i(ad-cb)^2(bx+a)}-\frac{A d^2\ln(bx+a)}{g^3 i(ad-cb)^3}-\frac{B d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2g^3 i(ad-cb)^3}-\frac{}{g^3}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{d^2 eAb^2}{2i(ad-cb)^4 g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{2d^3 Ab}{i(ad-cb)^4 g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^4 A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^4 g^3}+\frac{d^2 eBb^2\left(-\frac{\ln\left(\frac{be}{d}\right.}{2\left(\frac{be}{d}+\right.}\right.}{}\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{d^2 eAb^2}{2i(ad-cb)^4 g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{2d^3 Ab}{i(ad-cb)^4 g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^4 A\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^4 g^3}+\frac{d^2 eBb^2\left(-\frac{\ln\left(\frac{be}{d}\right.}{2\left(\frac{be}{d}+\right.}\right.}{}\right.}{d^2}$ |
| parallelrisch | $-\frac{2Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2 a^4b^2c^2d^2+4Ax^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4b^2c^2d^2+6Bx^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4b^2c^2d^2+4Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2 a^5bc^2d}{}$ |
| norman | $\frac{\frac{6Aab^2d-2Ab^3c+7Bab^2d-Bb^3c}{4gi(ad-cb)^2b^2}-\frac{\left(2Aa^2d^2+4Babcd-Bb^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2ig\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{\left(2Ab^2d+3Bb^2d\right)x}{2ig\left(a^2d^2-2abcd+b^2c^2\right)b}-\frac{Ba^2d^2\ln\left(\frac{e(b}{d}\right.}{2ig\left(a^3d^3-3a^2bcd^2+\right.}}{}$ |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x,method=_RETURN
VERBOSE)
```

output
```
A/g^3/i*(d^2/(a*d-b*c)^3*ln(d*x+c)+1/2/(a*d-b*c)/(b*x+a)^2+d/(a*d-b*c)^2/(
b*x+a)-d^2/(a*d-b*c)^3*ln(b*x+a))-B/g^3/i/d*(1/2*d^3/(a*d-b*c)^3*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2-2*d^2/(a*d-b*c)^3*b*e*(-1/(b*e/d+(a*d-b*c)*e/d/(d
*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+d/
(a*d-b*c)^3*e^2*b^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*
c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

## 3.37. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx$

### 3.37.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 349, normalized size of antiderivative = 1.37

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{(2A+B)b^2c^2-8(A+B)abcd+(6A+7B)a^2d^2-2\left(Bb^2d^2x^2+2Babd^2x+Ba^2d^2\right)\log\left(\frac{bex+ae}{dx+c}\right)^2-}{4\left((b^5c^3-3ab^4c^2d+3a^2b^3cd^2-a^3b^2d^3)g^3ix^2+2(a\right.}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x, algori
thm="fricas")
```

output
```
-1/4*((2*A + B)*b^2*c^2 - 8*(A + B)*a*b*c*d + (6*A + 7*B)*a^2*d^2 - 2*(B*b
^2*d^2*x^2 + 2*B*a*b*d^2*x + B*a^2*d^2)*log((b*e*x + a*e)/(d*x + c))^2 - 2
*((2*A + 3*B)*b^2*c*d - (2*A + 3*B)*a*b*d^2)*x - 2*((2*A + 3*B)*b^2*d^2*x^
2 - B*b^2*c^2 + 4*B*a*b*c*d + 2*A*a^2*d^2 + 2*(B*b^2*c*d + 2*(A + B)*a*b*d
^2)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3
*c*d^2 - a^3*b^2*d^3)*g^3*i*x^2 + 2*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b
^2*c*d^2 - a^4*b*d^3)*g^3*i*x + (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c
*d^2 - a^5*d^3)*g^3*i)
```

### 3.37.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. 2(221) = 442.

Time = 2.59 (sec) , antiderivative size = 889, normalized size of antiderivative = 3.49

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{Bd^2\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^3d^3g^3i-6a^2bcd^2g^3i+6ab^2c^2dg^3i-2b^3c^3g^3i}$$
$$+\frac{d^2\cdot(2A+3B)\log\left(x+\frac{2Aad^3+2Abcd^2+3Bad^3+3Bbcd^2-\frac{a^4d^6\cdot(2A+3B)}{(ad-bc)^3}+\frac{4a^3bcd^5\cdot(2A+3B)}{(ad-bc)^3}-\frac{6a^2b^2c^2d^4\cdot(2A+3B)}{(ad-bc)^3}+\frac{4ab^3c^3d^3\cdot(2A+3}{(ad-bc)^3}}{4Abd^3+6Bbd^3}\right.}{2g^3i\,(ad-bc)^3}$$
$$-\frac{d^2\cdot(2A+3B)\log\left(x+\frac{2Aad^3+2Abcd^2+3Bad^3+3Bbcd^2+\frac{a^4d^6\cdot(2A+3B)}{(ad-bc)^3}-\frac{4a^3bcd^5\cdot(2A+3B)}{(ad-bc)^3}+\frac{6a^2b^2c^2d^4\cdot(2A+3B)}{(ad-bc)^3}-\frac{4ab^3c^3d^3\cdot(2A+3}{(ad-bc)^3}}{4Abd^3+6Bbd^3}\right.}{2g^3i\,(ad-bc)^3}$$
$$+\frac{(3Bad-Bbc+2Bbdx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^4d^2g^3i-4a^3bcdg^3i+4a^3bd^2g^3ix+2a^2b^2c^2g^3i-8a^2b^2cdg^3ix+2a^2b^2d^2g^3ix^2+4ab^3c^2g^3ix-4ab^3cdg}$$
$$+\frac{6Aad-2Abc+7Bad-Bbc+x(4Abd+6Bbd)}{4a^4d^2g^3i-8a^3bcdg^3i+4a^2b^2c^2g^3i+x^2\cdot\left(4a^2b^2d^2g^3i-8ab^3cdg^3i+4b^4c^2g^3i\right)+x\left(8a^3bd^2g^3i-16a^2b^2c\right.}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3/(d*i*x+c*i),x)
```

output
```
-B*d**2*log(e*(a + b*x)/(c + d*x))**2/(2*a**3*d**3*g**3*i - 6*a**2*b*c*d**
2*g**3*i + 6*a*b**2*c**2*d*g**3*i - 2*b**3*c**3*g**3*i) + d**2*(2*A + 3*B)
*log(x + (2*A*a*d**3 + 2*A*b*c*d**2 + 3*B*a*d**3 + 3*B*b*c*d**2 - a**4*d**
6*(2*A + 3*B)/(a*d - b*c)**3 + 4*a**3*b*c*d**5*(2*A + 3*B)/(a*d - b*c)**3
- 6*a**2*b**2*c**2*d**4*(2*A + 3*B)/(a*d - b*c)**3 + 4*a*b**3*c**3*d**3*(2
*A + 3*B)/(a*d - b*c)**3 - b**4*c**4*d**2*(2*A + 3*B)/(a*d - b*c)**3)/(4*A
*b*d**3 + 6*B*b*d**3))/(2*g**3*i*(a*d - b*c)**3) - d**2*(2*A + 3*B)*log(x
+ (2*A*a*d**3 + 2*A*b*c*d**2 + 3*B*a*d**3 + 3*B*b*c*d**2 + a**4*d**6*(2*A
+ 3*B)/(a*d - b*c)**3 - 4*a**3*b*c*d**5*(2*A + 3*B)/(a*d - b*c)**3 + 6*a**
2*b**2*c**2*d**4*(2*A + 3*B)/(a*d - b*c)**3 - 4*a*b**3*c**3*d**3*(2*A + 3*
B)/(a*d - b*c)**3 + b**4*c**4*d**2*(2*A + 3*B)/(a*d - b*c)**3)/(4*A*b*d**3
 + 6*B*b*d**3))/(2*g**3*i*(a*d - b*c)**3) + (3*B*a*d - B*b*c + 2*B*b*d*x)*
log(e*(a + b*x)/(c + d*x))/(2*a**4*d**2*g**3*i - 4*a**3*b*c*d*g**3*i + 4*a
**3*b*d**2*g**3*i*x + 2*a**2*b**2*c**2*g**3*i - 8*a**2*b**2*c*d*g**3*i*x +
 2*a**2*b**2*d**2*g**3*i*x**2 + 4*a*b**3*c**2*g**3*i*x - 4*a*b**3*c*d*g**3
*i*x**2 + 2*b**4*c**2*g**3*i*x**2) + (6*A*a*d - 2*A*b*c + 7*B*a*d - B*b*c
+ x*(4*A*b*d + 6*B*b*d))/(4*a**4*d**2*g**3*i - 8*a**3*b*c*d*g**3*i + 4*a**
2*b**2*c**2*g**3*i + x**2*(4*a**2*b**2*d**2*g**3*i - 8*a*b**3*c*d*g**3*i +
 4*b**4*c**2*g**3*i) + x*(8*a**3*b*d**2*g**3*i - 16*a**2*b**2*c*d*g**3*i +
 8*a*b**3*c**2*g**3*i))
```

### 3.37.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(249) = 498$.

Time = 0.24 (sec) , antiderivative size = 885, normalized size of antiderivative = 3.47

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \frac{1}{2}B\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c^2 - 2\,ab^3cd + a^2b^2d^2)g^3ix^2 + 2\,(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)g^3ix + (a^2b^2c^2 - 2\,a^3bcd + a^4d^2)g^3i} + \cdots + \frac{ae}{dx+c}\right)$$
$$+ \frac{1}{2}A\left(\frac{2\,bdx - bc + 3\,ad}{(b^4c^2 - 2\,ab^3cd + a^2b^2d^2)g^3ix^2 + 2\,(ab^3c^2 - 2\,a^2b^2cd + a^3bd^2)g^3ix + (a^2b^2c^2 - 2\,a^3bcd + a^4d^2)g^3i} \cdots\right.$$
$$- \frac{(b^2c^2 - 8\,abcd + 7\,a^2d^2 + 2\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log{(bx+a)}^2 + 2\,(b^2d^2x^2 + 2\,abd^2x + a^2d^2)\log{(d}\cdots}{4\,(a^2b^3c^3g^3i - 3\,a^3b^2c^2dg^3i + 3\,a^4bcd^2g^3i - a^5d^3g^3i + \cdots}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x, algori
thm="maxima")
```

output
```
1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3*
i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2 -
 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) + 1/2*A*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^
2*d^2)*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^
2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i)) - 1/4*(b^2*c^2 -
8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x +
a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d
 - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(
3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a
^2*d^2)*log(b*x + a))*log(d*x + c))*B/(a^2*b^3*c^3*g^3*i - 3*a^3*b^2*c^2*d
*g^3*i + 3*a^4*b*c*d^2*g^3*i - a^5*d^3*g^3*i + (b^5*c^3*g^3*i - 3*a*b^4*c^
2*d*g^3*i + 3*a^2*b^3*c*d^2*g^3*i - a^3*b^2*d^3*g^3*i)*x^2 + 2*(a*b^4*c^3*
g^3*i - 3*a^2*b^3*c^2*d*g^3*i + 3*a^3*b^2*c*d^2*g^3*i - a^4*b*d^3*g^3*i)*x
)
```

### 3.37.8 Giac [A] (verification not implemented)

Time = 42.82 (sec) , antiderivative size = 139, normalized size of antiderivative = 0.55

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{1}{4}\left(\frac{2\,(dx+c)^2Be^3\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^2g^3i}+\frac{(2\,Ae^3+Be^3)(dx+c)^2}{(bex+ae)^2g^3i}\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i),x, algori
thm="giac")
```

output
```
-1/4*(2*(d*x + c)^2*B*e^3*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^2*g^
3*i) + (2*A*e^3 + B*e^3)*(d*x + c)^2/((b*e*x + a*e)^2*g^3*i))*(b*c/((b*c*e
 - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

### 3.37.9 Mupad [B] (verification not implemented)

Time = 3.50 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.14

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)}\,dx = \frac{3\,A\,a\,d}{2\,g^3\,i\,(a\,d-b\,c)^2\,(a+b\,x)^2}-\frac{B\,d^2\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{2\,g^3\,i\,(a\,d-b\,c)^3}-\frac{A\,b\,c}{2\,g^3\,i\,(a\,d-b\,c)^2\,(a+b\,x)^2}+\frac{7\,B\,a\,d}{4\,g^3\,i\,(a\,d-b\,c)^2\,(a+b\,x)^2}-\frac{B\,b\,c}{4\,g^3\,i\,(a\,d-b\,c)^2\,(a+b\,x)^2}+\frac{3\,B\,a^2\,d^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g^3\,i\,(a\,d-b\,c)^3\,(a+b\,x)^2}+\frac{B\,b^2\,c^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g^3\,i\,(a\,d-b\,c)^3\,(a+b\,x)^2}+\frac{A\,b\,d\,x}{g^3\,i\,(a\,d-b\,c)^2\,(a+b\,x)^2}+\frac{3\,B\,b\,d\,x}{2\,g^3\,i\,(a\,d-b\,c)^2\,(a+b\,x)^2}+\frac{B\,a\,b\,d^2\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i\,(a\,d-b\,c)^3\,(a+b\,x)^2}-\frac{B\,b^2\,c\,d\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i\,(a\,d-b\,c)^3\,(a+b\,x)^2}-\frac{2\,B\,a\,b\,c\,d\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i\,(a\,d-b\,c)^3\,(a+b\,x)^2}+\frac{A\,d^2\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g^3\,i\,(a\,d-b\,c)^3}+\frac{B\,d^2\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,3\mathrm{i}}{g^3\,i\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^3*(c*i + d*i*x)),x
)
```

output
```
(A*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g^3*i*(a*d - b*
c)^3) + (B*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*3i)/(g^3*i*(
a*d - b*c)^3) - (B*d^2*log((e*(a + b*x))/(c + d*x))^2)/(2*g^3*i*(a*d - b*c
)^3) + (3*A*a*d)/(2*g^3*i*(a*d - b*c)^2*(a + b*x)^2) - (A*b*c)/(2*g^3*i*(a
*d - b*c)^2*(a + b*x)^2) + (7*B*a*d)/(4*g^3*i*(a*d - b*c)^2*(a + b*x)^2) -
 (B*b*c)/(4*g^3*i*(a*d - b*c)^2*(a + b*x)^2) + (3*B*a^2*d^2*log((e*(a + b*
x))/(c + d*x)))/(2*g^3*i*(a*d - b*c)^3*(a + b*x)^2) + (B*b^2*c^2*log((e*(a
 + b*x))/(c + d*x)))/(2*g^3*i*(a*d - b*c)^3*(a + b*x)^2) + (A*b*d*x)/(g^3*
i*(a*d - b*c)^2*(a + b*x)^2) + (3*B*b*d*x)/(2*g^3*i*(a*d - b*c)^2*(a + b*x
)^2) + (B*a*b*d^2*x*log((e*(a + b*x))/(c + d*x)))/(g^3*i*(a*d - b*c)^3*(a
+ b*x)^2) - (B*b^2*c*d*x*log((e*(a + b*x))/(c + d*x)))/(g^3*i*(a*d - b*c)^
3*(a + b*x)^2) - (2*B*a*b*c*d*log((e*(a + b*x))/(c + d*x)))/(g^3*i*(a*d -
b*c)^3*(a + b*x)^2)
```

# 3.38 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx$



## 3.38.1 Optimal result

Integrand size = 40, antiderivative size = 373

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = & -\frac{3bBd^2(c+dx)}{(bc-ad)^4g^4i(a+bx)}+\frac{3b^2Bd(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2} \\ & -\frac{b^3B(c+dx)^3}{9(bc-ad)^4g^4i(a+bx)^3}+\frac{Bd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^4i} \\ & -\frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i(a+bx)} \\ & +\frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2} \\ & -\frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3} \\ & -\frac{d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i}\end{aligned}$$

output
```
-3*b*B*d^2*(d*x+c)/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/4*b^2*B*d*(d*x+c)^2/(-a*d+
b*c)^4/g^4/i/(b*x+a)^2-1/9*b^3*B*(d*x+c)^3/(-a*d+b*c)^4/g^4/i/(b*x+a)^3+1/
2*B*d^3*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^4/i-3*b*d^2*(d*x+c)*(A+B*ln(e
*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/2*b^2*d*(d*x+c)^2*(A+B*ln(
e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*x+a)^2-1/3*b^3*(d*x+c)^3*(A+B*ln
(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*x+a)^3-d^3*ln((b*x+a)/(d*x+c))*
(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i
```

## 3.38.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 492, normalized size of antiderivative = 1.32

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx$$
$$= \frac{-\frac{12A(bc-ad)^3}{(a+bx)^3}-\frac{4B(bc-ad)^3}{(a+bx)^3}+\frac{18Ad(bc-ad)^2}{(a+bx)^2}+\frac{15Bd(bc-ad)^2}{(a+bx)^2}+\frac{36Ad^2(-bc+ad)}{a+bx}+\frac{66Bd^2(-bc+ad)}{a+bx}-36Ad^3\log(a+bx)}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i
*x)),x]
```

output
```
((-12*A*(b*c - a*d)^3)/(a + b*x)^3 - (4*B*(b*c - a*d)^3)/(a + b*x)^3 + (18
*A*d*(b*c - a*d)^2)/(a + b*x)^2 + (15*B*d*(b*c - a*d)^2)/(a + b*x)^2 + (36
*A*d^2*(-(b*c) + a*d))/(a + b*x) + (66*B*d^2*(-(b*c) + a*d))/(a + b*x) - 3
6*A*d^3*Log[a + b*x] - 66*B*d^3*Log[a + b*x] + 18*B*d^3*Log[a + b*x]^2 - (
12*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^3 + (18*B*d*(b*
c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x)^2 + (36*B*d^2*(-(b*c) +
 a*d)*Log[(e*(a + b*x))/(c + d*x)])/(a + b*x) - 36*B*d^3*Log[a + b*x]*Log[
(e*(a + b*x))/(c + d*x)] + 36*A*d^3*Log[c + d*x] + 66*B*d^3*Log[c + d*x] -
 36*B*d^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] + 36*B*d^3*Log[(e
*(a + b*x))/(c + d*x)]*Log[c + d*x] + 18*B*d^3*Log[c + d*x]^2 - 36*B*d^3*L
og[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 36*B*d^3*PolyLog[2, (d*(a + b
*x))/(-(b*c) + a*d)] - 36*B*d^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])/(36
*(b*c - a*d)^4*g^4*i)
```

### 3.38.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 262, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2962, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^4(ci+dix)}\,dx$$

$$\Big\downarrow \; 2962$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i(bc-ad)^4}$$

$$\Big\downarrow \; 2772$$

$$\frac{-B\int -\frac{(c+dx)^4\left(2b^3-\frac{9d(a+bx)b^2}{c+dx}+\frac{18d^2(a+bx)^2b}{(c+dx)^2}+\frac{6d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^3}\right)}{6(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\cdots\right.\right.}{2(a+b\cdots}\cdots}{g^4i(bc-ad)^4}$$

$$\Big\downarrow \; 27$$

$$\frac{\frac{1}{6}B\int \frac{(c+dx)^4\left(2b^3-\frac{9d(a+bx)b^2}{c+dx}+\frac{18d^2(a+bx)^2b}{(c+dx)^2}+\frac{6d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^3}\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\cdots\right.\right.}{2(a+bx)\cdots}\cdots}{g^4i(bc-ad)^4}$$

$$\Big\downarrow \; 2010$$

$$\frac{\frac{1}{6}B\int \left(\frac{b\left(2b^2-\frac{9d(a+bx)b}{c+dx}+\frac{18d^2(a+bx)^2}{(c+dx)^2}\right)(c+dx)^4}{(a+bx)^4}+\frac{6d^3\log\left(\frac{a+bx}{c+dx}\right)(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(\cdots\right.}{2\cdots}\cdots}{g^4i(bc-ad)^4}$$

$$\Big\downarrow \; 2009$$

$$\frac{-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+d^3\left(-\log\left(\frac{a+bx}{c+dx}\right)\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-\frac{3bd^2(c+\cdots}{\cdots}\cdots}{g^4i(bc-ad)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i*x)),x
]
```

output
```
((B*((-18*b*d^2*(c + d*x))/(a + b*x) + (9*b^2*d*(c + d*x)^2)/(2*(a + b*x)^
2) - (2*b^3*(c + d*x)^3)/(3*(a + b*x)^3) + 3*d^3*Log[(a + b*x)/(c + d*x)]^
2))/6 - (3*b*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)
 + (3*b^2*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)
^2) - (b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(a + b*x)^
3) - d^3*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((
b*c - a*d)^4*g^4*i)
```

### 3.38.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.38.4 Maple [A] (verified)

Time = 1.63 (sec) , antiderivative size = 500, normalized size of antiderivative = 1.34

| method | result |
|---|---|
| parts | $\frac{A\left(\frac{d^3\ln(dx+c)}{(ad-cb)^4}+\frac{1}{3(ad-cb)(bx+a)^3}+\frac{d}{2(ad-cb)^2(bx+a)^2}+\frac{d^2}{(ad-cb)^3(bx+a)}-\frac{d^3\ln(bx+a)}{(ad-cb)^4}\right)}{g^4 i}-\frac{B\left(\frac{d^4\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^4}-\frac{3c}{}\right.}{}$ |
| risch | $\frac{A\,d^3\ln(dx+c)}{g^4 i(ad-cb)^4}+\frac{A}{3g^4 i(ad-cb)(bx+a)^3}+\frac{Ad}{2g^4 i(ad-cb)^2(bx+a)^2}+\frac{A\,d^2}{g^4 i(ad-cb)^3(bx+a)}-\frac{A\,d^3\ln(bx+a)}{g^4 i(ad-cb)^4}-\frac{B\,d^3}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{d^2e^2A\,b^3}{3i(ad-cb)^5g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{3d^3eA\,b^2}{2i(ad-cb)^5g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{3d^4Ab}{i(ad-cb)^5g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^5A\ln\left(\frac{be}{d}+\right.}{ei(ad-}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(\frac{d^2e^2A\,b^3}{3i(ad-cb)^5g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{3d^3eA\,b^2}{2i(ad-cb)^5g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{3d^4Ab}{i(ad-cb)^5g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^5A\ln\left(\frac{be}{d}+\right.}{ei(ad-}\right.}{}$ |
| parallelrisch | $-\frac{-288A\,x^2a^5b^3c^3d^2+162A\,x^2a^4b^4c^4d-12A\,x^3a^2b^6c^5-4B\,x^3a^2b^6c^5-36A\,x^2a^3b^5c^5+189B\,x^2a^6b^2c^2d^3-258B\,x^2a^5b^3}{}$ |
| norman | Expression too large to display |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x,method=_RETURN
VERBOSE)
```

output
```
A/g^4/i*(d^3/(a*d-b*c)^4*ln(d*x+c)+1/3/(a*d-b*c)/(b*x+a)^3+1/2*d/(a*d-b*c)
^2/(b*x+a)^2+d^2/(a*d-b*c)^3/(b*x+a)-d^3/(a*d-b*c)^4*ln(b*x+a))-B/g^4/i/d*
(1/2*d^4/(a*d-b*c)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-3*d^3/(a*d-b*c)^4*b
*e*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*
e/d+(a*d-b*c)*e/d/(d*x+c)))+3*d^2/(a*d-b*c)^4*b^2*e^2*(-1/2/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^2)-d/(a*d-b*c)^4*b^3*e^3*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

### 3.38.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 611, normalized size of antiderivative = 1.64

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx =$$
$$-\frac{4\,(3\,A+B)b^3c^3-27\,(2\,A+B)ab^2c^2d+108\,(A+B)a^2bcd^2-(66\,A+85\,B)a^3d^3+6\,((6\,A+11\,B)b^3cd}{36\,((b^7}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x, algori
thm="fricas")
```

output
```
-1/36*(4*(3*A + B)*b^3*c^3 - 27*(2*A + B)*a*b^2*c^2*d + 108*(A + B)*a^2*b*
c*d^2 - (66*A + 85*B)*a^3*d^3 + 6*((6*A + 11*B)*b^3*c*d^2 - (6*A + 11*B)*a
*b^2*d^3)*x^2 + 18*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x +
B*a^3*d^3)*log((b*e*x + a*e)/(d*x + c))^2 - 3*((6*A + 5*B)*b^3*c^2*d - 18*
(2*A + 3*B)*a*b^2*c*d^2 + (30*A + 49*B)*a^2*b*d^3)*x + 6*((6*A + 11*B)*b^3
*d^3*x^3 + 2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 + 6*A*a^3*d^3
+ 3*(2*B*b^3*c*d^2 + 3*(2*A + 3*B)*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*d - 6*B*a
*b^2*c*d^2 - 6*(A + B)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c
^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^4)*g^
4*i*x^3 + 3*(a*b^6*c^4 - 4*a^2*b^5*c^3*d + 6*a^3*b^4*c^2*d^2 - 4*a^4*b^3*c
*d^3 + a^5*b^2*d^4)*g^4*i*x^2 + 3*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*a^4*b
^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4)*g^4*i*x + (a^3*b^4*c^4 - 4*a^4*b
^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*g^4*i)
```

### 3.38.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1392 vs. 2(332) = 664.

Time = 9.88 (sec) , antiderivative size = 1392, normalized size of antiderivative = 3.73

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4/(d*i*x+c*i),x)
```

output
```
-B*d**3*log(e*(a + b*x)/(c + d*x))**2/(2*a**4*d**4*g**4*i - 8*a**3*b*c*d**
3*g**4*i + 12*a**2*b**2*c**2*d**2*g**4*i - 8*a*b**3*c**3*d*g**4*i + 2*b**4
*c**4*g**4*i) + d**3*(6*A + 11*B)*log(x + (6*A*a*d**4 + 6*A*b*c*d**3 + 11*
B*a*d**4 + 11*B*b*c*d**3 - a**5*d**8*(6*A + 11*B)/(a*d - b*c)**4 + 5*a**4*
b*c*d**7*(6*A + 11*B)/(a*d - b*c)**4 - 10*a**3*b**2*c**2*d**6*(6*A + 11*B)
/(a*d - b*c)**4 + 10*a**2*b**3*c**3*d**5*(6*A + 11*B)/(a*d - b*c)**4 - 5*a
*b**4*c**4*d**4*(6*A + 11*B)/(a*d - b*c)**4 + b**5*c**5*d**3*(6*A + 11*B)/
(a*d - b*c)**4)/(12*A*b*d**4 + 22*B*b*d**4))/(6*g**4*i*(a*d - b*c)**4) - d
**3*(6*A + 11*B)*log(x + (6*A*a*d**4 + 6*A*b*c*d**3 + 11*B*a*d**4 + 11*B*b
*c*d**3 + a**5*d**8*(6*A + 11*B)/(a*d - b*c)**4 - 5*a**4*b*c*d**7*(6*A + 1
1*B)/(a*d - b*c)**4 + 10*a**3*b**2*c**2*d**6*(6*A + 11*B)/(a*d - b*c)**4 -
 10*a**2*b**3*c**3*d**5*(6*A + 11*B)/(a*d - b*c)**4 + 5*a*b**4*c**4*d**4*(
6*A + 11*B)/(a*d - b*c)**4 - b**5*c**5*d**3*(6*A + 11*B)/(a*d - b*c)**4)/(
12*A*b*d**4 + 22*B*b*d**4))/(6*g**4*i*(a*d - b*c)**4) + (11*B*a**2*d**2 -
7*B*a*b*c*d + 15*B*a*b*d**2*x + 2*B*b**2*c**2 - 3*B*b**2*c*d*x + 6*B*b**2*
d**2*x**2)*log(e*(a + b*x)/(c + d*x))/(6*a**6*d**3*g**4*i - 18*a**5*b*c*d*
*2*g**4*i + 18*a**5*b*d**3*g**4*i*x + 18*a**4*b**2*c**2*d*g**4*i - 54*a**4
*b**2*c*d**2*g**4*i*x + 18*a**4*b**2*d**3*g**4*i*x**2 - 6*a**3*b**3*c**3*g
**4*i + 54*a**3*b**3*c**2*d*g**4*i*x - 54*a**3*b**3*c*d**2*g**4*i*x**2 + 6
*a**3*b**3*d**3*g**4*i*x**3 - 18*a**2*b**4*c**3*g**4*i*x + 54*a**2*b**4...
```

### 3.38.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1469 vs. 2(363) = 726.

Time = 0.31 (sec) , antiderivative size = 1469, normalized size of antiderivative = 3.94

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x, algori
thm="maxima")
```

output
```
-1/6*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*
g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3
)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d
^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*g^
4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))*log(b*e*x/(d*x
 + c) + a*e/(d*x + c)) - 1/6*A*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 1
1*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b
^4*c*d^2 - a^3*b^3*d^3)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3
*b^3*c*d^2 - a^4*b^2*d^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3
*a^4*b^2*c*d^2 - a^5*b*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a
^5*b*c*d^2 - a^6*d^3)*g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*
d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x +
c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4
)*g^4*i)) - 1/36*(4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^
3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3
*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2
 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d - 54*a*b^2*...
```

### 3.38.8 Giac [A] (verification not implemented)

Time = 55.04 (sec) , antiderivative size = 270, normalized size of antiderivative = 0.72

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx =$$

$$-\frac{1}{36}\left(\frac{6\left(2\,Bbe^4-\frac{3\,(bex+ae)Bde^3}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^3bcg^4i}{(dx+c)^3}-\frac{(bex+ae)^3adg^4i}{(dx+c)^3}}+\frac{12\,Abe^4+4\,Bbe^4-\frac{18\,(bex+ae)Ade^3}{dx+c}-\frac{9\,(bex+ae)Bde^3}{dx+c}}{\frac{(bex+ae)^3bcg^4i}{(dx+c)^3}-\frac{(bex+ae)^3adg^4i}{(dx+c)^3}}\right)\Bigg(\frac{}{(bc}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i),x, algori
thm="giac")
```

output
```
-1/36*(6*(2*B*b*e^4 - 3*(b*e*x + a*e)*B*d*e^3/(d*x + c))*log((b*e*x + a*e)
/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4*i/(d*x + c)^3 - (b*e*x + a*e)^3*a*d*g
^4*i/(d*x + c)^3) + (12*A*b*e^4 + 4*B*b*e^4 - 18*(b*e*x + a*e)*A*d*e^3/(d*
x + c) - 9*(b*e*x + a*e)*B*d*e^3/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4*i/(d*
x + c)^3 - (b*e*x + a*e)^3*a*d*g^4*i/(d*x + c)^3))*(b*c/((b*c*e - a*d*e)*(
b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

3.38. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx$

### 3.38.9 Mupad [B] (verification not implemented)

Time = 5.99 (sec) , antiderivative size = 970, normalized size of antiderivative = 2.60

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx = \frac{11\,A\,a^2\,d^2}{6\,g^4\,i\,(a\,d-b\,c)^3\,(a+b\,x)^3} - \frac{B\,d^3\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{2\,g^4\,i\,(a\,d-b\,c)^4} + \frac{A\,b^2\,c^2}{3\,g^4\,i\,(a\,d-b\,c)^3\,(a+b\,x)^3} + \frac{85\,B\,a^2\,d^2}{36\,g^4\,i\,(a\,d-b\,c)^3\,(a+b\,x)^3} + \frac{B\,b^2\,c^2}{9\,g^4\,i\,(a\,d-b\,c)^3\,(a+b\,x)^3} + \frac{11\,B\,a^3\,d^3\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{6\,g^4\,i\,(a\,d-b\,c)^4\,(a+b\,x)^3} - \frac{B\,b^3\,c^3\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{3\,g^4\,i\,(a\,d-b\,c)^4\,(a+b\,x)^3} + \frac{A\,b^2\,d^2\,x^2}{g^4\,i\,(a\,d-b\,c)^3\,(a+b\,x)^3} + \frac{11\,B\,b^2\,d^2\,x^2}{6\,g^4\,i\,(a\,d-b\,c)^3\,(a+b\,x)^3} - \frac{7\,A\,a\,b\,c\,d}{6\,g^4\,i\,(a\,d-b\,c)^3\,(a+b\,x)^3} - \frac{23\,B\,a\,b\,c\,d}{36\,g^4\,i\,(a\,d-b\,c)^3\,(a+b\,x)^3} + \frac{5\,A\,a\,b\,d^2\,x}{2\,g^4\,i\,(a\,d-b\,c)^3\,(a+b\,x)^3} + \frac{49\,B\,a\,b\,d^2\,x}{12\,g^4\,i\,(a\,d-b\,c)^3\,(a+b\,x)^3} - \frac{A\,b^2\,c\,d\,x}{2\,g^4\,i\,(a\,d-b\,c)^3\,(a+b\,x)^3} - \frac{5\,B\,b^2\,c\,d\,x}{12\,g^4\,i\,(a\,d-b\,c)^3\,(a+b\,x)^3} + \frac{3\,B\,a\,b^2\,c^2\,d\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g^4\,i\,(a\,d-b\,c)^4\,(a+b\,x)^3} - \frac{3\,B\,a^2\,b\,c\,d^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^4\,i\,(a\,d-b\,c)^4\,(a+b\,x)^3} + \frac{5\,B\,a^2\,b\,d^3\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g^4\,i\,(a\,d-b\,c)^4\,(a+b\,x)^3} + \frac{B\,b^3\,c^2\,d\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g^4\,i\,(a\,d-b\,c)^4\,(a+b\,x)^3} + \frac{B\,a\,b^2\,d^3\,x^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^4\,i\,(a\,d-b\,c)^4\,(a+b\,x)^3} - \frac{B\,b^3\,c\,d^2\,x^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^4\,i\,(a\,d-b\,c)^4\,(a+b\,x)^3} - \frac{3\,B\,a\,b^2\,c\,d^2\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^4\,i\,(a\,d-b\,c)^4\,(a+b\,x)^3} + \frac{A\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g^4\,i\,(a\,d-b\,c)^4} + \frac{B\,d^3\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,11\mathrm{i}}{3\,g^4\,i\,(a\,d-b\,c)^4}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^4*(c*i + d*i*x)),x
)
```

output
```
(A*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g^4*i*(a*d - b*
c)^4) + (B*d^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*11i)/(3*g^4*
i*(a*d - b*c)^4) - (B*d^3*log((e*(a + b*x))/(c + d*x))^2)/(2*g^4*i*(a*d -
b*c)^4) + (11*A*a^2*d^2)/(6*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (A*b^2*c^2)
/(3*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (85*B*a^2*d^2)/(36*g^4*i*(a*d - b*c
)^3*(a + b*x)^3) + (B*b^2*c^2)/(9*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (11*B
*a^3*d^3*log((e*(a + b*x))/(c + d*x)))/(6*g^4*i*(a*d - b*c)^4*(a + b*x)^3)
 - (B*b^3*c^3*log((e*(a + b*x))/(c + d*x)))/(3*g^4*i*(a*d - b*c)^4*(a + b*
x)^3) + (A*b^2*d^2*x^2)/(g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (11*B*b^2*d^2*
x^2)/(6*g^4*i*(a*d - b*c)^3*(a + b*x)^3) - (7*A*a*b*c*d)/(6*g^4*i*(a*d - b
*c)^3*(a + b*x)^3) - (23*B*a*b*c*d)/(36*g^4*i*(a*d - b*c)^3*(a + b*x)^3) +
 (5*A*a*b*d^2*x)/(2*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (49*B*a*b*d^2*x)/(1
2*g^4*i*(a*d - b*c)^3*(a + b*x)^3) - (A*b^2*c*d*x)/(2*g^4*i*(a*d - b*c)^3*
(a + b*x)^3) - (5*B*b^2*c*d*x)/(12*g^4*i*(a*d - b*c)^3*(a + b*x)^3) + (3*B
*a*b^2*c^2*d*log((e*(a + b*x))/(c + d*x)))/(2*g^4*i*(a*d - b*c)^4*(a + b*x
)^3) - (3*B*a^2*b*c*d^2*log((e*(a + b*x))/(c + d*x)))/(g^4*i*(a*d - b*c)^4
*(a + b*x)^3) + (5*B*a^2*b*d^3*x*log((e*(a + b*x))/(c + d*x)))/(2*g^4*i*(a
*d - b*c)^4*(a + b*x)^3) + (B*b^3*c^2*d*x*log((e*(a + b*x))/(c + d*x)))/(2
*g^4*i*(a*d - b*c)^4*(a + b*x)^3) + (B*a*b^2*d^3*x^2*log((e*(a + b*x))/(c
+ d*x)))/(g^4*i*(a*d - b*c)^4*(a + b*x)^3) - (B*b^3*c*d^2*x^2*log((e*(a...
```

3.38. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)}\,dx$

# 3.39 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$



## 3.39.1 Optimal result

Integrand size = 40, antiderivative size = 341

$$\begin{aligned}\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx &= \frac{3B(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)} - \frac{(6A+5B)(bc-ad)^2g^3(a+bx)}{2d^3i^2(c+dx)} \\ &\quad - \frac{3B(bc-ad)^2g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^2(c+dx)} + \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^2(c+dx)} \\ &\quad - \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^2(c+dx)} \\ &\quad - \frac{b(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^4i^2} \\ &\quad - \frac{3bB(bc-ad)^2g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}\end{aligned}$$

output
```
3*B*(-a*d+b*c)^2*g^3*(b*x+a)/d^3/i^2/(d*x+c)-1/2*(6*A+5*B)*(-a*d+b*c)^2*g^
3*(b*x+a)/d^3/i^2/(d*x+c)-3*B*(-a*d+b*c)^2*g^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c
))/d^3/i^2/(d*x+c)+1/2*g^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^2/(d*
x+c)-1/2*(-a*d+b*c)*g^3*(b*x+a)^2*(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/d^2/i^
2/(d*x+c)-1/2*b*(-a*d+b*c)^2*g^3*ln((-a*d+b*c)/b/(d*x+c))*(6*A+5*B+6*B*ln(
e*(b*x+a)/(d*x+c)))/d^4/i^2-3*b*B*(-a*d+b*c)^2*g^3*polylog(2,d*(b*x+a)/b/(
d*x+c))/d^4/i^2
```

### 3.39.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 359, normalized size of antiderivative = 1.05

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$$
$$= \frac{g^3\left(-2Ab^2d(2bc-3ad)x-2bBd(2bc-3ad)(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+b^3d^2x^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{2(bc\cdots}{\cdots}\cdots\right.}{\cdots}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*
i*x)^2,x]
```

output
```
(g^3*(-2*A*b^2*d*(2*b*c - 3*a*d)*x - 2*b*B*d*(2*b*c - 3*a*d)*(a + b*x)*Log
[(e*(a + b*x))/(c + d*x)] + b^3*d^2*x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
]) + (2*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) + 2*
b*B*(2*b*c - 3*a*d)*(b*c - a*d)*Log[c + d*x] + 6*b*(b*c - a*d)^2*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 2*B*(b*c - a*d)^2*((b*c - a*d)/
(c + d*x) + b*Log[a + b*x] - b*Log[c + d*x]) + b*B*(-(a^2*d^2*Log[a + b*x]
) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 3*b*B*(b*c - a*d)^2*((2
*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)])))/(2*d^4*i^2)
```

### 3.39.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 304, normalized size of antiderivative = 0.89, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {2962, 2784, 2784, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ci+dix)^2}\,dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{g^3(bc-ad)^2\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i^2}$$

$\Big\downarrow$ 2784

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{i^2}$$

$\Big\downarrow$ 2784

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{d}}{2d}\right)}{i^2}$$

$\Big\downarrow$ 2793

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\left(-\frac{6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)}{d}-\frac{b\left(6A+5B+6B\log\left(\frac{e(a+}{c+d}\right.\right.}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right.}{d}}{2d}\right.}{i^2}$$

$\Big\downarrow$ 2009

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A+5B\right)}{d^2}-\frac{(6A+5}{d(}}{2d}\right.}{i^2}$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^2
,x]
```

## 3.39. $\int\frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$

output
```
((b*c - a*d)^2*g^3*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*
d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)^2*(3*A + B +
3*B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)^2*(b - (d*(a + b*x))/(c +
d*x))) - ((6*B*(a + b*x))/(d*(c + d*x)) - ((6*A + 5*B)*(a + b*x))/(d*(c +
d*x)) - (6*B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d*(c + d*x)) - (b*(6
*A + 5*B + 6*B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c +
 d*x))])/d^2 - (6*b*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^2)/d)/(2*
d)))/i^2
```

### 3.39.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.39. $\int \frac{(agx+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$

### 3.39.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 845 vs. $2(333) = 666$.

Time = 1.67 (sec) , antiderivative size = 846, normalized size of antiderivative = 2.48

| method | result |
|---|---|
| derivativedivides | $-\dfrac{e(ad-cb)\left(-\dfrac{A\,d^2g^3(ad-cb)\left(-\dfrac{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}{d^3}+\dfrac{b^3e^3}{2d^4\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}-\dfrac{3be\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^4}-\dfrac{}{d^4\left(be-\left(\frac{be}{d}\right.\right.}\right.}{e^2i^2}\right.}{}$ |
| default | $-\dfrac{e(ad-cb)\left(-\dfrac{A\,d^2g^3(ad-cb)\left(-\dfrac{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}{d^3}+\dfrac{b^3e^3}{2d^4\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)^2}-\dfrac{3be\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^4}-\dfrac{}{d^4\left(be-\left(\frac{be}{d}\right.\right.}\right.}{e^2i^2}\right.}{}$ |
| parts | $\dfrac{g^3A\left(\dfrac{b^2\left(\frac{1}{2}bd\,x^2+3xad-2bcx\right)}{d^3}+\dfrac{3b\left(a^2d^2-2abcd+b^2c^2\right)\ln(dx+c)}{d^4}-\dfrac{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{d^4(dx+c)}\right)}{i^2}-\dfrac{g^3B\left(\dfrac{\left(a^3d^3-3a^2bc\,d\right.}{}\right.}{}$ |
| risch | Expression too large to display |

3.39. $\int\frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x,method=_RETU
RNVERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(-A*d^2*g^3*(a*d-b*c)/e^2/i^2*(-(b*e/d+(a*d-b*c)*e/d/(d
*x+c))/d^3+1/2*b^3*e^3/d^4/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2-3/d^4*b
*e*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-3/d^4*b^2*e^2/(b*e-(b*e/d+(a*d-
b*c)*e/d/(d*x+c))*d))-B*d^2*g^3*(a*d-b*c)/e^2/i^2*(-1/d^3*((b*e/d+(a*d-b*c
)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d
)-b^3*e^3/d^3*(-1/2/b^2/e^2/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)+1/2/
b/e/d/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)-1/2*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(2*b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d
)/b^2/e^2/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)^2)-3/d^3*b*e*(dilog(-((b*e
/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln
(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)-3/d^3*b^2*e^2*(1/b/e/d*ln(
b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d)+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/
d+(a*d-b*c)*e/d/(d*x+c))/b/e/(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d))))
```

### 3.39.5 Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output
```
integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
+ (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((b
*e*x + a*e)/(d*x + c)))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

### 3.39.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

### 3.39.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1341 vs. $2(332) = 664$.

Time = 0.27 (sec) , antiderivative size = 1341, normalized size of antiderivative = 3.93

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output
```
1/2*(2*c^3/(d^5*i^2*x + c*d^4*i^2) + 6*c^2*log(d*x + c)/(d^4*i^2) + (d*x^2
 - 4*c*x)/(d^3*i^2))*A*b^3*g^3 - 3*A*a*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) -
x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i^2))*g^3 + 3*A*a^2*b*g^3*(c/(d^3*i^2*
x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - B*a^3*g^3*(log(b*e*x/(d*x + c)
+ a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2) - 1/(d^2*i^2*x + c*d*i^2) - b*log(b
*x + a)/((b*c*d - a*d^2)*i^2) + b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A*
a^3*g^3/(d^2*i^2*x + c*d*i^2) - 1/2*(6*a^3*b*d^3*g^3*log(e) - (6*g^3*log(e
) + 7*g^3)*b^4*c^3 + (18*g^3*log(e) + 17*g^3)*a*b^3*c^2*d - 6*(3*g^3*log(e
) + 2*g^3)*a^2*b^2*c*d^2)*B*log(d*x + c)/(b*c*d^4*i^2 - a*d^5*i^2) + 1/2*(
(b^4*c*d^3*g^3*log(e) - a*b^3*d^4*g^3*log(e))*B*x^3 - ((3*g^3*log(e) + g^3
)*b^4*c^2*d^2 - (9*g^3*log(e) + 2*g^3)*a*b^3*c*d^3 + (6*g^3*log(e) + g^3)*
a^2*b^2*d^4)*B*x^2 - ((4*g^3*log(e) + g^3)*b^4*c^3*d - 2*(5*g^3*log(e) + g
^3)*a*b^3*c^2*d^2 + (6*g^3*log(e) + g^3)*a^2*b^2*c*d^3)*B*x - 3*((b^4*c^3*
d*g^3 - 3*a*b^3*c^2*d^2*g^3 + 3*a^2*b^2*c*d^3*g^3 - a^3*b*d^4*g^3)*B*x + (
b^4*c^4*g^3 - 3*a*b^3*c^3*d*g^3 + 3*a^2*b^2*c^2*d^2*g^3 - a^3*b*c*d^3*g^3)
*B)*log(d*x + c)^2 + 2*((g^3*log(e) - g^3)*b^4*c^4 - 4*(g^3*log(e) - g^3)*
a*b^3*c^3*d + 6*(g^3*log(e) - g^3)*a^2*b^2*c^2*d^2 - 3*(g^3*log(e) - g^3)*
a^3*b*c*d^3)*B + ((b^4*c*d^3*g^3 - a*b^3*d^4*g^3)*B*x^3 - 3*(b^4*c^2*d^2*g
^3 - 3*a*b^3*c*d^3*g^3 + 2*a^2*b^2*d^4*g^3)*B*x^2 - (6*b^4*c^3*d*g^3 - 12*
a*b^3*c^2*d^2*g^3 + 3*a^2*b^2*c*d^3*g^3 + 5*a^3*b*d^4*g^3)*B*x - (6*a*b...
```

### 3.39.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2814 vs. $2(332) = 664$.

Time = 72.93 (sec) , antiderivative size = 2814, normalized size of antiderivative = 8.25

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output
```
-1/24*(6*(B*b^8*c^5*e^5*g^3 - 5*B*a*b^7*c^4*d*e^5*g^3 + 10*B*a^2*b^6*c^3*d
^2*e^5*g^3 - 10*B*a^3*b^5*c^2*d^3*e^5*g^3 + 5*B*a^4*b^4*c*d^4*e^5*g^3 - B*
a^5*b^3*d^5*e^5*g^3 - 4*(b*e*x + a*e)*B*b^7*c^5*d*e^4*g^3/(d*x + c) + 20*(
b*e*x + a*e)*B*a*b^6*c^4*d^2*e^4*g^3/(d*x + c) - 40*(b*e*x + a*e)*B*a^2*b^
5*c^3*d^3*e^4*g^3/(d*x + c) + 40*(b*e*x + a*e)*B*a^3*b^4*c^2*d^4*e^4*g^3/(
d*x + c) - 20*(b*e*x + a*e)*B*a^4*b^3*c*d^5*e^4*g^3/(d*x + c) + 4*(b*e*x +
 a*e)*B*a^5*b^2*d^6*e^4*g^3/(d*x + c) + 6*(b*e*x + a*e)^2*B*b^6*c^5*d^2*e^
3*g^3/(d*x + c)^2 - 30*(b*e*x + a*e)^2*B*a*b^5*c^4*d^3*e^3*g^3/(d*x + c)^2
 + 60*(b*e*x + a*e)^2*B*a^2*b^4*c^3*d^4*e^3*g^3/(d*x + c)^2 - 60*(b*e*x +
a*e)^2*B*a^3*b^3*c^2*d^5*e^3*g^3/(d*x + c)^2 + 30*(b*e*x + a*e)^2*B*a^4*b^
2*c*d^6*e^3*g^3/(d*x + c)^2 - 6*(b*e*x + a*e)^2*B*a^5*b*d^7*e^3*g^3/(d*x +
 c)^2 - 4*(b*e*x + a*e)^3*B*b^5*c^5*d^3*e^2*g^3/(d*x + c)^3 + 20*(b*e*x +
a*e)^3*B*a*b^4*c^4*d^4*e^2*g^3/(d*x + c)^3 - 40*(b*e*x + a*e)^3*B*a^2*b^3*
c^3*d^5*e^2*g^3/(d*x + c)^3 + 40*(b*e*x + a*e)^3*B*a^3*b^2*c^2*d^6*e^2*g^3
/(d*x + c)^3 - 20*(b*e*x + a*e)^3*B*a^4*b*c*d^7*e^2*g^3/(d*x + c)^3 + 4*(b
*e*x + a*e)^3*B*a^5*d^8*e^2*g^3/(d*x + c)^3)*log((b*e*x + a*e)/(d*x + c))/
(b^4*d^4*e^4*i^2 - 4*(b*e*x + a*e)*b^3*d^5*e^3*i^2/(d*x + c) + 6*(b*e*x +
a*e)^2*b^2*d^6*e^2*i^2/(d*x + c)^2 - 4*(b*e*x + a*e)^3*b*d^7*e*i^2/(d*x +
c)^3 + (b*e*x + a*e)^4*d^8*i^2/(d*x + c)^4) + (6*A*b^8*c^5*e^5*g^3 + 11*B*
b^8*c^5*e^5*g^3 - 30*A*a*b^7*c^4*d*e^5*g^3 - 55*B*a*b^7*c^4*d*e^5*g^3 +...
```

## 3.39.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2
,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2
, x)
```

# 3.40 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$



## 3.40.1 Optimal result

Integrand size = 40, antiderivative size = 260

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx \\
&= -\frac{2B(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)}+\frac{(2A+B)(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)} \\
&\quad+\frac{2B(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^2(c+dx)}+\frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^2(c+dx)} \\
&\quad+\frac{b(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2} \\
&\quad+\frac{2bB(bc-ad)g^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}
\end{aligned}$$

output
```
-2*B*(-a*d+b*c)*g^2*(b*x+a)/d^2/i^2/(d*x+c)+(2*A+B)*(-a*d+b*c)*g^2*(b*x+a)
/d^2/i^2/(d*x+c)+2*B*(-a*d+b*c)*g^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^2/i^2/
(d*x+c)+g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^2/(d*x+c)+b*(-a*d+b*
c)*g^2*ln((-a*d+b*c)/b/(d*x+c))*(2*A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/d^3/i^2+
2*b*B*(-a*d+b*c)*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^2
```

## 3.40.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.92

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$$
$$= \frac{g^2\left(Ab^2dx+\frac{B(bc-ad)^2}{c+dx}+bB(bc-ad)\log(a+bx)+bBd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)-\frac{(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*
i*x)^2,x]
```

output
```
(g^2*(A*b^2*d*x + (B*(b*c - a*d)^2)/(c + d*x) + b*B*(b*c - a*d)*Log[a + b*
x] + b*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - ((b*c - a*d)^2*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) - 2*b*B*(b*c - a*d)*Log[c + d*x]
- 2*b*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + b*B*
(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(d^3*i^2)
```

## 3.40.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 227, normalized size of antiderivative = 0.87, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2962, 2784, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ci+dix)^2}\,dx$$

$$\Big\downarrow \; 2962$$

$$\frac{g^2(bc-ad)\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \; 2784$$

$$\frac{g^2(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{d}\right)}{i^2}$$

$$\Big\downarrow \text{2793}$$

$$\frac{g^2(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\left(-\frac{2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)}{d}-\frac{b\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{d}\right)}{i^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^2(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d^2}-\frac{(2A+B)(a+bx)}{d(c+dx)}-\frac{2bB\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2}-\ldots}{d}\right)}{i^2}$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^2
,x]
```

output
```
((b*c - a*d)*g^2*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c
 + d*x)^2*(b - (d*(a + b*x))/(c + d*x))) - ((2*B*(a + b*x))/(d*(c + d*x))
- ((2*A + B)*(a + b*x))/(d*(c + d*x)) - (2*B*(a + b*x)*Log[(e*(a + b*x))/(
c + d*x)])/(d*(c + d*x)) - (b*(2*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)])
*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 - (2*b*B*PolyLog[2, (d*(a + b*x
))/(b*(c + d*x))])/d^2)/d))/i^2
```

### 3.40.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.40.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 531 vs. 2(260) = 520.

Time = 1.49 (sec) , antiderivative size = 532, normalized size of antiderivative = 2.05

| method | result |
|---|---|
| parts | $\frac{g^2A\left(\frac{x\,b^2}{d^2}+\frac{2b(ad-cb)\ln(dx+c)}{d^3}-\frac{a^2d^2-2abcd+b^2c^2}{d^3(dx+c)}\right)}{i^2}-\frac{g^2B\left(\frac{\left(a^2d^2-2abcd+b^2c^2\right)\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\cdots\right.}{d^2}\cdots\right.}{\cdots}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{g^2d^2A\left(\frac{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}{d^2}+\frac{b^2e^2}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{2be\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^3}\right)}{e^2i^2}+\frac{g^2d^2B\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c)}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$ |
| default | $-\frac{e(ad-cb)\left(\frac{g^2d^2A\left(\frac{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}{d^2}+\frac{b^2e^2}{d^3\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{2be\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^3}\right)}{e^2i^2}+\frac{g^2d^2B\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c)}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}$ |
| risch | Expression too large to display |

3.40. $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x,method=_RETU
RNVERBOSE)
```

output
```
g^2*A/i^2*(x*b^2/d^2+2*b/d^3*(a*d-b*c)*ln(d*x+c)-1/d^3*(a^2*d^2-2*a*b*c*d+
b^2*c^2)/(d*x+c))-g^2*B/i^2/(a*d-b*c)/e*(1/d^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)
*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*
e/d/(d*x+c)-b*e/d)+2*b/d^2*e*(a^2*d^2-2*a*b*c*d+b^2*c^2)*(dilog(-((b*e/d+(
a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((
b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d)+e^2*b^2*(a^2*d^2-2*a*b*c*d+b^2
*c^2)/d^2*(1/b/e/d*ln((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)-ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/((b*e/d+(a*d-b*c)*e/d/(
d*x+c))*d-b*e)))
```

### 3.40.5 Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output
```
integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B
*a*b*g^2*x + B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)))/(d^2*i^2*x^2 + 2*c*d
*i^2*x + c^2*i^2), x)
```

### 3.40.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

### 3.40.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 886 vs. 2(259) = 518.

Time = 0.27 (sec) , antiderivative size = 886, normalized size of antiderivative = 3.41

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$$
$$= -Ab^2\left(\frac{c^2}{d^4i^2x+cd^3i^2}-\frac{x}{d^2i^2}+\frac{2\,c\log(dx+c)}{d^3i^2}\right)g^2$$
$$+2\,Aabg^2\left(\frac{c}{d^3i^2x+cd^2i^2}+\frac{\log(dx+c)}{d^2i^2}\right)$$
$$-Ba^2g^2\left(\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^2i^2x+cdi^2}-\frac{1}{d^2i^2x+cdi^2}-\frac{b\log(bx+a)}{(bcd-ad^2)i^2}+\frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right)$$
$$-\frac{Aa^2g^2}{d^2i^2x+cdi^2}$$
$$-\frac{(2\,a^2bd^2g^2\log(e)+2\,(g^2\log(e)+g^2)b^3c^2-(4\,g^2\log(e)+3\,g^2)ab^2cd)B\log(dx+c)}{bcd^3i^2-ad^4i^2}$$
$$+\frac{(b^3cd^2g^2\log(e)-ab^2d^3g^2\log(e))Bx^2+(b^3c^2dg^2\log(e)-ab^2cd^2g^2\log(e))Bx+((b^3c^2dg^2-2\,ab^2cd^2g^2}{}$$
$$-\frac{2\,(b^2cg^2-abdg^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^3i^2}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output
```
-A*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*
i^2))*g^2 + 2*A*a*b*g^2*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2
)) - B*a^2*g^2*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2)
 - 1/(d^2*i^2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log(
d*x + c)/((b*c*d - a*d^2)*i^2)) - A*a^2*g^2/(d^2*i^2*x + c*d*i^2) - (2*a^2
*b*d^2*g^2*log(e) + 2*(g^2*log(e) + g^2)*b^3*c^2 - (4*g^2*log(e) + 3*g^2)*
a*b^2*c*d)*B*log(d*x + c)/(b*c*d^3*i^2 - a*d^4*i^2) + ((b^3*c*d^2*g^2*log(
e) - a*b^2*d^3*g^2*log(e))*B*x^2 + (b^3*c^2*d*g^2*log(e) - a*b^2*c*d^2*g^2
*log(e))*B*x + ((b^3*c^2*d*g^2 - 2*a*b^2*c*d^2*g^2 + a^2*b*d^3*g^2)*B*x +
(b^3*c^3*g^2 - 2*a*b^2*c^2*d*g^2 + a^2*b*c*d^2*g^2)*B)*log(d*x + c)^2 - ((
g^2*log(e) - g^2)*b^3*c^3 - 3*(g^2*log(e) - g^2)*a*b^2*c^2*d + 2*(g^2*log(
e) - g^2)*a^2*b*c*d^2)*B + ((b^3*c*d^2*g^2 - a*b^2*d^3*g^2)*B*x^2 + (2*b^3
*c^2*d*g^2 - 2*a*b^2*c*d^2*g^2 - a^2*b*d^3*g^2)*B*x + (2*a*b^2*c^2*d*g^2 -
 3*a^2*b*c*d^2*g^2)*B)*log(b*x + a) - ((b^3*c*d^2*g^2 - a*b^2*d^3*g^2)*B*x
^2 + (b^3*c^2*d*g^2 - a*b^2*c*d^2*g^2)*B*x - (b^3*c^3*g^2 - 3*a*b^2*c^2*d*
g^2 + 2*a^2*b*c*d^2*g^2)*B)*log(d*x + c))/(b*c^2*d^3*i^2 - a*c*d^4*i^2 + (
b*c*d^4*i^2 - a*d^5*i^2)*x) - 2*(b^2*c*g^2 - a*b*d*g^2)*(log(b*x + a)*log(
(b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^3
*i^2)
```

### 3.40.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1774 vs. $2(259) = 518$.

Time = 63.25 (sec) , antiderivative size = 1774, normalized size of antiderivative = 6.82

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output
```
1/6*(2*(B*b^6*c^4*e^4*g^2 - 4*B*a*b^5*c^3*d*e^4*g^2 + 6*B*a^2*b^4*c^2*d^2*
e^4*g^2 - 4*B*a^3*b^3*c*d^3*e^4*g^2 + B*a^4*b^2*d^4*e^4*g^2 - 3*(b*e*x + a
*e)*B*b^5*c^4*d*e^3*g^2/(d*x + c) + 12*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e^3*g
^2/(d*x + c) - 18*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c) + 12*(
b*e*x + a*e)*B*a^3*b^2*c*d^4*e^3*g^2/(d*x + c) - 3*(b*e*x + a*e)*B*a^4*b*d
^5*e^3*g^2/(d*x + c) + 3*(b*e*x + a*e)^2*B*b^4*c^4*d^2*e^2*g^2/(d*x + c)^2
 - 12*(b*e*x + a*e)^2*B*a*b^3*c^3*d^3*e^2*g^2/(d*x + c)^2 + 18*(b*e*x + a*
e)^2*B*a^2*b^2*c^2*d^4*e^2*g^2/(d*x + c)^2 - 12*(b*e*x + a*e)^2*B*a^3*b*c*
d^5*e^2*g^2/(d*x + c)^2 + 3*(b*e*x + a*e)^2*B*a^4*d^6*e^2*g^2/(d*x + c)^2)
*log((b*e*x + a*e)/(d*x + c))/(b^3*d^3*e^3*i^2 - 3*(b*e*x + a*e)*b^2*d^4*e
^2*i^2/(d*x + c) + 3*(b*e*x + a*e)^2*b*d^5*e*i^2/(d*x + c)^2 - (b*e*x + a*
e)^3*d^6*i^2/(d*x + c)^3) + (2*A*b^6*c^4*e^4*g^2 + 3*B*b^6*c^4*e^4*g^2 - 8
*A*a*b^5*c^3*d*e^4*g^2 - 12*B*a*b^5*c^3*d*e^4*g^2 + 12*A*a^2*b^4*c^2*d^2*e
^4*g^2 + 18*B*a^2*b^4*c^2*d^2*e^4*g^2 - 8*A*a^3*b^3*c*d^3*e^4*g^2 - 12*B*a
^3*b^3*c*d^3*e^4*g^2 + 2*A*a^4*b^2*d^4*e^4*g^2 + 3*B*a^4*b^2*d^4*e^4*g^2 -
 6*(b*e*x + a*e)*A*b^5*c^4*d*e^3*g^2/(d*x + c) - 7*(b*e*x + a*e)*B*b^5*c^4
*d*e^3*g^2/(d*x + c) + 24*(b*e*x + a*e)*A*a*b^4*c^3*d^2*e^3*g^2/(d*x + c)
+ 28*(b*e*x + a*e)*B*a*b^4*c^3*d^2*e^3*g^2/(d*x + c) - 36*(b*e*x + a*e)*A*
a^2*b^3*c^2*d^3*e^3*g^2/(d*x + c) - 42*(b*e*x + a*e)*B*a^2*b^3*c^2*d^3*e^3
*g^2/(d*x + c) + 24*(b*e*x + a*e)*A*a^3*b^2*c*d^4*e^3*g^2/(d*x + c) + 2...
```

### 3.40.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2
,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2
, x)
```

# 3.41 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$



## 3.41.1 Optimal result

Integrand size = 38, antiderivative size = 160

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx = -\frac{Ag(a+bx)}{di^2(c+dx)}+\frac{Bg(a+bx)}{di^2(c+dx)} - \frac{Bg(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{di^2(c+dx)} - \frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i^2} - \frac{bBg\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}$$

output
```
-A*g*(b*x+a)/d/i^2/(d*x+c)+B*g*(b*x+a)/d/i^2/(d*x+c)-B*g*(b*x+a)*ln(e*(b*x
+a)/(d*x+c))/d/i^2/(d*x+c)-b*g*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/
(d*x+c)))/d^2/i^2-b*B*g*polylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i^2
```

## 3.41.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.09

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$$
$$= \frac{g\left(\frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}+2b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log(c+dx)-2B\left(\frac{bc-ad}{c+dx}+b\log(a+bx)-b\log(c+\right.}{2d^2i^2}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*
x)^2,x]
```

output
```
(g*((2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) + 2*b*(
A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 2*B*((b*c - a*d)/(c + d
*x) + b*Log[a + b*x] - b*Log[c + d*x]) - b*B*((2*Log[(d*(a + b*x))/(-(b*c)
 + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)])))/(2*d^2*i^2)
```

## 3.41.3 Rubi [A] (verified)

Time = 0.34 (sec) , antiderivative size = 146, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2962, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ci+dix)^2}\,dx$$
$$\Big\downarrow \text{2962}$$
$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^2}$$
$$\Big\downarrow \text{2793}$$
$$\frac{g\int \left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{d}-\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g\left(-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2}-\frac{A(a+bx)}{d(c+dx)}-\frac{bB\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2}-\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d(c+dx)}+\frac{B(a+bx)}{d(c+dx)}\right)}{i^2}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^2,x
]
```

output
```
(g*(-((A*(a + b*x))/(d*(c + d*x))) + (B*(a + b*x))/(d*(c + d*x)) - (B*(a +
 b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d*(c + d*x)) - (b*(A + B*Log[(e*(a +
b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 - (b*B*PolyLog
[2, (d*(a + b*x))/(b*(c + d*x))])/d^2))/i^2
```

### 3.41.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.41.4 Maple [A] (verified)

Time = 1.48 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.84

| method | result |
|---|---|
| parts | $\frac{gA\left(\frac{b\ln(dx+c)}{d^2}-\frac{ad-cb}{d^2(dx+c)}\right)}{i^2}-\frac{gB\left(\frac{(ad-cb)\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{d}+\frac{be(ad-cb)\left(\mathrm{dilog}\left(-\frac{\left(\frac{b}{}\right.}{}\right.\right.}{}\right.}{i^2(ad-cb)e}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{g\,d^2A\left(-\frac{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}{d}-\frac{be\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^2}\right)}{(ad-cb)e^2i^2}-\frac{g\,d^2B\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad}{d(}}{d}\right.}{}\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{g\,d^2A\left(-\frac{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}{d}-\frac{be\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^2}\right)}{(ad-cb)e^2i^2}-\frac{g\,d^2B\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad}{d(}}{d}\right.}{}\right.}{d^2}$ |
| risch | $\frac{gAb\ln(dx+c)}{i^2d^2}-\frac{gAa}{i^2d(dx+c)}+\frac{gAcb}{i^2d^2(dx+c)}-\frac{gB\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)ba}{i^2(ad-cb)d}+\frac{gB\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)b^2c}{i^2(ad-cb)d^2}-\frac{gB\ln\left(\frac{be}{d}+\right.}{i^2(ad-c}$ |

3.41. $\int\frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x,method=_RETURN
VERBOSE)
```

output
```
g*A/i^2*(b/d^2*ln(d*x+c)-(a*d-b*c)/d^2/(d*x+c))-g*B/i^2/(a*d-b*c)/e*((a*d-
b*c)/d*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d
-b*c)*e/d/(d*x+c)-b*e/d)+b*e*(a*d-b*c)/d*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*
x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*
e/d/(d*x+c))*d-b*e)/b/e)/d))
```

## 3.41.5 Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algori
thm="fricas")
```

output
```
integral((A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log((b*e*x + a*e)/(d*x + c))
)/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

## 3.41.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

### 3.41.7 Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algori
thm="maxima")
```

output
```
-1/2*B*b*g*(((d*x + c)*log(d*x + c)^2 + 2*c*log(d*x + c))/(d^3*i^2*x + c*d
^2*i^2) - 2*integrate((d*x*log(b*x + a) + d*x*log(e) + c)/(d^3*i^2*x^2 + 2
*c*d^2*i^2*x + c^2*d*i^2), x)) + A*b*g*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*
x + c)/(d^2*i^2)) - B*a*g*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x
 + c*d*i^2) - 1/(d^2*i^2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^
2) + b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A*a*g/(d^2*i^2*x + c*d*i^2)
```

### 3.41.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 895 vs. $2(159) = 318$.

Time = 48.78 (sec) , antiderivative size = 895, normalized size of antiderivative = 5.59

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=$$
$$-\frac{1}{2}\left(\frac{\left(Bb^4c^3e^3g-3\,Bab^3c^2de^3g+3\,Ba^2b^2cd^2e^3g-Ba^3bd^3e^3g-\frac{2\,(bex+ae)Bb^3c^3de^2g}{dx+c}+\frac{6\,(bex+ae)Bab^2c^2d^2e^2g}{dx+c}\right.}{b^2d^2e^2i^2-\frac{2\,(bex+ae)bd^3ei^2}{dx+c}+\frac{(bex+ae)^2d^4i^2}{(dx+c)^2}}\right.$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algori
thm="giac")
```

output
```
-1/2*((B*b^4*c^3*e^3*g - 3*B*a*b^3*c^2*d*e^3*g + 3*B*a^2*b^2*c*d^2*e^3*g -
 B*a^3*b*d^3*e^3*g - 2*(b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d*x + c) + 6*(b*e*
x + a*e)*B*a*b^2*c^2*d^2*e^2*g/(d*x + c) - 6*(b*e*x + a*e)*B*a^2*b*c*d^3*e
^2*g/(d*x + c) + 2*(b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))*log((b*e*x + a
*e)/(d*x + c))/(b^2*d^2*e^2*i^2 - 2*(b*e*x + a*e)*b*d^3*e*i^2/(d*x + c) +
(b*e*x + a*e)^2*d^4*i^2/(d*x + c)^2) + (A*b^4*c^3*e^3*g + B*b^4*c^3*e^3*g
- 3*A*a*b^3*c^2*d*e^3*g - 3*B*a*b^3*c^2*d*e^3*g + 3*A*a^2*b^2*c*d^2*e^3*g
+ 3*B*a^2*b^2*c*d^2*e^3*g - A*a^3*b*d^3*e^3*g - B*a^3*b*d^3*e^3*g - 2*(b*e
*x + a*e)*A*b^3*c^3*d*e^2*g/(d*x + c) - (b*e*x + a*e)*B*b^3*c^3*d*e^2*g/(d
*x + c) + 6*(b*e*x + a*e)*A*a*b^2*c^2*d^2*e^2*g/(d*x + c) + 3*(b*e*x + a*e
)*B*a*b^2*c^2*d^2*e^2*g/(d*x + c) - 6*(b*e*x + a*e)*A*a^2*b*c*d^3*e^2*g/(d
*x + c) - 3*(b*e*x + a*e)*B*a^2*b*c*d^3*e^2*g/(d*x + c) + 2*(b*e*x + a*e)*
A*a^3*d^4*e^2*g/(d*x + c) + (b*e*x + a*e)*B*a^3*d^4*e^2*g/(d*x + c))/(b^2*
d^2*e^2*i^2 - 2*(b*e*x + a*e)*b*d^3*e*i^2/(d*x + c) + (b*e*x + a*e)^2*d^4*
i^2/(d*x + c)^2) + (B*b^3*c^3*e*g - 3*B*a*b^2*c^2*d*e*g + 3*B*a^2*b*c*d^2*
e*g - B*a^3*d^3*e*g)*log(-b*e + (b*e*x + a*e)*d/(d*x + c))/(b*d^2*i^2) - (
B*b^3*c^3*e*g - 3*B*a*b^2*c^2*d*e*g + 3*B*a^2*b*c*d^2*e*g - B*a^3*d^3*e*g)
*log((b*e*x + a*e)/(d*x + c))/(b*d^2*i^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*
d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

### 3.41.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{{(c\,i+d\,i\,x)}^2}\,dx$$

input
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2,x
)
```

output
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^2,
x)
```

# 3.42 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx$



## 3.42.1 Optimal result

Integrand size = 30, antiderivative size = 98

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = \frac{A(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{B(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)i^2(c+dx)}$$

output
```
A*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)-B*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)+B*(b*x+a
)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)/i^2/(d*x+c)
```

## 3.42.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 104, normalized size of antiderivative = 1.06

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx$$
$$= \frac{Abc-bBc-aAd+aBd-bB(c+dx)\log(a+bx)+B(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right)+bBc\log(c+dx)+bBdx}{d(-bc+ad)i^2(c+dx)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x)^2,x]
```

output
```
(A*b*c - b*B*c - a*A*d + a*B*d - b*B*(c + d*x)*Log[a + b*x] + B*(b*c - a*d
)*Log[(e*(a + b*x))/(c + d*x)] + b*B*c*Log[c + d*x] + b*B*d*x*Log[c + d*x]
)/(d*(-(b*c) + a*d)*i^2*(c + d*x))
```

## 3.42.3 Rubi [A] (verified)

Time = 0.20 (sec) , antiderivative size = 73, normalized size of antiderivative = 0.74, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2952, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(\frac{e(a+bx)}{c+dx}\right)+A}{(ci+dix)^2} \, dx$$

$$\Big\downarrow \text{2952}$$

$$\frac{\int \left(A+B \log \left(\frac{e(a+bx)}{c+dx}\right)\right) d\frac{a+bx}{c+dx}}{i^2 (bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{A(a+bx)}{c+dx}+\frac{B(a+bx) \log \left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}-\frac{B(a+bx)}{c+dx}}{i^2 (bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x)^2,x]
```

output
```
((A*(a + b*x))/(c + d*x) - (B*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[(e*(
a + b*x))/(c + d*x)])/(c + d*x))/((b*c - a*d)*i^2)
```

### 3.42.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

### 3.42.4 Maple [A] (verified)

Time = 0.60 (sec) , antiderivative size = 82, normalized size of antiderivative = 0.84

| method | result | siz |
|---|---|---|
| parts | $-\frac{A}{i^2(dx+c)d}-\frac{B\left(\frac{e(bx+a)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{dx+c}-\frac{e(bx+a)}{dx+c}\right)}{i^2e(ad-cb)}$ | 82 |
| norman | $\frac{\frac{(-B+A)x}{ic}-\frac{aB\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(ad-cb)i}-\frac{Bbx\ln\left(\frac{e(bx+a)}{dx+c}\right)}{(ad-cb)i}}{i(dx+c)}$ | 91 |
| parallelrisch | $-\frac{Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^3+B\ln\left(\frac{e(bx+a)}{dx+c}\right)ab\,d^3-Bab\,d^3+B\,b^2c\,d^2+Aab\,d^3-A\,b^2c\,d^2}{i^2(dx+c)b\,d^3(ad-cb)}$ | 11 |
| risch | $-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{d\,i^2(dx+c)}-\frac{B\ln(bx+a)bdx-B\ln(-dx-c)bdx+B\ln(bx+a)bc-B\ln(-dx-c)bc+Aad-Abc-Bad+Bbc}{i^2(dx+c)d(ad-cb)}$ | 12 |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{d^2A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^2e^2i^2}+\frac{d^2B\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{(ad-cb)^2e^2i^2}\right)}{d^2}$ | 17 |
| default | $-\frac{e(ad-cb)\left(\frac{d^2A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^2e^2i^2}+\frac{d^2B\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{(ad-cb)^2e^2i^2}\right)}{d^2}$ | 17 |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/i^2/(d*x+c)/d-B/i^2/e/(a*d-b*c)*(e*(b*x+a)/(d*x+c)*ln(e*(b*x+a)/(d*x+c)
)-e*(b*x+a)/(d*x+c))
```

### 3.42.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 88, normalized size of antiderivative = 0.90

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = -\frac{(A-B)bc-(A-B)ad-(Bbdx+Bad)\log\left(\frac{bex+ae}{dx+c}\right)}{(bcd^2-ad^3)i^2x+(bc^2d-acd^2)i^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorithm="fricas"
)
```

output
```
-((A - B)*b*c - (A - B)*a*d - (B*b*d*x + B*a*d)*log((b*e*x + a*e)/(d*x + c
)))/((b*c*d^2 - a*d^3)*i^2*x + (b*c^2*d - a*c*d^2)*i^2)
```

### 3.42.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(78) = 156$.

Time = 0.61 (sec) , antiderivative size = 231, normalized size of antiderivative = 2.36

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = \frac{Bb\log\left(x+\frac{-\frac{Ba^2bd^2}{ad-bc}+\frac{2Bab^2cd}{ad-bc}+Babd-\frac{Bb^3c^2}{ad-bc}+Bb^2c}{2Bb^2d}\right)}{di^2\,(ad-bc)} - \frac{Bb\log\left(x+\frac{\frac{Ba^2bd^2}{ad-bc}-\frac{2Bab^2cd}{ad-bc}+Babd+\frac{Bb^3c^2}{ad-bc}+Bb^2c}{2Bb^2d}\right)}{di^2\,(ad-bc)} - \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{cdi^2+d^2i^2x} + \frac{-A+B}{cdi^2+d^2i^2x}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**2,x)
```

output
```
B*b*log(x + (-B*a**2*b*d**2/(a*d - b*c) + 2*B*a*b**2*c*d/(a*d - b*c) + B*a
*b*d - B*b**3*c**2/(a*d - b*c) + B*b**2*c)/(2*B*b**2*d))/(d*i**2*(a*d - b*
c)) - B*b*log(x + (B*a**2*b*d**2/(a*d - b*c) - 2*B*a*b**2*c*d/(a*d - b*c)
+ B*a*b*d + B*b**3*c**2/(a*d - b*c) + B*b**2*c)/(2*B*b**2*d))/(d*i**2*(a*d
 - b*c)) - B*log(e*(a + b*x)/(c + d*x))/(c*d*i**2 + d**2*i**2*x) + (-A + B
)/(c*d*i**2 + d**2*i**2*x)
```

### 3.42.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.37

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx$$
$$= -B\left(\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^2i^2x+cdi^2}-\frac{1}{d^2i^2x+cdi^2}-\frac{b\log(bx+a)}{(bcd-ad^2)i^2}+\frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right)-\frac{A}{d^2i^2x+cdi^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorithm="maxima"
)
```

output
```
-B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2) - 1/(d^2*i^
2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log(d*x + c)/((b
*c*d - a*d^2)*i^2)) - A/(d^2*i^2*x + c*d*i^2)
```

### 3.42.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 115, normalized size of antiderivative = 1.17

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx$$
$$= \left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{(bex+ae)B\log\left(\frac{bex+ae}{dx+c}\right)}{(dx+c)i^2}+\frac{(bex+ae)(A-B)}{(dx+c)i^2}\right)$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^2,x, algorithm="giac")
```

output
```
(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))*((
b*e*x + a*e)*B*log((b*e*x + a*e)/(d*x + c))/((d*x + c)*i^2) + (b*e*x + a*e
)*(A - B)/((d*x + c)*i^2))
```

### 3.42.9 Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 106, normalized size of antiderivative = 1.08

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^2}\,dx = -\frac{A-B}{x\,d^2\,i^2+c\,d\,i^2}-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{d^2\,i^2\,\left(x+\frac{c}{d}\right)}+\frac{B\,b\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{d\,i^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(c*i + d*i*x)^2,x)
```

output
```
(B*b*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(d*i^2*(a*d - b*c)) -
(B*log((e*(a + b*x))/(c + d*x)))/(d^2*i^2*(x + c/d)) - (A - B)/(d^2*i^2*x
+ c*d*i^2)
```

# 3.43 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$



## 3.43.1 Optimal result

Integrand size = 40, antiderivative size = 156

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx = -\frac{Ad(a+bx)}{(bc-ad)^2gi^2(c+dx)}+\frac{Bd(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^2gi^2(c+dx)}+\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2B(bc-ad)^2gi^2}$$

output
```
-A*d*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*x+c)+B*d*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*
x+c)-B*d*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^2/g/i^2/(d*x+c)+1/2*b*(A
+B*ln(e*(b*x+a)/(d*x+c)))^2/B/(-a*d+b*c)^2/g/i^2
```

## 3.43.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 292, normalized size of antiderivative = 1.87

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2b(c+dx)\left(A+B\,\mathrm{l}\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x
)^2),x]
```

output
```
(2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*b*(c + d*x)*Log[a
+ b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b*(c + d*x)*(A + B*Log[(e*
(a + b*x))/(c + d*x)])*Log[c + d*x] - 2*B*(b*c - a*d + b*(c + d*x)*Log[a +
 b*x] - b*(c + d*x)*Log[c + d*x]) - b*B*(c + d*x)*(Log[a + b*x]*(Log[a + b
*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*
c) + a*d)]) + b*B*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c
+ d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(2*(b*c -
 a*d)^2*g*i^2*(c + d*x))
```

### 3.43.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2962, 2788, 2009, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)(ci+dix)^2}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{2788}$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}d\frac{a+bx}{c+dx}-d\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}-\frac{B(a+bx)}{c+dx}\right)}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{2738}$$

$$\frac{\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2B}-d\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}-\frac{B(a+bx)}{c+dx}\right)}{gi^2(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x)^2),x
]
```

output
```
((b*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*B) - d*((A*(a + b*x))/(c +
d*x) - (B*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)]
)/(c + d*x)))/((b*c - a*d)^2*g*i^2)
```

#### 3.43.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.43.4 Maple [A] (verified)

Time = 0.90 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.48

| method | result |
|---|---|
| parts | $\frac{A\left(-\frac{1}{(ad-cb)(dx+c)}-\frac{b\ln(dx+c)}{(ad-cb)^2}+\frac{b\ln(bx+a)}{(ad-cb)^2}\right)}{g\,i^2} - \frac{B\left(\frac{d\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{ad-cb}-\frac{be\ln\left(\frac{be}{d}+\ldots\right.}{2(ad\ldots}\right.}{g\,i^2(ad-cb)e}$ |
| parallelrisch | $\frac{2Ba\,b^2d^4-2B\,b^3c\,d^3-2Aa\,b^2d^4+2A\,b^3c\,d^3+Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^3d^4+2Ax\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^4-2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^4+B\,l\ldots}{2i^2g(dx+c)b^2d^3(a^2d^2-2abcd+b^2c^2)}$ |
| norman | $\frac{\frac{(Abc-Bad)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{d(Ab-Bb)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{(-B+A)dx}{gic(ad-cb)}+\frac{Bbc\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{bBdx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2gi\left(a^2d^2-2abcd+b^2c^2\right)}}{i(dx+c)}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{d^2Ab\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^3g}+\frac{d^3A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^3g}-\frac{d^2Bb\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e\,i^2(ad-cb)^3g}+\frac{d^3B\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\ldots\right.\right.}{e^2i^2(ad-\ldots}\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{d^2Ab\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^3g}+\frac{d^3A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^3g}-\frac{d^2Bb\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e\,i^2(ad-cb)^3g}+\frac{d^3B\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\ldots\right.\right.}{e^2i^2(ad-\ldots}\right.}{d^2}$ |
| risch | $-\frac{A}{g\,i^2(ad-cb)(dx+c)}-\frac{Ab\ln(dx+c)}{g\,i^2(ad-cb)^2}+\frac{Ab\ln(bx+a)}{g\,i^2(ad-cb)^2}-\frac{B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)b}{g\,i^2(ad-cb)^2}-\frac{Bd\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)a}{g\,i^2(ad-cb)^2(dx+c)}+\frac{B\ln\ldots}{g\,i^2\ldots}$ |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^2,x,method=_RETURN
VERBOSE)
```

output
```
A/g/i^2*(-1/(a*d-b*c)/(d*x+c)-b/(a*d-b*c)^2*ln(d*x+c)+b/(a*d-b*c)^2*ln(b*x
+a))-B/g/i^2/(a*d-b*c)/e*(d/(a*d-b*c)*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-1/2*b*e/(a*d-b*c)*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)
```

### 3.43.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.97

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{2\,(A-B)bc-2\,(A-B)ad+(Bbdx+Bbc)\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,((A-B)bdx+Abc-Bad)\log\left(\frac{bex+ae}{dx+c}\right)}{2\,((b^2c^2d-2\,abcd^2+a^2d^3)gi^2x+(b^2c^3-2\,abc^2d+a^2cd^2)gi^2)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algori
thm="fricas")
```

output
```
1/2*(2*(A - B)*b*c - 2*(A - B)*a*d + (B*b*d*x + B*b*c)*log((b*e*x + a*e)/(
d*x + c))^2 + 2*((A - B)*b*d*x + A*b*c - B*a*d)*log((b*e*x + a*e)/(d*x + c
)))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g*i^2*x + (b^2*c^3 - 2*a*b*c^2*d
+ a^2*c*d^2)*g*i^2)
```

### 3.43.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 386 vs. $2(131) = 262$.

Time = 0.60 (sec) , antiderivative size = 386, normalized size of antiderivative = 2.47

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx = \frac{Bb\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^2d^2gi^2-4abcdgi^2+2b^2c^2gi^2} - \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{acdgi^2+ad^2gi^2x-bc^2gi^2-bcdgi^2x} + (A-B)\left(-\frac{b\log\left(x+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{gi^2(ad-bc)^2} + \frac{b\log\left(x+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{gi^2(ad-bc)^2} - \frac{1}{acdgi^2-bc^2gi^2+x(ad^2gi^2-bcdgi^2)}\right)$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)**2,x)
```

output
```
B*b*log(e*(a + b*x)/(c + d*x))**2/(2*a**2*d**2*g*i**2 - 4*a*b*c*d*g*i**2 +
 2*b**2*c**2*g*i**2) - B*log(e*(a + b*x)/(c + d*x))/(a*c*d*g*i**2 + a*d**2
*g*i**2*x - b*c**2*g*i**2 - b*c*d*g*i**2*x) + (A - B)*(-b*log(x + (-a**3*b
*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d**2/(a*d - b*c)**2 - 3*a*b**3*c**2*d
/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(
g*i**2*(a*d - b*c)**2) + b*log(x + (a**3*b*d**3/(a*d - b*c)**2 - 3*a**2*b*
*2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d - b**4*c
**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d))/(g*i**2*(a*d - b*c)**2) - 1/(a*c*
d*g*i**2 - b*c**2*g*i**2 + x*(a*d**2*g*i**2 - b*c*d*g*i**2)))
```

### 3.43.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 421 vs. $2(154) = 308$.

Time = 0.22 (sec) , antiderivative size = 421, normalized size of antiderivative = 2.70

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= B\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2} + \frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2} - \frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)\log\left(\frac{be}{dx+\dots} + \frac{ae}{dx+c}\right)$$
$$+ A\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2} + \frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2} - \frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)$$
$$- \frac{\left((bdx+bc)\log(bx+a)^2 + (bdx+bc)\log(dx+c)^2 + 2\,bc - 2\,ad + 2\,(bdx+bc)\log(bx+a) - 2\,(bdx+\dots\right.}{2\,(b^2c^3gi^2 - 2\,abc^2dgi^2 + a^2cd^2gi^2 + (b^2c^2dgi^2 - 2\,abcd^2gi^2 + a^2d^{\dots}}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algori
thm="maxima")
```

output
```
B*(1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*g*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + A*(1/((b*c*d
- a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c^2 - 2*a
*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)
*g*i^2)) - 1/2*((b*d*x + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^
2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x
 + b*c)*log(b*x + a))*log(d*x + c))*B/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 +
 a^2*c*d^2*g*i^2 + (b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x
)
```

### 3.43.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 233, normalized size of antiderivative = 1.49

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=\frac{1}{2}\left(\frac{Bbe\log\left(\frac{bex+ae}{dx+c}\right)^2}{bcgi^2-adgi^2}+\frac{2\,Abe\log\left(\frac{bex+ae}{dx+c}\right)}{bcgi^2-adgi^2}-\frac{2\,(bex+ae)Bd\log\left(\frac{bex+ae}{dx+c}\right)}{(bcgi^2-adgi^2)(dx+c)}-\frac{2\,(bex+ae)(Ad-Bd)}{(bcgi^2-adgi^2)(dx+c)}\right)\Bigg(\frac{}{(}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algori
thm="giac")
```

output
```
1/2*(B*b*e*log((b*e*x + a*e)/(d*x + c))^2/(b*c*g*i^2 - a*d*g*i^2) + 2*A*b*
e*log((b*e*x + a*e)/(d*x + c))/(b*c*g*i^2 - a*d*g*i^2) - 2*(b*e*x + a*e)*B
*d*log((b*e*x + a*e)/(d*x + c))/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) - 2*(b
*e*x + a*e)*(A*d - B*d)/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)))*(b*c/((b*c*e
- a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.43.9 Mupad [B] (verification not implemented)

Time = 2.50 (sec) , antiderivative size = 247, normalized size of antiderivative = 1.58

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx=\frac{B\,b\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{2\,g\,i^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{A-B}{(a\,d-b\,c)\,\left(c\,g\,i^2+d\,g\,i^2\,x\right)}$$
$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,(a\,d-b\,c)}{b\,d\,g\,i^2\,\left(\frac{x}{b}+\frac{c}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}$$
$$-\frac{b\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a^2\,d^2\,g\,i^2-b^2\,c^2\,g\,i^2}{g\,i^2\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A-B)\,2\mathrm{i}}{g\,i^2\,{(a\,d-b\,c)}^2}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)*(c*i + d*i*x)^2),x
)
```

output
```
(B*b*log((e*(a + b*x))/(c + d*x))^2)/(2*g*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d)) - (b*atan(((2*b*d*x + (a^2*d^2*g*i^2 - b^2*c^2*g*i^2)/(g*i^2*(a*d - b
*c)))*1i)/(a*d - b*c))*(A - B)*2i)/(g*i^2*(a*d - b*c)^2) - (A - B)/((a*d -
 b*c)*(c*g*i^2 + d*g*i^2*x)) - (B*log((e*(a + b*x))/(c + d*x))*(a*d - b*c)
)/(b*d*g*i^2*(x/b + c/(b*d))*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
```

---

3.43. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^2}\,dx$

# 3.44 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$



## 3.44.1 Optimal result

Integrand size = 40, antiderivative size = 261

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = -\frac{Bd^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2B(c+dx)}{(bc-ad)^3g^2i^2(a+bx)} + \frac{bBd\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2} + \frac{d^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(c+dx)} - \frac{b^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(a+bx)} - \frac{2bd\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2}$$

output
```
-B*d^2*(b*x+a)/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-b^2*B*(d*x+c)/(-a*d+b*c)^3/g^2
/i^2/(b*x+a)+b*B*d*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^2/i^2+d^2*(b*x+a)*
(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-b^2*(d*x+c)*(A+B*
ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^2/i^2/(b*x+a)-2*b*d*ln((b*x+a)/(d*x+
c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^2/i^2
```

### 3.44.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 324, normalized size of antiderivative = 1.24

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$$
$$= \frac{-\frac{b^2Bc}{a+bx}+\frac{abBd}{a+bx}+\frac{bBcd}{c+dx}-\frac{aBd^2}{c+dx}-\frac{b(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}+\frac{d(-bc+ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}-2bd\log(a+bx)\left(A\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i
*x)^2),x]
```

output
```
(-((b^2*B*c)/(a + b*x)) + (a*b*B*d)/(a + b*x) + (b*B*c*d)/(c + d*x) - (a*B
*d^2)/(c + d*x) - (b*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a
+ b*x) + (d*(-(b*c) + a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x)
 - 2*b*d*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*b*d*(A + B*
Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + b*B*d*(Log[a + b*x]*(Log[a +
b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)]) - b*B*d*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^3*g
^2*i^2)
```

### 3.44.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2962, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^2(ci+dix)^2}dx$$
$$\Big\downarrow \text{ 2962}$$
$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i^2(bc-ad)^3}$$

$$\Big\downarrow \text{2772}$$

$$\frac{-B\int\left(d^2-\frac{2b(c+dx)\log\left(\frac{a+bx}{c+dx}\right)d}{a+bx}-\frac{b^2(c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}+\frac{d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{c+dx}-2bd}{g^2i^2(bc-ad)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}+\frac{d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{c+dx}-2bd\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\left(\frac{b^2(c+dx)}{a+bx}+\right.}{g^2i^2(bc-ad)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i*x)^2)
,x]
```

output
```
(-(B*((d^2*(a + b*x))/(c + d*x) + (b^2*(c + d*x))/(a + b*x) - b*d*Log[(a +
 b*x)/(c + d*x)]^2)) + (d^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(c + d*x) - (b^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*
x) - 2*b*d*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
((b*c - a*d)^3*g^2*i^2)
```

### 3.44.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.44.4 Maple [A] (verified)

Time = 1.14 (sec) , antiderivative size = 366, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| parts | $\frac{A\left(-\frac{d}{(ad-cb)^2(dx+c)}-\frac{2db\ln(dx+c)}{(ad-cb)^3}-\frac{b}{(ad-cb)^2(bx+a)}+\frac{2db\ln(bx+a)}{(ad-cb)^3}\right)}{g^2i^2}-\frac{B\left(\frac{d^2\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-}{d(dx}\right.}{(ad-cb)^2}\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{d^2Ab^2}{i^2(ad-cb)^4g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{2d^3Ab\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^4g^2}+\frac{d^4A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^4g^2}+\frac{d^2Bb^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb}{d(dx+c}\right.}{\frac{be}{d}+\frac{(ad-cb)}{d(dx+c)}}\right.}{i^2(ad-cb}\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{d^2Ab^2}{i^2(ad-cb)^4g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{2d^3Ab\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^4g^2}+\frac{d^4A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^4g^2}+\frac{d^2Bb^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb}{d(dx+c}\right.}{\frac{be}{d}+\frac{(ad-cb)}{d(dx+c)}}\right.}{i^2(ad-cb}\right.}{d^2}$ |
| risch | $-\frac{Ad}{g^2i^2(ad-cb)^2(dx+c)}-\frac{2Adb\ln(dx+c)}{g^2i^2(ad-cb)^3}-\frac{Ab}{g^2i^2(ad-cb)^2(bx+a)}+\frac{2Adb\ln(bx+a)}{g^2i^2(ad-cb)^3}-\frac{Bd\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)b}{g^2i^2(ad-cb)^3}-$ |
| norman | $\frac{\left(2Aabcd-Ba^2d^2+Bb^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{\left(2Aab\,d^2+2A\,b^2cd-2Bab\,d^2+2B\,b^2cd\right)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{2b^2d^2A\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right.}{gi\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-\right.}$ |
| parallelrisch | $2Ax\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4bc^3d^2+2Ax\ln\left(\frac{e(bx+a)}{dx+c}\right)a^3b^2c^4d-2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^4bc^3d^2+2Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^3b^2c^4d+B\,x^2\ln($ |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,method=_RETU
RNVERBOSE)
```

output
```
A/g^2/i^2*(-d/(a*d-b*c)^2/(d*x+c)-2*d/(a*d-b*c)^3*b*ln(d*x+c)-b/(a*d-b*c)^
2/(b*x+a)+2*d/(a*d-b*c)^3*b*ln(b*x+a))-B/g^2/i^2/(a*d-b*c)/e*(d^2/(a*d-b*c
)^2*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*
c)*e/d/(d*x+c)-b*e/d)-1/(a*d-b*c)^2*b*d*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
2+1/(a*d-b*c)^2*e^2*b^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*
c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

### 3.44.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 334, normalized size of antiderivative = 1.28

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx =$$
$$-\frac{(A+B)b^2c^2-2\,Babcd-(A-B)a^2d^2+(Bb^2d^2x^2+Babcd+(Bb^2cd+Babd^2)x)\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,(A}{(b^4c^3d-3\,ab^3c^2d^2+3\,a^2b^2cd^3-a^3bd^4)g^2i^2x^2+(b^4c^4-2\,ab^3c^3d}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output
```
-((A + B)*b^2*c^2 - 2*B*a*b*c*d - (A - B)*a^2*d^2 + (B*b^2*d^2*x^2 + B*a*b
*c*d + (B*b^2*c*d + B*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*b^
2*c*d - A*a*b*d^2)*x + (2*A*b^2*d^2*x^2 + B*b^2*c^2 + 2*A*a*b*c*d - B*a^2*
d^2 + 2*((A + B)*b^2*c*d + (A - B)*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c)
))/((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*g^2*i^2*x^
2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*g^2*i^2*x + (a*b^3
*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*g^2*i^2)
```

### 3.44.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(235) = 470$.

Time = 1.90 (sec) , antiderivative size = 828, normalized size of antiderivative = 3.17

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$$
$$= -\frac{2Abd\log\left(x + \frac{-\frac{2Aa^4bd^5}{(ad-bc)^3} + \frac{8Aa^3b^2cd^4}{(ad-bc)^3} - \frac{12Aa^2b^3c^2d^3}{(ad-bc)^3} + \frac{8Aab^4c^3d^2}{(ad-bc)^3} + 2Aabd^2 - \frac{2Ab^5c^4d}{(ad-bc)^3} + 2Ab^2cd}{4Ab^2d^2}\right)}{g^2i^2\,(ad-bc)^3}$$
$$+ \frac{2Abd\log\left(x + \frac{\frac{2Aa^4bd^5}{(ad-bc)^3} - \frac{8Aa^3b^2cd^4}{(ad-bc)^3} + \frac{12Aa^2b^3c^2d^3}{(ad-bc)^3} - \frac{8Aab^4c^3d^2}{(ad-bc)^3} + 2Aabd^2 + \frac{2Ab^5c^4d}{(ad-bc)^3} + 2Ab^2cd}{4Ab^2d^2}\right)}{g^2i^2\,(ad-bc)^3}$$
$$+ \frac{Bbd\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{a^3d^3g^2i^2 - 3a^2bcd^2g^2i^2 + 3ab^2c^2dg^2i^2 - b^3c^3g^2i^2}$$
$$+ \frac{(-Bad - Bbc - 2Bbdx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{a^3cd^2g^2i^2 + a^3d^3g^2i^2x - 2a^2bc^2dg^2i^2 - a^2bcd^2g^2i^2x + a^2bd^3g^2i^2x^2 + ab^2c^3g^2i^2 - ab^2c^2dg^2i^2x - 2ab^2cd^2}$$
$$- \frac{Aad + Abc + 2Abdx - Bad + Bbc}{a^3cd^2g^2i^2 - 2a^2bc^2dg^2i^2 + ab^2c^3g^2i^2 + x^2\left(a^2bd^3g^2i^2 - 2ab^2cd^2g^2i^2 + b^3c^2dg^2i^2\right) + x\left(a^3d^3g^2i^2 - a^2bcd^2\right.}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2/(d*i*x+c*i)**2,x)
```

output
```
-2*A*b*d*log(x + (-2*A*a**4*b*d**5/(a*d - b*c)**3 + 8*A*a**3*b**2*c*d**4/(
a*d - b*c)**3 - 12*A*a**2*b**3*c**2*d**3/(a*d - b*c)**3 + 8*A*a*b**4*c**3*
d**2/(a*d - b*c)**3 + 2*A*a*b*d**2 - 2*A*b**5*c**4*d/(a*d - b*c)**3 + 2*A*
b**2*c*d)/(4*A*b**2*d**2))/(g**2*i**2*(a*d - b*c)**3) + 2*A*b*d*log(x + (2
*A*a**4*b*d**5/(a*d - b*c)**3 - 8*A*a**3*b**2*c*d**4/(a*d - b*c)**3 + 12*A
*a**2*b**3*c**2*d**3/(a*d - b*c)**3 - 8*A*a*b**4*c**3*d**2/(a*d - b*c)**3
+ 2*A*a*b*d**2 + 2*A*b**5*c**4*d/(a*d - b*c)**3 + 2*A*b**2*c*d)/(4*A*b**2*
d**2))/(g**2*i**2*(a*d - b*c)**3) + B*b*d*log(e*(a + b*x)/(c + d*x))**2/(a
**3*d**3*g**2*i**2 - 3*a**2*b*c*d**2*g**2*i**2 + 3*a*b**2*c**2*d*g**2*i**2
 - b**3*c**3*g**2*i**2) + (-B*a*d - B*b*c - 2*B*b*d*x)*log(e*(a + b*x)/(c
+ d*x))/(a**3*c*d**2*g**2*i**2 + a**3*d**3*g**2*i**2*x - 2*a**2*b*c**2*d*g
**2*i**2 - a**2*b*c*d**2*g**2*i**2*x + a**2*b*d**3*g**2*i**2*x**2 + a*b**2
*c**3*g**2*i**2 - a*b**2*c**2*d*g**2*i**2*x - 2*a*b**2*c*d**2*g**2*i**2*x*
*2 + b**3*c**3*g**2*i**2*x + b**3*c**2*d*g**2*i**2*x**2) - (A*a*d + A*b*c
+ 2*A*b*d*x - B*a*d + B*b*c)/(a**3*c*d**2*g**2*i**2 - 2*a**2*b*c**2*d*g**2
*i**2 + a*b**2*c**3*g**2*i**2 + x**2*(a**2*b*d**3*g**2*i**2 - 2*a*b**2*c*d
**2*g**2*i**2 + b**3*c**2*d*g**2*i**2) + x*(a**3*d**3*g**2*i**2 - a**2*b*c
*d**2*g**2*i**2 - a*b**2*c**2*d*g**2*i**2 + b**3*c**3*g**2*i**2))
```

### 3.44.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(261) = 522$.

Time = 0.24 (sec) , antiderivative size = 859, normalized size of antiderivative = 3.29

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^2(ci + dix)^2}\,dx =$$
$$-B\left(\frac{2\,bdx + bc + ad}{(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)g^2i^2x^2 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)g^2i^2x + (ab^2c^3 - 2\,a^2bc^2d + a^3cd\right.$$
$$\left. + \frac{ae}{dx + c}\right)$$
$$-A\left(\frac{2\,bdx + bc + ad}{(b^3c^2d - 2\,ab^2cd^2 + a^2bd^3)g^2i^2x^2 + (b^3c^3 - ab^2c^2d - a^2bcd^2 + a^3d^3)g^2i^2x + (ab^2c^3 - 2\,a^2bc^2d + a^3cd\right.$$
$$-\frac{\left(b^2c^2 - 2\,abcd + a^2d^2 - (b^2d^2x^2 + abcd + (b^2cd + abd^2)x)\log\left(bx + a\right)^2 + 2\,(b^2d^2x^2 + abcd + (b^2cd + ab\right.}{ab^3c^4g^2i^2 - 3\,a^2b^2c^3dg^2i^2 + 3\,a^3bc^2d^2g^2i^2 - a^4cd^3g^2i^2 + (b^4c^3dg^2i^2 - 3\,ab^3c^2d^2g^2i^2 + 3\,a^2b^2cd^3g}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output
```
-B*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2
*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*
c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/((
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) - A*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c
*d^2 + a^2*b*d^3)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3
*d^3)*g^2*i^2*x + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d
*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2
) - 2*b*d*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3
)*g^2*i^2)) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2 + a*b*c*d + (b
^2*c*d + a*b*d^2)*x)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d
+ a*b*d^2)*x)*log(b*x + a)*log(d*x + c) - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*
d + a*b*d^2)*x)*log(d*x + c)^2)*B/(a*b^3*c^4*g^2*i^2 - 3*a^2*b^2*c^3*d*g^2
*i^2 + 3*a^3*b*c^2*d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*c^3*d*g^2*i^2 -
3*a*b^3*c^2*d^2*g^2*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*d^4*g^2*i^2)*x^2
 + (b^4*c^4*g^2*i^2 - 2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d^3*g^2*i^2 - a^4*
d^4*g^2*i^2)*x)
```

### 3.44.8 Giac [A] (verification not implemented)

Time = 44.47 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.51

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = -\left(\frac{bc}{(bce-ade)(bc-ad)} - \frac{ad}{(bce-ade)(bc-ad)}\right)^2\left(\frac{(dx+c)Be^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)g^2i^2} + \frac{(Ae^2+Be^2)(dx+c)}{(bex+ae)g^2i^2}\right)$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output
```
-(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
*((d*x + c)*B*e^2*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)*g^2*i^2) + (
A*e^2 + B*e^2)*(d*x + c)/((b*e*x + a*e)*g^2*i^2))
```

### 3.44.9 Mupad [B] (verification not implemented)

Time = 2.93 (sec) , antiderivative size = 415, normalized size of antiderivative = 1.59

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = \frac{B\,b\,d\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{g^2\,i^2\,(a\,d-b\,c)^3} - \frac{A\,a\,d}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{A\,b\,c}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} + \frac{B\,a\,d}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,b\,c}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{2\,A\,b\,d\,x}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,a\,d\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,b\,c\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{2\,B\,b\,d\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{A\,b\,d\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,4\mathrm{i}}{g^2\,i^2\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^2*(c*i + d*i*x)^2)
,x)
```

output
```
(B*b*d*log((e*(a + b*x))/(c + d*x))^2)/(g^2*i^2*(a*d - b*c)^3) - (A*b*d*at
an((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*4i)/(g^2*i^2*(a*d - b*c)^3) -
 (A*a*d)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (A*b*c)/(g^2*i^2*(a
*d - b*c)^2*(a + b*x)*(c + d*x)) + (B*a*d)/(g^2*i^2*(a*d - b*c)^2*(a + b*x
)*(c + d*x)) - (B*b*c)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (2*A*
b*d*x)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (B*a*d*log((e*(a + b*
x))/(c + d*x)))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (B*b*c*log((
e*(a + b*x))/(c + d*x)))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (2*
B*b*d*x*log((e*(a + b*x))/(c + d*x)))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c
+ d*x))
```

# 3.45 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$



## 3.45.1 Optimal result

Integrand size = 40, antiderivative size = 364

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx &= \frac{Bd^3(a+bx)}{(bc-ad)^4g^3i^2(c+dx)}+\frac{3b^2Bd(c+dx)}{(bc-ad)^4g^3i^2(a+bx)}\\
&\quad-\frac{b^3B(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2}-\frac{3bBd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2}\\
&\quad-\frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(c+dx)}\\
&\quad+\frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)}\\
&\quad-\frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2}\\
&\quad+\frac{3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2}\end{aligned}$$

output
```
B*d^3*(b*x+a)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+3*b^2*B*d*(d*x+c)/(-a*d+b*c)^4/
g^3/i^2/(b*x+a)-1/4*b^3*B*(d*x+c)^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2-3/2*b*B
*d^2*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^3/i^2-d^3*(b*x+a)*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+3*b^2*d*(d*x+c)*(A+B*ln(e*(b*x+
a)/(d*x+c)))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/2*b^3*(d*x+c)^2*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2+3*b*d^2*ln((b*x+a)/(d*x+c))*(
A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^3/i^2
```

### 3.45.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.37 (sec) , antiderivative size = 453, normalized size of antiderivative = 1.24

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$$
$$= \frac{-\frac{bB(bc-ad)^2}{(a+bx)^2}+\frac{8b^2Bcd}{a+bx}-\frac{8abBd^2}{a+bx}+\frac{2bBd(bc-ad)}{a+bx}-\frac{4bBcd^2}{c+dx}+\frac{4aBd^3}{c+dx}+6bBd^2\log(a+bx)-\frac{2b(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{(a+bx)^2}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i
*x)^2),x]
```

output
```
(-((b*B*(b*c - a*d)^2)/(a + b*x)^2) + (8*b^2*B*c*d)/(a + b*x) - (8*a*b*B*d
^2)/(a + b*x) + (2*b*B*d*(b*c - a*d))/(a + b*x) - (4*b*B*c*d^2)/(c + d*x)
+ (4*a*B*d^3)/(c + d*x) + 6*b*B*d^2*Log[a + b*x] - (2*b*(b*c - a*d)^2*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^2 + (8*b*d*(b*c - a*d)*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + (4*d^2*(b*c - a*d)*(A + B*Log[(
e*(a + b*x))/(c + d*x)]))/(c + d*x) + 12*b*d^2*Log[a + b*x]*(A + B*Log[(e*
(a + b*x))/(c + d*x)]) - 6*b*B*d^2*Log[c + d*x] - 12*b*d^2*(A + B*Log[(e*(
a + b*x))/(c + d*x)])*Log[c + d*x] - 6*b*B*d^2*(Log[a + b*x]*(Log[a + b*x]
 - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
+ a*d)]) + 6*b*B*d^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])
*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c - a*d)^4
*g^3*i^2)
```

### 3.45.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^3(ci+dix)^2}dx$$

$$\Big\downarrow \ 2962$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i^2(bc-ad)^4}$$

$$\Big\downarrow \ 2772$$

$$\frac{-B\int -\frac{(c+dx)^3\left(b^3-\frac{6d(a+bx)b^2}{c+dx}-\frac{6d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)b}{(c+dx)^2}+\frac{2d^3(a+bx)^3}{(c+dx)^3}\right)}{2(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\left(\frac{e}{}\right.\right.}{a+bx}}{g^3i^2(bc-ad)^4}$$

$$\Big\downarrow \ 27$$

$$\frac{\frac{1}{2}B\int \frac{(c+dx)^3\left(b^3-\frac{6d(a+bx)b^2}{c+dx}-\frac{6d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)b}{(c+dx)^2}+\frac{2d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\left(\frac{e(a}{c+}\right.\right.}{a+bx}}{g^3i^2(bc-ad)^4}$$

$$\Big\downarrow \ 2010$$

$$\frac{\frac{1}{2}B\int \left(\frac{(c+dx)^3\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{2d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)^3}-\frac{6bd^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\right.}{a}}{g^3i^2(bc-ad)^4}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{b^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{d^3(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{c+dx}+3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\right.\right.}{g^3i^2(bc-ad)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i*x)^2)
,x]
```

output
```
((B*((2*d^3*(a + b*x))/(c + d*x) + (6*b^2*d*(c + d*x))/(a + b*x) - (b^3*(c
 + d*x)^2)/(2*(a + b*x)^2) - 3*b*d^2*Log[(a + b*x)/(c + d*x)]^2))/2 - (d^3
*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) + (3*b^2*d*(c +
 d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - (b^3*(c + d*x)^2*(
A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) + 3*b*d^2*Log[(a + b*
x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^4*g^3*i^2
)
```

### 3.45.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.45.4 Maple [A] (verified)

Time = 2.88 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.39

| method | result |
|---|---|
| parts | $\frac{A\left(-\frac{d^{2}}{(ad-cb)^{3}(dx+c)}-\frac{3d^{2}b\ln(dx+c)}{(ad-cb)^{4}}-\frac{b}{2(ad-cb)^{2}(bx+a)^{2}}+\frac{3d^{2}b\ln(bx+a)}{(ad-cb)^{4}}-\frac{2bd}{(ad-cb)^{3}(bx+a)}\right)}{g^{3}i^{2}}-\frac{B\left(\frac{d^{3}\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\right.}{}\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{d^{2}eA\,b^{3}}{2i^{2}(ad-cb)^{5}g^{3}\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}-\frac{3d^{3}A\,b^{2}}{i^{2}(ad-cb)^{5}g^{3}\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{3d^{4}Ab\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^{2}(ad-cb)^{5}g^{3}}+\frac{d^{5}A\left(\frac{be}{d}+\frac{(ad-c}{d(dx+}\right.}{e^{2}i^{2}(ad-cb)^{5}}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(\frac{d^{2}eA\,b^{3}}{2i^{2}(ad-cb)^{5}g^{3}\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}-\frac{3d^{3}A\,b^{2}}{i^{2}(ad-cb)^{5}g^{3}\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{3d^{4}Ab\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^{2}(ad-cb)^{5}g^{3}}+\frac{d^{5}A\left(\frac{be}{d}+\frac{(ad-c}{d(dx+}\right.}{e^{2}i^{2}(ad-cb)^{5}}\right.}{}$ |
| risch | $-\frac{A\,d^{2}}{g^{3}i^{2}(ad-cb)^{3}(dx+c)}-\frac{3A\,d^{2}b\ln(dx+c)}{g^{3}i^{2}(ad-cb)^{4}}-\frac{Ab}{2g^{3}i^{2}(ad-cb)^{2}(bx+a)^{2}}+\frac{3A\,d^{2}b\ln(bx+a)}{g^{3}i^{2}(ad-cb)^{4}}-\frac{2Abd}{g^{3}i^{2}(ad-cb)^{3}(bx+a)}$ |
| parallelrisch | $\frac{6B\,x^{3}\ln\left(\frac{e(bx+a)}{dx+c}\right)^{2}b^{7}d^{6}+12A\,x^{3}\ln\left(\frac{e(bx+a)}{dx+c}\right)b^{7}d^{6}+6B\,x^{3}\ln\left(\frac{e(bx+a)}{dx+c}\right)b^{7}d^{6}-12A\,x^{2}a\,b^{6}d^{6}+12A\,x^{2}b^{7}c\,d^{5}-6B\,x^{2}a\,b^{6}d}{}$ |
| norman | Expression too large to display |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,method=_RETU
RNVERBOSE)
```

output
```
A/g^3/i^2*(-d^2/(a*d-b*c)^3/(d*x+c)-3*d^2/(a*d-b*c)^4*b*ln(d*x+c)-1/2*b/(a
*d-b*c)^2/(b*x+a)^2+3*d^2/(a*d-b*c)^4*b*ln(b*x+a)-2*b/(a*d-b*c)^3*d/(b*x+a
))-B/g^3/i^2/(a*d-b*c)/e*(d^3/(a*d-b*c)^3*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-3/2/(a*d-b*c)^
3*b*d^2*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+3/(a*d-b*c)^3*b^2*d*e^2*(-1/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-
b*c)*e/d/(d*x+c)))-1/(a*d-b*c)^3*b^3*e^3*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

### 3.45.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 664, normalized size of antiderivative = 1.82

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx =$$
$$-\frac{(2\,A+B)b^3c^3-12\,(A+B)ab^2c^2d+3\,(2\,A+5\,B)a^2bcd^2+4\,(A-B)a^3d^3-6\,((2\,A+B)b^3cd^2-(2\,A}{4\,((b^6c^4d-4\,ab}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output
```
-1/4*((2*A + B)*b^3*c^3 - 12*(A + B)*a*b^2*c^2*d + 3*(2*A + 5*B)*a^2*b*c*d
^2 + 4*(A - B)*a^3*d^3 - 6*((2*A + B)*b^3*c*d^2 - (2*A + B)*a*b^2*d^3)*x^2
 - 6*(B*b^3*d^3*x^3 + B*a^2*b*c*d^2 + (B*b^3*c*d^2 + 2*B*a*b^2*d^3)*x^2 +
(2*B*a*b^2*c*d^2 + B*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 3*((2*
A + 3*B)*b^3*c^2*d + 2*(2*A - B)*a*b^2*c*d^2 - (6*A + B)*a^2*b*d^3)*x - 2*
(3*(2*A + B)*b^3*d^3*x^3 - B*b^3*c^3 + 6*B*a*b^2*c^2*d + 6*A*a^2*b*c*d^2 -
 2*B*a^3*d^3 + 3*((2*A + 3*B)*b^3*c*d^2 + 4*A*a*b^2*d^3)*x^2 + 3*(B*b^3*c^
2*d + 4*(A + B)*a*b^2*c*d^2 + 2*(A - B)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d
*x + c)))/((b^6*c^4*d - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*
d^4 + a^4*b^2*d^5)*g^3*i^2*x^3 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*
d^2 + 8*a^3*b^3*c^2*d^3 - 7*a^4*b^2*c*d^4 + 2*a^5*b*d^5)*g^3*i^2*x^2 + (2*
a*b^5*c^5 - 7*a^2*b^4*c^4*d + 8*a^3*b^3*c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^
5*b*c*d^4 + a^6*d^5)*g^3*i^2*x + (a^2*b^4*c^5 - 4*a^3*b^3*c^4*d + 6*a^4*b^
2*c^3*d^2 - 4*a^5*b*c^2*d^3 + a^6*c*d^4)*g^3*i^2)
```

### 3.45.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

### 3.45.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1721 vs. $2(358) = 716$.

Time = 0.33 (sec) , antiderivative size = 1721, normalized size of antiderivative = 4.73

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^3(ci + dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output
```
1/2*B*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a
*b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*
g^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3
 - 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c
^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d
 + 3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) -
6*b*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)*g^3*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 1/2*A
*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*d^
2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*g^3*i
^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 - 2*
a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*d^
2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d + 3*
a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 - 4*a
*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6*b*d
^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*
d^3 + a^4*d^4)*g^3*i^2)) - 1/4*(b^3*c^3 - 12*a*b^2*c^2*d + 15*a^2*b*c*d^2
- 4*a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2
 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b...
```

### 3.45.8 Giac [A] (verification not implemented)

Time = 54.53 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.76

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = -\frac{1}{4}\left(\frac{2\left(Bbe^3-\frac{2\,(bex+ae)Bde^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^2bcg^3i^2}{(dx+c)^2}-\frac{(bex+ae)^2adg^3i^2}{(dx+c)^2}}+\frac{2\,Abe^3+Bbe^3-\frac{4\,(bex+ae)Ade^2}{dx+c}-\frac{4\,(bex+ae)Bde^2}{dx+c}}{\frac{(bex+ae)^2bcg^3i^2}{(dx+c)^2}-\frac{(bex+ae)^2adg^3i^2}{(dx+c)^2}}\right)\left(\frac{}{(bce-ad}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output
```
-1/4*(2*(B*b*e^3 - 2*(b*e*x + a*e)*B*d*e^2/(d*x + c))*log((b*e*x + a*e)/(d
*x + c))/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*e*x + a*e)^2*a*d*g^
3*i^2/(d*x + c)^2) + (2*A*b*e^3 + B*b*e^3 - 4*(b*e*x + a*e)*A*d*e^2/(d*x +
 c) - 4*(b*e*x + a*e)*B*d*e^2/(d*x + c))/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x
 + c)^2 - (b*e*x + a*e)^2*a*d*g^3*i^2/(d*x + c)^2))*(b*c/((b*c*e - a*d*e)*
(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

### 3.45.9 Mupad [B] (verification not implemented)

Time = 5.64 (sec) , antiderivative size = 984, normalized size of antiderivative = 2.70

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \frac{3\,B\,b\,d^2\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{2\,g^3\,i^2\,(a\,d-b\,c)^4} - \frac{A\,a^2\,d^2}{g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$+ \frac{A\,b^2\,c^2}{2\,g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$+ \frac{B\,a^2\,d^2}{g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$+ \frac{B\,b^2\,c^2}{4\,g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{B\,a\,d\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{B\,b\,c\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g^3\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{3\,A\,b^2\,d^2\,x^2}{g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{3\,B\,b^2\,d^2\,x^2}{2\,g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{5\,A\,a\,b\,c\,d}{2\,g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{11\,B\,a\,b\,c\,d}{4\,g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{3\,B\,b\,d\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g^3\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{3\,B\,b^2\,d^2\,x^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{9\,A\,a\,b\,d^2\,x}{2\,g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{3\,B\,a\,b\,d^2\,x}{4\,g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{3\,A\,b^2\,c\,d\,x}{2\,g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{9\,B\,b^2\,c\,d\,x}{4\,g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{3\,B\,a\,b\,c\,d\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$
$$- \frac{3\,B\,a\,b\,d^2\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g^3\,i^2\,(a\,d-b\,c)^3\,(a+b\,x)^2\,(c+d\,x)}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^3*(c*i + d*i*x)^2)
,x)
```

output
```
(3*B*b*d^2*log((e*(a + b*x))/(c + d*x))^2)/(2*g^3*i^2*(a*d - b*c)^4) - (A*
b*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*6i)/(g^3*i^2*(a*d - b
*c)^4) - (B*b*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*3i)/(g^3*
i^2*(a*d - b*c)^4) - (A*a^2*d^2)/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d
*x)) + (A*b^2*c^2)/(2*g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) + (B*a^
2*d^2)/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) + (B*b^2*c^2)/(4*g^3*
i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (B*a*d*log((e*(a + b*x))/(c + d
*x)))/(g^3*i^2*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)) - (B*b*c*log((e*(a + b
*x))/(c + d*x)))/(2*g^3*i^2*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)) - (3*A*b^
2*d^2*x^2)/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*b^2*d^2*x^
2)/(2*g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (5*A*a*b*c*d)/(2*g^3*
i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (11*B*a*b*c*d)/(4*g^3*i^2*(a*d
- b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*b*d*x*log((e*(a + b*x))/(c + d*x)))
/(2*g^3*i^2*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)) - (3*B*b^2*d^2*x^2*log((e
*(a + b*x))/(c + d*x)))/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (9
*A*a*b*d^2*x)/(2*g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*a*b*d
^2*x)/(4*g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*A*b^2*c*d*x)/(2
*g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (9*B*b^2*c*d*x)/(4*g^3*i^2
*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*a*b*c*d*log((e*(a + b*x))/(c
+ d*x)))/(g^3*i^2*(a*d - b*c)^3*(a + b*x)^2*(c + d*x)) - (3*B*a*b*d^2*x...
```

## 3.45. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$

# 3.46 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx$



## 3.46.1 Optimal result

Integrand size = 40, antiderivative size = 457

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx =& -\frac{Bd^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)}-\frac{6b^2Bd^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)} \\ &+\frac{b^3Bd(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2}-\frac{b^4B(c+dx)^3}{9(bc-ad)^5g^4i^2(a+bx)^3} \\ &+\frac{2bBd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}+\frac{d^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(c+dx)} \\ &-\frac{6b^2d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} \\ &+\frac{2b^3d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} \\ &-\frac{b^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^5g^4i^2(a+bx)^3} \\ &-\frac{4bd^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2}\end{aligned}$$

output
```
-B*d^4*(b*x+a)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-6*b^2*B*d^2*(d*x+c)/(-a*d+b*c)
^5/g^4/i^2/(b*x+a)+b^3*B*d*(d*x+c)^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-1/9*b^
4*B*(d*x+c)^3/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3+2*b*B*d^3*ln((b*x+a)/(d*x+c))
^2/(-a*d+b*c)^5/g^4/i^2+d^4*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)
^5/g^4/i^2/(d*x+c)-6*b^2*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c
)^5/g^4/i^2/(b*x+a)+2*b^3*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*
c)^5/g^4/i^2/(b*x+a)^2-1/3*b^4*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d
+b*c)^5/g^4/i^2/(b*x+a)^3-4*b*d^3*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/(d
*x+c)))/(-a*d+b*c)^5/g^4/i^2
```

## 3.46.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.71 (sec) , antiderivative size = 520, normalized size of antiderivative = 1.14

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx =$$

$$-\frac{\frac{bB(bc-ad)^3}{(a+bx)^3}-\frac{6bBd(bc-ad)^2}{(a+bx)^2}+\frac{27b^2Bcd^2}{a+bx}-\frac{27abBd^3}{a+bx}+\frac{12bBd^2(bc-ad)}{a+bx}-\frac{9bBcd^3}{c+dx}+\frac{9aBd^4}{c+dx}+30bBd^3\log(a+bx)+\frac{3b}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i
*x)^2),x]
```

output
```
-1/9*((b*B*(b*c - a*d)^3)/(a + b*x)^3 - (6*b*B*d*(b*c - a*d)^2)/(a + b*x)^
2 + (27*b^2*B*c*d^2)/(a + b*x) - (27*a*b*B*d^3)/(a + b*x) + (12*b*B*d^2*(b
*c - a*d))/(a + b*x) - (9*b*B*c*d^3)/(c + d*x) + (9*a*B*d^4)/(c + d*x) + 3
0*b*B*d^3*Log[a + b*x] + (3*b*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(a + b*x)^3 - (9*b*d*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d
*x)]))/(a + b*x)^2 + (27*b*d^2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d
*x)]))/(a + b*x) - (9*d^3*(-(b*c) + a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(c + d*x) + 36*b*d^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]
) - 30*b*B*d^3*Log[c + d*x] - 36*b*d^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)*Log[c + d*x] - 18*b*B*d^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*
x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 18*b*B*d
^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^5*g^4*i^2)
```

### 3.46.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 313, normalized size of antiderivative = 0.68, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2962, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^4(ci+dix)^2}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i^2(bc-ad)^5}$$

$$\Big\downarrow \text{2772}$$

$$\frac{-B\int\left(d^4-\frac{4b(c+dx)\log\left(\frac{a+bx}{c+dx}\right)d^3}{a+bx}-\frac{6b^2(c+dx)^2d^2}{(a+bx)^2}+\frac{2b^3(c+dx)^3d}{(a+bx)^3}-\frac{b^4(c+dx)^4}{3(a+bx)^4}\right)d\frac{a+bx}{c+dx}-\frac{b^4(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{2b^3 \cdots}{\cdots}}{g \cdots}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^4(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{2b^3d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(a+bx)^2}-\frac{6b^2d^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}+\frac{d^4(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right) \cdots\right.}{c+dx}\cdots}{g^4i^2 \cdots}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i*x)^2)
,x]
```

output
```
(-(B*((d^4*(a + b*x))/(c + d*x) + (6*b^2*d^2*(c + d*x))/(a + b*x) - (b^3*d
*(c + d*x)^2)/(a + b*x)^2 + (b^4*(c + d*x)^3)/(9*(a + b*x)^3) - 2*b*d^3*Lo
g[(a + b*x)/(c + d*x)]^2)) + (d^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(c + d*x) - (6*b^2*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(a + b*x) + (2*b^3*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(a + b*x)^2 - (b^4*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*
(a + b*x)^3) - 4*b*d^3*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(
c + d*x)]))/((b*c - a*d)^5*g^4*i^2)
```

### 3.46.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.46.4 Maple [A] (verified)

Time = 2.64 (sec) , antiderivative size = 640, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| parts | $\frac{A\left(-\frac{d^3}{(ad-cb)^4(dx+c)}-\frac{4d^3b\ln(dx+c)}{(ad-cb)^5}-\frac{b}{3(ad-cb)^2(bx+a)^3}+\frac{4d^3b\ln(bx+a)}{(ad-cb)^5}-\frac{3b\,d^2}{(ad-cb)^4(bx+a)}-\frac{bd}{(ad-cb)^3(bx+a)^2}\right)}{g^4i^2}-\frac{B\left(\frac{d^4}{\cdots}\right.}{\cdots}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{d^2e^2A\,b^4}{3i^2(ad-cb)^6g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{2d^3eA\,b^3}{i^2(ad-cb)^6g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{6d^4A\,b^2}{i^2(ad-cb)^6g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{4d^5Ab\ln}{e\,i}\cdots\right.}{\cdots}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{d^2e^2A\,b^4}{3i^2(ad-cb)^6g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{2d^3eA\,b^3}{i^2(ad-cb)^6g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{6d^4A\,b^2}{i^2(ad-cb)^6g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{4d^5Ab\ln}{e\,i}\cdots\right.}{\cdots}$ |
| risch | $-\frac{A\,d^3}{g^4i^2(ad-cb)^4(dx+c)}-\frac{4A\,d^3b\ln(dx+c)}{g^4i^2(ad-cb)^5}-\frac{Ab}{3g^4i^2(ad-cb)^2(bx+a)^3}+\frac{4A\,d^3b\ln(bx+a)}{g^4i^2(ad-cb)^5}-\frac{3Ab\,d^2}{g^4i^2(ad-cb)^4(bx+a)}\cdots$ |
| parallelrisch | Expression too large to display |
| norman | Expression too large to display |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,method=_RETU
RNVERBOSE)
```

output
```
A/g^4/i^2*(-d^3/(a*d-b*c)^4/(d*x+c)-4*d^3/(a*d-b*c)^5*b*ln(d*x+c)-1/3*b/(a
*d-b*c)^2/(b*x+a)^3+4*d^3/(a*d-b*c)^5*b*ln(b*x+a)-3*b/(a*d-b*c)^4*d^2/(b*x
+a)-b/(a*d-b*c)^3*d/(b*x+a)^2)-B/g^4/i^2/(a*d-b*c)/e*(d^4/(a*d-b*c)^4*((b*
e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(
d*x+c)-b*e/d)-2/(a*d-b*c)^4*b*d^3*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+6/(a
*d-b*c)^4*b^2*d^2*e^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)
*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-4/(a*d-b*c)^4*b^3*d*e^3*(-1
/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^2)+1/(a*d-b*c)^4*b^4*e^4*(-1/3/(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^3))
```

3.46. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx$

### 3.46.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1019 vs. $2(453) = 906$.

Time = 0.32 (sec) , antiderivative size = 1019, normalized size of antiderivative = 2.23

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx =$$
$$-\frac{(3\,A+B)b^4c^4-9\,(2\,A+B)ab^3c^3d+54\,(A+B)a^2b^2c^2d^2-5\,(6\,A+11\,B)a^3bcd^3-9\,(A-B)a^4d^4+6}{}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output
```
-1/9*((3*A + B)*b^4*c^4 - 9*(2*A + B)*a*b^3*c^3*d + 54*(A + B)*a^2*b^2*c^2
*d^2 - 5*(6*A + 11*B)*a^3*b*c*d^3 - 9*(A - B)*a^4*d^4 + 6*((6*A + 5*B)*b^4
*c*d^3 - (6*A + 5*B)*a*b^3*d^4)*x^3 + 3*((6*A + 11*B)*b^4*c^2*d^2 + 8*(3*A
 + B)*a*b^3*c*d^3 - (30*A + 19*B)*a^2*b^2*d^4)*x^2 + 18*(B*b^4*d^4*x^4 + B
*a^3*b*c*d^3 + (B*b^4*c*d^3 + 3*B*a*b^3*d^4)*x^3 + 3*(B*a*b^3*c*d^3 + B*a^
2*b^2*d^4)*x^2 + (3*B*a^2*b^2*c*d^3 + B*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d
*x + c))^2 - ((6*A + 5*B)*b^4*c^3*d - 27*(2*A + 3*B)*a*b^3*c^2*d^2 - 3*(6*
A - 19*B)*a^2*b^2*c*d^3 + (66*A + 19*B)*a^3*b*d^4)*x + 3*(2*(6*A + 5*B)*b^
4*d^4*x^4 + B*b^4*c^4 - 6*B*a*b^3*c^3*d + 18*B*a^2*b^2*c^2*d^2 + 12*A*a^3*
b*c*d^3 - 3*B*a^4*d^4 + 2*((6*A + 11*B)*b^4*c*d^3 + 9*(2*A + B)*a*b^3*d^4)
*x^3 + 6*(B*b^4*c^2*d^2 + 3*(2*A + 3*B)*a*b^3*c*d^3 + 6*A*a^2*b^2*d^4)*x^2
 - 2*(B*b^4*c^3*d - 9*B*a*b^3*c^2*d^2 - 18*(A + B)*a^2*b^2*c*d^3 - 6*(A -
B)*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^8*c^5*d - 5*a*b^7*c^4*d
^2 + 10*a^2*b^6*c^3*d^3 - 10*a^3*b^5*c^2*d^4 + 5*a^4*b^4*c*d^5 - a^5*b^3*d
^6)*g^4*i^2*x^4 + (b^8*c^6 - 2*a*b^7*c^5*d - 5*a^2*b^6*c^4*d^2 + 20*a^3*b^
5*c^3*d^3 - 25*a^4*b^4*c^2*d^4 + 14*a^5*b^3*c*d^5 - 3*a^6*b^2*d^6)*g^4*i^2
*x^3 + 3*(a*b^7*c^6 - 4*a^2*b^6*c^5*d + 5*a^3*b^5*c^4*d^2 - 5*a^5*b^3*c^2*
d^4 + 4*a^6*b^2*c*d^5 - a^7*b*d^6)*g^4*i^2*x^2 + (3*a^2*b^6*c^6 - 14*a^3*b
^5*c^5*d + 25*a^4*b^4*c^4*d^2 - 20*a^5*b^3*c^3*d^3 + 5*a^6*b^2*c^2*d^4 + 2
*a^7*b*c*d^5 - a^8*d^6)*g^4*i^2*x + (a^3*b^5*c^6 - 5*a^4*b^4*c^5*d + 10...
```

### 3.46.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

### 3.46.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2560 vs. $2(453) = 906$.

Time = 0.43 (sec) , antiderivative size = 2560, normalized size of antiderivative = 5.60

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output
```
-1/3*B*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a^3
*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11
*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b
^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c
^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^
3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3
 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^4*
c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*g^
4*i^2*x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2
*d^3 + a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4
*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^
4*i^2) - 12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*
d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(b*e*x/(d
*x + c) + a*e/(d*x + c)) - 1/3*A*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*
d + 13*a^2*b*c*d^2 + 3*a^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*
c^2*d - 8*a*b^2*c*d^2 - 11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6
*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 -
 a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 +
 3*a^5*b^2*d^5)*g^4*i^2*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c
^3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 +...
```

### 3.46.8 Giac [A] (verification not implemented)

Time = 64.99 (sec) , antiderivative size = 436, normalized size of antiderivative = 0.95

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx =$$

$$-\frac{1}{18}\left(\frac{6\left(Bb^2e^4-\frac{3\,(bex+ae)Bbde^3}{dx+c}+\frac{3\,(bex+ae)^2Bd^2e^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)}{\frac{(bex+ae)^3b^2c^2g^4i^2}{(dx+c)^3}-\frac{2\,(bex+ae)^3abcdg^4i^2}{(dx+c)^3}+\frac{(bex+ae)^3a^2d^2g^4i^2}{(dx+c)^3}}+\frac{6\,Ab^2e^4+2\,Bb^2e^4-\frac{18\,(bex+ae)Abde^3}{dx+c}-\frac{9}{}}{\frac{(bex+ae)^3b^2c^2g^4i^2}{(dx+c)^3}-\frac{2\,(be}{}}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output
```
-1/18*(6*(B*b^2*e^4 - 3*(b*e*x + a*e)*B*b*d*e^3/(d*x + c) + 3*(b*e*x + a*e
)^2*B*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^3*b
^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4*i^2/(d*x + c)^3
 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d*x + c)^3) + (6*A*b^2*e^4 + 2*B*b^2*e
^4 - 18*(b*e*x + a*e)*A*b*d*e^3/(d*x + c) - 9*(b*e*x + a*e)*B*b*d*e^3/(d*x
 + c) + 18*(b*e*x + a*e)^2*A*d^2*e^2/(d*x + c)^2 + 18*(b*e*x + a*e)^2*B*d^
2*e^2/(d*x + c)^2)/((b*e*x + a*e)^3*b^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x
 + a*e)^3*a*b*c*d*g^4*i^2/(d*x + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d
*x + c)^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
 - a*d)))^2
```

### 3.46.9 Mupad [B] (verification not implemented)

Time = 9.16 (sec) , antiderivative size = 1679, normalized size of antiderivative = 3.67

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^4*(c*i + d*i*x)^2)
,x)
```

output
```
(2*B*b*d^3*log((e*(a + b*x))/(c + d*x))^2)/(g^4*i^2*(a*d - b*c)^2*(a^3*d^3
 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (log((e*(a + b*x))/(c + d*x
))*(x*((4*B)/(3*g^4*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (4*B*b*d^3*(((2
*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2))*(a*
d + b*c) + (a*c*(a*d - b*c))/d^2))/(g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) + (B*(3*a*d + b*c))/(3*g^4*i^2*(a^2*
b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (4*B*b^2*d^2*x^3)/(g^4*i^2*(a*d - b*
c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (4*B*b*d^3*x^2*(
b*d*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^
2)) + ((a*d + b*c)*(a*d - b*c))/d^2))/(g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^
3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (4*B*a*b*c*d^3*((2*a^2*d^2 + b^2
*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(g^4*i^2*(a*d -
b*c)^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/(b^2*x^4 + (
a^3*c)/(b*d) + (x*(a^3*d + 3*a^2*b*c))/(b*d) + (x^3*(b^3*c + 3*a*b^2*d))/(
b*d) + (x^2*(3*a*b^2*c + 3*a^2*b*d))/(b*d)) - (b*d^3*atan((b*d^3*((a^5*d^5
*g^4*i^2 + b^5*c^5*g^4*i^2 - 3*a*b^4*c^4*d*g^4*i^2 - 3*a^4*b*c*d^4*g^4*i^2
 + 2*a^2*b^3*c^3*d^2*g^4*i^2 + 2*a^3*b^2*c^2*d^3*g^4*i^2)/(a^4*d^4*g^4*i^2
 + b^4*c^4*g^4*i^2 - 4*a*b^3*c^3*d*g^4*i^2 - 4*a^3*b*c*d^3*g^4*i^2 + 6*a^2
*b^2*c^2*d^2*g^4*i^2) + 2*b*d*x)*(6*A + 5*B)*(a^4*d^4*g^4*i^2 + b^4*c^4*g^
4*i^2 - 4*a*b^3*c^3*d*g^4*i^2 - 4*a^3*b*c*d^3*g^4*i^2 + 6*a^2*b^2*c^2*d...
```

3.46. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^2}\,dx$

# 3.47 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$



## 3.47.1 Optimal result

Integrand size = 40, antiderivative size = 361

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = -\frac{3B(bc-ad)g^3(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{3bB(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} + \frac{b(3A+B)(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} + \frac{3bB(bc-ad)g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^3(c+dx)} + \frac{g^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{di^3(c+dx)^2} + \frac{(bc-ad)g^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^3(c+dx)^2} + \frac{b^2(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^3} + \frac{3b^2B(bc-ad)g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}$$

output
```
-3/4*B*(-a*d+b*c)*g^3*(b*x+a)^2/d^2/i^3/(d*x+c)^2-3*b*B*(-a*d+b*c)*g^3*(b*
x+a)/d^3/i^3/(d*x+c)+b*(3*A+B)*(-a*d+b*c)*g^3*(b*x+a)/d^3/i^3/(d*x+c)+3*b*
B*(-a*d+b*c)*g^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^3/i^3/(d*x+c)+g^3*(b*x+a)
^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^3/(d*x+c)^2+1/2*(-a*d+b*c)*g^3*(b*x+a)^
2*(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/d^2/i^3/(d*x+c)^2+b^2*(-a*d+b*c)*g^3*l
n((-a*d+b*c)/b/(d*x+c))*(3*A+B+3*B*ln(e*(b*x+a)/(d*x+c)))/d^4/i^3+3*b^2*B*
(-a*d+b*c)*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^3
```

### 3.47.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 317, normalized size of antiderivative = 0.88

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \frac{g^3\left(4Ab^3dx-\frac{B(bc-ad)^3}{(c+dx)^2}+\frac{10bB(bc-ad)^2}{c+dx}+10b^2B(bc-ad)\log(a+bx)+4b^2Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+\frac{2(bc-}{}\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*
i*x)^3,x]
```

output
```
(g^3*(4*A*b^3*d*x - (B*(b*c - a*d)^3)/(c + d*x)^2 + (10*b*B*(b*c - a*d)^2)
/(c + d*x) + 10*b^2*B*(b*c - a*d)*Log[a + b*x] + 4*b^2*B*d*(a + b*x)*Log[(
e*(a + b*x))/(c + d*x)] + (2*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d
*x)]))/(c + d*x)^2 - (12*b*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(c + d*x) - 14*b^2*B*(b*c - a*d)*Log[c + d*x] - 12*b^2*(b*c - a*d)*(A
 + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 6*b^2*B*(b*c - a*d)*((2*
Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog
[2, (b*(c + d*x))/(b*c - a*d)])))/(4*d^4*i^3)
```

### 3.47.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 300, normalized size of antiderivative = 0.83, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2962, 2784, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ci+dix)^3}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{g^3(bc-ad)\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^3}$$

$$\Big\downarrow \text{2784}$$

$$\frac{g^3(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{d}\right)}{i^3}$$

$\Big\downarrow$ 2793

$$\frac{g^3(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\left(-\frac{\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}-\frac{\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{d^2}-\frac{(a+bx)\left(3A+B+3B\log\left(\frac{e(a-}{c+}\right.\right.}{d(c+dx)}\right.}{d}\right.}{i^3}$$

$\Big\downarrow$ 2009

$$\frac{g^3(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{d^3}-\frac{b(3A+B)(a+bx)}{d^2(c+dx)}-\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{2d(c+dx)^2}}{d}\right.}{i^3}$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^3
,x]
```

output
```
((b*c - a*d)*g^3*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c
 + d*x)^3*(b - (d*(a + b*x))/(c + d*x))) - ((3*B*(a + b*x)^2)/(4*d*(c + d*
x)^2) + (3*b*B*(a + b*x))/(d^2*(c + d*x)) - (b*(3*A + B)*(a + b*x))/(d^2*(
c + d*x)) - (3*b*B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d^2*(c + d*x))
 - ((a + b*x)^2*(3*A + B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*
x)^2) - (b^2*(3*A + B + 3*B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a +
b*x))/(b*(c + d*x))])/d^3 - (3*b^2*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x)
)])/d^3)/d))/i^3
```

## 3.47. $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$

#### 3.47.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.47.4 Maple [A] (verified)

Time = 1.60 (sec) , antiderivative size = 681, normalized size of antiderivative = 1.89

| method | result |
|---|---|
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{g^3d^2A\left(\frac{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)be+\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2d}{2}}{d^3}+\frac{b^3e^3}{d^4\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{3b^2e^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right.}{d^4}\right.}{e^3i^3}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(\frac{g^3d^2A\left(\frac{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)be+\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2d}{2}}{d^3}+\frac{b^3e^3}{d^4\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}+\frac{3b^2e^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right.}{d^4}\right.}{e^3i^3}\right.}{}$ |
| parts | $\frac{g^3A\left(\frac{x\,b^3}{d^3}+\frac{3b^2(ad-cb)\ln(dx+c)}{d^4}-\frac{3b\left(a^2d^2-2abcd+b^2c^2\right)}{d^4(dx+c)}-\frac{a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3}{2d^4(dx+c)^2}\right)}{i^3}-\frac{g^3Bd\left(\frac{\left(a^3d^3-3a^2bc\,d^2+3a\right.}{}\right.}{}$ |
| risch | Expression too large to display |

3.47. $\int\frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(g^3*d^2/e^3/i^3*A*(1/d^3*(2*(b*e/d+(a*d-b*c)*e/d/(d*x+
c))*b*e+1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*d)+b^3*e^3/d^4/(b*e-(b*e/d+(a*
d-b*c)*e/d/(d*x+c))*d)+3/d^4*b^2*e^2*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*
d))+g^3*d^2/e^3/i^3*B*(1/d^2*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d
+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+2*b*e/d^3*((b
*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/
(d*x+c)-b*e/d)+b^3*e^3/d^3*(1/b/e/d*ln(b*e-(b*e/d+(a*d-b*c)*e/d/(d*x+c))*d
)+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*(b*e/d+(a*d-b*c)*e/d/(d*x+c))/b/e/(b*e-(
b*e/d+(a*d-b*c)*e/d/(d*x+c))*d))+3/d^3*b^2*e^2*(dilog(-((b*e/d+(a*d-b*c)*e
/d/(d*x+c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d
-b*c)*e/d/(d*x+c))*d-b*e)/b/e)/d))
```

### 3.47.5 Fricas [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output
```
integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
+ (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log((b
*e*x + a*e)/(d*x + c)))/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c
^3*i^3), x)
```

### 3.47.6 Sympy [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$=\frac{g^3\left(\int \frac{Aa^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{Ab^3x^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{Ba^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{3Aab^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right.}{}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**3,x)
```

output
```
g**3*(Integral(A*a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
+ Integral(A*b**3*x**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
 + Integral(B*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*A*a*b**2*x**2/(c**3 + 3*c**
2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*A*a**2*b*x/(c**3 + 3*c
**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(B*b**3*x**3*log(a*e/(c
 + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3)
, x) + Integral(3*B*a*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*B*a**2*b*x*log
(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**
3*x**3), x))/i**3
```

### 3.47.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2037 vs. $2(356) = 712$.

Time = 0.32 (sec) , antiderivative size = 2037, normalized size of antiderivative = 5.64

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output
```
-3/4*B*a^2*b*g^3*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^4*
i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a
*d^2)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c
^3*d^2 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2
- 2*a*b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2
*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3)) - 1/2*A*b^3*g^3*((6*c^2*d*x + 5*c^3)/(
d^6*i^3*x^2 + 2*c*d^5*i^3*x + c^2*d^4*i^3) - 2*x/(d^3*i^3) + 6*c*log(d*x +
 c)/(d^4*i^3)) + 1/4*B*a^3*g^3*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)
*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) - 2*
log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*
i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^
2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 3/2*A*a*b^2*g^
3*((4*c*d*x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d
*x + c)/(d^3*i^3)) - 3/2*(2*d*x + c)*A*a^2*b*g^3/(d^4*i^3*x^2 + 2*c*d^3*i^
3*x + c^2*d^2*i^3) - 1/2*A*a^3*g^3/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^
3) + 1/2*(6*a^3*b^2*d^3*g^3*log(e) - (6*g^3*log(e) + 7*g^3)*b^5*c^3 + (18*
g^3*log(e) + 19*g^3)*a*b^4*c^2*d - 2*(9*g^3*log(e) + 7*g^3)*a^2*b^3*c*d^2)
*B*log(d*x + c)/(b^2*c^2*d^4*i^3 - 2*a*b*c*d^5*i^3 + a^2*d^6*i^3) + 1/4*(4
*(b^5*c^2*d^3*g^3*log(e) - 2*a*b^4*c*d^4*g^3*log(e) + a^2*b^3*d^5*g^3*log(
e))*B*x^3 + 8*(b^5*c^3*d^2*g^3*log(e) - 2*a*b^4*c^2*d^3*g^3*log(e) + a^...
```

## 3.47.8 Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)/(d*i*x + c*i)
^3, x)
```

### 3.47.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(c\,i+d\,i\,x)^3}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3
,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3
, x)
```

# 3.48 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$



## 3.48.1 Optimal result

Integrand size = 40, antiderivative size = 251

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \frac{Bg^2(a+bx)^2}{4di^3(c+dx)^2}-\frac{Abg^2(a+bx)}{d^2i^3(c+dx)} +\frac{bBg^2(a+bx)}{d^2i^3(c+dx)}-\frac{bBg^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^3(c+dx)} -\frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^3(c+dx)^2} -\frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^3} -\frac{b^2Bg^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}$$

output
```
1/4*B*g^2*(b*x+a)^2/d/i^3/(d*x+c)^2-A*b*g^2*(b*x+a)/d^2/i^3/(d*x+c)+b*B*g^
2*(b*x+a)/d^2/i^3/(d*x+c)-b*B*g^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^2/i^3/(d
*x+c)-1/2*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^3/(d*x+c)^2-b^2*g^
2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3/i^3-b^2*B*g^2*p
olylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^3
```

### 3.48.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 245, normalized size of antiderivative = 0.98

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{g^2\left(\frac{B(bc-ad)^2}{(c+dx)^2} - \frac{6bB(bc-ad)}{c+dx} - 6b^2B\log(a+bx) - \frac{2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2} + \frac{8b(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx} + 6b^2\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*
i*x)^3,x]
```

output
```
(g^2*((B*(b*c - a*d)^2)/(c + d*x)^2 - (6*b*B*(b*c - a*d))/(c + d*x) - 6*b^
2*B*Log[a + b*x] - (2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
(c + d*x)^2 + (8*b*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c +
d*x) + 6*b^2*B*Log[c + d*x] + 4*b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*L
og[c + d*x] - 2*b^2*B*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x]
)*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(4*d^3*i^3)
```

### 3.48.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 217, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2962, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ci+dix)^3}\,dx$$
$$\Big\downarrow \; 2962$$
$$\frac{g^2 \int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^3}$$
$$\Big\downarrow \; 2793$$

$$\frac{g^2 \int \left(-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{d^2} - \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d(c+dx)}\right)d\frac{a+bx}{c+dx}}{i^3}$$

$$\Big\downarrow \ 2009$$

$$\frac{g^2\left(-\frac{b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3} - \frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2} - \frac{Ab(a+bx)}{d^2(c+dx)} - \frac{b^2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3} - \frac{bB(a+bx)\text{lo}}{d^2(c}\right.}{i^3}$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^3
,x]
```

output
```
(g^2*((B*(a + b*x)^2)/(4*d*(c + d*x)^2) - (A*b*(a + b*x))/(d^2*(c + d*x))
+ (b*B*(a + b*x))/(d^2*(c + d*x)) - (b*B*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)])/(d^2*(c + d*x)) - ((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(2*d*(c + d*x)^2) - (b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d
*(a + b*x))/(b*(c + d*x))])/d^3 - (b^2*B*PolyLog[2, (d*(a + b*x))/(b*(c +
d*x))])/d^3))/i^3
```

### 3.48.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

3.48. $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$

### 3.48.4 Maple [A] (verified)

Time = 1.56 (sec) , antiderivative size = 463, normalized size of antiderivative = 1.84

| method | result |
|---|---|
| parts | $\frac{g^2A\left(\frac{b^2\ln(dx+c)}{d^3}-\frac{2b(ad-cb)}{d^3(dx+c)}-\frac{a^2d^2-2abcd+b^2c^2}{2d^3(dx+c)^2}\right)}{i^3}-\frac{g^2Bd\left(\frac{(a^2d^2-2abcd+b^2c^2)\left(d\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.}{2}\right.\right.}{}\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{g^2d^2A\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)be+\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2d}{2}}{d^2}-\frac{b^2e^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^3}\right)}{(ad-cb)e^3i^3}-\frac{g^2d^2B\left(-\frac{\left(\frac{be}{d}+\frac{(ad}{d(d}\right.}{}\right.}{}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{g^2d^2A\left(-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)be+\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2d}{2}}{d^2}-\frac{b^2e^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{d^3}\right)}{(ad-cb)e^3i^3}-\frac{g^2d^2B\left(-\frac{\left(\frac{be}{d}+\frac{(ad}{d(d}\right.}{}\right.}{}\right.}{}$ |
| risch | Expression too large to display |

3.48. $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output
```
g^2*A/i^3*(b^2/d^3*ln(d*x+c)-2*b/d^3*(a*d-b*c)/(d*x+c)-1/2*(a^2*d^2-2*a*b*
c*d+b^2*c^2)/d^3/(d*x+c)^2)-g^2*B/i^3*d/(a*d-b*c)^2/e^2*((a^2*d^2-2*a*b*c*
d+b^2*c^2)/d^3*(d*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)+b*e*((b*e/d+(a*d-b*c)*e/
d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d))+e
^2*b^2*(a^2*d^2-2*a*b*c*d+b^2*c^2)/d^3*(dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+
c))*d-b*e)/b/e)/d+ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/
d/(d*x+c))*d-b*e)/b/e)/d))
```

### 3.48.5 Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output
```
integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B
*a*b*g^2*x + B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)))/(d^3*i^3*x^3 + 3*c*d
^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

### 3.48.6 Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$=\frac{g^2\left(\int \frac{Aa^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{Ab^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{Ba^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{2Aabx}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\right.}{i^3}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**3,x)
```

output
```
g**2*(Integral(A*a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
+ Integral(A*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
 + Integral(B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*a*b*x/(c**3 + 3*c**2*d*x
+ 3*c*d**2*x**2 + d**3*x**3), x) + Integral(B*b**2*x**2*log(a*e/(c + d*x)
+ b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + I
ntegral(2*B*a*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x
+ 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

### 3.48.7 Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output
```
-1/2*B*a*b*g^2*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^4*i^
3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d
^2)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3
*d^2 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 -
2*a*b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d
^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3)) + 1/4*B*a^2*g^2*((2*b*d*x + 3*b*c - a*d)
/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a
*c^2*d^2)*i^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c
*d^2*i^3*x + c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3
)) + 1/2*A*b^2*g^2*((4*c*d*x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d
^3*i^3) + 2*log(d*x + c)/(d^3*i^3)) - 1/2*B*b^2*g^2*(((d^2*x^2 + 2*c*d*x +
 c^2)*log(d*x + c)^2 + (4*c*d*x + 3*c^2)*log(d*x + c))/(d^5*i^3*x^2 + 2*c*
d^4*i^3*x + c^2*d^3*i^3) - 2*integrate(1/2*(2*d^2*x^2*log(b*x + a) + 2*d^2
*x^2*log(e) + 4*c*d*x + 3*c^2)/(d^5*i^3*x^3 + 3*c*d^4*i^3*x^2 + 3*c^2*d^3*
i^3*x + c^3*d^2*i^3), x)) - (2*d*x + c)*A*a*b*g^2/(d^4*i^3*x^2 + 2*c*d^3*i
^3*x + c^2*d^2*i^3) - 1/2*A*a^2*g^2/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i
^3)
```

### 3.48.8 Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)/(d*i*x + c*i)
^3, x)
```

### 3.48.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)}{(c\,i+d\,i\,x)^3}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3
,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3
, x)
```

# 3.49 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$



## 3.49.1 Optimal result

Integrand size = 38, antiderivative size = 85

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = -\frac{Bg(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)i^3(c+dx)^2}$$

output
```
-1/4*B*g*(b*x+a)^2/(-a*d+b*c)/i^3/(d*x+c)^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x
+a)/(d*x+c)))/(-a*d+b*c)/i^3/(d*x+c)^2
```

## 3.49.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 207 vs. 2(85) = 170.

Time = 0.10 (sec) , antiderivative size = 207, normalized size of antiderivative = 2.44

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = \frac{g\left(\frac{(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2(c+dx)^2} - \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2(c+dx)} + \frac{bB\left(\frac{1}{c+dx}+\frac{b\log(a+bx)}{bc-ad}-\frac{b\log(c+dx)}{bc-ad}\right)}{d^2} - \frac{B\left(\frac{bc-ad}{(c+dx)^2}+\frac{2b}{c+dx}+\frac{2b^2\log(a+bx)}{bc-ad} - \right.}{4d^2}\right.}{i^3}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*
x)^3,x]
```

output
```
(g*(((b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d^2*(c + d*x)^2)
 - (b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d^2*(c + d*x)) + (b*B*((c + d
*x)^(-1) + (b*Log[a + b*x])/(b*c - a*d) - (b*Log[c + d*x])/(b*c - a*d)))/d
^2 - (B*((b*c - a*d)/(c + d*x)^2 + (2*b)/(c + d*x) + (2*b^2*Log[a + b*x])/
(b*c - a*d) - (2*b^2*Log[c + d*x])/(b*c - a*d)))/(4*d^2)))/i^3
```

## 3.49.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 72, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {2962, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{(ci+dix)^3}dx$$

$$\Big\downarrow \text{ 2962}$$

$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}d\frac{a+bx}{c+dx}}{i^3(bc-ad)}$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{g\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2}-\frac{B(a+bx)^2}{4(c+dx)^2}\right)}{i^3(bc-ad)}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c*i + d*i*x)^3,x
]
```

output
```
(g*(-1/4*(B*(a + b*x)^2)/(c + d*x)^2 + ((a + b*x)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(2*(c + d*x)^2)))/((b*c - a*d)*i^3)
```

### 3.49.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.49.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 173 vs. $2(81) = 162$.

Time = 0.81 (sec) , antiderivative size = 174, normalized size of antiderivative = 2.05

| method | result |
|---|---|
| norman | $\frac{-\frac{2Aadg+2Abcg-Badg-Bbcg}{4i\,d^2}-\frac{(2Abg-Bbg)x}{2id}-\frac{B\,a^2g\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2i(ad-cb)}-\frac{B\,b^2g\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2(ad-cb)i}-\frac{Babgx\ln\left(\frac{e(bx+a)}{dx+c}\right)}{i(ad-cb)}}{i^2(dx+c)^2}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{g\,d^2A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^2e^3i^3}+\frac{g\,d^2B\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}\right)}{(ad-cb)^2e^3i^3}\right)}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(\frac{g\,d^2A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^2e^3i^3}+\frac{g\,d^2B\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}\right)}{(ad-cb)^2e^3i^3}\right)}{d^2}$ |
| parallelrisch | $-\frac{-B\,a^2b\,d^4g+B\,b^3c^2d^2g+4Axa\,b^2d^4g-4Ax\,b^3c\,d^3g-2Bxa\,b^2d^4g+2Bx\,b^3c\,d^3g+2B\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b\,d^4g+2B\,x^2\ln\left(\frac{e}{}\right.}{4i^3(dx+c)^2(ad-cb)b\,d^4}$ |
| risch | $-\frac{Bg(2bdx+ad+cb)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2d^2i^3(dx+c)^2}-\frac{g(-2B\ln(-dx-c)b^2d^2x^2+2B\ln(bx+a)b^2d^2x^2-4B\ln(-dx-c)b^2cdx+4B\ln(bx+}{}$ |
| parts | $\frac{gA\left(-\frac{b}{d^2(dx+c)}-\frac{ad-cb}{2d^2(dx+c)^2}\right)}{i^3}-\frac{gBd\left(a\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}\right)-\frac{cb\left(\left(\frac{be}{d}+\frac{(ad-c}{d(dx+}\right.\right.}{}\right.}{i^3(ad-cb)^2e^2}$ |

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x,method=_RETURN
VERBOSE)
```

output
```
(-1/4*(2*A*a*d*g+2*A*b*c*g-B*a*d*g-B*b*c*g)/i/d^2-1/2*(2*A*b*g-B*b*g)/i/d*
x-1/2*B*a^2*g/i/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))-1/2*B*b^2*g/(a*d-b*c)/i*x^
2*ln(e*(b*x+a)/(d*x+c))-B*a*b*g/i/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c)))/i^2/(
d*x+c)^2
```

## 3.49. $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$

### 3.49.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 185 vs. $2(81) = 162$.

Time = 0.28 (sec) , antiderivative size = 185, normalized size of antiderivative = 2.18

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx = -\frac{2\left((2A-B)b^2cd-(2A-B)abd^2\right)gx+\left((2A-B)b^2c^2-(2A-B)a^2d^2\right)g-2\left(Bb^2d^2gx^2+2Babd^2gx\right.}{4\left((bcd^4-ad^5)i^3x^2+2\,(bc^2d^3-acd^4)i^3x+(bc^3d^2-ac^2d^3)i^3\right)}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algori
thm="fricas")
```

output
```
-1/4*(2*((2*A - B)*b^2*c*d - (2*A - B)*a*b*d^2)*g*x + ((2*A - B)*b^2*c^2 -
 (2*A - B)*a^2*d^2)*g - 2*(B*b^2*d^2*g*x^2 + 2*B*a*b*d^2*g*x + B*a^2*d^2*g
)*log((b*e*x + a*e)/(d*x + c)))/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3
- a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3)
```

### 3.49.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(71) = 142$.

Time = 2.32 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.49

$$\begin{aligned}\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx &= \frac{Bb^2g\log\left(x+\frac{-\frac{Ba^2b^2d^2g}{ad-bc}+\frac{2Bab^3cdg}{ad-bc}+Bab^2dg-\frac{Bb^4c^2g}{ad-bc}+Bb^3cg}{2Bb^3dg}\right)}{2d^2i^3\,(ad-bc)} \\ &\quad - \frac{Bb^2g\log\left(x+\frac{\frac{Ba^2b^2d^2g}{ad-bc}-\frac{2Bab^3cdg}{ad-bc}+Bab^2dg+\frac{Bb^4c^2g}{ad-bc}+Bb^3cg}{2Bb^3dg}\right)}{2d^2i^3\,(ad-bc)} \\ &\quad + \frac{-2Aadg-2Abcg+Badg+Bbcg+x(-4Abdg+2Bbdg)}{4c^2d^2i^3+8cd^3i^3x+4d^4i^3x^2} \\ &\quad + \frac{(-Badg-Bbcg-2Bbdgx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2c^2d^2i^3+4cd^3i^3x+2d^4i^3x^2}\end{aligned}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**3,x)
```

output
```
B*b**2*g*log(x + (-B*a**2*b**2*d**2*g/(a*d - b*c) + 2*B*a*b**3*c*d*g/(a*d
- b*c) + B*a*b**2*d*g - B*b**4*c**2*g/(a*d - b*c) + B*b**3*c*g)/(2*B*b**3*
d*g))/(2*d**2*i**3*(a*d - b*c)) - B*b**2*g*log(x + (B*a**2*b**2*d**2*g/(a*
d - b*c) - 2*B*a*b**3*c*d*g/(a*d - b*c) + B*a*b**2*d*g + B*b**4*c**2*g/(a*
d - b*c) + B*b**3*c*g)/(2*B*b**3*d*g))/(2*d**2*i**3*(a*d - b*c)) + (-2*A*a
*d*g - 2*A*b*c*g + B*a*d*g + B*b*c*g + x*(-4*A*b*d*g + 2*B*b*d*g))/(4*c**2
*d**2*i**3 + 8*c*d**3*i**3*x + 4*d**4*i**3*x**2) + (-B*a*d*g - B*b*c*g - 2
*B*b*d*g*x)*log(e*(a + b*x)/(c + d*x))/(2*c**2*d**2*i**3 + 4*c*d**3*i**3*x
 + 2*d**4*i**3*x**2)
```

## 3.49.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(81) = 162$.

Time = 0.22 (sec) , antiderivative size = 567, normalized size of antiderivative = 6.67

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx =$$

$$-\frac{1}{4}\,Bbg\left(\frac{2\,(2\,dx+c)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^4i^3x^2+2\,cd^3i^3x+c^2d^2i^3}-\frac{bc^2-3\,acd+2\,(bcd-2\,ad^2)x}{(bcd^4-ad^5)i^3x^2+2\,(bc^2d^3-acd^4)i^3x+(bc^3d^2-ac^2d^3)i^3}-\right.$$

$$+\frac{1}{4}\,Bag\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}-\frac{2\,\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^3i^3x^2+2\,cd^2i^3x+c^2di^3}+\frac{}{(b^2c^2}\right.$$

$$-\frac{(2\,dx+c)Abg}{2\,(d^4i^3x^2+2\,cd^3i^3x+c^2d^2i^3)}-\frac{Aag}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algori
thm="maxima")
```

output
```
-1/4*B*b*g*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^4*i^3*x^
2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*
x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2
 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*
b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*i^3)) + 1/4*B*a*g*((2*b*d*x + 3*b*c - a*d)/((b*c*d
^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2
)*i^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3
*x + c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*
i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2
*(2*d*x + c)*A*b*g/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*A*a*g
/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
```

### 3.49.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 132, normalized size of antiderivative = 1.55

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)\left(\frac{2\,(bex+ae)^2Bg\log\left(\frac{bex+ae}{dx+c}\right)}{(dx+c)^2ei^3}+\frac{(bex+ae)^2(2\,Ag-I}{(dx+c)^2ei^3}\right.$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algori
thm="giac")
```

output
```
1/4*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d))
)*(2*(b*e*x + a*e)^2*B*g*log((b*e*x + a*e)/(d*x + c))/((d*x + c)^2*e*i^3)
+ (b*e*x + a*e)^2*(2*A*g - B*g)/((d*x + c)^2*e*i^3))
```

### 3.49.9 Mupad [B] (verification not implemented)

Time = 2.20 (sec) , antiderivative size = 198, normalized size of antiderivative = 2.33

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(ci+dix)^3}\,dx$$
$$= -\frac{x\,(2\,A\,b\,d\,g-B\,b\,d\,g)+A\,a\,d\,g+A\,b\,c\,g-\frac{B\,a\,d\,g}{2}-\frac{B\,b\,c\,g}{2}}{2\,c^2\,d^2\,i^3+4\,c\,d^3\,i^3\,x+2\,d^4\,i^3\,x^2}$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{B\,a\,g}{2\,d^2\,i^3}+\frac{B\,b\,c\,g}{2\,d^3\,i^3}+\frac{B\,b\,g\,x}{d^2\,i^3}\right)}{2\,c\,x+d\,x^2+\frac{c^2}{d}}+\frac{B\,b^2\,g\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,1\mathrm{i}}{d^2\,i^3\,(a\,d-b\,c)}$$

input
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x))))/(c*i + d*i*x)^3,x
)
```

output
```
(B*b^2*g*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*1i)/(d^2*i^3*(a*d - b*
c)) - (log((e*(a + b*x))/(c + d*x))*((B*a*g)/(2*d^2*i^3) + (B*b*c*g)/(2*d^
3*i^3) + (B*b*g*x)/(d^2*i^3)))/(2*c*x + d*x^2 + c^2/d) - (x*(2*A*b*d*g - B
*b*d*g) + A*a*d*g + A*b*c*g - (B*a*d*g)/2 - (B*b*c*g)/2)/(2*c^2*d^2*i^3 +
2*d^4*i^3*x^2 + 4*c*d^3*i^3*x)
```

# 3.50 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx$



## 3.50.1 Optimal result

Integrand size = 30, antiderivative size = 144

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx = \frac{B}{4di^3(c+dx)^2}+\frac{bB}{2d(bc-ad)i^3(c+dx)}+\frac{b^2B\log(a+bx)}{2d(bc-ad)^2i^3} - \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2di^3(c+dx)^2}-\frac{b^2B\log(c+dx)}{2d(bc-ad)^2i^3}$$

output
```
1/4*B/d/i^3/(d*x+c)^2+1/2*b*B/d/(-a*d+b*c)/i^3/(d*x+c)+1/2*b^2*B*ln(b*x+a)
/d/(-a*d+b*c)^2/i^3+1/2*(-A-B*ln(e*(b*x+a)/(d*x+c)))/d/i^3/(d*x+c)^2-1/2*b
^2*B*ln(d*x+c)/d/(-a*d+b*c)^2/i^3
```

## 3.50.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.77

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx = \frac{-2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+\frac{B\left((bc-ad)(3bc-ad+2bdx)+2b^2(c+dx)^2\log(a+bx)-2b^2(c+dx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4di^3(c+dx)^2}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x)^3,x]
```

output
```
(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + (B*((b*c - a*d)*(3*b*c - a*d +
2*b*d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x]
))/(b*c - a*d)^2)/(4*d*i^3*(c + d*x)^2)
```

### 3.50.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 138, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.133$, Rules used = {2948, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ci+dix)^3}dx$$

$$\Big\downarrow \text{2948}$$

$$\frac{B(bc-ad)\int \frac{1}{i^2(a+bx)(c+dx)^3}dx}{2di}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2di^3(c+dx)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{B(bc-ad)\int \frac{1}{(a+bx)(c+dx)^3}dx}{2di^3}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2di^3(c+dx)^2}$$

$$\Big\downarrow \text{54}$$

$$\frac{B(bc-ad)\int \left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{db^2}{(bc-ad)^3(c+dx)}-\frac{db}{(bc-ad)^2(c+dx)^2}-\frac{d}{(bc-ad)(c+dx)^3}\right)dx}{2di^3}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2di^3(c+dx)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{B(bc-ad)\left(\frac{b^2\log(a+bx)}{(bc-ad)^3}-\frac{b^2\log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}\right)}{2di^3}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2di^3(c+dx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/(c*i + d*i*x)^3,x]
```

output
```
-1/2*(A + B*Log[(e*(a + b*x))/(c + d*x)])/(d*i^3*(c + d*x)^2) + (B*(b*c -
a*d)*(1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b^2*L
og[a + b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3))/(2*d*i^3)
```

#### 3.50.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2948
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*(
(A + B*Log[e*((a + b*x)^n/(c + d*x)^n)])/(g*(m + 1))), x] - Simp[B*n*((b*c
- a*d)/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /
; FreeQ[{a, b, c, d, e, f, g, A, B, m, n}, x] && EqQ[n + mn, 0] && NeQ[b*c
- a*d, 0] && NeQ[m, -1] &&  !(EqQ[m, -2] && IntegerQ[n])
```

## 3.50.4 Maple [A] (verified)

Time = 0.77 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.53

| method | result |
|---|---|
| parts | $-\frac{A}{2i^3(dx+c)^2 d}-\frac{Bd\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}-\frac{be\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\right.}{d}\right.}{i^3(ad-cb)^2e^2}$ |
| norman | $\frac{\frac{B\,b^2cx\ln\left(\frac{e(bx+a)}{dx+c}\right)}{\left(a^2d^2-2abcd+b^2c^2\right)i}-\frac{2Aa\,d^2-2Abcd-Ba\,d^2+3Bbcd}{4i\,d^2(ad-cb)}-\frac{Bbx}{2i(ad-cb)}-\frac{Ba(ad-2cb)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2i\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{b^2Bd\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2\left(a^2d^2-2abcd+b^2c^2\right)i}}{i^2(dx+c)^2}$ |
| parallelrisch | $-\frac{-4Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3c\,d^4-4B\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^2c\,d^4-2B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3d^5+2Bxa\,b^2d^5-2Bx\,b^3c\,d^4+2B\ln\left(\frac{e(bx+c}{dx+c}\right.}{4i^3(dx+c)^2(a^2d^2-2abcd+b^2c^2)b\,d^4}$ |
| risch | $-\frac{B\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2d\,i^3(dx+c)^2}-\frac{2B\ln(dx+c)b^2d^2x^2-2B\ln(-bx-a)b^2d^2x^2+4B\ln(dx+c)b^2cdx-4B\ln(-bx-a)b^2cdx+2B\ln(dx-}{4(a^2d^2-2a}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{d^2Ab\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^3e^2i^3}+\frac{d^3A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^3e^3i^3}-\frac{d^2Bb\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{(ad-cb)^3e^2i^3}\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{d^2Ab\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^3e^2i^3}+\frac{d^3A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^3e^3i^3}-\frac{d^2Bb\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{(ad-cb)^3e^2i^3}\right.}{d^2}$ |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/2*A/i^3/(d*x+c)^2/d-B/i^3*d/(a*d-b*c)^2/e^2*(1/2*(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^2-b*e/d*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(
a*d-b*c)*e/d/(d*x+c)-b*e/d))
```

## 3.50.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.53

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx = -\frac{(2A-3B)b^2c^2-4(A-B)abcd+(2A-B)a^2d^2-2(Bb^2cd-Babd^2)x-2(Bb^2d^2x^2+2Bb^2cdx+}{4((b^2c^2d^3-2abcd^4+a^2d^5)i^3x^2+2(b^2c^3d^2-2abc^2d^3+a^2cd^4)i^3x+(b^2c^4d-2abc^3d^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algorithm="fricas"
)
```

output
```
-1/4*((2*A - 3*B)*b^2*c^2 - 4*(A - B)*a*b*c*d + (2*A - B)*a^2*d^2 - 2*(B*b
^2*c*d - B*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*b^2*c*d*x + 2*B*a*b*c*d - B
*a^2*d^2)*log((b*e*x + a*e)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*
d^5)*i^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*i^3*x + (b^2*c^
4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*i^3)
```

### 3.50.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(124) = 248$.

Time = 1.08 (sec) , antiderivative size = 422, normalized size of antiderivative = 2.93

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx = -\frac{Bb^2\log\left(x + \frac{-\frac{Ba^3b^2d^3}{(ad-bc)^2} + \frac{3Ba^2b^3cd^2}{(ad-bc)^2} - \frac{3Bab^4c^2d}{(ad-bc)^2} + Bab^2d + \frac{Bb^5c^3}{(ad-bc)^2} + Bb^3c}{2Bb^3d}\right)}{2di^3\,(ad-bc)^2} + \frac{Bb^2\log\left(x + \frac{\frac{Ba^3b^2d^3}{(ad-bc)^2} - \frac{3Ba^2b^3cd^2}{(ad-bc)^2} + \frac{3Bab^4c^2d}{(ad-bc)^2} + Bab^2d - \frac{Bb^5c^3}{(ad-bc)^2} + Bb^3c}{2Bb^3d}\right)}{2di^3\,(ad-bc)^2} - \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2c^2di^3 + 4cd^2i^3x + 2d^3i^3x^2} + \frac{-2Aad + 2Abc + Bad - 3Bbc - 2Bbdx}{4ac^2d^2i^3 - 4bc^3di^3 + x^2\cdot(4ad^4i^3 - 4bcd^3i^3) + x\,(8acd^3i^3 - 8bc^2d^2i^3)}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)**3,x)
```

output
```
-B*b**2*log(x + (-B*a**3*b**2*d**3/(a*d - b*c)**2 + 3*B*a**2*b**3*c*d**2/(
a*d - b*c)**2 - 3*B*a*b**4*c**2*d/(a*d - b*c)**2 + B*a*b**2*d + B*b**5*c**
3/(a*d - b*c)**2 + B*b**3*c)/(2*B*b**3*d))/(2*d*i**3*(a*d - b*c)**2) + B*b
**2*log(x + (B*a**3*b**2*d**3/(a*d - b*c)**2 - 3*B*a**2*b**3*c*d**2/(a*d -
 b*c)**2 + 3*B*a*b**4*c**2*d/(a*d - b*c)**2 + B*a*b**2*d - B*b**5*c**3/(a*
d - b*c)**2 + B*b**3*c)/(2*B*b**3*d))/(2*d*i**3*(a*d - b*c)**2) - B*log(e*
(a + b*x)/(c + d*x))/(2*c**2*d*i**3 + 4*c*d**2*i**3*x + 2*d**3*i**3*x**2)
+ (-2*A*a*d + 2*A*b*c + B*a*d - 3*B*b*c - 2*B*b*d*x)/(4*a*c**2*d**2*i**3 -
 4*b*c**3*d*i**3 + x**2*(4*a*d**4*i**3 - 4*b*c*d**3*i**3) + x*(8*a*c*d**3*
i**3 - 8*b*c**2*d**2*i**3))
```

### 3.50.7 Maxima [A] (verification not implemented)

Time = 0.21 (sec) , antiderivative size = 255, normalized size of antiderivative = 1.77

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}B\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}-\frac{2\,\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^3i^3x^2+2\,cd^2i^3x+c^2di^3}+\frac{2\,b^2\,l\ldots}{(b^2c^2d-2\ldots}\right.$$
$$-\frac{A}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algorithm="maxima"
)
```

output
```
1/4*B*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 -
 a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) - 2*log(b*e*x/(d*x + c) + a*e
/(d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 2*b^2*log(b*x + a)
/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*
d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*A/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^
2*d*i^3)
```

### 3.50.8 Giac [A] (verification not implemented)

Time = 0.39 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.64

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bex+ae)Bb}{(bci^3-adi^3)(dx+c)}-\frac{(bex+ae)^2Bd}{(bcei^3-adei^3)(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)-\frac{(bex+ae)^2(2\,Ad-Bd)}{(bcei^3-adei^3)(dx+c)^2}+\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(d*i*x+c*i)^3,x, algorithm="giac")
```

output
```
1/4*(2*(2*(b*e*x + a*e)*B*b/((b*c*i^3 - a*d*i^3)*(d*x + c)) - (b*e*x + a*e
)^2*B*d/((b*c*e*i^3 - a*d*e*i^3)*(d*x + c)^2))*log((b*e*x + a*e)/(d*x + c)
) - (b*e*x + a*e)^2*(2*A*d - B*d)/((b*c*e*i^3 - a*d*e*i^3)*(d*x + c)^2) +
4*(b*e*x + a*e)*(A*b - B*b)/((b*c*i^3 - a*d*i^3)*(d*x + c)))*(b*c/((b*c*e
- a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.50.9 Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.44

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ci+dix)^3}\,dx=\frac{B\,b^2\,\mathrm{atanh}\left(\frac{2\,a^2\,d^3\,i^3-2\,b^2\,c^2\,d\,i^3}{2\,d\,i^3\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{d\,i^3\,{\left(a\,d-b\,c\right)}^2}$$
$$-\frac{B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,d^2\,i^3\,\left(2\,c\,x+d\,x^2+\frac{c^2}{d}\right)}-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c-B\,a\,d+3\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{B\,b\,d\,x}{a\,d-b\,c}}{2\,c^2\,d\,i^3+4\,c\,d^2\,i^3\,x+2\,d^3\,i^3\,x^2}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/(c*i + d*i*x)^3,x)
```

output
```
(B*b^2*atanh((2*a^2*d^3*i^3 - 2*b^2*c^2*d*i^3)/(2*d*i^3*(a*d - b*c)^2) + (
2*b*d*x)/(a*d - b*c)))/(d*i^3*(a*d - b*c)^2) - (B*log((e*(a + b*x))/(c + d
*x)))/(2*d^2*i^3*(2*c*x + d*x^2 + c^2/d)) - ((2*A*a*d - 2*A*b*c - B*a*d +
3*B*b*c)/(2*(a*d - b*c)) + (B*b*d*x)/(a*d - b*c))/(2*c^2*d*i^3 + 2*d^3*i^3
*x^2 + 4*c*d^2*i^3*x)
```

# 3.51 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$



## 3.51.1 Optimal result

Integrand size = 40, antiderivative size = 243

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx = -\frac{B\left(4b-\frac{d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3gi^3} - \frac{b^2B\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3gi^3} + \frac{d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2} - \frac{2bd(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3gi^3(c+dx)} + \frac{b^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3gi^3}$$

output
```
-1/4*B*(4*b-d*(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g/i^3-1/2*b^2*B*ln((b*x+a)/(
d*x+c))^2/(-a*d+b*c)^3/g/i^3+1/2*d^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))
/(-a*d+b*c)^3/g/i^3/(d*x+c)^2-2*b*d*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-
a*d+b*c)^3/g/i^3/(d*x+c)+b^2*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)
))/(-a*d+b*c)^3/g/i^3
```

### 3.51.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.27 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.72

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4b(bc-ad)(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4b^2(c+dx)^2\log(a+b}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x
)^3),x]
```

output
```
(2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*b*(b*c - a*d)*(c
 + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*b^2*(c + d*x)^2*Log[a + b
*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*b^2*(c + d*x)^2*(A + B*Log[(e
*(a + b*x))/(c + d*x)])*Log[c + d*x] - 4*b*B*(c + d*x)*(b*c - a*d + b*(c +
 d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - B*((b*c - a*d)^2 + 2*b*(b
*c - a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*L
og[c + d*x]) - 2*b^2*B*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*
(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2
*b^2*B*(c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*L
og[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c - a*d)^3*g
*i^3*(c + d*x)^2)
```

### 3.51.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 182, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)(ci+dix)^3}\,dx$$

$$\Big\downarrow \ 2962$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}d\frac{a+bx}{c+dx}}{gi^3(bc-ad)^3}$$

$$\Big\downarrow \text{2772}$$

$$\frac{-B\int \left(\frac{b^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}-\frac{1}{2}d\left(4b-\frac{d(a+bx)}{c+dx}\right)\right)d\frac{a+bx}{c+dx}+b^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+\frac{d^2(a+bx)^2\left(B\log\left(\frac{e(a}{c-}\right.\right.}{2(c+dx)^2}}{gi^3(bc-ad)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+\frac{d^2(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2}-\frac{2bd(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{c+dx}-B\left(\frac{1}{2}b^2\log^2\left(\frac{a+}{c+}\right.\right.}{gi^3(bc-ad)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)*(c*i + d*i*x)^3),x
]
```

output
```
(-(B*((4*b - (d*(a + b*x))/(c + d*x))^2/4 + (b^2*Log[(a + b*x)/(c + d*x)]^
2)/2)) + (d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(c + d*
x)^2) - (2*b*d*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) +
 b^2*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c
- a*d)^3*g*i^3)
```

### 3.51.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.51.4 Maple [A] (verified)

Time = 1.22 (sec) , antiderivative size = 367, normalized size of antiderivative = 1.51

| method | result |
|---|---|
| parts | $\frac{A\left(-\frac{1}{2(ad-cb)(dx+c)^2}+\frac{b^2\ln(dx+c)}{(ad-cb)^3}+\frac{b}{(ad-cb)^2(dx+c)}-\frac{b^2\ln(bx+a)}{(ad-cb)^3}\right)}{g\,i^3}-\frac{Bd\left(\frac{d\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\left(\frac{b}{}\right.\right.}{ad-cb}\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{d^2Ab^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^4g}-\frac{2d^3Ab\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^3(ad-cb)^4g}+\frac{d^4A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e^3i^3(ad-cb)^4g}+\frac{d^2Bb^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e\,i^3(ad-cb)^4g}-\frac{2d^3}{}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(\frac{d^2Ab^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^4g}-\frac{2d^3Ab\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^3(ad-cb)^4g}+\frac{d^4A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e^3i^3(ad-cb)^4g}+\frac{d^2Bb^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e\,i^3(ad-cb)^4g}-\frac{2d^3}{}\right.}{}$ |
| parallelrisch | $-\frac{-4Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^3bc^4d^2-8Bx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^2b^2c^5d+12Axa^3bc^4d^2-8Axa^2b^2c^5d-10Bxa^3bc^4d^2+8Bxa^2b^2c^5d}{}$ |
| norman | $\frac{-\frac{2Aad^3-6Abcd^2-Bad^3+7Bbcd^2}{4gi(ad-cb)^2d^2}-\frac{\left(2Ab^2c^2+Ba^2d^2-4Babcd\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{2gi\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}+\frac{\left(2Abd^2-3Bbd^2\right)x}{2ig\left(a^2d^2-2abcd+b^2c^2\right)d}-\frac{Bb^2c^2\ln\left(\frac{e}{}\right.}{2gi\left(a^3d^3-3a^2bcd^2\right.}}{}$ |
| risch | $-\frac{A}{2g\,i^3(ad-cb)(dx+c)^2}+\frac{Ab^2\ln(dx+c)}{g\,i^3(ad-cb)^3}+\frac{Ab}{g\,i^3(ad-cb)^2(dx+c)}-\frac{Ab^2\ln(bx+a)}{g\,i^3(ad-cb)^3}+\frac{3B\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)b^2}{2g\,i^3(ad-cb)^3}+$ |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^3,x,method=_RETURN
VERBOSE)
```

output
```
A/g/i^3*(-1/2/(a*d-b*c)/(d*x+c)^2+b^2/(a*d-b*c)^3*ln(d*x+c)+b/(a*d-b*c)^2/
(d*x+c)-b^2/(a*d-b*c)^3*ln(b*x+a))-B/g/i^3*d/(a*d-b*c)^2/e^2*(d/(a*d-b*c)*
(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*b*e/(a*d-b*c)*((b*e/d+(a*d-b*c)*e/d/(d*x
+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+1/2/d/(a
*d-b*c)*e^2*b^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)
```

### 3.51.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 355, normalized size of antiderivative = 1.46

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{(6\,A-7\,B)b^2c^2-8\,(A-B)abcd+(2\,A-B)a^2d^2+2\,(Bb^2d^2x^2+2\,Bb^2cdx+Bb^2c^2)\log\left(\frac{bex+ae}{dx+c}\right)^2+2\,((}{4\,((b^3c^3d^2-3\,ab^2c^2d^3+3\,a^2bcd^4-a^3d^5)gi^3x^2+2\,(b^3c^4}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algori
thm="fricas")
```

output
```
1/4*((6*A - 7*B)*b^2*c^2 - 8*(A - B)*a*b*c*d + (2*A - B)*a^2*d^2 + 2*(B*b^
2*d^2*x^2 + 2*B*b^2*c*d*x + B*b^2*c^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*
((2*A - 3*B)*b^2*c*d - (2*A - 3*B)*a*b*d^2)*x + 2*((2*A - 3*B)*b^2*d^2*x^2
 + 2*A*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2 + 2*(2*(A - B)*b^2*c*d - B*a*b*d^
2)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^
2*b*c*d^4 - a^3*d^5)*g*i^3*x^2 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*
c^2*d^3 - a^3*c*d^4)*g*i^3*x + (b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2
- a^3*c^2*d^3)*g*i^3)
```

### 3.51.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(207) = 414$.

Time = 2.51 (sec) , antiderivative size = 889, normalized size of antiderivative = 3.66

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx = -\frac{Bb^2\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^3d^3gi^3 - 6a^2bcd^2gi^3 + 6ab^2c^2dgi^3 - 2b^3c^3gi^3} + \frac{b^2\cdot(2A-3B)\log\left(x + \frac{2Aab^2d+2Ab^3c-3Bab^2d-3Bb^3c-\frac{a^4b^2d^4\cdot(2A-3B)}{(ad-bc)^3}+\frac{4a^3b^3cd^3\cdot(2A-3B)}{(ad-bc)^3}-\frac{6a^2b^4c^2d^2\cdot(2A-3B)}{(ad-bc)^3}+\frac{4ab^5c^3d(2A-3}{(ad-bc)^3}}{4Ab^3d-6Bb^3d}\right)}{2gi^3\,(ad-bc)^3} - \frac{b^2\cdot(2A-3B)\log\left(x + \frac{2Aab^2d+2Ab^3c-3Bab^2d-3Bb^3c+\frac{a^4b^2d^4\cdot(2A-3B)}{(ad-bc)^3}-\frac{4a^3b^3cd^3\cdot(2A-3B)}{(ad-bc)^3}+\frac{6a^2b^4c^2d^2\cdot(2A-3B)}{(ad-bc)^3}-\frac{4ab^5c^3d(2A-3}{(ad-bc)^3}}{4Ab^3d-6Bb^3d}\right)}{2gi^3\,(ad-bc)^3} + \frac{(-Bad+3Bbc+2Bbdx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2c^2d^2gi^3+4a^2cd^3gi^3x+2a^2d^4gi^3x^2-4abc^3dgi^3-8abc^2d^2gi^3x-4abcd^3gi^3x^2+2b^2c^4gi^3+4b^2c^3dgi} + \frac{-2Aad+6Abc+Bad-7Bbc+x(4Abd-6Bbd)}{4a^2c^2d^2gi^3-8abc^3dgi^3+4b^2c^4gi^3+x^2\cdot(4a^2d^4gi^3-8abcd^3gi^3+4b^2c^2d^2gi^3)+x\,(8a^2cd^3gi^3-16abc^2d}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)**3,x)
```

output
```
-B*b**2*log(e*(a + b*x)/(c + d*x))**2/(2*a**3*d**3*g*i**3 - 6*a**2*b*c*d**
2*g*i**3 + 6*a*b**2*c**2*d*g*i**3 - 2*b**3*c**3*g*i**3) + b**2*(2*A - 3*B)
*log(x + (2*A*a*b**2*d + 2*A*b**3*c - 3*B*a*b**2*d - 3*B*b**3*c - a**4*b**
2*d**4*(2*A - 3*B)/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3*(2*A - 3*B)/(a*d -
b*c)**3 - 6*a**2*b**4*c**2*d**2*(2*A - 3*B)/(a*d - b*c)**3 + 4*a*b**5*c**3
*d*(2*A - 3*B)/(a*d - b*c)**3 - b**6*c**4*(2*A - 3*B)/(a*d - b*c)**3)/(4*A
*b**3*d - 6*B*b**3*d))/(2*g*i**3*(a*d - b*c)**3) - b**2*(2*A - 3*B)*log(x
+ (2*A*a*b**2*d + 2*A*b**3*c - 3*B*a*b**2*d - 3*B*b**3*c + a**4*b**2*d**4*
(2*A - 3*B)/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3*(2*A - 3*B)/(a*d - b*c)**3
 + 6*a**2*b**4*c**2*d**2*(2*A - 3*B)/(a*d - b*c)**3 - 4*a*b**5*c**3*d*(2*A
 - 3*B)/(a*d - b*c)**3 + b**6*c**4*(2*A - 3*B)/(a*d - b*c)**3)/(4*A*b**3*d
 - 6*B*b**3*d))/(2*g*i**3*(a*d - b*c)**3) + (-B*a*d + 3*B*b*c + 2*B*b*d*x)
*log(e*(a + b*x)/(c + d*x))/(2*a**2*c**2*d**2*g*i**3 + 4*a**2*c*d**3*g*i**
3*x + 2*a**2*d**4*g*i**3*x**2 - 4*a*b*c**3*d*g*i**3 - 8*a*b*c**2*d**2*g*i*
*3*x - 4*a*b*c*d**3*g*i**3*x**2 + 2*b**2*c**4*g*i**3 + 4*b**2*c**3*d*g*i**
3*x + 2*b**2*c**2*d**2*g*i**3*x**2) + (-2*A*a*d + 6*A*b*c + B*a*d - 7*B*b*
c + x*(4*A*b*d - 6*B*b*d))/(4*a**2*c**2*d**2*g*i**3 - 8*a*b*c**3*d*g*i**3
+ 4*b**2*c**4*g*i**3 + x**2*(4*a**2*d**4*g*i**3 - 8*a*b*c*d**3*g*i**3 + 4*
b**2*c**2*d**2*g*i**3) + x*(8*a**2*c*d**3*g*i**3 - 16*a*b*c**2*d**2*g*i**3
 + 8*b**2*c**3*d*g*i**3))
```

### 3.51.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 885 vs. $2(237) = 474$.

Time = 0.26 (sec) , antiderivative size = 885, normalized size of antiderivative = 3.64

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)(ci + dix)^3}\,dx$$

$$= \frac{1}{2}B\left(\frac{2\,bdx + 3\,bc - ad}{(b^2c^2d^2 - 2\,abcd^3 + a^2d^4)gi^3x^2 + 2\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3)gi^3x + (b^2c^4 - 2\,abc^3d + a^2c^2d^2)gi^3} + \right.$$

$$\left. + \frac{ae}{dx + c}\right)$$

$$+ \frac{1}{2}A\left(\frac{2\,bdx + 3\,bc - ad}{(b^2c^2d^2 - 2\,abcd^3 + a^2d^4)gi^3x^2 + 2\,(b^2c^3d - 2\,abc^2d^2 + a^2cd^3)gi^3x + (b^2c^4 - 2\,abc^3d + a^2c^2d^2)gi^3}\right.$$

$$- \frac{\left(7\,b^2c^2 - 8\,abcd + a^2d^2 + 2\,(b^2d^2x^2 + 2\,b^2cdx + b^2c^2)\log(bx + a)^2 + 2\,(b^2d^2x^2 + 2\,b^2cdx + b^2c^2)\log(dx\right.}{4\,(b^3c^5gi^3 - 3\,ab^2c^4dgi^3 + 3\,a^2bc^3d^2gi^3 - a^3c^2d^3gi^3 + (b}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algori
thm="maxima")
```

output
```
1/2*B*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*i^
3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2*a
*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) + 1/2*A*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^
2*d^4)*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^
2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^
3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3)) - 1/4*(7*b^2*c^2
- 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x +
a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d
 - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(
3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*log(b*x + a))*log(d*x + c))*B/(b^3*c^5*g*i^3 - 3*a*b^2*c^4*d*g*i^3
 + 3*a^2*b*c^3*d^2*g*i^3 - a^3*c^2*d^3*g*i^3 + (b^3*c^3*d^2*g*i^3 - 3*a*b^
2*c^2*d^3*g*i^3 + 3*a^2*b*c*d^4*g*i^3 - a^3*d^5*g*i^3)*x^2 + 2*(b^3*c^4*d*
g*i^3 - 3*a*b^2*c^3*d^2*g*i^3 + 3*a^2*b*c^2*d^3*g*i^3 - a^3*c*d^4*g*i^3)*x
)
```

### 3.51.8 Giac [A] (verification not implemented)

Time = 0.41 (sec) , antiderivative size = 442, normalized size of antiderivative = 1.82

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\,Bb^2e\log\left(\frac{bex+ae}{dx+c}\right)^2}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}+\frac{4\,Ab^2e\log\left(\frac{bex+ae}{dx+c}\right)}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}-2\left(\frac{4\,(bex+ae)Bbd}{(b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3)(c\right.\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algori
thm="giac")
```

output
```
1/4*(2*B*b^2*e*log((b*e*x + a*e)/(d*x + c))^2/(b^2*c^2*g*i^3 - 2*a*b*c*d*g
*i^3 + a^2*d^2*g*i^3) + 4*A*b^2*e*log((b*e*x + a*e)/(d*x + c))/(b^2*c^2*g*
i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3) - 2*(4*(b*e*x + a*e)*B*b*d/((b^2*c^
2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)) - (b*e*x + a*e)^2*B*
d^2/((b^2*c^2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a^2*d^2*e*g*i^3)*(d*x + c)^2))
*log((b*e*x + a*e)/(d*x + c)) + (2*A*d^2 - B*d^2)*(b*e*x + a*e)^2/((b^2*c^
2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a^2*d^2*e*g*i^3)*(d*x + c)^2) - 8*(A*b*d -
 B*b*d)*(b*e*x + a*e)/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(
d*x + c)))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
- a*d)))
```

### 3.51.9 Mupad [B] (verification not implemented)

Time = 3.61 (sec) , antiderivative size = 545, normalized size of antiderivative = 2.24

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx = \frac{3\,A\,b\,c}{2\,g\,i^3\,(a\,d-b\,c)^2\,(c+d\,x)^2}-\frac{A\,a\,d}{2\,g\,i^3\,(a\,d-b\,c)^2\,(c+d\,x)^2}-\frac{B\,b^2\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2}{2\,g\,i^3\,(a\,d-b\,c)^3}+\frac{B\,a\,d}{4\,g\,i^3\,(a\,d-b\,c)^2\,(c+d\,x)^2}-\frac{7\,B\,b\,c}{4\,g\,i^3\,(a\,d-b\,c)^2\,(c+d\,x)^2}-\frac{B\,a^2\,d^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g\,i^3\,(a\,d-b\,c)^3\,(c+d\,x)^2}-\frac{3\,B\,b^2\,c^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{2\,g\,i^3\,(a\,d-b\,c)^3\,(c+d\,x)^2}+\frac{A\,b\,d\,x}{g\,i^3\,(a\,d-b\,c)^2\,(c+d\,x)^2}-\frac{3\,B\,b\,d\,x}{2\,g\,i^3\,(a\,d-b\,c)^2\,(c+d\,x)^2}+\frac{B\,a\,b\,d^2\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g\,i^3\,(a\,d-b\,c)^3\,(c+d\,x)^2}-\frac{B\,b^2\,c\,d\,x\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g\,i^3\,(a\,d-b\,c)^3\,(c+d\,x)^2}+\frac{2\,B\,a\,b\,c\,d\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)}{g\,i^3\,(a\,d-b\,c)^3\,(c+d\,x)^2}+\frac{A\,b^2\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g\,i^3\,(a\,d-b\,c)^3}-\frac{B\,b^2\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,3\mathrm{i}}{g\,i^3\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)*(c*i + d*i*x)^3),x
)
```

output
```
(A*b^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g*i^3*(a*d - b*
c)^3) - (B*b^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*3i)/(g*i^3*(
a*d - b*c)^3) - (B*b^2*log((e*(a + b*x))/(c + d*x))^2)/(2*g*i^3*(a*d - b*c
)^3) - (A*a*d)/(2*g*i^3*(a*d - b*c)^2*(c + d*x)^2) + (3*A*b*c)/(2*g*i^3*(a
*d - b*c)^2*(c + d*x)^2) + (B*a*d)/(4*g*i^3*(a*d - b*c)^2*(c + d*x)^2) - (
7*B*b*c)/(4*g*i^3*(a*d - b*c)^2*(c + d*x)^2) - (B*a^2*d^2*log((e*(a + b*x)
)/(c + d*x)))/(2*g*i^3*(a*d - b*c)^3*(c + d*x)^2) - (3*B*b^2*c^2*log((e*(a
 + b*x))/(c + d*x)))/(2*g*i^3*(a*d - b*c)^3*(c + d*x)^2) + (A*b*d*x)/(g*i^
3*(a*d - b*c)^2*(c + d*x)^2) - (3*B*b*d*x)/(2*g*i^3*(a*d - b*c)^2*(c + d*x
)^2) + (B*a*b*d^2*x*log((e*(a + b*x))/(c + d*x)))/(g*i^3*(a*d - b*c)^3*(c
+ d*x)^2) - (B*b^2*c*d*x*log((e*(a + b*x))/(c + d*x)))/(g*i^3*(a*d - b*c)^
3*(c + d*x)^2) + (2*B*a*b*c*d*log((e*(a + b*x))/(c + d*x)))/(g*i^3*(a*d -
b*c)^3*(c + d*x)^2)
```

---

3.51. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)(ci+dix)^3}\,dx$

# 3.52 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$



## 3.52.1 Optimal result

Integrand size = 40, antiderivative size = 365

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx &= \frac{Bd^3(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2}-\frac{3bBd^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)} \\ &\quad-\frac{b^3B(c+dx)}{(bc-ad)^4g^2i^3(a+bx)}+\frac{3b^2Bd\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^2i^3} \\ &\quad-\frac{d^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2} \\ &\quad+\frac{3bd^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(c+dx)} \\ &\quad-\frac{b^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(a+bx)} \\ &\quad-\frac{3b^2d\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3}\end{aligned}$$

output
```
1/4*B*d^3*(b*x+a)^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-3*b*B*d^2*(b*x+a)/(-a*d
+b*c)^4/g^2/i^3/(d*x+c)-b^3*B*(d*x+c)/(-a*d+b*c)^4/g^2/i^3/(b*x+a)+3/2*b^2
*B*d*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^2/i^3-1/2*d^3*(b*x+a)^2*(A+B*ln(
e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2+3*b*d^2*(b*x+a)*(A+B*ln
(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)-b^3*(d*x+c)*(A+B*ln(e*(b
*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-3*b^2*d*ln((b*x+a)/(d*x+c))*(
A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3
```

## 3.52.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 452, normalized size of antiderivative = 1.24

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$$

$$= \frac{-\frac{4b^3Bc}{a+bx}+\frac{4ab^2Bd}{a+bx}+\frac{Bd(bc-ad)^2}{(c+dx)^2}+\frac{8b^2Bcd}{c+dx}-\frac{8abBd^2}{c+dx}+\frac{2bBd(bc-ad)}{c+dx}+6b^2Bd\log(a+bx)-\frac{4b^2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{a+bx}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i
*x)^3),x]
```

output
```
((-4*b^3*B*c)/(a + b*x) + (4*a*b^2*B*d)/(a + b*x) + (B*d*(b*c - a*d)^2)/(c
 + d*x)^2 + (8*b^2*B*c*d)/(c + d*x) - (8*a*b*B*d^2)/(c + d*x) + (2*b*B*d*(
b*c - a*d))/(c + d*x) + 6*b^2*B*d*Log[a + b*x] - (4*b^2*(b*c - a*d)*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - (2*d*(b*c - a*d)^2*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(c + d*x)^2 - (8*b*d*(b*c - a*d)*(A + B*Log[(e*
(a + b*x))/(c + d*x)]))/(c + d*x) - 12*b^2*d*Log[a + b*x]*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]) - 6*b^2*B*d*Log[c + d*x] + 12*b^2*d*(A + B*Log[(e*(a
+ b*x))/(c + d*x)])*Log[c + d*x] + 6*b^2*B*d*(Log[a + b*x]*(Log[a + b*x] -
 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)]) - 6*b^2*B*d*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*L
og[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4*(b*c - a*d)^4*g
^2*i^3)
```

### 3.52.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 253, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.075$, Rules used = {2962, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^2(ci+dix)^3}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i^3(bc-ad)^4}$$

$$\Big\downarrow \text{2772}$$

$$\frac{-B\int\left(-\frac{(c+dx)^2b^3}{(a+bx)^2}-\frac{3d(c+dx)\log\left(\frac{a+bx}{c+dx}\right)b^2}{a+bx}+3d^2b-\frac{d^3(a+bx)}{2(c+dx)}\right)d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-3b^2d\log\left(\frac{a+bx}{c+dx}\right)}{g^2i^3(bc-ad)^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-3b^2d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-\frac{d^3(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2}+\frac{3bd^2(a+bx)\left(B\right.}{c}}{g^2i^3(bc-ad)^4}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^2*(c*i + d*i*x)^3)
,x]
```

output
```
(-(B*(-1/4*(d^3*(a + b*x)^2)/(c + d*x)^2 + (3*b*d^2*(a + b*x))/(c + d*x) +
 (b^3*(c + d*x))/(a + b*x) - (3*b^2*d*Log[(a + b*x)/(c + d*x)]^2)/2)) - (d
^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(c + d*x)^2) + (3*
b*d^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) - (b^3*(c
+ d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - 3*b^2*d*Log[(a +
b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^4*g^2*i
^3)
```

### 3.52.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.52.4 Maple [A] (verified)

Time = 2.83 (sec) , antiderivative size = 506, normalized size of antiderivative = 1.39

| method | result |
|---|---|
| parts | $\frac{A\left(-\frac{d}{2(ad-cb)^2(dx+c)^2}+\frac{3d\,b^2\ln(dx+c)}{(ad-cb)^4}+\frac{2db}{(ad-cb)^3(dx+c)}+\frac{b^2}{(ad-cb)^3(bx+a)}-\frac{3d\,b^2\ln(bx+a)}{(ad-cb)^4}\right)}{g^2 i^3} - \frac{Bd\left(d^2\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.}{}\right.\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{d^2 A\,b^3}{i^3(ad-cb)^5 g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{3d^3 A\,b^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^5 g^2}-\frac{3d^4 Ab\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2 i^3(ad-cb)^5 g^2}+\frac{d^5 A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e^3 i^3(ad-cb)^5 g^2}-\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(\frac{d^2 A\,b^3}{i^3(ad-cb)^5 g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{3d^3 A\,b^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^5 g^2}-\frac{3d^4 Ab\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2 i^3(ad-cb)^5 g^2}+\frac{d^5 A\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e^3 i^3(ad-cb)^5 g^2}-\right.}{}$ |
| risch | $-\frac{Ad}{2g^2 i^3(ad-cb)^2(dx+c)^2}+\frac{3Ad\,b^2\ln(dx+c)}{g^2 i^3(ad-cb)^4}+\frac{2Adb}{g^2 i^3(ad-cb)^3(dx+c)}+\frac{A\,b^2}{g^2 i^3(ad-cb)^3(bx+a)}-\frac{3Ad\,b^2\ln(bx+a)}{g^2 i^3(ad-cb)^4}$ |
| parallelrisch | $-\frac{6B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2 a\,b^5 d^7+12B\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2 b^6 c\,d^6+12A\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^5 d^7+24A\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^6 c\,d^6-18}{}$ |
| norman | Expression too large to display |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output
```
A/g^2/i^3*(-1/2*d/(a*d-b*c)^2/(d*x+c)^2+3*d/(a*d-b*c)^4*b^2*ln(d*x+c)+2*d/
(a*d-b*c)^3*b/(d*x+c)+b^2/(a*d-b*c)^3/(b*x+a)-3*d/(a*d-b*c)^4*b^2*ln(b*x+a
))-B/g^2/i^3*d/(a*d-b*c)^2/e^2*(d^2/(a*d-b*c)^2*(1/2*(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^2)-3*d/(a*d-b*c)^2*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c
)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+3/2/(a*d-b*c)^2*b^2*e^2*ln(b*e
/d+(a*d-b*c)*e/d/(d*x+c))^2-1/d/(a*d-b*c)^2*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)))
```

3.52. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$

### 3.52.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.84

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = -\frac{4(A+B)b^3c^3+3(2A-5B)ab^2c^2d-12(A-B)a^2bcd^2+(2A-B)a^3d^3+6((2A-B)b^3cd^2-(2A}{4((b^5c^4d^2-4a}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output
```
-1/4*(4*(A + B)*b^3*c^3 + 3*(2*A - 5*B)*a*b^2*c^2*d - 12*(A - B)*a^2*b*c*d
^2 + (2*A - B)*a^3*d^3 + 6*((2*A - B)*b^3*c*d^2 - (2*A - B)*a*b^2*d^3)*x^2
 + 6*(B*b^3*d^3*x^3 + B*a*b^2*c^2*d + (2*B*b^3*c*d^2 + B*a*b^2*d^3)*x^2 +
(B*b^3*c^2*d + 2*B*a*b^2*c*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 + 3*((6*
A - B)*b^3*c^2*d - 2*(2*A + B)*a*b^2*c*d^2 - (2*A - 3*B)*a^2*b*d^3)*x + 2*
(3*(2*A - B)*b^3*d^3*x^3 + 2*B*b^3*c^3 + 6*A*a*b^2*c^2*d - 6*B*a^2*b*c*d^2
 + B*a^3*d^3 + 3*(4*A*b^3*c*d^2 + (2*A - 3*B)*a*b^2*d^3)*x^2 + 3*(2*(A + B
)*b^3*c^2*d + 4*(A - B)*a*b^2*c*d^2 - B*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d
*x + c)))/((b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*
c*d^5 + a^4*b*d^6)*g^2*i^3*x^3 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^
3*c^3*d^3 - 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*g^2*i^3*x^2 + (b^
5*c^6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^
2*d^4 + 2*a^5*c*d^5)*g^2*i^3*x + (a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*
c^4*d^2 - 4*a^4*b*c^3*d^3 + a^5*c^2*d^4)*g^2*i^3)
```

### 3.52.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**2/(d*i*x+c*i)**3,x)
```

output
```
Timed out
```

### 3.52.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1721 vs. $2(359) = 718$.

Time = 0.33 (sec) , antiderivative size = 1721, normalized size of antiderivative = 4.72

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output
```
-1/2*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d +
a*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)
*g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*
c*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2
+ 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d
+ 3*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4
 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) -
 6*b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*g^2*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/2*
A*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b*d
^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*g^2*
i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d^4
 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5*a
^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3*a
^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4 - 4*
a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) - 6*b^
2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c
*d^3 + a^4*d^4)*g^2*i^3)) - 1/4*(4*b^3*c^3 - 15*a*b^2*c^2*d + 12*a^2*b*c*d
^2 - a^3*d^3 - 6*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*
d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(...
```

### 3.52.8 Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{(bgx+ag)^2(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)/((b*g*x + a*g)^2*(d*i*x + c*i
)^3), x)
```

### 3.52.9 Mupad [B] (verification not implemented)

Time = 5.71 (sec) , antiderivative size = 983, normalized size of antiderivative = 2.69

$$
\begin{aligned}
\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = {} & \frac{A\,b^2\,c^2}{g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& - \frac{A\,a^2\,d^2}{2\,g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& - \frac{3\,B\,b^2\,d\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)^2}{2\,g^2\,i^3\,\left(a\,d-b\,c\right)^4} \\
& + \frac{B\,a^2\,d^2}{4\,g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& + \frac{B\,b^2\,c^2}{g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& - \frac{B\,a\,d\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{2\,g^2\,i^3\,\left(a\,d-b\,c\right)^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& - \frac{B\,b\,c\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{g^2\,i^3\,\left(a\,d-b\,c\right)^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& + \frac{3\,A\,b^2\,d^2\,x^2}{g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& - \frac{3\,B\,b^2\,d^2\,x^2}{2\,g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& + \frac{5\,A\,a\,b\,c\,d}{2\,g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& - \frac{11\,B\,a\,b\,c\,d}{4\,g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& - \frac{3\,B\,b\,d\,x\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{2\,g^2\,i^3\,\left(a\,d-b\,c\right)^2\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& + \frac{3\,B\,b^2\,d^2\,x^2\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& + \frac{3\,A\,a\,b\,d^2\,x}{2\,g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& - \frac{9\,B\,a\,b\,d^2\,x}{4\,g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& + \frac{9\,A\,b^2\,c\,d\,x}{2\,g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& - \frac{3\,B\,b^2\,c\,d\,x}{4\,g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2} \\
& + \frac{3\,B\,a\,b\,c\,d\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)}{g^2\,i^3\,\left(a\,d-b\,c\right)^3\,\left(a+b\,x\right)\,\left(c+d\,x\right)^2}
\end{aligned}
$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^2*(c*i + d*i*x)^3)
,x)
```

output
```
(A*b^2*d*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*6i)/(g^2*i^3*(a*d
- b*c)^4) - (3*B*b^2*d*log((e*(a + b*x))/(c + d*x))^2)/(2*g^2*i^3*(a*d - b
*c)^4) - (B*b^2*d*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*3i)/(g^2*
i^3*(a*d - b*c)^4) - (A*a^2*d^2)/(2*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d
*x)^2) + (A*b^2*c^2)/(g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (B*a^
2*d^2)/(4*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (B*b^2*c^2)/(g^2*
i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) - (B*a*d*log((e*(a + b*x))/(c + d
*x)))/(2*g^2*i^3*(a*d - b*c)^2*(a + b*x)*(c + d*x)^2) - (B*b*c*log((e*(a +
 b*x))/(c + d*x)))/(g^2*i^3*(a*d - b*c)^2*(a + b*x)*(c + d*x)^2) + (3*A*b^
2*d^2*x^2)/(g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) - (3*B*b^2*d^2*x^
2)/(2*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (5*A*a*b*c*d)/(2*g^2*
i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) - (11*B*a*b*c*d)/(4*g^2*i^3*(a*d
- b*c)^3*(a + b*x)*(c + d*x)^2) - (3*B*b*d*x*log((e*(a + b*x))/(c + d*x)))
/(2*g^2*i^3*(a*d - b*c)^2*(a + b*x)*(c + d*x)^2) + (3*B*b^2*d^2*x^2*log((e
*(a + b*x))/(c + d*x)))/(g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (3
*A*a*b*d^2*x)/(2*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) - (9*B*a*b*d
^2*x)/(4*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (9*A*b^2*c*d*x)/(2
*g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) - (3*B*b^2*c*d*x)/(4*g^2*i^3
*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (3*B*a*b*c*d*log((e*(a + b*x))/(c
+ d*x)))/(g^2*i^3*(a*d - b*c)^3*(a + b*x)*(c + d*x)^2) + (3*B*a*b*d^2*x...
```

3.52. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$

# 3.53 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx$



## 3.53.1 Optimal result

Integrand size = 40, antiderivative size = 463

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = -\frac{Bd^4(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2}+\frac{4bBd^3(a+bx)}{(bc-ad)^5g^3i^3(c+dx)}+\frac{4b^3Bd(c+dx)}{(bc-ad)^5g^3i^3(a+bx)}-\frac{b^4B(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2}-\frac{3b^2Bd^2\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}+\frac{d^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2}-\frac{4bd^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(c+dx)}+\frac{4b^3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(a+bx)}-\frac{b^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2}+\frac{6b^2d^2\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3}$$

output
```
-1/4*B*d^4*(b*x+a)^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+4*b*B*d^3*(b*x+a)/(-a*
d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*B*d*(d*x+c)/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/
4*b^4*B*(d*x+c)^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2-3*b^2*B*d^2*ln((b*x+a)/(d
*x+c))^2/(-a*d+b*c)^5/g^3/i^3+1/2*d^4*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c))
)/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2-4*b*d^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)
))/(-a*d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c))
)/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*b^4*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)
))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+6*b^2*d^2*ln((b*x+a)/(d*x+c))*(A+B*ln(e*
(b*x+a)/(d*x+c)))/(-a*d+b*c)^5/g^3/i^3
```

### 3.53.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.63 (sec) , antiderivative size = 533, normalized size of antiderivative = 1.15

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx =$$

$$-\frac{\frac{b^2B(bc-ad)^2}{(a+bx)^2}-\frac{12b^3Bcd}{a+bx}+\frac{12ab^2Bd^2}{a+bx}-\frac{2b^2Bd(bc-ad)}{a+bx}+\frac{Bd^2(bc-ad)^2}{(c+dx)^2}+\frac{12b^2Bcd^2}{c+dx}-\frac{12abBd^3}{c+dx}+\frac{2bBd^2(bc-ad)}{c+dx}+\frac{2b^2(bc-ac}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i
*x)^3),x]
```

output
```
-1/4*((b^2*B*(b*c - a*d)^2)/(a + b*x)^2 - (12*b^3*B*c*d)/(a + b*x) + (12*a
*b^2*B*d^2)/(a + b*x) - (2*b^2*B*d*(b*c - a*d))/(a + b*x) + (B*d^2*(b*c -
a*d)^2)/(c + d*x)^2 + (12*b^2*B*c*d^2)/(c + d*x) - (12*a*b*B*d^3)/(c + d*x
) + (2*b*B*d^2*(b*c - a*d))/(c + d*x) + (2*b^2*(b*c - a*d)^2*(A + B*Log[(e
*(a + b*x))/(c + d*x)]))/(a + b*x)^2 - (12*b^2*d*(b*c - a*d)*(A + B*Log[(e
*(a + b*x))/(c + d*x)]))/(a + b*x) - (2*d^2*(b*c - a*d)^2*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(c + d*x)^2 - (12*b*d^2*(b*c - a*d)*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(c + d*x) - 24*b^2*d^2*Log[a + b*x]*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]) + 24*b^2*d^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log
[c + d*x] + 12*b^2*B*d^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))
/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 12*b^2*B*d^
2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*P
olyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^5*g^3*i^3)
```

### 3.53.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 319, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2962, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^3(ci+dix)^3}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i^3(bc-ad)^5}$$

$$\Big\downarrow \text{2772}$$

$$-B\int\left(\frac{6b^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)d^2}{a+bx}+\frac{1}{2}\left(-\frac{(c+dx)^3b^4}{(a+bx)^3}+\frac{8d(c+dx)^2b^3}{(a+bx)^2}-8d^3b+\frac{d^4(a+bx)}{c+dx}\right)\right)d\frac{a+bx}{c+dx}-\frac{b^4(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^4(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{4b^3d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}+6b^2d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+\frac{d^4(a+bx)^2(}{}}{g^3i^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^3*(c*i + d*i*x)^3)
,x]
```

output
```
(-(B*((d^4*(a + b*x)^2)/(4*(c + d*x)^2) - (4*b*d^3*(a + b*x))/(c + d*x) -
(4*b^3*d*(c + d*x))/(a + b*x) + (b^4*(c + d*x)^2)/(4*(a + b*x)^2) + 3*b^2*
d^2*Log[(a + b*x)/(c + d*x)]^2)) + (d^4*(a + b*x)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(2*(c + d*x)^2) - (4*b*d^3*(a + b*x)*(A + B*Log[(e*(a + b*
x))/(c + d*x)]))/(c + d*x) + (4*b^3*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(
c + d*x)]))/(a + b*x) - (b^4*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(2*(a + b*x)^2) + 6*b^2*d^2*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/((b*c - a*d)^5*g^3*i^3)
```

### 3.53.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.53.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 649, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| parts | $\frac{A\left(-\frac{d^2}{2(ad-cb)^3(dx+c)^2}+\frac{6d^2b^2\ln(dx+c)}{(ad-cb)^5}+\frac{3d^2b}{(ad-cb)^4(dx+c)}+\frac{b^2}{2(ad-cb)^3(bx+a)^2}-\frac{6d^2b^2\ln(bx+a)}{(ad-cb)^5}+\frac{3b^2d}{(ad-cb)^4(bx+a)}\right)}{g^3i^3} - \frac{Bd}{\ldots}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{d^2eAb^4}{2i^3(ad-cb)^6g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{4d^3Ab^3}{i^3(ad-cb)^6g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{6d^4Ab^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^6g^3}-\frac{4d^5Ab\left(\frac{be}{d}+\ldots\right.}{e^2i^3(ad-\ldots}\right.}{\ldots}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{d^2eAb^4}{2i^3(ad-cb)^6g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{4d^3Ab^3}{i^3(ad-cb)^6g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{6d^4Ab^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^6g^3}-\frac{4d^5Ab\left(\frac{be}{d}+\ldots\right.}{e^2i^3(ad-\ldots}\right.}{\ldots}$ |
| risch | Expression too large to display |
| parallelrisch | Expression too large to display |
| norman | Expression too large to display |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output
```
A/g^3/i^3*(-1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*d^2/(a*d-b*c)^5*b^2*ln(d*x+c)+
3*d^2/(a*d-b*c)^4*b/(d*x+c)+1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*d^2/(a*d-b*c)^
5*b^2*ln(b*x+a)+3*b^2/(a*d-b*c)^4*d/(b*x+a))-B/g^3/i^3*d/(a*d-b*c)^2/e^2*(
d^3/(a*d-b*c)^3*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-4*d^2/(a*d-b*c)^3*b*e*((b*
e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(
d*x+c)-b*e/d)+3*d/(a*d-b*c)^3*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-4/
(a*d-b*c)^3*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+1/d/(a*d-b*c)^3*b^4*e^4*(-1/2
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/
d+(a*d-b*c)*e/d/(d*x+c))^2))
```

3.53. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx$

### 3.53.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1011 vs. $2(455) = 910$.

Time = 0.33 (sec) , antiderivative size = 1011, normalized size of antiderivative = 2.18

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag + bgx)^3(ci + dix)^3}\,dx = -\frac{(2\,A + B)b^4c^4 - 16\,(A + B)ab^3c^3d + 30\,Ba^2b^2c^2d^2 + 16\,(A - B)a^3bcd^3 - (2\,A - B)a^4d^4 - 24\,(Ab^4cd^3}{}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output
```
-1/4*((2*A + B)*b^4*c^4 - 16*(A + B)*a*b^3*c^3*d + 30*B*a^2*b^2*c^2*d^2 +
16*(A - B)*a^3*b*c*d^3 - (2*A - B)*a^4*d^4 - 24*(A*b^4*c*d^3 - A*a*b^3*d^4
)*x^3 - 12*((3*A + B)*b^4*c^2*d^2 - 2*B*a*b^3*c*d^3 - (3*A - B)*a^2*b^2*d^
4)*x^2 - 12*(B*b^4*d^4*x^4 + B*a^2*b^2*c^2*d^2 + 2*(B*b^4*c*d^3 + B*a*b^3*
d^4)*x^3 + (B*b^4*c^2*d^2 + 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*x^2 + 2*(B*a*
b^3*c^2*d^2 + B*a^2*b^2*c*d^3)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 4*((2*A
 + 3*B)*b^4*c^3*d + 3*(4*A - B)*a*b^3*c^2*d^2 - 3*(4*A + B)*a^2*b^2*c*d^3
- (2*A - 3*B)*a^3*b*d^4)*x - 2*(12*A*b^4*d^4*x^4 - B*b^4*c^4 + 8*B*a*b^3*c
^3*d + 12*A*a^2*b^2*c^2*d^2 - 8*B*a^3*b*c*d^3 + B*a^4*d^4 + 12*((2*A + B)*
b^4*c*d^3 + (2*A - B)*a*b^3*d^4)*x^3 + 6*((2*A + 3*B)*b^4*c^2*d^2 + 8*A*a*
b^3*c*d^3 + (2*A - 3*B)*a^2*b^2*d^4)*x^2 + 4*(B*b^4*c^3*d + 6*(A + B)*a*b^
3*c^2*d^2 + 6*(A - B)*a^2*b^2*c*d^3 - B*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d
*x + c)))/((b^7*c^5*d^2 - 5*a*b^6*c^4*d^3 + 10*a^2*b^5*c^3*d^4 - 10*a^3*b^
4*c^2*d^5 + 5*a^4*b^3*c*d^6 - a^5*b^2*d^7)*g^3*i^3*x^4 + 2*(b^7*c^6*d - 4*
a*b^6*c^5*d^2 + 5*a^2*b^5*c^4*d^3 - 5*a^4*b^3*c^2*d^5 + 4*a^5*b^2*c*d^6 -
a^6*b*d^7)*g^3*i^3*x^3 + (b^7*c^7 - a*b^6*c^6*d - 9*a^2*b^5*c^5*d^2 + 25*a
^3*b^4*c^4*d^3 - 25*a^4*b^3*c^3*d^4 + 9*a^5*b^2*c^2*d^5 + a^6*b*c*d^6 - a^
7*d^7)*g^3*i^3*x^2 + 2*(a*b^6*c^7 - 4*a^2*b^5*c^6*d + 5*a^3*b^4*c^5*d^2 -
5*a^5*b^2*c^3*d^4 + 4*a^6*b*c^2*d^5 - a^7*c*d^6)*g^3*i^3*x + (a^2*b^5*c^7
- 5*a^3*b^4*c^6*d + 10*a^4*b^3*c^5*d^2 - 10*a^5*b^2*c^4*d^3 + 5*a^6*b*c...
```

### 3.53.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2106 vs. $2(430) = 860$.

Time = 138.12 (sec) , antiderivative size = 2106, normalized size of antiderivative = 4.55

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**3/(d*i*x+c*i)**3,x)
```

output
```
6*A*b**2*d**2*log(x + (-6*A*a**6*b**2*d**8/(a*d - b*c)**5 + 36*A*a**5*b**3
*c*d**7/(a*d - b*c)**5 - 90*A*a**4*b**4*c**2*d**6/(a*d - b*c)**5 + 120*A*a
**3*b**5*c**3*d**5/(a*d - b*c)**5 - 90*A*a**2*b**6*c**4*d**4/(a*d - b*c)**
5 + 36*A*a*b**7*c**5*d**3/(a*d - b*c)**5 + 6*A*a*b**2*d**3 - 6*A*b**8*c**6
*d**2/(a*d - b*c)**5 + 6*A*b**3*c*d**2)/(12*A*b**3*d**3))/(g**3*i**3*(a*d
- b*c)**5) - 6*A*b**2*d**2*log(x + (6*A*a**6*b**2*d**8/(a*d - b*c)**5 - 36
*A*a**5*b**3*c*d**7/(a*d - b*c)**5 + 90*A*a**4*b**4*c**2*d**6/(a*d - b*c)*
*5 - 120*A*a**3*b**5*c**3*d**5/(a*d - b*c)**5 + 90*A*a**2*b**6*c**4*d**4/(
a*d - b*c)**5 - 36*A*a*b**7*c**5*d**3/(a*d - b*c)**5 + 6*A*a*b**2*d**3 + 6
*A*b**8*c**6*d**2/(a*d - b*c)**5 + 6*A*b**3*c*d**2)/(12*A*b**3*d**3))/(g**
3*i**3*(a*d - b*c)**5) - 3*B*b**2*d**2*log(e*(a + b*x)/(c + d*x))**2/(a**5
*d**5*g**3*i**3 - 5*a**4*b*c*d**4*g**3*i**3 + 10*a**3*b**2*c**2*d**3*g**3*
i**3 - 10*a**2*b**3*c**3*d**2*g**3*i**3 + 5*a*b**4*c**4*d*g**3*i**3 - b**5
*c**5*g**3*i**3) + (-B*a**3*d**3 + 7*B*a**2*b*c*d**2 + 4*B*a**2*b*d**3*x +
 7*B*a*b**2*c**2*d + 28*B*a*b**2*c*d**2*x + 18*B*a*b**2*d**3*x**2 - B*b**3
*c**3 + 4*B*b**3*c**2*d*x + 18*B*b**3*c*d**2*x**2 + 12*B*b**3*d**3*x**3)*l
og(e*(a + b*x)/(c + d*x))/(2*a**6*c**2*d**4*g**3*i**3 + 4*a**6*c*d**5*g**3
*i**3*x + 2*a**6*d**6*g**3*i**3*x**2 - 8*a**5*b*c**3*d**3*g**3*i**3 - 12*a
**5*b*c**2*d**4*g**3*i**3*x + 4*a**5*b*d**6*g**3*i**3*x**3 + 12*a**4*b**2*
c**4*d**2*g**3*i**3 + 8*a**4*b**2*c**3*d**3*g**3*i**3*x - 18*a**4*b**2*...
```

### 3.53.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2380 vs. $2(455) = 910$.

Time = 0.35 (sec) , antiderivative size = 2380, normalized size of antiderivative = 5.14

$$\int \frac{A + B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output
```
1/2*B*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3
 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d
^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d
^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4
*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 +
(b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^
6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 +
2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6
- 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3
*i^3) + 12*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3
*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2
*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*
c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + 1/2*A*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2
 - a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2
 + a^2*b*d^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a
^3*b^3*c*d^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 +
 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*
i^3*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^
2*d^4 + a^6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b...
```

### 3.53.8 Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{(bgx+ag)^3(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)/((b*g*x + a*g)^3*(d*i*x + c*i
)^3), x)
```

### 3.53.9 Mupad [B] (verification not implemented)

Time = 9.10 (sec) , antiderivative size = 1443, normalized size of antiderivative = 3.12

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^3*(c*i + d*i*x)^3)
,x)
```

output
```
(A*b^2*d^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*12i)/(g^3*i^3*(a
*d - b*c)^5) - (3*B*b^2*d^2*log((e*(a + b*x))/(c + d*x))^2)/(g^3*i^3*(a*d
- b*c)^5) - (A*a^3*d^3)/(2*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2)
- (A*b^3*c^3)/(2*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (B*a^3*d
^3)/(4*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (B*b^3*c^3)/(4*g^3
*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (B*a*d*log((e*(a + b*x))/(c
+ d*x)))/(2*g^3*i^3*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)^2) - (B*b*c*log((e
*(a + b*x))/(c + d*x)))/(2*g^3*i^3*(a*d - b*c)^2*(a + b*x)^2*(c + d*x)^2)
+ (6*A*b^3*d^3*x^3)/(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (7*A
*a*b^2*c^2*d)/(2*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (7*A*a^2
*b*c*d^2)/(2*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (15*B*a*b^2*
c^2*d)/(4*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (15*B*a^2*b*c*d
^2)/(4*g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (2*A*a^2*b*d^3*x)/
(g^3*i^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) - (3*B*a^2*b*d^3*x)/(g^3*i
^3*(a*d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (2*A*b^3*c^2*d*x)/(g^3*i^3*(a*
d - b*c)^4*(a + b*x)^2*(c + d*x)^2) + (3*B*b^3*c^2*d*x)/(g^3*i^3*(a*d - b*
c)^4*(a + b*x)^2*(c + d*x)^2) + (9*A*a*b^2*d^3*x^2)/(g^3*i^3*(a*d - b*c)^4
*(a + b*x)^2*(c + d*x)^2) - (3*B*a*b^2*d^3*x^2)/(g^3*i^3*(a*d - b*c)^4*(a
+ b*x)^2*(c + d*x)^2) + (9*A*b^3*c*d^2*x^2)/(g^3*i^3*(a*d - b*c)^4*(a + b*
x)^2*(c + d*x)^2) + (3*B*b^3*c*d^2*x^2)/(g^3*i^3*(a*d - b*c)^4*(a + b*x...
```

# 3.54 $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx$



## 3.54.1 Optimal result

Integrand size = 40, antiderivative size = 563

$$\begin{aligned}\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx &= \frac{Bd^5(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2}-\frac{5bBd^4(a+bx)}{(bc-ad)^6g^4i^3(c+dx)}\\
&-\frac{10b^3Bd^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)}+\frac{5b^4Bd(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2}\\
&-\frac{b^5B(c+dx)^3}{9(bc-ad)^6g^4i^3(a+bx)^3}+\frac{5b^2Bd^3\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}\\
&-\frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2}\\
&+\frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(c+dx)}\\
&-\frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)}\\
&+\frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2}\\
&-\frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3}\\
&-\frac{10b^2d^3\log\left(\frac{a+bx}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3}\end{aligned}$$

output
```
1/4*B*d^5*(b*x+a)^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-5*b*B*d^4*(b*x+a)/(-a*d
+b*c)^6/g^4/i^3/(d*x+c)-10*b^3*B*d^2*(d*x+c)/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+
5/4*b^4*B*d*(d*x+c)^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/9*b^5*B*(d*x+c)^3/(
-a*d+b*c)^6/g^4/i^3/(b*x+a)^3+5*b^2*B*d^3*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)
^6/g^4/i^3-1/2*d^5*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^6/g^4/
i^3/(d*x+c)^2+5*b*d^4*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^6/g^4
/i^3/(d*x+c)-10*b^3*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^6/g
^4/i^3/(b*x+a)+5/2*b^4*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^
6/g^4/i^3/(b*x+a)^2-1/3*b^5*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*
c)^6/g^4/i^3/(b*x+a)^3-10*b^2*d^3*ln((b*x+a)/(d*x+c))*(A+B*ln(e*(b*x+a)/(d
*x+c)))/(-a*d+b*c)^6/g^4/i^3
```

### 3.54.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.89 (sec) , antiderivative size = 637, normalized size of antiderivative = 1.13

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx =$$

$$-\frac{\frac{4b^2B(bc-ad)^3}{(a+bx)^3}-\frac{33b^2Bd(bc-ad)^2}{(a+bx)^2}+\frac{216b^3Bcd^2}{a+bx}-\frac{216ab^2Bd^3}{a+bx}+\frac{66b^2Bd^2(bc-ad)}{a+bx}-\frac{9Bd^3(bc-ad)^2}{(c+dx)^2}-\frac{144b^2Bcd^3}{c+dx}+\frac{144abBd^4}{c+dx}\cdots}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i
*x)^3),x]
```

output
```
-1/36*((4*b^2*B*(b*c - a*d)^3)/(a + b*x)^3 - (33*b^2*B*d*(b*c - a*d)^2)/(a
 + b*x)^2 + (216*b^3*B*c*d^2)/(a + b*x) - (216*a*b^2*B*d^3)/(a + b*x) + (6
6*b^2*B*d^2*(b*c - a*d))/(a + b*x) - (9*B*d^3*(b*c - a*d)^2)/(c + d*x)^2 -
 (144*b^2*B*c*d^3)/(c + d*x) + (144*a*b*B*d^4)/(c + d*x) - (18*b*B*d^3*(b*
c - a*d))/(c + d*x) + 120*b^2*B*d^3*Log[a + b*x] + (12*b^2*(b*c - a*d)^3*(
A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^3 - (54*b^2*d*(b*c - a*d)^2
*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)^2 + (216*b^2*d^2*(b*c - a
*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + (18*d^3*(b*c - a*d)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x)^2 + (144*b*d^3*(b*c - a*
d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d*x) + 360*b^2*d^3*Log[a + b
*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 120*b^2*B*d^3*Log[c + d*x] - 36
0*b^2*d^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 180*b^2*B*d^
3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Poly
Log[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 180*b^2*B*d^3*((2*Log[(d*(a + b*x)
)/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x)
)/(b*c - a*d)]))/((b*c - a*d)^6*g^4*i^3)
```

### 3.54.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.69, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2962, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{(ag+bgx)^4(ci+dix)^3}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i^3(bc-ad)^6}$$

$$\Big\downarrow \text{2772}$$

$$\frac{-B\int -\frac{(c+dx)^4\left(2b^5-\frac{15d(a+bx)b^4}{c+dx}+\frac{60d^2(a+bx)^2b^3}{(c+dx)^2}+\frac{60d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)b^2}{(c+dx)^3}-\frac{30d^4(a+bx)^4b}{(c+dx)^4}+\frac{3d^5(a+bx)^5}{(c+dx)^5}\right)}{6(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^5(c+dx)^3\left(B\log\left(\frac{e(a\cdots}{c+\cdots}\right.\right.}{3(a+bx)^3}\cdots}{\cdots}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}B\int \frac{(c+dx)^4\left(2b^5-\frac{15d(a+bx)b^4}{c+dx}+\frac{60d^2(a+bx)^2b^3}{(c+dx)^2}+\frac{60d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)b^2}{(c+dx)^3}-\frac{30d^4(a+bx)^4b}{(c+dx)^4}+\frac{3d^5(a+bx)^5}{(c+dx)^5}\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^5(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right.\right.}{3(a+bx)^3}$$

$\Big\downarrow$ 2010

$$\frac{1}{6}B\int\left(\frac{\left(2b^5-\frac{15d(a+bx)b^4}{c+dx}+\frac{60d^2(a+bx)^2b^3}{(c+dx)^2}-\frac{30d^4(a+bx)^4b}{(c+dx)^4}+\frac{3d^5(a+bx)^5}{(c+dx)^5}\right)(c+dx)^4}{(a+bx)^4}+\frac{60b^2d^3\log\left(\frac{a+bx}{c+dx}\right)(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^5(c+dx)^3\left(B\log\right.}{3(a+}$$

$\Big\downarrow$ 2009

$$-\frac{b^5(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}+\frac{5b^4d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{10b^3d^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-10b^2d^3\log\left(\frac{a+bx}{c+dx}\right)\Big($$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])/((a*g + b*g*x)^4*(c*i + d*i*x)^3)
,x]
```

output
```
((B*((3*d^5*(a + b*x)^2)/(2*(c + d*x)^2) - (30*b*d^4*(a + b*x))/(c + d*x)
- (60*b^3*d^2*(c + d*x))/(a + b*x) + (15*b^4*d*(c + d*x)^2)/(2*(a + b*x)^2
) - (2*b^5*(c + d*x)^3)/(3*(a + b*x)^3) + 30*b^2*d^3*Log[(a + b*x)/(c + d*
x)]^2))/6 - (d^5*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(c +
 d*x)^2) + (5*b*d^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(c + d
*x) - (10*b^3*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x
) + (5*b^4*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x
)^2) - (b^5*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*(a + b*x)
^3) - 10*b^2*d^3*Log[(a + b*x)/(c + d*x)]*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/((b*c - a*d)^6*g^4*i^3)
```

### 3.54.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.54. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx$

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.54.4 Maple [A] (verified)

Time = 7.99 (sec) , antiderivative size = 787, normalized size of antiderivative = 1.40

| method | result |
|---|---|
| parts | $\frac{A\left(-\frac{d^3}{2(ad-cb)^4(dx+c)^2}+\frac{10d^3b^2\ln(dx+c)}{(ad-cb)^6}+\frac{4d^3b}{(ad-cb)^5(dx+c)}+\frac{b^2}{3(ad-cb)^3(bx+a)^3}-\frac{10d^3b^2\ln(bx+a)}{(ad-cb)^6}+\frac{6b^2d^2}{(ad-cb)^5(bx+a)}+\frac{}{2(ad-}\right.}{g^4i^3}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{d^2e^2A\,b^5}{3i^3(ad-cb)^7g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{5d^3eA\,b^4}{2i^3(ad-cb)^7g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{10d^4A\,b^3}{i^3(ad-cb)^7g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{10d^5A\,b^2}{e\,i}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(\frac{d^2e^2A\,b^5}{3i^3(ad-cb)^7g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{5d^3eA\,b^4}{2i^3(ad-cb)^7g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{10d^4A\,b^3}{i^3(ad-cb)^7g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{10d^5A\,b^2}{e\,i}\right.}{}$ |
| risch | Expression too large to display |
| parallelrisch | Expression too large to display |
| norman | Expression too large to display |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output
```
A/g^4/i^3*(-1/2*d^3/(a*d-b*c)^4/(d*x+c)^2+10*d^3/(a*d-b*c)^6*b^2*ln(d*x+c)
+4*d^3/(a*d-b*c)^5*b/(d*x+c)+1/3*b^2/(a*d-b*c)^3/(b*x+a)^3-10*d^3/(a*d-b*c
)^6*b^2*ln(b*x+a)+6*b^2/(a*d-b*c)^5*d^2/(b*x+a)+3/2*b^2/(a*d-b*c)^4*d/(b*x
+a)^2)-B/g^4/i^3*d/(a*d-b*c)^2/e^2*(d^4/(a*d-b*c)^4*(1/2*(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d
*x+c))^2)-5*d^3/(a*d-b*c)^4*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+5*d^2/(a*d-b*c)^4*b^2*e^
2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-10*d/(a*d-b*c)^4*b^3*e^3*(-1/(b*e/d+(a
*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/
d/(d*x+c)))+5/(a*d-b*c)^4*b^4*e^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln
(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-1/d/(a*
d-b*c)^4*b^5*e^5*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

## 3.54. $\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx$

### 3.54.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1509 vs. 2(551) = 1102.

Time = 0.35 (sec) , antiderivative size = 1509, normalized size of antiderivative = 2.68

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output
```
-1/36*(4*(3*A + B)*b^5*c^5 - 45*(2*A + B)*a*b^4*c^4*d + 360*(A + B)*a^2*b^
3*c^3*d^2 - 10*(12*A + 49*B)*a^3*b^2*c^2*d^3 - 180*(A - B)*a^4*b*c*d^4 + 9
*(2*A - B)*a^5*d^5 + 120*((3*A + B)*b^5*c*d^4 - (3*A + B)*a*b^4*d^5)*x^4 +
 60*(3*(3*A + 2*B)*b^5*c^2*d^3 + 2*(3*A - 2*B)*a*b^4*c*d^4 - (15*A + 2*B)*
a^2*b^3*d^5)*x^3 + 20*((6*A + 11*B)*b^5*c^3*d^2 + 21*(3*A + B)*a*b^4*c^2*d
^3 - 3*(12*A + 13*B)*a^2*b^3*c*d^4 - (33*A - 7*B)*a^3*b^2*d^5)*x^2 + 180*(
B*b^5*d^5*x^5 + B*a^3*b^2*c^2*d^3 + (2*B*b^5*c*d^4 + 3*B*a*b^4*d^5)*x^4 +
(B*b^5*c^2*d^3 + 6*B*a*b^4*c*d^4 + 3*B*a^2*b^3*d^5)*x^3 + (3*B*a*b^4*c^2*d
^3 + 6*B*a^2*b^3*c*d^4 + B*a^3*b^2*d^5)*x^2 + (3*B*a^2*b^3*c^2*d^3 + 2*B*a
^3*b^2*c*d^4)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 5*((6*A + 5*B)*b^5*c^4*d
 - 36*(2*A + 3*B)*a*b^4*c^3*d^2 - 6*(24*A - 13*B)*a^2*b^3*c^2*d^3 + 4*(48*
A + 13*B)*a^3*b^2*c*d^4 + 9*(2*A - 3*B)*a^4*b*d^5)*x + 6*(20*(3*A + B)*b^5
*d^5*x^5 + 2*B*b^5*c^5 - 15*B*a*b^4*c^4*d + 60*B*a^2*b^3*c^3*d^2 + 60*A*a^
3*b^2*c^2*d^3 - 30*B*a^4*b*c*d^4 + 3*B*a^5*d^5 + 20*((6*A + 5*B)*b^5*c*d^4
 + 9*A*a*b^4*d^5)*x^4 + 10*((6*A + 11*B)*b^5*c^2*d^3 + 18*(2*A + B)*a*b^4*
c*d^4 + 9*(2*A - B)*a^2*b^3*d^5)*x^3 + 10*(2*B*b^5*c^3*d^2 + 9*(2*A + 3*B)
*a*b^4*c^2*d^3 + 36*A*a^2*b^3*c*d^4 + 3*(2*A - 3*B)*a^3*b^2*d^5)*x^2 - 5*(
B*b^5*c^4*d - 12*B*a*b^4*c^3*d^2 - 36*(A + B)*a^2*b^3*c^2*d^3 - 24*(A - B)
*a^3*b^2*c*d^4 + 3*B*a^4*b*d^5)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^6
*d^2 - 6*a*b^8*c^5*d^3 + 15*a^2*b^7*c^4*d^4 - 20*a^3*b^6*c^3*d^5 + 15*a...
```

### 3.54.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)**4/(d*i*x+c*i)**3,x)
```

output
```
Timed out
```

### 3.54.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3816 vs. $2(551)=1102$.

Time = 0.57 (sec) , antiderivative size = 3816, normalized size of antiderivative = 6.78

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output
```
-1/6*B*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^2
+ 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(2*
b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a*b
^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*c^
4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^
3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 +
10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7
)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c
^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*
d^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a
^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a
^8*d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d
^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^
6)*g^4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^
6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log(b
*x + a)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^
3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*lo
g(d*x + c)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3
*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) - 1/6*A*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b...
```

### 3.54.8 Giac [F]

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx=\int \frac{B\log\left(\frac{(bx+a)e}{dx+c}\right)+A}{(bgx+ag)^4(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)/((b*g*x + a*g)^4*(d*i*x + c*i
)^3), x)
```

### 3.54.9 Mupad [B] (verification not implemented)

Time = 13.33 (sec) , antiderivative size = 2291, normalized size of antiderivative = 4.07

$$\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))/((a*g + b*g*x)^4*(c*i + d*i*x)^3)
,x)
```

output
```
((12*A*b^4*c^4 - 18*A*a^4*d^4 + 9*B*a^4*d^4 + 4*B*b^4*c^4 + 282*A*a^2*b^2*
c^2*d^2 + 319*B*a^2*b^2*c^2*d^2 - 78*A*a*b^3*c^3*d + 162*A*a^3*b*c*d^3 - 4
1*B*a*b^3*c^3*d - 171*B*a^3*b*c*d^3)/(6*(a*d - b*c)) + (10*x^2*(33*A*a^2*b
^2*d^4 - 7*B*a^2*b^2*d^4 + 6*A*b^4*c^2*d^2 + 11*B*b^4*c^2*d^2 + 69*A*a*b^3
*c*d^3 + 32*B*a*b^3*c*d^3))/(3*(a*d - b*c)) + (5*x*(18*A*a^3*b*d^4 - 27*B*
a^3*b*d^4 - 6*A*b^4*c^3*d - 5*B*b^4*c^3*d + 66*A*a*b^3*c^2*d^2 + 210*A*a^2
*b^2*c*d^3 + 103*B*a*b^3*c^2*d^2 + 25*B*a^2*b^2*c*d^3))/(6*(a*d - b*c)) +
(10*x^3*(15*A*a*b^3*d^4 + 2*B*a*b^3*d^4 + 9*A*b^4*c*d^3 + 6*B*b^4*c*d^3))/
(a*d - b*c) + (20*x^4*(3*A*b^4*d^4 + B*b^4*d^4))/(a*d - b*c))/(x^5*(6*a^4*
b^3*d^6*g^4*i^3 + 6*b^7*c^4*d^2*g^4*i^3 - 24*a*b^6*c^3*d^3*g^4*i^3 - 24*a^
3*b^4*c*d^5*g^4*i^3 + 36*a^2*b^5*c^2*d^4*g^4*i^3) + x*(18*a^2*b^5*c^6*g^4*
i^3 + 12*a^7*c*d^5*g^4*i^3 - 60*a^3*b^4*c^5*d*g^4*i^3 - 30*a^6*b*c^2*d^4*g
^4*i^3 + 60*a^4*b^3*c^4*d^2*g^4*i^3) + x^2*(6*a^7*d^6*g^4*i^3 + 18*a*b^6*c
^6*g^4*i^3 + 12*a^6*b*c*d^5*g^4*i^3 - 36*a^2*b^5*c^5*d*g^4*i^3 - 30*a^3*b^
4*c^4*d^2*g^4*i^3 + 120*a^4*b^3*c^3*d^3*g^4*i^3 - 90*a^5*b^2*c^2*d^4*g^4*i
^3) + x^3*(6*b^7*c^6*g^4*i^3 + 18*a^6*b*d^6*g^4*i^3 + 12*a*b^6*c^5*d*g^4*i
^3 - 36*a^5*b^2*c*d^5*g^4*i^3 - 90*a^2*b^5*c^4*d^2*g^4*i^3 + 120*a^3*b^4*c
^3*d^3*g^4*i^3 - 30*a^4*b^3*c^2*d^4*g^4*i^3) + x^4*(18*a^5*b^2*d^6*g^4*i^3
 + 12*b^7*c^5*d*g^4*i^3 - 30*a*b^6*c^4*d^2*g^4*i^3 - 60*a^4*b^3*c*d^5*g^4*
i^3 + 60*a^3*b^4*c^2*d^4*g^4*i^3) + 6*a^3*b^4*c^6*g^4*i^3 + 6*a^7*c^2*d...
```

# 3.55 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$



## 3.55.1 Optimal result

Integrand size = 40, antiderivative size = 539

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx\\
&=\frac{3B^2(bc-ad)^4g^3ix}{10bd^3}-\frac{3B^2(bc-ad)^3g^3i(c+dx)^2}{20d^4}+\frac{bB^2(bc-ad)^2g^3i(c+dx)^3}{30d^4}\\
&-\frac{B(bc-ad)^2g^3i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^2d}\\
&-\frac{B(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2}\\
&+\frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2}\\
&+\frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b}\\
&+\frac{B(bc-ad)^3g^3i(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^2}\\
&-\frac{B(bc-ad)^4g^3i(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^3}\\
&-\frac{B(bc-ad)^5g^3i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^2d^4}\\
&-\frac{B^2(bc-ad)^5g^3i\log(c+dx)}{10b^2d^4}-\frac{B^2(bc-ad)^5g^3i\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^2d^4}
\end{aligned}$$

output
```
3/10*B^2*(-a*d+b*c)^4*g^3*i*x/b/d^3-3/20*B^2*(-a*d+b*c)^3*g^3*i*(d*x+c)^2/
d^4+1/30*b*B^2*(-a*d+b*c)^2*g^3*i*(d*x+c)^3/d^4-1/30*B*(-a*d+b*c)^2*g^3*i*
(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d-1/10*B*(-a*d+b*c)*g^3*i*(b*x+a
)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2+1/20*(-a*d+b*c)*g^3*i*(b*x+a)^4*(A+B*l
n(e*(b*x+a)/(d*x+c)))^2/b^2+1/5*g^3*i*(b*x+a)^4*(d*x+c)*(A+B*ln(e*(b*x+a)/
(d*x+c)))^2/b+1/60*B*(-a*d+b*c)^3*g^3*i*(b*x+a)^2*(3*A+B+3*B*ln(e*(b*x+a)/
(d*x+c)))/b^2/d^2-1/60*B*(-a*d+b*c)^4*g^3*i*(b*x+a)*(6*A+5*B+6*B*ln(e*(b*x
+a)/(d*x+c)))/b^2/d^3-1/60*B*(-a*d+b*c)^5*g^3*i*ln((-a*d+b*c)/b/(d*x+c))*(
6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^4-1/10*B^2*(-a*d+b*c)^5*g^3*i*ln
(d*x+c)/b^2/d^4-1/10*B^2*(-a*d+b*c)^5*g^3*i*polylog(2,d*(b*x+a)/b/(d*x+c))
/b^2/d^4
```

## 3.55.2 Mathematica [A] (verified)

Time = 0.45 (sec) , antiderivative size = 905, normalized size of antiderivative = 1.68

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$

$$=\frac{g^3i\left(5(bc-ad)(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+4d(a+bx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{5B(bc-ad)^2\left(6Abd(\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2,x]
```

output
```
(g^3*i*(5*(b*c - a*d)*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 +
 4*d*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (5*B*(b*c - a*d)
^2*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*
x))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*
B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c
 + d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x
)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) +
 a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*
d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
)))/(3*d^4) + (B*(b*c - a*d)*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d
)^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x
)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(
A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 24*(b*c - a*d)^4*(A
 + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2*b*d
*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c
- a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(
a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*d*x + (
-(b*c) + a*d)*Log[c + d*x]) + 12*B*(b*c - a*d)^4*((2*Log[(d*(a + b*x))/...
```

### 3.55.3 Rubi [A] (verified)

Time = 1.35 (sec) , antiderivative size = 617, normalized size of antiderivative = 1.14, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.275$, Rules used = {2962, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2962}$$

$$g^3i(bc-ad)^5\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$g^3 i(bc - ad)^5 \left( -\frac{2B \int \frac{(a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{\int \frac{(a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right) + \right.}{5b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)}\right.$$

$\big\downarrow$ 2773

$$g^3 i(bc - ad)^5 \left( -\frac{2B \left( \frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\int \frac{(a+bx)^3}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{4b} \right)}{5b} + \frac{\int \frac{(a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{(a -}{}\right.$$

$\big\downarrow$ 49

$$g^3 i(bc - ad)^5 \left( -\frac{2B \left( \frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\int \left(\frac{b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{3b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{3b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{1}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right) d\frac{a+bx}{c+dx}}{4b} \right)}{5b}\right.$$

$\big\downarrow$ 2009

$$g^3 i(bc - ad)^5 \left( \frac{\int \frac{(a+bx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} - \frac{2B \left( \frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\left(\frac{b^3}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{3b^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2} + \frac{}{d^4 (}\right.}{4b}\right.}{5b}\right.$$

$\big\downarrow$ 2781

3.55. $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$g^3 i(bc-ad)^5 \left( \frac{\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}}{5b} - \frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{b^{\cdots}}{3d^4\left(b-\frac{d(\cdots}{\cdots}\right.}\right.}{\cdots}\right.}{\cdots} \right.$$

$\big\downarrow$ 2784

$$g^3 i(bc-ad)^5 \left( \frac{\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int \frac{(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{2b}}{5b} - \frac{2B\left(\frac{(a+bx)^4 \cdots}{4b(c+\cdots}\right.}{\cdots} \right.$$

$\big\downarrow$ 2784

$$g^3 i(bc-ad)^5 \left( \frac{\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int \frac{(a+bx)\left(6A+5B+6B\log\left(\frac{e(a\cdots}{c+\cdots}\right.\right.}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\cdots}}{2d}}{3d}\cdots\right.}{2b}}{5b} \cdots \right.$$

3.55. $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$\Big\downarrow \; 2784$$

$$g^3 i(bc-ad)^5 \left( \frac{\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\frac{(a+bx)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{}}{3d}\right)}{2b}}{5b} \right.$$

$$\Big\downarrow \; 2754$$

$$g^3 i(bc-ad)^5 \left( \frac{\frac{(a+bx)^4 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\frac{(a+bx)^2\left(3B\log\left(\frac{e(a+bx)}{c+dx}\right)+3A+B\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\frac{(a+bx)\left(6B\log\left(\frac{e(a+bx)}{c+dx}\right)+6A}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{}}{}\right)}{2b}}{5b} \right.$$

---

3.55. $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$\downarrow$ 2838

$$g^3 i(bc - ad)^5 \left( -\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\left(\frac{b^3}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{3b^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2} + \frac{3b}{d^4\left(b-\frac{d(a+bx)}{c+dx}\right)} + \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^4}\right)}{4b}\right)}{5b} + \frac{(a+ \cdots}{\cdots} \right.$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x
]
```

output
```
(b*c - a*d)^5*g^3*i*(((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(5*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*(((a + b*x)^4*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]))/(4*b*(c + d*x)^4*(b - (d*(a + b*x))/(c
+ d*x))^4) - (B*(b^3/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) - (3*b^2)/(2*
d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3*b)/(d^4*(b - (d*(a + b*x))/(c +
d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]/d^4))/(4*b)))/(5*b) + (((a + b*x
)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*b*(c + d*x)^4*(b - (d*(a +
b*x))/(c + d*x))^4) - (B*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2*(3*A
+ B + 3*B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x
))/(c + d*x))^2) - (((a + b*x)*(6*A + 5*B + 6*B*Log[(e*(a + b*x))/(c + d*x
)]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((6*A + 11*B + 6*B*L
og[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (6
*B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d))/(3*d)))/(2*b))/(5
*b))
```

## 3.55. $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

### 3.55.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

## 3.55. $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.55.4 Maple [F]

$$\int (bgx+ag)^3\,(dix+ci)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

### 3.55.5 Fricas [F]

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^3(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="fricas")
```

output
```
integral(A^2*b^3*d*g^3*i*x^4 + A^2*a^3*c*g^3*i + (A^2*b^3*c + 3*A^2*a*b^2*
d)*g^3*i*x^3 + 3*(A^2*a*b^2*c + A^2*a^2*b*d)*g^3*i*x^2 + (3*A^2*a^2*b*c +
A^2*a^3*d)*g^3*i*x + (B^2*b^3*d*g^3*i*x^4 + B^2*a^3*c*g^3*i + (B^2*b^3*c +
 3*B^2*a*b^2*d)*g^3*i*x^3 + 3*(B^2*a*b^2*c + B^2*a^2*b*d)*g^3*i*x^2 + (3*B
^2*a^2*b*c + B^2*a^3*d)*g^3*i*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b
^3*d*g^3*i*x^4 + A*B*a^3*c*g^3*i + (A*B*b^3*c + 3*A*B*a*b^2*d)*g^3*i*x^3 +
 3*(A*B*a*b^2*c + A*B*a^2*b*d)*g^3*i*x^2 + (3*A*B*a^2*b*c + A*B*a^3*d)*g^3
*i*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

### 3.55.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

### 3.55.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3186 vs. 2(514) = 1028.

Time = 0.33 (sec) , antiderivative size = 3186, normalized size of antiderivative = 5.91

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="maxima")
```

output
```
1/5*A^2*b^3*d*g^3*i*x^5 + 1/4*A^2*b^3*c*g^3*i*x^4 + 3/4*A^2*a*b^2*d*g^3*i*
x^4 + A^2*a*b^2*c*g^3*i*x^3 + A^2*a^2*b*d*g^3*i*x^3 + 3/2*A^2*a^2*b*c*g^3*
i*x^2 + 1/2*A^2*a^3*d*g^3*i*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^3*c*g^3*i + 3*(x^2*log(b*e*
x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
 - (b*c - a*d)*x/(b*d))*A*B*a^2*b*c*g^3*i + (2*x^3*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b^2*c*g^3*i +
 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4
 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d
- a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*c*g^3*i + (
x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(
d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^3*d*g^3*i + (2*x^3*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3
- ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*b
*d*g^3*i + 1/4*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x
 + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b
^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*b^2*
d*g^3*i + 1/30*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b
*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 ...
```

## 3.55.8 Giac [F]

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (bgx+ag)^3(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)
^2, x)
```

### 3.55.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x
)
```

output
```
int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
x)
```

# 3.56 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$



## 3.56.1 Optimal result

Integrand size = 40, antiderivative size = 450

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx \\
&= -\frac{B^2(bc-ad)^3g^2ix}{3bd^2}+\frac{B^2(bc-ad)^2g^2i(c+dx)^2}{12d^3} \\
&\quad-\frac{B(bc-ad)^2g^2i(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d} \\
&\quad-\frac{B(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^2} \\
&\quad+\frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^2} \\
&\quad+\frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b} \\
&\quad+\frac{B(bc-ad)^3g^2i(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d^2} \\
&\quad+\frac{B(bc-ad)^4g^2i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{12b^2d^3} \\
&\quad+\frac{B^2(bc-ad)^4g^2i\log(c+dx)}{6b^2d^3}+\frac{B^2(bc-ad)^4g^2i\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^2d^3}
\end{aligned}$$

output
```
-1/3*B^2*(-a*d+b*c)^3*g^2*i*x/b/d^2+1/12*B^2*(-a*d+b*c)^2*g^2*i*(d*x+c)^2/
d^3-1/12*B*(-a*d+b*c)^2*g^2*i*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d-
1/6*B*(-a*d+b*c)*g^2*i*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2+1/12*(-a*
d+b*c)*g^2*i*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2+1/4*g^2*i*(b*x+a)
^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/12*B*(-a*d+b*c)^3*g^2*i*(b*x+
a)*(2*A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^2+1/12*B*(-a*d+b*c)^4*g^2*i*ln(
(-a*d+b*c)/b/(d*x+c))*(2*A+3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^3+1/6*B^2*
(-a*d+b*c)^4*g^2*i*ln(d*x+c)/b^2/d^3+1/6*B^2*(-a*d+b*c)^4*g^2*i*polylog(2,
d*(b*x+a)/b/(d*x+c))/b^2/d^3
```

### 3.56.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 680, normalized size of antiderivative = 1.51

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$

$$= \frac{g^2 i\Bigg(4(bc-ad)(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+3d(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{4B(bc-ad)^2\big(2Abd(\ldots}{\ldots}}{\ldots}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2,x]
```

output
```
(g^2*i*(4*(b*c - a*d)*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 +
 3*d*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (4*B*(b*c - a*d)
^2*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/
(c + d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b
*c - a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d
*x)])*Log[c + d*x] + B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) +
 B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log
[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d^3 - (B*(b*c - a*d
)*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x
))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/
(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B
*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c
+ d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)
^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*c) +
a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
))/d^3))/(12*b^2)
```

### 3.56.3 Rubi [A] (verified)

Time = 1.15 (sec) , antiderivative size = 516, normalized size of antiderivative = 1.15, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2962, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{ 2962}$$

$$g^2i(bc-ad)^4\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2783}$$

$$g^2i(bc-ad)^4\left(-\frac{B\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}+\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{4b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right.$$

$$\Big\downarrow \text{ 2773}$$

$$g^2i(bc-ad)^4\left(-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\int \frac{(a+bx)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right)}{2b}+\frac{\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}+\frac{(a+}{4}\right.$$

$$\Big\downarrow \text{ 49}$$

$$g^2 i(bc-ad)^4\left(-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\int\left(\frac{b^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{1}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}}{3b}\right)}{2b}+\frac{\int\frac{(a+bx)^2\left(A+B\right.}{(c+dx)^2\left(b\right.}}{}\right.$$

$\Big\downarrow$ 2009

$$g^2 i(bc-ad)^4\left(\frac{\int\frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\left(\frac{b^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\log\left(b-\right.}{}\right.}{3b}\right.}{2b}\right.$$

$\Big\downarrow$ 2781

$$g^2 i(bc-ad)^4\left(\frac{\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\int\frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}}{4b}-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\left(\frac{b^2}{2d^3\left(b-\frac{d(}{}\right.}\right.}{}\right.}{2b}\right.$$

$\Big\downarrow$ 2784

---

3.56. $\quad \int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$g^2 i(bc - ad)^4 \left( \frac{\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\int \frac{(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{3b}}{4b} - \frac{B\left(\frac{(a+bx)^3\left(\right.}{3b(c+d}\right.}{}\right.$$

$\downarrow$ 2784

$$g^2 i(bc - ad)^4 \left( \frac{\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)} - \frac{\int \frac{2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}d}{d}}{2d}\right.}{3b}}{4b}\right.$$

$\downarrow$ 2754

---

3.56. $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$g^2 i(bc - ad)^4 \left( \frac{\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\frac{(a+bx)\left(2B\log\left(\frac{e(a+bx)}{c+dx}\right)+2A+B\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)} - \frac{2B\int \frac{\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}}{d}}{}}{2d}}{3b}}{4b}$$

$\downarrow$ 2838

$$g^2 i(bc - ad)^4 \left( -\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{B\left(\frac{b^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^3}\right)}{3b}\right)}{2b} + \frac{\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right.}}{}\right.$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x
]
```

## 3.56. $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

output
```
(b*c - a*d)^4*g^2*i*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(4*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^4) - (B*(((a + b*x)^3*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c +
d*x))^3) - (B*(b^2/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*b)/(d^3*(b
 - (d*(a + b*x))/(c + d*x))) - Log[b - (d*(a + b*x))/(c + d*x)]/d^3))/(3*b
)))/(2*b) + (((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b*(c
+ d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*(((a + b*x)^2*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^
2) - (((a + b*x)*(2*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x
)*(b - (d*(a + b*x))/(c + d*x))) - (-(((2*A + 3*B + 2*B*Log[(e*(a + b*x))/
(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (2*B*PolyLog[2, (d*
(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d)))/(3*b))/(4*b))
```

### 3.56.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.56.4 Maple [F]

$$\int (bgx+ag)^2 (dix+ci) \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

### 3.56.5 Fricas [F]

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (bgx+ag)^2(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="fricas")
```

output
```
integral(A^2*b^2*d*g^2*i*x^3 + A^2*a^2*c*g^2*i + (A^2*b^2*c + 2*A^2*a*b*d)
*g^2*i*x^2 + (2*A^2*a*b*c + A^2*a^2*d)*g^2*i*x + (B^2*b^2*d*g^2*i*x^3 + B^
2*a^2*c*g^2*i + (B^2*b^2*c + 2*B^2*a*b*d)*g^2*i*x^2 + (2*B^2*a*b*c + B^2*a
^2*d)*g^2*i*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^2*d*g^2*i*x^3 + A
*B*a^2*c*g^2*i + (A*B*b^2*c + 2*A*B*a*b*d)*g^2*i*x^2 + (2*A*B*a*b*c + A*B*
a^2*d)*g^2*i*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

### 3.56.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

### 3.56.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2243 vs. $2(429) = 858$.

Time = 0.31 (sec) , antiderivative size = 2243, normalized size of antiderivative = 4.98

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="maxima")
```

output
```
1/4*A^2*b^2*d*g^2*i*x^4 + 1/3*A^2*b^2*c*g^2*i*x^3 + 2/3*A^2*a*b*d*g^2*i*x^
3 + A^2*a*b*c*g^2*i*x^2 + 1/2*A^2*a^2*d*g^2*i*x^2 + 2*(x*log(b*e*x/(d*x +
c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^2*c*g^2*i
 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^
2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a*b*c*g^2*i + 1/3*(2*x^3*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x
 + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))
*A*B*b^2*c*g^2*i + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x
 + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*d*g^2*i +
2/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 -
 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)
*x)/(b^2*d^2))*A*B*a*b*d*g^2*i + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2
- a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x
)/(b^3*d^3))*A*B*b^2*d*g^2*i + A^2*a^2*c*g^2*i*x - 1/12*(2*a^3*c*d^3*g^2*i
 + (2*g^2*i*log(e) + g^2*i)*b^3*c^4 - 2*(4*g^2*i*log(e) + g^2*i)*a*b^2*c^3
*d + (12*g^2*i*log(e) - g^2*i)*a^2*b*c^2*d^2)*B^2*log(d*x + c)/(b*d^3) - 1
/6*(b^4*c^4*g^2*i - 4*a*b^3*c^3*d*g^2*i + 6*a^2*b^2*c^2*d^2*g^2*i - 4*a^3*
b*c*d^3*g^2*i + a^4*d^4*g^2*i)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d)
 + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^3) + 1/12*(3*B^2*...
```

## 3.56.8 Giac [F]

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^2(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)
^2, x)
```

### 3.56.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x
)
```

output
```
int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
x)
```

# 3.57 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$



## 3.57.1 Optimal result

Integrand size = 38, antiderivative size = 343

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2gix}{3bd} - \frac{B(bc-ad)^2gi(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2d} \\
&\quad - \frac{B(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2} \\
&\quad + \frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2} \\
&\quad + \frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3b} \\
&\quad - \frac{B(bc-ad)^3gi\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2d^2} \\
&\quad - \frac{B^2(bc-ad)^3gi\log(c+dx)}{3b^2d^2} - \frac{B^2(bc-ad)^3gi\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^2d^2}
\end{aligned}$$

output
```
1/3*B^2*(-a*d+b*c)^2*g*i*x/b/d-1/3*B*(-a*d+b*c)^2*g*i*(b*x+a)*(A+B*ln(e*(b
*x+a)/(d*x+c)))/b^2/d-1/3*B*(-a*d+b*c)*g*i*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*
x+c)))/b^2+1/6*(-a*d+b*c)*g*i*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2+
1/3*g*i*(b*x+a)^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b-1/3*B*(-a*d+b*c)
^3*g*i*ln((-a*d+b*c)/b/(d*x+c))*(A+B+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d^2-1/3*
B^2*(-a*d+b*c)^3*g*i*ln(d*x+c)/b^2/d^2-1/3*B^2*(-a*d+b*c)^3*g*i*polylog(2,
d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

### 3.57.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 869 vs. 2(343) = 686.

Time = 0.41 (sec) , antiderivative size = 869, normalized size of antiderivative = 2.53

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$= \frac{gi\Big(-6Ab^2Bcd(bc-ad)x+6aAbBd^2(-bc+ad)x+4AbBd(bc-ad)(bc+ad)x-6bB^2cd(bc-ad)(a+b}{}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^2,x]
```

output
```
(g*i*(-6*A*b^2*B*c*d*(b*c - a*d)*x + 6*a*A*b*B*d^2*(-(b*c) + a*d)*x + 4*A*
b*B*d*(b*c - a*d)*(b*c + a*d)*x - 6*b*B^2*c*d*(b*c - a*d)*(a + b*x)*Log[(e
*(a + b*x))/(c + d*x)] + 6*a*B^2*d^2*(-(b*c) + a*d)*(a + b*x)*Log[(e*(a +
b*x))/(c + d*x)] + 4*B^2*d*(b*c - a*d)*(b*c + a*d)*(a + b*x)*Log[(e*(a + b
*x))/(c + d*x)] - 2*b^2*B*d^2*(b*c - a*d)*x^2*(A + B*Log[(e*(a + b*x))/(c
+ d*x)]) + 6*a^2*b*B*c*d^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)
]) - 2*a^3*B*d^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*a*b
^2*c*d^2*x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 3*b^2*d^2*(b*c + a*d)*
x^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 2*b^3*d^3*x^3*(A + B*Log[(e*(
a + b*x))/(c + d*x)])^2 + 6*b*B^2*c*(b*c - a*d)^2*Log[c + d*x] + 6*a*B^2*d
*(b*c - a*d)^2*Log[c + d*x] - 4*B^2*(b*c - a*d)^2*(b*c + a*d)*Log[c + d*x]
 + 2*b^3*B*c^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 6*a*b^2
*B*c^2*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*B^2*(b*c -
a*d)*(a^2*d^2*Log[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x]))
- 3*a^2*b*B^2*c*d^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
 - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + a^3*B^2*d^3*(Log
[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2,
 (d*(a + b*x))/(-(b*c) + a*d)]) - b^3*B^2*c^3*((2*Log[(d*(a + b*x))/(-(b*c
) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)]) + 3*a*b^2*B^2*c^2*d*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log...
```

### 3.57.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 405, normalized size of antiderivative = 1.18, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.237, Rules used = {2962, 2783, 2773, 49, 2009, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2962}$$

$$gi(bc-ad)^3\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$ad)^3\left(-\frac{2B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right.}{3b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right.$$

$gi(bc-$

$\downarrow$ 2773

$gi(bc-$

$$ad)^3\left(-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int \frac{a+bx}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+}{3b}\right.$$

$\downarrow$ 49

$gi(bc-$

$$ad)^3\left(-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int \left(\frac{b}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}+\frac{1}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a}{c}}{3b}\right.$$

$\downarrow$ 2009

$gi(bc-$

$$ad)^3\left(\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^2}\right)}{2b}\right)}{3b}+\right.$$

$\downarrow$ 2781

---

3.57. $\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$gi(bc-ad)^3\left(\frac{\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{3b}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{b}{d^2\left(b-\frac{d(a-}{c+}\right.}\right.}{}\right.}{3b}\right.$$

$\big\downarrow$ 2784

$$gi(bc-ad)^3\left(\frac{\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}}{3b}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+b}{c+d}\right.\right.}{2b(c+dx)^2\left(b-\frac{d(a+}{c+}\right.}\right.}{}\right.$$

$\big\downarrow$ 2754

$$gi(bc-ad)^3\left(\frac{\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx}{c+dx}\right.\right.}{d}}{d}\right.}{b}}{3b}\right.$$

$\big\downarrow$ 2838

---

3.57. $\int(ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$gi(bc-ad)^3\left(-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^2}\right)}{2b}\right)}{3b}+\frac{\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-B\left(\frac{(a+\cdots}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^3*g*i*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3
*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*(((a + b*x)^2*(A +
B*Log[(e*(a + b*x))/(c + d*x)]))/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c +
d*x))^2) - (B*(b/(d^2*(b - (d*(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x
))/(c + d*x)]/d^2))/(2*b)))/(3*b) + (((a + b*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)])^2)/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(((
a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b
*x))/(c + d*x))) - (-(((A + B + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d
*(a + b*x))/(b*(c + d*x))])/d) - (B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))
])/d)/d))/b)/(3*b))
```

### 3.57.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.57. $\int (ag + bgx)(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

### 3.57.4 Maple [F]

$$\int (bgx + ag)(dix + ci)\left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

### 3.57.5 Fricas [F]

$$\int (ag + bgx)(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx + ag)(dix + ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algori
thm="fricas")
```

output
```
integral(A^2*b*d*g*i*x^2 + A^2*a*c*g*i + (A^2*b*c + A^2*a*d)*g*i*x + (B^2*
b*d*g*i*x^2 + B^2*a*c*g*i + (B^2*b*c + B^2*a*d)*g*i*x)*log((b*e*x + a*e)/(
d*x + c))^2 + 2*(A*B*b*d*g*i*x^2 + A*B*a*c*g*i + (A*B*b*c + A*B*a*d)*g*i*x
)*log((b*e*x + a*e)/(d*x + c)), x)
```

### 3.57.6 Sympy [F(-1)]

Timed out.

$$\int (ag + bgx)(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

### 3.57.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1252 vs. 2(326) = 652.

Time = 0.30 (sec) , antiderivative size = 1252, normalized size of antiderivative = 3.65

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algori
thm="maxima")
```

output
```
1/3*A^2*b*d*g*i*x^3 + 1/2*A^2*b*c*g*i*x^2 + 1/2*A^2*a*d*g*i*x^2 + 2*(x*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A
*B*a*c*g*i + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/
b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*b*c*g*i + (x^2*log(b
*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/
d^2 - (b*c - a*d)*x/(b*d))*A*B*a*d*g*i + 1/3*(2*x^3*log(b*e*x/(d*x + c) +
a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c
*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b*d*g*i + A^2*
a*c*g*i*x + 1/3*(b^2*c^3*g*i*log(e) - a^2*c*d^2*g*i - (3*g*i*log(e) - g*i)
*a*b*c^2*d)*B^2*log(d*x + c)/(b*d^2) + 1/3*(b^3*c^3*g*i - 3*a*b^2*c^2*d*g*
i + 3*a^2*b*c*d^2*g*i - a^3*d^3*g*i)*(log(b*x + a)*log((b*d*x + a*d)/(b*c
- a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d^2) + 1/6*(2*B^
2*b^3*d^3*g*i*x^3*log(e)^2 + ((3*g*i*log(e)^2 - 2*g*i*log(e))*b^3*c*d^2 +
(3*g*i*log(e)^2 + 2*g*i*log(e))*a*b^2*d^3)*B^2*x^2 - 2*((g*i*log(e) - g*i)
*b^3*c^2*d - (3*g*i*log(e)^2 - 2*g*i)*a*b^2*c*d^2 - (g*i*log(e) + g*i)*a^2
*b*d^3)*B^2*x + (2*B^2*b^3*d^3*g*i*x^3 + 6*B^2*a*b^2*c*d^2*g*i*x + 3*(b^3*
c*d^2*g*i + a*b^2*d^3*g*i)*B^2*x^2 + (3*a^2*b*c*d^2*g*i - a^3*d^3*g*i)*B^2
)*log(b*x + a)^2 + (2*B^2*b^3*d^3*g*i*x^3 + 6*B^2*a*b^2*c*d^2*g*i*x + 3*(b
^3*c*d^2*g*i + a*b^2*d^3*g*i)*B^2*x^2 - (b^3*c^3*g*i - 3*a*b^2*c^2*d*g*i)*
B^2)*log(d*x + c)^2 + 2*(2*B^2*b^3*d^3*g*i*x^3*log(e) + ((3*g*i*log(e) ...
```

### 3.57.8 Giac [F]

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (bgx+ag)(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algori
thm="giac")
```

output
```
integrate((b*g*x + a*g)*(d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^2
, x)
```

### 3.57.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$=\int (a\,g+b\,g\,x)\,(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

# 3.58 $\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$



## 3.58.1 Optimal result

Integrand size = 30, antiderivative size = 203

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = -\frac{B(bc-ad)i(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2}+\frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d}+\frac{B^2(bc-ad)^2i\log(c+dx)}{b^2d}+\frac{B(bc-ad)^2i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}-\frac{B^2(bc-ad)^2i\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}$$

output
```
-B*(-a*d+b*c)*i*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2+1/2*i*(d*x+c)^2*(A
+B*ln(e*(b*x+a)/(d*x+c)))^2/d+B^2*(-a*d+b*c)^2*i*ln(d*x+c)/b^2/d+B*(-a*d+b
*c)^2*i*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/d-B^2*(-
a*d+b*c)^2*i*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/d
```

### 3.58.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 205, normalized size of antiderivative = 1.01

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\frac{i\left((c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{B(bc-ad)\left((-bBc+aBd)\log^2(a+bx)+2\left(Abdx+Bd(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)+(-bBc+aBd)\log\right.\right.}{}\right.}{2d}$$

input
```
Integrate[(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(i*((c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*((
-(b*B*c) + a*B*d)*Log[a + b*x]^2 + 2*(A*b*d*x + B*d*(a + b*x)*Log[(e*(a +
b*x))/(c + d*x)] + (-(b*B*c) + a*B*d)*Log[c + d*x]) + 2*(b*c - a*d)*Log[a
+ b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)] + B*Log[(b*(c + d*x))/(b*c - a*
d)]) + 2*B*(b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))/b^2))/(2
*d)
```

### 3.58.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 227, normalized size of antiderivative = 1.12, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.233, Rules used = {2952, 2756, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$

$$\Big\downarrow \ 2952$$

$$i(bc-ad)^2\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2756$$

$$i(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{d}\right)$$

$\downarrow$ 2789

$$i(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{d}\right)$$

$\downarrow$ 2751

$$i(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{d}\right)$$

$\downarrow$ 16

$$i(bc-ad)^2\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}\right)}{d}\right)$$

$\downarrow$ 2779

3.58. $\int(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$i(bc- ad)^2\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\frac{B\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\cdots\right.\right.}{b(c+dx)\left(b-\cdots\right.}\right.}{\cdots}\right)}{d}\right.$$

$\downarrow$ 2838

$$i(bc- ad)^2\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\frac{B\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right.}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.}{b}\right.}{d}\right.$$

input
```
Int[(c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^2*i*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(2*d*(b - (d*(a +
b*x))/(c + d*x))^2) - (B*((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*(a + b*x)
)/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 -
 (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(a + b*
x))])/b)/b))/d)
```

3.58. $\int(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

### 3.58.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

## 3.58. $\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

### 3.58.4 Maple [F]

$$\int (dix+ci)\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2dx$$

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

### 3.58.5 Fricas [F]

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx=\int (dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="fricas"
)
```

output
```
integral(A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log((b*e*x + a*e)/(d*
x + c))^2 + 2*(A*B*d*i*x + A*B*c*i)*log((b*e*x + a*e)/(d*x + c)), x)
```

### 3.58.6 Sympy [F(-1)]

Timed out.

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

### 3.58.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(200) = 400$.

Time = 0.28 (sec) , antiderivative size = 633, normalized size of antiderivative = 3.12

$$\int (ci + dix) \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{1}{2}A^2dix^2 + 2\left(x\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) + \frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right)ABci$$
$$+ \left(x^2\log\left(\frac{bex}{dx+c} + \frac{ae}{dx+c}\right) - \frac{a^2\log(bx+a)}{b^2} + \frac{c^2\log(dx+c)}{d^2} - \frac{(bc-ad)x}{bd}\right)ABdi$$
$$+ A^2cix - \frac{((i\log(e)-i)bc^2 + acdi)B^2\log(dx+c)}{bd}$$
$$- \frac{(b^2c^2i - 2\,abcdi + a^2d^2i)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d}$$
$$+ \frac{B^2b^2d^2ix^2\log(e)^2 + 2\left(abd^2i\log(e) + \left(i\log(e)^2 - i\log(e)\right)b^2cd\right)B^2x + \left(B^2b^2d^2ix^2 + 2\,B^2b^2cdix + (2\,c\right.}{}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxima"
)
```

output
```
1/2*A^2*d*i*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x +
a)/b - c*log(d*x + c)/d)*A*B*c*i + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c
)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*
B*d*i + A^2*c*i*x - ((i*log(e) - i)*b*c^2 + a*c*d*i)*B^2*log(d*x + c)/(b*d
) - (b^2*c^2*i - 2*a*b*c*d*i + a^2*d^2*i)*(log(b*x + a)*log((b*d*x + a*d)/
(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d) + 1/2*(B
^2*b^2*d^2*i*x^2*log(e)^2 + 2*(a*b*d^2*i*log(e) + (i*log(e)^2 - i*log(e))*
b^2*c*d)*B^2*x + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + (2*a*b*c*d*i - a
^2*d^2*i)*B^2)*log(b*x + a)^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + B
^2*b^2*c^2*i)*log(d*x + c)^2 + 2*(B^2*b^2*d^2*i*x^2*log(e) + ((2*i*log(e)
- i)*b^2*c*d + a*b*d^2*i)*B^2*x + ((2*i*log(e) - i)*a*b*c*d - (i*log(e) -
i)*a^2*d^2)*B^2)*log(b*x + a) - 2*(B^2*b^2*d^2*i*x^2*log(e) + ((2*i*log(e)
 - i)*b^2*c*d + a*b*d^2*i)*B^2*x + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x
+ (2*a*b*c*d*i - a^2*d^2*i)*B^2)*log(b*x + a))*log(d*x + c))/(b^2*d)
```

### 3.58.8 Giac [F]

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

### 3.58.9 Mupad [F(-1)]

Timed out.

$$\int (ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

# 3.59 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$



## 3.59.1 Optimal result

Integrand size = 40, antiderivative size = 286

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx &= \frac{2B(bc-ad)i\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g}+\frac{di(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g}\\ &\quad-\frac{(bc-ad)i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}\\ &\quad+\frac{2B^2(bc-ad)i\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2g}\\ &\quad+\frac{2B(bc-ad)i\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}\\ &\quad+\frac{2B^2(bc-ad)i\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}\end{aligned}$$

output
```
2*B*(-a*d+b*c)*i*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/
g+d*i*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2/g-(-a*d+b*c)*i*(A+B*ln(e*(
b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/g+2*B^2*(-a*d+b*c)*i*poly
log(2,d*(b*x+a)/b/(d*x+c))/b^2/g+2*B*(-a*d+b*c)*i*(A+B*ln(e*(b*x+a)/(d*x+c
)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g+2*B^2*(-a*d+b*c)*i*polylog(3,b*(d
*x+c)/d/(b*x+a))/b^2/g
```

### 3.59.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1214 vs. $2(286) = 572$.

Time = 0.82 (sec) , antiderivative size = 1214, normalized size of antiderivative = 4.24

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \text{Too large to display}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*
g*x),x]
```

output
```
(i*(3*A^2*b*d*x + 3*A^2*(b*c - a*d)*Log[a + b*x] - 3*A*B*(a*d*Log[a/b + x]
^2 - 2*a*d*Log[a/b + x]*(1 + Log[a + b*x]) + 2*(-(b*c) + a*d + Log[c/d + x
]*(b*c + a*d*Log[a + b*x] - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) + (-(b*
d*x) + a*d*Log[a + b*x])*Log[(e*(a + b*x))/(c + d*x)]) - 2*a*d*PolyLog[2,
(b*(c + d*x))/(b*c - a*d)]) + 3*A*b*B*c*(Log[a/b + x]^2 - 2*Log[a + b*x]*(
Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)]) - 2*(Log[c/d +
 x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*
d)])) - B^2*(a*d*Log[a/b + x]^3 - 3*d*(2*b*x - 2*(a + b*x)*Log[a/b + x] +
(a + b*x)*Log[a/b + x]^2) - 3*b*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (c + d
*x)*Log[c/d + x]^2) - 3*d*(b*x - a*Log[a + b*x])*(-Log[a/b + x] + Log[c/d
+ x] + Log[(e*(a + b*x))/(c + d*x)])^2 + 6*(a*d + 2*b*d*x - b*d*x*Log[c/d
+ x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c
/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLo
g[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 3*(Log[a/b + x] - Log[c/d + x] - Log
[(e*(a + b*x))/(c + d*x)])*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a/b + x] +
a*d*Log[a/b + x]^2 + 2*Log[c/d + x]*(b*(c + d*x) - a*d*Log[(d*(a + b*x))/(
-(b*c) + a*d)]) - 2*a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 3*a*d*(Lo
g[a/b + x]^2*(Log[c/d + x] - Log[(b*(c + d*x))/(b*c - a*d)]) - 2*Log[a/b +
 x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*PolyLog[3, (d*(a + b*x))/
(-(b*c) + a*d)]) + 3*a*d*(Log[c/d + x]^2*Log[(d*(a + b*x))/(-(b*c) + a*...
```

### 3.59.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 272, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2962, 2789, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ag+bgx}dx$$

$$\Big\downarrow \ 2962$$

$$\frac{i(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \ 2789$$

$$\frac{i(bc-ad)\left(\frac{d\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \ 2755$$

$$\frac{i(bc-ad)\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2B\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \ 2754$$

$$i(bc-ad)\left(\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2B\left(\frac{B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+b}{b(c+d}\right.}{}\right.}{b}\right.}{b}\right.$$
$$\overline{\qquad g \qquad}$$

$\downarrow$ 2779

$$i(bc-ad)\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2B\left(\frac{B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}\right)}{b}\right)}{b}+\frac{2B\int\frac{(c+}{}}{}\right.$$
$$\overline{\qquad g \qquad}$$

$\downarrow$ 2821

$$i(bc-ad)\left(\frac{\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\int\frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}}{b}+\right.$$
$$\overline{\qquad g \qquad}$$

---

3.59. $\displaystyle\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$\Big\downarrow$ 2838

$$\frac{i(bc-ad)\left(\frac{\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\int\frac{(c+dx)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}}{b}+\right.}{g}$$

$\Big\downarrow$ 7143

$$\frac{i(bc-ad)\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2B\left(-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d}-\frac{B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d}\right)}{b}\right)}{b}+\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c-}{d(a-}\right.\right.}{}\right.}{g}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x),x
]
```

output
```
((b*c - a*d)*i*((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(b*
(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (2*B*(-(((A + B*Log[(e*(a + b*x
))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*PolyLog[2, (d
*(a + b*x))/(b*(c + d*x))])/d))/b))/b + (-(((A + B*Log[(e*(a + b*x))/(c +
d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (2*B*((A + B*Log[(e*(a
 + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + B*PolyLog[3
, (b*(c + d*x))/(d*(a + b*x))]))/b)/b))/g
```

## 3.59. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

### 3.59.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

## 3.59. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.59.4 Maple [F]

$$\int \frac{(dix + ci)\left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{bgx + ag}dx$$

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

### 3.59.5 Fricas [F]

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag + bgx}\,dx = \int \frac{(dix + ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{bgx + ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algori
thm="fricas")
```

output
```
integral((A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log((b*e*x + a*e)/(d
*x + c))^2 + 2*(A*B*d*i*x + A*B*c*i)*log((b*e*x + a*e)/(d*x + c)))/(b*g*x
+ a*g), x)
```

### 3.59.6 Sympy [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \frac{i\left(\int \frac{A^2c}{a+bx}\,dx+\int \frac{A^2dx}{a+bx}\,dx+\int \frac{B^2c\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx+\int \frac{2ABc\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx+\int \frac{B^2dx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx\ \cdots}{g}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)
```

output
```
i*(Integral(A**2*c/(a + b*x), x) + Integral(A**2*d*x/(a + b*x), x) + Integ
ral(B**2*c*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integra
l(2*A*B*c*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(B*
*2*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(2*
A*B*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g
```

### 3.59.7 Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algori
thm="maxima")
```

output
```
A^2*d*i*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + A^2*c*i*log(b*g*x + a*g)/(b*g
) + (B^2*b*d*i*x + (b*c*i - a*d*i)*B^2*log(b*x + a))*log(d*x + c)^2/(b^2*g
) - integrate(-(B^2*b^2*c^2*i*log(e)^2 + 2*A*B*b^2*c^2*i*log(e) + (B^2*b^2
*d^2*i*log(e)^2 + 2*A*B*b^2*d^2*i*log(e))*x^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2
*b^2*c*d*i*x + B^2*b^2*c^2*i)*log(b*x + a)^2 + 2*(B^2*b^2*c*d*i*log(e)^2 +
 2*A*B*b^2*c*d*i*log(e))*x + 2*(B^2*b^2*c^2*i*log(e) + A*B*b^2*c^2*i + (B^
2*b^2*d^2*i*log(e) + A*B*b^2*d^2*i)*x^2 + 2*(B^2*b^2*c*d*i*log(e) + A*B*b^
2*c*d*i)*x)*log(b*x + a) - 2*(B^2*b^2*c^2*i*log(e) + A*B*b^2*c^2*i + ((i*l
og(e) + i)*B^2*b^2*d^2 + A*B*b^2*d^2*i)*x^2 + (2*A*B*b^2*c*d*i + (2*b^2*c*
d*i*log(e) + a*b*d^2*i)*B^2)*x + (B^2*b^2*d^2*i*x^2 + (3*b^2*c*d*i - a*b*d
^2*i)*B^2*x + (b^2*c^2*i + a*b*c*d*i - a^2*d^2*i)*B^2)*log(b*x + a))*log(d
*x + c))/(b^3*d*g*x^2 + a*b^2*c*g + (b^3*c*g + a*b^2*d*g)*x), x)
```

### 3.59.8 Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algori
thm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g)
, x)
```

### 3.59.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x),x
)
```

output
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x),
x)
```

# 3.60 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$



## 3.60.1 Optimal result

Integrand size = 40, antiderivative size = 241

$$\begin{aligned}&\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx\\
&=-\frac{2B^2i(c+dx)}{bg^2(a+bx)}-\frac{2Bi(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{bg^2(a+bx)}\\
&\quad-\frac{i(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{bg^2(a+bx)}-\frac{di\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}\\
&\quad+\frac{2Bdi\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}+\frac{2B^2di\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}\end{aligned}$$

output
```
-2*B^2*i*(d*x+c)/b/g^2/(b*x+a)-2*B*i*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b
/g^2/(b*x+a)-i*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b/g^2/(b*x+a)-d*i*(A+
B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/g^2+2*B*d*i*(A+B*
ln(e*(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g^2+2*B^2*d*i*po
lylog(3,b*(d*x+c)/d/(b*x+a))/b^2/g^2
```

### 3.60.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1407 vs. $2(241) = 482$.

Time = 1.21 (sec) , antiderivative size = 1407, normalized size of antiderivative = 5.84

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*
g*x)^2,x]
```

output
```
(i*((3*A^2*(-(b*c) + a*d))/(a + b*x) + 3*A^2*d*Log[a + b*x] - (6*A*b*B*c*(
-(d*(a + b*x)*Log[c/d + x]) + d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)
] + (b*c - a*d)*(1 + Log[(e*(a + b*x))/(c + d*x)])))/((b*c - a*d)*(a + b*x
)) + (3*b*B^2*c*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a + b*x] - 2*(b*c - a*
d)*Log[(e*(a + b*x))/(c + d*x)] - 2*d*(a + b*x)*Log[a + b*x]*Log[(e*(a + b
*x))/(c + d*x)] - (b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)]^2 + 2*d*(a + b*
x)*Log[c + d*x] - 2*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*
d)/(b*c + b*d*x)] + d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
+ d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + d*(a
 + b*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)])))/((b*c - a*d)*(a + b*x)) + 3*A*B*d*(Log[a/b + x]^2 - 2*Log[a/b + x]
*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a
 + b*x]*((a*d)/(b*c - a*d) + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])
+ 2*a*((a + b*x)^(-1) + Log[(e*(a + b*x))/(c + d*x)]/(a + b*x) + (d*Log[c
+ d*x])/(-(b*c) + a*d)) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + (B^2*
d*((b*c - a*d)*(a + b*x)*Log[a/b + x]^3 + 3*a*(b*c - a*d)*(2 + 2*Log[a/b +
 x] + Log[a/b + x]^2) + 3*(b*c - a*d)*(a + (a + b*x)*Log[a + b*x])*(-Log[a
/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2 + 3*a*(d*(a + b*x
)*Log[a/b + x]^2 + 2*((-(b*c) + a*d)*Log[c/d + x] + d*(a + b*x)*(Log[a ...
```

### 3.60.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.175, Rules used = {2962, 2780, 2742, 2741, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{ 2962}$$

$$\frac{i\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{ 2780}$$

$$\frac{i\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^2}$$

$$\Big\downarrow \text{ 2742}$$

$$\frac{i\left(\frac{2B\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^2}$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{i\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{2B\left(-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}}{b}\right)}{g^2}$$

$$\Big\downarrow \text{ 2779}$$

$$\frac{i\left(\frac{d\left(\frac{2B\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}\right)}{b}+\frac{2B\left(-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}\right.}{}\right.}{g^2}$$

$\downarrow$ 2821

$$\frac{i\left(\frac{d\left(\frac{2B\left(\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\int\frac{(c+dx)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}\right)}{b}+\frac{2B\left(\right.}{}\right.}{g^2}$$

$\downarrow$ 7143

$$\frac{i\left(\frac{d\left(\frac{2B\left(\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+B\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}\right)}{b}+\frac{2B\left(-\frac{(c+dx)\left(B\log\left(\right.\right.}{a+}\right.}{}\right.}{g^2}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^2
,x]
```

output
```
(i*((-(((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x)) + 2*B
*(-((B*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(a + b*x)))/b + (d*(-(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1
- (b*(c + d*x))/(d*(a + b*x))])/b) + (2*B*((A + B*Log[(e*(a + b*x))/(c + d
*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + B*PolyLog[3, (b*(c + d*x))
/(d*(a + b*x))]))/b))/b))/g^2
```

## 3.60. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

### 3.60.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

---

## 3.60. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.60.4 Maple [F]

$$\int \frac{(dix + ci)\left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx + ag)^2}dx$$

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

### 3.60.5 Fricas [F]

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^2}dx = \int \frac{(dix + ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{(bgx + ag)^2}dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algo
rithm="fricas")
```

output
```
integral((A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log((b*e*x + a*e)/(d
*x + c))^2 + 2*(A*B*d*i*x + A*B*c*i)*log((b*e*x + a*e)/(d*x + c)))/(b^2*g^
2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

### 3.60.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^2}dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

### 3.60.7 Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algo
rithm="maxima")
```

output
```
A^2*d*i*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - 2*A*B*c*i*(
log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^2*x
+ a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c
 - a*b*d)*g^2)) - A^2*c*i/(b^2*g^2*x + a*b*g^2) - ((b*c*i - a*d*i)*B^2 - (
B^2*b*d*i*x + B^2*a*d*i)*log(b*x + a))*log(d*x + c)^2/(b^3*g^2*x + a*b^2*g
^2) - integrate(-(B^2*b^2*c^2*i*log(e)^2 + (B^2*b^2*d^2*i*log(e)^2 + 2*A*B
*b^2*d^2*i*log(e))*x^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + B^2*b^2*
c^2*i)*log(b*x + a)^2 + 2*(B^2*b^2*c*d*i*log(e)^2 + A*B*b^2*c*d*i*log(e))*
x + 2*(B^2*b^2*c^2*i*log(e) + (B^2*b^2*d^2*i*log(e) + A*B*b^2*d^2*i)*x^2 +
 (2*B^2*b^2*c*d*i*log(e) + A*B*b^2*c*d*i)*x)*log(b*x + a) - 2*((b^2*c^2*i*
log(e) - a*b*c*d*i + a^2*d^2*i)*B^2 + (B^2*b^2*d^2*i*log(e) + A*B*b^2*d^2*
i)*x^2 + (A*B*b^2*c*d*i + ((2*i*log(e) - i)*b^2*c*d + a*b*d^2*i)*B^2)*x +
(2*B^2*b^2*d^2*i*x^2 + 2*(b^2*c*d*i + a*b*d^2*i)*B^2*x + (b^2*c^2*i + a^2*
d^2*i)*B^2)*log(b*x + a))*log(d*x + c))/(b^4*d*g^2*x^3 + a^2*b^2*c*g^2 + (
b^4*c*g^2 + 2*a*b^3*d*g^2)*x^2 + (2*a*b^3*c*g^2 + a^2*b^2*d*g^2)*x), x)
```

### 3.60.8 Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, algo
rithm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*g)
^2, x)
```

### 3.60.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{{(a\,g+b\,g\,x)}^2}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^2
,x)
```

output
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^2
, x)
```

# 3.61 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$



## 3.61.1 Optimal result

Integrand size = 40, antiderivative size = 141

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{B^2i(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{Bi(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)g^3(a+bx)^2}$$

output
```
-1/4*B^2*i*(d*x+c)^2/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*B*i*(d*x+c)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*i*(d*x+c)^2*(A+B*ln(e*(b*x+
a)/(d*x+c)))^2/(-a*d+b*c)/g^3/(b*x+a)^2
```

### 3.61.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.53 (sec) , antiderivative size = 765, normalized size of antiderivative = 5.43

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{i\left(2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-4d(-bc+ad)(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+4Bd(a+bx)\right.}{}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*
g*x)^3,x]
```

output
```
-1/4*(i*(2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 4*d*(-(b
*c) + a*d)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 4*B*d*(a + b
*x)*(2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*d*(a + b*x)*Lo
g[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(a + b*x)*(A + B*Log
[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*B*(b*c - a*d + d*(a + b*x)*Log
[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*(a + b*x)*(Log[a + b*x]*(Log[a
 + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(
-(b*c) + a*d)]) + B*d*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Lo
g[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B*(
2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d*(-(b*c) + a*d)*
(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*d^2*(a + b*x)^2*Log[a +
 b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d^2*(a + b*x)^2*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 4*B*d*(a + b*x)*(b*c - a*d + d*(a
 + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*((b*c - a*d)^2 + 2*d*
(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d^2*(a + b*x
)^2*Log[c + d*x]) + 2*B*d^2*(a + b*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Lo
g[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]
) - 2*B*d^2*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*
x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))))/(b^2*(b*c -
 a*d)*g^3*(a + b*x)^2)
```

### 3.61.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 114, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2962, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3}dx$$

$$\Big\downarrow \; 2962$$

$$\frac{i\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)}$$

$$\Big\downarrow \; 2742$$

$$\frac{i\left(B\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}\right)}{g^3(bc-ad)}$$

$$\Big\downarrow \; 2741$$

$$\frac{i\left(B\left(-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}\right)-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}\right)}{g^3(bc-ad)}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^3
,x]
```

output
```
(i*(-1/2*((c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x)^2
+ B*(-1/4*(B*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[(e*(a + b*
x))/(c + d*x)]))/(2*(a + b*x)^2))))/((b*c - a*d)*g^3)
```

### 3.61.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.61.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 329 vs. 2(135) = 270.

Time = 0.80 (sec) , antiderivative size = 330, normalized size of antiderivative = 2.34

| method | result |
|---|---|
| parts | $\frac{iA^2\left(-\frac{-ad+cb}{2b^2(bx+a)^2}-\frac{d}{b^2(bx+a)}\right)}{g^3}-\frac{iB^2e^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{g^3(ad-cb)}-\frac{2iBA}{}$ |
| norman | $\frac{\frac{B^2cdix\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}+\frac{ciBd(2A+B)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{g(ad-cb)}-\frac{2A^2aid+2A^2bci+2adiBA+2bciBA+adiB^2+B^2bci}{4gb^2}-\frac{(2A^2id+2diBA+B^2}{2gb}}{(bx+a)^2g^2}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{id^2eA^2}{2(ad-cb)^2g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{2id^2eAB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{(ad-cb)^2g^3}+\frac{id^2eB^2\left(-\frac{\ln\left(\frac{be}{d}\right.}{2\left(\frac{be}{d}\right.}\right.}{}\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{id^2eA^2}{2(ad-cb)^2g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{2id^2eAB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}\right)}{(ad-cb)^2g^3}+\frac{id^2eB^2\left(-\frac{\ln\left(\frac{be}{d}\right.}{2\left(\frac{be}{d}\right.}\right.}{}\right.}{d^2}$ |
| risch | $\frac{iA^2ad}{2g^3b^2(bx+a)^2}-\frac{iA^2c}{2g^3b(bx+a)^2}-\frac{iA^2d}{g^3b^2(bx+a)}+\frac{iB^2e^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2g^3(ad-cb)\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ecb}{d(dx+c)}\right)^2}+\frac{iB^2e^2\ln\left(\frac{be}{d}+\frac{(ad-cb}{d(dx+}\right.}{2g^3(ad-cb)\left(\frac{be}{d}+\frac{ea}{dx+c}-\right.}$ |
| parallelrisch | $-\frac{2A^2a^2b^2d^3i+B^2a^2b^2d^3i-B^2b^4c^2di-2A^2c^2ib^4d+2AB\,a^2b^2d^3i-2AB\,b^4c^2di-8ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^4cd^2i-4AB\,x^2\ln}{}$ |

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x,method=_RETU
RNVERBOSE)
```

output
```
i*A^2/g^3*(-1/2*(-a*d+b*c)/b^2/(b*x+a)^2-d/b^2/(b*x+a))-i*B^2/g^3/(a*d-b*c
)*e^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/
4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*i*B*A/g^3/(a*d-b*c)*e^2*(-1/2/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2)
```

## 3.61. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

### 3.61.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 289 vs. 2(135) = 270.

Time = 0.31 (sec) , antiderivative size = 289, normalized size of antiderivative = 2.05

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{2\left((2A^2+2AB+B^2)b^2cd-(2A^2+2AB+B^2)abd^2\right)ix+2\left(B^2b^2d^2ix^2+2B^2b^2cdix+B^2b^2c^2i\right)\log(}{4\left((b^5c-\right.}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algo
rithm="fricas")
```

output
```
-1/4*(2*((2*A^2 + 2*A*B + B^2)*b^2*c*d - (2*A^2 + 2*A*B + B^2)*a*b*d^2)*i*
x + 2*(B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x + B^2*b^2*c^2*i)*log((b*e*x +
 a*e)/(d*x + c))^2 + ((2*A^2 + 2*A*B + B^2)*b^2*c^2 - (2*A^2 + 2*A*B + B^2
)*a^2*d^2)*i + 2*((2*A*B + B^2)*b^2*d^2*i*x^2 + 2*(2*A*B + B^2)*b^2*c*d*i*
x + (2*A*B + B^2)*b^2*c^2*i)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c - a*b^4
*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3)
```

### 3.61.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 714 vs. 2(122) = 244.

Time = 5.02 (sec) , antiderivative size = 714, normalized size of antiderivative = 5.06

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx =$$
$$-\frac{Bd^2i(2A+B)\log\left(x+\frac{2ABad^3i+2ABbcd^2i+B^2ad^3i+B^2bcd^2i-\frac{Ba^2d^4i(2A+B)}{ad-bc}+\frac{2Babcd^3i(2A+B)}{ad-bc}-\frac{Bb^2c^2d^2i(2A+B)}{ad-bc}}{4ABbd^3i+2B^2bd^3i}\right)}{2b^2g^3(ad-bc)}$$
$$+\frac{Bd^2i(2A+B)\log\left(x+\frac{2ABad^3i+2ABbcd^2i+B^2ad^3i+B^2bcd^2i+\frac{Ba^2d^4i(2A+B)}{ad-bc}-\frac{2Babcd^3i(2A+B)}{ad-bc}+\frac{Bb^2c^2d^2i(2A+B)}{ad-bc}}{4ABbd^3i+2B^2bd^3i}\right)}{2b^2g^3(ad-bc)}$$
$$+\frac{(B^2c^2i+2B^2cdix+B^2d^2ix^2)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^3dg^3-2a^2bcg^3+4a^2bdg^3x-4ab^2cg^3x+2ab^2dg^3x^2-2b^3cg^3x^2}$$
$$+\frac{-2A^2adi-2A^2bci-2ABadi-2ABbci-B^2adi-B^2bci+x(-4A^2bdi-4ABbdi-2B^2bdi)}{4a^2b^2g^3+8ab^3g^3x+4b^4g^3x^2}$$
$$+\frac{(-2ABadi-2ABbci-4ABbdix-B^2adi-B^2bci-2B^2bdix)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2a^2b^2g^3+4ab^3g^3x+2b^4g^3x^2}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)
```

output
```
-B*d**2*i*(2*A + B)*log(x + (2*A*B*a*d**3*i + 2*A*B*b*c*d**2*i + B**2*a*d*
*3*i + B**2*b*c*d**2*i - B*a**2*d**4*i*(2*A + B)/(a*d - b*c) + 2*B*a*b*c*d
**3*i*(2*A + B)/(a*d - b*c) - B*b**2*c**2*d**2*i*(2*A + B)/(a*d - b*c))/(4
*A*B*b*d**3*i + 2*B**2*b*d**3*i))/(2*b**2*g**3*(a*d - b*c)) + B*d**2*i*(2*
A + B)*log(x + (2*A*B*a*d**3*i + 2*A*B*b*c*d**2*i + B**2*a*d**3*i + B**2*b
*c*d**2*i + B*a**2*d**4*i*(2*A + B)/(a*d - b*c) - 2*B*a*b*c*d**3*i*(2*A +
B)/(a*d - b*c) + B*b**2*c**2*d**2*i*(2*A + B)/(a*d - b*c))/(4*A*B*b*d**3*i
 + 2*B**2*b*d**3*i))/(2*b**2*g**3*(a*d - b*c)) + (B**2*c**2*i + 2*B**2*c*d
*i*x + B**2*d**2*i*x**2)*log(e*(a + b*x)/(c + d*x))**2/(2*a**3*d*g**3 - 2*
a**2*b*c*g**3 + 4*a**2*b*d*g**3*x - 4*a*b**2*c*g**3*x + 2*a*b**2*d*g**3*x*
*2 - 2*b**3*c*g**3*x**2) + (-2*A**2*a*d*i - 2*A**2*b*c*i - 2*A*B*a*d*i - 2
*A*B*b*c*i - B**2*a*d*i - B**2*b*c*i + x*(-4*A**2*b*d*i - 4*A*B*b*d*i - 2*
B**2*b*d*i))/(4*a**2*b**2*g**3 + 8*a*b**3*g**3*x + 4*b**4*g**3*x**2) + (-2
*A*B*a*d*i - 2*A*B*b*c*i - 4*A*B*b*d*i*x - B**2*a*d*i - B**2*b*c*i - 2*B**
2*b*d*i*x)*log(e*(a + b*x)/(c + d*x))/(2*a**2*b**2*g**3 + 4*a*b**3*g**3*x
+ 2*b**4*g**3*x**2)
```

## 3.61. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

### 3.61.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1987 vs. $2(135) = 270$.

Time = 0.30 (sec) , antiderivative size = 1987, normalized size of antiderivative = 14.09

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algo
rithm="maxima")
```

output
```
-1/2*(2*b*x + a)*B^2*d*i*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^4*g^3*x
^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + 1/4*(2*((2*b*d*x - b*c + 3*a*d)/((b^4*
c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*
d)*g^3) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2
*d^2*log(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3))*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) - (b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^
2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x +
 a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*
b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d
^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))/(
a^2*b^3*c^2*g^3 - 2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b
^4*c*d*g^3 + a^2*b^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 +
 a^3*b^2*d^2*g^3)*x))*B^2*c*i - 1/4*(2*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*
b*d)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*
b^3*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a
*b^3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2
- 2*a*b^3*c*d + a^2*b^2*d^2)*g^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) +
(7*a*b^2*c^2 - 8*a^2*b*c*d + a^3*d^2 - 2*(2*a^2*b*c*d - a^3*d^2 + (2*b^3*c
*d - a*b^2*d^2)*x^2 + 2*(2*a*b^2*c*d - a^2*b*d^2)*x)*log(b*x + a)^2 - 2*(2
*a^2*b*c*d - a^3*d^2 + (2*b^3*c*d - a*b^2*d^2)*x^2 + 2*(2*a*b^2*c*d - a...
```

### 3.61.8 Giac [A] (verification not implemented)

Time = 0.45 (sec) , antiderivative size = 206, normalized size of antiderivative = 1.46

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(\frac{2\,(dx+c)^2B^2e^3i\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)^2g^3}+\frac{2\,(2\,ABe^3i+B^2e^3i)(dx+c)^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^2g^3}+\frac{(2\,A^2e^3i+2\,ABe^3i+\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, algo
rithm="giac")
```

output
```
-1/4*(2*(d*x + c)^2*B^2*e^3*i*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e
)^2*g^3) + 2*(2*A*B*e^3*i + B^2*e^3*i)*(d*x + c)^2*log((b*e*x + a*e)/(d*x
+ c))/((b*e*x + a*e)^2*g^3) + (2*A^2*e^3*i + 2*A*B*e^3*i + B^2*e^3*i)*(d*x
 + c)^2/((b*e*x + a*e)^2*g^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((
b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.61.9 Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 469, normalized size of antiderivative = 3.33

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{x\,(2\,b\,d\,i\,A^2+2\,b\,d\,i\,A\,B+b\,d\,i\,B^2)+A^2\,a\,d\,i+A^2\,b\,c\,i+\frac{B^2\,a\,d\,i}{2}+\frac{B^2\,b\,c\,i}{2}+A\,B\,a\,d\,i+A\,B\,b\,c\,i}{2\,a^2\,b^2\,g^3+4\,a\,b^3\,g^3\,x+2\,b^4\,g^3\,x^2}$$
$$-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{\frac{B^2\,c\,i}{2\,b^2\,g^3}+\frac{B^2\,a\,d\,i}{2\,b^3\,g^3}+\frac{B^2\,d\,i\,x}{b^2\,g^3}}{2\,a\,x+b\,x^2+\frac{a^2}{b}}-\frac{B^2\,d^2\,i}{2\,b^2\,g^3\,(a\,d-b\,c)}\right)$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(x\left(\frac{B^2\,i}{b^2\,g^3}+\frac{2\,A\,B\,i}{b^2\,g^3}\right)+\frac{A\,B\,a\,i}{b^3\,g^3}+\frac{B\,i\,(A\,b\,c-B\,a\,d+B\,b\,c)}{b^3\,d\,g^3}+\frac{B^2\,d^2\,i\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{2\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{2\,b\,d^2}\right)}{b^2\,g^3\,(a\,d-b\,c)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}$$
$$-\frac{B\,d^2\,i\,\mathrm{atan}\left(\frac{\left(\frac{2\,c\,b^3\,g^3+2\,a\,d\,b^2\,g^3}{2\,b^2\,g^3}+2\,b\,d\,x\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(2\,A+B)\,1\mathrm{i}}{b^2\,g^3\,(a\,d-b\,c)}$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^3
,x)
```

output
```
- (x*(2*A^2*b*d*i + B^2*b*d*i + 2*A*B*b*d*i) + A^2*a*d*i + A^2*b*c*i + (B^
2*a*d*i)/2 + (B^2*b*c*i)/2 + A*B*a*d*i + A*B*b*c*i)/(2*a^2*b^2*g^3 + 2*b^4
*g^3*x^2 + 4*a*b^3*g^3*x) - log((e*(a + b*x))/(c + d*x))^2*(((B^2*c*i)/(2*
b^2*g^3) + (B^2*a*d*i)/(2*b^3*g^3) + (B^2*d*i*x)/(b^2*g^3))/(2*a*x + b*x^2
 + a^2/b) - (B^2*d^2*i)/(2*b^2*g^3*(a*d - b*c))) - (log((e*(a + b*x))/(c +
 d*x))*(x*((B^2*i)/(b^2*g^3) + (2*A*B*i)/(b^2*g^3)) + (A*B*a*i)/(b^3*g^3)
+ (B*i*(A*b*c - B*a*d + B*b*c))/(b^3*d*g^3) + (B^2*d^2*i*((2*a^2*d^2 + b^2
*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(b^2*g^3*(a*d -
b*c))))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d) - (B*d^2*i*atan((((2*b^3*c*g^3
 + 2*a*b^2*d*g^3)/(2*b^2*g^3) + 2*b*d*x)*1i)/(a*d - b*c))*(2*A + B)*1i)/(b
^2*g^3*(a*d - b*c))
```

3.61. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

# 3.62 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$



## 3.62.1 Optimal result

Integrand size = 40, antiderivative size = 287

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = {} & \frac{B^2di(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2} \\ & - \frac{2bB^2i(c+dx)^3}{27(bc-ad)^2g^4(a+bx)^3} \\ & + \frac{Bdi(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} \\ & - \frac{2bBi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^2g^4(a+bx)^3} \\ & + \frac{di(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2g^4(a+bx)^2} \\ & - \frac{bi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^2g^4(a+bx)^3}\end{aligned}$$

output
```
1/4*B^2*d*i*(d*x+c)^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-2/27*b*B^2*i*(d*x+c)^3/(-
a*d+b*c)^2/g^4/(b*x+a)^3+1/2*B*d*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(
-a*d+b*c)^2/g^4/(b*x+a)^2-2/9*b*B*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/
(-a*d+b*c)^2/g^4/(b*x+a)^3+1/2*d*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2
/(-a*d+b*c)^2/g^4/(b*x+a)^2-1/3*b*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^
2/(-a*d+b*c)^2/g^4/(b*x+a)^3
```

## 3.62.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.60 (sec) , antiderivative size = 1032, normalized size of antiderivative = 3.60

$$\int \frac{(ci + dix) \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^4}\,dx =$$
$$-\frac{i\left(36(bc-ad)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + 54d(bc-ad)^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + 2B\left(12A(bc\right.\right.}{}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*
g*x)^4,x]
```

output
```
-1/108*(i*(36*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 54*d*
(b*c - a*d)^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 2*B*(12*A
*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B
*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(
b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*(a +
 b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a
*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e
*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(a + b*x)
)/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*
x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*Log[c + d*x
] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] -
36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - 18*B*d^3*
(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c +
 d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c
) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) +
27*B*d*(a + b*x)*(2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4
*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 4*d^2*(
a + b*x)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*d^2*(a +
b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - 4*B*d*(a + b*x)
*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) + B*...
```

### 3.62.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 218, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{i\int \left(\frac{b(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}-\frac{d(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}}{g^4(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{b(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}-\frac{2bB(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}+\frac{Bd(c+dx)^2\left(B\log\left(\frac{e(a+b\ldots}{c+dx}\right.\right.}{2(a+bx)^2}\right.}{g^4(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^4
,x]
```

output
```
(i*((B^2*d*(c + d*x)^2)/(4*(a + b*x)^2) - (2*b*B^2*(c + d*x)^3)/(27*(a + b
*x)^3) + (B*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*
x)^2) - (2*b*B*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(a + b
*x)^3) + (d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*
x)^2) - (b*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(a + b*x
)^3)))/((b*c - a*d)^2*g^4)
```

#### 3.62.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.62.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 642 vs. $2(275) = 550$.

Time = 0.99 (sec) , antiderivative size = 643, normalized size of antiderivative = 2.24

| method | result |
|---|---|
| parts | $\frac{iA^2\left(-\frac{-ad+cb}{3b^2(bx+a)^3}-\frac{d}{2b^2(bx+a)^2}\right)}{g^4} - \frac{iB^2(ad-cb)^2e^2\left(\frac{d^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.}\right.}{(ad-cb)^4}\right.}{g^4 c}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{i\,d^2e^2A^2b}{3(ad-cb)^3g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i\,d^3eA^2}{2(ad-cb)^3g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{2i\,d^2e^2ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{}{9\left(\frac{be}{d}+\right.}\right.}{(ad-cb)^3g^4}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(\frac{i\,d^2e^2A^2b}{3(ad-cb)^3g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{i\,d^3eA^2}{2(ad-cb)^3g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{2i\,d^2e^2ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{}{9\left(\frac{be}{d}+\right.}\right.}{(ad-cb)^3g^4}\right.}{}$ |
| norman | $-\frac{\frac{18A^2a^2b\,d^2i+18A^2a\,b^2cdi-36A^2b^3c^2i+30AB\,a^2b\,d^2i+30ABa\,b^2cdi-24AB\,b^3c^2i+19B^2a^2b\,d^2i+19B^2a\,b^2cdi-8B^2b^3c^2i}{108g\,b^3(ad-cb)}-\frac{(18A}{}}{}$ |
| parallelrisch | $-\frac{-216ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^5c\,d^3i-27B^2a\,b^5c^2d^2i-54ABa\,b^5c^2d^2i-18B^2x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^6d^4i-30B^2x^3\ln\left(\frac{e(bx+a)}{dx+c}\right.}{}$ |
| risch | Expression too large to display |

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x,method=_RETU
RNVERBOSE)
```

output
```
i*A^2/g^4*(-1/3*(-a*d+b*c)/b^2/(b*x+a)^3-1/2*d/b^2/(b*x+a)^2)-i*B^2/g^4/d^
3*(a*d-b*c)^2*e^2*(d^4/(a*d-b*c)^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-d^3/(a*d-b
*c)^4*b*e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2*i*B*A/g^4/d^3*(a*d-b*c)^2*e^2*(
d^4/(a*d-b*c)^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-d^3/(a*d-b*c)^4*b*e*(-1/3
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/
d+(a*d-b*c)*e/d/(d*x+c))^3))
```

## 3.62. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

### 3.62.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(275) = 550$.

Time = 0.31 (sec) , antiderivative size = 601, normalized size of antiderivative = 2.09

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$$
$$= \frac{6\left((6\,AB+5\,B^2)b^3cd^2-(6\,AB+5\,B^2)ab^2d^3\right)ix^2-3\left((18\,A^2+6\,AB-B^2)b^3c^2d-18\,(2\,A^2+2\,AB+B^2\right.}{}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algo
rithm="fricas")
```

output
```
1/108*(6*((6*A*B + 5*B^2)*b^3*c*d^2 - (6*A*B + 5*B^2)*a*b^2*d^3)*i*x^2 - 3
*((18*A^2 + 6*A*B - B^2)*b^3*c^2*d - 18*(2*A^2 + 2*A*B + B^2)*a*b^2*c*d^2
+ (18*A^2 + 30*A*B + 19*B^2)*a^2*b*d^3)*i*x + 18*(B^2*b^3*d^3*i*x^3 + 3*B^
2*a*b^2*d^3*i*x^2 - 3*(B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^2)*i*x - (2*B^2*b^3
*c^3 - 3*B^2*a*b^2*c^2*d)*i)*log((b*e*x + a*e)/(d*x + c))^2 - (4*(9*A^2 +
6*A*B + 2*B^2)*b^3*c^3 - 27*(2*A^2 + 2*A*B + B^2)*a*b^2*c^2*d + (18*A^2 +
30*A*B + 19*B^2)*a^3*d^3)*i + 6*((6*A*B + 5*B^2)*b^3*d^3*i*x^3 + 3*(2*B^2*
b^3*c*d^2 + 3*(2*A*B + B^2)*a*b^2*d^3)*i*x^2 - 3*((6*A*B + B^2)*b^3*c^2*d
- 6*(2*A*B + B^2)*a*b^2*c*d^2)*i*x - (4*(3*A*B + B^2)*b^3*c^3 - 9*(2*A*B +
 B^2)*a*b^2*c^2*d)*i)*log((b*e*x + a*e)/(d*x + c)))/((b^7*c^2 - 2*a*b^6*c*
d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4
*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2
- 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4)
```

### 3.62.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1387 vs. $2(267) = 534$.

Time = 10.34 (sec) , antiderivative size = 1387, normalized size of antiderivative = 4.83

$$\int \frac{(ci + dix)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^4}\,dx =$$

$$-\frac{Bd^3i(6A+5B)\log\left(x + \frac{6ABad^4i+6ABbcd^3i+5B^2ad^4i+5B^2bcd^3i-\frac{Ba^3d^6i(6A+5B)}{(ad-bc)^2}+\frac{3Ba^2bcd^5i(6A+5B)}{(ad-bc)^2}-\frac{3Bab^2c^2d^4i(6A+5B)}{(ad-bc)^2}+\cdots}{12ABbd^4i+10B^2bd^4i}\right.}{18b^2g^4\,(ad-bc)^2}$$

$$+\frac{Bd^3i(6A+5B)\log\left(x + \frac{6ABad^4i+6ABbcd^3i+5B^2ad^4i+5B^2bcd^3i+\frac{Ba^3d^6i(6A+5B)}{(ad-bc)^2}-\frac{3Ba^2bcd^5i(6A+5B)}{(ad-bc)^2}+\frac{3Bab^2c^2d^4i(6A+5B)}{(ad-bc)^2}-\cdots}{12ABbd^4i+10B^2bd^4i}\right.}{18b^2g^4\,(ad-bc)^2}$$

$$+\frac{(3B^2ac^2di+6B^2acd^2ix+3B^2ad^3ix^2-2B^2bc^3i-3B^2bc^2dix+\cdots}{6a^5d^2g^4-12a^4bcdg^4+18a^4bd^2g^4x+6a^3b^2c^2g^4-36a^3b^2cdg^4x+18a^3b^2d^2g^4x^2+18a^2b^3c^2g^4x-36a^2b^3\cdots}$$

$$+\frac{(-6ABa^2d^2i-6ABabcdi-18ABabd^2ix+12ABb^2c^2i+18ABb^2cdix-5B^2a^2d^2i-5B^2abcdi-15B^2\cdots}{18a^4b^2dg^4-18a^3b^3cg^4+54a^3b^3dg^4x-54a^2b^4cg^4x+54a^2b^4dg^4x^2-54ab^5\cdots}$$

$$+\frac{-18A^2a^2d^2i-18A^2abcdi+36A^2b^2c^2i-30ABa^2d^2i-30ABabcdi+24ABb^2c^2i-19B^2a^2d^2i-19B^2a\cdots}{108a^4b^2dg^4-108a^3b^3cg^4+x^3\cdot(108ab^5dg^4-\cdots}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4,x)
```

output
```
-B*d**3*i*(6*A + 5*B)*log(x + (6*A*B*a*d**4*i + 6*A*B*b*c*d**3*i + 5*B**2*
a*d**4*i + 5*B**2*b*c*d**3*i - B*a**3*d**6*i*(6*A + 5*B)/(a*d - b*c)**2 +
3*B*a**2*b*c*d**5*i*(6*A + 5*B)/(a*d - b*c)**2 - 3*B*a*b**2*c**2*d**4*i*(6
*A + 5*B)/(a*d - b*c)**2 + B*b**3*c**3*d**3*i*(6*A + 5*B)/(a*d - b*c)**2)/
(12*A*B*b*d**4*i + 10*B**2*b*d**4*i))/(18*b**2*g**4*(a*d - b*c)**2) + B*d*
*3*i*(6*A + 5*B)*log(x + (6*A*B*a*d**4*i + 6*A*B*b*c*d**3*i + 5*B**2*a*d**
4*i + 5*B**2*b*c*d**3*i + B*a**3*d**6*i*(6*A + 5*B)/(a*d - b*c)**2 - 3*B*a
**2*b*c*d**5*i*(6*A + 5*B)/(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**4*i*(6*A +
5*B)/(a*d - b*c)**2 - B*b**3*c**3*d**3*i*(6*A + 5*B)/(a*d - b*c)**2)/(12*A
*B*b*d**4*i + 10*B**2*b*d**4*i))/(18*b**2*g**4*(a*d - b*c)**2) + (3*B**2*a
*c**2*d*i + 6*B**2*a*c*d**2*i*x + 3*B**2*a*d**3*i*x**2 - 2*B**2*b*c**3*i -
 3*B**2*b*c**2*d*i*x + B**2*b*d**3*i*x**3)*log(e*(a + b*x)/(c + d*x))**2/(
6*a**5*d**2*g**4 - 12*a**4*b*c*d*g**4 + 18*a**4*b*d**2*g**4*x + 6*a**3*b**
2*c**2*g**4 - 36*a**3*b**2*c*d*g**4*x + 18*a**3*b**2*d**2*g**4*x**2 + 18*a
**2*b**3*c**2*g**4*x - 36*a**2*b**3*c*d*g**4*x**2 + 6*a**2*b**3*d**2*g**4*
x**3 + 18*a*b**4*c**2*g**4*x**2 - 12*a*b**4*c*d*g**4*x**3 + 6*b**5*c**2*g*
*4*x**3) + (-6*A*B*a**2*d**2*i - 6*A*B*a*b*c*d*i - 18*A*B*a*b*d**2*i*x + 1
2*A*B*b**2*c**2*i + 18*A*B*b**2*c*d*i*x - 5*B**2*a**2*d**2*i - 5*B**2*a*b*
c*d*i - 15*B**2*a*b*d**2*i*x + 4*B**2*b**2*c**2*i + 3*B**2*b**2*c*d*i*x -
6*B**2*b**2*d**2*i*x**2)*log(e*(a + b*x)/(c + d*x))/(18*a**4*b**2*d*g**...
```

### 3.62.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3282 vs. $2(275) = 550$.

Time = 0.42 (sec) , antiderivative size = 3282, normalized size of antiderivative = 11.44

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algo
rithm="maxima")
```

output
```
-1/6*(3*b*x + a)*B^2*d*i*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^5*g^4*x
^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/54*(6*((6*b^2*d^
2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((
b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*
d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g
^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)
/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log
(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4))*l
og(b*e*x/(d*x + c) + a*e/(d*x + c)) + (4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^
2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3
+ 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*
x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3
*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*
x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2
*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2
+ 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g^4 -
3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*g^4 -
 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + 3*(a*b^6
*c^3*g^4 - 3*a^2*b^5*c^2*d*g^4 + 3*a^3*b^4*c*d^2*g^4 - a^4*b^3*d^3*g^4)*x^
2 + 3*(a^2*b^5*c^3*g^4 - 3*a^3*b^4*c^2*d*g^4 + 3*a^4*b^3*c*d^2*g^4 - a^...
```

### 3.62.8 Giac [A] (verification not implemented)

Time = 0.51 (sec) , antiderivative size = 455, normalized size of antiderivative = 1.59

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{1}{108}\left(\frac{18\left(2\,B^2be^4i-\frac{3\,(bex+ae)B^2de^3i}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^3bcg^4}{(dx+c)^3}-\frac{(bex+ae)^3adg^4}{(dx+c)^3}}+\frac{6\left(12\,ABbe^4i+4\,B^2be^4i-\frac{18\,(bex+ae)ABde^3i}{dx+c}-\frac{9\,(be\cdots}{\cdots}\right.}{\frac{(bex+ae)^3bcg^4}{(dx+c)^3}-\frac{(bex+ae)^3adg^4}{(dx+c)^3}}\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, algo
rithm="giac")
```

output
```
-1/108*(18*(2*B^2*b*e^4*i - 3*(b*e*x + a*e)*B^2*d*e^3*i/(d*x + c))*log((b*
e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^3*b*c*g^4/(d*x + c)^3 - (b*e*x + a*
e)^3*a*d*g^4/(d*x + c)^3) + 6*(12*A*B*b*e^4*i + 4*B^2*b*e^4*i - 18*(b*e*x
+ a*e)*A*B*d*e^3*i/(d*x + c) - 9*(b*e*x + a*e)*B^2*d*e^3*i/(d*x + c))*log(
(b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4/(d*x + c)^3 - (b*e*x + a
*e)^3*a*d*g^4/(d*x + c)^3) + (36*A^2*b*e^4*i + 24*A*B*b*e^4*i + 8*B^2*b*e^
4*i - 54*(b*e*x + a*e)*A^2*d*e^3*i/(d*x + c) - 54*(b*e*x + a*e)*A*B*d*e^3*
i/(d*x + c) - 27*(b*e*x + a*e)*B^2*d*e^3*i/(d*x + c))/((b*e*x + a*e)^3*b*c
*g^4/(d*x + c)^3 - (b*e*x + a*e)^3*a*d*g^4/(d*x + c)^3))*(b*c/((b*c*e - a*
d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.62.9 Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 955, normalized size of antiderivative = 3.33

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$= -\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{\frac{B^2\,c\,i}{3\,b^2\,g^4}+\frac{B^2\,a\,d\,i}{6\,b^3\,g^4}+\frac{B^2\,d\,i\,x}{2\,b^2\,g^4}}{3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2}-\frac{B^2\,d^3\,i}{6\,b^2\,g^4\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)$$

$$-\frac{\frac{18\,i\,A^2\,a^2\,d^2+18\,i\,A^2\,a\,b\,c\,d-36\,i\,A^2\,b^2\,c^2+30\,i\,A\,B\,a^2\,d^2+30\,i\,A\,B\,a\,b\,c\,d-24\,i\,A\,B\,b^2\,c^2+19\,i\,B^2\,a^2\,d^2+19\,i\,B^2\,a\,b\,c\,d-8\,i\,B^2\,b^2\,c^2}{6\,(a\,d-b\,c)}+\frac{x^2\,(5\,i\,\ldots}{\ldots}}{18\,a^3\,b^2\,g^4+54\,a^2\,b^3\,g^4\,x+54\,a\,b^4\,g\ldots}$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(x\left(\frac{A\,B\,i}{b^2\,g^4}+\frac{B^2\,d^3\,i\left(b\left(\frac{3\,a^2\,d^2-4\,a\,b\,c\,d+b^2\,c^2}{6\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{3\,b\,d^2}\right)+\frac{3\,a^2\,d^2-4\,a\,b\,c\,d+b^2\,c^2}{3\,d^3}+\frac{2\,a\,(a\,d-b\,c)}{3\,d^2}\right)}{3\,b^2\,g^4\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)+\frac{A\,B\,a\,i}{3\,b^3\,g^4}+\frac{B\,i\,\ldots}{\ldots}}{\frac{3\,a^2\,x}{d}+\frac{a\ldots}{b\ldots}}$$

$$-\frac{B\,d^3\,i\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x-\frac{18\,b^4\,c^2\,g^4-18\,a^2\,b^2\,d^2\,g^4}{18\,b^2\,g^4\,(a\,d-b\,c)}\right)1\mathrm{i}}{a\,d-b\,c}\right)\,(6\,A+5\,B)\,1\mathrm{i}}{9\,b^2\,g^4\,{(a\,d-b\,c)}^2}$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^4
,x)
```

output
```
- log((e*(a + b*x))/(c + d*x))^2*(((B^2*c*i)/(3*b^2*g^4) + (B^2*a*d*i)/(6*
b^3*g^4) + (B^2*d*i*x)/(2*b^2*g^4))/(3*a^2*x + a^3/b + b^2*x^3 + 3*a*b*x^2
) - (B^2*d^3*i)/(6*b^2*g^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((18*A^2*a^
2*d^2*i - 36*A^2*b^2*c^2*i + 19*B^2*a^2*d^2*i - 8*B^2*b^2*c^2*i + 30*A*B*a
^2*d^2*i - 24*A*B*b^2*c^2*i + 18*A^2*a*b*c*d*i + 19*B^2*a*b*c*d*i + 30*A*B
*a*b*c*d*i)/(6*(a*d - b*c)) + (x^2*(5*B^2*b^2*d^2*i + 6*A*B*b^2*d^2*i))/(a
*d - b*c) + (x*(18*A^2*a*b*d^2*i + 19*B^2*a*b*d^2*i - 18*A^2*b^2*c*d*i + B
^2*b^2*c*d*i + 30*A*B*a*b*d^2*i - 6*A*B*b^2*c*d*i))/(2*(a*d - b*c)))/(18*a
^3*b^2*g^4 + 18*b^5*g^4*x^3 + 54*a^2*b^3*g^4*x + 54*a*b^4*g^4*x^2) - (log(
(e*(a + b*x))/(c + d*x))*(x*((A*B*i)/(b^2*g^4) + (B^2*d^3*i*(b*((3*a^2*d^2
 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^2*d^
2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)))/(3*b^2*g^4*
(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) + (A*B*a*i)/(3*b^3*g^4) + (B*i*(2*A*b*c
- B*a*d + B*b*c))/(3*b^3*d*g^4) + (B^2*d^3*i*(a*((3*a^2*d^2 + b^2*c^2 - 4*
a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4
*a*b^2*c^2*d - 6*a^2*b*c*d^2)/(3*b*d^4)))/(3*b^2*g^4*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d)) - (B^2*d^3*i*x^2*((b^2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c))/
(3*d^2)))/(3*b^2*g^4*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/((3*a^2*x)/d + a^3
/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (B*d^3*i*atan(((2*b*d*x - (18*b^4*
c^2*g^4 - 18*a^2*b^2*d^2*g^4)/(18*b^2*g^4*(a*d - b*c)))*1i)/(a*d - b*c)...
```

3.62. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

# 3.63 $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$



## 3.63.1 Optimal result

Integrand size = 40, antiderivative size = 445

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{B^2d^2i(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2} + \frac{4bB^2di(c+dx)^3}{27(bc-ad)^3g^5(a+bx)^3} - \frac{b^2B^2i(c+dx)^4}{32(bc-ad)^3g^5(a+bx)^4} - \frac{Bd^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^5(a+bx)^2} + \frac{4bBdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^3g^5(a+bx)^3} - \frac{b^2Bi(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^3g^5(a+bx)^4} - \frac{d^2i(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^3g^5(a+bx)^4}$$

output
```
-1/4*B^2*d^2*i*(d*x+c)^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+4/27*b*B^2*d*i*(d*x+c)
^3/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/32*b^2*B^2*i*(d*x+c)^4/(-a*d+b*c)^3/g^5/(b
*x+a)^4-1/2*B*d^2*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^5
/(b*x+a)^2+4/9*b*B*d*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/
g^5/(b*x+a)^3-1/8*b^2*B*i*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)
^3/g^5/(b*x+a)^4-1/2*d^2*i*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b
*c)^3/g^5/(b*x+a)^2+2/3*b*d*i*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*
d+b*c)^3/g^5/(b*x+a)^3-1/4*b^2*i*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(
-a*d+b*c)^3/g^5/(b*x+a)^4
```

### 3.63.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.69 (sec) , antiderivative size = 1255, normalized size of antiderivative = 2.82

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{i\left(216(bc-ad)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-288d(-bc+ad)^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+16Bd(c\right.}{}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*
g*x)^5,x]
```

output
```
-1/864*(i*(216*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 288*
d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 16*B
*d*(a + b*x)*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^
2*(a + b*x) - 15*B*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b
*x)^2 + 66*B*d^2*(b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*
x] + 66*B*d^3*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]
^2 + 12*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b*c - a*d)^
2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^
2*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*
(a + b*x))/(c + d*x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a +
b*x)^3*Log[c + d*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d
)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[c
+ d*x] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a
+ b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d
*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x)
)/(b*c - a*d)]) + 3*B*(36*A*(b*c - a*d)^4 + 9*B*(b*c - a*d)^4 + 48*A*d*(-(
b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c) + a*d)^3*(a + b*x) + 72*A*d^2*(b*
c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c - a*d)^2*(a + b*x)^2 + 144*A*d^3*(-
(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 144*A*d^
4*(a + b*x)^4*Log[a + b*x] - 300*B*d^4*(a + b*x)^4*Log[a + b*x] + 72*B*...
```

### 3.63.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{i\int \left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^5}{(a+bx)^5}-\frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^4}{(a+bx)^4}+\frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^3}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}}{g^5(bc-ad)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{b^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}-\frac{b^2B(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}-\frac{Bd^2(c+dx)^2\left(B\log\left(\frac{e(c}{c}\right.\right.}{2(a+bx)^2}\right.}{g^5(bc}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)^5
,x]
```

output
```
(i*(-1/4*(B^2*d^2*(c + d*x)^2)/(a + b*x)^2 + (4*b*B^2*d*(c + d*x)^3)/(27*(
a + b*x)^3) - (b^2*B^2*(c + d*x)^4)/(32*(a + b*x)^4) - (B*d^2*(c + d*x)^2*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) + (4*b*B*d*(c + d*x)
^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(a + b*x)^3) - (b^2*B*(c + d*x
)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(a + b*x)^4) - (d^2*(c + d*x)
^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*x)^2) + (2*b*d*(c + d
*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(a + b*x)^3) - (b^2*(c +
d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*(a + b*x)^4)))/((b*c - a
*d)^3*g^5)
```

### 3.63.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.63.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 925 vs. $2(427) = 854$.

Time = 1.61 (sec) , antiderivative size = 926, normalized size of antiderivative = 2.08

| method | result |
|---|---|
| parts | $\frac{i A^2\left(-\frac{d}{3b^2(bx+a)^3}-\frac{-ad+cb}{4b^2(bx+a)^4}\right)}{g^5} - \frac{i B^2(ad-cb)^2 e^2\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}-\frac{1}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}\right)}{(ad-cb)^5}\right)}{}$ |
| derivativedivides | Expression too large to display |
| default | Expression too large to display |
| norman | Expression too large to display |
| parallelrisch | Expression too large to display |
| risch | Expression too large to display |

input
```
int((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x,method=_RETU
RNVERBOSE)
```

output
```
i*A^2/g^5*(-1/3*d/b^2/(b*x+a)^3-1/4*(-a*d+b*c)/b^2/(b*x+a)^4)-i*B^2/g^5/d^
3*(a*d-b*c)^2*e^2*(d^5/(a*d-b*c)^5*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*d^4/(a*d
-b*c)^5*b*e*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)+d^3/(a*d-b*c)^5*e^2*b^2*(-1/4/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d
+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^4))-2*i*B*A/g^5/d^3*(a*d-b*c)^2*e^2*(d^5/(a*d-b*c)^5*(-
1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*d^4/(a*d-b*c)^5*b*e*(-1/3/(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^3)+d^3/(a*d-b*c)^5*e^2*b^2*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

### 3.63.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 985 vs. $2(427) = 854$.

Time = 0.31 (sec) , antiderivative size = 985, normalized size of antiderivative = 2.21

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$
$$-\frac{12\left((12\,AB+13\,B^2)b^4cd^3-(12\,AB+13\,B^2)ab^3d^4\right)ix^3-6\left((12\,AB+B^2)b^4c^2d^2-16\,(6\,AB+5\,B^2)ab\right.}{}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algo
rithm="fricas")
```

output
```
-1/864*(12*((12*A*B + 13*B^2)*b^4*c*d^3 - (12*A*B + 13*B^2)*a*b^3*d^4)*i*x
^3 - 6*((12*A*B + B^2)*b^4*c^2*d^2 - 16*(6*A*B + 5*B^2)*a*b^3*c*d^3 + (84*
A*B + 79*B^2)*a^2*b^2*d^4)*i*x^2 + 4*((72*A^2 + 12*A*B - 5*B^2)*b^4*c^3*d
- 12*(18*A^2 + 6*A*B - B^2)*a*b^3*c^2*d^2 + 108*(2*A^2 + 2*A*B + B^2)*a^2*
b^2*c*d^3 - (72*A^2 + 156*A*B + 115*B^2)*a^3*b*d^4)*i*x + 72*(B^2*b^4*d^4*
i*x^4 + 4*B^2*a*b^3*d^4*i*x^3 + 6*B^2*a^2*b^2*d^4*i*x^2 + 4*(B^2*b^4*c^3*d
 - 3*B^2*a*b^3*c^2*d^2 + 3*B^2*a^2*b^2*c*d^3)*i*x + (3*B^2*b^4*c^4 - 8*B^2
*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2)*i)*log((b*e*x + a*e)/(d*x + c))^2 +
(27*(8*A^2 + 4*A*B + B^2)*b^4*c^4 - 64*(9*A^2 + 6*A*B + 2*B^2)*a*b^3*c^3*d
 + 216*(2*A^2 + 2*A*B + B^2)*a^2*b^2*c^2*d^2 - (72*A^2 + 156*A*B + 115*B^2
)*a^4*d^4)*i + 12*((12*A*B + 13*B^2)*b^4*d^4*i*x^4 + 4*(3*B^2*b^4*c*d^3 +
2*(6*A*B + 5*B^2)*a*b^3*d^4)*i*x^3 - 6*(B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^
3 - 6*(2*A*B + B^2)*a^2*b^2*d^4)*i*x^2 + 4*((12*A*B + B^2)*b^4*c^3*d - 6*(
6*A*B + B^2)*a*b^3*c^2*d^2 + 18*(2*A*B + B^2)*a^2*b^2*c*d^3)*i*x + (9*(4*A
*B + B^2)*b^4*c^4 - 32*(3*A*B + B^2)*a*b^3*c^3*d + 36*(2*A*B + B^2)*a^2*b^
2*c^2*d^2)*i)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^3 - 3*a*b^8*c^2*d + 3*
a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*
a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d +
3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d
+ 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d...
```

### 3.63.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
```

output
```
Timed out
```

### 3.63.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4808 vs. $2(427) = 854$.

Time = 0.57 (sec) , antiderivative size = 4808, normalized size of antiderivative = 10.80

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algo
rithm="maxima")
```

output
```
-1/12*(4*b*x + a)*B^2*d*i*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^6*g^5*
x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g^5)
 + 1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^
2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*
c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3
*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4
*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a
^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 -
 a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a
^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3
*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c
^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)
)*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216
*a^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*
d^4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72
*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*
d^4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x^
2 + 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - 4*(7*b^4*c^3*d - 60*a*b^3*c^
2*d^2 + 324*a^2*b^2*c*d^3 - 271*a^3*b*d^4)*x - 300*(b^4*d^4*x^4 + 4*a*b^3*
d^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a) + 1...
```

### 3.63.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 744, normalized size of antiderivative = 1.67

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{864}\left(\frac{72\left(3B^2b^2e^5i-\frac{8(bex+ae)B^2bde^4i}{dx+c}+\frac{6(bex+ae)^2B^2d^2e^3i}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^4b^2c^2g^5}{(dx+c)^4}-\frac{2(bex+ae)^4abcdg^5}{(dx+c)^4}+\frac{(bex+ae)^4a^2d^2g^5}{(dx+c)^4}}+\frac{12\left(36ABb^2e^5i+9B^2b^2e^5i-\right.}{}\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, algo
rithm="giac")
```

output
```
-1/864*(72*(3*B^2*b^2*e^5*i - 8*(b*e*x + a*e)*B^2*b*d*e^4*i/(d*x + c) + 6*
(b*e*x + a*e)^2*B^2*d^2*e^3*i/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))^2/
((b*e*x + a*e)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*e*x + a*e)^4*a*b*c*d*g^5/(
d*x + c)^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d*x + c)^4) + 12*(36*A*B*b^2*e^5
*i + 9*B^2*b^2*e^5*i - 96*(b*e*x + a*e)*A*B*b*d*e^4*i/(d*x + c) - 32*(b*e*
x + a*e)*B^2*b*d*e^4*i/(d*x + c) + 72*(b*e*x + a*e)^2*A*B*d^2*e^3*i/(d*x +
 c)^2 + 36*(b*e*x + a*e)^2*B^2*d^2*e^3*i/(d*x + c)^2)*log((b*e*x + a*e)/(d
*x + c))/((b*e*x + a*e)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*e*x + a*e)^4*a*b*
c*d*g^5/(d*x + c)^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d*x + c)^4) + (216*A^2*
b^2*e^5*i + 108*A*B*b^2*e^5*i + 27*B^2*b^2*e^5*i - 576*(b*e*x + a*e)*A^2*b
*d*e^4*i/(d*x + c) - 384*(b*e*x + a*e)*A*B*b*d*e^4*i/(d*x + c) - 128*(b*e*
x + a*e)*B^2*b*d*e^4*i/(d*x + c) + 432*(b*e*x + a*e)^2*A^2*d^2*e^3*i/(d*x
+ c)^2 + 432*(b*e*x + a*e)^2*A*B*d^2*e^3*i/(d*x + c)^2 + 216*(b*e*x + a*e)
^2*B^2*d^2*e^3*i/(d*x + c)^2)/((b*e*x + a*e)^4*b^2*c^2*g^5/(d*x + c)^4 - 2
*(b*e*x + a*e)^4*a*b*c*d*g^5/(d*x + c)^4 + (b*e*x + a*e)^4*a^2*d^2*g^5/(d*
x + c)^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
- a*d)))
```

### 3.63.9 Mupad [B] (verification not implemented)

Time = 7.44 (sec) , antiderivative size = 1870, normalized size of antiderivative = 4.20

$$\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)^5
,x)
```

output
```
((72*A^2*a^3*d^3*i + 216*A^2*b^3*c^3*i + 115*B^2*a^3*d^3*i + 27*B^2*b^3*c^
3*i + 156*A*B*a^3*d^3*i + 108*A*B*b^3*c^3*i - 360*A^2*a*b^2*c^2*d*i + 72*A
^2*a^2*b*c*d^2*i - 101*B^2*a*b^2*c^2*d*i + 115*B^2*a^2*b*c*d^2*i - 276*A*B
*a*b^2*c^2*d*i + 156*A*B*a^2*b*c*d^2*i)/(12*(a*d - b*c)) + (x^2*(79*B^2*a*
b^2*d^3*i - B^2*b^3*c*d^2*i + 84*A*B*a*b^2*d^3*i - 12*A*B*b^3*c*d^2*i))/(2
*(a*d - b*c)) + (x*(72*A^2*a^2*b*d^3*i + 115*B^2*a^2*b*d^3*i + 72*A^2*b^3*
c^2*d*i - 5*B^2*b^3*c^2*d*i + 156*A*B*a^2*b*d^3*i + 12*A*B*b^3*c^2*d*i - 1
44*A^2*a*b^2*c*d^2*i + 7*B^2*a*b^2*c*d^2*i - 60*A*B*a*b^2*c*d^2*i))/(3*(a*
d - b*c)) + (d*x^3*(13*B^2*b^3*d^2*i + 12*A*B*b^3*d^2*i))/(a*d - b*c))/(x*
(288*a^3*b^4*c*g^5 - 288*a^4*b^3*d*g^5) - x^3*(288*a^2*b^5*d*g^5 - 288*a*b
^6*c*g^5) + x^4*(72*b^7*c*g^5 - 72*a*b^6*d*g^5) + x^2*(432*a^2*b^5*c*g^5 -
 432*a^3*b^4*d*g^5) + 72*a^4*b^3*c*g^5 - 72*a^5*b^2*d*g^5) - log((e*(a + b
*x))/(c + d*x))^2*(((B^2*c*i)/(4*b^2*g^5) + (B^2*a*d*i)/(12*b^3*g^5) + (B^
2*d*i*x)/(3*b^2*g^5))/(4*a^3*x + a^4/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x
^3) - (B^2*d^4*i)/(12*b^2*g^5*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2))) - (log((e*(a + b*x))/(c + d*x))*(x*((2*A*B*i)/(3*b^2*g^5) + (B^2
*d^4*i*(b*(a*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c
))/(4*b*d^2)) + (6*a^3*d^3 - b^3*c^3 + 5*a*b^2*c^2*d - 10*a^2*b*c*d^2)/(12
*b*d^4)) + a*(b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d -
b*c))/(4*b*d^2)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d ...
```

3.63. $\int \frac{(ci+dix)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

# 3.64 $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

### 3.64.1 Optimal result

Integrand size = 42, antiderivative size = 711

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\frac{3B^2(bc-ad)^5g^3i^2x}{20b^2d^3}+\frac{B^2(bc-ad)^2g^3i^2(a+bx)^4}{60b^3}-\frac{3B^2(bc-ad)^4g^3i^2(c+dx)^2}{40bd^4}$$
$$+\frac{B^2(bc-ad)^3g^3i^2(c+dx)^3}{60d^4}-\frac{B(bc-ad)^3g^3i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{90b^3d}$$
$$-\frac{B(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^3}$$
$$-\frac{B(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^2}$$
$$+\frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^3}$$
$$+\frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{15b^2}$$
$$+\frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b}$$
$$+\frac{B(bc-ad)^4g^3i^2(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^2}$$
$$-\frac{B(bc-ad)^5g^3i^2(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^3}$$
$$-\frac{B(bc-ad)^6g^3i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{180b^3d^4}$$
$$-\frac{B^2(bc-ad)^6g^3i^2\log(c+dx)}{20b^3d^4}-\frac{B^2(bc-ad)^6g^3i^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{30b^3d^4}$$

output
```
3/20*B^2*(-a*d+b*c)^5*g^3*i^2*x/b^2/d^3+1/60*B^2*(-a*d+b*c)^2*g^3*i^2*(b*x
+a)^4/b^3-3/40*B^2*(-a*d+b*c)^4*g^3*i^2*(d*x+c)^2/b/d^4+1/60*B^2*(-a*d+b*c
)^3*g^3*i^2*(d*x+c)^3/d^4-1/90*B*(-a*d+b*c)^3*g^3*i^2*(b*x+a)^3*(A+B*ln(e*
(b*x+a)/(d*x+c)))/b^3/d-1/20*B*(-a*d+b*c)^2*g^3*i^2*(b*x+a)^4*(A+B*ln(e*(b
*x+a)/(d*x+c)))/b^3-1/15*B*(-a*d+b*c)*g^3*i^2*(b*x+a)^4*(d*x+c)*(A+B*ln(e*
(b*x+a)/(d*x+c)))/b^2+1/60*(-a*d+b*c)^2*g^3*i^2*(b*x+a)^4*(A+B*ln(e*(b*x+a
)/(d*x+c)))^2/b^3+1/15*(-a*d+b*c)*g^3*i^2*(b*x+a)^4*(d*x+c)*(A+B*ln(e*(b*x
+a)/(d*x+c)))^2/b^2+1/6*g^3*i^2*(b*x+a)^4*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x
+c)))^2/b+1/180*B*(-a*d+b*c)^4*g^3*i^2*(b*x+a)^2*(3*A+B+3*B*ln(e*(b*x+a)/(
d*x+c)))/b^3/d^2-1/180*B*(-a*d+b*c)^5*g^3*i^2*(b*x+a)*(6*A+5*B+6*B*ln(e*(b
*x+a)/(d*x+c)))/b^3/d^3-1/180*B*(-a*d+b*c)^6*g^3*i^2*ln((-a*d+b*c)/b/(d*x+
c))*(6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b^3/d^4-1/20*B^2*(-a*d+b*c)^6*g^3
*i^2*ln(d*x+c)/b^3/d^4-1/30*B^2*(-a*d+b*c)^6*g^3*i^2*polylog(2,d*(b*x+a)/b
/(d*x+c))/b^3/d^4
```

### 3.64.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1559 vs. 2(711) = 1422.

Time = 0.79 (sec) , antiderivative size = 1559, normalized size of antiderivative = 2.19

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2,x]
```

output
```
(g^3*i^2*(15*(b*c - a*d)^2*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2 + 24*d*(b*c - a*d)*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
+ 10*d^2*(a + b*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (5*B*(b*c -
a*d)^3*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a
+ b*x))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b
*x))/(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])
- 6*B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a +
 b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b*
c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c)
+ a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)])))/d^4 + (2*B*(b*c - a*d)^2*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c -
 a*d)^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a +
 b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)
^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*d^4*(a + b*x)^4*(A + B*Log[(e*
(a + b*x))/(c + d*x)]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 24*(b*c - a*d)^
4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 4*B*(b*c - a*d)^2*(2
*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(
b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d
^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^3*(b*...
```

### 3.64.3 Rubi [A] (verified)

Time = 2.05 (sec) , antiderivative size = 963, normalized size of antiderivative = 1.35, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.405$, Rules used = {2962, 2783, 2782, 27, 87, 49, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2962}$$

$$g^3i^2(bc-ad)^6\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$g^3i^2(bc-ad)^6\left(-\frac{B\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{3b}+\frac{\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}\right.$$

$\downarrow$ 2782

$$g^3i^2(bc-ad)^6\left(-\frac{B\left(-B\int \frac{(a+bx)^3\left(5b-\frac{d(a+bx)}{c+dx}\right)}{20b^2(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)}{3b}+\frac{\int \frac{(a+b}{(c}}{}\right.$$

$\downarrow$ 27

$$g^3i^2(bc-ad)^6\left(-\frac{B\left(-\frac{B\int \frac{(a+bx)^3\left(5b-\frac{d(a+bx)}{c+dx}\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{20b^2}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)}{3b}+\frac{\int \frac{(a+bx)^3\left(A+\right.}{(c+dx)^3}}{}\right.$$

$\downarrow$ 87

$$g^3i^2(bc-ad)^6\left(-\frac{B\left(-\frac{B\left(\int \frac{(a+bx)^3}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}+\frac{(a+bx)^4}{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{20b^2}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{3b}\right.$$

$\downarrow$ 49

3.64. $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$g^3i^2(bc-ad)^6\left(-\frac{B\left(-\frac{B\left(\int\left(\frac{b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{1}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}+\frac{(a+bx)^4}{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{20b^2}+\frac{(a+bx}{20b^2}\right.}{3b}\right.$$

$\downarrow$ 2009

$$g^3i^2(bc-ad)^6\left(\frac{\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{3b}-\frac{B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{B\left(\frac{b^3}{3d^4\left(b-\frac{d(}{c}\right.}\right.}{}\right.}{}\right.$$

$\downarrow$ 2783

$$g^3i^2(bc-ad)^6\left(\frac{-\frac{2B\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}}{3b}-\frac{B\left(\frac{(a-}{2(}\right.}{}\right.$$

$\downarrow$ 2773

---

3.64. $\int(ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$g^3i^2(bc-ad)^6\left(\frac{-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\frac{(a+bx)^3}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}\right)}{5b}+\frac{\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^4\left(B\log\right.}{5b(c+dx)^4\left(\right.}}{3b}\right.$$

$\downarrow$ 49

$$g^3i^2(bc-ad)^6\left(\frac{-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int\left(\frac{b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{1}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}}{4b}\right)}{5b}+\frac{\int\frac{(a-}{}}{}}{3b}\right.$$

$\downarrow$ 2009

$$g^3i^2(bc-ad)^6\left(\frac{\frac{\int\frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}-\frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{b^3}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3b^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3b}{d^4\left(b-\frac{d(a+bx}{c+dx}\right.}\right.}{4b}\right.}{5b}}{3b}\right.$$

$\downarrow$ 2781

---

3.64. $\int(ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$g^3 i^2 (bc - ad)^6 \left( \frac{\frac{\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{2b}}{5b} - \frac{2B\left(\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{B\left(\frac{b^3}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \right.}{}\right.}{5b}}{3b} \right.$$

$$\Big\downarrow \text{ 2784}$$

$$(bc - ad)^6 g^3 i^2 \left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{B\left(\frac{}{(c+d}\right.}{}\right.}{} \right.$$

$$\Big\downarrow \text{ 2784}$$

---

3.64. $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$(bc-ad)^6 g^3 i^2 \left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{B\left(\frac{}{(c+d}\right.}{}\right.}{}\right.$$

$\downarrow$ 2784

$$(bc-ad)^6 g^3 i^2 \left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{B\left(\frac{}{(c+d}\right.}{}\right.}{}\right.$$

3.64. $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$\Big\downarrow \text{2754}$$

$$(bc-ad)^6 g^3 i^2 \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{B\left(\frac{}{(c+d}\right.}{}\right.}{}\right.$$

$$\Big\downarrow \text{2838}$$

---

3.64. $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$(bc-ad)^6g^3i^2\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{B\left(\frac{}{(c+d}\right.}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
,x]
```

## 3.64. $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

output
```
(b*c - a*d)^6*g^3*i^2*(((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(6*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^6) - (B*(((a + b*x)^4*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]))/(5*b*(c + d*x)^4*(b - (d*(a + b*x))/(c
+ d*x))^5) + ((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(20*b^2*(c
 + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*((a + b*x)^4/((c + d*x)^4*
(b - (d*(a + b*x))/(c + d*x))^4) + b^3/(3*d^4*(b - (d*(a + b*x))/(c + d*x)
)^3) - (3*b^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3*b)/(d^4*(b - (
d*(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]/d^4))/(20*b^2)
))/(3*b) + (((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*b*(c +
 d*x)^4*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*(((a + b*x)^4*(A + B*Log[(
e*(a + b*x))/(c + d*x)]))/(4*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4
) - (B*(b^3/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) - (3*b^2)/(2*d^4*(b -
(d*(a + b*x))/(c + d*x))^2) + (3*b)/(d^4*(b - (d*(a + b*x))/(c + d*x))) +
Log[b - (d*(a + b*x))/(c + d*x)]/d^4))/(4*b)))/(5*b) + (((a + b*x)^4*(A +
B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*b*(c + d*x)^4*(b - (d*(a + b*x))/(c
+ d*x))^4) - (B*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*d*(
c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2*(3*A + B + 3*B
*Log[(e*(a + b*x))/(c + d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d
*x))^2) - (((a + b*x)*(6*A + 5*B + 6*B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(
c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((6*A + 11*B + 6*B*Log[(e*...
```

#### 3.64.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

## 3.64. $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.64.4 Maple [F]

$$\int (bgx+ag)^3 (dix+ci)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

## 3.64.5 Fricas [F]

$$\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^3(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="fricas")
```

output
```
integral(A^2*b^3*d^2*g^3*i^2*x^5 + A^2*a^3*c^2*g^3*i^2 + (2*A^2*b^3*c*d +
3*A^2*a*b^2*d^2)*g^3*i^2*x^4 + (A^2*b^3*c^2 + 6*A^2*a*b^2*c*d + 3*A^2*a^2*
b*d^2)*g^3*i^2*x^3 + (3*A^2*a*b^2*c^2 + 6*A^2*a^2*b*c*d + A^2*a^3*d^2)*g^3
*i^2*x^2 + (3*A^2*a^2*b*c^2 + 2*A^2*a^3*c*d)*g^3*i^2*x + (B^2*b^3*d^2*g^3*
i^2*x^5 + B^2*a^3*c^2*g^3*i^2 + (2*B^2*b^3*c*d + 3*B^2*a*b^2*d^2)*g^3*i^2*
x^4 + (B^2*b^3*c^2 + 6*B^2*a*b^2*c*d + 3*B^2*a^2*b*d^2)*g^3*i^2*x^3 + (3*B
^2*a*b^2*c^2 + 6*B^2*a^2*b*c*d + B^2*a^3*d^2)*g^3*i^2*x^2 + (3*B^2*a^2*b*c
^2 + 2*B^2*a^3*c*d)*g^3*i^2*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3
*d^2*g^3*i^2*x^5 + A*B*a^3*c^2*g^3*i^2 + (2*A*B*b^3*c*d + 3*A*B*a*b^2*d^2)
*g^3*i^2*x^4 + (A*B*b^3*c^2 + 6*A*B*a*b^2*c*d + 3*A*B*a^2*b*d^2)*g^3*i^2*x
^3 + (3*A*B*a*b^2*c^2 + 6*A*B*a^2*b*c*d + A*B*a^3*d^2)*g^3*i^2*x^2 + (3*A*
B*a^2*b*c^2 + 2*A*B*a^3*c*d)*g^3*i^2*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

### 3.64.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

### 3.64.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5178 vs. 2(680) = 1360.

Time = 0.37 (sec) , antiderivative size = 5178, normalized size of antiderivative = 7.28

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="maxima")
```

output
```
1/6*A^2*b^3*d^2*g^3*i^2*x^6 + 2/5*A^2*b^3*c*d*g^3*i^2*x^5 + 3/5*A^2*a*b^2*
d^2*g^3*i^2*x^5 + 1/4*A^2*b^3*c^2*g^3*i^2*x^4 + 3/2*A^2*a*b^2*c*d*g^3*i^2*
x^4 + 3/4*A^2*a^2*b*d^2*g^3*i^2*x^4 + A^2*a*b^2*c^2*g^3*i^2*x^3 + 2*A^2*a^
2*b*c*d*g^3*i^2*x^3 + 1/3*A^2*a^3*d^2*g^3*i^2*x^3 + 3/2*A^2*a^2*b*c^2*g^3*
i^2*x^2 + A^2*a^3*c*d*g^3*i^2*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x +
c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^3*c^2*g^3*i^2 + 3*(x^2*lo
g(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x +
c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*b*c^2*g^3*i^2 + (2*x^3*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3
- ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b^2
*c^2*g^3*i^2 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*lo
g(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 -
 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b
^3*c^2*g^3*i^2 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x
 + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^3*c*d*g^3*i^
2 + 2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3
 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^
2)*x)/(b^2*d^2))*A*B*a^2*b*c*d*g^3*i^2 + 1/2*(6*x^4*log(b*e*x/(d*x + c) +
a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3
*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - ...
```

### 3.64.8 Giac [F]

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (bgx+ag)^3(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) +
A)^2, x)
```

### 3.64.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2
,x)
```

output
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2
, x)
```

# 3.65 $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

### 3.65.1 Optimal result

Integrand size = 42, antiderivative size = 761

$$
\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx \\
&= -\frac{B^2(bc-ad)^4g^2i^2x}{10b^2d^2}-\frac{B^2(bc-ad)^3g^2i^2(c+dx)^2}{20bd^3}+\frac{B^2(bc-ad)^2g^2i^2(c+dx)^3}{30d^3} \\
&\quad+\frac{B^2(bc-ad)^5g^2i^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3}-\frac{B(bc-ad)^3g^2i^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d} \\
&\quad-\frac{B(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15b^3} \\
&\quad-\frac{B(bc-ad)^3g^2i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{5bd^3} \\
&\quad+\frac{4B(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3} \\
&\quad-\frac{bB(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^3} \\
&\quad+\frac{(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{30b^3} \\
&\quad+\frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2} \\
&\quad+\frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b} \\
&\quad+\frac{B(bc-ad)^4g^2i^2(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d^2} \\
&\quad+\frac{B(bc-ad)^5g^2i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^3d^3} \\
&\quad+\frac{B^2(bc-ad)^5g^2i^2\log(c+dx)}{10b^3d^3}+\frac{B^2(bc-ad)^5g^2i^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{15b^3d^3}
\end{aligned}
$$

output
```
-1/10*B^2*(-a*d+b*c)^4*g^2*i^2*x/b^2/d^2-1/20*B^2*(-a*d+b*c)^3*g^2*i^2*(d*
x+c)^2/b/d^3+1/30*B^2*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3/d^3+1/30*B^2*(-a*d+b*
c)^5*g^2*i^2*ln((b*x+a)/(d*x+c))/b^3/d^3-1/30*B*(-a*d+b*c)^3*g^2*i^2*(b*x+
a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/d-1/15*B*(-a*d+b*c)^2*g^2*i^2*(b*x+a)
^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3-1/5*B*(-a*d+b*c)^3*g^2*i^2*(d*x+c)^2*(A
+B*ln(e*(b*x+a)/(d*x+c)))/b/d^3+4/15*B*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3*(A+B
*ln(e*(b*x+a)/(d*x+c)))/d^3-1/10*b*B*(-a*d+b*c)*g^2*i^2*(d*x+c)^4*(A+B*ln(
e*(b*x+a)/(d*x+c)))/d^3+1/30*(-a*d+b*c)^2*g^2*i^2*(b*x+a)^3*(A+B*ln(e*(b*x
+a)/(d*x+c)))^2/b^3+1/10*(-a*d+b*c)*g^2*i^2*(b*x+a)^3*(d*x+c)*(A+B*ln(e*(b
*x+a)/(d*x+c)))^2/b^2+1/5*g^2*i^2*(b*x+a)^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/b+1/30*B*(-a*d+b*c)^4*g^2*i^2*(b*x+a)*(2*A+B+2*B*ln(e*(b*x+a)/(d
*x+c)))/b^3/d^2+1/30*B*(-a*d+b*c)^5*g^2*i^2*ln((-a*d+b*c)/b/(d*x+c))*(2*A+
3*B+2*B*ln(e*(b*x+a)/(d*x+c)))/b^3/d^3+1/10*B^2*(-a*d+b*c)^5*g^2*i^2*ln(d*
x+c)/b^3/d^3+1/15*B^2*(-a*d+b*c)^5*g^2*i^2*polylog(2,d*(b*x+a)/b/(d*x+c))/
b^3/d^3
```

### 3.65.2 Mathematica [A] (verified)

Time = 0.54 (sec) , antiderivative size = 1194, normalized size of antiderivative = 1.57

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$

$$=\frac{g^2i^2\left(20d^3(bc-ad)^2(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+30d^4(bc-ad)(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 +\cdots\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2,x]
```

output
```
(g^2*i^2*(20*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d
*x)])^2 + 30*d^4*(b*c - a*d)*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2 + 12*d^5*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 20*B*(
b*c - a*d)^3*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(
a + b*x))/(c + d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
) - 2*B*(b*c - a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*
x))/(c + d*x)])*Log[c + d*x] + B*(b*c - a*d)*(b*d*x + (-(b*c) + a*d)*Log[c
 + d*x]) + B*(b*c - a*d)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
 d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 10*B*(b*
c - a*d)^2*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e
*(a + b*x))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x
)]) - 6*B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a +
b*x))/(c + d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*
(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (
-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b
*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c
 - a*d)])) + B*(b*c - a*d)*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^
3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 12*d^2*(b*c - a*d)^2*(a + b*x)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*...
```

### 3.65.3 Rubi [A] (verified)

Time = 1.94 (sec) , antiderivative size = 852, normalized size of antiderivative = 1.12, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {2962, 2783, 2782, 27, 1195, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{ 2962}$$

$$g^2i^2(bc-ad)^5\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 2783}$$

$$g^2i^2(bc-ad)^5\left(-\frac{2B\int\frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{2\int\frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.$$

$\downarrow$ 2782

$$g^2i^2(bc-ad)^5\left(-\frac{2B\left(-B\int\frac{(c+dx)\left(b^2-\frac{4d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{12d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{2b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right.}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{5b}\right.$$

$\downarrow$ 27

$$g^2i^2(bc-ad)^5\left(-\frac{2B\left(-\frac{B\int\frac{(c+dx)\left(b^2-\frac{4d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{12d^3}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{2b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right.}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{5b}\right.$$

$\downarrow$ 1195

$$g^2i^2(bc-ad)^5\left(-\frac{2B\left(-\frac{B\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{5d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{12d^3}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\right.}{5b}\right.$$

$\downarrow$ 2009

3.65. $\int(ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$g^2 i^2 (bc - ad)^5 \left( \frac{2\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b} - \frac{2B\left(\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \right.}{} \right.$$

$\downarrow$ 2783

$$g^2 i^2 (bc - ad)^5 \left( \frac{2\left(-\frac{B\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b} + \frac{\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b} + \frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b} - \frac{2B}{} \right.$$

$\downarrow$ 2773

$$g^2 i^2 (bc - ad)^5 \left( \frac{2\left(-\frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{B\int \frac{(a+bx)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right)}{2b} + \frac{\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b} + \frac{(a+bx)^3\left(B\right.}{4b(c+dx}\right.}{5b} \right.$$

$\downarrow$ 49

3.65. $\int (ag + bgx)^2 (ci + dix)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$g^2 i^2 (bc-ad)^5 \left( \frac{2\left( -\frac{B\left( \frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{B\int \left( \frac{b^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} + \frac{1}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)} \right) d\frac{a+bx}{c+dx}}{3b} \right)}{2b} + \frac{\int \frac{(a+bx)^2\left(A+B\log\left(\cdots\right.\right.}{(c+dx)^2\left(b-\frac{d(a\cdots}{c\cdots}\right.}}{4b} \cdots \right.}{5b} \cdots \right.$$

$\downarrow$ 2009

$$g^2 i^2 (bc-ad)^5 \left( \frac{2\left( \frac{\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{4b} - \frac{B\left( \frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{B\left( \frac{b^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)} - \frac{\log\left(b-\frac{d(a+b\cdots}{c+d\cdots}\right.}{d^3} \cdots \right.}{3b} \cdots \right.}{2b} \cdots \right.}{5b} \cdots \right.$$

$\downarrow$ 2781

## 3.65. $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$g^2 i^2 (bc - ad)^5 \left( \frac{2\left( \frac{\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}}{4b} - \frac{B\left(\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{B\left(\frac{b^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)\cdots}\right.}{\cdots}\right.}{2b} \cdots \right)}{5b} \cdots \right.$$

$\downarrow$ 2784

$$g^2 i^2 (bc - ad)^5 \left( \frac{2\left( \frac{\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\int \frac{(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{3b}}{4b} - \frac{B\left(\frac{(a+bx)^3\left(B\log\cdots\right.}{3b(c+dx)^3\cdots}\right.}{\cdots} \cdots \right)}{5b} \cdots \right.$$

---

3.65. $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$\Big\downarrow \text{2784}$$

$$(bc-ad)^5g^2i^2\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(a+bx)^3}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right.}{}\right.$$

$$\Big\downarrow \text{2754}$$

3.65. $\int(ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$(bc - ad)^5 g^2 i^2 \left( \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{5b(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right.}{}\right.$$

$\downarrow$ 2838

3.65. $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$(bc-ad)^5 g^2 i^2 \left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
,x]
```

## 3.65. $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

output
```
(b*c - a*d)^5*g^2*i^2*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(5*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*((b^2*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]))/(4*d^3*(b - (d*(a + b*x))/(c + d*x))^4) - (2*
b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*d^3*(b - (d*(a + b*x))/(c + d*x
))^3) + (A + B*Log[(e*(a + b*x))/(c + d*x)])/(2*d^3*(b - (d*(a + b*x))/(c
+ d*x))^2) - (B*(b/(b - (d*(a + b*x))/(c + d*x))^3 - 5/(2*(b - (d*(a + b*x
))/(c + d*x))^2) + 1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c
+ d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(12*d^3)))/(5*b) + (2
*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*b*(c + d*x)^3*(b
 - (d*(a + b*x))/(c + d*x))^4) - (B*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))
/(c + d*x)]))/(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (B*(b^2/
(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*b)/(d^3*(b - (d*(a + b*x))/(c
 + d*x))) - Log[b - (d*(a + b*x))/(c + d*x)]/d^3))/(3*b)))/(2*b) + (((a +
b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b*(c + d*x)^3*(b - (d*(a
 + b*x))/(c + d*x))^3) - (2*B*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c +
d*x)]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(2
*A + B + 2*B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b*x)
)/(c + d*x))) - (-(((2*A + 3*B + 2*B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 -
 (d*(a + b*x))/(b*(c + d*x))])/d) - (2*B*PolyLog[2, (d*(a + b*x))/(b*(c +
d*x))])/d)/d)/(2*d)))/(3*b))/(4*b)))/(5*b))
```

### 3.65.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

## 3.65. $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.65.4 Maple [F]

$$\int (bgx+ag)^2 (dix+ci)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

### 3.65.5 Fricas [F]

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^2(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="fricas")
```

output
```
integral(A^2*b^2*d^2*g^2*i^2*x^4 + A^2*a^2*c^2*g^2*i^2 + 2*(A^2*b^2*c*d +
A^2*a*b*d^2)*g^2*i^2*x^3 + (A^2*b^2*c^2 + 4*A^2*a*b*c*d + A^2*a^2*d^2)*g^2
*i^2*x^2 + 2*(A^2*a*b*c^2 + A^2*a^2*c*d)*g^2*i^2*x + (B^2*b^2*d^2*g^2*i^2*
x^4 + B^2*a^2*c^2*g^2*i^2 + 2*(B^2*b^2*c*d + B^2*a*b*d^2)*g^2*i^2*x^3 + (B
^2*b^2*c^2 + 4*B^2*a*b*c*d + B^2*a^2*d^2)*g^2*i^2*x^2 + 2*(B^2*a*b*c^2 + B
^2*a^2*c*d)*g^2*i^2*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^2*d^2*g^2
*i^2*x^4 + A*B*a^2*c^2*g^2*i^2 + 2*(A*B*b^2*c*d + A*B*a*b*d^2)*g^2*i^2*x^3
 + (A*B*b^2*c^2 + 4*A*B*a*b*c*d + A*B*a^2*d^2)*g^2*i^2*x^2 + 2*(A*B*a*b*c^
2 + A*B*a^2*c*d)*g^2*i^2*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

### 3.65.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

### 3.65.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3656 vs. 2(728) = 1456.

Time = 0.34 (sec) , antiderivative size = 3656, normalized size of antiderivative = 4.80

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="maxima")
```

output
```
1/5*A^2*b^2*d^2*g^2*i^2*x^5 + 1/2*A^2*b^2*c*d*g^2*i^2*x^4 + 1/2*A^2*a*b*d^
2*g^2*i^2*x^4 + 1/3*A^2*b^2*c^2*g^2*i^2*x^3 + 4/3*A^2*a*b*c*d*g^2*i^2*x^3
+ 1/3*A^2*a^2*d^2*g^2*i^2*x^3 + A^2*a*b*c^2*g^2*i^2*x^2 + A^2*a^2*c*d*g^2*
i^2*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c
*log(d*x + c)/d)*A*B*a^2*c^2*g^2*i^2 + 2*(x^2*log(b*e*x/(d*x + c) + a*e/(d
*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*
d))*A*B*a*b*c^2*g^2*i^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))
+ 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x
^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2*c^2*g^2*i^2 + 2*(x^2*log(
b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)
/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*c*d*g^2*i^2 + 4/3*(2*x^3*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3
- ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b*c
*d*g^2*i^2 + 1/6*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b
*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*
(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^2*
c*d*g^2*i^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(
b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*
c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*d^2*g^2*i^2 + 1/6*(6*x^4*log(b*e*x/(d
*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/...
```

## 3.65.8 Giac [F]

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (bgx+ag)^2(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) +
A)^2, x)
```

### 3.65.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2
,x)
```

output
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2
, x)
```

# 3.66 $\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

### 3.66.1 Optimal result

Integrand size = 40, antiderivative size = 589

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\frac{B^2(bc-ad)^3gi^2x}{12b^2d}+\frac{B^2(bc-ad)^2gi^2(c+dx)^2}{12bd^2}-\frac{B^2(bc-ad)^4gi^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2}$$
$$-\frac{B(bc-ad)^3gi^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3d}$$
$$-\frac{B(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3}$$
$$+\frac{B(bc-ad)^2gi^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4bd^2}$$
$$-\frac{B(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6d^2}$$
$$+\frac{(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{12b^3}$$
$$+\frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b^2}$$
$$+\frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4b}$$
$$-\frac{B(bc-ad)^4gi^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6b^3d^2}$$
$$-\frac{B^2(bc-ad)^4gi^2\log(c+dx)}{4b^3d^2}-\frac{B^2(bc-ad)^4gi^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^3d^2}$$

output
```
1/12*B^2*(-a*d+b*c)^3*g*i^2*x/b^2/d+1/12*B^2*(-a*d+b*c)^2*g*i^2*(d*x+c)^2/
b/d^2-1/12*B^2*(-a*d+b*c)^4*g*i^2*ln((b*x+a)/(d*x+c))/b^3/d^2-1/6*B*(-a*d+
b*c)^3*g*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/d-1/6*B*(-a*d+b*c)^2*
g*i^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3+1/4*B*(-a*d+b*c)^2*g*i^2*(
d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d^2-1/6*B*(-a*d+b*c)*g*i^2*(d*x+c)^
3*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2+1/12*(-a*d+b*c)^2*g*i^2*(b*x+a)^2*(A+B*l
n(e*(b*x+a)/(d*x+c)))^2/b^3+1/6*(-a*d+b*c)*g*i^2*(b*x+a)^2*(d*x+c)*(A+B*ln
(e*(b*x+a)/(d*x+c)))^2/b^2+1/4*g*i^2*(b*x+a)^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)
/(d*x+c)))^2/b-1/6*B*(-a*d+b*c)^4*g*i^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B+B*ln
(e*(b*x+a)/(d*x+c)))/b^3/d^2-1/4*B^2*(-a*d+b*c)^4*g*i^2*ln(d*x+c)/b^3/d^2-
1/6*B^2*(-a*d+b*c)^4*g*i^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^2
```

### 3.66.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 677, normalized size of antiderivative = 1.15

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$

$$= \frac{gi^2\Bigl(-4(bc-ad)(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+3b(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{4B(bc-ad)^2\bigl(2Ab}{}}{}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2,x]
```

output
```
(g*i^2*(-4*(b*c - a*d)*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
+ 3*b*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (4*B*(b*c - a*d
)^2*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*(b*d*x + (b*c - a*d)*Log[a + b*
x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*(c +
d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)^2*Log[a + b*x]
*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a*d)^2*Log[c + d*x] - B
*(b*c - a*d)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*
d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3 - (B*(b*c - a*d)*
(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*Log[a +
b*x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*
d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
 + 2*b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a*d)^
3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d)^3*Lo
g[c + d*x] - 3*B*(b*c - a*d)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))
/(12*d^2)
```

### 3.66.3 Rubi [A] (verified)

Time = 1.44 (sec) , antiderivative size = 670, normalized size of antiderivative = 1.14, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.350$, Rules used = {2962, 2783, 2782, 27, 86, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2962}$$

$$gi^2(bc-ad)^4\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$gi^2(bc-ad)^4\left(-\frac{B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right.$$

$$\Big\downarrow \text{2782}$$

$$gi^2(bc-ad)^4\left(-\frac{B\left(-B\int -\frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{6d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+b}{c+d}\right.\right.}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right.}}{2b}\right.$$

$$\Big\downarrow \text{27}$$

$$gi^2(bc-ad)^4\left(-\frac{B\left(\frac{B\int \frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a-}{c-}}{2b}\right.$$

$$\Big\downarrow \text{86}$$

$$gi^2(bc-ad)^4\left(-\frac{B\left(\frac{B\int \left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right.}{2b}\right.$$

$$\Big\downarrow \text{2009}$$

3.66. $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$gi^2(bc-ad)^4\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}-\frac{B\left(-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}\right.}{}\right.}{2b}\right.$$

$\downarrow$ 2783

$$gi^2(bc-ad)^4\left(\frac{-\frac{2B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{2b}-\frac{B\left(-\frac{B\log}{2d}\right.}{}\right.$$

$\downarrow$ 2773

$$gi^2(bc-ad)^4\left(\frac{-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{a+bx}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(B\log\left(\right.\right.}{3b(c+dx)^2\left(b-\right.}}{2b}\right.$$

$\downarrow$ 49

---

3.66. $\int(ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$gi^2(bc-ad)^4\left(\frac{-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\left(\frac{b}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}+\frac{1}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a\cdots}{\cdots}}{2b}\right.$$

$\downarrow$ 2009

$$gi^2(bc-ad)^4\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^2}\right)}{2b}\right)}{3b}+\frac{(a+bx)^2\left(B\cdots\right.}{3b(c+dx\cdots}}{2b}\right.$$

$\downarrow$ 2781

$$gi^2(bc-ad)^4\left(\frac{\frac{\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{3b}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\cdots}{\cdots}\right.}{2b}\right.}{3b}}{2b}\right.$$

$\downarrow$ 2784

---

3.66. $\int(ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$gi^2(bc-ad)^4\left(\frac{\frac{\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}}{3b}-\frac{2B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right.}{}}{2b}\right.$$

$\downarrow$ 2754

$$gi^2(bc-ad)^4\left(\frac{\frac{\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{B\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{d}}{d}\right.}{b}}{3b}}{2b}\right.$$

$\downarrow$ 2838

$$gi^2(bc-ad)^4\left(-\frac{B\left(-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{1}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right)}{6d^2}\right)}{2b}+\right.$$

---

3.66. $\int(ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x
]
```

output
```
(b*c - a*d)^4*g*i^2*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(4*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^4) - (B*((b*(A + B*Log[(e*(
a + b*x))/(c + d*x)]))/(3*d^2*(b - (d*(a + b*x))/(c + d*x))^3) - (A + B*Lo
g[(e*(a + b*x))/(c + d*x)])/(2*d^2*(b - (d*(a + b*x))/(c + d*x))^2) + (B*(
-(b - (d*(a + b*x))/(c + d*x))^(-2) + 1/(b*(b - (d*(a + b*x))/(c + d*x)))
+ Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(6
*d^2)))/(2*b) + (((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b
*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*(((a + b*x)^2*(A + B*
Log[(e*(a + b*x))/(c + d*x)]))/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*
x))^2) - (B*(b/(d^2*(b - (d*(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x))
/(c + d*x)]/d^2))/(2*b)))/(3*b) + (((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(
c + d*x)])^2)/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(((a
+ b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b*x
))/(c + d*x))) - (-(((A + B + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(
a + b*x))/(b*(c + d*x))])/d) - (B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])
/d)/d))/b)/(3*b))/(2*b))
```

#### 3.66.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

## 3.66. $\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.66.4 Maple [F]

$$\int (bgx+ag)\,(dix+ci)^{2}\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^{2}dx$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

### 3.66.5 Fricas [F]

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx = \int (bgx+ag)(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="fricas")
```

output
```
integral(A^2*b*d^2*g*i^2*x^3 + A^2*a*c^2*g*i^2 + (2*A^2*b*c*d + A^2*a*d^2)
*g*i^2*x^2 + (A^2*b*c^2 + 2*A^2*a*c*d)*g*i^2*x + (B^2*b*d^2*g*i^2*x^3 + B^
2*a*c^2*g*i^2 + (2*B^2*b*c*d + B^2*a*d^2)*g*i^2*x^2 + (B^2*b*c^2 + 2*B^2*a
*c*d)*g*i^2*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b*d^2*g*i^2*x^3 + A
*B*a*c^2*g*i^2 + (2*A*B*b*c*d + A*B*a*d^2)*g*i^2*x^2 + (A*B*b*c^2 + 2*A*B*
a*c*d)*g*i^2*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

### 3.66.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

### 3.66.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2259 vs. $2(562) = 1124$.

Time = 0.32 (sec) , antiderivative size = 2259, normalized size of antiderivative = 3.84

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="maxima")
```

output
```
1/4*A^2*b*d^2*g*i^2*x^4 + 2/3*A^2*b*c*d*g*i^2*x^3 + 1/3*A^2*a*d^2*g*i^2*x^
3 + 1/2*A^2*b*c^2*g*i^2*x^2 + A^2*a*c*d*g*i^2*x^2 + 2*(x*log(b*e*x/(d*x +
c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a*c^2*g*i^2
 + (x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*
log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*b*c^2*g*i^2 + 2*(x^2*log(b*e*x
/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2
- (b*c - a*d)*x/(b*d))*A*B*a*c*d*g*i^2 + 2/3*(2*x^3*log(b*e*x/(d*x + c) +
a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c
*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b*c*d*g*i^2 +
1/3*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 -
 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)
*x)/(b^2*d^2))*A*B*a*d^2*g*i^2 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2
- a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x
)/(b^3*d^3))*A*B*b*d^2*g*i^2 + A^2*a*c^2*g*i^2*x - 1/12*(7*a^2*b*c^2*d^2*g
*i^2 - 2*a^3*c*d^3*g*i^2 - (2*g*i^2*log(e) - g*i^2)*b^3*c^4 + 2*(4*g*i^2*l
og(e) - 3*g*i^2)*a*b^2*c^3*d)*B^2*log(d*x + c)/(b^2*d^2) + 1/6*(b^4*c^4*g*
i^2 - 4*a*b^3*c^3*d*g*i^2 + 6*a^2*b^2*c^2*d^2*g*i^2 - 4*a^3*b*c*d^3*g*i^2
+ a^4*d^4*g*i^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(
-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d^2) + 1/12*(3*B^2*b^4*d^4*g*i^2*...
```

### 3.66.8 Giac [F]

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (bgx+ag)(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="giac")
```

output
```
integrate((b*g*x + a*g)*(d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)
^2, x)
```

### 3.66.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)\,\left(c\,i+d\,i\,x\right)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x
)
```

output
```
int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
x)
```

# 3.67 $\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$



## 3.67.1 Optimal result

Integrand size = 32, antiderivative size = 334

$$\begin{aligned}
&\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2i^2x}{3b^2}+\frac{B^2(bc-ad)^3i^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d} \\
&\quad -\frac{2B(bc-ad)^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^3} \\
&\quad -\frac{B(bc-ad)i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3bd}+\frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d} \\
&\quad +\frac{B^2(bc-ad)^3i^2\log(c+dx)}{b^3d}+\frac{2B(bc-ad)^3i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d} \\
&\quad -\frac{2B^2(bc-ad)^3i^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}
\end{aligned}$$

output
```
1/3*B^2*(-a*d+b*c)^2*i^2*x/b^2+1/3*B^2*(-a*d+b*c)^3*i^2*ln((b*x+a)/(d*x+c)
)/b^3/d-2/3*B*(-a*d+b*c)^2*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3-1/3
*B*(-a*d+b*c)*i^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d+1/3*i^2*(d*x+c
)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d+B^2*(-a*d+b*c)^3*i^2*ln(d*x+c)/b^3/d+2
/3*B*(-a*d+b*c)^3*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a)
)/b^3/d-2/3*B^2*(-a*d+b*c)^3*i^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/d
```

### 3.67.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 287, normalized size of antiderivative = 0.86

$$\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$= \frac{i^2\left((c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B(bc-ad)\left(2Abd(bc-ad)x-B(bc-ad)(bdx+(bc-ad)\log(a+bx))+2Bd(bc-ad)(a+bx)\log(\cdots\right.}{\cdots}}{\cdots}$$

input
```
Integrate[(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(i^2*((c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*
(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*(b*d*x + (b*c - a*d)*Log[a + b*x])
+ 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + b^2*(c + d*x)
^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*(b*c - a*d)^2*Log[a + b*x]*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(b*c - a*d)^2*Log[c + d*x] - B*(b*
c - a*d)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
 - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(3*d)
```

### 3.67.3 Rubi [A] (verified)

Time = 0.93 (sec) , antiderivative size = 360, normalized size of antiderivative = 1.08, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.344$, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$
$$\Big\downarrow \text{2952}$$
$$i^2(bc-ad)^3\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$
$$\Big\downarrow \text{2756}$$

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)$$

$\downarrow$ 2789

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{3d}\right)$$

$\downarrow$ 2756

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{3d}\right)$$

$\downarrow$ 54

3.67. $\int(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+\cdots}{(\cdots}}{\cdots}\cdots}{3d}\cdots$$

$$\Big\downarrow \; 2009$$

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}\cdots\right)}{2d}\cdots\right)}{b}\cdots\right)}{3d}\cdots$$

$$\Big\downarrow \; 2789$$

---

3.67. $\int (ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right.}{}\right.}{3d}\right.$$

$\downarrow$ 2751

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right.}{3d}\right.$$

$\downarrow$ 16

3.67. $\int(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}}{b}+\right.}{3d}\right.$$

$\downarrow$ 2779

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{\frac{\frac{B\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e}{}\right.\right.}{b(c+dx)\left(b-\right.}\right.}{}}{b}\right.}{}\right.$$

$\downarrow$ 2838

3.67. $\int(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{\frac{B\,\mathrm{PolyLog}\left(2,\right.}{b}}{}\right.}{}\right.$$

input
```
Int[(c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(b*c - a*d)^3*i^2*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(3*d*(b - (d*(a
+ b*x))/(c + d*x))^3) - (2*B*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])/(2*
d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(1/(b*(b - (d*(a + b*x))/(c + d*x)
)) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))
/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b*(c +
 d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*(a + b*x))/(c + d*x)]
)/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x
))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b)/
b))/(3*d))
```

### 3.67.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

## 3.67. $\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

### 3.67.4 Maple [F]

$$\int (dix+ci)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

### 3.67.5 Fricas [F]

$$\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (dix+ci)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x^
2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B
*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))
, x)
```

### 3.67.6 Sympy [F(-1)]

Timed out.

$$\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

### 3.67.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1202 vs. $2(319) = 638$.

Time = 0.28 (sec) , antiderivative size = 1202, normalized size of antiderivative = 3.60

$$\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output
```
1/3*A^2*d^2*i^2*x^3 + A^2*c*d*i^2*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*
x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*c^2*i^2 + 2*(x^2*log(b*
e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d
^2 - (b*c - a*d)*x/(b*d))*A*B*c*d*i^2 + 1/3*(2*x^3*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*d^2*i^2 + A^2*c
^2*i^2*x - 1/3*(5*a*b*c^2*d*i^2 - 2*a^2*c*d^2*i^2 + (2*i^2*log(e) - 3*i^2)
*b^2*c^3)*B^2*log(d*x + c)/(b^2*d) - 2/3*(b^3*c^3*i^2 - 3*a*b^2*c^2*d*i^2
+ 3*a^2*b*c*d^2*i^2 - a^3*d^3*i^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c -
a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d) + 1/3*(B^2*b^3*
d^3*i^2*x^3*log(e)^2 + (a*b^2*d^3*i^2*log(e) + (3*i^2*log(e)^2 - i^2*log(e
))*b^3*c*d^2)*B^2*x^2 + ((3*i^2*log(e)^2 - 4*i^2*log(e) + i^2)*b^3*c^2*d +
 2*(3*i^2*log(e) - i^2)*a*b^2*c*d^2 - (2*i^2*log(e) - i^2)*a^2*b*d^3)*B^2*
x + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x
 + (3*a*b^2*c^2*d*i^2 - 3*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B^2)*log(b*x + a)
^2 + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*
x + B^2*b^3*c^3*i^2)*log(d*x + c)^2 + (2*B^2*b^3*d^3*i^2*x^3*log(e) + (a*b
^2*d^3*i^2 + (6*i^2*log(e) - i^2)*b^3*c*d^2)*B^2*x^2 + 2*(3*a*b^2*c*d^2*i^
2 - a^2*b*d^3*i^2 + (3*i^2*log(e) - 2*i^2)*b^3*c^2*d)*B^2*x + (2*(3*i^2*lo
g(e) - 2*i^2)*a*b^2*c^2*d - (6*i^2*log(e) - 7*i^2)*a^2*b*c*d^2 + (2*i^2...
```

### 3.67.8 Giac [F]

$$\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (dix+ci)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output
```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

### 3.67.9 Mupad [F(-1)]

Timed out.

$$\int (ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (c\,i+d\,i\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

# 3.68 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$



## 3.68.1 Optimal result

Integrand size = 42, antiderivative size = 535

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx &= -\frac{Bd(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g}\\
&+\frac{2B(bc-ad)^2i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g}\\
&+\frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g}+\frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg}\\
&+\frac{B^2(bc-ad)^2i^2\log(c+dx)}{b^3g}+\frac{B(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}\\
&-\frac{(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}\\
&+\frac{2B^2(bc-ad)^2i^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g}-\frac{B^2(bc-ad)^2i^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}\\
&+\frac{2B(bc-ad)^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}\\
&+\frac{2B^2(bc-ad)^2i^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}\end{aligned}$$

output
```
-B*d*(-a*d+b*c)*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g+2*B*(-a*d+b*
c)^2*i^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g+d*(-a*
d+b*c)*i^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^3/g+1/2*i^2*(d*x+c)^2*(
A+B*ln(e*(b*x+a)/(d*x+c)))^2/b/g+B^2*(-a*d+b*c)^2*i^2*ln(d*x+c)/b^3/g+B*(-
a*d+b*c)^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g
-(-a*d+b*c)^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))/
b^3/g+2*B^2*(-a*d+b*c)^2*i^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/g-B^2*(-a*
d+b*c)^2*i^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g+2*B*(-a*d+b*c)^2*i^2*(A+
B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g+2*B^2*(-a*d+
b*c)^2*i^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^3/g
```

### 3.68.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2615 vs. $2(535) = 1070$.

Time = 2.44 (sec) , antiderivative size = 2615, normalized size of antiderivative = 4.89

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x),x]
```

output
```
(i^2*(12*A^2*b*d*(2*b*c - a*d)*x + 6*A^2*b^2*d^2*x^2 + 12*A^2*(b*c - a*d)^
2*Log[a + b*x] - 24*A*b*B*c*(a*d*Log[a/b + x]^2 - 2*a*d*Log[a/b + x]*(1 +
Log[a + b*x]) + 2*(-(b*c) + a*d + Log[c/d + x]*(b*c + a*d*Log[a + b*x] - a
*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) + (-(b*d*x) + a*d*Log[a + b*x])*Log[
(e*(a + b*x))/(c + d*x)]) - 2*a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) +
 12*A*b^2*B*c^2*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x] - Log[c/d +
 x] - Log[(e*(a + b*x))/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-
(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 6*A*B*(-4*a*d^2*
(a + b*x)*(-1 + Log[a/b + x]) + 2*a^2*d^2*Log[a/b + x]^2 + 4*a*b*d*(c + d*
x)*(-1 + Log[c/d + x]) + d^2*(b*x*(2*a - b*x) + 2*b^2*x^2*Log[a/b + x] - 2
*a^2*Log[a + b*x]) - 2*d^2*(b*x*(-2*a + b*x) + 2*a^2*Log[a + b*x])*(Log[a/
b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)]) + b^2*(d*x*(-2*c + d
*x) - 2*d^2*x^2*Log[c/d + x] + 2*c^2*Log[c + d*x]) - 4*a^2*d^2*(Log[c/d +
x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d
)])) - 8*b*B^2*c*(a*d*Log[a/b + x]^3 - 3*d*(2*b*x - 2*(a + b*x)*Log[a/b +
x] + (a + b*x)*Log[a/b + x]^2) - 3*b*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (
c + d*x)*Log[c/d + x]^2) - 3*d*(b*x - a*Log[a + b*x])*(-Log[a/b + x] + Log
[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2 + 6*(a*d + 2*b*d*x - b*d*x*Log
[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*
Log[c/d + x] + (b*c - a*d)*Log[(b*(c + d*x))/(b*c - a*d)]) + (b*c - a*d...
```

### 3.68.3 Rubi [A] (verified)

Time = 1.97 (sec) , antiderivative size = 501, normalized size of antiderivative = 0.94, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {2962, 2789, 2756, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ag+bgx}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i^2(bc-ad)^2\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \text{2789}$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \text{2756}$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \text{2789}$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{d\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{d}\right)}{b}+\frac{d\int \frac{\left(A+B\log\left(\frac{e(a+\ldots}{c+\ldots}\right.\right.}{\left(b-\frac{d(a+bx)}{c+dx}\ldots\right.}}{b}\ldots\right.}{g}$$

$$\Big\downarrow \text{2751}$$

---

3.68. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{d}\right)}{b}}{g}$$

$\downarrow$ 16

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}\right)}{d}\right)}{b}\right)}{g}$$

$\downarrow$ 2755

---

3.68. $\int\frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}\right)}{d}\right)}{b}\right.$$
$$\overline{\qquad\qquad\qquad g \qquad}$$

$\downarrow$ 2754

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}\right)}{d}\right)}{b}\right.$$

$\downarrow$ 2779

---

3.68. $$\int\frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$

$$i^2(bc-ad)^2 \left( \frac{d\left( \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B\left( \frac{\frac{B\int \frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b} - \frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}}{b} + \frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(}{c}\right.\right.}{b(c+dx)\left(b-\frac{a}{}\right.}\right.}{}\right.}{d}\right.}{b}\right.$$

$\downarrow$ 2821

$$i^2(bc-ad)^2 \left( \frac{\frac{\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\int \frac{(c+dx)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}}{b} - \frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}}{b}\right.$$

$\downarrow$ 2838

---

3.68. $\displaystyle\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$i^2(bc-ad)^2 \left( \frac{\frac{\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\int \frac{(c+dx)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}}{b} - \frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}}{b}\right.$$

$\downarrow$ 7143

$$i^2(bc-ad)^2 \left( \frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B\left(\frac{\frac{B\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.}{b}\right.}{d}\right.}{b}\right.$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
,x]
```

## 3.68. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

output
```
((b*c - a*d)^2*i^2*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(2*d*(b - (
d*(a + b*x))/(c + d*x))^2) - (B*((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(
c + d*x)]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*(a
 + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*
Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*
(a + b*x))])/b)/b))/d))/b + ((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c +
d*x)])^2)/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (2*B*(-(((A + B*Lo
g[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*
PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d))/b))/b + (-(((A + B*Log[(e*(a
+ b*x))/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (2*B*((A
+ B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))]
+ B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b)/b)/b))/g
```

#### 3.68.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.68. $\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx} \, dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.68.4 Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{bgx+ag}dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

### 3.68.5 Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algo
rithm="fricas")
```

output
```
integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x
^2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*
B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)
))/(b*g*x + a*g), x)
```

### 3.68.6 Sympy [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$

$$= \frac{i^2\left(\int \frac{A^2c^2}{a+bx}\,dx + \int \frac{A^2d^2x^2}{a+bx}\,dx + \int \frac{B^2c^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx + \int \frac{2ABc^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx + \int \frac{2A^2cdx}{a+bx}\,dx + \int \frac{B^2d^2x \cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)
```

output
```
i**2*(Integral(A**2*c**2/(a + b*x), x) + Integral(A**2*d**2*x**2/(a + b*x)
, x) + Integral(B**2*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x
), x) + Integral(2*A*B*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x)
, x) + Integral(2*A**2*c*d*x/(a + b*x), x) + Integral(B**2*d**2*x**2*log(a
*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(2*A*B*d**2*x**
2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(2*B**2*c*d
*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(4*A*B*
c*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g
```

## 3.68.7 Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algo
rithm="maxima")
```

output
```
2*A^2*c*d*i^2*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + 1/2*A^2*d^2*i^2*(2*a^2*
log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A^2*c^2*i^2*log(b*g*x +
a*g)/(b*g) + 1/2*(B^2*b^2*d^2*i^2*x^2 + 2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B^
2*x + 2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B^2*log(b*x + a))*log(
d*x + c)^2/(b^3*g) - integrate(-(B^2*b^3*c^3*i^2*log(e)^2 + 2*A*B*b^3*c^3*
i^2*log(e) + (B^2*b^3*d^3*i^2*log(e)^2 + 2*A*B*b^3*d^3*i^2*log(e))*x^3 + 3
*(B^2*b^3*c*d^2*i^2*log(e)^2 + 2*A*B*b^3*c*d^2*i^2*log(e))*x^2 + (B^2*b^3*
d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^
3*i^2)*log(b*x + a)^2 + 3*(B^2*b^3*c^2*d*i^2*log(e)^2 + 2*A*B*b^3*c^2*d*i^
2*log(e))*x + 2*(B^2*b^3*c^3*i^2*log(e) + A*B*b^3*c^3*i^2 + (B^2*b^3*d^3*i
^2*log(e) + A*B*b^3*d^3*i^2)*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(e) + A*B*b^3*c
*d^2*i^2)*x^2 + 3*(B^2*b^3*c^2*d*i^2*log(e) + A*B*b^3*c^2*d*i^2)*x)*log(b*
x + a) - (2*B^2*b^3*c^3*i^2*log(e) + 2*A*B*b^3*c^3*i^2 + (2*A*B*b^3*d^3*i^
2 + (2*i^2*log(e) + i^2)*B^2*b^3*d^3)*x^3 + (6*A*B*b^3*c*d^2*i^2 - (a*b^2*
d^3*i^2 - 2*(3*i^2*log(e) + 2*i^2)*b^3*c*d^2)*B^2)*x^2 + 2*(3*A*B*b^3*c^2*
d*i^2 + (3*b^3*c^2*d*i^2*log(e) + 2*a*b^2*c*d^2*i^2 - a^2*b*d^3*i^2)*B^2)*
x + 2*(B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + (4*b^3*c^2*d*i^2 -
2*a*b^2*c*d^2*i^2 + a^2*b*d^3*i^2)*B^2*x + (b^3*c^3*i^2 + a*b^2*c^2*d*i^2
- 2*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B^2)*log(b*x + a))*log(d*x + c))/(b^4*d
*g*x^2 + a*b^3*c*g + (b^4*c*g + a*b^3*d*g)*x), x)
```

### 3.68.8 Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algo
rithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*
g), x)
```

### 3.68.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{(c\,i+d\,i\,x)^2 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
, x)
```

# 3.69 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$



## 3.69.1 Optimal result

Integrand size = 42, antiderivative size = 442

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx \\
&= -\frac{2B^2(bc-ad)i^2(c+dx)}{b^2g^2(a+bx)} - \frac{2B(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^2(a+bx)} \\
&\quad + \frac{2Bd(bc-ad)i^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2} \\
&\quad + \frac{d^2i^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^2} - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g^2(a+bx)} \\
&\quad - \frac{2d(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{2B^2d(bc-ad)i^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g^2} \\
&\quad + \frac{4Bd(bc-ad)i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{4B^2d(bc-ad)i^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}
\end{aligned}$$

output
```
-2*B^2*(-a*d+b*c)*i^2*(d*x+c)/b^2/g^2/(b*x+a)-2*B*(-a*d+b*c)*i^2*(d*x+c)*(
A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g^2/(b*x+a)+2*B*d*(-a*d+b*c)*i^2*ln((-a*d+b
*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3/g^2+d^2*i^2*(b*x+a)*(A+B*ln
(e*(b*x+a)/(d*x+c)))^2/b^3/g^2-(-a*d+b*c)*i^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/b^2/g^2/(b*x+a)-2*d*(-a*d+b*c)*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2
*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^2+2*B^2*d*(-a*d+b*c)*i^2*polylog(2,d*(b*x
+a)/b/(d*x+c))/b^3/g^2+4*B*d*(-a*d+b*c)*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*po
lylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^2+4*B^2*d*(-a*d+b*c)*i^2*polylog(3,b*(d
*x+c)/d/(b*x+a))/b^3/g^2
```

## 3.69.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2649 vs. $2(442) = 884$.

Time = 2.54 (sec) , antiderivative size = 2649, normalized size of antiderivative = 5.99

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^2,x]
```

output
```
(i^2*(3*A^2*b*d^2*x - (3*A^2*(b*c - a*d)^2)/(a + b*x) + 6*A^2*d*(b*c - a*d
)*Log[a + b*x] - (6*A*b^2*B*c^2*(-(d*(a + b*x)*Log[c/d + x]) + d*(a + b*x)
*Log[(d*(a + b*x))/(-(b*c) + a*d)] + (b*c - a*d)*(1 + Log[(e*(a + b*x))/(c
 + d*x)])))/((b*c - a*d)*(a + b*x)) + (3*b^2*B^2*c^2*(-2*b*c + 2*a*d - 2*d
*(a + b*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)] - 2*d
*(a + b*x)*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - (b*c - a*d)*Log[(e*
(a + b*x))/(c + d*x)]^2 + 2*d*(a + b*x)*Log[c + d*x] - 2*d*(a + b*x)*Log[(
e*(a + b*x))/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] + d*(a + b*x)*(Log[
a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)]) + d*(a + b*x)*(Log[(b*c - a*d)/(b*c + b*d*x
)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x)])
- 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(a + b*x)) + 6*A
*b*B*c*d*(Log[a/b + x]^2 - 2*Log[a/b + x]*Log[a + b*x] - 2*Log[c/d + x]*Lo
g[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a + b*x]*((a*d)/(b*c - a*d) + Log[
c/d + x] + Log[(e*(a + b*x))/(c + d*x)]) + 2*a*((a + b*x)^(-1) + Log[(e*(a
 + b*x))/(c + d*x)]/(a + b*x) + (d*Log[c + d*x])/(-(b*c) + a*d)) - 2*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)]) + 6*A*B*d^2*((a + b*x)*(-1 + Log[a/b + x
]) - a*Log[a/b + x]^2 - (a^2*(1 + Log[a/b + x]))/(a + b*x) - b*(c/d + x)*(
-1 + Log[c/d + x]) + (a^2*Log[c/d + x])/(a + b*x) + (b*x - a^2/(a + b*x) -
 2*a*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c...
```

### 3.69.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i^2(bc-ad)\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{i^2(bc-ad)\int \left(\frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2(a+bx)^2}+\frac{2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right)d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i^2(bc-ad)\left(\frac{d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^3(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{4Bd\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^3}+\frac{2Bd\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{b^3}\right.}{}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^2,x]
```

output
```
((b*c - a*d)*i^2*((-2*B^2*(c + d*x))/(b^2*(a + b*x)) - (2*B*(c + d*x)*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(b^2*(a + b*x)) - ((c + d*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)])^2)/(b^2*(a + b*x)) + (d^2*(a + b*x)*(A + B*Log[(
e*(a + b*x))/(c + d*x)])^2)/(b^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)))
+ (2*B*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c
+ d*x))])/b^3 - (2*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (b*(c
+ d*x))/(d*(a + b*x))])/b^3 + (2*B^2*d*PolyLog[2, (d*(a + b*x))/(b*(c + d*
x))])/b^3 + (4*B*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c +
 d*x))/(d*(a + b*x))])/b^3 + (4*B^2*d*PolyLog[3, (b*(c + d*x))/(d*(a + b*x
))])/b^3))/g^2
```

### 3.69.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.69.4 Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx+ag)^2}dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

### 3.69.5 Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, al
gorithm="fricas")
```

output
```
integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x
^2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*
B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)
))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

### 3.69.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

### 3.69.7 Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, al
gorithm="maxima")
```

output
```
-A^2*(a^2/(b^4*g^2*x + a*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^
2))*d^2*i^2 + 2*A^2*c*d*i^2*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2
*g^2)) - 2*A*B*c^2*i^2*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x +
a*b*g^2) + 1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2)
- d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - A^2*c^2*i^2/(b^2*g^2*x + a*b*g^2
) + (B^2*b^2*d^2*i^2*x^2 + B^2*a*b*d^2*i^2*x - (b^2*c^2*i^2 - 2*a*b*c*d*i^
2 + a^2*d^2*i^2)*B^2 + 2*((b^2*c*d*i^2 - a*b*d^2*i^2)*B^2*x + (a*b*c*d*i^2
 - a^2*d^2*i^2)*B^2)*log(b*x + a))*log(d*x + c)^2/(b^4*g^2*x + a*b^3*g^2)
- integrate(-(B^2*b^3*c^3*i^2*log(e)^2 + (B^2*b^3*d^3*i^2*log(e)^2 + 2*A*B
*b^3*d^3*i^2*log(e))*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(e)^2 + 2*A*B*b^3*c*d^2
*i^2*log(e))*x^2 + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*
b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^2)*log(b*x + a)^2 + (3*B^2*b^3*c^2*d*i^2*l
og(e)^2 + 4*A*B*b^3*c^2*d*i^2*log(e))*x + 2*(B^2*b^3*c^3*i^2*log(e) + (B^2
*b^3*d^3*i^2*log(e) + A*B*b^3*d^3*i^2)*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(e) +
 A*B*b^3*c*d^2*i^2)*x^2 + (3*B^2*b^3*c^2*d*i^2*log(e) + 2*A*B*b^3*c^2*d*i^
2)*x)*log(b*x + a) - 2*((A*B*b^3*d^3*i^2 + (i^2*log(e) + i^2)*B^2*b^3*d^3)
*x^3 + (b^3*c^3*i^2*log(e) - a*b^2*c^2*d*i^2 + 2*a^2*b*c*d^2*i^2 - a^3*d^3
*i^2)*B^2 + (3*A*B*b^3*c*d^2*i^2 + (3*b^3*c*d^2*i^2*log(e) + 2*a*b^2*d^3*i
^2)*B^2)*x^2 + (2*A*B*b^3*c^2*d*i^2 + (2*a*b^2*c*d^2*i^2 + (3*i^2*log(e) -
 i^2)*b^3*c^2*d)*B^2)*x + (B^2*b^3*d^3*i^2*x^3 + (5*b^3*c*d^2*i^2 - 2*a...
```

### 3.69.8 Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, al
gorithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*
g)^2, x)
```

### 3.69.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^2,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^2, x)
```

# 3.70 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$



## 3.70.1 Optimal result

Integrand size = 42, antiderivative size = 387

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx &= -\frac{2B^2di^2(c+dx)}{b^2g^3(a+bx)}-\frac{B^2i^2(c+dx)^2}{4bg^3(a+bx)^2}-\frac{2Bdi^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^2g^3(a+bx)}\\
&\quad-\frac{Bi^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2bg^3(a+bx)^2}-\frac{di^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^2g^3(a+bx)}\\
&\quad-\frac{i^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2bg^3(a+bx)^2}-\frac{d^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}\\
&\quad+\frac{2Bd^2i^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}+\frac{2B^2d^2i^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}\end{aligned}$$

output
```
-2*B^2*d*i^2*(d*x+c)/b^2/g^3/(b*x+a)-1/4*B^2*i^2*(d*x+c)^2/b/g^3/(b*x+a)^2
-2*B*d*i^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g^3/(b*x+a)-1/2*B*i^2*(
d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/g^3/(b*x+a)^2-d*i^2*(d*x+c)*(A+B*ln
(e*(b*x+a)/(d*x+c)))^2/b^2/g^3/(b*x+a)-1/2*i^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)
/(d*x+c)))^2/b/g^3/(b*x+a)^2-d^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*
(d*x+c)/d/(b*x+a))/b^3/g^3+2*B*d^2*i^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog
(2,b*(d*x+c)/d/(b*x+a))/b^3/g^3+2*B^2*d^2*i^2*polylog(3,b*(d*x+c)/d/(b*x+a
))/b^3/g^3
```

### 3.70.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3426 vs. $2(387) = 774$.

Time = 4.11 (sec) , antiderivative size = 3426, normalized size of antiderivative = 8.85

$$\int \frac{(ci + dix)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^3,x]
```

output
```
(i^2*((-6*A^2*(b*c - a*d)^2)/(a + b*x)^2 + (24*A^2*d*(-(b*c) + a*d))/(a +
b*x) + 12*A^2*d^2*Log[a + b*x] - (6*A*b^2*B*c^2*(b^2*c^2 - 4*a*b*c*d + a^2
*d^2 - 2*b^2*c*d*x - 2*a*b*d^2*x - 2*b^2*d^2*x^2 + 2*d^2*(a + b*x)^2*Log[c
/d + x] - 2*d^2*(a + b*x)^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*b^2*c^2*
Log[(e*(a + b*x))/(c + d*x)] - 4*a*b*c*d*Log[(e*(a + b*x))/(c + d*x)] + 2*
a^2*d^2*Log[(e*(a + b*x))/(c + d*x)]))/((b*c - a*d)^2*(a + b*x)^2) - (12*A
*b*B*c*d*(3*a*b^2*c^2 - 4*a^2*b*c*d + a^3*d^2 + 4*b^3*c^2*x - 6*a*b^2*c*d*
x + 2*a^2*b*d^2*x - 2*d*(-2*b*c + a*d)*(a + b*x)^2*Log[a + b*x] + 2*(b*c -
 a*d)^2*(a + 2*b*x)*Log[(e*(a + b*x))/(c + d*x)] - 4*a^2*b*c*d*Log[c + d*x
] + 2*a^3*d^2*Log[c + d*x] - 8*a*b^2*c*d*x*Log[c + d*x] + 4*a^2*b*d^2*x*Lo
g[c + d*x] - 4*b^3*c*d*x^2*Log[c + d*x] + 2*a*b^2*d^2*x^2*Log[c + d*x]))/(
(b*c - a*d)^2*(a + b*x)^2) + (3*b^2*B^2*c^2*(-(b*c - a*d)^2 + 6*d*(b*c - a
*d)*(a + b*x) + 6*d^2*(a + b*x)^2*Log[a + b*x] - 2*d^2*(a + b*x)^2*Log[a +
 b*x]^2 - 2*(b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)] + 4*d*(b*c - a*d)*(
a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 4*d^2*(a + b*x)^2*Log[a + b*x]*Log
[(e*(a + b*x))/(c + d*x)] - 2*(b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)]^2
 - 6*d^2*(a + b*x)^2*Log[c + d*x] + 4*d^2*(a + b*x)^2*Log[a + b*x]*Log[(b*
(c + d*x))/(b*c - a*d)] - 4*d^2*(a + b*x)^2*Log[(d*(a + b*x))/(-(b*c) + a*
d)]*Log[(b*c - a*d)/(b*c + b*d*x)] + 4*d^2*(a + b*x)^2*Log[(e*(a + b*x))/(
c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] - 2*d^2*(a + b*x)^2*Log[(b*c -...
```

### 3.70.3 Rubi [A] (verified)

Time = 1.37 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {2962, 2780, 2742, 2741, 2780, 2742, 2741, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i^2\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^3}$$

$$\Big\downarrow \text{2780}$$

$$\frac{i^2\left(\frac{\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^3}$$

$$\Big\downarrow \text{2742}$$

$$\frac{i^2\left(\frac{B\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}}{b}+\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^3}$$

$$\Big\downarrow \text{2741}$$

$$\frac{i^2\left(\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{B\left(-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}\right)-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}}{b}\right)}{g^3}$$

$$\Big\downarrow \text{2780}$$

$$\frac{i^2\left(\frac{d\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{B\left(-\frac{(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}-\frac{B(c+dx)^2}{4(a+bx)^2}\right)-\cdots}{b}\cdots\right.}{g^3}\cdots$$

$\big\downarrow$ 2742

$$\frac{i^2\left(\frac{d\left(\frac{2B\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{B\left(-\frac{(c+dx)^2\left(B\log\cdots\right.}{2\cdots}\cdots\right.}{\cdots}\cdots\right.}{g^3}\cdots$$

$\big\downarrow$ 2741

$$\frac{i^2\left(\frac{d\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{2B\left(-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{B(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}}{b}\right)}{b}+\frac{B\left(-\frac{(c+dx)\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right.}{g^3}\cdots$$

$\big\downarrow$ 2779

---

3.70. $\displaystyle\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

$$i^2\left(\frac{d\left(\frac{d\left(\frac{2B\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}\right)}{b}+\frac{2B\left(-\frac{(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right.}{a+bx}\right.}{}}{b}\right.}{g^3}$$

$\downarrow$ 2821

$$i^2\left(\frac{d\left(\frac{d\left(\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)-B\int\frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}\right)}{b}+\frac{2B}{}}{b}\right.$$

$\downarrow$ 7143

---

3.70. $\displaystyle\int\frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

$$i^2\left(\frac{d\left(\frac{d\left(\frac{2B\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)+B\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b}\right)}{b}+2B\left(-\frac{(c+dx)\left(B\log\right.}{a}\right.}{b}\right.$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^3,x]
```

output
```
(i^2*((-1/2*((c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x)
^2 + B*(-1/4*(B*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]))/(2*(a + b*x)^2)))/b + (d*((-(((c + d*x)*(A + B*Log[(e*
(a + b*x))/(c + d*x)])^2)/(a + b*x)) + 2*B*(-((B*(c + d*x))/(a + b*x)) - (
(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x)))/b + (d*(-(((A
+ B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/
b) + (2*B*((A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(
d*(a + b*x))] + B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b))/b))/b))/g^
3
```

### 3.70.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.70.4 Maple [F]

$$\int \frac{(dix + ci)^2 \left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx + ag)^3} dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

### 3.70.5 Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, al
gorithm="fricas")
```

output
```
integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x
^2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*
B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log((b*e*x + a*e)/(d*x + c)
))/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

### 3.70.6 Sympy [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{i^2\left(\int \frac{A^2c^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{A^2d^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{B^2c^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{2ABc^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right.}{}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)
```

output
```
i**2*(Integral(A**2*c**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
x) + Integral(A**2*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**
3), x) + Integral(B**2*c**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a**3
+ 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*c**2*log(a*
e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x
**3), x) + Integral(2*A**2*c*d*x/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3
*x**3), x) + Integral(B**2*d**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))*
*2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*d*
*2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2
*x**2 + b**3*x**3), x) + Integral(2*B**2*c*d*x*log(a*e/(c + d*x) + b*e*x/(
c + d*x))**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integra
l(4*A*B*c*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*
a*b**2*x**2 + b**3*x**3), x))/g**3
```

### 3.70.7 Maxima [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, al
gorithm="maxima")
```

output
```
-A*B*c*d*i^2*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^4*g^3*
x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d
)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3
*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^
3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2
*a*b^3*c*d + a^2*b^2*d^2)*g^3)) + 1/2*A^2*d^2*i^2*((4*a*b*x + 3*a^2)/(b^5*
g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) + 1/2*A
*B*c^2*i^2*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*
c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c)
+ a*e/(d*x + c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x
 + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - (2*b*x + a)*A^2*c*d*i^2/(b^4*g^3*x
^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*A^2*c^2*i^2/(b^3*g^3*x^2 + 2*a*b^2
*g^3*x + a^2*b*g^3) - 1/2*(4*(b^2*c*d*i^2 - a*b*d^2*i^2)*B^2*x + (b^2*c^2*
i^2 + 2*a*b*c*d*i^2 - 3*a^2*d^2*i^2)*B^2 - 2*(B^2*b^2*d^2*i^2*x^2 + 2*B^2*
a*b*d^2*i^2*x + B^2*a^2*d^2*i^2)*log(b*x + a))*log(d*x + c)^2/(b^5*g^3*x^2
 + 2*a*b^4*g^3*x + a^2*b^3*g^3) - integrate(-(3*B^2*b^3*c^2*d*i^2*x*log(e)
^2 + B^2*b^3*c^3*i^2*log(e)^2 + (B^2*b^3*d^3*i^2*log(e)^2 + 2*A*B*b^3*d^3*
i^2*log(e))*x^3 + (3*B^2*b^3*c*d^2*i^2*log(e)^2 + 2*A*B*b^3*c*d^2*i^2*log(
e))*x^2 + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^...
```

### 3.70.8 Giac [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, al
gorithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*
g)^3, x)
```

### 3.70.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{{\left(c\,i+d\,i\,x\right)}^2\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}{{\left(a\,g+b\,g\,x\right)}^3}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^3,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^3, x)
```

# 3.71 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$



## 3.71.1 Optimal result

Integrand size = 42, antiderivative size = 147

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{2B^2i^2(c+dx)^3}{27(bc-ad)g^4(a+bx)^3} - \frac{2Bi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)g^4(a+bx)^3}$$

output
```
-2/27*B^2*i^2*(d*x+c)^3/(-a*d+b*c)/g^4/(b*x+a)^3-2/9*B*i^2*(d*x+c)^3*(A+B*
ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^4/(b*x+a)^3-1/3*i^2*(d*x+c)^3*(A+B*ln(
e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)/g^4/(b*x+a)^3
```

## 3.71.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.08 (sec) , antiderivative size = 1352, normalized size of antiderivative = 9.20

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{i^2\left(18(bc-ad)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+54d(bc-ad)^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-54d^2(-bc\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^4,x]
```

output
```
-1/54*(i^2*(18*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 54*d
*(b*c - a*d)^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 54*d^2*(
-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + B*(12*A
*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B
*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(
b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*(a +
 b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a
*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e
*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[(e*(a + b*x)
)/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*
x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a + b*x)^3*Log[c + d*x
] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] -
36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x] - 18*B*d^3*
(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[(b*(c +
 d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c
) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) +
54*B*d^2*(a + b*x)^2*(2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) +
 2*d*(a + b*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*d*(a
+ b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 2*B*(b*c - a*d
+ d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*(a + b*x)*...
```

### 3.71.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2962, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^4} \, dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i^2 \int \frac{(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)}$$

$$\Big\downarrow \text{2742}$$

$$\frac{i^2\left(\frac{2}{3}B\int \frac{(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}\right)}{g^4(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{i^2\left(\frac{2}{3}B\left(-\frac{(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3(a+bx)^3}-\frac{B(c+dx)^3}{9(a+bx)^3}\right)-\frac{(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}\right)}{g^4(bc-ad)}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^4,x]
```

output
```
(i^2*(-1/3*((c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x)^
3 + (2*B*(-1/9*(B*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Log[(e*(a
 + b*x))/(c + d*x)]))/(3*(a + b*x)^3)))/3))/((b*c - a*d)*g^4)
```

### 3.71.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.71.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(141) = 282$.

Time = 1.05 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.50

| method | result |
|---|---|
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{i^2d^2e^2A^2}{3(ad-cb)^2g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{2i^2d^2e^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\right)}{(ad-cb)^2g^4}+\frac{i^2d^2e^2B^2\left(-\frac{\ln(}{3(}\right.}{}\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{i^2d^2e^2A^2}{3(ad-cb)^2g^4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}+\frac{2i^2d^2e^2AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{1}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}\right)}{(ad-cb)^2g^4}+\frac{i^2d^2e^2B^2\left(-\frac{\ln(}{3(}\right.}{}\right.}{d^2}$ |
| parts | $\frac{i^2A^2\left(-\frac{a^2d^2-2abcd+b^2c^2}{3b^3(bx+a)^3}+\frac{d(ad-cb)}{b^3(bx+a)^2}-\frac{d^2}{b^3(bx+a)}\right)}{g^4}-\frac{i^2B^2e^3\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{2}{27\left(\frac{be}{d}+\frac{(a}{d(}\right.}\right.}{g^4(ad-cb)}$ |
| risch | $-\frac{i^2A^2a^2d^2}{3g^4b^3(bx+a)^3}+\frac{2i^2A^2acd}{3g^4b^2(bx+a)^3}-\frac{i^2A^2c^2}{3g^4b(bx+a)^3}+\frac{i^2A^2d^2a}{g^4b^3(bx+a)^2}-\frac{i^2A^2dc}{g^4b^2(bx+a)^2}-\frac{i^2A^2d^2}{g^4b^3(bx+a)}+\frac{i^2B^2}{3g^4(ad-c}$ |
| norman | $\frac{B^2c\,d^2i^2x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}+\frac{B^2c^2d\,i^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}-\frac{9A^2a^2d^2i^2+9A^2abcd\,i^2+9A^2b^2c^2i^2+6AB\,a^2d^2i^2+6ABabcd\,i^2+6AB\,b^2}{27g\,b^3}$ |
| parallelrisch | $-\frac{-54AB\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5c\,d^3i^2-54ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5c^2d^2i^2-18AB\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)b^5d^4i^2-27B^2x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{}$ |

input

```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x,method=_RE
TURNVERBOSE)
```

output

```
-1/d^2*e*(a*d-b*c)*(-1/3*i^2*d^2*e^2/(a*d-b*c)^2/g^4*A^2/(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^3+2*i^2*d^2*e^2/(a*d-b*c)^2/g^4*A*B*(-1/3/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^3)+i^2*d^2*e^2/(a*d-b*c)^2/g^4*B^2*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

## 3.71. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

### 3.71.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(141) = 282$.

Time = 0.31 (sec) , antiderivative size = 444, normalized size of antiderivative = 3.02

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{3\left((9A^2+6AB+2B^2)b^3cd^2-(9A^2+6AB+2B^2)ab^2d^3\right)i^2x^2+3\left((9A^2+6AB+2B^2)b^3c^2d-(9A\right.}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, al
gorithm="fricas")
```

output
```
-1/27*(3*((9*A^2 + 6*A*B + 2*B^2)*b^3*c*d^2 - (9*A^2 + 6*A*B + 2*B^2)*a*b^
2*d^3)*i^2*x^2 + 3*((9*A^2 + 6*A*B + 2*B^2)*b^3*c^2*d - (9*A^2 + 6*A*B + 2
*B^2)*a^2*b*d^3)*i^2*x + ((9*A^2 + 6*A*B + 2*B^2)*b^3*c^3 - (9*A^2 + 6*A*B
 + 2*B^2)*a^3*d^3)*i^2 + 9*(B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2
+ 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^2)*log((b*e*x + a*e)/(d*x + c))^2
+ 6*((3*A*B + B^2)*b^3*d^3*i^2*x^3 + 3*(3*A*B + B^2)*b^3*c*d^2*i^2*x^2 + 3
*(3*A*B + B^2)*b^3*c^2*d*i^2*x + (3*A*B + B^2)*b^3*c^3*i^2)*log((b*e*x + a
*e)/(d*x + c)))/((b^7*c - a*b^6*d)*g^4*x^3 + 3*(a*b^6*c - a^2*b^5*d)*g^4*x
^2 + 3*(a^2*b^5*c - a^3*b^4*d)*g^4*x + (a^3*b^4*c - a^4*b^3*d)*g^4)
```

### 3.71.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1182 vs. $2(131) = 262$.

Time = 28.41 (sec) , antiderivative size = 1182, normalized size of antiderivative = 8.04

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx =$$
$$-\frac{2Bd^3i^2\cdot(3A+B)\log\left(x+\frac{6ABad^4i^2+6ABbcd^3i^2+2B^2ad^4i^2+2B^2bcd^3i^2-\frac{2Ba^2d^5i^2\cdot(3A+B)}{ad-bc}+\frac{4Babcd^4i^2\cdot(3A+B)}{ad-bc}-\frac{2Bb^2c^2d^3i^2\cdot(3}{ad-bc}}{12ABbd^4i^2+4B^2bd^4i^2}\right.}{9b^3g^4\,(ad-bc)}$$
$$+\frac{2Bd^3i^2\cdot(3A+B)\log\left(x+\frac{6ABad^4i^2+6ABbcd^3i^2+2B^2ad^4i^2+2B^2bcd^3i^2+\frac{2Ba^2d^5i^2\cdot(3A+B)}{ad-bc}-\frac{4Babcd^4i^2\cdot(3A+B)}{ad-bc}+\frac{2Bb^2c^2d^3i^2\cdot(3}{ad-bc}}{12ABbd^4i^2+4B^2bd^4i^2}\right.}{9b^3g^4\,(ad-bc)}$$
$$+\frac{(B^2c^3i^2+3B^2c^2di^2x+3B^2cd^2i^2x^2+B^2d^3i^2x^3)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{3a^4dg^4-3a^3bcg^4+9a^3bdg^4x-9a^2b^2cg^4x+9a^2b^2dg^4x^2-9ab^3cg^4x^2+3ab^3dg^4x^3-3b^4cg^4x^3}$$
$$+\frac{-9A^2a^2d^2i^2-9A^2abcdi^2-9A^2b^2c^2i^2-6ABa^2d^2i^2-6ABabcdi^2-6ABb^2c^2i^2-2B^2a^2d^2i^2-2B^2abc}{}$$
$$+\frac{(-6ABa^2d^2i^2-6ABabcdi^2-18ABabd^2i^2x-6ABb^2c^2i^2-18ABb^2cdi^2x-18ABb^2d^2i^2x^2-2B^2a^2d^2}{9a^3b^3g^4+27a^2b^4g^4x+27ab^5g}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4,x)
```

output
```
-2*B*d**3*i**2*(3*A + B)*log(x + (6*A*B*a*d**4*i**2 + 6*A*B*b*c*d**3*i**2
+ 2*B**2*a*d**4*i**2 + 2*B**2*b*c*d**3*i**2 - 2*B*a**2*d**5*i**2*(3*A + B)
/(a*d - b*c) + 4*B*a*b*c*d**4*i**2*(3*A + B)/(a*d - b*c) - 2*B*b**2*c**2*d
**3*i**2*(3*A + B)/(a*d - b*c))/(12*A*B*b*d**4*i**2 + 4*B**2*b*d**4*i**2))
/(9*b**3*g**4*(a*d - b*c)) + 2*B*d**3*i**2*(3*A + B)*log(x + (6*A*B*a*d**4
*i**2 + 6*A*B*b*c*d**3*i**2 + 2*B**2*a*d**4*i**2 + 2*B**2*b*c*d**3*i**2 +
2*B*a**2*d**5*i**2*(3*A + B)/(a*d - b*c) - 4*B*a*b*c*d**4*i**2*(3*A + B)/(
a*d - b*c) + 2*B*b**2*c**2*d**3*i**2*(3*A + B)/(a*d - b*c))/(12*A*B*b*d**4
*i**2 + 4*B**2*b*d**4*i**2))/(9*b**3*g**4*(a*d - b*c)) + (B**2*c**3*i**2 +
 3*B**2*c**2*d*i**2*x + 3*B**2*c*d**2*i**2*x**2 + B**2*d**3*i**2*x**3)*log
(e*(a + b*x)/(c + d*x))**2/(3*a**4*d*g**4 - 3*a**3*b*c*g**4 + 9*a**3*b*d*g
**4*x - 9*a**2*b**2*c*g**4*x + 9*a**2*b**2*d*g**4*x**2 - 9*a*b**3*c*g**4*x
**2 + 3*a*b**3*d*g**4*x**3 - 3*b**4*c*g**4*x**3) + (-9*A**2*a**2*d**2*i**2
 - 9*A**2*a*b*c*d*i**2 - 9*A**2*b**2*c**2*i**2 - 6*A*B*a**2*d**2*i**2 - 6*
A*B*a*b*c*d*i**2 - 6*A*B*b**2*c**2*i**2 - 2*B**2*a**2*d**2*i**2 - 2*B**2*a
*b*c*d*i**2 - 2*B**2*b**2*c**2*i**2 + x**2*(-27*A**2*b**2*d**2*i**2 - 18*A
*B*b**2*d**2*i**2 - 6*B**2*b**2*d**2*i**2) + x*(-27*A**2*a*b*d**2*i**2 - 2
7*A**2*b**2*c*d*i**2 - 18*A*B*a*b*d**2*i**2 - 18*A*B*b**2*c*d*i**2 - 6*B**
2*a*b*d**2*i**2 - 6*B**2*b**2*c*d*i**2))/(27*a**3*b**3*g**4 + 81*a**2*b**4
*g**4*x + 81*a*b**5*g**4*x**2 + 27*b**6*g**4*x**3) + (-6*A*B*a**2*d**2*...
```

## 3.71. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

### 3.71.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5532 vs. 2(141) = 282.

Time = 0.58 (sec) , antiderivative size = 5532, normalized size of antiderivative = 37.63

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, al
gorithm="maxima")
```

output
```
-1/3*(3*b*x + a)*B^2*c*d*i^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^5*g
^4*x^3 + 3*a*b^4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*(3*b^2*x^2
 + 3*a*b*x + a^2)*B^2*d^2*i^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^6*
g^4*x^3 + 3*a*b^5*g^4*x^2 + 3*a^2*b^4*g^4*x + a^3*b^3*g^4) - 1/54*(6*((6*b
^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*
x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b
^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d
^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x
 + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^
3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^
4))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + (4*b^3*c^3 - 27*a*b^2*c^2*d + 1
08*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3
*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3
*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(
5*b^3*c^2*d - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2
*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*
a*b^2*d^3*x^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3
*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))/(a^3*b^4*c^3*g
^4 - 3*a^4*b^3*c^2*d*g^4 + 3*a^5*b^2*c*d^2*g^4 - a^6*b*d^3*g^4 + (b^7*c^3*
g^4 - 3*a*b^6*c^2*d*g^4 + 3*a^2*b^5*c*d^2*g^4 - a^3*b^4*d^3*g^4)*x^3 + ...
```

### 3.71.8 Giac [A] (verification not implemented)

Time = 0.52 (sec) , antiderivative size = 219, normalized size of antiderivative = 1.49

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{1}{27}\left(\frac{9\,(dx+c)^3B^2e^4i^2\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)^3g^4}+\frac{6\,(3\,ABe^4i^2+B^2e^4i^2)(dx+c)^3\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^3g^4}+\frac{(9\,A^2e^4i^2+6\,AB}{(be}\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4,x, al
gorithm="giac")
```

output
```
-1/27*(9*(d*x + c)^3*B^2*e^4*i^2*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x +
a*e)^3*g^4) + 6*(3*A*B*e^4*i^2 + B^2*e^4*i^2)*(d*x + c)^3*log((b*e*x + a*e
)/(d*x + c))/((b*e*x + a*e)^3*g^4) + (9*A^2*e^4*i^2 + 6*A*B*e^4*i^2 + 2*B^
2*e^4*i^2)*(d*x + c)^3/((b*e*x + a*e)^3*g^4))*(b*c/((b*c*e - a*d*e)*(b*c -
 a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.71.9 Mupad [B] (verification not implemented)

Time = 3.99 (sec) , antiderivative size = 1153, normalized size of antiderivative = 7.84

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{x^2\,(9\,A^2\,b^2\,d^2\,i^2+6\,A\,B\,b^2\,d^2\,i^2+2\,B^2\,b^2\,d^2\,i^2)+x\,(9\,c\,A^2\,b^2\,d\,i^2+9\,a\,A^2\,b\,d^2\,i^2+6\,c\,A\,B\,b^2\,d\,i^2+6\,a\,A}{}$$

$$-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{x\left(b\left(\frac{B^2\,c\,d\,i^2}{3\,b^3\,g^4}+\frac{B^2\,a\,d^2\,i^2}{3\,b^4\,g^4}\right)+\frac{2\,B^2\,c\,d\,i^2}{3\,b^2\,g^4}+\frac{2\,B^2\,a\,d^2\,i^2}{3\,b^3\,g^4}\right)+a\left(\frac{B^2\,c\,d\,i^2}{3\,b^3\,g^4}+\frac{B^2\,a\,d^2\,i^2}{3\,b^4\,g^4}\right)+\frac{B^2\,c^2\,i^2}{3\,b^2\,g^4}}{3\,a^2\,x+\frac{a^3}{b}+b^2\,x^3+3\,a\,b\,x^2} - \frac{B^2\,d^3\,i^2}{3\,b^3\,g^4\,(a\,d-b\,c)}\right)$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(x\left(b\left(\frac{B\,i^2\,(2\,A\,b\,c-B\,a\,d+B\,b\,c)}{3\,b^4\,g^4}+\frac{2\,A\,B\,a\,d\,i^2}{3\,b^4\,g^4}\right)+\frac{2\,B\,i^2\,(2\,A\,b\,c-B\,a\,d+B\,b\,c)}{3\,b^3\,g^4}+\frac{2\,B^2\,d^3\,i^2\left(b\left(\frac{3\,a^2\,d^2-4\,a\,b\,c\,d+}{6\,b\,d^3}\right.\right.}{}\right.\right.}{}$$

$$-\frac{B\,d^3\,i^2\,\mathrm{atan}\left(\frac{\left(\frac{9\,c\,b^4\,g^4+9\,a\,d\,b^3\,g^4}{9\,b^3\,g^4}+2\,b\,d\,x\right)1\mathrm{i}}{a\,d-b\,c}\right)(3\,A+B)\,4\mathrm{i}}{9\,b^3\,g^4\,(a\,d-b\,c)}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^4,x)
```

output
```
- (x^2*(9*A^2*b^2*d^2*i^2 + 2*B^2*b^2*d^2*i^2 + 6*A*B*b^2*d^2*i^2) + x*(9*
A^2*a*b*d^2*i^2 + 2*B^2*a*b*d^2*i^2 + 9*A^2*b^2*c*d*i^2 + 2*B^2*b^2*c*d*i^
2 + 6*A*B*a*b*d^2*i^2 + 6*A*B*b^2*c*d*i^2) + 3*A^2*a^2*d^2*i^2 + 3*A^2*b^2
*c^2*i^2 + (2*B^2*a^2*d^2*i^2)/3 + (2*B^2*b^2*c^2*i^2)/3 + 2*A*B*a^2*d^2*i
^2 + 2*A*B*b^2*c^2*i^2 + 3*A^2*a*b*c*d*i^2 + (2*B^2*a*b*c*d*i^2)/3 + 2*A*B
*a*b*c*d*i^2)/(9*a^3*b^3*g^4 + 9*b^6*g^4*x^3 + 27*a^2*b^4*g^4*x + 27*a*b^5
*g^4*x^2) - log((e*(a + b*x))/(c + d*x))^2*((x*(b*((B^2*c*d*i^2)/(3*b^3*g^
4) + (B^2*a*d^2*i^2)/(3*b^4*g^4)) + (2*B^2*c*d*i^2)/(3*b^2*g^4) + (2*B^2*a
*d^2*i^2)/(3*b^3*g^4)) + a*((B^2*c*d*i^2)/(3*b^3*g^4) + (B^2*a*d^2*i^2)/(3
*b^4*g^4)) + (B^2*c^2*i^2)/(3*b^2*g^4) + (B^2*d^2*i^2*x^2)/(b^2*g^4))/(3*a
^2*x + a^3/b + b^2*x^3 + 3*a*b*x^2) - (B^2*d^3*i^2)/(3*b^3*g^4*(a*d - b*c)
)) - (log((e*(a + b*x))/(c + d*x))*(x*(b*((B*i^2*(2*A*b*c - B*a*d + B*b*c)
)/(3*b^4*g^4) + (2*A*B*a*d*i^2)/(3*b^4*g^4)) + (2*B*i^2*(2*A*b*c - B*a*d +
 B*b*c))/(3*b^3*g^4) + (2*B^2*d^3*i^2*(b*((3*a^2*d^2 + b^2*c^2 - 4*a*b*c*d
)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^2*d^2 + b^2*c^2 - 4*a*b*c*
d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)))/(3*b^3*g^4*(a*d - b*c)) + (4*A*B*
a*d*i^2)/(3*b^3*g^4)) + x^2*((2*A*B*d*i^2)/(b^2*g^4) - (2*B^2*d^3*i^2*((b^
2*c - a*b*d)/(3*d^2) - (2*b*(a*d - b*c))/(3*d^2)))/(3*b^3*g^4*(a*d - b*c))
) + a*((B*i^2*(2*A*b*c - B*a*d + B*b*c))/(3*b^4*g^4) + (2*A*B*a*d*i^2)/(3*
b^4*g^4)) + (2*B*i^2*(A*b^2*c^2 - B*a^2*d^2 + B*a*b*c*d))/(3*b^4*d*g^4)...
```

## 3.71. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4}\,dx$

# 3.72 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$



## 3.72.1 Optimal result

Integrand size = 42, antiderivative size = 299

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{2B^2di^2(c+dx)^3}{27(bc-ad)^2g^5(a+bx)^3} - \frac{bB^2i^2(c+dx)^4}{32(bc-ad)^2g^5(a+bx)^4} + \frac{2Bdi^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^2g^5(a+bx)^3} - \frac{bBi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^2g^5(a+bx)^4} + \frac{di^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^2g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^2g^5(a+bx)^4}$$

output
```
2/27*B^2*d*i^2*(d*x+c)^3/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/32*b*B^2*i^2*(d*x+c)
^4/(-a*d+b*c)^2/g^5/(b*x+a)^4+2/9*B*d*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x
+c)))/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/8*b*B*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(
d*x+c)))/(-a*d+b*c)^2/g^5/(b*x+a)^4+1/3*d*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/
(d*x+c)))^2/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/4*b*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+
a)/(d*x+c)))^2/(-a*d+b*c)^2/g^5/(b*x+a)^4
```

### 3.72.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.46 (sec) , antiderivative size = 1703, normalized size of antiderivative = 5.70

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx=\text{Too large to display}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^5,x]
```

output
```
-1/864*(i^2*(216*(b*c - a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 57
6*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 43
2*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 3
2*B*d*(a + b*x)*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18*A*d*(b*c - a*
d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a
+ b*x)^2 + 66*B*d^2*(b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a +
 b*x] + 66*B*d^3*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*x)^3*Log[a + b
*x]^2 + 12*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18*B*d*(b*c - a*
d)^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c - a*d)*(a + b*
x)^2*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[
(e*(a + b*x))/(c + d*x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*(a
 + b*x)^3*Log[c + d*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) +
a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]*Log
[c + d*x] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(a + b*x)^3*Log
[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2,
 (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)]) + 3*B*(36*A*(b*c - a*d)^4 + 9*B*(b*c - a*d)^4 + 48*A*d*
(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c) + a*d)^3*(a + b*x) + 72*A*d^2*
(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c - a*d)^2*(a + b*x)^2 + 144*A*d^3
*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(b*c) + a*d)*(a + b*x)^3 - 14...
```

### 3.72.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{ 2962}$$

$$\frac{i^2\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^2}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{i^2\int \left(\frac{b(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^5}-\frac{d(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}}{g^5(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i^2\left(-\frac{b(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}-\frac{bB(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{8(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}+\frac{2Bd(c+dx)^3\left(B\log\left(\frac{e(a+\right.}{c+d}\right.}{9(a+bx)^3}\right.}{g^5(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^5,x]
```

output
```
(i^2*((2*B^2*d*(c + d*x)^3)/(27*(a + b*x)^3) - (b*B^2*(c + d*x)^4)/(32*(a
+ b*x)^4) + (2*B*d*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(a
 + b*x)^3) - (b*B*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(a
+ b*x)^4) + (d*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(a +
 b*x)^3) - (b*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*(a +
b*x)^4)))/((b*c - a*d)^2*g^5)
```

#### 3.72.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.72.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(287) = 574$.

Time = 1.64 (sec) , antiderivative size = 685, normalized size of antiderivative = 2.29

| method | result |
|---|---|
| parts | $\frac{i^2A^2\left(\frac{2d(ad-cb)}{3b^3(bx+a)^3}-\frac{a^2d^2-2abcd+b^2c^2}{4b^3(bx+a)^4}-\frac{d^2}{2b^3(bx+a)^2}\right)}{g^5}-\frac{i^2B^2(ad-cb)^3e^3\left(\frac{d^5\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{9\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}\right.}{(ad-cb)^5}\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{i^2d^2e^3A^2b}{4(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^2d^3e^2A^2}{3(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{2i^2d^2e^3ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{}{16\left(\frac{be}{d}+\right.}\right.}{(ad-cb)^3g^5}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(\frac{i^2d^2e^3A^2b}{4(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{i^2d^3e^2A^2}{3(ad-cb)^3g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}-\frac{2i^2d^2e^3ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{}{16\left(\frac{be}{d}+\right.}\right.}{(ad-cb)^3g^5}\right.}{}$ |
| norman | Expression too large to display |
| parallelrisch | Expression too large to display |
| risch | Expression too large to display |

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x,method=_RE
TURNVERBOSE)
```

output
```
i^2*A^2/g^5*(2/3*d*(a*d-b*c)/b^3/(b*x+a)^3-1/4*(a^2*d^2-2*a*b*c*d+b^2*c^2)
/b^3/(b*x+a)^4-1/2*d^2/b^3/(b*x+a)^2)-i^2*B^2/g^5/d^4*(a*d-b*c)^3*e^3*(d^5
/(a*d-b*c)^5*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x
+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^4/(a*d-b*c)^5*b*e*(-1/4/(b*e/
d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d+(a
*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))^4))-2*i^2*B*A/g^5/d^4*(a*d-b*c)^3*e^3*(d^5/(a*d-b*c)^5*(-1
/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^3)-d^4/(a*d-b*c)^5*b*e*(-1/4/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d
*x+c))^4))
```

## 3.72. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

### 3.72.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 837 vs. $2(287) = 574$.

Time = 0.30 (sec) , antiderivative size = 837, normalized size of antiderivative = 2.80

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$$

$$= \frac{12\left((12\,AB+7\,B^2)b^4cd^3-(12\,AB+7\,B^2)ab^3d^4\right)i^2x^3-6\left((72\,A^2+12\,AB-5\,B^2)b^4c^2d^2-16\,(9\,A^2+6\,A\right.}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, al
gorithm="fricas")
```

output
```
1/864*(12*((12*A*B + 7*B^2)*b^4*c*d^3 - (12*A*B + 7*B^2)*a*b^3*d^4)*i^2*x^
3 - 6*((72*A^2 + 12*A*B - 5*B^2)*b^4*c^2*d^2 - 16*(9*A^2 + 6*A*B + 2*B^2)*
a*b^3*c*d^3 + (72*A^2 + 84*A*B + 37*B^2)*a^2*b^2*d^4)*i^2*x^2 - 4*((144*A^
2 + 60*A*B + 11*B^2)*b^4*c^3*d - 24*(9*A^2 + 6*A*B + 2*B^2)*a*b^3*c^2*d^2
+ (72*A^2 + 84*A*B + 37*B^2)*a^3*b*d^4)*i^2*x - (27*(8*A^2 + 4*A*B + B^2)*
b^4*c^4 - 32*(9*A^2 + 6*A*B + 2*B^2)*a*b^3*c^3*d + (72*A^2 + 84*A*B + 37*B
^2)*a^4*d^4)*i^2 + 72*(B^2*b^4*d^4*i^2*x^4 + 4*B^2*a*b^3*d^4*i^2*x^3 - 6*(
B^2*b^4*c^2*d^2 - 2*B^2*a*b^3*c*d^3)*i^2*x^2 - 4*(2*B^2*b^4*c^3*d - 3*B^2*
a*b^3*c^2*d^2)*i^2*x - (3*B^2*b^4*c^4 - 4*B^2*a*b^3*c^3*d)*i^2)*log((b*e*x
 + a*e)/(d*x + c))^2 + 12*((12*A*B + 7*B^2)*b^4*d^4*i^2*x^4 + 4*(3*B^2*b^4
*c*d^3 + 4*(3*A*B + B^2)*a*b^3*d^4)*i^2*x^3 - 6*((12*A*B + B^2)*b^4*c^2*d^
2 - 8*(3*A*B + B^2)*a*b^3*c*d^3)*i^2*x^2 - 4*((24*A*B + 5*B^2)*b^4*c^3*d -
 12*(3*A*B + B^2)*a*b^3*c^2*d^2)*i^2*x - (9*(4*A*B + B^2)*b^4*c^4 - 16*(3*
A*B + B^2)*a*b^3*c^3*d)*i^2)*log((b*e*x + a*e)/(d*x + c)))/((b^9*c^2 - 2*a
*b^8*c*d + a^2*b^7*d^2)*g^5*x^4 + 4*(a*b^8*c^2 - 2*a^2*b^7*c*d + a^3*b^6*d
^2)*g^5*x^3 + 6*(a^2*b^7*c^2 - 2*a^3*b^6*c*d + a^4*b^5*d^2)*g^5*x^2 + 4*(a
^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*g^5*x + (a^4*b^5*c^2 - 2*a^5*b^4
*c*d + a^6*b^3*d^2)*g^5)
```

### 3.72.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2055 vs. $2(277) = 554$.

Time = 69.34 (sec) , antiderivative size = 2055, normalized size of antiderivative = 6.87

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
```

output
```
-B*d**4*i**2*(12*A + 7*B)*log(x + (12*A*B*a*d**5*i**2 + 12*A*B*b*c*d**4*i*
*2 + 7*B**2*a*d**5*i**2 + 7*B**2*b*c*d**4*i**2 - B*a**3*d**7*i**2*(12*A +
7*B)/(a*d - b*c)**2 + 3*B*a**2*b*c*d**6*i**2*(12*A + 7*B)/(a*d - b*c)**2 -
 3*B*a*b**2*c**2*d**5*i**2*(12*A + 7*B)/(a*d - b*c)**2 + B*b**3*c**3*d**4*
i**2*(12*A + 7*B)/(a*d - b*c)**2)/(24*A*B*b*d**5*i**2 + 14*B**2*b*d**5*i**
2))/(72*b**3*g**5*(a*d - b*c)**2) + B*d**4*i**2*(12*A + 7*B)*log(x + (12*A
*B*a*d**5*i**2 + 12*A*B*b*c*d**4*i**2 + 7*B**2*a*d**5*i**2 + 7*B**2*b*c*d*
*4*i**2 + B*a**3*d**7*i**2*(12*A + 7*B)/(a*d - b*c)**2 - 3*B*a**2*b*c*d**6
*i**2*(12*A + 7*B)/(a*d - b*c)**2 + 3*B*a*b**2*c**2*d**5*i**2*(12*A + 7*B)
/(a*d - b*c)**2 - B*b**3*c**3*d**4*i**2*(12*A + 7*B)/(a*d - b*c)**2)/(24*A
*B*b*d**5*i**2 + 14*B**2*b*d**5*i**2))/(72*b**3*g**5*(a*d - b*c)**2) + (4*
B**2*a*c**3*d*i**2 + 12*B**2*a*c**2*d**2*i**2*x + 12*B**2*a*c*d**3*i**2*x*
*2 + 4*B**2*a*d**4*i**2*x**3 - 3*B**2*b*c**4*i**2 - 8*B**2*b*c**3*d*i**2*x
 - 6*B**2*b*c**2*d**2*i**2*x**2 + B**2*b*d**4*i**2*x**4)*log(e*(a + b*x)/(
c + d*x))**2/(12*a**6*d**2*g**5 - 24*a**5*b*c*d*g**5 + 48*a**5*b*d**2*g**5
*x + 12*a**4*b**2*c**2*g**5 - 96*a**4*b**2*c*d*g**5*x + 72*a**4*b**2*d**2*
g**5*x**2 + 48*a**3*b**3*c**2*g**5*x - 144*a**3*b**3*c*d*g**5*x**2 + 48*a*
*3*b**3*d**2*g**5*x**3 + 72*a**2*b**4*c**2*g**5*x**2 - 96*a**2*b**4*c*d*g*
*5*x**3 + 12*a**2*b**4*d**2*g**5*x**4 + 48*a*b**5*c**2*g**5*x**3 - 24*a*b*
*5*c*d*g**5*x**4 + 12*b**6*c**2*g**5*x**4) + (-12*A*B*a**3*d**3*i**2 - ...
```

### 3.72.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8031 vs. $2(287) = 574$.

Time = 0.79 (sec) , antiderivative size = 8031, normalized size of antiderivative = 26.86

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, al
gorithm="maxima")
```

output
```
-1/6*(4*b*x + a)*B^2*c*d*i^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^6*g
^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2*g
^5) - 1/12*(6*b^2*x^2 + 4*a*b*x + a^2)*B^2*d^2*i^2*log(b*e*x/(d*x + c) + a
*e/(d*x + c))^2/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*a^3
*b^4*g^5*x + a^4*b^3*g^5) + 1/288*(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*
b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2
+ 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*
d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*
d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^
2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*
c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^
2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 -
4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12
*d^4*log(d*x + c)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^
2*c*d^3 + a^4*b*d^4)*g^5))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (9*b^4*c
^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4
- 300*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 +
163*a^2*b^2*d^4)*x^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d^4*x^3 + 6*a^2*b^2*d^4*x
^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(b*x + a)^2 + 72*(b^4*d^4*x^4 + 4*a*b^3*d
^4*x^3 + 6*a^2*b^2*d^4*x^2 + 4*a^3*b*d^4*x + a^4*d^4)*log(d*x + c)^2 - ...
```

### 3.72.8 Giac [A] (verification not implemented)

Time = 0.58 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.60

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{864}\left(\frac{72\left(3\,B^2be^5i^2-\frac{4\,(bex+ae)B^2de^4i^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^4bcg^5}{(dx+c)^4}-\frac{(bex+ae)^4adg^5}{(dx+c)^4}}+\frac{12\left(36\,ABbe^5i^2+9\,B^2be^5i^2-\frac{48\,(bex+ae)ABde^4i^2}{dx+c}\right.}{\frac{(bex+ae)^4bcg^5}{(dx+c)^4}-\frac{(bex+ae)}{(dx+}}\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, al
gorithm="giac")
```

output
```
-1/864*(72*(3*B^2*b*e^5*i^2 - 4*(b*e*x + a*e)*B^2*d*e^4*i^2/(d*x + c))*log
((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^4*b*c*g^5/(d*x + c)^4 - (b*e*x
+ a*e)^4*a*d*g^5/(d*x + c)^4) + 12*(36*A*B*b*e^5*i^2 + 9*B^2*b*e^5*i^2 - 4
8*(b*e*x + a*e)*A*B*d*e^4*i^2/(d*x + c) - 16*(b*e*x + a*e)*B^2*d*e^4*i^2/(
d*x + c))*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^4*b*c*g^5/(d*x + c)^
4 - (b*e*x + a*e)^4*a*d*g^5/(d*x + c)^4) + (216*A^2*b*e^5*i^2 + 108*A*B*b*
e^5*i^2 + 27*B^2*b*e^5*i^2 - 288*(b*e*x + a*e)*A^2*d*e^4*i^2/(d*x + c) - 1
92*(b*e*x + a*e)*A*B*d*e^4*i^2/(d*x + c) - 64*(b*e*x + a*e)*B^2*d*e^4*i^2/
(d*x + c))/((b*e*x + a*e)^4*b*c*g^5/(d*x + c)^4 - (b*e*x + a*e)^4*a*d*g^5/
(d*x + c)^4))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b
*c - a*d)))
```

### 3.72.9 Mupad [B] (verification not implemented)

Time = 6.38 (sec) , antiderivative size = 1940, normalized size of antiderivative = 6.49

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^5,x)
```

output
```
- log((e*(a + b*x))/(c + d*x))^2*((x*(b*((B^2*c*d*i^2)/(6*b^3*g^5) + (B^2*
a*d^2*i^2)/(12*b^4*g^5)) + (B^2*c*d*i^2)/(2*b^2*g^5) + (B^2*a*d^2*i^2)/(4*
b^3*g^5)) + a*((B^2*c*d*i^2)/(6*b^3*g^5) + (B^2*a*d^2*i^2)/(12*b^4*g^5)) +
 (B^2*c^2*i^2)/(4*b^2*g^5) + (B^2*d^2*i^2*x^2)/(2*b^2*g^5))/(4*a^3*x + a^4
/b + b^3*x^4 + 6*a^2*b*x^2 + 4*a*b^2*x^3) - (B^2*d^4*i^2)/(12*b^3*g^5*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((72*A^2*a^3*d^3*i^2 - 216*A^2*b^3*c^3*i^2
 + 37*B^2*a^3*d^3*i^2 - 27*B^2*b^3*c^3*i^2 + 84*A*B*a^3*d^3*i^2 - 108*A*B*
b^3*c^3*i^2 + 72*A^2*a*b^2*c^2*d*i^2 + 72*A^2*a^2*b*c*d^2*i^2 + 37*B^2*a*b
^2*c^2*d*i^2 + 37*B^2*a^2*b*c*d^2*i^2 + 84*A*B*a*b^2*c^2*d*i^2 + 84*A*B*a^
2*b*c*d^2*i^2)/(12*(a*d - b*c)) + (x^3*(7*B^2*b^3*d^3*i^2 + 12*A*B*b^3*d^3
*i^2))/(a*d - b*c) + (x*(72*A^2*a^2*b*d^3*i^2 + 37*B^2*a^2*b*d^3*i^2 - 144
*A^2*b^3*c^2*d*i^2 - 11*B^2*b^3*c^2*d*i^2 + 72*A^2*a*b^2*c*d^2*i^2 + 37*B^
2*a*b^2*c*d^2*i^2 + 84*A*B*a^2*b*d^3*i^2 - 60*A*B*b^3*c^2*d*i^2 + 84*A*B*a
*b^2*c*d^2*i^2))/(3*(a*d - b*c)) + (x^2*(72*A^2*a*b^2*d^3*i^2 + 37*B^2*a*b
^2*d^3*i^2 - 72*A^2*b^3*c*d^2*i^2 + 5*B^2*b^3*c*d^2*i^2 + 84*A*B*a*b^2*d^3
*i^2 - 12*A*B*b^3*c*d^2*i^2))/(2*(a*d - b*c)))/(72*a^4*b^3*g^5 + 72*b^7*g^
5*x^4 + 288*a^3*b^4*g^5*x + 288*a*b^6*g^5*x^3 + 432*a^2*b^5*g^5*x^2) - (lo
g((e*(a + b*x))/(c + d*x))*(x^2*((A*B*d*i^2)/(b^2*g^5) + (B^2*d^4*i^2*(b*(
b*((4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(12*b*d^3) + (a*(a*d - b*c))/(4*b*d^2
)) + (4*a^2*d^2 + b^2*c^2 - 5*a*b*c*d)/(6*d^3) + (a*(a*d - b*c))/(2*d^2...
```

## 3.72. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

# 3.73 $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$



## 3.73.1 Optimal result

Integrand size = 42, antiderivative size = 463

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = &-\frac{2B^2d^2i^2(c+dx)^3}{27(bc-ad)^3g^6(a+bx)^3} \\ &+\frac{bB^2di^2(c+dx)^4}{16(bc-ad)^3g^6(a+bx)^4} \\ &-\frac{2b^2B^2i^2(c+dx)^5}{125(bc-ad)^3g^6(a+bx)^5} \\ &-\frac{2Bd^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^3g^6(a+bx)^3} \\ &+\frac{bBdi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4(bc-ad)^3g^6(a+bx)^4} \\ &-\frac{2b^2Bi^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{25(bc-ad)^3g^6(a+bx)^5} \\ &-\frac{d^2i^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^3g^6(a+bx)^3} \\ &+\frac{bdi^2(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3g^6(a+bx)^4} \\ &-\frac{b^2i^2(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^3g^6(a+bx)^5}\end{aligned}$$

output
```
-2/27*B^2*d^2*i^2*(d*x+c)^3/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/16*b*B^2*d*i^2*(d
*x+c)^4/(-a*d+b*c)^3/g^6/(b*x+a)^4-2/125*b^2*B^2*i^2*(d*x+c)^5/(-a*d+b*c)^
3/g^6/(b*x+a)^5-2/9*B*d^2*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+
b*c)^3/g^6/(b*x+a)^3+1/4*b*B*d*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(
-a*d+b*c)^3/g^6/(b*x+a)^4-2/25*b^2*B*i^2*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+
c)))/(-a*d+b*c)^3/g^6/(b*x+a)^5-1/3*d^2*i^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/2*b*d*i^2*(d*x+c)^4*(A+B*ln(e*(b*x+
a)/(d*x+c)))^2/(-a*d+b*c)^3/g^6/(b*x+a)^4-1/5*b^2*i^2*(d*x+c)^5*(A+B*ln(e*
(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^6/(b*x+a)^5
```

### 3.73.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.00 (sec) , antiderivative size = 2010, normalized size of antiderivative = 4.34

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^6,x]
```

output
```
-1/54000*(i^2*(10800*(b*c - a*d)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
+ 27000*d*(b*c - a*d)^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 -
 18000*d^2*(-(b*c) + a*d)^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2 + 1000*B*d^2*(a + b*x)^2*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 - 18
*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2*(
b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a +
b*x)^3*Log[a + b*x] + 66*B*d^3*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*(a + b*
x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)] - 18
*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*c
- a*d)*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^3*(a + b*x)^3*Log
[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*x]
 - 66*B*d^3*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a + b*
x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x))/
(c + d*x)]*Log[c + d*x] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*(
a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b*x
)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)]) + 375*B*d*(a + b*x)*(36*A*(b*c - a*d)^4 +
 9*B*(b*c - a*d)^4 + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c) +
a*d)^3*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c - a*
d)^2*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 300*B*d^3*(-(...
```

### 3.73.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^6}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i^2\int \frac{(c+dx)^6\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^6}d\frac{a+bx}{c+dx}}{g^6(bc-ad)^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{i^2 \int \left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^6}{(a+bx)^6}-\frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^5}{(a+bx)^5}+\frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^4}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}}{g^6(bc-ad)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i^2\left(-\frac{b^2(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5(a+bx)^5}-\frac{2b^2B(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{25(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}-\frac{2Bd^2(c+dx)^3\left(B\log\right.}{9(a+bx}\right.}{g^6(b}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^6,x]
```

output
```
(i^2*((-2*B^2*d^2*(c + d*x)^3)/(27*(a + b*x)^3) + (b*B^2*d*(c + d*x)^4)/(1
6*(a + b*x)^4) - (2*b^2*B^2*(c + d*x)^5)/(125*(a + b*x)^5) - (2*B*d^2*(c +
 d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(a + b*x)^3) + (b*B*d*(c
+ d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*(a + b*x)^4) - (2*b^2*B*
(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(25*(a + b*x)^5) - (d^2*
(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(a + b*x)^3) + (b*d
*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*x)^4) - (b^
2*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*(a + b*x)^5)))/((
b*c - a*d)^3*g^6)
```

### 3.73.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.73.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 967 vs. $2(445) = 890$.

Time = 1.81 (sec) , antiderivative size = 968, normalized size of antiderivative = 2.09

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 968 |
| derivativedivides | Expression too large to display | 1082 |
| default | Expression too large to display | 1082 |
| norman | Expression too large to display | 1977 |
| parallelrisch | Expression too large to display | 2225 |
| risch | Expression too large to display | 5135 |

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x,method=_RE
TURNVERBOSE)
```

output
```
i^2*A^2/g^6*(-1/3*d^2/b^3/(b*x+a)^3+1/2*d*(a*d-b*c)/b^3/(b*x+a)^4-1/5*(a^2
*d^2-2*a*b*c*d+b^2*c^2)/b^3/(b*x+a)^5)-i^2*B^2/g^6/d^4*(a*d-b*c)^3*e^3*(d^
6/(a*d-b*c)^6*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*
x+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-2*d^5/(a*d-b*c)^6*b*e*(-1/4/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d
+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^4)+d^4/(a*d-b*c)^6*e^2*b^2*(-1/5/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/25/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/125/(b*e/d+(a*d-b*c)*e/d/(d*x+c))
^5))-2*i^2*B*A/g^6/d^4*(a*d-b*c)^3*e^3*(d^6/(a*d-b*c)^6*(-1/3/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^3)-2*d^5/(a*d-b*c)^6*b*e*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)+d^4
/(a*d-b*c)^6*e^2*b^2*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5))
```

### 3.73.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1323 vs. 2(445) = 890.

Time = 0.34 (sec) , antiderivative size = 1323, normalized size of antiderivative = 2.86

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, al
gorithm="fricas")
```

output
```
-1/54000*(60*((60*A*B + 47*B^2)*b^5*c*d^4 - (60*A*B + 47*B^2)*a*b^4*d^5)*i
^2*x^4 - 30*((60*A*B - 13*B^2)*b^5*c^2*d^3 - 50*(12*A*B + 7*B^2)*a*b^4*c*d
^4 + 3*(180*A*B + 121*B^2)*a^2*b^3*d^5)*i^2*x^3 + 10*(2*(900*A^2 + 60*A*B
- 43*B^2)*b^5*c^3*d^2 - 75*(72*A^2 + 12*A*B - 5*B^2)*a*b^4*c^2*d^3 + 600*(
9*A^2 + 6*A*B + 2*B^2)*a^2*b^3*c*d^4 - (1800*A^2 + 2820*A*B + 1489*B^2)*a^
3*b^2*d^5)*i^2*x^2 + 5*(27*(200*A^2 + 60*A*B + 7*B^2)*b^5*c^4*d - 100*(144
*A^2 + 60*A*B + 11*B^2)*a*b^4*c^3*d^2 + 1200*(9*A^2 + 6*A*B + 2*B^2)*a^2*b
^3*c^2*d^3 - (1800*A^2 + 2820*A*B + 1489*B^2)*a^4*b*d^5)*i^2*x + (432*(25*
A^2 + 10*A*B + 2*B^2)*b^5*c^5 - 3375*(8*A^2 + 4*A*B + B^2)*a*b^4*c^4*d + 2
000*(9*A^2 + 6*A*B + 2*B^2)*a^2*b^3*c^3*d^2 - (1800*A^2 + 2820*A*B + 1489*
B^2)*a^5*d^5)*i^2 + 1800*(B^2*b^5*d^5*i^2*x^5 + 5*B^2*a*b^4*d^5*i^2*x^4 +
10*B^2*a^2*b^3*d^5*i^2*x^3 + 10*(B^2*b^5*c^3*d^2 - 3*B^2*a*b^4*c^2*d^3 + 3
*B^2*a^2*b^3*c*d^4)*i^2*x^2 + 5*(3*B^2*b^5*c^4*d - 8*B^2*a*b^4*c^3*d^2 + 6
*B^2*a^2*b^3*c^2*d^3)*i^2*x + (6*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*d + 10*B^2
*a^2*b^3*c^3*d^2)*i^2)*log((b*e*x + a*e)/(d*x + c))^2 + 60*((60*A*B + 47*B
^2)*b^5*d^5*i^2*x^5 + 5*(12*B^2*b^5*c*d^4 + 5*(12*A*B + 7*B^2)*a*b^4*d^5)*
i^2*x^4 - 10*(3*B^2*b^5*c^2*d^3 - 30*B^2*a*b^4*c*d^4 - 20*(3*A*B + B^2)*a^
2*b^3*d^5)*i^2*x^3 + 10*(2*(30*A*B + B^2)*b^5*c^3*d^2 - 15*(12*A*B + B^2)*
a*b^4*c^2*d^3 + 60*(3*A*B + B^2)*a^2*b^3*c*d^4)*i^2*x^2 + 5*(9*(20*A*B + 3
*B^2)*b^5*c^4*d - 20*(24*A*B + 5*B^2)*a*b^4*c^3*d^2 + 120*(3*A*B + B^2)...
```

### 3.73.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**6,x)
```

output
```
Timed out
```

### 3.73.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10880 vs. $2(445) = 890$.

Time = 1.13 (sec) , antiderivative size = 10880, normalized size of antiderivative = 23.50

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, al
gorithm="maxima")
```

output
```
-1/10*(5*b*x + a)*B^2*c*d*i^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^7*
g^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*a^
4*b^3*g^6*x + a^5*b^2*g^6) - 1/30*(10*b^2*x^2 + 5*a*b*x + a^2)*B^2*d^2*i^2
*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4 + 1
0*a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6) -
1/9000*(60*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a^2*b^2*c^
2*d^2 - 163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 +
 10*(2*b^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b^4*c^3*d
 - 17*a*b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c^4 - 4*a
*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^6*x^5 +
5*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5
*b^5*d^4)*g^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2
- 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4*b^6*c^3
*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2 + 5*(a^4*b
^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d
^4)*g^6*x + (a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2
*c*d^3 + a^9*b*d^4)*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b^5*c^4*d +
 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^
6) - 60*d^5*log(d*x + c)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^3*d^2 -
10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6))*log(b*e*x/(d*x ...
```

### 3.73.8 Giac [A] (verification not implemented)

Time = 0.57 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.68

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx =$$
$$-\frac{1}{54000}\left(\frac{1800\left(6\,B^2b^2e^6i^2-\frac{15\,(bex+ae)B^2bde^5i^2}{dx+c}+\frac{10\,(bex+ae)^2B^2d^2e^4i^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^5b^2c^2g^6}{(dx+c)^5}-\frac{2\,(bex+ae)^5abcdg^6}{(dx+c)^5}+\frac{(bex+ae)^5a^2d^2g^6}{(dx+c)^5}}+\frac{60\left(360\,ABb^2e^6i^2+\right.}{}\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, al
gorithm="giac")
```

output
```
-1/54000*(1800*(6*B^2*b^2*e^6*i^2 - 15*(b*e*x + a*e)*B^2*b*d*e^5*i^2/(d*x
+ c) + 10*(b*e*x + a*e)^2*B^2*d^2*e^4*i^2/(d*x + c)^2)*log((b*e*x + a*e)/(
d*x + c))^2/((b*e*x + a*e)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*e*x + a*e)^5*a
*b*c*d*g^6/(d*x + c)^5 + (b*e*x + a*e)^5*a^2*d^2*g^6/(d*x + c)^5) + 60*(36
0*A*B*b^2*e^6*i^2 + 72*B^2*b^2*e^6*i^2 - 900*(b*e*x + a*e)*A*B*b*d*e^5*i^2
/(d*x + c) - 225*(b*e*x + a*e)*B^2*b*d*e^5*i^2/(d*x + c) + 600*(b*e*x + a*
e)^2*A*B*d^2*e^4*i^2/(d*x + c)^2 + 200*(b*e*x + a*e)^2*B^2*d^2*e^4*i^2/(d*
x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^5*b^2*c^2*g^6/(d*x +
 c)^5 - 2*(b*e*x + a*e)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*e*x + a*e)^5*a^2*d^
2*g^6/(d*x + c)^5) + (10800*A^2*b^2*e^6*i^2 + 4320*A*B*b^2*e^6*i^2 + 864*B
^2*b^2*e^6*i^2 - 27000*(b*e*x + a*e)*A^2*b*d*e^5*i^2/(d*x + c) - 13500*(b*
e*x + a*e)*A*B*b*d*e^5*i^2/(d*x + c) - 3375*(b*e*x + a*e)*B^2*b*d*e^5*i^2/
(d*x + c) + 18000*(b*e*x + a*e)^2*A^2*d^2*e^4*i^2/(d*x + c)^2 + 12000*(b*e
*x + a*e)^2*A*B*d^2*e^4*i^2/(d*x + c)^2 + 4000*(b*e*x + a*e)^2*B^2*d^2*e^4
*i^2/(d*x + c)^2)/((b*e*x + a*e)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*e*x + a*
e)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*e*x + a*e)^5*a^2*d^2*g^6/(d*x + c)^5))*(
b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.73.9 Mupad [B] (verification not implemented)

Time = 9.11 (sec) , antiderivative size = 3434, normalized size of antiderivative = 7.42

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^6,x)
```

output
```
((1800*A^2*a^4*d^4*i^2 + 10800*A^2*b^4*c^4*i^2 + 1489*B^2*a^4*d^4*i^2 + 86
4*B^2*b^4*c^4*i^2 + 2820*A*B*a^4*d^4*i^2 + 4320*A*B*b^4*c^4*i^2 - 16200*A^
2*a*b^3*c^3*d*i^2 + 1800*A^2*a^3*b*c*d^3*i^2 - 2511*B^2*a*b^3*c^3*d*i^2 +
1489*B^2*a^3*b*c*d^3*i^2 + 1800*A^2*a^2*b^2*c^2*d^2*i^2 + 1489*B^2*a^2*b^2
*c^2*d^2*i^2 + 2820*A*B*a^2*b^2*c^2*d^2*i^2 - 9180*A*B*a*b^3*c^3*d*i^2 + 2
820*A*B*a^3*b*c*d^3*i^2)/(60*(a*d - b*c)) + (x^3*(363*B^2*a*b^3*d^4*i^2 +
13*B^2*b^4*c*d^3*i^2 + 540*A*B*a*b^3*d^4*i^2 - 60*A*B*b^4*c*d^3*i^2))/(2*(
a*d - b*c)) + (x*(1800*A^2*a^3*b*d^4*i^2 + 1489*B^2*a^3*b*d^4*i^2 + 5400*A
^2*b^4*c^3*d*i^2 + 189*B^2*b^4*c^3*d*i^2 - 9000*A^2*a*b^3*c^2*d^2*i^2 + 18
00*A^2*a^2*b^2*c*d^3*i^2 - 911*B^2*a*b^3*c^2*d^2*i^2 + 1489*B^2*a^2*b^2*c*
d^3*i^2 + 2820*A*B*a^3*b*d^4*i^2 + 1620*A*B*b^4*c^3*d*i^2 - 4380*A*B*a*b^3
*c^2*d^2*i^2 + 2820*A*B*a^2*b^2*c*d^3*i^2))/(12*(a*d - b*c)) + (x^2*(1800*
A^2*a^2*b^2*d^4*i^2 + 1489*B^2*a^2*b^2*d^4*i^2 + 1800*A^2*b^4*c^2*d^2*i^2
- 86*B^2*b^4*c^2*d^2*i^2 - 3600*A^2*a*b^3*c*d^3*i^2 + 289*B^2*a*b^3*c*d^3*
i^2 + 2820*A*B*a^2*b^2*d^4*i^2 + 120*A*B*b^4*c^2*d^2*i^2 - 780*A*B*a*b^3*c
*d^3*i^2))/(6*(a*d - b*c)) + (d*x^4*(47*B^2*b^4*d^3*i^2 + 60*A*B*b^4*d^3*i
^2))/(a*d - b*c))/(x*(4500*a^4*b^5*c*g^6 - 4500*a^5*b^4*d*g^6) - x^4*(4500
*a^2*b^7*d*g^6 - 4500*a*b^8*c*g^6) + x^5*(900*b^9*c*g^6 - 900*a*b^8*d*g^6)
 + x^2*(9000*a^3*b^6*c*g^6 - 9000*a^4*b^5*d*g^6) + x^3*(9000*a^2*b^7*c*g^6
 - 9000*a^3*b^6*d*g^6) + 900*a^5*b^4*c*g^6 - 900*a^6*b^3*d*g^6) - log((...
```

3.73. $\int \frac{(ci+dix)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$

# 3.74 $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

### 3.74.1 Optimal result

Integrand size = 42, antiderivative size = 1089

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$= \frac{5B^2(bc-ad)^6g^3i^3x}{84b^3d^3}+\frac{B^2(bc-ad)^3g^3i^3(a+bx)^4}{140b^4}-\frac{29B^2(bc-ad)^5g^3i^3(c+dx)^2}{840b^2d^4}$$
$$+\frac{47B^2(bc-ad)^4g^3i^3(c+dx)^3}{1260bd^4}-\frac{13B^2(bc-ad)^3g^3i^3(c+dx)^4}{420d^4}$$
$$+\frac{bB^2(bc-ad)^2g^3i^3(c+dx)^5}{105d^4}-\frac{B^2(bc-ad)^7g^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4}$$
$$-\frac{B(bc-ad)^4g^3i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{210b^4d}$$
$$-\frac{3B(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{140b^4}$$
$$-\frac{B(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35b^3}$$
$$+\frac{2B(bc-ad)^4g^3i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21bd^4}$$
$$-\frac{3B(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{14d^4}$$
$$+\frac{6bB(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{35d^4}$$
$$-\frac{b^2B(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{21d^4}$$
$$+\frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{140b^4}$$
$$+\frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{35b^3}$$
$$+\frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{14b^2}$$
$$+\frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{7b}$$
$$+\frac{B(bc-ad)^5g^3i^3(a+bx)^2\left(3A+B+3B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^2}$$
$$-\frac{B(bc-ad)^6g^3i^3(a+bx)\left(6A+5B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^3}$$
$$-\frac{B(bc-ad)^7g^3i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(6A+11B+6B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{420b^4d^4}$$

output
```
5/84*B^2*(-a*d+b*c)^6*g^3*i^3*x/b^3/d^3+1/140*B^2*(-a*d+b*c)^3*g^3*i^3*(b*
x+a)^4/b^4-29/840*B^2*(-a*d+b*c)^5*g^3*i^3*(d*x+c)^2/b^2/d^4+47/1260*B^2*(
-a*d+b*c)^4*g^3*i^3*(d*x+c)^3/b/d^4-13/420*B^2*(-a*d+b*c)^3*g^3*i^3*(d*x+c
)^4/d^4+1/105*b*B^2*(-a*d+b*c)^2*g^3*i^3*(d*x+c)^5/d^4-1/210*B^2*(-a*d+b*c
)^7*g^3*i^3*ln((b*x+a)/(d*x+c))/b^4/d^4-1/210*B*(-a*d+b*c)^4*g^3*i^3*(b*x+
a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/d-3/140*B*(-a*d+b*c)^3*g^3*i^3*(b*x+a
)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4-1/35*B*(-a*d+b*c)^2*g^3*i^3*(b*x+a)^4*
(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^3+2/21*B*(-a*d+b*c)^4*g^3*i^3*(d*x+c
)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/b/d^4-3/14*B*(-a*d+b*c)^3*g^3*i^3*(d*x+c)^
4*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4+6/35*b*B*(-a*d+b*c)^2*g^3*i^3*(d*x+c)^5*
(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4-1/21*b^2*B*(-a*d+b*c)*g^3*i^3*(d*x+c)^6*(A
+B*ln(e*(b*x+a)/(d*x+c)))/d^4+1/140*(-a*d+b*c)^3*g^3*i^3*(b*x+a)^4*(A+B*ln
(e*(b*x+a)/(d*x+c)))^2/b^4+1/35*(-a*d+b*c)^2*g^3*i^3*(b*x+a)^4*(d*x+c)*(A+
B*ln(e*(b*x+a)/(d*x+c)))^2/b^3+1/14*(-a*d+b*c)*g^3*i^3*(b*x+a)^4*(d*x+c)^2
*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2+1/7*g^3*i^3*(b*x+a)^4*(d*x+c)^3*(A+B*ln
(e*(b*x+a)/(d*x+c)))^2/b+1/420*B*(-a*d+b*c)^5*g^3*i^3*(b*x+a)^2*(3*A+B+3*B
*ln(e*(b*x+a)/(d*x+c)))/b^4/d^2-1/420*B*(-a*d+b*c)^6*g^3*i^3*(b*x+a)*(6*A+
5*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^3-1/420*B*(-a*d+b*c)^7*g^3*i^3*ln((-a
*d+b*c)/b/(d*x+c))*(6*A+11*B+6*B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^4-11/420*B^2
*(-a*d+b*c)^7*g^3*i^3*ln(d*x+c)/b^4/d^4-1/70*B^2*(-a*d+b*c)^7*g^3*i^3*p...
```

### 3.74.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2330 vs. $2(1089) = 2178$.

Time = 1.73 (sec) , antiderivative size = 2330, normalized size of antiderivative = 2.14

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Result too large to show}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2,x]
```

output
```
(g^3*i^3*(35*(b*c - a*d)^3*(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2 + 84*d*(b*c - a*d)^2*(a + b*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^
2 + 70*d^2*(b*c - a*d)*(a + b*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
+ 20*d^3*(a + b*x)^7*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (35*B*(b*c -
 a*d)^4*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a
 + b*x))/(c + d*x)] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a +
b*x))/(c + d*x)]) + 2*d^3*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])
 - 6*B*(b*c - a*d)^3*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[(e*(a + b*x
))/(c + d*x)])*Log[c + d*x] + B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x - d^2*(a
+ b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*(b*d*x + (-(b
*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*((2*Log[(d*(a + b*x))/(-(b*c)
 + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)])))/(3*d^4) + (7*B*(b*c - a*d)^3*(24*A*b*d*(b*c - a*d)^3*x + 24*B*d*(
b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 12*d^2*(b*c - a*d)^2
*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 8*d^3*(b*c - a*d)*(a +
 b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*d^4*(a + b*x)^4*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*c - a*d)^4*Log[c + d*x] - 24*(b*c -
a*d)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + 4*B*(b*c - a*d)
^2*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x])
+ B*(b*c - a*d)*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)...
```

### 3.74.3 Rubi [A] (verified)

Time = 3.48 (sec) , antiderivative size = 1398, normalized size of antiderivative = 1.28, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.524$, Rules used = {2962, 2783, 2782, 27, 2123, 2009, 2783, 2782, 27, 87, 49, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2962}$$

$$g^3i^3(bc-ad)^7\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^8}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$g^3i^3(bc-ad)^7\left(-\frac{2B\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}}{7b}+\frac{3\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}}{7b}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.$$

$\downarrow$ 2782

$$g^3i^3(bc-ad)^7\left(-\frac{2B\left(-B\int -\frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{60d^4(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}+\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)\right.}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.}{7b}\right.$$

$\downarrow$ 27

$$g^3i^3(bc-ad)^7\left(-\frac{2B\left(\frac{B\int \frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{60d^4}+\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\right.}{7b}\right.$$

$\downarrow$ 2123

$$g^3i^3(bc-ad)^7\left(-\frac{2B\left(\frac{B\int\left(-\frac{10db^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}+\frac{26db}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{19d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3b}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2b^2}+\frac{c+dx}{(a+bx)b^3}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)b^3}\right)d\frac{a}{c}}{60d^4}\right.}{7b}\right.$$

$\downarrow$ 2009

3.74. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$g^3i^3(bc-ad)^7\left(\frac{3\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}}{7b}-\frac{2B\left(\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right.}{}\right.$$

$\downarrow$ 2783

$$g^3i^3(bc-ad)^7\left(\frac{3\left(-\frac{B\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{3b}+\frac{\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}\right)}{7b}-\frac{2B}{}\right.$$

$\downarrow$ 2782

$$(bc-ad)^7g^3i^3\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\log\left(\frac{e(a+b}{c+d}\right.\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.$$

$\downarrow$ 27

3.74. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$g^3 i^3 (bc - ad)^7 \left( 3 \left( -\frac{B\left(-\frac{B\int \frac{(a+bx)^3\left(5b-\frac{d(a+bx)}{c+dx}\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{20b^2}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{(a+bx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)}{3b} + \frac{\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(\ldots}{\ldots}\right.\right.}{(c+dx)^3\left(b-\frac{d(a+\ldots}{c+\ldots}\right.}}{3b} \right. \right.}{7b}$$

$\downarrow$ 87

$$(bc - ad)^7 g^3 i^3 \left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a+b\ldots}{c+d\ldots}\right.\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.} \right.}{\ldots} \right.$$

$\downarrow$ 49

3.74. $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$(bc-ad)^7g^3i^3\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\log\left(\frac{e(a+b}{c+d}\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.\right.$$

$\downarrow$ 2009

$$(bc-ad)^7g^3i^3\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\log\left(\frac{e(a+b}{c+d}\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.\right.$$

$\downarrow$ 2783

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3.74. $\int(ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$(bc - ad)^7 g^3 i^3 \left( \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a+b}{c+d}\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.$$

$\downarrow$ 2773

$$(bc - ad)^7 g^3 i^3 \left( \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a+b}{c+d}\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.$$

$\downarrow$ 49

---

3.74. $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$(bc-$$

$$ad)^7g^3i^3\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\log\left(\frac{e(a+b}{c+d\right.}\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.$$

$\downarrow$ 2009

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3.74. $\int(ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$(bc-ad)^7g^3i^3\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\log\left(\frac{e(a+b}{c+d}\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.$$

$\downarrow$ 2781

3.74. $\int(ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$(bc-ad)^7g^3i^3\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\log\left(\frac{e(a+b}{c+d}\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.$$

$\downarrow$ 2784

3.74. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$(bc - ad)^7 g^3 i^3 \left( \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b - \frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a+b}{c+d}\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.$$

↓ 2784

3.74. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$(bc - ad)^7 g^3 i^3 \left( \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b - \frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a+b}{c+d}\right.\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.$$

$\downarrow$ 2784

---

3.74. $\int (ag+bgx)^3(ci+dix)^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 \, dx$

$$(bc - ad)^7 g^3 i^3 \left( \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^4}{7b(c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^7} - \frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\log\left(\frac{e(a+b}{c+d}\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.$$

3.74. $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$\Big\downarrow \text{2754}$$

$$(bc-ad)^7g^3i^3\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\log\left(\frac{e(a+b}{c+dx}\right.\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.$$

3.74. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$\downarrow 2838$$

$$(bc-ad)^7g^3i^3\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\log\left(\frac{e(a+b}{c+d}\right.\right.}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.\right.$$

3.74. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
,x]
```

output
```
(b*c - a*d)^7*g^3*i^3*(((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(7*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^7) - (2*B*((b^3*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]))/(6*d^4*(b - (d*(a + b*x))/(c + d*x))^6) - (3*
b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*d^4*(b - (d*(a + b*x))/(c + d
*x))^5) + (3*b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*d^4*(b - (d*(a + b
*x))/(c + d*x))^4) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*d^4*(b - (d*(
a + b*x))/(c + d*x))^3) + (B*((-2*b^2)/(b - (d*(a + b*x))/(c + d*x))^5 + (
13*b)/(2*(b - (d*(a + b*x))/(c + d*x))^4) - 19/(3*(b - (d*(a + b*x))/(c +
d*x))^3) + 1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b
*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c
 + d*x)]/b^3))/(60*d^4)))/(7*b) + (3*(((a + b*x)^4*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/(6*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^6) - (B*((
(a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(5*b*(c + d*x)^4*(b - (d
*(a + b*x))/(c + d*x))^5) + ((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x
)]))/(20*b^2*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*((a + b*x)^
4/((c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) + b^3/(3*d^4*(b - (d*(a +
b*x))/(c + d*x))^3) - (3*b^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3
*b)/(d^4*(b - (d*(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]
/d^4))/(20*b^2)))/(3*b) + (((a + b*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2)/(5*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*(((a + b...
```

### 3.74.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

## 3.74. $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.74.4 Maple [F]

$$\int (bgx+ag)^3 (dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

### 3.74.5 Fricas [F]

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (bgx+ag)^3(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="fricas")
```

output
```
integral(A^2*b^3*d^3*g^3*i^3*x^6 + A^2*a^3*c^3*g^3*i^3 + 3*(A^2*b^3*c*d^2
+ A^2*a*b^2*d^3)*g^3*i^3*x^5 + 3*(A^2*b^3*c^2*d + 3*A^2*a*b^2*c*d^2 + A^2*
a^2*b*d^3)*g^3*i^3*x^4 + (A^2*b^3*c^3 + 9*A^2*a*b^2*c^2*d + 9*A^2*a^2*b*c*
d^2 + A^2*a^3*d^3)*g^3*i^3*x^3 + 3*(A^2*a*b^2*c^3 + 3*A^2*a^2*b*c^2*d + A^
2*a^3*c*d^2)*g^3*i^3*x^2 + 3*(A^2*a^2*b*c^3 + A^2*a^3*c^2*d)*g^3*i^3*x + (
B^2*b^3*d^3*g^3*i^3*x^6 + B^2*a^3*c^3*g^3*i^3 + 3*(B^2*b^3*c*d^2 + B^2*a*b
^2*d^3)*g^3*i^3*x^5 + 3*(B^2*b^3*c^2*d + 3*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3
)*g^3*i^3*x^4 + (B^2*b^3*c^3 + 9*B^2*a*b^2*c^2*d + 9*B^2*a^2*b*c*d^2 + B^2
*a^3*d^3)*g^3*i^3*x^3 + 3*(B^2*a*b^2*c^3 + 3*B^2*a^2*b*c^2*d + B^2*a^3*c*d
^2)*g^3*i^3*x^2 + 3*(B^2*a^2*b*c^3 + B^2*a^3*c^2*d)*g^3*i^3*x)*log((b*e*x
+ a*e)/(d*x + c))^2 + 2*(A*B*b^3*d^3*g^3*i^3*x^6 + A*B*a^3*c^3*g^3*i^3 + 3
*(A*B*b^3*c*d^2 + A*B*a*b^2*d^3)*g^3*i^3*x^5 + 3*(A*B*b^3*c^2*d + 3*A*B*a*
b^2*c*d^2 + A*B*a^2*b*d^3)*g^3*i^3*x^4 + (A*B*b^3*c^3 + 9*A*B*a*b^2*c^2*d
+ 9*A*B*a^2*b*c*d^2 + A*B*a^3*d^3)*g^3*i^3*x^3 + 3*(A*B*a*b^2*c^3 + 3*A*B*
a^2*b*c^2*d + A*B*a^3*c*d^2)*g^3*i^3*x^2 + 3*(A*B*a^2*b*c^3 + A*B*a^3*c^2*
d)*g^3*i^3*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

### 3.74.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

### 3.74.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6921 vs. 2(1042) = 2084.

Time = 0.40 (sec) , antiderivative size = 6921, normalized size of antiderivative = 6.36

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="maxima")
```

output
```
1/7*A^2*b^3*d^3*g^3*i^3*x^7 + 1/2*A^2*b^3*c*d^2*g^3*i^3*x^6 + 1/2*A^2*a*b^
2*d^3*g^3*i^3*x^6 + 3/5*A^2*b^3*c^2*d*g^3*i^3*x^5 + 9/5*A^2*a*b^2*c*d^2*g^
3*i^3*x^5 + 3/5*A^2*a^2*b*d^3*g^3*i^3*x^5 + 1/4*A^2*b^3*c^3*g^3*i^3*x^4 +
9/4*A^2*a*b^2*c^2*d*g^3*i^3*x^4 + 9/4*A^2*a^2*b*c*d^2*g^3*i^3*x^4 + 1/4*A^
2*a^3*d^3*g^3*i^3*x^4 + A^2*a*b^2*c^3*g^3*i^3*x^3 + 3*A^2*a^2*b*c^2*d*g^3*
i^3*x^3 + A^2*a^3*c*d^2*g^3*i^3*x^3 + 3/2*A^2*a^2*b*c^3*g^3*i^3*x^2 + 3/2*
A^2*a^3*c^2*d*g^3*i^3*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*
log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^3*c^3*g^3*i^3 + 3*(x^2*log(b*e*x/
(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 -
 (b*c - a*d)*x/(b*d))*A*B*a^2*b*c^3*g^3*i^3 + (2*x^3*log(b*e*x/(d*x + c) +
 a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*
c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*b^2*c^3*g^3
*i^3 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b*x +
a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*
c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b^3*c^3*g
^3*i^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^
2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^3*c^2*d*g^3*i^3 + 3*
(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c
^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/
(b^2*d^2))*A*B*a^2*b*c^2*d*g^3*i^3 + 3/4*(6*x^4*log(b*e*x/(d*x + c) + a...
```

### 3.74.8 Giac [F]

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)^3(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) +
A)^2, x)
```

### 3.74.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2
,x)
```

output
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2
, x)
```

# 3.75 $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

### 3.75.1 Optimal result

Integrand size = 42, antiderivative size = 908

$$
\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx \\
&= -\frac{7B^2(bc-ad)^5g^2i^3x}{180b^3d^2}-\frac{7B^2(bc-ad)^4g^2i^3(c+dx)^2}{360b^2d^3} \\
&-\frac{B^2(bc-ad)^3g^2i^3(c+dx)^3}{60bd^3}+\frac{B^2(bc-ad)^2g^2i^3(c+dx)^4}{60d^3} \\
&+\frac{B^2(bc-ad)^6g^2i^3\log\left(\frac{a+bx}{c+dx}\right)}{36b^4d^3}-\frac{B(bc-ad)^4g^2i^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d} \\
&-\frac{B(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30b^4} \\
&-\frac{B(bc-ad)^4g^2i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^2d^3} \\
&+\frac{B(bc-ad)^3g^2i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{45bd^3} \\
&+\frac{7B(bc-ad)^2g^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60d^3} \\
&-\frac{bB(bc-ad)g^2i^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{15d^3} \\
&+\frac{(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{60b^4} \\
&+\frac{(bc-ad)^2g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^3} \\
&+\frac{(bc-ad)g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^2} \\
&+\frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6b} \\
&+\frac{B(bc-ad)^5g^2i^3(a+bx)\left(2A+B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d^2} \\
&+\frac{B(bc-ad)^6g^2i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(2A+3B+2B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{60b^4d^3} \\
&+\frac{11B^2(bc-ad)^6g^2i^3\log(c+dx)}{180b^4d^3}+\frac{B^2(bc-ad)^6g^2i^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{30b^4d^3}
\end{aligned}
$$

output
```
-7/180*B^2*(-a*d+b*c)^5*g^2*i^3*x/b^3/d^2-7/360*B^2*(-a*d+b*c)^4*g^2*i^3*(
d*x+c)^2/b^2/d^3-1/60*B^2*(-a*d+b*c)^3*g^2*i^3*(d*x+c)^3/b/d^3+1/60*B^2*(-
a*d+b*c)^2*g^2*i^3*(d*x+c)^4/d^3+1/36*B^2*(-a*d+b*c)^6*g^2*i^3*ln((b*x+a)/
(d*x+c))/b^4/d^3-1/60*B*(-a*d+b*c)^4*g^2*i^3*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(
d*x+c)))/b^4/d-1/30*B*(-a*d+b*c)^3*g^2*i^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*
x+c)))/b^4-1/10*B*(-a*d+b*c)^4*g^2*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)
))/b^2/d^3+1/45*B*(-a*d+b*c)^3*g^2*i^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)
))/b/d^3+7/60*B*(-a*d+b*c)^2*g^2*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))
/d^3-1/15*b*B*(-a*d+b*c)*g^2*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3
+1/60*(-a*d+b*c)^3*g^2*i^3*(b*x+a)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^4+1/2
0*(-a*d+b*c)^2*g^2*i^3*(b*x+a)^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^3
+1/10*(-a*d+b*c)*g^2*i^3*(b*x+a)^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2
/b^2+1/6*g^2*i^3*(b*x+a)^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b+1/60*
B*(-a*d+b*c)^5*g^2*i^3*(b*x+a)*(2*A+B+2*B*ln(e*(b*x+a)/(d*x+c)))/b^4/d^2+1
/60*B*(-a*d+b*c)^6*g^2*i^3*ln((-a*d+b*c)/b/(d*x+c))*(2*A+3*B+2*B*ln(e*(b*x
+a)/(d*x+c)))/b^4/d^3+11/180*B^2*(-a*d+b*c)^6*g^2*i^3*ln(d*x+c)/b^4/d^3+1/
30*B^2*(-a*d+b*c)^6*g^2*i^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^3
```

### 3.75.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 1555, normalized size of antiderivative = 1.71

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2,x]
```

output
```
(g^2*i^3*(15*(b*c - a*d)^2*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2 - 24*b*(b*c - a*d)*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
+ 10*b^2*(c + d*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (5*B*(b*c -
a*d)^3*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*L
og[a + b*x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b
*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x)
)/(c + d*x)] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c +
 d*x)]) + 2*b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c
- a*d)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*
d)^3*Log[c + d*x] - 3*B*(b*c - a*d)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[
(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]))
)/b^4 + (2*B*(b*c - a*d)^2*(24*A*b*d*(b*c - a*d)^3*x - 12*B*(b*c - a*d)^3*
(b*d*x + (b*c - a*d)*Log[a + b*x]) - 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*
x + b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x]) - B*(b*c - a*d)*(6*b*d
*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(
b*c - a*d)^3*Log[a + b*x]) + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*
x))/(c + d*x)] + 12*b^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))
/(c + d*x)]) + 8*b^3*(b*c - a*d)*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c +
 d*x)]) + 6*b^4*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 24*(b*c
 - a*d)^4*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*c...
```

### 3.75.3 Rubi [A] (verified)

Time = 3.00 (sec) , antiderivative size = 1213, normalized size of antiderivative = 1.34, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.476$, Rules used = {2962, 2783, 2782, 27, 1195, 2009, 2783, 2782, 27, 1195, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2dx$$

$$\Big\downarrow \text{2962}$$

$$g^2i^3(bc-ad)^6\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$g^2i^3(bc-ad)^6\left(-\frac{B\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{3b}+\frac{\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{2b}+\frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}\right.$$

$\downarrow$ 2782

$$g^2i^3(bc-ad)^6\left(-\frac{B\left(-B\int \frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{30d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{B\log\left(\frac{e(a+bx}{c+dx}\right.}{3d^3\left(b-\frac{d(a+b}{c+d}\right.}\right.}{3b}\right.$$

$\downarrow$ 27

$$g^2i^3(bc-ad)^6\left(-\frac{B\left(-\frac{B\int \frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{30d^3}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.}{3b}\right.$$

$\downarrow$ 1195

$$g^2i^3(bc-ad)^6\left(-\frac{B\left(-\frac{B\int \left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{9d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{6bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{30d^3}+\frac{b^2\left(B\log\right.}{5d^3\left(b-\right.}\right.}{3b}\right.$$

$\downarrow$ 2009

3.75. $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$g^2 i^3 (bc - ad)^6 \left( \frac{\int \frac{(a+bx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{2b} - \frac{B\left(\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{B\left(\right.}{}\right.}{}\right.$$

$\downarrow$ 2783

$$g^2 i^3 (bc - ad)^6 \left( \frac{-\frac{2B\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{2\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{(a+bx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}}{2b} - \frac{B\left(\frac{b^2}{}\right.}{}\right.$$

$\downarrow$ 2782

$$g^2 i^3 (bc - ad)^6 \left( \frac{-\frac{2B\left(-B\int \frac{(c+dx)\left(b^2-\frac{4d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{12d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx} + \frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right)}{5b} + \frac{2}{}}{2b}\right.$$

$\downarrow$ 27

3.75. $\int (ag+bgx)^2 (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$g^2 i^3 (bc - ad)^6 \left( \frac{-\frac{2B\left(-\frac{B\int \frac{(c+dx)\left(b^2-\frac{4d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{12d^3}+\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{2b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right)}{5b} + \frac{2}{}}{2b} \right.$$

$\Big\downarrow$ 1195

$$(bc - ad)^6 g^2 i^3 \left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \right.}{} \right.$$

$\Big\downarrow$ 2009

$$g^2 i^3 (bc - ad)^6 \left( \frac{\frac{2\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b} - \frac{2B\left(\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{2b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}\right.}{}\right.}{5b}}{2b} \right.$$

$\Big\downarrow$ 2783

---

3.75. $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$(bc -$$

$$ad)^6 g^2 i^3 \left( \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \right.}{}\right.$$

$\Big\downarrow$ 2773

$$(bc -$$

$$ad)^6 g^2 i^3 \left( \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \right.}{}\right.$$

$\Big\downarrow$ 49

3.75. $\int (ag+bgx)^2(ci+dix)^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$(bc - ad)^6 g^2 i^3 \left( \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} -\right.}{}\right.$$

$\downarrow$ 2009

$$(bc - ad)^6 g^2 i^3 \left( \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3 \left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} -\right.}{}\right.$$

---

3.75. $\int (ag + bgx)^2 (ci + dix)^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$\Big\downarrow \; 2781$$

$$(bc-ad)^6 g^2 i^3 \left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \right.}{} \right.$$

$$\Big\downarrow \; 2784$$

---

3.75. $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$(bc - ad)^6 g^2 i^3 \left( \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \right.}{} \right.$$

$\downarrow$ 2784

3.75. $\int (ag+bgx)^2(ci+dix)^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$(bc-$$

$$ad)^6 g^2 i^3 \left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \right.}{} \right.$$

$$\downarrow 2754$$

3.75. $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$(bc-$$

$$ad)^6 g^2 i^3 \left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \right.}{} \right.$$

$\downarrow$ 2838

---

3.75. $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$(bc-ad)^6 g^2 i^3 \left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{B\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
,x]
```

3.75. $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

output
```
(b*c - a*d)^6*g^2*i^3*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(6*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^6) - (B*((b^2*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(5*d^3*(b - (d*(a + b*x))/(c + d*x))^5) - (b*(A
 + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^4
) + (A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*d^3*(b - (d*(a + b*x))/(c + d*
x))^3) - (B*((3*b)/(2*(b - (d*(a + b*x))/(c + d*x))^4) - 3/(b - (d*(a + b*
x))/(c + d*x))^3 + 1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (
d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[b - (d*(a +
b*x))/(c + d*x)]/b^3))/(30*d^3)))/(3*b) + (((a + b*x)^3*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])^2)/(5*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^5) -
(2*B*((b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(4*d^3*(b - (d*(a + b*x))
/(c + d*x))^4) - (2*b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*d^3*(b - (d
*(a + b*x))/(c + d*x))^3) + (A + B*Log[(e*(a + b*x))/(c + d*x)])/(2*d^3*(b
 - (d*(a + b*x))/(c + d*x))^2) - (B*(b/(b - (d*(a + b*x))/(c + d*x))^3 - 5
/(2*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b*(b - (d*(a + b*x))/(c + d*x)))
 + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(
12*d^3)))/(5*b) + (2*(((a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)
/(4*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^4) - (B*(((a + b*x)^3*(A +
 B*Log[(e*(a + b*x))/(c + d*x)]))/(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c +
 d*x))^3) - (B*(b^2/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*b)/(d^...
```

### 3.75.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

## 3.75. $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.75.4 Maple [F]

$$\int (bgx+ag)^2 (dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

## 3.75.5 Fricas [F]

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (bgx+ag)^2(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="fricas")
```

output
```
integral(A^2*b^2*d^3*g^2*i^3*x^5 + A^2*a^2*c^3*g^2*i^3 + (3*A^2*b^2*c*d^2
+ 2*A^2*a*b*d^3)*g^2*i^3*x^4 + (3*A^2*b^2*c^2*d + 6*A^2*a*b*c*d^2 + A^2*a^
2*d^3)*g^2*i^3*x^3 + (A^2*b^2*c^3 + 6*A^2*a*b*c^2*d + 3*A^2*a^2*c*d^2)*g^2
*i^3*x^2 + (2*A^2*a*b*c^3 + 3*A^2*a^2*c^2*d)*g^2*i^3*x + (B^2*b^2*d^3*g^2*
i^3*x^5 + B^2*a^2*c^3*g^2*i^3 + (3*B^2*b^2*c*d^2 + 2*B^2*a*b*d^3)*g^2*i^3*
x^4 + (3*B^2*b^2*c^2*d + 6*B^2*a*b*c*d^2 + B^2*a^2*d^3)*g^2*i^3*x^3 + (B^2
*b^2*c^3 + 6*B^2*a*b*c^2*d + 3*B^2*a^2*c*d^2)*g^2*i^3*x^2 + (2*B^2*a*b*c^3
 + 3*B^2*a^2*c^2*d)*g^2*i^3*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^2
*d^3*g^2*i^3*x^5 + A*B*a^2*c^3*g^2*i^3 + (3*A*B*b^2*c*d^2 + 2*A*B*a*b*d^3)
*g^2*i^3*x^4 + (3*A*B*b^2*c^2*d + 6*A*B*a*b*c*d^2 + A*B*a^2*d^3)*g^2*i^3*x
^3 + (A*B*b^2*c^3 + 6*A*B*a*b*c^2*d + 3*A*B*a^2*c*d^2)*g^2*i^3*x^2 + (2*A*
B*a*b*c^3 + 3*A*B*a^2*c^2*d)*g^2*i^3*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

### 3.75.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

### 3.75.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5196 vs. $2(869)=1738$.

Time = 0.36 (sec) , antiderivative size = 5196, normalized size of antiderivative = 5.72

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="maxima")
```

output
```
1/6*A^2*b^2*d^3*g^2*i^3*x^6 + 3/5*A^2*b^2*c*d^2*g^2*i^3*x^5 + 2/5*A^2*a*b*
d^3*g^2*i^3*x^5 + 3/4*A^2*b^2*c^2*d*g^2*i^3*x^4 + 3/2*A^2*a*b*c*d^2*g^2*i^
3*x^4 + 1/4*A^2*a^2*d^3*g^2*i^3*x^4 + 1/3*A^2*b^2*c^3*g^2*i^3*x^3 + 2*A^2*
a*b*c^2*d*g^2*i^3*x^3 + A^2*a^2*c*d^2*g^2*i^3*x^3 + A^2*a*b*c^3*g^2*i^3*x^
2 + 3/2*A^2*a^2*c^2*d*g^2*i^3*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x +
c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a^2*c^3*g^2*i^3 + 2*(x^2*lo
g(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x +
c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a*b*c^3*g^2*i^3 + 1/3*(2*x^3*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b^2
*c^3*g^2*i^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x +
 a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*a^2*c^2*d*g^2*i^
3 + 2*(2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3
 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^
2)*x)/(b^2*d^2))*A*B*a*b*c^2*d*g^2*i^3 + 1/4*(6*x^4*log(b*e*x/(d*x + c) +
a*e/(d*x + c)) - 6*a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3
*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3
*d^3)*x)/(b^3*d^3))*A*B*b^2*c^2*d*g^2*i^3 + (2*x^3*log(b*e*x/(d*x + c) + a
*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*
d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a^2*c*d^2*g^...
```

## 3.75.8 Giac [F]

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx = \int (bgx+ag)^2(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) +
A)^2, x)
```

### 3.75.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2
,x)
```

output
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2
, x)
```

# 3.76 $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

### 3.76.1 Optimal result

Integrand size = 40, antiderivative size = 730

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$\begin{aligned}
&=\frac{B^2(bc-ad)^4gi^3x}{60b^3d}+\frac{B^2(bc-ad)^3gi^3(c+dx)^2}{30b^2d^2}+\frac{B^2(bc-ad)^2gi^3(c+dx)^3}{30bd^2}\\
&-\frac{B^2(bc-ad)^5gi^3\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d^2}-\frac{B(bc-ad)^4gi^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4d}\\
&-\frac{B(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4}\\
&+\frac{3B(bc-ad)^3gi^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{20b^2d^2}\\
&+\frac{B(bc-ad)^2gi^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{30bd^2}\\
&-\frac{B(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10d^2}\\
&+\frac{(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^4}\\
&+\frac{(bc-ad)^2gi^3(a+bx)^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{10b^3}\\
&+\frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{20b^2}\\
&+\frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b}\\
&-\frac{B(bc-ad)^5gi^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{10b^4d^2}\\
&-\frac{11B^2(bc-ad)^5gi^3\log(c+dx)}{60b^4d^2}-\frac{B^2(bc-ad)^5gi^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^4d^2}
\end{aligned}$$

output
```
1/60*B^2*(-a*d+b*c)^4*g*i^3*x/b^3/d+1/30*B^2*(-a*d+b*c)^3*g*i^3*(d*x+c)^2/
b^2/d^2+1/30*B^2*(-a*d+b*c)^2*g*i^3*(d*x+c)^3/b/d^2-1/12*B^2*(-a*d+b*c)^5*
g*i^3*ln((b*x+a)/(d*x+c))/b^4/d^2-1/10*B*(-a*d+b*c)^4*g*i^3*(b*x+a)*(A+B*l
n(e*(b*x+a)/(d*x+c)))/b^4/d-1/10*B*(-a*d+b*c)^3*g*i^3*(b*x+a)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))/b^4+3/20*B*(-a*d+b*c)^3*g*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a
)/(d*x+c)))/b^2/d^2+1/30*B*(-a*d+b*c)^2*g*i^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/
(d*x+c)))/b/d^2-1/10*B*(-a*d+b*c)*g*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c
)))/d^2+1/20*(-a*d+b*c)^3*g*i^3*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^
4+1/10*(-a*d+b*c)^2*g*i^3*(b*x+a)^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/
b^3+3/20*(-a*d+b*c)*g*i^3*(b*x+a)^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^
2/b^2+1/5*g*i^3*(b*x+a)^2*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b-1/10*B
*(-a*d+b*c)^5*g*i^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B+B*ln(e*(b*x+a)/(d*x+c)))
/b^4/d^2-11/60*B^2*(-a*d+b*c)^5*g*i^3*ln(d*x+c)/b^4/d^2-1/10*B^2*(-a*d+b*c
)^5*g*i^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^2
```

### 3.76.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.23

$$\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{gi^3\left(-5(bc-ad)(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+4b(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{5B(bc-ad)^2\left(6Ab\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2,x]
```

output
```
(g*i^3*(-5*(b*c - a*d)*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
+ 4*b*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (5*B*(b*c - a*d
)^2*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*Log[
a + b*x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c
- a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(
c + d*x)] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)]) + 2*b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a
*d)^3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d)^
3*Log[c + d*x] - 3*B*(b*c - a*d)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*
(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(
3*b^4) - (B*(b*c - a*d)*(24*A*b*d*(b*c - a*d)^3*x - 12*B*(b*c - a*d)^3*(b*
d*x + (b*c - a*d)*Log[a + b*x]) - 4*B*(b*c - a*d)^2*(2*b*d*(b*c - a*d)*x +
 b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x]) - B*(b*c - a*d)*(6*b*d*(b
*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c
 - a*d)^3*Log[a + b*x]) + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[(e*(a + b*x))
/(c + d*x)] + 12*b^2*(b*c - a*d)^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c
 + d*x)]) + 8*b^3*(b*c - a*d)*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)]) + 6*b^4*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 24*(b*c -
a*d)^4*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 24*B*(b*c - a*d
)^4*Log[c + d*x] - 12*B*(b*c - a*d)^4*(Log[a + b*x]*(Log[a + b*x] - 2*L...
```

### 3.76.3 Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 959, normalized size of antiderivative = 1.31, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.475, Rules used = {2962, 2783, 2782, 27, 86, 2009, 2783, 2782, 27, 86, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2962}$$

$$gi^3(bc-ad)^5\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$gi^3(bc-ad)^5\left(-\frac{2B\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{3\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+\right.}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.$$

$\downarrow$ 2782

$$gi^3(bc-ad)^5\left(-\frac{2B\left(-B\int-\frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{12d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b}+\frac{3\int\frac{(a+bx)\left(A+B\log\left(\right.\right.}{(c+dx)\left(b-\frac{d(a-}{c+}\right.}}{5b}\right.$$

$\downarrow$ 27

$$gi^3(bc-ad)^5\left(-\frac{2B\left(\frac{B\int\frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b}+\frac{3\int\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}}{5b}\right.$$

$\downarrow$ 86

$$gi^3(bc-ad)^5\left(-\frac{2B\left(\frac{B\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\right.}{4}\right.}{5b}\right.$$

$\downarrow$ 2009

---

3.76. $\int(ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$gi^3(bc-ad)^5\left(\frac{3\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}-\frac{2B\left(-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d(a+}{c+}\right.}{b^3}\right.}{}\right.}{5b}\right.$$

$\Big\downarrow$ 2783

$$gi^3(bc-ad)^5\left(\frac{3\left(-\frac{B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b}-\frac{2B\left(\right.}{}\right.$$

$\Big\downarrow$ 2782

$$gi^3(bc-ad)^5\left(\frac{3\left(-\frac{B\left(-B\int -\frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{6d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+}{c+}}{2b}\right.}{5b}\right.$$

$\Big\downarrow$ 27

3.76. $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$gi^3(bc-ad)^5\left(\frac{3\left(-\frac{B\left(\frac{B\int \frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\right.}{5b}\right.$$

$\downarrow$ 86

$$gi^3(bc-ad)^5\left(\frac{3\left(-\frac{B\left(\frac{B\int \left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\right.}{5b}\right.$$

$\downarrow$ 2009

---

3.76. $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$gi^3(bc-ad)^5 \left(3\left(\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}-\frac{B\left(-\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{}{b\left(b-\right.}\right.}{6d^2}\right.}{2b}\right.\right.}{5b}$$

$\downarrow$ 2783

$$gi^3(bc-ad)^5 \left(3\left(\frac{-\frac{2B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{2b}-\frac{B\left(-\frac{B\log\left(\frac{e(}{}\right.}{2d^2\left(b-\right.}\right.}{}\right.\right.}{}$$

$\downarrow$ 2773

---

3.76. $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$(bc - ad)^5 gi^3 \left( \frac{(a+bx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b-\frac{d}{}\right.}{b}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 49

$$(bc - ad)^5 gi^3 \left( \frac{(a+bx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b-\frac{d}{}\right.}{b}\right.}{}\right.}{5b}\right.$$

3.76. $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$\downarrow 2009$$

$$(bc-ad)^5gi^3\left(\frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2B\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d}{}\right.}{b}\right.}{}\right.}{5b}\right.$$

$$\downarrow 2781$$

---

3.76. $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$(bc-ad)^5 g i^3 \left( \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b-\frac{d}{}\right.}{b}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2784

---

3.76. $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$(bc-ad)^5gi^3\left(\frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2B\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d}{}\right.}{b}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2754

3.76. $\int(ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$(bc-ad)^5 g i^3 \left(\frac{(a+bx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b-\frac{d}{}\right.}{b}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2838

3.76. $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$(bc - ad)^5 g i^3 \left( \frac{(a+bx)^2 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2B\left(-\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3} - \frac{\log\left(b-\ldots\right.}{b\ldots}\right.}{\ldots}\right.}{5b\ldots}\right.$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x
]
```

## 3.76. $\int (ag+bgx)(ci+dix)^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

output
```
(b*c - a*d)^5*g*i^3*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/
(5*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*((b*(A + B*Log[(e
*(a + b*x))/(c + d*x)]))/(4*d^2*(b - (d*(a + b*x))/(c + d*x))^4) - (A + B*
Log[(e*(a + b*x))/(c + d*x)])/(3*d^2*(b - (d*(a + b*x))/(c + d*x))^3) + (B
*(-(b - (d*(a + b*x))/(c + d*x))^(-3) + 1/(2*b*(b - (d*(a + b*x))/(c + d*x
))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b
^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(12*d^2)))/(5*b) + (3*(((a + b
*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*b*(c + d*x)^2*(b - (d*(a
+ b*x))/(c + d*x))^4) - (B*((b*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(3*d^
2*(b - (d*(a + b*x))/(c + d*x))^3) - (A + B*Log[(e*(a + b*x))/(c + d*x)])/
(2*d^2*(b - (d*(a + b*x))/(c + d*x))^2) + (B*(-(b - (d*(a + b*x))/(c + d*x
))^(-2) + 1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b
^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(6*d^2)))/(2*b) + (((a + b*x)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*b*(c + d*x)^2*(b - (d*(a + b*
x))/(c + d*x))^3) - (2*B*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(b/(d^2*(b - (d*
(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]/d^2))/(2*b)))/(3
*b) + (((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b*(c + d*x)
^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(((a + b*x)*(A + B*Log[(e*(a + b*
x))/(c + d*x)]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((A +...
```

### 3.76.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

## 3.76. $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.76.4 Maple [F]

$$\int (bgx + ag) (dix + ci)^3 \left(A + B\ln\left(\frac{e(bx + a)}{dx + c}\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

## 3.76.5 Fricas [F]

$$\int (ag + bgx)(ci + dix)^3 \left(A + B\log\left(\frac{e(a + bx)}{c + dx}\right)\right)^2 dx = \int (bgx + ag)(dix + ci)^3 \left(B\log\left(\frac{(bx + a)e}{dx + c}\right) + A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="fricas")
```

output
```
integral(A^2*b*d^3*g*i^3*x^4 + A^2*a*c^3*g*i^3 + (3*A^2*b*c*d^2 + A^2*a*d^
3)*g*i^3*x^3 + 3*(A^2*b*c^2*d + A^2*a*c*d^2)*g*i^3*x^2 + (A^2*b*c^3 + 3*A^
2*a*c^2*d)*g*i^3*x + (B^2*b*d^3*g*i^3*x^4 + B^2*a*c^3*g*i^3 + (3*B^2*b*c*d
^2 + B^2*a*d^3)*g*i^3*x^3 + 3*(B^2*b*c^2*d + B^2*a*c*d^2)*g*i^3*x^2 + (B^2
*b*c^3 + 3*B^2*a*c^2*d)*g*i^3*x)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b
*d^3*g*i^3*x^4 + A*B*a*c^3*g*i^3 + (3*A*B*b*c*d^2 + A*B*a*d^3)*g*i^3*x^3 +
 3*(A*B*b*c^2*d + A*B*a*c*d^2)*g*i^3*x^2 + (A*B*b*c^3 + 3*A*B*a*c^2*d)*g*i
^3*x)*log((b*e*x + a*e)/(d*x + c)), x)
```

### 3.76.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

### 3.76.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3218 vs. 2(697) = 1394.

Time = 0.32 (sec) , antiderivative size = 3218, normalized size of antiderivative = 4.41

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="maxima")
```

output
```
1/5*A^2*b*d^3*g*i^3*x^5 + 3/4*A^2*b*c*d^2*g*i^3*x^4 + 1/4*A^2*a*d^3*g*i^3*
x^4 + A^2*b*c^2*d*g*i^3*x^3 + A^2*a*c*d^2*g*i^3*x^3 + 1/2*A^2*b*c^3*g*i^3*
x^2 + 3/2*A^2*a*c^2*d*g*i^3*x^2 + 2*(x*log(b*e*x/(d*x + c) + a*e/(d*x + c)
) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A*B*a*c^3*g*i^3 + (x^2*log(b*e*x/
(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 -
 (b*c - a*d)*x/(b*d))*A*B*b*c^3*g*i^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(
d*x + c)) - a^2*log(b*x + a)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b
*d))*A*B*a*c^2*d*g*i^3 + (2*x^3*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a
^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 -
2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*b*c^2*d*g*i^3 + (2*x^3*log(b*e*x/(
d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^
3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2))*A*B*a*c
*d^2*g*i^3 + 1/4*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b
*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*
(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*b*c*
d^2*g*i^3 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*a^4*log(b
*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*
(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))*A*B*a*d^
3*g*i^3 + 1/30*(12*x^5*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 12*a^5*log(b
*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 ...
```

### 3.76.8 Giac [F]

$$\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \int (bgx+ag)(dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algo
rithm="giac")
```

output
```
integrate((b*g*x + a*g)*(d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)
^2, x)
```

### 3.76.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)\,\left(c\,i+d\,i\,x\right)^3\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x
)
```

output
```
int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,
x)
```

# 3.77 $\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$



## 3.77.1 Optimal result

Integrand size = 32, antiderivative size = 420

$$\begin{aligned}
\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx
&= \frac{5B^2(bc-ad)^3i^3x}{12b^3} + \frac{B^2(bc-ad)^2i^3(c+dx)^2}{12b^2d} + \frac{5B^2(bc-ad)^4i^3\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d} \\
&\quad - \frac{B(bc-ad)^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^4} \\
&\quad - \frac{B(bc-ad)^2i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{4b^2d} \\
&\quad - \frac{B(bc-ad)i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{6bd} + \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4d} \\
&\quad + \frac{11B^2(bc-ad)^4i^3\log(c+dx)}{12b^4d} + \frac{B(bc-ad)^4i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} \\
&\quad - \frac{B^2(bc-ad)^4i^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d}
\end{aligned}$$

output
```
5/12*B^2*(-a*d+b*c)^3*i^3*x/b^3+1/12*B^2*(-a*d+b*c)^2*i^3*(d*x+c)^2/b^2/d+
5/12*B^2*(-a*d+b*c)^4*i^3*ln((b*x+a)/(d*x+c))/b^4/d-1/2*B*(-a*d+b*c)^3*i^3
*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4-1/4*B*(-a*d+b*c)^2*i^3*(d*x+c)^2*
(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/d-1/6*B*(-a*d+b*c)*i^3*(d*x+c)^3*(A+B*ln(e
*(b*x+a)/(d*x+c)))/b/d+1/4*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d+1
1/12*B^2*(-a*d+b*c)^4*i^3*ln(d*x+c)/b^4/d+1/2*B*(-a*d+b*c)^4*i^3*(A+B*ln(e
*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/d-1/2*B^2*(-a*d+b*c)^4*i^
3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/d
```

### 3.77.2 Mathematica [A] (verified)

Time = 0.19 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.93

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$$
$$= \frac{i^3\left((c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 - \frac{B(bc-ad)\left(6Abd(bc-ad)^2x-3B(bc-ad)^2(bdx+(bc-ad)\log(a+bx))-B(bc-ad)(2bd(bc-a\cdots\right.}{\cdots}\right.}{\cdots}$$

input
```
Integrate[(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

output
```
(i^3*((c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - (B*(b*c - a*d)*
(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*(b*d*x + (b*c - a*d)*Log[a +
b*x]) - B*(b*c - a*d)*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*
d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
 + 2*b^3*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 6*(b*c - a*d)^
3*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 6*B*(b*c - a*d)^3*Lo
g[c + d*x] - 3*B*(b*c - a*d)^3*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c +
 d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(3*b^
4)))/(4*d)
```

### 3.77.3 Rubi [A] (verified)

Time = 1.31 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.23, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.469, Rules used = {2952, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2952}$$

$$i^3(bc-ad)^4\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2756}$$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2d}\right)$$

$$\Big\downarrow \text{2789}$$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{d\int \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{2d}\right)$$

$$\Big\downarrow \text{2756}$$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+}{c+}}{b}\right.}{2d}\right.$$

$\Big\downarrow$ 54

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a}{c}}{3d}\right.}{b}\right.}{2d}\right.$$

$\Big\downarrow$ 2009

---

3.77. $\int(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right.}{b^3}\right.}{\phantom{x}}\right.}{b}\right.}{2d}\right.$$

$\downarrow$ 2789

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\right.}{\phantom{x}}\right.}{2d}\right.$$

$\downarrow$ 2756

3.77. $\int(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\cdots}{b}\cdots\right)}{\cdots}\right.$$

$$\Big\downarrow \; 54$$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(\cdots\right.}{(a+bx\cdots}}{\cdots}}{b}\cdots\right)}{\cdots}\right.$$

$$\Big\downarrow \; 2009$$

---

3.77. $\int(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\cdots\right)}{2d}\right)}{b}}{b}\cdots\right)}{\cdots}\right)$$

$\downarrow$ 2789

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\cdots}{\cdots}\right)}{b}\cdots\right)}{\cdots}\right)$$

$\downarrow$ 2751

---

3.77. $\int(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2dx$

$$ad)^4 \left( \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{i^3(bc- \; B\left( \frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)} - \frac{B\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b} + \frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}}{b}\right.}{} \right.$$

$\downarrow$ 16

$$ad)^4 \left( \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{i^3(bc- \; B\left( \frac{\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b} + \frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)} + \frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}}{b}}{b} + - \; d\right.}{} \right.$$

$\downarrow$ 2779

---

3.77. $\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{\frac{\frac{B\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b}}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a}{c\cdot}\right.\right.}{b(c+dx)\left(b-\frac{d(}{c}\right.}\right.}{}\right.}{}\right.$$

$\downarrow$ 2838

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{\frac{B\,\text{PolyLog}\left(2,\frac{b(c+}{d(a+}\right.}{b}}{}\right.}{}\right.$$

input
```
Int[(c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2,x]
```

3.77. $\int(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx$

output
```
(b*c - a*d)^4*i^3*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(4*d*(b - (d*(a
+ b*x))/(c + d*x))^4) - (B*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])/(3*d*
(b - (d*(a + b*x))/(c + d*x))^3) - (B*(1/(2*b*(b - (d*(a + b*x))/(c + d*x)
)^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^
3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(3*d)))/b + ((d*((A + B*Log[(e*
(a + b*x))/(c + d*x)])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(1/(b*(b
 - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(
a + b*x))/(c + d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b
- (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c +
d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*
x))/(d*(a + b*x))])/b)/b)/b)/b))/(2*d))
```

### 3.77.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

## 3.77. $\int (ci + dix)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx$

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

## 3.77.4 Maple [F]

$$\int (dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2 dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2,x)
```

### 3.77.5 Fricas [F]

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\int (dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="frica
s")
```

output
```
integral(A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*c
^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2*
c^3*i^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^2
*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log((b*e*x + a*e)/(d*x + c)),
x)
```

### 3.77.6 Sympy [F(-1)]

Timed out.

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2,x)
```

output
```
Timed out
```

### 3.77.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1789 vs. 2(399) = 798.

Time = 0.29 (sec) , antiderivative size = 1789, normalized size of antiderivative = 4.26

$$\int (ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="maxim
a")
```

output
```
1/4*A^2*d^3*i^3*x^4 + A^2*c*d^2*i^3*x^3 + 3/2*A^2*c^2*d*i^3*x^2 + 2*(x*log
(b*e*x/(d*x + c) + a*e/(d*x + c)) + a*log(b*x + a)/b - c*log(d*x + c)/d)*A
*B*c^3*i^3 + 3*(x^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - a^2*log(b*x + a
)/b^2 + c^2*log(d*x + c)/d^2 - (b*c - a*d)*x/(b*d))*A*B*c^2*d*i^3 + (2*x^3
*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 2*a^3*log(b*x + a)/b^3 - 2*c^3*log
(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d
^2))*A*B*c*d^2*i^3 + 1/12*(6*x^4*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 6*
a^4*log(b*x + a)/b^4 + 6*c^4*log(d*x + c)/d^4 - (2*(b^3*c*d^2 - a*b^2*d^3)
*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3))
*A*B*d^3*i^3 + A^2*c^3*i^3*x - 1/12*(26*a*b^2*c^3*d*i^3 - 21*a^2*b*c^2*d^2
*i^3 + 6*a^3*c*d^3*i^3 + (6*i^3*log(e) - 11*i^3)*b^3*c^4)*B^2*log(d*x + c)
/(b^3*d) - 1/2*(b^4*c^4*i^3 - 4*a*b^3*c^3*d*i^3 + 6*a^2*b^2*c^2*d^2*i^3 -
4*a^3*b*c*d^3*i^3 + a^4*d^4*i^3)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*
d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^4*d) + 1/12*(3*B^2*b^4
*d^4*i^3*x^4*log(e)^2 + 2*(a*b^3*d^4*i^3*log(e) + (6*i^3*log(e)^2 - i^3*lo
g(e))*b^4*c*d^3)*B^2*x^3 + ((18*i^3*log(e)^2 - 9*i^3*log(e) + i^3)*b^4*c^2
*d^2 + 2*(6*i^3*log(e) - i^3)*a*b^3*c*d^3 - (3*i^3*log(e) - i^3)*a^2*b^2*d
^4)*B^2*x^2 + ((12*i^3*log(e)^2 - 18*i^3*log(e) + 7*i^3)*b^4*c^3*d + (36*i
^3*log(e) - 19*i^3)*a*b^3*c^2*d^2 - (24*i^3*log(e) - 17*i^3)*a^2*b^2*c*d^3
 + (6*i^3*log(e) - 5*i^3)*a^3*b*d^4)*B^2*x + 3*(B^2*b^4*d^4*i^3*x^4 + 4...
```

### 3.77.8 Giac [F]

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2,x, algorithm="giac"
)
```

output
```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2, x)
```

### 3.77.9 Mupad [F(-1)]

Timed out.

$$\int (ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 dx = \int (c\,i+d\,i\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2 dx$$

input
```
int((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2,x)
```

output
```
int((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2, x)
```

# 3.78 $\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

### 3.78.1 Optimal result

Integrand size = 42, antiderivative size = 712

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx \\
&= \frac{B^2d(bc-ad)^2i^3x}{3b^3g}+\frac{B^2(bc-ad)^3i^3\log\left(\frac{a+bx}{c+dx}\right)}{3b^4g} \\
&\quad-\frac{5Bd(bc-ad)^2i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^4g} \\
&\quad-\frac{B(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3b^2g} \\
&\quad+\frac{2B(bc-ad)^3i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g} \\
&\quad+\frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g} \\
&\quad+\frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g}+\frac{i^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3bg} \\
&\quad+\frac{2B^2(bc-ad)^3i^3\log(c+dx)}{b^4g}+\frac{5B(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g} \\
&\quad-\frac{(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&\quad+\frac{2B^2(bc-ad)^3i^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g}-\frac{5B^2(bc-ad)^3i^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g} \\
&\quad+\frac{2B(bc-ad)^3i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&\quad+\frac{2B^2(bc-ad)^3i^3\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}
\end{aligned}
$$

output
```
1/3*B^2*d*(-a*d+b*c)^2*i^3*x/b^3/g+1/3*B^2*(-a*d+b*c)^3*i^3*ln((b*x+a)/(d*
x+c))/b^4/g-5/3*B*d*(-a*d+b*c)^2*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b
^4/g-1/3*B*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^2/g+2*B*
(-a*d+b*c)^3*i^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/
g+d*(-a*d+b*c)^2*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^4/g+1/2*(-a*d
+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^2/g+1/3*i^3*(d*x+c)^3*
(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b/g+2*B^2*(-a*d+b*c)^3*i^3*ln(d*x+c)/b^4/g+5
/3*B*(-a*d+b*c)^3*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a)
)/b^4/g-(-a*d+b*c)^3*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b
*x+a))/b^4/g+2*B^2*(-a*d+b*c)^3*i^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g-5
/3*B^2*(-a*d+b*c)^3*i^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g+2*B*(-a*d+b*c
)^3*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g+2
*B^2*(-a*d+b*c)^3*i^3*polylog(3,b*(d*x+c)/d/(b*x+a))/b^4/g
```

### 3.78.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5044 vs. $2(712) = 1424$.

Time = 4.05 (sec) , antiderivative size = 5044, normalized size of antiderivative = 7.08

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x),x]
```

output
```
Result too large to show
```

### 3.78.3 Rubi [A] (verified)

Time = 3.03 (sec) , antiderivative size = 857, normalized size of antiderivative = 1.20, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.381$, Rules used = {2962, 2789, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ag+bgx}\,dx$$

$$\Big\downarrow \; 2962$$

$$\frac{i^3(bc-ad)^3 \int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \; 2789$$

$$\frac{i^3(bc-ad)^3 \left(\frac{d\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \; 2756$$

$$\frac{i^3(bc-ad)^3 \left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \; 2789$$

3.78. $\displaystyle\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{3d}\right)}{b}+\frac{d\int\frac{\left(A+B\log\left(\frac{e(a\cdots}{c+\cdots}\right.\right.}{\left(b-\frac{d(a+bx\cdots}{c+dx}\right.}}{b}\cdots\right.}{g}$$

$\downarrow$ 2756

---

3.78. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+\cdots}{c+\cdots}}{b}\cdots}{3d}\cdots\right)}{b}\cdots\right)}{g\cdots}$$

$\big\downarrow$ 54

---

3.78. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\cdots}{(a+\cdots}\cdots}{\cdots}\right)}{3d}\cdots\right)}{b}\cdots\right)$$

$\downarrow$ 2009

---

3.78. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$\frac{i^3(bc-ad)^3\left(\frac{\left(d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}\right.}{2d}\right.}{b}\right.}{3d}\right.}{b}\right.}{g}$$

$\downarrow$ 2789

---

3.78. $\int\frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{\frac{d\int\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(\frac{e(a+bx)}{c+dx}\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\cdots\right.}{\cdots}\right.}{3d}\right.}{b}\right.$$

$\downarrow$ 2751

3.78. $\displaystyle\int\frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{B\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+b}{c+d}}{b}}{b}}{3d}\right)}{b}\right.$$

$\downarrow$ 16

3.78. $\int\frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}}{b}+\cdots\right)}{3d}\cdots\right)}{b}\cdots\right)$$

$$\Big\downarrow \; 2755$$

---

3.78. $$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2B\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{B\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}}{b}+\cdots}{3d}\right)}{b}\right.$$

$\downarrow$ 2754

3.78. $\displaystyle\int\frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$(bc-ad)^3 i^3 \left( \frac{d \left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left( \frac{d\left( \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B\left( \frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)} \right)}{2d} \right)}{b} + \frac{d\left( \frac{(a+bx)\left(A+B \cdots\right.}{b(c+dx)\left(\cdots\right.} \cdots\right.}{\cdots} \cdots \right)}{3d} \cdots \right)}{b} \cdots \right.$$

$\downarrow$ 2779

---

3.78. $\displaystyle\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$(bc-ad)^3 i^3 \left( \frac{d\left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{d\left(\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b} + \frac{d\left(\frac{(a+bx)\left(A+B \cdots\right.}{b(c+dx)\left(\cdots\right.}\right.}{\cdots}\right.}{\cdots}\right.}{b \cdots} \right.$$

$\downarrow$ 2821

3.78. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$(bc-ad)^3 i^3 \left( \frac{d \left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{d\left(\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b} + \frac{d\left(\frac{(a+bx)\left(A+B \cdots\right.}{b(c+dx)\left(\cdots\right.}\right.}{\cdots}\right.}{\cdots}\right.}{b} \cdots\right.$$

↓ 2838

3.78. $\displaystyle\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

$$(bc-ad)^3 i^3 \left( \frac{d\left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left(\frac{d\left(\frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b} + \frac{d\left(\frac{(a+bx)\left(A+B \cdots\right)}{b(c+dx)\left(\cdots\right)} \cdots\right)}{\cdots}}{\cdots} \cdots\right)}{b} \cdots \right)$$

$\downarrow$ 7143

3.78. $$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$

$$(bc-ad)^3 i^3 \left( \frac{d \left( \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2B\left( \frac{d\left( \frac{A+B\log\left(\frac{e(a+bx)}{c+dx}\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{B\left( \frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)} \right)}{2d} \right)}{b} + \frac{d\left( \frac{(a+bx)\left(A+B \cdots\right.}{b(c+dx)\left(\cdots\right.} \cdots \right.}{\cdots} \cdots \right)}{\cdots} \cdots \right)}{b} \cdots \right)$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
,x]
```

## 3.78. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

output
```
((b*c - a*d)^3*i^3*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(3*d*(b - (
d*(a + b*x))/(c + d*x))^3) - (2*B*((d*((A + B*Log[(e*(a + b*x))/(c + d*x)]
)/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*(1/(b*(b - (d*(a + b*x))/(c +
 d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/
b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(b
*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*(a + b*x))/(c +
d*x)])/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c
+ d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b
)/b)/b))/(3*d)))/b + ((d*((A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(2*d*(b -
 (d*(a + b*x))/(c + d*x))^2) - (B*((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))
/(c + d*x)]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*Log[b - (d*
(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[(e*(a + b*x))/(c + d*x)]
)*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*PolyLog[2, (b*(c + d*x))/(
d*(a + b*x))])/b)/b))/d))/b + ((d*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c
+ d*x)])^2)/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (2*B*(-(((A + B*
Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (
B*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d))/b))/b + (-(((A + B*Log[(e*(
a + b*x))/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (2*B*((
A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))
] + B*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b)/b)/b)/b))/g
```

### 3.78.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

---

## 3.78. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

## 3.78. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.78.4 Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{bgx+ag}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x)
```

### 3.78.5 Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algo
rithm="fricas")
```

output
```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^
2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log((b*e*x + a*e)/(d*x + c)))
/(b*g*x + a*g), x)
```

### 3.78.6 Sympy [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx$$
$$= \frac{i^3\left(\int \frac{A^2c^3}{a+bx}\,dx + \int \frac{A^2d^3x^3}{a+bx}\,dx + \int \frac{B^2c^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a+bx}\,dx + \int \frac{2ABc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a+bx}\,dx + \int \frac{3A^2cd^2x^2}{a+bx}\,dx + \int \frac{3A^2}{a+}\right.}{}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g),x)
```

output
```
i**3*(Integral(A**2*c**3/(a + b*x), x) + Integral(A**2*d**3*x**3/(a + b*x)
, x) + Integral(B**2*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a + b*x
), x) + Integral(2*A*B*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x)
, x) + Integral(3*A**2*c*d**2*x**2/(a + b*x), x) + Integral(3*A**2*c**2*d*
x/(a + b*x), x) + Integral(B**2*d**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d
*x))**2/(a + b*x), x) + Integral(2*A*B*d**3*x**3*log(a*e/(c + d*x) + b*e*x
/(c + d*x))/(a + b*x), x) + Integral(3*B**2*c*d**2*x**2*log(a*e/(c + d*x)
+ b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(3*B**2*c**2*d*x*log(a*e/(c
+ d*x) + b*e*x/(c + d*x))**2/(a + b*x), x) + Integral(6*A*B*c*d**2*x**2*lo
g(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x) + Integral(6*A*B*c**2*d*x
*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a + b*x), x))/g
```

### 3.78.7 Maxima [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algo
rithm="maxima")
```

output
```
3*A^2*c^2*d*i^3*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - 1/6*A^2*d^3*i^3*(6*a^
3*log(b*x + a)/(b^4*g) - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/(b^3*g)) + 3/2*
A^2*c*d^2*i^3*(2*a^2*log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A^2
*c^3*i^3*log(b*g*x + a*g)/(b*g) + 1/6*(2*B^2*b^3*d^3*i^3*x^3 + 3*(3*b^3*c*
d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2*c*d^2*i^3
+ a^2*b*d^3*i^3)*B^2*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^
2*i^3 - a^3*d^3*i^3)*B^2*log(b*x + a))*log(d*x + c)^2/(b^4*g) - integrate(
-1/3*(3*B^2*b^4*c^4*i^3*log(e)^2 + 6*A*B*b^4*c^4*i^3*log(e) + 3*(B^2*b^4*d
^4*i^3*log(e)^2 + 2*A*B*b^4*d^4*i^3*log(e))*x^4 + 12*(B^2*b^4*c*d^3*i^3*lo
g(e)^2 + 2*A*B*b^4*c*d^3*i^3*log(e))*x^3 + 18*(B^2*b^4*c^2*d^2*i^3*log(e)^
2 + 2*A*B*b^4*c^2*d^2*i^3*log(e))*x^2 + 3*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4
*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b
^4*c^4*i^3)*log(b*x + a)^2 + 12*(B^2*b^4*c^3*d*i^3*log(e)^2 + 2*A*B*b^4*c^
3*d*i^3*log(e))*x + 6*(B^2*b^4*c^4*i^3*log(e) + A*B*b^4*c^4*i^3 + (B^2*b^4
*d^4*i^3*log(e) + A*B*b^4*d^4*i^3)*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e) + A*B
*b^4*c*d^3*i^3)*x^3 + 6*(B^2*b^4*c^2*d^2*i^3*log(e) + A*B*b^4*c^2*d^2*i^3)
*x^2 + 4*(B^2*b^4*c^3*d*i^3*log(e) + A*B*b^4*c^3*d*i^3)*x)*log(b*x + a) -
(6*B^2*b^4*c^4*i^3*log(e) + 6*A*B*b^4*c^4*i^3 + 2*(3*A*B*b^4*d^4*i^3 + (3*
i^3*log(e) + i^3)*B^2*b^4*d^4)*x^4 + (24*A*B*b^4*c*d^3*i^3 - (a*b^3*d^4*i^
3 - 3*(8*i^3*log(e) + 3*i^3)*b^4*c*d^3)*B^2)*x^3 + 3*(12*A*B*b^4*c^2*d^...
```

## 3.78.8 Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g),x, algo
rithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*
g), x)
```

### 3.78.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
, x)
```

# 3.79 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

### 3.79.1 Optimal result

Integrand size = 42, antiderivative size = 692

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= -\frac{2B^2(bc-ad)^2i^3(c+dx)}{b^3g^2(a+bx)} - \frac{Bd^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2}$$
$$- \frac{2B(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^2(a+bx)}$$
$$+ \frac{4Bd(bc-ad)^2i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^2}$$
$$+ \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g^2}$$
$$- \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^2(a+bx)} + \frac{di^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g^2}$$
$$+ \frac{B^2d(bc-ad)^2i^3\log(c+dx)}{b^4g^2} + \frac{Bd(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$- \frac{3d(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{4B^2d(bc-ad)^2i^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^2} - \frac{B^2d(bc-ad)^2i^3\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{6Bd(bc-ad)^2i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{6B^2d(bc-ad)^2i^3\,\mathrm{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$

output
```
-2*B^2*(-a*d+b*c)^2*i^3*(d*x+c)/b^3/g^2/(b*x+a)-B*d^2*(-a*d+b*c)*i^3*(b*x+
a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^2-2*B*(-a*d+b*c)^2*i^3*(d*x+c)*(A+B*l
n(e*(b*x+a)/(d*x+c)))/b^3/g^2/(b*x+a)+4*B*d*(-a*d+b*c)^2*i^3*ln((-a*d+b*c)
/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^2+2*d^2*(-a*d+b*c)*i^3*(b*x+
a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/b^4/g^2-(-a*d+b*c)^2*i^3*(d*x+c)*(A+B*ln(
e*(b*x+a)/(d*x+c)))^2/b^3/g^2/(b*x+a)+1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a
)/(d*x+c)))^2/b^2/g^2+B^2*d*(-a*d+b*c)^2*i^3*ln(d*x+c)/b^4/g^2+B*d*(-a*d+b
*c)^2*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^2-3*
d*(-a*d+b*c)^2*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1-b*(d*x+c)/d/(b*x+a))
/b^4/g^2+4*B^2*d*(-a*d+b*c)^2*i^3*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g^2-B
^2*d*(-a*d+b*c)^2*i^3*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^2+6*B*d*(-a*d+b
*c)^2*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g
^2+6*B^2*d*(-a*d+b*c)^2*i^3*polylog(3,b*(d*x+c)/d/(b*x+a))/b^4/g^2
```

### 3.79.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4506 vs. $2(692) = 1384$.

Time = 5.81 (sec) , antiderivative size = 4506, normalized size of antiderivative = 6.51

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^2,x]
```

output
```
(i^3*(4*A^2*b*d^2*(3*b*c - 2*a*d)*x + 2*A^2*b^2*d^3*x^2 - (4*A^2*(b*c - a*
d)^3)/(a + b*x) + 12*A^2*d*(b*c - a*d)^2*Log[a + b*x] - (8*A*b^3*B*c^3*(-(
d*(a + b*x)*Log[c/d + x]) + d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)]
+ (b*c - a*d)*(1 + Log[(e*(a + b*x))/(c + d*x)])))/((b*c - a*d)*(a + b*x))
 + (4*b^3*B^2*c^3*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a + b*x] - 2*(b*c -
a*d)*Log[(e*(a + b*x))/(c + d*x)] - 2*d*(a + b*x)*Log[a + b*x]*Log[(e*(a +
 b*x))/(c + d*x)] - (b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)]^2 + 2*d*(a +
b*x)*Log[c + d*x] - 2*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c -
a*d)/(b*c + b*d*x)] + d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(
c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + d*
(a + b*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)])))/((b*c - a*d)*(a + b*x)) + 12*A*b^2*B*c^2*d*(Log[a/b + x]^2 - 2*L
og[a/b + x]*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)
] + 2*Log[a + b*x]*((a*d)/(b*c - a*d) + Log[c/d + x] + Log[(e*(a + b*x))/(
c + d*x)]) + 2*a*((a + b*x)^(-1) + Log[(e*(a + b*x))/(c + d*x)]/(a + b*x)
+ (d*Log[c + d*x])/(-(b*c) + a*d)) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d
)]) + 4*A*B*d^3*(4*a^2 - (4*a*b*c)/d + a*b*x - (b^2*c*x)/d + (2*a^3)/(a +
b*x) + 3*a^2*Log[a/b + x]^2 + (4*a*b*c*Log[c/d + x])/d - a^2*Log[a + b*x]
+ (2*a^3*d*Log[a + b*x])/(b*c - a*d) + 6*a^2*Log[c/d + x]*Log[a + b*x] ...
```

### 3.79.3 Rubi [A] (verified)

Time = 0.92 (sec) , antiderivative size = 577, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i^3(bc-ad)^2\int \frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{i^3(bc-ad)^2\int\left(\frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3(a+bx)^2}+\frac{3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d^2\left(A+B\log\left(\frac{e(a+b}{c+d}\right.\right.}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{g^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i^3(bc-ad)^2\left(-\frac{Bd^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{6Bd\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4}\right.}{}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^2,x]
```

output
```
((b*c - a*d)^2*i^3*((-2*B^2*(c + d*x))/(b^3*(a + b*x)) - (2*B*(c + d*x)*(A
 + B*Log[(e*(a + b*x))/(c + d*x)]))/(b^3*(a + b*x)) - (B*d^2*(a + b*x)*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]))/(b^4*(c + d*x)*(b - (d*(a + b*x))/(c +
d*x))) - ((c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(b^3*(a + b*x)
) + (d*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*b^2*(b - (d*(a + b*x))/(
c + d*x))^2) + (2*d^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(b
^4*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (B^2*d*Log[b - (d*(a + b*x))
/(c + d*x)])/b^4 + (4*B*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*
(a + b*x))/(b*(c + d*x))])/b^4 + (B*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])
*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b^4 - (3*d*(A + B*Log[(e*(a + b*x))
/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b^4 + (4*B^2*d*PolyLo
g[2, (d*(a + b*x))/(b*(c + d*x))])/b^4 - (B^2*d*PolyLog[2, (b*(c + d*x))/(
d*(a + b*x))])/b^4 + (6*B*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2
, (b*(c + d*x))/(d*(a + b*x))])/b^4 + (6*B^2*d*PolyLog[3, (b*(c + d*x))/(d
*(a + b*x))])/b^4))/g^2
```

3.79. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx$

#### 3.79.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.79.4 Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx+ag)^2}\,dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x)
```

### 3.79.5 Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, al
gorithm="fricas")
```

output
```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^
2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log((b*e*x + a*e)/(d*x + c)))
/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

## 3.79.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

## 3.79.7 Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, al
gorithm="maxima")
```

output
```
-3*A^2*(a^2/(b^4*g^2*x + a*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*
g^2))*c*d^2*i^3 + 1/2*(2*a^3/(b^5*g^2*x + a*b^4*g^2) + 6*a^2*log(b*x + a)/
(b^4*g^2) + (b*x^2 - 4*a*x)/(b^3*g^2))*A^2*d^3*i^3 + 3*A^2*c^2*d*i^3*(a/(b
^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - 2*A*B*c^3*i^3*(log(b*e*x
/(d*x + c) + a*e/(d*x + c))/(b^2*g^2*x + a*b*g^2) + 1/(b^2*g^2*x + a*b*g^2
) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)
*g^2)) - A^2*c^3*i^3/(b^2*g^2*x + a*b*g^2) + 1/2*(B^2*b^3*d^3*i^3*x^3 + 3*
(2*b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 2*(3*a*b^2*c*d^2*i^3 - 2*a^2*b
*d^3*i^3)*B^2*x - 2*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^2*i^3 -
 a^3*d^3*i^3)*B^2 + 6*((b^3*c^2*d*i^3 - 2*a*b^2*c*d^2*i^3 + a^2*b*d^3*i^3)
*B^2*x + (a*b^2*c^2*d*i^3 - 2*a^2*b*c*d^2*i^3 + a^3*d^3*i^3)*B^2)*log(b*x
+ a))*log(d*x + c)^2/(b^5*g^2*x + a*b^4*g^2) - integrate(-(B^2*b^4*c^4*i^3
*log(e)^2 + (B^2*b^4*d^4*i^3*log(e)^2 + 2*A*B*b^4*d^4*i^3*log(e))*x^4 + 4*
(B^2*b^4*c*d^3*i^3*log(e)^2 + 2*A*B*b^4*c*d^3*i^3*log(e))*x^3 + 6*(B^2*b^4
*c^2*d^2*i^3*log(e)^2 + 2*A*B*b^4*c^2*d^2*i^3*log(e))*x^2 + (B^2*b^4*d^4*i
^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c
^3*d*i^3*x + B^2*b^4*c^4*i^3)*log(b*x + a)^2 + 2*(2*B^2*b^4*c^3*d*i^3*log(
e)^2 + 3*A*B*b^4*c^3*d*i^3*log(e))*x + 2*(B^2*b^4*c^4*i^3*log(e) + (B^2*b^
4*d^4*i^3*log(e) + A*B*b^4*d^4*i^3)*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e) + A*
B*b^4*c*d^3*i^3)*x^3 + 6*(B^2*b^4*c^2*d^2*i^3*log(e) + A*B*b^4*c^2*d^2*...
```

## 3.79.8 Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2,x, al
gorithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*
g)^2, x)
```

### 3.79.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{{\left(c\,i+d\,i\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}{{\left(a\,g+b\,g\,x\right)}^2}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^2,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^2, x)
```

# 3.80 $\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

### 3.80.1 Optimal result

Integrand size = 42, antiderivative size = 604

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\frac{4B^2d(bc-ad)i^3(c+dx)}{b^3g^3(a+bx)} - \frac{B^2(bc-ad)i^3(c+dx)^2}{4b^2g^3(a+bx)^2}$$
$$- \frac{4Bd(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^3g^3(a+bx)}$$
$$- \frac{B(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2b^2g^3(a+bx)^2}$$
$$+ \frac{2Bd^2(bc-ad)i^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{b^4g^3}$$
$$+ \frac{d^3i^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^4g^3} - \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b^3g^3(a+bx)}$$
$$- \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2b^2g^3(a+bx)^2}$$
$$- \frac{3d^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$
$$+ \frac{2B^2d^2(bc-ad)i^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^3}$$
$$+ \frac{6Bd^2(bc-ad)i^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$
$$+ \frac{6B^2d^2(bc-ad)i^3\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$

output
```
-4*B^2*d*(-a*d+b*c)*i^3*(d*x+c)/b^3/g^3/(b*x+a)-1/4*B^2*(-a*d+b*c)*i^3*(d*
x+c)^2/b^2/g^3/(b*x+a)^2-4*B*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d
*x+c)))/b^3/g^3/(b*x+a)-1/2*B*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(
d*x+c)))/b^2/g^3/(b*x+a)^2+2*B*d^2*(-a*d+b*c)*i^3*ln((-a*d+b*c)/b/(d*x+c))
*(A+B*ln(e*(b*x+a)/(d*x+c)))/b^4/g^3+d^3*i^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*
x+c)))^2/b^4/g^3-2*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/
b^3/g^3/(b*x+a)-1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2
/b^2/g^3/(b*x+a)^2-3*d^2*(-a*d+b*c)*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2*ln(1
-b*(d*x+c)/d/(b*x+a))/b^4/g^3+2*B^2*d^2*(-a*d+b*c)*i^3*polylog(2,d*(b*x+a)
/b/(d*x+c))/b^4/g^3+6*B*d^2*(-a*d+b*c)*i^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*pol
ylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^3+6*B^2*d^2*(-a*d+b*c)*i^3*polylog(3,b*(
d*x+c)/d/(b*x+a))/b^4/g^3
```

### 3.80.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5989 vs. $2(604) = 1208$.

Time = 6.89 (sec) , antiderivative size = 5989, normalized size of antiderivative = 9.92

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^3,x]
```

output
```
Result too large to show
```

### 3.80.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 486, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3}\,dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i^3(bc-ad)\int \frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{i^3(bc-ad)\int \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d^3}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d^2}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d}{b^3(a+bx)^2}+\frac{(c+dx)^3\left(A+B\log\right.}{b^2(a+b}\right.}{g^3}$$

$$\Big\downarrow \text{2009}$$

$$i^3(bc-ad)\left(\frac{d^3(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{6Bd^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{b^4}+\frac{2Bd^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+b}{c+dx}\right.\right.}{b^4}\right.$$

input

```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^3,x]
```

output

```
((b*c - a*d)*i^3*((-4*B^2*d*(c + d*x))/(b^3*(a + b*x)) - (B^2*(c + d*x)^2)
/(4*b^2*(a + b*x)^2) - (4*B*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)
]))/(b^3*(a + b*x)) - (B*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(2*b^2*(a + b*x)^2) - (2*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^2)/(b^3*(a + b*x)) - ((c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)
/(2*b^2*(a + b*x)^2) + (d^3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^2)/(b^4*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (2*B*d^2*(A + B*Log[(e
*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/b^4 - (3*d^2
*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x)
)])/b^4 + (2*B^2*d^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/b^4 + (6*B*d
^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (b*(c + d*x))/(d*(a + b
*x))])/b^4 + (6*B^2*d^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/b^4))/g^3
```

## 3.80. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$

### 3.80.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.80.4 Maple [F]

$$\int \frac{(dix+ci)^3\left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(bgx+ag)^3}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x)
```

## 3.80.5 Fricas [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, al
gorithm="fricas")
```

output
```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d^
2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log((b*e*x + a*e)/(d*x + c)))
/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

### 3.80.6 Sympy [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{i^3\left(\int \frac{A^2c^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{A^2d^3x^3}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{B^2c^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2ABc^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\right.}{}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3,x)
```

output
```
i**3*(Integral(A**2*c**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
x) + Integral(A**2*d**3*x**3/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**
3), x) + Integral(B**2*c**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(a**3
+ 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*c**3*log(a*
e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x
**3), x) + Integral(3*A**2*c*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2
+ b**3*x**3), x) + Integral(3*A**2*c**2*d*x/(a**3 + 3*a**2*b*x + 3*a*b**2*
x**2 + b**3*x**3), x) + Integral(B**2*d**3*x**3*log(a*e/(c + d*x) + b*e*x/
(c + d*x))**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integr
al(2*A*B*d**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x
 + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(3*B**2*c*d**2*x**2*log(a*e/(c
 + d*x) + b*e*x/(c + d*x))**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x*
*3), x) + Integral(3*B**2*c**2*d*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2
/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(6*A*B*c*d*
*2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2
*x**2 + b**3*x**3), x) + Integral(6*A*B*c**2*d*x*log(a*e/(c + d*x) + b*e*x
/(c + d*x))/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

### 3.80.7 Maxima [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, al
gorithm="maxima")
```

output
```
-3/2*A*B*c^2*d*i^3*(2*(2*b*x + a)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^
4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) + (3*a*b*c - a^2*d + 2*(2*b^2*c -
 a*b*d)*x)/((b^5*c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a
^2*b^3*c - a^3*b^2*d)*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 -
2*a*b^3*c*d + a^2*b^2*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c
^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3)) - 1/2*A^2*d^3*i^3*((6*a^2*b*x + 5*a^
3)/(b^6*g^3*x^2 + 2*a*b^5*g^3*x + a^2*b^4*g^3) - 2*x/(b^3*g^3) + 6*a*log(b
*x + a)/(b^4*g^3)) + 3/2*A^2*c*d^2*i^3*((4*a*b*x + 3*a^2)/(b^5*g^3*x^2 + 2
*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) + 1/2*A*B*c^3*i^3*
((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2
*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x
+ c))/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 2*d^2*log(b*x + a)/((b^3
*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*c^2 - 2*a*
b^2*c*d + a^2*b*d^2)*g^3)) - 3/2*(2*b*x + a)*A^2*c^2*d*i^3/(b^4*g^3*x^2 +
2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*A^2*c^3*i^3/(b^3*g^3*x^2 + 2*a*b^2*g^3*
x + a^2*b*g^3) + 1/2*(2*B^2*b^3*d^3*i^3*x^3 + 4*B^2*a*b^2*d^3*i^3*x^2 - 2*
(3*b^3*c^2*d*i^3 - 6*a*b^2*c*d^2*i^3 + 2*a^2*b*d^3*i^3)*B^2*x - (b^3*c^3*i
^3 + 3*a*b^2*c^2*d*i^3 - 9*a^2*b*c*d^2*i^3 + 5*a^3*d^3*i^3)*B^2 + 6*((b^3*
c*d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 2*(a*b^2*c*d^2*i^3 - a^2*b*d^3*i^3)*B
^2*x + (a^2*b*c*d^2*i^3 - a^3*d^3*i^3)*B^2)*log(b*x + a))*log(d*x + c)^...
```

### 3.80.8 Giac [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3,x, al
gorithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(b*g*x + a*
g)^3, x)
```

### 3.80.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{\left(c\,i+d\,i\,x\right)^3\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}{{\left(a\,g+b\,g\,x\right)}^3}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^3,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^3, x)
```

# 3.81 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$



## 3.81.1 Optimal result

Integrand size = 42, antiderivative size = 147

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{B^2i^3(c+dx)^4}{32(bc-ad)g^5(a+bx)^4} - \frac{Bi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)g^5(a+bx)^4} - \frac{i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)g^5(a+bx)^4}$$

output
```
-1/32*B^2*i^3*(d*x+c)^4/(-a*d+b*c)/g^5/(b*x+a)^4-1/8*B*i^3*(d*x+c)^4*(A+B*
ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)/g^5/(b*x+a)^4-1/4*i^3*(d*x+c)^4*(A+B*ln(
e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)/g^5/(b*x+a)^4
```

### 3.81.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.71 (sec) , antiderivative size = 2401, normalized size of antiderivative = 16.33

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^5,x]
```

output
```
-1/4*((b*c - a*d)^3*i^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(b^4*g^5*(
a + b*x)^4) - (d*(b*c - a*d)^2*i^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)
/(b^4*g^5*(a + b*x)^3) - (3*d^2*(b*c - a*d)*i^3*(A + B*Log[(e*(a + b*x))/(
c + d*x)])^2)/(2*b^4*g^5*(a + b*x)^2) - (d^3*i^3*(A + B*Log[(e*(a + b*x))/
(c + d*x)])^2)/(b^4*g^5*(a + b*x)) + (B*(b*c - a*d)^4*i^3*(-1/4*A/((b*c -
a*d)*(a + b*x)^4) - B/(16*(b*c - a*d)*(a + b*x)^4) + (A*d)/(3*(b*c - a*d)^
2*(a + b*x)^3) + (7*B*d)/(36*(b*c - a*d)^2*(a + b*x)^3) - (A*d^2)/(2*(b*c
- a*d)^3*(a + b*x)^2) - (13*B*d^2)/(24*(b*c - a*d)^3*(a + b*x)^2) + (A*d^3
)/((b*c - a*d)^4*(a + b*x)) + (25*B*d^3)/(12*(b*c - a*d)^4*(a + b*x)) + (A
*d^4*Log[a + b*x])/(b*c - a*d)^5 + (25*B*d^4*Log[a + b*x])/(12*(b*c - a*d)
^5) - (B*d^4*Log[a + b*x]^2)/(2*(b*c - a*d)^5) - (B*Log[(e*(a + b*x))/(c +
 d*x)])/(4*(b*c - a*d)*(a + b*x)^4) + (B*d*Log[(e*(a + b*x))/(c + d*x)])/(
3*(b*c - a*d)^2*(a + b*x)^3) - (B*d^2*Log[(e*(a + b*x))/(c + d*x)])/(2*(b*
c - a*d)^3*(a + b*x)^2) + (B*d^3*Log[(e*(a + b*x))/(c + d*x)])/((b*c - a*d
)^4*(a + b*x)) + (B*d^4*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x)])/(b*c -
a*d)^5 - (A*d^4*Log[c + d*x])/(b*c - a*d)^5 - (25*B*d^4*Log[c + d*x])/(12*
(b*c - a*d)^5) + (B*d^4*Log[-((d*(a + b*x))/(b*c - a*d))]*Log[c + d*x])/(b
*c - a*d)^5 - (B*d^4*Log[(e*(a + b*x))/(c + d*x)]*Log[c + d*x])/(b*c - a*d
)^5 - (B*d^4*Log[c + d*x]^2)/(2*(b*c - a*d)^5) + (B*d^4*Log[a + b*x]*Log[(
b*(c + d*x))/(b*c - a*d)])/(b*c - a*d)^5 + (B*d^4*PolyLog[2, -((d*(a + ...
```

### 3.81.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2962, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^5}dx$$

$$\Big\downarrow \ 2962$$

$$\frac{i^3 \int \frac{(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)}$$

$$\Big\downarrow \ 2742$$

$$\frac{i^3\left(\frac{1}{2}B\int \frac{(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}\right)}{g^5(bc-ad)}$$

$$\Big\downarrow \ 2741$$

$$\frac{i^3\left(\frac{1}{2}B\left(-\frac{(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{4(a+bx)^4}-\frac{B(c+dx)^4}{16(a+bx)^4}\right)-\frac{(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}\right)}{g^5(bc-ad)}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^5,x]
```

output
```
(i^3*(-1/4*((c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x)^
4 + (B*(-1/16*(B*(c + d*x)^4)/(a + b*x)^4 - ((c + d*x)^4*(A + B*Log[(e*(a
+ b*x))/(c + d*x)]))/(4*(a + b*x)^4)))/2))/((b*c - a*d)*g^5)
```

### 3.81.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.81.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(141) = 282$.

Time = 1.65 (sec) , antiderivative size = 367, normalized size of antiderivative = 2.50

| method | result |
|---|---|
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{i^3d^2e^3A^2}{4(ad-cb)^2g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{2i^3d^2e^3AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}\right)}{(ad-cb)^2g^5}+\frac{i^3d^2e^3B^2\left(-\frac{\ln}{4}\cdots\right.}{\cdots}\cdots\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{i^3d^2e^3A^2}{4(ad-cb)^2g^5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}+\frac{2i^3d^2e^3AB\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{1}{16\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}\right)}{(ad-cb)^2g^5}+\frac{i^3d^2e^3B^2\left(-\frac{\ln}{4}\cdots\right.}{\cdots}\cdots\right.}{d^2}$ |
| parts | $\frac{i^3A^2\left(-\frac{d\left(a^2d^2-2abcd+b^2c^2\right)}{b^4(bx+a)^3}+\frac{3d^2(ad-cb)}{2b^4(bx+a)^2}-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{4b^4(bx+a)^4}-\frac{d^3}{b^4(bx+a)}\right)}{g^5}-\frac{i^3B^2e^4\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c}\cdots\right)}{4\left(\frac{be}{d}+\frac{(ad-cb)}{d(dx+c)}\cdots\right.}\cdots\right.}{\cdots}$ |
| risch | $-\frac{i^3A^2d^3a^2}{g^5b^4(bx+a)^3}+\frac{2i^3A^2d^2ac}{g^5b^3(bx+a)^3}-\frac{i^3A^2d\,c^2}{g^5b^2(bx+a)^3}+\frac{3i^3A^2d^3a}{2g^5b^4(bx+a)^2}-\frac{3i^3A^2d^2c}{2g^5b^3(bx+a)^2}+\frac{i^3A^2a^3d^3}{4g^5b^4(bx+a)^4}-\frac{3i^3A^2\cdots}{4g^5b^3(b\cdots}$ |
| norman | $\frac{B^2c\,d^3i^3x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{(ad-cb)g}+\frac{B^2c^3d\,i^3x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{g(ad-cb)}+\frac{\left(8i^3A^2c^3+4i^3c^3BA+i^3c^3B^2\right)x}{8ga}+\frac{3\left(8A^2a\,c^2d\,i^3+8A^2b\,c^3i^3+4ABa\,c^2d\cdots\right.}{16\cdots}$ |
| parallelrisch | $\frac{B^2x^4a^6c\,d^4i^3+64AB\,x^3\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c^2d^3i^3+96AB\,x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c^3d^2i^3+64ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^6c^4d\,i^3+16AB\,x\cdots}{\cdots}$ |

input

```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x,method=_RE
TURNVERBOSE)
```

output

```
-1/d^2*e*(a*d-b*c)*(-1/4*i^3*d^2*e^3/(a*d-b*c)^2/g^5*A^2/(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^4+2*i^3*d^2*e^3/(a*d-b*c)^2/g^5*A*B*(-1/4/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*
x+c))^4)+i^3*d^2*e^3/(a*d-b*c)^2/g^5*B^2*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

## 3.81. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

### 3.81.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 559 vs. $2(141) = 282$.

Time = 0.31 (sec) , antiderivative size = 559, normalized size of antiderivative = 3.80

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = -\frac{4\left((8A^2+4AB+B^2)b^4cd^3-(8A^2+4AB+B^2)ab^3d^4\right)i^3x^3+6\left((8A^2+4AB+B^2)b^4c^2d^2-(8A^2+\right.}{}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, al
gorithm="fricas")
```

output
```
-1/32*(4*((8*A^2 + 4*A*B + B^2)*b^4*c*d^3 - (8*A^2 + 4*A*B + B^2)*a*b^3*d^
4)*i^3*x^3 + 6*((8*A^2 + 4*A*B + B^2)*b^4*c^2*d^2 - (8*A^2 + 4*A*B + B^2)*
a^2*b^2*d^4)*i^3*x^2 + 4*((8*A^2 + 4*A*B + B^2)*b^4*c^3*d - (8*A^2 + 4*A*B
 + B^2)*a^3*b*d^4)*i^3*x + ((8*A^2 + 4*A*B + B^2)*b^4*c^4 - (8*A^2 + 4*A*B
 + B^2)*a^4*d^4)*i^3 + 8*(B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 +
6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b^4*c^4*i^3)*log((
b*e*x + a*e)/(d*x + c))^2 + 4*((4*A*B + B^2)*b^4*d^4*i^3*x^4 + 4*(4*A*B +
B^2)*b^4*c*d^3*i^3*x^3 + 6*(4*A*B + B^2)*b^4*c^2*d^2*i^3*x^2 + 4*(4*A*B +
B^2)*b^4*c^3*d*i^3*x + (4*A*B + B^2)*b^4*c^4*i^3)*log((b*e*x + a*e)/(d*x +
 c)))/((b^9*c - a*b^8*d)*g^5*x^4 + 4*(a*b^8*c - a^2*b^7*d)*g^5*x^3 + 6*(a^
2*b^7*c - a^3*b^6*d)*g^5*x^2 + 4*(a^3*b^6*c - a^4*b^5*d)*g^5*x + (a^4*b^5*
c - a^5*b^4*d)*g^5)
```

### 3.81.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**5,x)
```

output
```
Timed out
```

### 3.81.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 11688 vs. $2(141) = 282$.

Time = 1.03 (sec) , antiderivative size = 11688, normalized size of antiderivative = 79.51

$$\int \frac{(ci + dix)^3 \left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^5} dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, al
gorithm="maxima")
```

output
```
-1/4*(4*b*x + a)*B^2*c^2*d*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^6
*g^5*x^4 + 4*a*b^5*g^5*x^3 + 6*a^2*b^4*g^5*x^2 + 4*a^3*b^3*g^5*x + a^4*b^2
*g^5) - 1/4*(6*b^2*x^2 + 4*a*b*x + a^2)*B^2*c*d^2*i^3*log(b*e*x/(d*x + c)
+ a*e/(d*x + c))^2/(b^7*g^5*x^4 + 4*a*b^6*g^5*x^3 + 6*a^2*b^5*g^5*x^2 + 4*
a^3*b^4*g^5*x + a^4*b^3*g^5) - 1/4*(4*b^3*x^3 + 6*a*b^2*x^2 + 4*a^2*b*x +
a^3)*B^2*d^3*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^8*g^5*x^4 + 4*a
*b^7*g^5*x^3 + 6*a^2*b^6*g^5*x^2 + 4*a^3*b^5*g^5*x + a^4*b^4*g^5) + 1/288*
(12*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^2 + 25*a^
3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*c*d^2 + 1
3*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3*b^5*d^3)
*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4*b^4*d^3)
*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a^5*b^3*d^
3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 - a^6*b^2*
d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a^7*b*d^3)
*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2
- 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c^4 - 4*a*
b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5))*log(b*e
*x/(d*x + c) + a*e/(d*x + c)) - (9*b^4*c^4 - 64*a*b^3*c^3*d + 216*a^2*b^2*
c^2*d^2 - 576*a^3*b*c*d^3 + 415*a^4*d^4 - 300*(b^4*c*d^3 - a*b^3*d^4)*x^3
+ 6*(13*b^4*c^2*d^2 - 176*a*b^3*c*d^3 + 163*a^2*b^2*d^4)*x^2 + 72*(b^4*...
```

### 3.81.8 Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 218, normalized size of antiderivative = 1.48

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{32}\left(\frac{8\,(dx+c)^4B^2e^5i^3\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)^4g^5}+\frac{4\,(4\,ABe^5i^3+B^2e^5i^3)(dx+c)^4\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^4g^5}+\frac{(8\,A^2e^5i^3+4\,AB\right.$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^5,x, al
gorithm="giac")
```

output
```
-1/32*(8*(d*x + c)^4*B^2*e^5*i^3*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x +
a*e)^4*g^5) + 4*(4*A*B*e^5*i^3 + B^2*e^5*i^3)*(d*x + c)^4*log((b*e*x + a*e
)/(d*x + c))/((b*e*x + a*e)^4*g^5) + (8*A^2*e^5*i^3 + 4*A*B*e^5*i^3 + B^2*
e^5*i^3)*(d*x + c)^4/((b*e*x + a*e)^4*g^5))*(b*c/((b*c*e - a*d*e)*(b*c - a
*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.81.9 Mupad [B] (verification not implemented)

Time = 6.02 (sec) , antiderivative size = 1565, normalized size of antiderivative = 10.65

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^5,x)
```

output
```
-(24*A^2*a^4*d^4*i^3 - 24*A^2*b^4*c^4*i^3 + 3*B^2*a^4*d^4*i^3 - 3*B^2*b^4*
c^4*i^3 + 12*A*B*a^4*d^4*i^3 - 12*A*B*b^4*c^4*i^3 - 24*B^2*b^4*c^4*i^3*log
((e*(a + b*x))/(c + d*x))^2 + B^2*a^4*d^4*i^3*atan((a*d*1i + b*c*1i + b*d*
x*2i)/(a*d - b*c))*24i + 12*B^2*a^4*d^4*i^3*log((e*(a + b*x))/(c + d*x)) -
 12*B^2*b^4*c^4*i^3*log((e*(a + b*x))/(c + d*x)) - 24*B^2*b^4*d^4*i^3*x^4*
log((e*(a + b*x))/(c + d*x))^2 + 96*A^2*a^3*b*d^4*i^3*x + 12*B^2*a^3*b*d^4
*i^3*x - 96*A^2*b^4*c^3*d*i^3*x - 12*B^2*b^4*c^3*d*i^3*x + B^2*b^4*d^4*i^3
*x^4*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*24i + A*B*a^4*d^4*i^3*
atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*96i + 96*A^2*a*b^3*d^4*i^3*
x^3 + 12*B^2*a*b^3*d^4*i^3*x^3 - 96*A^2*b^4*c*d^3*i^3*x^3 - 12*B^2*b^4*c*d
^3*i^3*x^3 + 48*A*B*a^4*d^4*i^3*log((e*(a + b*x))/(c + d*x)) - 48*A*B*b^4*
c^4*i^3*log((e*(a + b*x))/(c + d*x)) + 144*A^2*a^2*b^2*d^4*i^3*x^2 + 18*B^
2*a^2*b^2*d^4*i^3*x^2 - 144*A^2*b^4*c^2*d^2*i^3*x^2 - 18*B^2*b^4*c^2*d^2*i
^3*x^2 + 48*A*B*a^3*b*d^4*i^3*x - 48*A*B*b^4*c^3*d*i^3*x + 48*B^2*a*b^3*d^
4*i^3*x^3*log((e*(a + b*x))/(c + d*x)) - 96*B^2*b^4*c^3*d*i^3*x*log((e*(a
+ b*x))/(c + d*x))^2 - 48*B^2*b^4*c*d^3*i^3*x^3*log((e*(a + b*x))/(c + d*x
)) + A*B*b^4*d^4*i^3*x^4*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*96
i + B^2*a^3*b*d^4*i^3*x*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*96i
 + 48*A*B*a*b^3*d^4*i^3*x^3 - 48*A*B*b^4*c*d^3*i^3*x^3 + 72*B^2*a^2*b^2*d^
4*i^3*x^2*log((e*(a + b*x))/(c + d*x)) - 72*B^2*b^4*c^2*d^2*i^3*x^2*log...
```

3.81. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^5}\,dx$

# 3.82 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$



## 3.82.1 Optimal result

Integrand size = 42, antiderivative size = 299

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \frac{B^2di^3(c+dx)^4}{32(bc-ad)^2g^6(a+bx)^4} - \frac{2bB^2i^3(c+dx)^5}{125(bc-ad)^2g^6(a+bx)^5} + \frac{Bdi^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^2g^6(a+bx)^4} - \frac{2bBi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{25(bc-ad)^2g^6(a+bx)^5} + \frac{di^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^2g^6(a+bx)^4} - \frac{bi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^2g^6(a+bx)^5}$$

output
```
1/32*B^2*d*i^3*(d*x+c)^4/(-a*d+b*c)^2/g^6/(b*x+a)^4-2/125*b*B^2*i^3*(d*x+c
)^5/(-a*d+b*c)^2/g^6/(b*x+a)^5+1/8*B*d*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*
x+c)))/(-a*d+b*c)^2/g^6/(b*x+a)^4-2/25*b*B*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)
/(d*x+c)))/(-a*d+b*c)^2/g^6/(b*x+a)^5+1/4*d*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a
)/(d*x+c)))^2/(-a*d+b*c)^2/g^6/(b*x+a)^4-1/5*b*i^3*(d*x+c)^5*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g^6/(b*x+a)^5
```

### 3.82.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.53 (sec) , antiderivative size = 2456, normalized size of antiderivative = 8.21

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^6,x]
```

output
```
-1/36000*(i^3*(7200*(b*c - a*d)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 +
 27000*d*(b*c - a*d)^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 -
36000*d^2*(-(b*c) + a*d)^3*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
)^2 + 18000*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2 + 2000*B*d^2*(a + b*x)^2*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3 -
18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*(a + b*x) + 36*A*d^2
*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*(a + b*x)^2 + 36*A*d^3*(a
+ b*x)^3*Log[a + b*x] + 66*B*d^3*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*(a +
b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)] -
18*B*d*(b*c - a*d)^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^2*(b*
c - a*d)*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)] + 36*B*d^3*(a + b*x)^3*L
og[a + b*x]*Log[(e*(a + b*x))/(c + d*x)] - 36*A*d^3*(a + b*x)^3*Log[c + d*
x] - 66*B*d^3*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*(a + b*x)^3*Log[(d*(a +
b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[(e*(a + b*x)
)/(c + d*x)]*Log[c + d*x] - 18*B*d^3*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3
*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*(a + b
*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*(a + b*x)^3*Poly
Log[2, (b*(c + d*x))/(b*c - a*d)]) + 375*B*d*(a + b*x)*(36*A*(b*c - a*d)^4
 + 9*B*(b*c - a*d)^4 + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c)
+ a*d)^3*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c...
```

### 3.82.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^6}\,dx$$

$$\Big\downarrow \text{ 2962}$$

$$\frac{i^3 \int \frac{(c+dx)^6\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^6}d\frac{a+bx}{c+dx}}{g^6(bc-ad)^2}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{i^3 \int \left(\frac{b(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^6}-\frac{d(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^5}\right)d\frac{a+bx}{c+dx}}{g^6(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i^3\left(-\frac{b(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{5(a+bx)^5}-\frac{2bB(c+dx)^5\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{25(a+bx)^5}+\frac{d(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}+\frac{Bd(c+dx)^4\left(B\log\left(\frac{e(a+}{c+d}\right.\right.}{8(a+bx)^4}\right.}{g^6(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^6,x]
```

output
```
(i^3*((B^2*d*(c + d*x)^4)/(32*(a + b*x)^4) - (2*b*B^2*(c + d*x)^5)/(125*(a
 + b*x)^5) + (B*d*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(a
+ b*x)^4) - (2*b*B*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(25*(
a + b*x)^5) + (d*(c + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*(a
 + b*x)^4) - (b*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*(a
+ b*x)^5)))/((b*c - a*d)^2*g^6)
```

#### 3.82.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.82.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(287) = 574$.

Time = 1.86 (sec) , antiderivative size = 722, normalized size of antiderivative = 2.41

| method | result |
|---|---|
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{i^3d^2e^4A^2b}{5(ad-cb)^3g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{i^3d^3e^3A^2}{4(ad-cb)^3g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{2i^3d^2e^4ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{}{25\left(\frac{be}{d}+\cdots\right.}\cdots\right.}{(ad-cb)^3g^6}\cdots\right.}{\cdots}$ |
| default | $-\frac{e(ad-cb)\left(\frac{i^3d^2e^4A^2b}{5(ad-cb)^3g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{i^3d^3e^3A^2}{4(ad-cb)^3g^6\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^4}-\frac{2i^3d^2e^4ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{5\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^5}-\frac{}{25\left(\frac{be}{d}+\cdots\right.}\cdots\right.}{(ad-cb)^3g^6}\cdots\right.}{\cdots}$ |
| parts | $\frac{i^3A^2\left(-\frac{-a^3d^3+3a^2bc\,d^2-3a\,b^2c^2d+b^3c^3}{5b^4(bx+a)^5}+\frac{d^2(ad-cb)}{b^4(bx+a)^3}-\frac{d^3}{2b^4(bx+a)^2}-\frac{3d\left(a^2d^2-2abcd+b^2c^2\right)}{4b^4(bx+a)^4}\right)}{g^6}-\frac{i^3B^2(ad-cb)^4e^4\left(\frac{d^6\left(-\cdots\right.}{\cdots}\cdots\right.}{\cdots}$ |
| norman | Expression too large to display |
| parallelrisch | Expression too large to display |
| risch | Expression too large to display |

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x,method=_RE
TURNVERBOSE)
```

output
```
-1/d^2*e*(a*d-b*c)*(1/5*i^3*d^2*e^4/(a*d-b*c)^3/g^6*A^2*b/(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^5-1/4*i^3*d^3*e^3/(a*d-b*c)^3/g^6*A^2/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^4-2*i^3*d^2*e^4/(a*d-b*c)^3/g^6*A*B*b*(-1/5/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))^5)+2*i^3*d^3*e^3/(a*d-b*c)^3/g^6*A*B*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)-
i^3*d^2*e^4/(a*d-b*c)^3/g^6*B^2*b*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*ln
(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-2/125/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5)+i^3*d^3*
e^3/(a*d-b*c)^3/g^6*B^2*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^2-1/8/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*
c)*e/d/(d*x+c))-1/32/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4))
```

## 3.82. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$

### 3.82.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1045 vs. $2(287) = 574$.

Time = 0.34 (sec) , antiderivative size = 1045, normalized size of antiderivative = 3.49

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$$
$$= \frac{20\,((20\,AB+9\,B^2)b^5cd^4-(20\,AB+9\,B^2)ab^4d^5)i^3x^4-10\,((200\,A^2+20\,AB-11\,B^2)b^5c^2d^3-50\,(8\,A^2+\ldots}{}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, al
gorithm="fricas")
```

output
```
1/4000*(20*((20*A*B + 9*B^2)*b^5*c*d^4 - (20*A*B + 9*B^2)*a*b^4*d^5)*i^3*x
^4 - 10*((200*A^2 + 20*A*B - 11*B^2)*b^5*c^2*d^3 - 50*(8*A^2 + 4*A*B + B^2
)*a*b^4*c*d^4 + (200*A^2 + 180*A*B + 61*B^2)*a^2*b^3*d^5)*i^3*x^3 - 10*(2*
(200*A^2 + 60*A*B + 7*B^2)*b^5*c^3*d^2 - 75*(8*A^2 + 4*A*B + B^2)*a*b^4*c^
2*d^3 + (200*A^2 + 180*A*B + 61*B^2)*a^3*b^2*d^5)*i^3*x^2 - 5*((600*A^2 +
220*A*B + 39*B^2)*b^5*c^4*d - 100*(8*A^2 + 4*A*B + B^2)*a*b^4*c^3*d^2 + (2
00*A^2 + 180*A*B + 61*B^2)*a^4*b*d^5)*i^3*x - (32*(25*A^2 + 10*A*B + 2*B^2
)*b^5*c^5 - 125*(8*A^2 + 4*A*B + B^2)*a*b^4*c^4*d + (200*A^2 + 180*A*B + 6
1*B^2)*a^5*d^5)*i^3 + 200*(B^2*b^5*d^5*i^3*x^5 + 5*B^2*a*b^4*d^5*i^3*x^4 -
 10*(B^2*b^5*c^2*d^3 - 2*B^2*a*b^4*c*d^4)*i^3*x^3 - 10*(2*B^2*b^5*c^3*d^2
- 3*B^2*a*b^4*c^2*d^3)*i^3*x^2 - 5*(3*B^2*b^5*c^4*d - 4*B^2*a*b^4*c^3*d^2)
*i^3*x - (4*B^2*b^5*c^5 - 5*B^2*a*b^4*c^4*d)*i^3)*log((b*e*x + a*e)/(d*x +
 c))^2 + 20*((20*A*B + 9*B^2)*b^5*d^5*i^3*x^5 + 5*(4*B^2*b^5*c*d^4 + 5*(4*
A*B + B^2)*a*b^4*d^5)*i^3*x^4 - 10*((20*A*B + B^2)*b^5*c^2*d^3 - 10*(4*A*B
 + B^2)*a*b^4*c*d^4)*i^3*x^3 - 10*(2*(20*A*B + 3*B^2)*b^5*c^3*d^2 - 15*(4*
A*B + B^2)*a*b^4*c^2*d^3)*i^3*x^2 - 5*((60*A*B + 11*B^2)*b^5*c^4*d - 20*(4
*A*B + B^2)*a*b^4*c^3*d^2)*i^3*x - (16*(5*A*B + B^2)*b^5*c^5 - 25*(4*A*B +
 B^2)*a*b^4*c^4*d)*i^3)*log((b*e*x + a*e)/(d*x + c)))/((b^11*c^2 - 2*a*b^1
0*c*d + a^2*b^9*d^2)*g^6*x^5 + 5*(a*b^10*c^2 - 2*a^2*b^9*c*d + a^3*b^8*d^2
)*g^6*x^4 + 10*(a^2*b^9*c^2 - 2*a^3*b^8*c*d + a^4*b^7*d^2)*g^6*x^3 + 10...
```

### 3.82.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**6,x)
```

output
```
Timed out
```

### 3.82.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 15765 vs. $2(287) = 574$.

Time = 1.50 (sec) , antiderivative size = 15765, normalized size of antiderivative = 52.73

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, al
gorithm="maxima")
```

output
```
-3/20*(5*b*x + a)*B^2*c^2*d*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^
7*g^6*x^5 + 5*a*b^6*g^6*x^4 + 10*a^2*b^5*g^6*x^3 + 10*a^3*b^4*g^6*x^2 + 5*
a^4*b^3*g^6*x + a^5*b^2*g^6) - 1/10*(10*b^2*x^2 + 5*a*b*x + a^2)*B^2*c*d^2
*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^8*g^6*x^5 + 5*a*b^7*g^6*x^4
 + 10*a^2*b^6*g^6*x^3 + 10*a^3*b^5*g^6*x^2 + 5*a^4*b^4*g^6*x + a^5*b^3*g^6
) - 1/20*(10*b^3*x^3 + 10*a*b^2*x^2 + 5*a^2*b*x + a^3)*B^2*d^3*i^3*log(b*e
*x/(d*x + c) + a*e/(d*x + c))^2/(b^9*g^6*x^5 + 5*a*b^8*g^6*x^4 + 10*a^2*b^
7*g^6*x^3 + 10*a^3*b^6*g^6*x^2 + 5*a^4*b^5*g^6*x + a^5*b^4*g^6) - 1/9000*(
60*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a^2*b^2*c^2*d^2 -
163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^4)*x^3 + 10*(2*b
^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b^4*c^3*d - 17*a*
b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c^4 - 4*a*b^9*c^3
*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^6*x^5 + 5*(a*b^9
*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d^3 + a^5*b^5*d^4
)*g^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*c^2*d^2 - 4*a^5*
b^5*c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4*b^6*c^3*d + 6*a
^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2 + 5*(a^4*b^6*c^4 -
 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a^8*b^2*d^4)*g^6*
x + (a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4*a^8*b^2*c*d^3 +
 a^9*b*d^4)*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a...
```

### 3.82.8 Giac [A] (verification not implemented)

Time = 0.64 (sec) , antiderivative size = 479, normalized size of antiderivative = 1.60

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx =$$

$$-\frac{1}{4000}\left(\frac{200\left(4\,B^2be^6i^3-\frac{5\,(bex+ae)B^2de^5i^3}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^5bcg^6}{(dx+c)^5}-\frac{(bex+ae)^5adg^6}{(dx+c)^5}}+\frac{20\left(80\,ABbe^6i^3+16\,B^2be^6i^3-\frac{100\,(bex+ae)ABd}{dx+c}\right.}{\frac{(bex+ae)^5bcg^6}{(dx+c)^5}-\frac{(bex}{(}}\right.$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^6,x, al
gorithm="giac")
```

output
```
-1/4000*(200*(4*B^2*b*e^6*i^3 - 5*(b*e*x + a*e)*B^2*d*e^5*i^3/(d*x + c))*l
og((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^5*b*c*g^6/(d*x + c)^5 - (b*e*
x + a*e)^5*a*d*g^6/(d*x + c)^5) + 20*(80*A*B*b*e^6*i^3 + 16*B^2*b*e^6*i^3
- 100*(b*e*x + a*e)*A*B*d*e^5*i^3/(d*x + c) - 25*(b*e*x + a*e)*B^2*d*e^5*i
^3/(d*x + c))*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^5*b*c*g^6/(d*x +
 c)^5 - (b*e*x + a*e)^5*a*d*g^6/(d*x + c)^5) + (800*A^2*b*e^6*i^3 + 320*A*
B*b*e^6*i^3 + 64*B^2*b*e^6*i^3 - 1000*(b*e*x + a*e)*A^2*d*e^5*i^3/(d*x + c
) - 500*(b*e*x + a*e)*A*B*d*e^5*i^3/(d*x + c) - 125*(b*e*x + a*e)*B^2*d*e^
5*i^3/(d*x + c))/((b*e*x + a*e)^5*b*c*g^6/(d*x + c)^5 - (b*e*x + a*e)^5*a*
d*g^6/(d*x + c)^5))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d
*e)*(b*c - a*d)))
```

### 3.82.9 Mupad [B] (verification not implemented)

Time = 8.87 (sec) , antiderivative size = 3720, normalized size of antiderivative = 12.44

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^6,x)
```

output
```
- log((e*(a + b*x))/(c + d*x))^2*((x*(a*(b*((B^2*a*d^3*i^3)/(20*b^5*g^6) +
 (B^2*c*d^2*i^3)/(10*b^4*g^6)) + (3*B^2*a*d^3*i^3)/(20*b^4*g^6) + (3*B^2*c
*d^2*i^3)/(10*b^3*g^6)) + b*(a*((B^2*a*d^3*i^3)/(20*b^5*g^6) + (B^2*c*d^2*
i^3)/(10*b^4*g^6)) + (3*B^2*c^2*d*i^3)/(20*b^3*g^6)) + (3*B^2*c^2*d*i^3)/(
5*b^2*g^6)) + x^2*(b*(b*((B^2*a*d^3*i^3)/(20*b^5*g^6) + (B^2*c*d^2*i^3)/(1
0*b^4*g^6)) + (3*B^2*a*d^3*i^3)/(20*b^4*g^6) + (3*B^2*c*d^2*i^3)/(10*b^3*g
^6)) + (3*B^2*a*d^3*i^3)/(10*b^3*g^6) + (3*B^2*c*d^2*i^3)/(5*b^2*g^6)) + a
*(a*((B^2*a*d^3*i^3)/(20*b^5*g^6) + (B^2*c*d^2*i^3)/(10*b^4*g^6)) + (3*B^2
*c^2*d*i^3)/(20*b^3*g^6)) + (B^2*c^3*i^3)/(5*b^2*g^6) + (B^2*d^3*i^3*x^3)/
(2*b^2*g^6))/(5*a^4*x + a^5/b + b^4*x^5 + 10*a^3*b*x^2 + 5*a*b^3*x^4 + 10*
a^2*b^2*x^3) - (B^2*d^5*i^3)/(20*b^4*g^6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))
 - ((200*A^2*a^4*d^4*i^3 - 800*A^2*b^4*c^4*i^3 + 61*B^2*a^4*d^4*i^3 - 64*B
^2*b^4*c^4*i^3 + 180*A*B*a^4*d^4*i^3 - 320*A*B*b^4*c^4*i^3 + 200*A^2*a*b^3
*c^3*d*i^3 + 200*A^2*a^3*b*c*d^3*i^3 + 61*B^2*a*b^3*c^3*d*i^3 + 61*B^2*a^3
*b*c*d^3*i^3 + 200*A^2*a^2*b^2*c^2*d^2*i^3 + 61*B^2*a^2*b^2*c^2*d^2*i^3 +
180*A*B*a^2*b^2*c^2*d^2*i^3 + 180*A*B*a*b^3*c^3*d*i^3 + 180*A*B*a^3*b*c*d^
3*i^3)/(20*(a*d - b*c)) + (x^4*(9*B^2*b^4*d^4*i^3 + 20*A*B*b^4*d^4*i^3))/(
a*d - b*c) + (x^3*(200*A^2*a*b^3*d^4*i^3 + 61*B^2*a*b^3*d^4*i^3 - 200*A^2*
b^4*c*d^3*i^3 + 11*B^2*b^4*c*d^3*i^3 + 180*A*B*a*b^3*d^4*i^3 - 20*A*B*b^4*
c*d^3*i^3))/(2*(a*d - b*c)) + (x*(200*A^2*a^3*b*d^4*i^3 + 61*B^2*a^3*b*...
```

3.82. $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^6}\,dx$

# 3.83 $\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx$



## 3.83.1 Optimal result

Integrand size = 42, antiderivative size = 463

$$\begin{aligned}\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx &= -\frac{B^2d^2i^3(c+dx)^4}{32(bc-ad)^3g^7(a+bx)^4} \\ &+ \frac{4bB^2di^3(c+dx)^5}{125(bc-ad)^3g^7(a+bx)^5} \\ &- \frac{b^2B^2i^3(c+dx)^6}{108(bc-ad)^3g^7(a+bx)^6} \\ &- \frac{Bd^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{8(bc-ad)^3g^7(a+bx)^4} \\ &+ \frac{4bBdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{25(bc-ad)^3g^7(a+bx)^5} \\ &- \frac{b^2Bi^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{18(bc-ad)^3g^7(a+bx)^6} \\ &- \frac{d^2i^3(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{4(bc-ad)^3g^7(a+bx)^4} \\ &+ \frac{2bdi^3(c+dx)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{5(bc-ad)^3g^7(a+bx)^5} \\ &- \frac{b^2i^3(c+dx)^6\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{6(bc-ad)^3g^7(a+bx)^6}\end{aligned}$$

output
```
-1/32*B^2*d^2*i^3*(d*x+c)^4/(-a*d+b*c)^3/g^7/(b*x+a)^4+4/125*b*B^2*d*i^3*(
d*x+c)^5/(-a*d+b*c)^3/g^7/(b*x+a)^5-1/108*b^2*B^2*i^3*(d*x+c)^6/(-a*d+b*c)
^3/g^7/(b*x+a)^6-1/8*B*d^2*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d
+b*c)^3/g^7/(b*x+a)^4+4/25*b*B*d*i^3*(d*x+c)^5*(A+B*ln(e*(b*x+a)/(d*x+c)))
/(-a*d+b*c)^3/g^7/(b*x+a)^5-1/18*b^2*B*i^3*(d*x+c)^6*(A+B*ln(e*(b*x+a)/(d*
x+c)))/(-a*d+b*c)^3/g^7/(b*x+a)^6-1/4*d^2*i^3*(d*x+c)^4*(A+B*ln(e*(b*x+a)/
(d*x+c)))^2/(-a*d+b*c)^3/g^7/(b*x+a)^4+2/5*b*d*i^3*(d*x+c)^5*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^7/(b*x+a)^5-1/6*b^2*i^3*(d*x+c)^6*(A+B*ln(
e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^7/(b*x+a)^6
```

### 3.83.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 2.69 (sec) , antiderivative size = 2606, normalized size of antiderivative = 5.63

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx=\text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g +
b*g*x)^7,x]
```

output
```
(i^3*(-6000*A*B*(b*c - a*d)^6 - 1000*B^2*(b*c - a*d)^6 + 25920*a*A*B*d*(-(
b*c) + a*d)^5 + 5184*a*B^2*d*(-(b*c) + a*d)^5 - 25920*A*b*B*d*(b*c - a*d)^
5*x - 5184*b*B^2*d*(b*c - a*d)^5*x + 32400*a*A*B*d^2*(b*c - a*d)^4*(a + b*
x) + 14580*a*B^2*d^2*(b*c - a*d)^4*(a + b*x) + 7200*A*B*d*(b*c - a*d)^5*(a
 + b*x) + 2640*B^2*d*(b*c - a*d)^5*(a + b*x) + 32400*A*b*B*d^2*(b*c - a*d)
^4*x*(a + b*x) + 14580*b*B^2*d^2*(b*c - a*d)^4*x*(a + b*x) - 49500*A*B*d^2
*(b*c - a*d)^4*(a + b*x)^2 - 15675*B^2*d^2*(b*c - a*d)^4*(a + b*x)^2 + 432
00*a*A*B*d^3*(-(b*c) + a*d)^3*(a + b*x)^2 + 33840*a*B^2*d^3*(-(b*c) + a*d)
^3*(a + b*x)^2 - 43200*A*b*B*d^3*(b*c - a*d)^3*x*(a + b*x)^2 - 33840*b*B^2
*d^3*(b*c - a*d)^3*x*(a + b*x)^2 + 64800*a*A*B*d^4*(b*c - a*d)^2*(a + b*x)
^3 + 83160*a*B^2*d^4*(b*c - a*d)^2*(a + b*x)^3 + 42000*A*B*d^3*(b*c - a*d)
^3*(a + b*x)^3 + 34900*B^2*d^3*(b*c - a*d)^3*(a + b*x)^3 + 64800*A*b*B*d^4
*(b*c - a*d)^2*x*(a + b*x)^3 + 83160*b*B^2*d^4*(b*c - a*d)^2*x*(a + b*x)^3
 - 63000*A*B*d^4*(b*c - a*d)^2*(a + b*x)^4 - 83850*B^2*d^4*(b*c - a*d)^2*(
a + b*x)^4 + 129600*a*A*B*d^5*(-(b*c) + a*d)*(a + b*x)^4 + 295920*a*B^2*d^
5*(-(b*c) + a*d)*(a + b*x)^4 - 129600*A*b*B*d^5*(b*c - a*d)*x*(a + b*x)^4
- 295920*b*B^2*d^5*(b*c - a*d)*x*(a + b*x)^4 + 126000*A*B*d^5*(b*c - a*d)*
(a + b*x)^5 + 293700*B^2*d^5*(b*c - a*d)*(a + b*x)^5 - 129600*a*A*B*d^6*(a
 + b*x)^5*Log[a + b*x] - 295920*a*B^2*d^6*(a + b*x)^5*Log[a + b*x] - 12960
0*A*b*B*d^6*x*(a + b*x)^5*Log[a + b*x] - 295920*b*B^2*d^6*x*(a + b*x)^5...
```

### 3.83.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 336, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^7}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{i^3\int \frac{(c+dx)^7\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^7}d\frac{a+bx}{c+dx}}{g^7(bc-ad)^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{i^3 \int \left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^7}{(a+bx)^7}-\frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^6}{(a+bx)^6}+\frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^5}{(a+bx)^5}\right)d\frac{a+bx}{c+dx}}{g^7(bc-ad)^3}$$

$$\downarrow 2009$$

$$\frac{i^3\left(-\frac{b^2(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{6(a+bx)^6}-\frac{b^2B(c+dx)^6\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{18(a+bx)^6}-\frac{d^2(c+dx)^4\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{4(a+bx)^4}-\frac{Bd^2(c+dx)^4\left(B\log\left(\frac{e}{}\right.\right.}{8(a+bx)^4}\right.}{g^7(b}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a*g + b*g*x)
^7,x]
```

output
```
(i^3*(-1/32*(B^2*d^2*(c + d*x)^4)/(a + b*x)^4 + (4*b*B^2*d*(c + d*x)^5)/(1
25*(a + b*x)^5) - (b^2*B^2*(c + d*x)^6)/(108*(a + b*x)^6) - (B*d^2*(c + d*
x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(8*(a + b*x)^4) + (4*b*B*d*(c +
 d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(25*(a + b*x)^5) - (b^2*B*(c
 + d*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(18*(a + b*x)^6) - (d^2*(c
 + d*x)^4*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(4*(a + b*x)^4) + (2*b*d
*(c + d*x)^5*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(5*(a + b*x)^5) - (b^
2*(c + d*x)^6*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(6*(a + b*x)^6)))/((
b*c - a*d)^3*g^7)
```

### 3.83.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.83.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(445) = 890$.

Time = 2.52 (sec) , antiderivative size = 1019, normalized size of antiderivative = 2.20

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1019 |
| derivativedivides | Expression too large to display | 1082 |
| default | Expression too large to display | 1082 |
| norman | Expression too large to display | 2555 |
| parallelrisch | Expression too large to display | 2803 |
| risch | Expression too large to display | 6548 |

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^7,x,method=_RE
TURNVERBOSE)
```

output
```
i^3*A^2/g^7*(-3/5*d*(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^4/(b*x+a)^5-1/3*d^3/b^4/
(b*x+a)^3+3/4*d^2*(a*d-b*c)/b^4/(b*x+a)^4-1/6*(-a^3*d^3+3*a^2*b*c*d^2-3*a*
b^2*c^2*d+b^3*c^3)/b^4/(b*x+a)^6)-i^3*B^2/g^7/d^5*(a*d-b*c)^4*e^4*(d^7/(a*
d-b*c)^7*(-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^2-1/8/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))
-1/32/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^4)-2*d^6/(a*d-b*c)^7*b*e*(-1/5/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/25/(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^5*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/125/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))^5)+d^5/(a*d-b*c)^7*e^2*b^2*(-1/6/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^6*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/18/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^6*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/108/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^6)
)-2*i^3*B*A/g^7/d^5*(a*d-b*c)^4*e^4*(d^7/(a*d-b*c)^7*(-1/4/(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/16/(b*e/d+(a*d-b*c)*e/d
/(d*x+c))^4)-2*d^6/(a*d-b*c)^7*b*e*(-1/5/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/25/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^5)+d^5/(
a*d-b*c)^7*e^2*b^2*(-1/6/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^6*ln(b*e/d+(a*d-b*c
)*e/d/(d*x+c))-1/36/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^6))
```

### 3.83.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1610 vs. $2(445) = 890$.

Time = 0.37 (sec) , antiderivative size = 1610, normalized size of antiderivative = 3.48

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^7,x, al
gorithm="fricas")
```

output
```
-1/108000*(60*((60*A*B + 37*B^2)*b^6*c*d^5 - (60*A*B + 37*B^2)*a*b^5*d^6)*
i^3*x^5 - 30*((60*A*B - 23*B^2)*b^6*c^2*d^4 - 36*(20*A*B + 9*B^2)*a*b^5*c*
d^5 + (660*A*B + 347*B^2)*a^2*b^4*d^6)*i^3*x^4 + 20*((1800*A^2 + 60*A*B -
53*B^2)*b^6*c^3*d^3 - 27*(200*A^2 + 20*A*B - 11*B^2)*a*b^5*c^2*d^4 + 675*(
8*A^2 + 4*A*B + B^2)*a^2*b^4*c*d^5 - (1800*A^2 + 2220*A*B + 919*B^2)*a^3*b
^3*d^6)*i^3*x^3 + 15*((5400*A^2 + 1140*A*B + 73*B^2)*b^6*c^4*d^2 - 72*(200
*A^2 + 60*A*B + 7*B^2)*a*b^5*c^3*d^3 + 1350*(8*A^2 + 4*A*B + B^2)*a^2*b^4*
c^2*d^4 - (1800*A^2 + 2220*A*B + 919*B^2)*a^4*b^2*d^6)*i^3*x^2 + 6*(8*(135
0*A^2 + 390*A*B + 53*B^2)*b^6*c^5*d - 45*(600*A^2 + 220*A*B + 39*B^2)*a*b^
5*c^4*d^2 + 2250*(8*A^2 + 4*A*B + B^2)*a^2*b^4*c^3*d^3 - (1800*A^2 + 2220*
A*B + 919*B^2)*a^5*b*d^6)*i^3*x + (1000*(18*A^2 + 6*A*B + B^2)*b^6*c^6 - 1
728*(25*A^2 + 10*A*B + 2*B^2)*a*b^5*c^5*d + 3375*(8*A^2 + 4*A*B + B^2)*a^2
*b^4*c^4*d^2 - (1800*A^2 + 2220*A*B + 919*B^2)*a^6*d^6)*i^3 + 1800*(B^2*b^
6*d^6*i^3*x^6 + 6*B^2*a*b^5*d^6*i^3*x^5 + 15*B^2*a^2*b^4*d^6*i^3*x^4 + 20*
(B^2*b^6*c^3*d^3 - 3*B^2*a*b^5*c^2*d^4 + 3*B^2*a^2*b^4*c*d^5)*i^3*x^3 + 15
*(3*B^2*b^6*c^4*d^2 - 8*B^2*a*b^5*c^3*d^3 + 6*B^2*a^2*b^4*c^2*d^4)*i^3*x^2
 + 6*(6*B^2*b^6*c^5*d - 15*B^2*a*b^5*c^4*d^2 + 10*B^2*a^2*b^4*c^3*d^3)*i^3
*x + (10*B^2*b^6*c^6 - 24*B^2*a*b^5*c^5*d + 15*B^2*a^2*b^4*c^4*d^2)*i^3)*l
og((b*e*x + a*e)/(d*x + c))^2 + 60*((60*A*B + 37*B^2)*b^6*d^6*i^3*x^6 + 6*
(10*B^2*b^6*c*d^5 + 3*(20*A*B + 9*B^2)*a*b^5*d^6)*i^3*x^5 - 15*(2*B^2*b...
```

### 3.83.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**7,x)
```

output
```
Timed out
```

### 3.83.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 20330 vs. $2(445) = 890$.

Time = 2.08 (sec) , antiderivative size = 20330, normalized size of antiderivative = 43.91

$$\int \frac{(ci + dix)^3 \left(A + B \log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag + bgx)^7} dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^7,x, al
gorithm="maxima")
```

output
```
-1/10*(6*b*x + a)*B^2*c^2*d*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^
8*g^7*x^6 + 6*a*b^7*g^7*x^5 + 15*a^2*b^6*g^7*x^4 + 20*a^3*b^5*g^7*x^3 + 15
*a^4*b^4*g^7*x^2 + 6*a^5*b^3*g^7*x + a^6*b^2*g^7) - 1/20*(15*b^2*x^2 + 6*a
*b*x + a^2)*B^2*c*d^2*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(b^9*g^7*
x^6 + 6*a*b^8*g^7*x^5 + 15*a^2*b^7*g^7*x^4 + 20*a^3*b^6*g^7*x^3 + 15*a^4*b
^5*g^7*x^2 + 6*a^5*b^4*g^7*x + a^6*b^3*g^7) - 1/60*(20*b^3*x^3 + 15*a*b^2*
x^2 + 6*a^2*b*x + a^3)*B^2*d^3*i^3*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/
(b^10*g^7*x^6 + 6*a*b^9*g^7*x^5 + 15*a^2*b^8*g^7*x^4 + 20*a^3*b^7*g^7*x^3
+ 15*a^4*b^6*g^7*x^2 + 6*a^5*b^5*g^7*x + a^6*b^4*g^7) + 1/10800*(60*((60*b
^5*d^5*x^5 - 10*b^5*c^5 + 62*a*b^4*c^4*d - 163*a^2*b^3*c^3*d^2 + 237*a^3*b
^2*c^2*d^3 - 213*a^4*b*c*d^4 + 147*a^5*d^5 - 30*(b^5*c*d^4 - 11*a*b^4*d^5)
*x^4 + 20*(b^5*c^2*d^3 - 8*a*b^4*c*d^4 + 37*a^2*b^3*d^5)*x^3 - 15*(b^5*c^3
*d^2 - 7*a*b^4*c^2*d^3 + 23*a^2*b^3*c*d^4 - 57*a^3*b^2*d^5)*x^2 + 6*(2*b^5
*c^4*d - 13*a*b^4*c^3*d^2 + 37*a^2*b^3*c^2*d^3 - 63*a^3*b^2*c*d^4 + 87*a^4
*b*d^5)*x)/((b^12*c^5 - 5*a*b^11*c^4*d + 10*a^2*b^10*c^3*d^2 - 10*a^3*b^9*
c^2*d^3 + 5*a^4*b^8*c*d^4 - a^5*b^7*d^5)*g^7*x^6 + 6*(a*b^11*c^5 - 5*a^2*b
^10*c^4*d + 10*a^3*b^9*c^3*d^2 - 10*a^4*b^8*c^2*d^3 + 5*a^5*b^7*c*d^4 - a^
6*b^6*d^5)*g^7*x^5 + 15*(a^2*b^10*c^5 - 5*a^3*b^9*c^4*d + 10*a^4*b^8*c^3*d
^2 - 10*a^5*b^7*c^2*d^3 + 5*a^6*b^6*c*d^4 - a^7*b^5*d^5)*g^7*x^4 + 20*(a^3
*b^9*c^5 - 5*a^4*b^8*c^4*d + 10*a^5*b^7*c^3*d^2 - 10*a^6*b^6*c^2*d^3 + ...
```

### 3.83.8 Giac [A] (verification not implemented)

Time = 0.66 (sec) , antiderivative size = 780, normalized size of antiderivative = 1.68

$$\int \frac{(ci+dix)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx =$$

$$-\frac{1}{108000}\left(\frac{1800\left(10\,B^2b^2e^7i^3-\frac{24\,(bex+ae)B^2bde^6i^3}{dx+c}+\frac{15\,(bex+ae)^2B^2d^2e^5i^3}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^6b^2c^2g^7}{(dx+c)^6}-\frac{2\,(bex+ae)^6abcdg^7}{(dx+c)^6}+\frac{(bex+ae)^6a^2d^2g^7}{(dx+c)^6}}+\frac{60\left(600\,ABb^2e^7i^3\right.}{}\right.$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^7,x, al
gorithm="giac")
```

output
```
-1/108000*(1800*(10*B^2*b^2*e^7*i^3 - 24*(b*e*x + a*e)*B^2*b*d*e^6*i^3/(d*
x + c) + 15*(b*e*x + a*e)^2*B^2*d^2*e^5*i^3/(d*x + c)^2)*log((b*e*x + a*e)
/(d*x + c))^2/((b*e*x + a*e)^6*b^2*c^2*g^7/(d*x + c)^6 - 2*(b*e*x + a*e)^6
*a*b*c*d*g^7/(d*x + c)^6 + (b*e*x + a*e)^6*a^2*d^2*g^7/(d*x + c)^6) + 60*(
600*A*B*b^2*e^7*i^3 + 100*B^2*b^2*e^7*i^3 - 1440*(b*e*x + a*e)*A*B*b*d*e^6
*i^3/(d*x + c) - 288*(b*e*x + a*e)*B^2*b*d*e^6*i^3/(d*x + c) + 900*(b*e*x
+ a*e)^2*A*B*d^2*e^5*i^3/(d*x + c)^2 + 225*(b*e*x + a*e)^2*B^2*d^2*e^5*i^3
/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b*e*x + a*e)^6*b^2*c^2*g^7/(d
*x + c)^6 - 2*(b*e*x + a*e)^6*a*b*c*d*g^7/(d*x + c)^6 + (b*e*x + a*e)^6*a^
2*d^2*g^7/(d*x + c)^6) + (18000*A^2*b^2*e^7*i^3 + 6000*A*B*b^2*e^7*i^3 + 1
000*B^2*b^2*e^7*i^3 - 43200*(b*e*x + a*e)*A^2*b*d*e^6*i^3/(d*x + c) - 1728
0*(b*e*x + a*e)*A*B*b*d*e^6*i^3/(d*x + c) - 3456*(b*e*x + a*e)*B^2*b*d*e^6
*i^3/(d*x + c) + 27000*(b*e*x + a*e)^2*A^2*d^2*e^5*i^3/(d*x + c)^2 + 13500
*(b*e*x + a*e)^2*A*B*d^2*e^5*i^3/(d*x + c)^2 + 3375*(b*e*x + a*e)^2*B^2*d^
2*e^5*i^3/(d*x + c)^2)/((b*e*x + a*e)^6*b^2*c^2*g^7/(d*x + c)^6 - 2*(b*e*x
 + a*e)^6*a*b*c*d*g^7/(d*x + c)^6 + (b*e*x + a*e)^6*a^2*d^2*g^7/(d*x + c)^
6))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d))
)
```

### 3.83.9 Mupad [B] (verification not implemented)

Time = 10.60 (sec) , antiderivative size = 6275, normalized size of antiderivative = 13.55

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^7}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(a*g + b*g*x)
^7,x)
```

output
```
((1800*A^2*a^5*d^5*i^3 + 18000*A^2*b^5*c^5*i^3 + 919*B^2*a^5*d^5*i^3 + 100
0*B^2*b^5*c^5*i^3 + 2220*A*B*a^5*d^5*i^3 + 6000*A*B*b^5*c^5*i^3 - 25200*A^
2*a*b^4*c^4*d*i^3 + 1800*A^2*a^4*b*c*d^4*i^3 - 2456*B^2*a*b^4*c^4*d*i^3 +
919*B^2*a^4*b*c*d^4*i^3 + 1800*A^2*a^2*b^3*c^3*d^2*i^3 + 1800*A^2*a^3*b^2*
c^2*d^3*i^3 + 919*B^2*a^2*b^3*c^3*d^2*i^3 + 919*B^2*a^3*b^2*c^2*d^3*i^3 +
2220*A*B*a^2*b^3*c^3*d^2*i^3 + 2220*A*B*a^3*b^2*c^2*d^3*i^3 - 11280*A*B*a*
b^4*c^4*d*i^3 + 2220*A*B*a^4*b*c*d^4*i^3)/(60*(a*d - b*c)) + (x^4*(347*B^2
*a*b^4*d^5*i^3 + 23*B^2*b^5*c*d^4*i^3 + 660*A*B*a*b^4*d^5*i^3 - 60*A*B*b^5
*c*d^4*i^3))/(2*(a*d - b*c)) + (x^2*(1800*A^2*a^3*b^2*d^5*i^3 + 919*B^2*a^
3*b^2*d^5*i^3 + 5400*A^2*b^5*c^3*d^2*i^3 + 73*B^2*b^5*c^3*d^2*i^3 - 9000*A
^2*a*b^4*c^2*d^3*i^3 + 1800*A^2*a^2*b^3*c*d^4*i^3 - 431*B^2*a*b^4*c^2*d^3*
i^3 + 919*B^2*a^2*b^3*c*d^4*i^3 + 2220*A*B*a^3*b^2*d^5*i^3 + 1140*A*B*b^5*
c^3*d^2*i^3 - 3180*A*B*a*b^4*c^2*d^3*i^3 + 2220*A*B*a^2*b^3*c*d^4*i^3))/(4
*(a*d - b*c)) + (x^3*(1800*A^2*a^2*b^3*d^5*i^3 + 919*B^2*a^2*b^3*d^5*i^3 +
 1800*A^2*b^5*c^2*d^3*i^3 - 53*B^2*b^5*c^2*d^3*i^3 - 3600*A^2*a*b^4*c*d^4*
i^3 + 244*B^2*a*b^4*c*d^4*i^3 + 2220*A*B*a^2*b^3*d^5*i^3 + 60*A*B*b^5*c^2*
d^3*i^3 - 480*A*B*a*b^4*c*d^4*i^3))/(3*(a*d - b*c)) + (x*(1800*A^2*a^4*b*d
^5*i^3 + 919*B^2*a^4*b*d^5*i^3 + 10800*A^2*b^5*c^4*d*i^3 + 424*B^2*b^5*c^4
*d*i^3 - 16200*A^2*a*b^4*c^3*d^2*i^3 + 1800*A^2*a^3*b^2*c*d^4*i^3 - 1331*B
^2*a*b^4*c^3*d^2*i^3 + 919*B^2*a^3*b^2*c*d^4*i^3 + 2220*A*B*a^4*b*d^5*i...
```

# 3.84 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$

### 3.84.1 Optimal result

Integrand size = 42, antiderivative size = 718

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{bB^2(bc-ad)^2g^3x}{3d^3i} + \frac{B^2(bc-ad)^3g^3\log\left(\frac{a+bx}{c+dx}\right)}{3d^4i}$$
$$+ \frac{7B(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^3i}$$
$$- \frac{b^2B(bc-ad)g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{3d^4i}$$
$$+ \frac{6B(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i}$$
$$+ \frac{3(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i}$$
$$- \frac{3b^2(bc-ad)g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4i}$$
$$+ \frac{b^3g^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3d^4i}$$
$$+ \frac{(bc-ad)^3g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i} - \frac{2B^2(bc-ad)^3g^3\log(c+dx)}{d^4i}$$
$$- \frac{7B(bc-ad)^3g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i}$$
$$+ \frac{6B^2(bc-ad)^3g^3\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$
$$+ \frac{2B(bc-ad)^3g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$
$$+ \frac{7B^2(bc-ad)^3g^3\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} - \frac{2B^2(bc-ad)^3g^3\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$

output
```
1/3*b*B^2*(-a*d+b*c)^2*g^3*x/d^3/i+1/3*B^2*(-a*d+b*c)^3*g^3*ln((b*x+a)/(d*
x+c))/d^4/i+7/3*B*(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3
/i-1/3*b^2*B*(-a*d+b*c)*g^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4/i+6*
B*(-a*d+b*c)^3*g^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^
4/i+3*(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i-3/2*b^2
*(-a*d+b*c)*g^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i+1/3*b^3*g^3*
(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i+(-a*d+b*c)^3*g^3*ln((-a*d+b*
c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i-2*B^2*(-a*d+b*c)^3*g^3*l
n(d*x+c)/d^4/i-7/3*B*(-a*d+b*c)^3*g^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(
d*x+c)/d/(b*x+a))/d^4/i+6*B^2*(-a*d+b*c)^3*g^3*polylog(2,d*(b*x+a)/b/(d*x+
c))/d^4/i+2*B*(-a*d+b*c)^3*g^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*
x+a)/b/(d*x+c))/d^4/i+7/3*B^2*(-a*d+b*c)^3*g^3*polylog(2,b*(d*x+c)/d/(b*x+
a))/d^4/i-2*B^2*(-a*d+b*c)^3*g^3*polylog(3,d*(b*x+a)/b/(d*x+c))/d^4/i
```

### 3.84.2 Mathematica [A] (verified)

Time = 1.09 (sec) , antiderivative size = 1020, normalized size of antiderivative = 1.42

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{g^3\left(6bd(bc-ad)^2x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+2d^3(a+b\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i +
d*i*x),x]
```

output
```
(g^3*(6*b*d*(b*c - a*d)^2*x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 3*d^2
*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 2*d^3
*(a + b*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 6*A^2*(b*c - a*d)^3*
Log[c + d*x] + 12*A*B*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c
- a*d)/(b*c + b*d*x)] + 6*B^2*(b*c - a*d)^3*Log[(e*(a + b*x))/(c + d*x)]^2
*Log[(b*c - a*d)/(b*c + b*d*x)] - 6*A*B*(b*c - a*d)^3*(Log[(b*c - a*d)/(b*
c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c +
b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 6*B*(b*c - a*d)^2*(2
*a*d*Log[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b*c*(A + B*Log[
(e*(a + b*x))/(c + d*x)])*Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x]
 - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
+ a*d)]) + b*B*c*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log
[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 3*B*(b*c - a*d)^2*
(2*A*b*d*x + 2*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c - a*d
)*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c
+ d*x] + B*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x
])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + 2*B*(b*c - a
*d)*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[(e*(a + b*x))
/(c + d*x)] - d^2*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*B*(
b*c - a*d)^2*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c...
```

### 3.84.3 Rubi [A] (verified)

Time = 1.09 (sec) , antiderivative size = 629, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ci+dix}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{g^3(bc-ad)^3\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \text{2795}$$

$$\frac{g^3(bc-ad)^3\int\left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+b}{c+d}}{i}$$

$\downarrow$ 2009

$$g^3(bc-ad)^3\left(\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{b^2B\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{2B\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e}{}\right.\right.}{d^4}\right.$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)
,x]
```

output
```
((b*c - a*d)^3*g^3*((b*B^2)/(3*d^4*(b - (d*(a + b*x))/(c + d*x))) + (B^2*L
og[(a + b*x)/(c + d*x)])/(3*d^4) - (b^2*B*(A + B*Log[(e*(a + b*x))/(c + d*
x)]))/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (7*B*(a + b*x)*(A + B*Log[
(e*(a + b*x))/(c + d*x)]))/(3*d^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)))
 + (b^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*d^4*(b - (d*(a + b*x))/
(c + d*x))^3) - (3*b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*d^4*(b -
 (d*(a + b*x))/(c + d*x))^2) + (3*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c +
d*x)])^2)/(d^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (2*B^2*Log[b - (
d*(a + b*x))/(c + d*x)])/d^4 + (6*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*L
og[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 + ((A + B*Log[(e*(a + b*x))/(c +
d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 - (7*B*(A + B*Log[(e*(a
 + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/(3*d^4) + (6*B^
2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 + (2*B*(A + B*Log[(e*(a + b
*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 + (7*B^2*Pol
yLog[2, (b*(c + d*x))/(d*(a + b*x))])/(3*d^4) - (2*B^2*PolyLog[3, (d*(a +
b*x))/(b*(c + d*x))])/d^4))/i
```

## 3.84. $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$

### 3.84.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.84.4 Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{dix+ci}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

## 3.84.5 Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algo
rithm="fricas")
```

output
```
integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*
a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^
2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)))
/(d*i*x + c*i), x)
```

## 3.84.6 Sympy [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{g^3\left(\int \frac{A^2a^3}{c+dx}\,dx + \int \frac{A^2b^3x^3}{c+dx}\,dx + \int \frac{B^2a^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx + \int \frac{2ABa^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx + \int \frac{3A^2ab^2x^2}{c+dx}\,dx + \int \frac{3A}{c}\right.}{}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i),x)
```

output
```
g**3*(Integral(A**2*a**3/(c + d*x), x) + Integral(A**2*b**3*x**3/(c + d*x)
, x) + Integral(B**2*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x
), x) + Integral(2*A*B*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x)
, x) + Integral(3*A**2*a*b**2*x**2/(c + d*x), x) + Integral(3*A**2*a**2*b*
x/(c + d*x), x) + Integral(B**2*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d
*x))**2/(c + d*x), x) + Integral(2*A*B*b**3*x**3*log(a*e/(c + d*x) + b*e*x
/(c + d*x))/(c + d*x), x) + Integral(3*B**2*a*b**2*x**2*log(a*e/(c + d*x)
+ b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(3*B**2*a**2*b*x*log(a*e/(c
+ d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(6*A*B*a*b**2*x**2*lo
g(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integral(6*A*B*a**2*b*x
*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i
```

## 3.84.7 Maxima [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algo
rithm="maxima")
```

output
```
3*A^2*a^2*b*g^3*(x/(d*i) - c*log(d*x + c)/(d^2*i)) - 1/6*A^2*b^3*g^3*(6*c^
3*log(d*x + c)/(d^4*i) - (2*d^2*x^3 - 3*c*d*x^2 + 6*c^2*x)/(d^3*i)) + 3/2*
A^2*a*b^2*g^3*(2*c^2*log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A^2
*a^3*g^3*log(d*i*x + c*i)/(d*i) - 1/6*(2*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3
+ 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*B^2*log(d*x + c)^3 - (2*B^2*b^3*d^3*g^3
*x^3 - 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B^2*x^2 + 6*(b^3*c^2*d*g^3 - 3*
a*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B^2*x)*log(d*x + c)^2)/(d^4*i) - integr
ate(-1/3*(3*B^2*a^3*d^2*g^3*log(e)^2 + 6*A*B*a^3*d^2*g^3*log(e) + 3*(B^2*b
^3*d^2*g^3*log(e)^2 + 2*A*B*b^3*d^2*g^3*log(e))*x^3 + 9*(B^2*a*b^2*d^2*g^3
*log(e)^2 + 2*A*B*a*b^2*d^2*g^3*log(e))*x^2 + 3*(B^2*b^3*d^2*g^3*x^3 + 3*B
^2*a*b^2*d^2*g^3*x^2 + 3*B^2*a^2*b*d^2*g^3*x + B^2*a^3*d^2*g^3)*log(b*x +
a)^2 + 9*(B^2*a^2*b*d^2*g^3*log(e)^2 + 2*A*B*a^2*b*d^2*g^3*log(e))*x + 6*(
B^2*a^3*d^2*g^3*log(e) + A*B*a^3*d^2*g^3 + (B^2*b^3*d^2*g^3*log(e) + A*B*b
^3*d^2*g^3)*x^3 + 3*(B^2*a*b^2*d^2*g^3*log(e) + A*B*a*b^2*d^2*g^3)*x^2 + 3
*(B^2*a^2*b*d^2*g^3*log(e) + A*B*a^2*b*d^2*g^3)*x)*log(b*x + a) - (6*B^2*a
^3*d^2*g^3*log(e) + 6*A*B*a^3*d^2*g^3 + 2*(3*A*B*b^3*d^2*g^3 + (3*g^3*log(
e) + g^3)*B^2*b^3*d^2)*x^3 + 3*(6*A*B*a*b^2*d^2*g^3 - (b^3*c*d*g^3 - 3*(2*
g^3*log(e) + g^3)*a*b^2*d^2)*B^2)*x^2 + 6*(3*A*B*a^2*b*d^2*g^3 + (b^3*c^2*
g^3 - 3*a*b^2*c*d*g^3 + 3*(g^3*log(e) + g^3)*a^2*b*d^2)*B^2)*x + 6*(B^2*b^
3*d^2*g^3*x^3 + 3*B^2*a*b^2*d^2*g^3*x^2 + 3*B^2*a^2*b*d^2*g^3*x + B^2*a...
```

### 3.84.8 Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algo
rithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*
i), x)
```

## 3.84.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{{\left(a\,g+b\,g\,x\right)}^3\,{\left(A+B\,\ln\left(\frac{e\,\left(a+b\,x\right)}{c+d\,x}\right)\right)}^2}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
, x)
```

# 3.85 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$



## 3.85.1 Optimal result

Integrand size = 42, antiderivative size = 536

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx\\
&= -\frac{B(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i}\\
&\quad -\frac{4B(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i}\\
&\quad -\frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i}+\frac{b^2g^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^3i}\\
&\quad -\frac{(bc-ad)^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i}+\frac{B^2(bc-ad)^2g^2\log(c+dx)}{d^3i}\\
&\quad +\frac{B(bc-ad)^2g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i}\\
&\quad -\frac{4B^2(bc-ad)^2g^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}\\
&\quad -\frac{2B(bc-ad)^2g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}\\
&\quad -\frac{B^2(bc-ad)^2g^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i}+\frac{2B^2(bc-ad)^2g^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}
\end{aligned}$$

output
```
-B*(-a*d+b*c)*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2/i-4*B*(-a*d+b*c)
^2*g^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3/i-2*(-a*d+
b*c)*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i+1/2*b^2*g^2*(d*x+c)^2
*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i-(-a*d+b*c)^2*g^2*ln((-a*d+b*c)/b/(d*x
+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i+B^2*(-a*d+b*c)^2*g^2*ln(d*x+c)/d^
3/i+B*(-a*d+b*c)^2*g^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a
))/d^3/i-4*B^2*(-a*d+b*c)^2*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i-2*B*(
-a*d+b*c)^2*g^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))
/d^3/i-B^2*(-a*d+b*c)^2*g^2*polylog(2,b*(d*x+c)/d/(b*x+a))/d^3/i+2*B^2*(-a
*d+b*c)^2*g^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^3/i
```

### 3.85.2 Mathematica [A] (verified)

Time = 0.87 (sec) , antiderivative size = 727, normalized size of antiderivative = 1.36

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{g^2\left(-2bd(bc-ad)x\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+2A^2(bc-ad)^2\log(c\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i +
d*i*x),x]
```

output
```
(g^2*(-2*b*d*(b*c - a*d)*x*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + d^2*(a
 + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 2*A^2*(b*c - a*d)^2*Log
[c + d*x] - 4*A*B*(b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*
d)/(b*c + b*d*x)] - 2*B^2*(b*c - a*d)^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log
[(b*c - a*d)/(b*c + b*d*x)] + 2*A*B*(b*c - a*d)^2*(Log[(b*c - a*d)/(b*c +
b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*
x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b*c - a*d)*(2*a*d*L
og[a + b*x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b*c*(A + B*Log[(e*(a
+ b*x))/(c + d*x)])*Log[c + d*x] - a*B*d*(Log[a + b*x]*(Log[a + b*x] - 2*L
og[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)
]) + b*B*c*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d
*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - B*(b*c - a*d)*(2*A*b*d*x
 + 2*B*d*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)] - 2*B*(b*c - a*d)*Log[c +
d*x] - 2*(b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c + d*x] + B
*(b*c - a*d)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - 4*B^2*(b*c - a*d)^2*(L
og[(e*(a + b*x))/(c + d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - Poly
Log[3, (d*(a + b*x))/(b*(c + d*x))])))/(2*d^3*i)
```

### 3.85.3 Rubi [A] (verified)

Time = 0.79 (sec) , antiderivative size = 470, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ci+dix}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{g^2(bc-ad)^2\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \text{2795}$$

$$\frac{g^2(bc-ad)^2\int \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \text{2009}$$

$$g^2(bc-ad)^2\left(\frac{b^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2B\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3}-\frac{4B\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3}+\right.$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)
,x]
```

output
```
((b*c - a*d)^2*g^2*(-((B*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(
d^2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)))) + (b^2*(A + B*Log[(e*(a + b*
x))/(c + d*x)])^2)/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*(a + b*x)*
(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^2*(c + d*x)*(b - (d*(a + b*x))/
(c + d*x))) - (B^2*Log[b - (d*(a + b*x))/(c + d*x)])/d^3 - (4*B*(A + B*Log
[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 - ((A
 + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])
/d^3 + (B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a
 + b*x))])/d^3 - (4*B^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 - (2*
B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*
x))])/d^3 - (B^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/d^3 + (2*B^2*Pol
yLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^3))/i
```

### 3.85.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.85.4 Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{dix+ci}dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

### 3.85.5 Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^2 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algo
rithm="fricas")
```

output
```
integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x
^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*
B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)
))/(d*i*x + c*i), x)
```

### 3.85.6 Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{g^2\left(\int \frac{A^2a^2}{c+dx}\,dx + \int \frac{A^2b^2x^2}{c+dx}\,dx + \int \frac{B^2a^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx + \int \frac{2ABa^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx + \int \frac{2A^2abx}{c+dx}\,dx + \int \frac{B^2b^2 \cdots}{\cdots}\right.}{\cdots}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i),x)
```

output
```
g**2*(Integral(A**2*a**2/(c + d*x), x) + Integral(A**2*b**2*x**2/(c + d*x)
, x) + Integral(B**2*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x
), x) + Integral(2*A*B*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x)
, x) + Integral(2*A**2*a*b*x/(c + d*x), x) + Integral(B**2*b**2*x**2*log(a
*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(2*A*B*b**2*x**
2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integral(2*B**2*a*b
*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(4*A*B*
a*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i
```

### 3.85.7 Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algo
rithm="maxima")
```

output
```
2*A^2*a*b*g^2*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + 1/2*A^2*b^2*g^2*(2*c^2*
log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A^2*a^2*g^2*log(d*i*x +
c*i)/(d*i) + 1/6*(2*(b^2*c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B^2*log(d*
x + c)^3 + 3*(B^2*b^2*d^2*g^2*x^2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B^2*x)
*log(d*x + c)^2)/(d^3*i) - integrate(-(B^2*a^2*d*g^2*log(e)^2 + 2*A*B*a^2*
d*g^2*log(e) + (B^2*b^2*d*g^2*log(e)^2 + 2*A*B*b^2*d*g^2*log(e))*x^2 + (B^
2*b^2*d*g^2*x^2 + 2*B^2*a*b*d*g^2*x + B^2*a^2*d*g^2)*log(b*x + a)^2 + 2*(B
^2*a*b*d*g^2*log(e)^2 + 2*A*B*a*b*d*g^2*log(e))*x + 2*(B^2*a^2*d*g^2*log(e
) + A*B*a^2*d*g^2 + (B^2*b^2*d*g^2*log(e) + A*B*b^2*d*g^2)*x^2 + 2*(B^2*a*
b*d*g^2*log(e) + A*B*a*b*d*g^2)*x)*log(b*x + a) - (2*B^2*a^2*d*g^2*log(e)
+ 2*A*B*a^2*d*g^2 + (2*A*B*b^2*d*g^2 + (2*g^2*log(e) + g^2)*B^2*b^2*d)*x^2
 + 2*(2*A*B*a*b*d*g^2 - (b^2*c*g^2 - 2*(g^2*log(e) + g^2)*a*b*d)*B^2)*x +
2*(B^2*b^2*d*g^2*x^2 + 2*B^2*a*b*d*g^2*x + B^2*a^2*d*g^2)*log(b*x + a))*lo
g(d*x + c))/(d^2*i*x + c*d*i), x)
```

### 3.85.8 Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algo
rithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*
i), x)
```

### 3.85.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
, x)
```

3.85. $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$

# 3.86 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$



## 3.86.1 Optimal result

Integrand size = 40, antiderivative size = 283

$$\begin{aligned}\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx &= \frac{2B(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^2i}+\frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di}\\
&+\frac{(bc-ad)g\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i}+\frac{2B^2(bc-ad)g\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}\\
&+\frac{2B(bc-ad)g\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}\\
&-\frac{2B^2(bc-ad)g\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}\end{aligned}$$

output
```
2*B*(-a*d+b*c)*g*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2/
i+g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d/i+(-a*d+b*c)*g*ln((-a*d+b*c)/b
/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i+2*B^2*(-a*d+b*c)*g*polylog(2
,d*(b*x+a)/b/(d*x+c))/d^2/i+2*B*(-a*d+b*c)*g*(A+B*ln(e*(b*x+a)/(d*x+c)))*p
olylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i-2*B^2*(-a*d+b*c)*g*polylog(3,d*(b*x+a)
/b/(d*x+c))/d^2/i
```

### 3.86.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1227 vs. $2(283) = 566$.

Time = 0.66 (sec) , antiderivative size = 1227, normalized size of antiderivative = 4.34

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \text{Too large to display}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*
i*x),x]
```

output
```
(g*(3*A^2*b*d*x - 3*A^2*(b*c - a*d)*Log[c + d*x] - 3*a*A*B*d*(Log[c/d + x]
^2 + 2*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)])*Log[c
+ d*x] - 2*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)])) - 3*A*B*(-2*d*(a + b*x)*(-1 + Log[a/b + x]) + 2
*b*(c + d*x)*(-1 + Log[c/d + x]) - b*c*Log[c/d + x]^2 + 2*b*(Log[a/b + x]
- Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)])*(d*x - c*Log[c + d*x]) + 2*
b*c*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)])) + a*B^2*d*(Log[c/d + x]^3 + 3*Log[c/d + x]^2*(-Log[a/b
 + x] + Log[(d*(a + b*x))/(-(b*c) + a*d)]) + 3*(-Log[a/b + x] + Log[c/d +
x] + Log[(e*(a + b*x))/(c + d*x)])^2*Log[c + d*x] + 3*Log[a/b + x]^2*Log[(
b*(c + d*x))/(b*c - a*d)] + 6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c
) + a*d)] + 3*(Log[a/b + x] - Log[c/d + x] - Log[(e*(a + b*x))/(c + d*x)])
*(Log[c/d + x]^2 - 2*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLo
g[2, (d*(a + b*x))/(-(b*c) + a*d)])) + 6*Log[c/d + x]*PolyLog[2, (b*(c + d
*x))/(b*c - a*d)] - 6*PolyLog[3, (d*(a + b*x))/(-(b*c) + a*d)] - 6*PolyLog
[3, (b*(c + d*x))/(b*c - a*d)]) + B^2*(3*d*(2*b*x - 2*(a + b*x)*Log[a/b +
x] + (a + b*x)*Log[a/b + x]^2) - b*c*Log[c/d + x]^3 + 3*b*(2*d*x - 2*(c +
d*x)*Log[c/d + x] + (c + d*x)*Log[c/d + x]^2) + 3*b*(-Log[a/b + x] + Log[c
/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2*(d*x - c*Log[c + d*x]) - 6*(a*d
+ 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Log[a/b + x]*(-(d*(...
```

### 3.86.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ci+dix}dx$$

$$\Big\downarrow \textbf{2962}$$

$$\frac{g(bc-ad)\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \textbf{2795}$$

$$\frac{g(bc-ad)\int \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}\right)d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \textbf{2009}$$

$$\frac{g(bc-ad)\left(\frac{2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2}+\frac{2B\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2}+\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+b}{c+d}\right.\right.}{d^2}\right.}{i}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x),x
]
```

output
```
((b*c - a*d)*g*(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d*(c +
 d*x)*(b - (d*(a + b*x))/(c + d*x))) + (2*B*(A + B*Log[(e*(a + b*x))/(c +
d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 + ((A + B*Log[(e*(a + b*x
))/(c + d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 + (2*B^2*PolyLo
g[2, (d*(a + b*x))/(b*(c + d*x))])/d^2 + (2*B*(A + B*Log[(e*(a + b*x))/(c
+ d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^2 - (2*B^2*PolyLog[3,
(d*(a + b*x))/(b*(c + d*x))])/d^2))/i
```

### 3.86.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.86.4 Maple [F]

$$\int \frac{(bgx + ag)\left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{dix + ci}\,dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x)
```

## 3.86.5 Fricas [F]

$$\int \frac{(ag + bgx)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci + dix}\,dx = \int \frac{(bgx + ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{dix + ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algori
thm="fricas")
```

output
```
integral((A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/(d
*x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c)))/(d*i*x
+ c*i), x)
```

### 3.86.6 Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \frac{g\left(\int \frac{A^2a}{c+dx}\,dx+\int \frac{A^2bx}{c+dx}\,dx+\int \frac{B^2a\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx+\int \frac{2ABa\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx+\int \frac{B^2bx\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx}{i}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i),x)
```

output
```
g*(Integral(A**2*a/(c + d*x), x) + Integral(A**2*b*x/(c + d*x), x) + Integ
ral(B**2*a*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integra
l(2*A*B*a*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x) + Integral(B*
*2*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c + d*x), x) + Integral(2*
A*B*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c + d*x), x))/i
```

### 3.86.7 Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algori
thm="maxima")
```

output
```
A^2*b*g*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + A^2*a*g*log(d*i*x + c*i)/(d*i
) + 1/3*(3*B^2*b*d*g*x*log(d*x + c)^2 - (b*c*g - a*d*g)*B^2*log(d*x + c)^3
)/(d^2*i) - integrate(-(B^2*a*g*log(e)^2 + 2*A*B*a*g*log(e) + (B^2*b*g*x +
 B^2*a*g)*log(b*x + a)^2 + (B^2*b*g*log(e)^2 + 2*A*B*b*g*log(e))*x + 2*(B^
2*a*g*log(e) + A*B*a*g + (B^2*b*g*log(e) + A*B*b*g)*x)*log(b*x + a) - 2*(B
^2*a*g*log(e) + A*B*a*g + ((g*log(e) + g)*B^2*b + A*B*b*g)*x + (B^2*b*g*x
+ B^2*a*g)*log(b*x + a))*log(d*x + c))/(d*i*x + c*i), x)
```

### 3.86.8 Giac [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algori
thm="giac")
```

output
```
integrate((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i)
, x)
```

### 3.86.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x),x
)
```

output
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x),
x)
```

# 3.87 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$



## 3.87.1 Optimal result

Integrand size = 32, antiderivative size = 127

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx = -\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di} - \frac{2B\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{di} + \frac{2B^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

output
```
-ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d/i-2*B*(A+B*ln(e*
(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d/i+2*B^2*polylog(3,d*(b*
x+a)/b/(d*x+c))/d/i
```

## 3.87.2 Mathematica [A] (verified)

Time = 0.58 (sec) , antiderivative size = 252, normalized size of antiderivative = 1.98

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{A^2\log(c+dx)+2AB\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-2AB\log\left(\frac{e(a+bx)}{c+dx}\right)\log\left(\frac{bc-ad}{bc+bdx}\right)-B^2\log^2\left(\frac{e(a+bx)}{c+dx}\right)\log\left(\right.}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x),x]
```

output
```
(A^2*Log[c + d*x] + 2*A*B*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*c - a*d
)/(b*c + b*d*x)] - 2*A*B*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*d)/(b*c
 + b*d*x)] - B^2*Log[(e*(a + b*x))/(c + d*x)]^2*Log[(b*c - a*d)/(b*c + b*d
*x)] + A*B*Log[(b*c - a*d)/(b*c + b*d*x)]^2 - 2*B^2*Log[(e*(a + b*x))/(c +
 d*x)]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*A*B*PolyLog[2, (b*(c +
d*x))/(b*c - a*d)] + 2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/(d*i)
```

## 3.87.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.94, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {2952, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{ci+dix}dx$$

$$\Big\downarrow \; 2952$$

$$\frac{\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \; 2754$$

$$\frac{\frac{2B\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d}}{i}$$

$$\Big\downarrow \; 2821$$

$$\frac{\frac{2B\left(B\int \frac{(c+dx)\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d}}{i}$$

$$\Big\downarrow \; 7143$$

$$\frac{\frac{2B\left(B\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)-\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d}}{i}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x),x]
```

output
```
(-(((A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d
*x))])/d) + (2*B*(-((A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a
+ b*x))/(b*(c + d*x))]) + B*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]))/d)/i
```

### 3.87.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.87.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 337 vs. $2(127) = 254$.

Time = 1.41 (sec) , antiderivative size = 338, normalized size of antiderivative = 2.66

| method | result |
|---|---|
| parts | $\frac{A^2\ln(dx+c)}{id} - \frac{B^2\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)+2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\mathrm{Li}_2\left(\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)-2\right.}{id}$ |
| risch | $\frac{A^2\ln(dx+c)}{id} - \frac{B^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{di} - \frac{2B^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\mathrm{Li}_2\left(\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)}{di}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{dA^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ie(ad-cb)}+\frac{dB^2\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)+2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\mathrm{Li}\right.}{ie(ad-cb)}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(\frac{dA^2\ln\left(be-\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)d\right)}{ie(ad-cb)}+\frac{dB^2\left(\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(1-\frac{d\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{be}\right)+2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\mathrm{Li}\right.}{ie(ad-cb)}\right.}{}$ |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x,method=_RETURNVERBOSE)
```

output
```
A^2/i*ln(d*x+c)/d-B^2/i/d*(ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(1-1/b/e*d*
(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*polylog(2
,1/b/e*d*(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-2*polylog(3,1/b/e*d*(b*e/d+(a*d-b*
c)*e/d/(d*x+c))))+2*B*A/i*(-dilog(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d-b*e)/b
/e)/d-ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(-((b*e/d+(a*d-b*c)*e/d/(d*x+c))*d
-b*e)/b/e)/d)
```

### 3.87.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorithm="fricas"
)
```

output
```
integral((B^2*log((b*e*x + a*e)/(d*x + c))^2 + 2*A*B*log((b*e*x + a*e)/(d*
x + c)) + A^2)/(d*i*x + c*i), x)
```

### 3.87.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\frac{\int \frac{A^2}{c+dx}\,dx+\int \frac{B^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c+dx}\,dx+\int \frac{2AB\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c+dx}\,dx}{i}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i),x)
```

output
```
(Integral(A**2/(c + d*x), x) + Integral(B**2*log(a*e/(c + d*x) + b*e*x/(c
+ d*x))**2/(c + d*x), x) + Integral(2*A*B*log(a*e/(c + d*x) + b*e*x/(c + d
*x))/(c + d*x), x))/i
```

### 3.87.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorithm="maxima"
)
```

output
```
1/3*B^2*log(d*x + c)^3/(d*i) + A^2*log(d*i*x + c*i)/(d*i) - integrate(-(B^
2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*log(e) + A*B)*log(
b*x + a) - 2*(B^2*log(b*x + a) + B^2*log(e) + A*B)*log(d*x + c))/(d*i*x +
c*i), x)
```

### 3.87.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i), x)
```

### 3.87.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(c*i + d*i*x),x)
```

output
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(c*i + d*i*x), x)
```

# 3.88 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$



## 3.88.1 Optimal result

Integrand size = 42, antiderivative size = 44

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)gi}$$

output
```
1/3*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)/g/i
```

## 3.88.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 79, normalized size of antiderivative = 1.80

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{3A^2\log\left(\frac{e(a+bx)}{c+dx}\right)+3AB\log^2\left(\frac{e(a+bx)}{c+dx}\right)+B^2\log^3\left(\frac{e(a+bx)}{c+dx}\right)}{3bcgi-3adgi}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i
*x)),x]
```

output
```
(3*A^2*Log[(e*(a + b*x))/(c + d*x)] + 3*A*B*Log[(e*(a + b*x))/(c + d*x)]^2
 + B^2*Log[(e*(a + b*x))/(c + d*x)]^3)/(3*b*c*g*i - 3*a*d*g*i)
```

### 3.88.3 Rubi [A] (warning: unable to verify)

Time = 0.33 (sec) , antiderivative size = 37, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2962, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)(ci+dix)}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}}{gi(bc-ad)}$$

$$\Big\downarrow \text{2739}$$

$$\frac{\int \frac{(a+bx)^2}{(c+dx)^2}d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{Bgi(bc-ad)}$$

$$\Big\downarrow \text{15}$$

$$\frac{(a+bx)^3}{3Bgi(c+dx)^3(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i*x)),x
]
```

output
```
(a + b*x)^3/(3*B*(b*c - a*d)*g*i*(c + d*x)^3)
```

#### 3.88.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.88.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 105 vs. $2(42) = 84$.

Time = 0.66 (sec) , antiderivative size = 106, normalized size of antiderivative = 2.41

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{B^{2} \ln \left(\frac{e(bx+a)}{dx+c}\right)^{3} b^{2} d^{2}+3 A B \ln \left(\frac{e(bx+a)}{dx+c}\right)^{2} b^{2} d^{2}+3 A^{2} \ln \left(\frac{e(bx+a)}{dx+c}\right) b^{2} d^{2}}{3 b^{2} d^{2} g i (ad-cb)}$ | 106 |
| norman | $-\frac{B^{2} \ln \left(\frac{e(bx+a)}{dx+c}\right)^{3}}{3 g i (ad-cb)}-\frac{A^{2} \ln \left(\frac{e(bx+a)}{dx+c}\right)}{g i (ad-cb)}-\frac{B A \ln \left(\frac{e(bx+a)}{dx+c}\right)^{2}}{g i (ad-cb)}$ | 113 |
| parts | $\frac{A^{2}\left(\frac{\ln(dx+c)}{ad-cb}-\frac{\ln(bx+a)}{ad-cb}\right)}{g i}-\frac{B^{2} \ln \left(\frac{e(bx+a)}{dx+c}\right)^{3}}{3 g i (ad-cb)}-\frac{B A \ln \left(\frac{e(bx+a)}{dx+c}\right)^{2}}{g i (ad-cb)}$ | 123 |
| risch | $\frac{A^{2} \ln(dx+c)}{g i (ad-cb)}-\frac{A^{2} \ln(bx+a)}{g i (ad-cb)}-\frac{B^{2} \ln \left(\frac{e(bx+a)}{dx+c}\right)^{3}}{3 g i (ad-cb)}-\frac{B A \ln \left(\frac{e(bx+a)}{dx+c}\right)^{2}}{g i (ad-cb)}$ | 130 |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{d^{2} A^{2} \ln \left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e i (ad-cb)^{2} g}+\frac{d^{2} A B \ln \left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}{e i (ad-cb)^{2} g}+\frac{d^{2} B^{2} \ln \left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{3}}{3 e i (ad-cb)^{2} g}\right)}{d^{2}}$ | 182 |
| default | $-\frac{e(ad-cb)\left(\frac{d^{2} A^{2} \ln \left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e i (ad-cb)^{2} g}+\frac{d^{2} A B \ln \left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{2}}{e i (ad-cb)^{2} g}+\frac{d^{2} B^{2} \ln \left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^{3}}{3 e i (ad-cb)^{2} g}\right)}{d^{2}}$ | 182 |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i),x,method=_RETURN
VERBOSE)
```

output
```
-1/3*(B^2*ln(e*(b*x+a)/(d*x+c))^3*b^2*d^2+3*A*B*ln(e*(b*x+a)/(d*x+c))^2*b^
2*d^2+3*A^2*ln(e*(b*x+a)/(d*x+c))*b^2*d^2)/b^2/d^2/g/i/(a*d-b*c)
```

### 3.88.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 87 vs. $2(42) = 84$.

Time = 0.31 (sec) , antiderivative size = 87, normalized size of antiderivative = 1.98

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{B^2\log\left(\frac{bex+ae}{dx+c}\right)^3+3\,AB\log\left(\frac{bex+ae}{dx+c}\right)^2+3\,A^2\log\left(\frac{bex+ae}{dx+c}\right)}{3\,(bc-ad)gi}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i),x, algori
thm="fricas")
```

output
```
1/3*(B^2*log((b*e*x + a*e)/(d*x + c))^3 + 3*A*B*log((b*e*x + a*e)/(d*x + c
))^2 + 3*A^2*log((b*e*x + a*e)/(d*x + c)))/((b*c - a*d)*g*i)
```

### 3.88.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 206 vs. $2(31) = 62$.

Time = 0.38 (sec) , antiderivative size = 206, normalized size of antiderivative = 4.68

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = A^2\left(\frac{\log\left(x+\frac{-\frac{a^2d^2}{ad-bc}+\frac{2abcd}{ad-bc}+ad-\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{gi\,(ad-bc)} - \frac{\log\left(x+\frac{\frac{a^2d^2}{ad-bc}-\frac{2abcd}{ad-bc}+ad+\frac{b^2c^2}{ad-bc}+bc}{2bd}\right)}{gi\,(ad-bc)}\right) - \frac{AB\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{adgi-bcgi} - \frac{B^2\log\left(\frac{e(a+bx)}{c+dx}\right)^3}{3adgi-3bcgi}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)/(d*i*x+c*i),x)
```

output
```
A**2*(log(x + (-a**2*d**2/(a*d - b*c) + 2*a*b*c*d/(a*d - b*c) + a*d - b**2
*c**2/(a*d - b*c) + b*c)/(2*b*d))/(g*i*(a*d - b*c)) - log(x + (a**2*d**2/(
a*d - b*c) - 2*a*b*c*d/(a*d - b*c) + a*d + b**2*c**2/(a*d - b*c) + b*c)/(2
*b*d))/(g*i*(a*d - b*c))) - A*B*log(e*(a + b*x)/(c + d*x))**2/(a*d*g*i - b
*c*g*i) - B**2*log(e*(a + b*x)/(c + d*x))**3/(3*a*d*g*i - 3*b*c*g*i)
```

### 3.88.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 397 vs. 2(42) = 84.

Time = 0.22 (sec) , antiderivative size = 397, normalized size of antiderivative = 9.02

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = B^2\left(\frac{\log(bx+a)}{(bc-ad)gi}-\frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)^2$$
$$+2\,AB\left(\frac{\log(bx+a)}{(bc-ad)gi}-\frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)$$
$$-\frac{1}{3}B^2\left(\frac{3\left(\log(bx+a)^2-2\log(bx+a)\log(dx+c)+\log(dx+c)^2\right)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{bcgi-adgi}-\frac{\log(bx+a)^3}{}\right.$$
$$+A^2\left(\frac{\log(bx+a)}{(bc-ad)gi}-\frac{\log(dx+c)}{(bc-ad)gi}\right)$$
$$-\frac{\left(\log(bx+a)^2-2\log(bx+a)\log(dx+c)+\log(dx+c)^2\right)AB}{bcgi-adgi}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i),x, algori
thm="maxima")
```

output
```
B^2*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))*log(
b*e*x/(d*x + c) + a*e/(d*x + c))^2 + 2*A*B*(log(b*x + a)/((b*c - a*d)*g*i)
 - log(d*x + c)/((b*c - a*d)*g*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) -
1/3*B^2*(3*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c) + log(d*x + c)^2)
*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b*c*g*i - a*d*g*i) - (log(b*x + a)^
3 - 3*log(b*x + a)^2*log(d*x + c) + 3*log(b*x + a)*log(d*x + c)^2 - log(d*
x + c)^3)/(b*c*g*i - a*d*g*i)) + A^2*(log(b*x + a)/((b*c - a*d)*g*i) - log
(d*x + c)/((b*c - a*d)*g*i)) - (log(b*x + a)^2 - 2*log(b*x + a)*log(d*x +
c) + log(d*x + c)^2)*A*B/(b*c*g*i - a*d*g*i)
```

### 3.88.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 132 vs. 2(42) = 84.

Time = 0.47 (sec) , antiderivative size = 132, normalized size of antiderivative = 3.00

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$$
$$=\frac{\left(B^2e\log\left(\frac{bex+ae}{dx+c}\right)^3+3\,ABe\log\left(\frac{bex+ae}{dx+c}\right)^2+3\,A^2e\log\left(\frac{bex+ae}{dx+c}\right)\right)\left(\frac{bc}{(bce-ade)(bc-ad)}-\frac{ad}{(bce-ade)(bc-ad)}\right)}{3\,gi}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i),x, algori
thm="giac")
```

output
```
1/3*(B^2*e*log((b*e*x + a*e)/(d*x + c))^3 + 3*A*B*e*log((b*e*x + a*e)/(d*x
 + c))^2 + 3*A^2*e*log((b*e*x + a*e)/(d*x + c)))*(b*c/((b*c*e - a*d*e)*(b*
c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))/(g*i)
```

### 3.88.9 Mupad [B] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 96, normalized size of antiderivative = 2.18

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$$
$$= -\frac{-6\mathrm{i}\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,A^2+3\,A\,B\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2+B^2\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g\,i\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)*(c*i + d*i*x)),x
)
```

output
```
-(B^2*log((e*(a + b*x))/(c + d*x))^3 - A^2*atan((a*d*1i + b*c*1i + b*d*x*2
i)/(a*d - b*c))*6i + 3*A*B*log((e*(a + b*x))/(c + d*x))^2)/(3*g*i*(a*d - b
*c))
```

# 3.89 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$



## 3.89.1 Optimal result

Integrand size = 42, antiderivative size = 183

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{2bB^2(c+dx)}{(bc-ad)^2g^2i(a+bx)} - \frac{2bB(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2g^2i(a+bx)} - \frac{d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^2g^2i}$$

output
```
-2*b*B^2*(d*x+c)/(-a*d+b*c)^2/g^2/i/(b*x+a)-2*b*B*(d*x+c)*(A+B*ln(e*(b*x+a
)/(d*x+c)))/(-a*d+b*c)^2/g^2/i/(b*x+a)-b*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)
))^2/(-a*d+b*c)^2/g^2/i/(b*x+a)-1/3*d*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*
d+b*c)^2/g^2/i
```

### 3.89.2 Mathematica [A] (verified)

Time = 0.43 (sec) , antiderivative size = 186, normalized size of antiderivative = 1.02

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-\frac{3(A^2+2AB+2B^2)\,d(a+bx)\log(a+bx)+6B(A+B)(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right)+3B(aAd+Abdx+b}{3(bc-}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d
*i*x)),x]
```

output
```
-1/3*(3*(A^2 + 2*A*B + 2*B^2)*d*(a + b*x)*Log[a + b*x] + 6*B*(A + B)*(b*c
- a*d)*Log[(e*(a + b*x))/(c + d*x)] + 3*B*(a*A*d + A*b*d*x + b*B*(c + d*x)
)*Log[(e*(a + b*x))/(c + d*x)]^2 + B^2*d*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)]^3 + 3*(A^2 + 2*A*B + 2*B^2)*(b*c - a*d - d*(a + b*x)*Log[c + d*x]))/
((b*c - a*d)^2*g^2*i*(a + b*x))
```

### 3.89.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 136, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2(ci+dix)}dx$$
$$\Big\downarrow \text{2962}$$
$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i(bc-ad)^2}$$
$$\Big\downarrow \text{2795}$$
$$\frac{\int \left(\frac{b(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}-\frac{d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}\right)d\frac{a+bx}{c+dx}}{g^2i(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{d\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{3B}-\frac{b(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}-\frac{2bB(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{2bB^2(c+dx)}{a+bx}}{g^2i(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d*i*x))
,x]
```

output
```
((-2*b*B^2*(c + d*x))/(a + b*x) - (2*b*B*(c + d*x)*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(a + b*x) - (b*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2)/(a + b*x) - (d*(A + B*Log[(e*(a + b*x))/(c + d*x)])^3)/(3*B))/((b*c
- a*d)^2*g^2*i)
```

### 3.89.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.89.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(181) = 362$.

Time = 0.98 (sec) , antiderivative size = 400, normalized size of antiderivative = 2.19

| method | result |
|---|---|
| norman | $\frac{-\frac{\left(A^2ad+2ABbc+2B^2bc\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{B(Aad+Bbc)\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{B^2ad\ln\left(\frac{e(bx+a)}{dx+c}\right)^3}{3gi\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{b\left(A^2d+2ABd+2B^2d\right)x\ln\left(\frac{e(\cdots}{\cdots}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}}{g(bx+a)}$ |
| parts | $\frac{A^2\left(\frac{d\ln(dx+c)}{(ad-cb)^2}+\frac{1}{(bx+a)(ad-cb)}-\frac{d\ln(bx+a)}{(ad-cb)^2}\right)}{g^2i}-\frac{B^2\left(\frac{d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3(ad-cb)^2}-\frac{dbe\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{2\ln\left(\frac{be}{d}+\frac{(ad-\cdots}{d(dx\cdots}\right)}{\frac{be}{d}+\frac{(ad-cb\cdots}{d(dx+\cdots}}\cdots\right)}{(ad-cb)^2}\cdots\right)}{g^2id}$ |
| parallelrisch | $-\frac{3ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a^3bc^2d+6ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)a^3bc^2d+B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^3a^4c^2d+3B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a^3bc^3-3A^2xa\cdots}{\cdots}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{d^2A^2b}{i(ad-cb)^3g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^3A^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^3g^2}-\frac{2d^2ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{i(ad-cb)^3g^2}+\frac{d^3\cdots}{\cdots}\cdots\right)}{\cdots}$ |
| default | $-\frac{e(ad-cb)\left(\frac{d^2A^2b}{i(ad-cb)^3g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^3A^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^3g^2}-\frac{2d^2ABb\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}-\frac{1}{\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}}\right)}{i(ad-cb)^3g^2}+\frac{d^3\cdots}{\cdots}\cdots\right)}{\cdots}$ |
| risch | $\frac{A^2d\ln(dx+c)}{g^2i(ad-cb)^2}+\frac{A^2}{g^2i(bx+a)(ad-cb)}-\frac{A^2d\ln(bx+a)}{g^2i(ad-cb)^2}-\frac{B^2d\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3g^2i(ad-cb)^2}-\frac{B^2be\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{g^2i(ad-cb)^2\left(\frac{be}{d}+\frac{ea}{dx+c}-\frac{ec\cdots}{d(dx\cdots}\right)}\cdots$ |

input

```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x,method=_RETU
RNVERBOSE)
```

output

```
(-(A^2*a*d+2*A*B*b*c+2*B^2*b*c)/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+
a)/(d*x+c))-B*(A*a*d+B*b*c)/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(
d*x+c))^2-1/3*B^2*a*d/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(d*x+c)
)^3-1/g/i*b*(A^2*d+2*A*B*d+2*B^2*d)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*
x+a)/(d*x+c))-(A^2+2*A*B+2*B^2)*b/g/i/a/(a*d-b*c)*x-1/3*b*B^2*d/g/i/(a^2*d
^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^3-B*b*d*(A+B)/g/i/(a^2*d^2-2
*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^2)/g/(b*x+a)
```

### 3.89.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 231, normalized size of antiderivative = 1.26

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{(B^2bdx+B^2ad)\log\left(\frac{bex+ae}{dx+c}\right)^3+3\,(A^2+2\,AB+2\,B^2)bc-3\,(A^2+2\,AB+2\,B^2)ad+3\,(B^2bc+ABad}{3\,((b^3c^2-2\,ab^2cd+a^2bd^2)g^2ix+}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x, algo
rithm="fricas")
```

output
```
-1/3*((B^2*b*d*x + B^2*a*d)*log((b*e*x + a*e)/(d*x + c))^3 + 3*(A^2 + 2*A*
B + 2*B^2)*b*c - 3*(A^2 + 2*A*B + 2*B^2)*a*d + 3*(B^2*b*c + A*B*a*d + (A*B
 + B^2)*b*d*x)*log((b*e*x + a*e)/(d*x + c))^2 + 3*(A^2*a*d + (A^2 + 2*A*B
+ 2*B^2)*b*d*x + 2*(A*B + B^2)*b*c)*log((b*e*x + a*e)/(d*x + c)))/((b^3*c^
2 - 2*a*b^2*c*d + a^2*b*d^2)*g^2*i*x + (a*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)
*g^2*i)
```

### 3.89.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 541 vs. $2(158) = 316$.

Time = 0.74 (sec) , antiderivative size = 541, normalized size of antiderivative = 2.96

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$
$$= -\frac{B^2d\log\left(\frac{e(a+bx)}{c+dx}\right)^3}{3a^2d^2g^2i-6abcdg^2i+3b^2c^2g^2i}+\frac{(2AB+2B^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{a^2dg^2i-abcg^2i+abdg^2ix-b^2cg^2ix}$$
$$+\left(A^2+2AB+2B^2\right)\left(\frac{d\log\left(x+\frac{-\frac{a^3d^4}{(ad-bc)^2}+\frac{3a^2bcd^3}{(ad-bc)^2}-\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2+\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{g^2i\,(ad-bc)^2}\right.$$
$$-\frac{d\log\left(x+\frac{\frac{a^3d^4}{(ad-bc)^2}-\frac{3a^2bcd^3}{(ad-bc)^2}+\frac{3ab^2c^2d^2}{(ad-bc)^2}+ad^2-\frac{b^3c^3d}{(ad-bc)^2}+bcd}{2bd^2}\right)}{g^2i\,(ad-bc)^2}$$
$$\left.+\frac{1}{a^2dg^2i-abcg^2i+x\left(abdg^2i-b^2cg^2i\right)}\right)$$
$$+\frac{(-ABad-ABbdx-B^2bc-B^2bdx)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{a^3d^2g^2i-2a^2bcdg^2i+a^2bd^2g^2ix+ab^2c^2g^2i-2ab^2cdg^2ix+b^3c^2g^2ix}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2/(d*i*x+c*i),x)
```

output
```
-B**2*d*log(e*(a + b*x)/(c + d*x))**3/(3*a**2*d**2*g**2*i - 6*a*b*c*d*g**2
*i + 3*b**2*c**2*g**2*i) + (2*A*B + 2*B**2)*log(e*(a + b*x)/(c + d*x))/(a*
*2*d*g**2*i - a*b*c*g**2*i + a*b*d*g**2*i*x - b**2*c*g**2*i*x) + (A**2 + 2
*A*B + 2*B**2)*(d*log(x + (-a**3*d**4/(a*d - b*c)**2 + 3*a**2*b*c*d**3/(a*
d - b*c)**2 - 3*a*b**2*c**2*d**2/(a*d - b*c)**2 + a*d**2 + b**3*c**3*d/(a*
d - b*c)**2 + b*c*d)/(2*b*d**2))/(g**2*i*(a*d - b*c)**2) - d*log(x + (a**3
*d**4/(a*d - b*c)**2 - 3*a**2*b*c*d**3/(a*d - b*c)**2 + 3*a*b**2*c**2*d**2
/(a*d - b*c)**2 + a*d**2 - b**3*c**3*d/(a*d - b*c)**2 + b*c*d)/(2*b*d**2))
/(g**2*i*(a*d - b*c)**2) + 1/(a**2*d*g**2*i - a*b*c*g**2*i + x*(a*b*d*g**2
*i - b**2*c*g**2*i))) + (-A*B*a*d - A*B*b*d*x - B**2*b*c - B**2*b*d*x)*log
(e*(a + b*x)/(c + d*x))**2/(a**3*d**2*g**2*i - 2*a**2*b*c*d*g**2*i + a**2*
b*d**2*g**2*i*x + a*b**2*c**2*g**2*i - 2*a*b**2*c*d*g**2*i*x + b**3*c**2*g
**2*i*x)
```

3.89. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$

### 3.89.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1008 vs. $2(181) = 362$.

Time = 0.26 (sec) , antiderivative size = 1008, normalized size of antiderivative = 5.51

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx =$$

$$-B^2\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log\Big(\ldots+\frac{ae}{dx+c}\Big)^2$$

$$-2\,AB\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log\Big(\ldots+\frac{ae}{dx+c}\Big)$$

$$+\frac{1}{3}B^2\left(\frac{3\left((bdx+ad)\log(bx+a)^2+(bdx+ad)\log(dx+c)^2-2\,bc+2\,ad-2\,(bdx+ad)\log(bx+a)+\ldots\right.}{ab^2c^2g^2i-2\,a^2bcdg^2i+a^3d^2g^2i+(b^3c^2g^2i-2\ldots}\right.$$

$$-A^2\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)$$

$$+\frac{\left((bdx+ad)\log(bx+a)^2+(bdx+ad)\log(dx+c)^2-2\,bc+2\,ad-2\,(bdx+ad)\log(bx+a)+2\,(bdx+\ldots\right.}{ab^2c^2g^2i-2\,a^2bcdg^2i+a^3d^2g^2i+(b^3c^2g^2i-2\,ab^2cdg^2i+a^2b\ldots}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x, algo
rithm="maxima")
```

output
```
-B^2*(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)
/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*g^2*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2 - 2*A*B*(1
/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*
c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d +
 a^2*d^2)*g^2*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) + 1/3*B^2*(3*((b*d*
x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2
*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))
*log(d*x + c))*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(a*b^2*c^2*g^2*i - 2*a
^2*b*c*d*g^2*i + a^3*d^2*g^2*i + (b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*
b*d^2*g^2*i)*x) - ((b*d*x + a*d)*log(b*x + a)^3 - (b*d*x + a*d)*log(d*x +
c)^3 - 3*(b*d*x + a*d)*log(b*x + a)^2 - 3*(b*d*x + a*d - (b*d*x + a*d)*log
(b*x + a))*log(d*x + c)^2 + 6*b*c - 6*a*d + 6*(b*d*x + a*d)*log(b*x + a) -
 3*(2*b*d*x + (b*d*x + a*d)*log(b*x + a)^2 + 2*a*d - 2*(b*d*x + a*d)*log(b
*x + a))*log(d*x + c))/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^2*g^2*
i + (b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x)) - A^2*(1/((b
^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*c^2
- 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2
*d^2)*g^2*i)) + ((b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)
^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b...
```

### 3.89.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2(dix+ci)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x, algo
rithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/((b*g*x + a*g)^2*(d*i*x + c
*i)), x)
```

### 3.89.9 Mupad [B] (verification not implemented)

Time = 2.97 (sec) , antiderivative size = 419, normalized size of antiderivative = 2.29

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$
$$= \frac{A^2+2\,A\,B+2\,B^2}{(a\,d-b\,c)\,\left(a\,g^2\,i+b\,g^2\,i\,x\right)} - \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B\,d\,(A+B)}{g^2\,i\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)} - \frac{B^2\,(a\,d-b\,c)}{b\,d\,g^2\,i\,\left(\frac{x}{d}+\frac{a}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)$$
$$- \frac{B^2\,d\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g^2\,i\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)} + \frac{2\,B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,(a\,d-b\,c)\,(A+B)}{b\,d\,g^2\,i\,\left(\frac{x}{d}+\frac{a}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}$$
$$+ \frac{d\,\mathrm{atan}\left(\frac{d\left(2\,b\,d\,x+\frac{a^2\,d^2\,g^2\,i-b^2\,c^2\,g^2\,i}{g^2\,i\,(a\,d-b\,c)}\right)\,\left(A^2+2\,A\,B+2\,B^2\right)\,1\mathrm{i}}{(a\,d-b\,c)\,\left(d\,A^2+2\,d\,A\,B+2\,d\,B^2\right)}\right)\,\left(A^2+2\,A\,B+2\,B^2\right)\,2\mathrm{i}}{g^2\,i\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^2*(c*i + d*i*x))
,x)
```

output
```
(A^2 + 2*B^2 + 2*A*B)/((a*d - b*c)*(a*g^2*i + b*g^2*i*x)) - log((e*(a + b*
x))/(c + d*x))^2*((B*d*(A + B))/(g^2*i*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) -
(B^2*(a*d - b*c))/(b*d*g^2*i*(x/d + a/(b*d))*(a^2*d^2 + b^2*c^2 - 2*a*b*c*
d))) - (B^2*d*log((e*(a + b*x))/(c + d*x))^3)/(3*g^2*i*(a^2*d^2 + b^2*c^2
- 2*a*b*c*d)) + (d*atan((d*(2*b*d*x + (a^2*d^2*g^2*i - b^2*c^2*g^2*i)/(g^2
*i*(a*d - b*c)))*(A^2 + 2*B^2 + 2*A*B)*1i)/((a*d - b*c)*(A^2*d + 2*B^2*d +
 2*A*B*d)))*(A^2 + 2*B^2 + 2*A*B)*2i)/(g^2*i*(a*d - b*c)^2) + (2*B*log((e*
(a + b*x))/(c + d*x))*(a*d - b*c)*(A + B))/(b*d*g^2*i*(x/d + a/(b*d))*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d))
```

# 3.90 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$



## 3.90.1 Optimal result

Integrand size = 42, antiderivative size = 343

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx &= \frac{4bB^2d(c+dx)}{(bc-ad)^3g^3i(a+bx)}-\frac{b^2B^2(c+dx)^2}{4(bc-ad)^3g^3i(a+bx)^2}\\
&+\frac{4bBd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^3i(a+bx)}\\
&-\frac{b^2B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2}\\
&+\frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^3i(a+bx)}\\
&-\frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3g^3i(a+bx)^2}\\
&+\frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3g^3i}\end{aligned}$$

output
```
4*b*B^2*d*(d*x+c)/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/4*b^2*B^2*(d*x+c)^2/(-a*d+b
*c)^3/g^3/i/(b*x+a)^2+4*b*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*
c)^3/g^3/i/(b*x+a)-1/2*b^2*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b
*c)^3/g^3/i/(b*x+a)^2+2*b*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*
c)^3/g^3/i/(b*x+a)-1/2*b^2*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b
*c)^3/g^3/i/(b*x+a)^2+1/3*d^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^3
/g^3/i
```

### 3.90.2 Mathematica [A] (verified)

Time = 0.65 (sec) , antiderivative size = 318, normalized size of antiderivative = 0.93

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \frac{-3(2A^2+2AB+B^2)(bc-ad)^2+6(2A^2+6AB+7B^2)\,d(bc-ad)(a+bx)+6(2A^2+6AB+7B^2)\,d^2(}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d
*i*x)),x]
```

output
```
(-3*(2*A^2 + 2*A*B + B^2)*(b*c - a*d)^2 + 6*(2*A^2 + 6*A*B + 7*B^2)*d*(b*c
 - a*d)*(a + b*x) + 6*(2*A^2 + 6*A*B + 7*B^2)*d^2*(a + b*x)^2*Log[a + b*x]
 - 6*B*(b*c - a*d)*(-6*a*A*d - 7*a*B*d + b*B*(c - 6*d*x) + 2*A*b*(c - 2*d*
x))*Log[(e*(a + b*x))/(c + d*x)] - 6*B*(-2*a^2*A*d^2 - 4*a*b*d*(A*d*x + B*
(c + d*x)) + b^2*(-2*A*d^2*x^2 + B*(c^2 - 2*c*d*x - 3*d^2*x^2)))*Log[(e*(a
 + b*x))/(c + d*x)]^2 + 4*B^2*d^2*(a + b*x)^2*Log[(e*(a + b*x))/(c + d*x)]
^3 - 6*(2*A^2 + 6*A*B + 7*B^2)*d^2*(a + b*x)^2*Log[c + d*x])/(12*(b*c - a*
d)^3*g^3*i*(a + b*x)^2)
```

### 3.90.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$$

$$\Big\downarrow \text{ 2962}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i(bc-ad)^3}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^3}{(a+bx)^3}-\frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^2}{(a+bx)^2}+\frac{d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx}}{g^3i(bc-ad)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{b^2(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}-\frac{b^2B(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{d^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{3B}+\frac{2bd(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}}{g^3i(bc-ad)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d*i*x))
,x]
```

output
```
((4*b*B^2*d*(c + d*x))/(a + b*x) - (b^2*B^2*(c + d*x)^2)/(4*(a + b*x)^2) +
 (4*b*B*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - (b^2
*B*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) + (2*
b*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x) - (b^2*(c
+ d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*x)^2) + (d^2*(A
 + B*Log[(e*(a + b*x))/(c + d*x)])^3)/(3*B))/((b*c - a*d)^3*g^3*i)
```

### 3.90.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.90.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. $2(335) = 670$.

Time = 1.31 (sec) , antiderivative size = 767, normalized size of antiderivative = 2.24

| method | result |
|---|---|
| parts | $\frac{A^2\left(\frac{d^2\ln(dx+c)}{(ad-cb)^3}+\frac{1}{2(ad-cb)(bx+a)^2}+\frac{d}{(ad-cb)^2(bx+a)}-\frac{d^2\ln(bx+a)}{(ad-cb)^3}\right)}{g^3 i}-\frac{B^2\left(\frac{d^3\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3(ad-cb)^3}-\frac{2d^2be\left(-\frac{\ln\left(\frac{be}{d}+\frac{(ad}{d(}\right.}{\frac{be}{d}+\frac{(ad}{d(}}\right.}{}\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{d^2eA^2b^2}{2i(ad-cb)^4g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{2d^3A^2b}{i(ad-cb)^4g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^4A^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^4g^3}+\frac{2d^2eABb^2\left(-\frac{\ln(}{2(}\right.}{}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{d^2eA^2b^2}{2i(ad-cb)^4g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}+\frac{2d^3A^2b}{i(ad-cb)^4g^3\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}+\frac{d^4A^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{ei(ad-cb)^4g^3}+\frac{2d^2eABb^2\left(-\frac{\ln(}{2(}\right.}{}\right.}{}$ |
| norman | $\frac{6A^2ab^2d-2A^2b^3c+14ABab^2d-2ABb^3c+15B^2ab^2d-B^2b^3c}{4gib^2\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{\left(2A^2a^2d^2+8ABabcd-2ABb^2c^2+8B^2abcd-B^2b^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right.}{2ig\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}$ |
| risch | $\frac{A^2d^2\ln(dx+c)}{g^3i(ad-cb)^3}+\frac{A^2}{2g^3i(ad-cb)(bx+a)^2}+\frac{A^2d}{g^3i(ad-cb)^2(bx+a)}-\frac{A^2d^2\ln(bx+a)}{g^3i(ad-cb)^3}-\frac{B^2d^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^3}{3g^3i(ad-cb)^3}-$ |
| parallelrisch | Expression too large to display |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x,method=_RETU
RNVERBOSE)
```

output
```
A^2/g^3/i*(d^2/(a*d-b*c)^3*ln(d*x+c)+1/2/(a*d-b*c)/(b*x+a)^2+d/(a*d-b*c)^2
/(b*x+a)-d^2/(a*d-b*c)^3*ln(b*x+a))-B^2/g^3/i/d*(1/3*d^3/(a*d-b*c)^3*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^3-2*d^2/(a*d-b*c)^3*b*e*(-1/(b*e/d+(a*d-b*c)*e/
d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c
))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+d/(a*d
-b*c)^3*e^2*b^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))-2*B*A/g^3/i/d*(1/2*d^3/(a*d-
b*c)^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*d^2/(a*d-b*c)^3*b*e*(-1/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*
e/d/(d*x+c)))+d/(a*d-b*c)^3*e^2*b^2*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

### 3.90.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 540, normalized size of antiderivative = 1.57

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx =$$
$$-\frac{3\,(2\,A^2+2\,AB+B^2)b^2c^2-24\,(A^2+2\,AB+2\,B^2)abcd+3\,(6\,A^2+14\,AB+15\,B^2)a^2d^2-4\,(B^2b^2d^2x}{}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, algo
rithm="fricas")
```

output
```
-1/12*(3*(2*A^2 + 2*A*B + B^2)*b^2*c^2 - 24*(A^2 + 2*A*B + 2*B^2)*a*b*c*d
+ 3*(6*A^2 + 14*A*B + 15*B^2)*a^2*d^2 - 4*(B^2*b^2*d^2*x^2 + 2*B^2*a*b*d^2
*x + B^2*a^2*d^2)*log((b*e*x + a*e)/(d*x + c))^3 - 6*((2*A*B + 3*B^2)*b^2*
d^2*x^2 - B^2*b^2*c^2 + 4*B^2*a*b*c*d + 2*A*B*a^2*d^2 + 2*(B^2*b^2*c*d + 2
*(A*B + B^2)*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 6*((2*A^2 + 6*A*
B + 7*B^2)*b^2*c*d - (2*A^2 + 6*A*B + 7*B^2)*a*b*d^2)*x - 6*((2*A^2 + 6*A*
B + 7*B^2)*b^2*d^2*x^2 + 2*A^2*a^2*d^2 - (2*A*B + B^2)*b^2*c^2 + 8*(A*B +
B^2)*a*b*c*d + 2*((2*A*B + 3*B^2)*b^2*c*d + 2*(A^2 + 2*A*B + 2*B^2)*a*b*d^
2)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*
c*d^2 - a^3*b^2*d^3)*g^3*i*x^2 + 2*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^
2*c*d^2 - a^4*b*d^3)*g^3*i*x + (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*
d^2 - a^5*d^3)*g^3*i)
```

### 3.90.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1488 vs. $2(303) = 606$.

Time = 4.24 (sec) , antiderivative size = 1488, normalized size of antiderivative = 4.34

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3/(d*i*x+c*i),x)
```

output
```
-B**2*d**2*log(e*(a + b*x)/(c + d*x))**3/(3*a**3*d**3*g**3*i - 9*a**2*b*c*
d**2*g**3*i + 9*a*b**2*c**2*d*g**3*i - 3*b**3*c**3*g**3*i) + d**2*(2*A**2
+ 6*A*B + 7*B**2)*log(x + (2*A**2*a*d**3 + 2*A**2*b*c*d**2 + 6*A*B*a*d**3
+ 6*A*B*b*c*d**2 + 7*B**2*a*d**3 + 7*B**2*b*c*d**2 - a**4*d**6*(2*A**2 + 6
*A*B + 7*B**2)/(a*d - b*c)**3 + 4*a**3*b*c*d**5*(2*A**2 + 6*A*B + 7*B**2)/
(a*d - b*c)**3 - 6*a**2*b**2*c**2*d**4*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*
c)**3 + 4*a*b**3*c**3*d**3*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3 - b**4
*c**4*d**2*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3)/(4*A**2*b*d**3 + 12*A
*B*b*d**3 + 14*B**2*b*d**3))/(2*g**3*i*(a*d - b*c)**3) - d**2*(2*A**2 + 6*
A*B + 7*B**2)*log(x + (2*A**2*a*d**3 + 2*A**2*b*c*d**2 + 6*A*B*a*d**3 + 6*
A*B*b*c*d**2 + 7*B**2*a*d**3 + 7*B**2*b*c*d**2 + a**4*d**6*(2*A**2 + 6*A*B
 + 7*B**2)/(a*d - b*c)**3 - 4*a**3*b*c*d**5*(2*A**2 + 6*A*B + 7*B**2)/(a*d
 - b*c)**3 + 6*a**2*b**2*c**2*d**4*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**
3 - 4*a*b**3*c**3*d**3*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3 + b**4*c**
4*d**2*(2*A**2 + 6*A*B + 7*B**2)/(a*d - b*c)**3)/(4*A**2*b*d**3 + 12*A*B*b
*d**3 + 14*B**2*b*d**3))/(2*g**3*i*(a*d - b*c)**3) + (6*A*B*a*d - 2*A*B*b*
c + 4*A*B*b*d*x + 7*B**2*a*d - B**2*b*c + 6*B**2*b*d*x)*log(e*(a + b*x)/(c
 + d*x))/(2*a**4*d**2*g**3*i - 4*a**3*b*c*d*g**3*i + 4*a**3*b*d**2*g**3*i*
x + 2*a**2*b**2*c**2*g**3*i - 8*a**2*b**2*c*d*g**3*i*x + 2*a**2*b**2*d**2*
g**3*i*x**2 + 4*a*b**3*c**2*g**3*i*x - 4*a*b**3*c*d*g**3*i*x**2 + 2*b**...
```

### 3.90.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2115 vs. $2(335) = 670$.

Time = 0.35 (sec) , antiderivative size = 2115, normalized size of antiderivative = 6.17

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, algo
rithm="maxima")
```

output
```
1/2*B^2*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^
3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(b*e*x/(d*x + c) + a*e/
(d*x + c))^2 + A*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*
b^2*d^2)*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (
a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(b*e*x/(d*x
+ c) + a*e/(d*x + c)) - 1/12*B^2*(6*(b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(
b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a
*b*d^2*x + a^2*d^2)*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*
x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x
 + 3*a^2*d^2 - 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d
*x + c))*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(a^2*b^3*c^3*g^3*i - 3*a^3*b
^2*c^2*d*g^3*i + 3*a^4*b*c*d^2*g^3*i - a^5*d^3*g^3*i + (b^5*c^3*g^3*i - 3*
a*b^4*c^2*d*g^3*i + 3*a^2*b^3*c*d^2*g^3*i - a^3*b^2*d^3*g^3*i)*x^2 + 2*(a*
b^4*c^3*g^3*i - 3*a^2*b^3*c^2*d*g^3*i + 3*a^3*b^2*c*d^2*g^3*i - a^4*b*d^3*
g^3*i)*x) + (3*b^2*c^2 - 48*a*b*c*d + 45*a^2*d^2 - 4*(b^2*d^2*x^2 + 2*a...
```

### 3.90.8 Giac [A] (verification not implemented)

Time = 58.16 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.62

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\,(dx+c)^2B^2e^3\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)^2g^3i}+\frac{2\,(2\,ABe^3+B^2e^3)(dx+c)^2\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)^2g^3i}+\frac{(2\,A^2e^3+2\,ABe^3+B^2}{(bex+ae)^2g}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x, algo
rithm="giac")
```

output
```
-1/4*(2*(d*x + c)^2*B^2*e^3*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)^
2*g^3*i) + 2*(2*A*B*e^3 + B^2*e^3)*(d*x + c)^2*log((b*e*x + a*e)/(d*x + c)
)/((b*e*x + a*e)^2*g^3*i) + (2*A^2*e^3 + 2*A*B*e^3 + B^2*e^3)*(d*x + c)^2/
((b*e*x + a*e)^2*g^3*i))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e
- a*d*e)*(b*c - a*d)))^2
```

### 3.90.9 Mupad [B] (verification not implemented)

Time = 4.84 (sec) , antiderivative size = 981, normalized size of antiderivative = 2.86

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{\frac{B^2\,d^2\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{2\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{2\,b\,d^2}\right)}{g^3\,i\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}+\frac{B^2\,x\,(a\,d-b\,c)}{g^3\,i\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}} - \frac{B\,d^2\,(2\,A+3\,B)}{2\,g^3\,i\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)$$
$$- \frac{\frac{6\,A^2\,a\,d-2\,A^2\,b\,c+15\,B^2\,a\,d-B^2\,b\,c+14\,A\,B\,a\,d-2\,A\,B\,b\,c}{2\,(a\,d-b\,c)}+\frac{x\,(2\,b\,d\,A^2+6\,b\,d\,A\,B+7\,b\,d\,B^2)}{a\,d-b\,c}}{x^2\,(2\,b^3\,c\,g^3\,i-2\,a\,b^2\,d\,g^3\,i)+x\,(4\,a\,b^2\,c\,g^3\,i-4\,a^2\,b\,d\,g^3\,i)-2\,a^3\,d\,g^3\,i+2\,a^2\,b\,c\,g^3\,i}$$
$$+ \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B\,d^2\left(\frac{2\,a^2\,d^2-3\,a\,b\,c\,d+b^2\,c^2}{2\,b\,d^3}+\frac{a\,(a\,d-b\,c)}{2\,b\,d^2}\right)(2\,A+3\,B)}{g^3\,i\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}-\frac{B^2}{b\,d\,g^3\,i\,(a\,d-b\,c)}+\frac{B\,x\,(2\,A+3\,B)\,(a\,d-b\,c)}{g^3\,i\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)}{\frac{b\,x^2}{d}+\frac{a^2}{b\,d}+\frac{2\,a\,x}{d}}$$
$$- \frac{B^2\,d^2\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g^3\,i\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}$$
$$+ \frac{d^2\,\mathrm{atan}\left(\frac{d^2\left(A^2+3\,A\,B+\frac{7\,B^2}{2}\right)(2\,i\,a^3\,d^3\,g^3-2\,i\,a^2\,b\,c\,d^2\,g^3-2\,i\,a\,b^2\,c^2\,d\,g^3+2\,i\,b^3\,c^3\,g^3)\,1\mathrm{i}}{g^3\,i\,(a\,d-b\,c)^3\,(2\,A^2\,d^2+6\,A\,B\,d^2+7\,B^2\,d^2)}+\frac{b\,d^3\,x\,(i\,a^2\,d^2\,g^3-2\,i\,a\,b\,c\,d\,g^3+i\,b^2\,c^2\,g^3)\,(A}{g^3\,i\,(a\,d-b\,c)^3\,(2\,A^2\,d^2+6\,A\,B\,d^2-}\right.}{g^3\,i\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^3*(c*i + d*i*x))
,x)
```

output
```
log((e*(a + b*x))/(c + d*x))^2*(((B^2*d^2*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*
d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2)))/(g^3*i*(a^3*d^3 - b^3*c^3 + 3*a
*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B^2*x*(a*d - b*c))/(g^3*i*(a^3*d^3 - b^3*c
^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/((b*x^2)/d + a^2/(b*d) + (2*a*x)/d)
- (B*d^2*(2*A + 3*B))/(2*g^3*i*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*
b*c*d^2))) - ((6*A^2*a*d - 2*A^2*b*c + 15*B^2*a*d - B^2*b*c + 14*A*B*a*d -
 2*A*B*b*c)/(2*(a*d - b*c)) + (x*(2*A^2*b*d + 7*B^2*b*d + 6*A*B*b*d))/(a*d
 - b*c))/(x^2*(2*b^3*c*g^3*i - 2*a*b^2*d*g^3*i) + x*(4*a*b^2*c*g^3*i - 4*a
^2*b*d*g^3*i) - 2*a^3*d*g^3*i + 2*a^2*b*c*g^3*i) + (log((e*(a + b*x))/(c +
 d*x))*((B*d^2*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*
c))/(2*b*d^2))*(2*A + 3*B))/(g^3*i*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*
a^2*b*c*d^2)) - B^2/(b*d*g^3*i*(a*d - b*c)) + (B*x*(2*A + 3*B)*(a*d - b*c)
)/(g^3*i*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((b*x^2)/d
 + a^2/(b*d) + (2*a*x)/d) - (B^2*d^2*log((e*(a + b*x))/(c + d*x))^3)/(3*g^
3*i*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (d^2*atan((d^2*
(A^2 + (7*B^2)/2 + 3*A*B)*(2*a^3*d^3*g^3*i + 2*b^3*c^3*g^3*i - 2*a*b^2*c^2
*d*g^3*i - 2*a^2*b*c*d^2*g^3*i)*1i)/(g^3*i*(a*d - b*c)^3*(2*A^2*d^2 + 7*B^
2*d^2 + 6*A*B*d^2)) + (b*d^3*x*(a^2*d^2*g^3*i + b^2*c^2*g^3*i - 2*a*b*c*d*
g^3*i)*(A^2 + (7*B^2)/2 + 3*A*B)*4i)/(g^3*i*(a*d - b*c)^3*(2*A^2*d^2 + 7*B
^2*d^2 + 6*A*B*d^2)))*(A^2 + (7*B^2)/2 + 3*A*B)*2i)/(g^3*i*(a*d - b*c)^...
```

3.90. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$

# 3.91 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$



## 3.91.1 Optimal result

Integrand size = 42, antiderivative size = 507

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = &-\frac{6bB^2d^2(c+dx)}{(bc-ad)^4g^4i(a+bx)}+\frac{3b^2B^2d(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2}\\
&-\frac{2b^3B^2(c+dx)^3}{27(bc-ad)^4g^4i(a+bx)^3}\\
&-\frac{6bBd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^4i(a+bx)}\\
&+\frac{3b^2Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2}\\
&-\frac{2b^3B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^4g^4i(a+bx)^3}\\
&-\frac{3bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^4i(a+bx)}\\
&+\frac{3b^2d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^4i(a+bx)^2}\\
&-\frac{b^3(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^4g^4i(a+bx)^3}\\
&-\frac{d^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^4g^4i}\end{aligned}$$

output
```
-6*b*B^2*d^2*(d*x+c)/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/4*b^2*B^2*d*(d*x+c)^2/(-
a*d+b*c)^4/g^4/i/(b*x+a)^2-2/27*b^3*B^2*(d*x+c)^3/(-a*d+b*c)^4/g^4/i/(b*x+
a)^3-6*b*B*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(b*x
+a)+3/2*b^2*B*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/i/(
b*x+a)^2-2/9*b^3*B*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^4/
i/(b*x+a)^3-3*b*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^4
/i/(b*x+a)+3/2*b^2*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/
g^4/i/(b*x+a)^2-1/3*b^3*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)
^4/g^4/i/(b*x+a)^3-1/3*d^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^4/g^
4/i
```

### 3.91.2 Mathematica [A] (verified)

Time = 0.80 (sec) , antiderivative size = 442, normalized size of antiderivative = 0.87

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx =$$

$$-\frac{4(9A^2+6AB+2B^2)(bc-ad)^3-3(18A^2+30AB+19B^2)\,d(bc-ad)^2(a+bx)-6(18A^2+66AB+$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d
*i*x)),x]
```

output
```
-1/108*(4*(9*A^2 + 6*A*B + 2*B^2)*(b*c - a*d)^3 - 3*(18*A^2 + 30*A*B + 19*
B^2)*d*(b*c - a*d)^2*(a + b*x) - 6*(18*A^2 + 66*A*B + 85*B^2)*d^2*(-(b*c)
+ a*d)*(a + b*x)^2 + 6*(18*A^2 + 66*A*B + 85*B^2)*d^3*(a + b*x)^3*Log[a +
b*x] + 6*B*(b*c - a*d)*(4*(3*A + B)*(b*c - a*d)^2 + 3*(6*A + 5*B)*d*(-(b*c
) + a*d)*(a + b*x) + 6*(6*A + 11*B)*d^2*(a + b*x)^2)*Log[(e*(a + b*x))/(c
+ d*x)] + 18*B*(6*a^3*A*d^3 + 18*a^2*b*d^2*(A*d*x + B*(c + d*x)) + 9*a*b^2
*d*(2*A*d^2*x^2 + B*(-c^2 + 2*c*d*x + 3*d^2*x^2)) + b^3*(6*A*d^3*x^3 + B*(
2*c^3 - 3*c^2*d*x + 6*c*d^2*x^2 + 11*d^3*x^3)))*Log[(e*(a + b*x))/(c + d*x
)]^2 + 36*B^2*d^3*(a + b*x)^3*Log[(e*(a + b*x))/(c + d*x)]^3 - 6*(18*A^2 +
 66*A*B + 85*B^2)*d^3*(a + b*x)^3*Log[c + d*x])/((b*c - a*d)^4*g^4*i*(a +
b*x)^3)
```

### 3.91.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^4(ci+dix)}dx$$

$$\Big\downarrow \ 2962$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i(bc-ad)^4}$$

$$\Big\downarrow \ 2795$$

$$\frac{\int \left(\frac{b^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^4}{(a+bx)^4}-\frac{3b^2d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^3}{(a+bx)^3}+\frac{3bd^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2(c+dx)^2}{(a+bx)^2}-\frac{d^3\left(A+B\log\left(\frac{e(a+b}{c+dx}\right.\right.}{a+bx}\right.}{g^4i(bc-ad)^4}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{b^3(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}-\frac{2b^3B(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}+\frac{3b^2Bd(c+dx)^2\left(B\log\left(\right.\right.}{2(a+bx)^2}}{}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d*i*x))
,x]
```

output
```
((-6*b*B^2*d^2*(c + d*x))/(a + b*x) + (3*b^2*B^2*d*(c + d*x)^2)/(4*(a + b*
x)^2) - (2*b^3*B^2*(c + d*x)^3)/(27*(a + b*x)^3) - (6*b*B*d^2*(c + d*x)*(A
 + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) + (3*b^2*B*d*(c + d*x)^2*(A
+ B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) - (2*b^3*B*(c + d*x)^3*
(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(9*(a + b*x)^3) - (3*b*d^2*(c + d*x)
*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x) + (3*b^2*d*(c + d*x)^2*
(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*x)^2) - (b^3*(c + d*x)^3
*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(3*(a + b*x)^3) - (d^3*(A + B*Log
[(e*(a + b*x))/(c + d*x)])^3)/(3*B))/((b*c - a*d)^4*g^4*i)
```

#### 3.91.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.91.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1075 vs. $2(493) = 986$.

Time = 1.84 (sec) , antiderivative size = 1076, normalized size of antiderivative = 2.12

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1076 |
| derivativedivides | Expression too large to display | 1244 |
| default | Expression too large to display | 1244 |
| risch | Expression too large to display | 1422 |
| norman | Expression too large to display | 1693 |
| parallelrisch | Expression too large to display | 1967 |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x,method=_RETU
RNVERBOSE)
```

output
```
A^2/g^4/i*(d^3/(a*d-b*c)^4*ln(d*x+c)+1/3/(a*d-b*c)/(b*x+a)^3+1/2*d/(a*d-b*
c)^2/(b*x+a)^2+d^2/(a*d-b*c)^3/(b*x+a)-d^3/(a*d-b*c)^4*ln(b*x+a))-B^2/g^4/
i/d*(1/3*d^4/(a*d-b*c)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3-3*d^3/(a*d-b*c)
^4*b*e*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2
-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+
(a*d-b*c)*e/d/(d*x+c)))+3*d^2/(a*d-b*c)^4*b^2*e^2*(-1/2/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^2)-d/(a*d-b*c)^4*b^3*e^3*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d
-b*c)*e/d/(d*x+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2*B*A/g^4/i/d*(1
/2*d^4/(a*d-b*c)^4*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-3*d^3/(a*d-b*c)^4*b*e
*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/
d+(a*d-b*c)*e/d/(d*x+c)))+3*d^2/(a*d-b*c)^4*b^2*e^2*(-1/2/(b*e/d+(a*d-b*c)
*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2)-d/(a*d-b*c)^4*b^3*e^3*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(
b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))
```

## 3.91.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 940, normalized size of antiderivative = 1.85

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx =$$
$$-\frac{4\left(9A^2+6AB+2B^2\right)b^3c^3-81\left(2A^2+2AB+B^2\right)ab^2c^2d+324\left(A^2+2AB+2B^2\right)a^2bcd^2-\left(198A^2\right.}{}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x, algo
rithm="fricas")
```

output
```
-1/108*(4*(9*A^2 + 6*A*B + 2*B^2)*b^3*c^3 - 81*(2*A^2 + 2*A*B + B^2)*a*b^2
*c^2*d + 324*(A^2 + 2*A*B + 2*B^2)*a^2*b*c*d^2 - (198*A^2 + 510*A*B + 575*
B^2)*a^3*d^3 + 36*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3
*x + B^2*a^3*d^3)*log((b*e*x + a*e)/(d*x + c))^3 + 6*((18*A^2 + 66*A*B + 8
5*B^2)*b^3*c*d^2 - (18*A^2 + 66*A*B + 85*B^2)*a*b^2*d^3)*x^2 + 18*((6*A*B
+ 11*B^2)*b^3*d^3*x^3 + 2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 18*B^2*a^2*b*c
*d^2 + 6*A*B*a^3*d^3 + 3*(2*B^2*b^3*c*d^2 + 3*(2*A*B + 3*B^2)*a*b^2*d^3)*x
^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 - 6*(A*B + B^2)*a^2*b*d^3)*x)*lo
g((b*e*x + a*e)/(d*x + c))^2 - 3*((18*A^2 + 30*A*B + 19*B^2)*b^3*c^2*d - 5
4*(2*A^2 + 6*A*B + 7*B^2)*a*b^2*c*d^2 + (90*A^2 + 294*A*B + 359*B^2)*a^2*b
*d^3)*x + 6*((18*A^2 + 66*A*B + 85*B^2)*b^3*d^3*x^3 + 18*A^2*a^3*d^3 + 4*(
3*A*B + B^2)*b^3*c^3 - 27*(2*A*B + B^2)*a*b^2*c^2*d + 108*(A*B + B^2)*a^2*
b*c*d^2 + 3*(2*(6*A*B + 11*B^2)*b^3*c*d^2 + 9*(2*A^2 + 6*A*B + 7*B^2)*a*b^
2*d^3)*x^2 - 3*((6*A*B + 5*B^2)*b^3*c^2*d - 18*(2*A*B + 3*B^2)*a*b^2*c*d^2
 - 18*(A^2 + 2*A*B + 2*B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c)))/((
b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3 + a^4*b^3*d^
4)*g^4*i*x^3 + 3*(a*b^6*c^4 - 4*a^2*b^5*c^3*d + 6*a^3*b^4*c^2*d^2 - 4*a^4*
b^3*c*d^3 + a^5*b^2*d^4)*g^4*i*x^2 + 3*(a^2*b^5*c^4 - 4*a^3*b^4*c^3*d + 6*
a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4)*g^4*i*x + (a^3*b^4*c^4 - 4*
a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*g^4*i)
```

### 3.91.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2388 vs. 2(459) = 918.

Time = 29.82 (sec) , antiderivative size = 2388, normalized size of antiderivative = 4.71

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4/(d*i*x+c*i),x)
```

output
```
-B**2*d**3*log(e*(a + b*x)/(c + d*x))**3/(3*a**4*d**4*g**4*i - 12*a**3*b*c
*d**3*g**4*i + 18*a**2*b**2*c**2*d**2*g**4*i - 12*a*b**3*c**3*d*g**4*i + 3
*b**4*c**4*g**4*i) + d**3*(18*A**2 + 66*A*B + 85*B**2)*log(x + (18*A**2*a*
d**4 + 18*A**2*b*c*d**3 + 66*A*B*a*d**4 + 66*A*B*b*c*d**3 + 85*B**2*a*d**4
 + 85*B**2*b*c*d**3 - a**5*d**8*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**
4 + 5*a**4*b*c*d**7*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 - 10*a**3*
b**2*c**2*d**6*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 + 10*a**2*b**3*
c**3*d**5*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 - 5*a*b**4*c**4*d**4
*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 + b**5*c**5*d**3*(18*A**2 + 6
6*A*B + 85*B**2)/(a*d - b*c)**4)/(36*A**2*b*d**4 + 132*A*B*b*d**4 + 170*B*
*2*b*d**4))/(18*g**4*i*(a*d - b*c)**4) - d**3*(18*A**2 + 66*A*B + 85*B**2)
*log(x + (18*A**2*a*d**4 + 18*A**2*b*c*d**3 + 66*A*B*a*d**4 + 66*A*B*b*c*d
**3 + 85*B**2*a*d**4 + 85*B**2*b*c*d**3 + a**5*d**8*(18*A**2 + 66*A*B + 85
*B**2)/(a*d - b*c)**4 - 5*a**4*b*c*d**7*(18*A**2 + 66*A*B + 85*B**2)/(a*d
- b*c)**4 + 10*a**3*b**2*c**2*d**6*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c
)**4 - 10*a**2*b**3*c**3*d**5*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4
+ 5*a*b**4*c**4*d**4*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4 - b**5*c*
*5*d**3*(18*A**2 + 66*A*B + 85*B**2)/(a*d - b*c)**4)/(36*A**2*b*d**4 + 132
*A*B*b*d**4 + 170*B**2*b*d**4))/(18*g**4*i*(a*d - b*c)**4) + (66*A*B*a**2*
d**2 - 42*A*B*a*b*c*d + 90*A*B*a*b*d**2*x + 12*A*B*b**2*c**2 - 18*A*B*b...
```

### 3.91.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3434 vs. $2(493) = 986$.

Time = 0.48 (sec) , antiderivative size = 3434, normalized size of antiderivative = 6.77

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x, algo
rithm="maxima")
```

output
```
-1/6*B^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
 - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3
)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d
^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b
*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*
g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3
*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))*log(b*e*x/(d
*x + c) + a*e/(d*x + c))^2 - 1/3*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c
*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3
*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d +
 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2
*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d
 + 3*a^5*b*c*d^2 - a^6*d^3)*g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^
3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(
d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a
^4*d^4)*g^4*i))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/108*B^2*(6*(4*b^3
*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b
^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)
*log(b*x + a)^2 - 18*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a...
```

## 3.91.8 Giac [A] (verification not implemented)

Time = 71.59 (sec) , antiderivative size = 451, normalized size of antiderivative = 0.89

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx =$$

$$-\frac{1}{108}\left(\frac{18\left(2\,B^2be^4-\frac{3\,(bex+ae)B^2de^3}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^3bcg^4i}{(dx+c)^3}-\frac{(bex+ae)^3adg^4i}{(dx+c)^3}}+\frac{6\left(12\,ABbe^4+4\,B^2be^4-\frac{18\,(bex+ae)ABde^3}{dx+c}-\frac{9\,(bex+ae}{dx-}\right.}{\frac{(bex+ae)^3bcg^4i}{(dx+c)^3}-\frac{(bex+ae)^3adg^4i}{(dx+c)^3}}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x, algo
rithm="giac")
```

output
```
-1/108*(18*(2*B^2*b*e^4 - 3*(b*e*x + a*e)*B^2*d*e^3/(d*x + c))*log((b*e*x
+ a*e)/(d*x + c))^2/((b*e*x + a*e)^3*b*c*g^4*i/(d*x + c)^3 - (b*e*x + a*e)
^3*a*d*g^4*i/(d*x + c)^3) + 6*(12*A*B*b*e^4 + 4*B^2*b*e^4 - 18*(b*e*x + a*
e)*A*B*d*e^3/(d*x + c) - 9*(b*e*x + a*e)*B^2*d*e^3/(d*x + c))*log((b*e*x +
 a*e)/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4*i/(d*x + c)^3 - (b*e*x + a*e)^3*
a*d*g^4*i/(d*x + c)^3) + (36*A^2*b*e^4 + 24*A*B*b*e^4 + 8*B^2*b*e^4 - 54*(
b*e*x + a*e)*A^2*d*e^3/(d*x + c) - 54*(b*e*x + a*e)*A*B*d*e^3/(d*x + c) -
27*(b*e*x + a*e)*B^2*d*e^3/(d*x + c))/((b*e*x + a*e)^3*b*c*g^4*i/(d*x + c)
^3 - (b*e*x + a*e)^3*a*d*g^4*i/(d*x + c)^3))*(b*c/((b*c*e - a*d*e)*(b*c -
a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

### 3.91.9 Mupad [B] (verification not implemented)

Time = 8.27 (sec) , antiderivative size = 1882, normalized size of antiderivative = 3.71

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^4*(c*i + d*i*x))
,x)
```

output
```
log((e*(a + b*x))/(c + d*x))^2*(((B^2*d^3*(a*((3*a^2*d^2 + b^2*c^2 - 4*a*b
*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^3*d^3 - b^3*c^3 + 4*a*
b^2*c^2*d - 6*a^2*b*c*d^2)/(3*b*d^4)))/(g^4*i*(a^4*d^4 + b^4*c^4 + 6*a^2*b
^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) - (B^2*d^3*x^2*((b^2*c - a*b*
d)/(3*d^2) - (2*b*(a*d - b*c))/(3*d^2)))/(g^4*i*(a^4*d^4 + b^4*c^4 + 6*a^2
*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)) + (B^2*d^3*x*(b*((3*a^2*d^2
 + b^2*c^2 - 4*a*b*c*d)/(6*b*d^3) + (a*(a*d - b*c))/(3*b*d^2)) + (3*a^2*d^
2 + b^2*c^2 - 4*a*b*c*d)/(3*d^3) + (2*a*(a*d - b*c))/(3*d^2)))/(g^4*i*(a^4
*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3*b*c*d^3)))/((3*
a^2*x)/d + a^3/(b*d) + (b^2*x^3)/d + (3*a*b*x^2)/d) - (d^3*(11*B^2 + 6*A*B
))/(6*g^4*i*(a^4*d^4 + b^4*c^4 + 6*a^2*b^2*c^2*d^2 - 4*a*b^3*c^3*d - 4*a^3
*b*c*d^3))) + ((198*A^2*a^2*d^2 + 36*A^2*b^2*c^2 + 575*B^2*a^2*d^2 + 8*B^2
*b^2*c^2 + 510*A*B*a^2*d^2 + 24*A*B*b^2*c^2 - 126*A^2*a*b*c*d - 73*B^2*a*b
*c*d - 138*A*B*a*b*c*d)/(6*(a*d - b*c)) + (x*(90*A^2*a*b*d^2 + 359*B^2*a*b
*d^2 - 18*A^2*b^2*c*d - 19*B^2*b^2*c*d + 294*A*B*a*b*d^2 - 30*A*B*b^2*c*d)
)/(2*(a*d - b*c)) + (d*x^2*(18*A^2*b^2*d + 85*B^2*b^2*d + 66*A*B*b^2*d))/(
a*d - b*c))/(x*(54*a^4*b*d^2*g^4*i + 54*a^2*b^3*c^2*g^4*i - 108*a^3*b^2*c*
d*g^4*i) + x^2*(54*a*b^4*c^2*g^4*i + 54*a^3*b^2*d^2*g^4*i - 108*a^2*b^3*c*
d*g^4*i) + x^3*(18*b^5*c^2*g^4*i + 18*a^2*b^3*d^2*g^4*i - 36*a*b^4*c*d*g^4
*i) + 18*a^5*d^2*g^4*i + 18*a^3*b^2*c^2*g^4*i - 36*a^4*b*c*d*g^4*i) - (...
```

3.91. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$

# 3.92 $\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2} \, dx$

### 3.92.1 Optimal result

Integrand size = 42, antiderivative size = 722

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \frac{2AB(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)} - \frac{2B^2(bc-ad)^2g^3(a+bx)}{d^3i^2(c+dx)}$$
$$+ \frac{2B^2(bc-ad)^2g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^2(c+dx)}$$
$$- \frac{bB(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2}$$
$$- \frac{6bB(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^2}$$
$$- \frac{3b(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2}$$
$$- \frac{(bc-ad)^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2(c+dx)} + \frac{b^3g^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^4i^2}$$
$$- \frac{3b(bc-ad)^2g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i^2} + \frac{bB^2(bc-ad)^2g^3\log(c+dx)}{d^4i^2}$$
$$+ \frac{bB(bc-ad)^2g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2}$$
$$- \frac{6bB^2(bc-ad)^2g^3\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$
$$- \frac{6bB(bc-ad)^2g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$
$$- \frac{bB^2(bc-ad)^2g^3\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} + \frac{6bB^2(bc-ad)^2g^3\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$

output
```
2*A*B*(-a*d+b*c)^2*g^3*(b*x+a)/d^3/i^2/(d*x+c)-2*B^2*(-a*d+b*c)^2*g^3*(b*x
+a)/d^3/i^2/(d*x+c)+2*B^2*(-a*d+b*c)^2*g^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d
^3/i^2/(d*x+c)-b*B*(-a*d+b*c)*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^3/
i^2-6*b*B*(-a*d+b*c)^2*g^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x
+c)))/d^4/i^2-3*b*(-a*d+b*c)*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3
/i^2-(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^2/(d*x+c
)+1/2*b^3*g^3*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i^2-3*b*(-a*d+b*
c)^2*g^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i^2+b*
B^2*(-a*d+b*c)^2*g^3*ln(d*x+c)/d^4/i^2+b*B*(-a*d+b*c)^2*g^3*(A+B*ln(e*(b*x
+a)/(d*x+c)))*ln(1-b*(d*x+c)/d/(b*x+a))/d^4/i^2-6*b*B^2*(-a*d+b*c)^2*g^3*p
olylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^2-6*b*B*(-a*d+b*c)^2*g^3*(A+B*ln(e*(b*
x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^2-b*B^2*(-a*d+b*c)^2*g
^3*polylog(2,b*(d*x+c)/d/(b*x+a))/d^4/i^2+6*b*B^2*(-a*d+b*c)^2*g^3*polylog
(3,d*(b*x+a)/b/(d*x+c))/d^4/i^2
```

### 3.92.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4443 vs. 2(722) = 1444.

Time = 6.29 (sec) , antiderivative size = 4443, normalized size of antiderivative = 6.15

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i +
d*i*x)^2,x]
```

output
```
(g^3*(-4*A^2*b^2*d*(2*b*c - 3*a*d)*x + 2*A^2*b^3*d^2*x^2 + (4*A^2*(b*c - a
*d)^3)/(c + d*x) + 12*A^2*b*(b*c - a*d)^2*Log[c + d*x] + (8*a^3*A*B*d^3*(b
*c - a*d + b*(c + d*x)*Log[a/b + x] + (-(b*c) + a*d)*Log[(e*(a + b*x))/(c
+ d*x)] - b*c*Log[(b*(c + d*x))/(b*c - a*d)] - b*d*x*Log[(b*(c + d*x))/(b*
c - a*d)]))/((b*c - a*d)*(c + d*x)) + 12*a^2*A*b*B*d^2*(-Log[c/d + x]^2 +
2*Log[c/d + x]*Log[c + d*x] + 2*(-(c/(c + d*x)) + (b*c*Log[a + b*x])/(-(b*
c) + a*d) + (b*c*Log[c + d*x])/(b*c - a*d) - Log[a/b + x]*Log[c + d*x] + L
og[(e*(a + b*x))/(c + d*x)]*(c/(c + d*x) + Log[c + d*x]) + Log[a/b + x]*Lo
g[(b*(c + d*x))/(b*c - a*d)]) + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]
) + 4*A*b^3*B*(-4*c^2 + (4*a*c*d)/b - c*d*x + (a*d^2*x)/b - (2*c^3)/(c + d
*x) + 4*c^2*Log[c/d + x] - 3*c^2*Log[c/d + x]^2 - (a^2*d^2*Log[a + b*x])/b
^2 + (2*b*c^3*Log[a + b*x])/(-(b*c) + a*d) - 4*c*d*x*Log[(e*(a + b*x))/(c
+ d*x)] + d^2*x^2*Log[(e*(a + b*x))/(c + d*x)] + (2*c^3*Log[(e*(a + b*x))/
(c + d*x)])/(c + d*x) + c^2*Log[c + d*x] + (2*b*c^3*Log[c + d*x])/(b*c - a
*d) + 6*c^2*Log[c/d + x]*Log[c + d*x] + 6*c^2*Log[(e*(a + b*x))/(c + d*x)]
*Log[c + d*x] - (2*c*Log[a/b + x]*(2*a*d + 3*b*c*Log[c + d*x] - 3*b*c*Log[
(b*(c + d*x))/(b*c - a*d)]))/b + 6*c^2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
a*d)]) + 24*a*A*b^2*B*d*(d*(a/b + x)*(-1 + Log[a/b + x]) - (c^2*Log[a/b +
x])/(c + d*x) - (c + d*x)*(-1 + Log[c/d + x]) + c*Log[c/d + x]^2 + (c^2*(1
 + Log[c/d + x]))/(c + d*x) + (b*c^2*(Log[a + b*x] - Log[c + d*x]))/(b*...
```

### 3.92.3 Rubi [A] (verified)

Time = 0.84 (sec) , antiderivative size = 592, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^2}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{g^3(bc-ad)^2 \int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{g^3(bc-ad)^2 \int \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2} + \frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)} - \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3}\right) d\frac{a+b}{c+d}}{i^2}$$

$\downarrow$ 2009

$$g^3(bc-ad)^2\left(\frac{b^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{6bB\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^4} - \frac{3b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^4}\right.$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)
^2,x]
```

output
```
((b*c - a*d)^2*g^3*((2*A*B*(a + b*x))/(d^3*(c + d*x)) - (2*B^2*(a + b*x))/
(d^3*(c + d*x)) + (2*B^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d^3*(c +
 d*x)) - (b*B*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(d^3*(c + d*
x)*(b - (d*(a + b*x))/(c + d*x))) - ((a + b*x)*(A + B*Log[(e*(a + b*x))/(c
 + d*x)])^2)/(d^3*(c + d*x)) + (b^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) - (3*b*(a + b*x)*(A + B*Log[(e*(
a + b*x))/(c + d*x)])^2)/(d^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (
b*B^2*Log[b - (d*(a + b*x))/(c + d*x)])/d^4 - (6*b*B*(A + B*Log[(e*(a + b*
x))/(c + d*x)])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 - (3*b*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 +
(b*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (b*(c + d*x))/(d*(a + b*
x))])/d^4 - (6*b*B^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 - (6*b*B
*(A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x
))])/d^4 - (b*B^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/d^4 + (6*b*B^2*
PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^4))/i^2
```

### 3.92.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.92.4 Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix+ci)^2}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

### 3.92.5 Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output
```
integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*
a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^
2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)))
/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

### 3.92.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

### 3.92.7 Maxima [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output
```
1/2*(2*c^3/(d^5*i^2*x + c*d^4*i^2) + 6*c^2*log(d*x + c)/(d^4*i^2) + (d*x^2
 - 4*c*x)/(d^3*i^2))*A^2*b^3*g^3 - 3*A^2*a*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2
) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i^2))*g^3 + 3*A^2*a^2*b*g^3*(c/(d^
3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - 2*A*B*a^3*g^3*(log(b*e*x/
(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2) - 1/(d^2*i^2*x + c*d*i^2)
 - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log(d*x + c)/((b*c*d - a*d^2)*
i^2)) - A^2*a^3*g^3/(d^2*i^2*x + c*d*i^2) + 1/2*(2*((b^3*c^2*d*g^3 - 2*a*b
^2*c*d^2*g^3 + a^2*b*d^3*g^3)*B^2*x + (b^3*c^3*g^3 - 2*a*b^2*c^2*d*g^3 + a
^2*b*c*d^2*g^3)*B^2)*log(d*x + c)^3 + (B^2*b^3*d^3*g^3*x^3 - 3*(b^3*c*d^2*
g^3 - 2*a*b^2*d^3*g^3)*B^2*x^2 - 2*(2*b^3*c^2*d*g^3 - 3*a*b^2*c*d^2*g^3)*B
^2*x + 2*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^
3)*B^2)*log(d*x + c)^2)/(d^5*i^2*x + c*d^4*i^2) - integrate(-(B^2*a^3*d^3*
g^3*log(e)^2 + (B^2*b^3*d^3*g^3*log(e)^2 + 2*A*B*b^3*d^3*g^3*log(e))*x^3 +
 3*(B^2*a*b^2*d^3*g^3*log(e)^2 + 2*A*B*a*b^2*d^3*g^3*log(e))*x^2 + (B^2*b^
3*d^3*g^3*x^3 + 3*B^2*a*b^2*d^3*g^3*x^2 + 3*B^2*a^2*b*d^3*g^3*x + B^2*a^3*
d^3*g^3)*log(b*x + a)^2 + 3*(B^2*a^2*b*d^3*g^3*log(e)^2 + 2*A*B*a^2*b*d^3*
g^3*log(e))*x + 2*(B^2*a^3*d^3*g^3*log(e) + (B^2*b^3*d^3*g^3*log(e) + A*B*
b^3*d^3*g^3)*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e) + A*B*a*b^2*d^3*g^3)*x^2 +
3*(B^2*a^2*b*d^3*g^3*log(e) + A*B*a^2*b*d^3*g^3)*x)*log(b*x + a) - ((2*A*B
*b^3*d^3*g^3 + (2*g^3*log(e) + g^3)*B^2*b^3*d^3)*x^3 + 2*(b^3*c^3*g^3 -...
```

### 3.92.8 Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*
i)^2, x)
```

### 3.92.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^2,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^2, x)
```

# 3.93 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$



## 3.93.1 Optimal result

Integrand size = 42, antiderivative size = 469

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx \\
&= -\frac{2AB(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)}+\frac{2B^2(bc-ad)g^2(a+bx)}{d^2i^2(c+dx)} \\
&\quad -\frac{2B^2(bc-ad)g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^2(c+dx)} \\
&\quad +\frac{2bB(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^3i^2} \\
&\quad +\frac{bg^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2}+\frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2(c+dx)} \\
&\quad +\frac{2b(bc-ad)g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^2} \\
&\quad +\frac{2bB^2(bc-ad)g^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&\quad +\frac{4bB(bc-ad)g^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&\quad -\frac{4bB^2(bc-ad)g^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}
\end{aligned}$$

output
```
-2*A*B*(-a*d+b*c)*g^2*(b*x+a)/d^2/i^2/(d*x+c)+2*B^2*(-a*d+b*c)*g^2*(b*x+a)
/d^2/i^2/(d*x+c)-2*B^2*(-a*d+b*c)*g^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^2/i^
2/(d*x+c)+2*b*B*(-a*d+b*c)*g^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/
(d*x+c)))/d^3/i^2+b*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i^2+(-a*
d+b*c)*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i^2/(d*x+c)+2*b*(-a*d
+b*c)*g^2*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^2+2
*b*B^2*(-a*d+b*c)*g^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^2+4*b*B*(-a*d+b
*c)*g^2*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^2
-4*b*B^2*(-a*d+b*c)*g^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^3/i^2
```

### 3.93.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2622 vs. $2(469) = 938$.

Time = 3.22 (sec) , antiderivative size = 2622, normalized size of antiderivative = 5.59

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i +
d*i*x)^2,x]
```

output
```
(g^2*(3*A^2*b^2*d*x - (3*A^2*(b*c - a*d)^2)/(c + d*x) + 6*A^2*b*(-(b*c) +
a*d)*Log[c + d*x] + (6*a^2*A*B*d^2*(b*c - a*d + b*(c + d*x)*Log[a/b + x] +
 (-(b*c) + a*d)*Log[(e*(a + b*x))/(c + d*x)] - b*c*Log[(b*(c + d*x))/(b*c
- a*d)] - b*d*x*Log[(b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)*(c + d*x)) +
 6*a*A*b*B*d*(-Log[c/d + x]^2 + 2*Log[c/d + x]*Log[c + d*x] + 2*(-(c/(c +
d*x)) + (b*c*Log[a + b*x])/(-(b*c) + a*d) + (b*c*Log[c + d*x])/(b*c - a*d)
 - Log[a/b + x]*Log[c + d*x] + Log[(e*(a + b*x))/(c + d*x)]*(c/(c + d*x) +
 Log[c + d*x]) + Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*PolyLog[
2, (d*(a + b*x))/(-(b*c) + a*d)]) + 6*A*b^2*B*(d*(a/b + x)*(-1 + Log[a/b +
 x]) - (c^2*Log[a/b + x])/(c + d*x) - (c + d*x)*(-1 + Log[c/d + x]) + c*Lo
g[c/d + x]^2 + (c^2*(1 + Log[c/d + x]))/(c + d*x) + (b*c^2*(Log[a + b*x] -
 Log[c + d*x]))/(b*c - a*d) + (-Log[a/b + x] + Log[c/d + x] + Log[(e*(a +
b*x))/(c + d*x)])*(d*x - c^2/(c + d*x) - 2*c*Log[c + d*x]) - 2*c*(Log[a/b
+ x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a
*d)])) - (3*a^2*B^2*d^2*(2*b*c - 2*a*d + 2*b*(c + d*x)*Log[a + b*x] - 2*(b
*c - a*d)*Log[(e*(a + b*x))/(c + d*x)] - 2*b*(c + d*x)*Log[a + b*x]*Log[(e
*(a + b*x))/(c + d*x)] + (b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)]^2 - 2*b*
(c + d*x)*Log[c + d*x] - 2*b*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)]*Log[(b
*c - a*d)/(b*c + b*d*x)] + b*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log
[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d...
```

### 3.93.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 380, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^2}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{g^2(bc-ad)\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{g^2(bc-ad)\int \left(\frac{2b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2}+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)^2}\right)d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{ 2009}$$

$$g^2(bc-ad)\left(\frac{4bB\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3}+\frac{2bB\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3}+\frac{2b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\right.\right.}{d^3}\right.$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)
^2,x]
```

output
```
((b*c - a*d)*g^2*((-2*A*B*(a + b*x))/(d^2*(c + d*x)) + (2*B^2*(a + b*x))/(
d^2*(c + d*x)) - (2*B^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d^2*(c +
d*x)) + ((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^2*(c + d*x))
 + (b*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^2*(c + d*x)*(b
- (d*(a + b*x))/(c + d*x))) + (2*b*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*
Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 + (2*b*(A + B*Log[(e*(a + b*x))/
(c + d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 + (2*b*B^2*PolyLog
[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 + (4*b*B*(A + B*Log[(e*(a + b*x))/(c
 + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 - (4*b*B^2*PolyLog[
3, (d*(a + b*x))/(b*(c + d*x))])/d^3))/i^2
```

### 3.93.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.93.4 Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix+ci)^2}dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

### 3.93.5 Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output
```
integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x
^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*
B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)
))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

### 3.93.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

### 3.93.7 Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output
```
-A^2*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^
3*i^2))*g^2 + 2*A^2*a*b*g^2*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2
*i^2)) - 2*A*B*a^2*g^2*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x +
c*d*i^2) - 1/(d^2*i^2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2)
+ b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A^2*a^2*g^2/(d^2*i^2*x + c*d*i^2
) - 1/3*(2*((b^2*c*d*g^2 - a*b*d^2*g^2)*B^2*x + (b^2*c^2*g^2 - a*b*c*d*g^2
)*B^2)*log(d*x + c)^3 - 3*(B^2*b^2*d^2*g^2*x^2 + B^2*b^2*c*d*g^2*x - (b^2*
c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B^2)*log(d*x + c)^2)/(d^4*i^2*x + c
*d^3*i^2) - integrate(-(B^2*a^2*d^2*g^2*log(e)^2 + (B^2*b^2*d^2*g^2*log(e)
^2 + 2*A*B*b^2*d^2*g^2*log(e))*x^2 + (B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d^2*
g^2*x + B^2*a^2*d^2*g^2)*log(b*x + a)^2 + 2*(B^2*a*b*d^2*g^2*log(e)^2 + 2*
A*B*a*b*d^2*g^2*log(e))*x + 2*(B^2*a^2*d^2*g^2*log(e) + (B^2*b^2*d^2*g^2*l
og(e) + A*B*b^2*d^2*g^2)*x^2 + 2*(B^2*a*b*d^2*g^2*log(e) + A*B*a*b*d^2*g^2
)*x)*log(b*x + a) + 2*((b^2*c^2*g^2 - 2*a*b*c*d*g^2 - (g^2*log(e) - g^2)*a
^2*d^2)*B^2 - (A*B*b^2*d^2*g^2 + (g^2*log(e) + g^2)*B^2*b^2*d^2)*x^2 - (2*
A*B*a*b*d^2*g^2 + (2*a*b*d^2*g^2*log(e) + b^2*c*d*g^2)*B^2)*x - (B^2*b^2*d
^2*g^2*x^2 + 2*B^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log(b*x + a))*log(d*x
+ c))/(d^4*i^2*x^2 + 2*c*d^3*i^2*x + c^2*d^2*i^2), x)
```

### 3.93.8 Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*
i)^2, x)
```

### 3.93.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^2,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^2, x)
```

# 3.94 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$



## 3.94.1 Optimal result

Integrand size = 40, antiderivative size = 261

$$\begin{aligned}
&\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx \\
&= \frac{2ABg(a+bx)}{di^2(c+dx)} - \frac{2B^2g(a+bx)}{di^2(c+dx)} + \frac{2B^2g(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{di^2(c+dx)} \\
&\quad - \frac{g(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{di^2(c+dx)} - \frac{bg\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^2} \\
&\quad - \frac{2bBg\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2} + \frac{2bB^2g\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}
\end{aligned}$$

output
```
2*A*B*g*(b*x+a)/d/i^2/(d*x+c)-2*B^2*g*(b*x+a)/d/i^2/(d*x+c)+2*B^2*g*(b*x+a
)*ln(e*(b*x+a)/(d*x+c))/d/i^2/(d*x+c)-g*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c))
)^2/d/i^2/(d*x+c)-b*g*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))
^2/d^2/i^2-2*b*B*g*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+
c))/d^2/i^2+2*b*B^2*g*polylog(3,d*(b*x+a)/b/(d*x+c))/d^2/i^2
```

### 3.94.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1412 vs. $2(261) = 522$.

Time = 1.02 (sec) , antiderivative size = 1412, normalized size of antiderivative = 5.41

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*
i*x)^2,x]
```

output
```
(g*((3*A^2*(b*c - a*d))/(c + d*x) + 3*A^2*b*Log[c + d*x] + (6*a*A*B*d*(b*c
 - a*d + b*(c + d*x)*Log[a/b + x] + (-(b*c) + a*d)*Log[(e*(a + b*x))/(c +
d*x)] - b*c*Log[(b*(c + d*x))/(b*c - a*d)] - b*d*x*Log[(b*(c + d*x))/(b*c
- a*d)]))/((b*c - a*d)*(c + d*x)) + 3*A*b*B*(-Log[c/d + x]^2 + 2*Log[c/d +
 x]*Log[c + d*x] + 2*(-(c/(c + d*x)) + (b*c*Log[a + b*x])/(-(b*c) + a*d) +
 (b*c*Log[c + d*x])/(b*c - a*d) - Log[a/b + x]*Log[c + d*x] + Log[(e*(a +
b*x))/(c + d*x)]*(c/(c + d*x) + Log[c + d*x]) + Log[a/b + x]*Log[(b*(c + d
*x))/(b*c - a*d)]) + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - (3*a*B^
2*d*(2*b*c - 2*a*d + 2*b*(c + d*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(e*(a
+ b*x))/(c + d*x)] - 2*b*(c + d*x)*Log[a + b*x]*Log[(e*(a + b*x))/(c + d*x
)] + (b*c - a*d)*Log[(e*(a + b*x))/(c + d*x)]^2 - 2*b*(c + d*x)*Log[c + d*
x] - 2*b*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d
*x)] + b*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c
- a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*(c + d*x)*(Log[
(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c
 - a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/((b*c
 - a*d)*(c + d*x)) + (b*B^2*((b*c - a*d)*(c + d*x)*Log[c/d + x]^3 + 3*c*(b
*c - a*d)*(2 + 2*Log[c/d + x] + Log[c/d + x]^2) + 3*(b*c - a*d)*(-Log[a/b
+ x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])^2*(c + (c + d*x)*Log[c
 + d*x]) + 3*c*Log[a/b + x]*(-(d*(a + b*x)*Log[a/b + x]) + 2*b*(c + d*x...
```

### 3.94.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 239, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^2}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{g\int \left(-\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d}-\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g\left(-\frac{2bB\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^2}-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^2}-\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d(c+dx)}+\frac{2AB(}{d(c}\right.}{i^2}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)^2
,x]
```

output
```
(g*((2*A*B*(a + b*x))/(d*(c + d*x)) - (2*B^2*(a + b*x))/(d*(c + d*x)) + (2
*B^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d*(c + d*x)) - ((a + b*x)*(A
 + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d*(c + d*x)) - (b*(A + B*Log[(e*(a
+ b*x))/(c + d*x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 - (2*b*B*(
A + B*Log[(e*(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))
])/d^2 + (2*b*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^2))/i^2
```

#### 3.94.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.94.4 Maple [F]

$$\int \frac{(bgx + ag)\left(A + B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix + ci)^2}dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x)
```

### 3.94.5 Fricas [F]

$$\int \frac{(ag + bgx)\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci + dix)^2}\,dx = \int \frac{(bgx + ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right) + A\right)^2}{(dix + ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output
```
integral((A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log((b*e*x + a*e)/(d
*x + c))^2 + 2*(A*B*b*g*x + A*B*a*g)*log((b*e*x + a*e)/(d*x + c)))/(d^2*i^
2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

### 3.94.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

### 3.94.7 Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output
```
A^2*b*g*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - 2*A*B*a*g*(
log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2) - 1/(d^2*i^2*x
+ c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log(d*x + c)/((b*c*d
 - a*d^2)*i^2)) - A^2*a*g/(d^2*i^2*x + c*d*i^2) + 1/3*(3*(b*c*g - a*d*g)*B
^2*log(d*x + c)^2 + (B^2*b*d*g*x + B^2*b*c*g)*log(d*x + c)^3)/(d^3*i^2*x +
 c*d^2*i^2) - integrate(-(B^2*a*d*g*log(e)^2 + (B^2*b*d*g*x + B^2*a*d*g)*l
og(b*x + a)^2 + (B^2*b*d*g*log(e)^2 + 2*A*B*b*d*g*log(e))*x + 2*(B^2*a*d*g
*log(e) + (B^2*b*d*g*log(e) + A*B*b*d*g)*x)*log(b*x + a) - 2*(((g*log(e) -
 g)*a*d + b*c*g)*B^2 + (B^2*b*d*g*log(e) + A*B*b*d*g)*x + (B^2*b*d*g*x + B
^2*a*d*g)*log(b*x + a))*log(d*x + c))/(d^3*i^2*x^2 + 2*c*d^2*i^2*x + c^2*d
*i^2), x)
```

### 3.94.8 Giac [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output
```
integrate((b*g*x + a*g)*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*i)
^2, x)
```

### 3.94.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^2
,x)
```

output
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^2
, x)
```

# 3.95 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$



## 3.95.1 Optimal result

Integrand size = 32, antiderivative size = 152

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = -\frac{2AB(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{2B^2(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{2B^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)i^2(c+dx)} + \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)i^2(c+dx)}$$

output
```
-2*A*B*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)+2*B^2*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)
-2*B^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)/i^2/(d*x+c)+(b*x+a)*(A+B*l
n(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)/i^2/(d*x+c)
```

### 3.95.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.24 (sec) , antiderivative size = 315, normalized size of antiderivative = 2.07

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \frac{-\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B\left(2(bc-ad)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)-2b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right) \cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x)^2,x]
```

output
```
(-(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)*(A + B*Log[(e
*(a + b*x))/(c + d*x)]) + 2*b*(c + d*x)*Log[a + b*x]*(A + B*Log[(e*(a + b*
x))/(c + d*x)]) - 2*b*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[c
 + d*x] - 2*B*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c +
d*x]) - b*B*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b
*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + b*B*(c + d*x)*
((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(d*i^2*(c + d*x))
```

### 3.95.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 111, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2952, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^2}\,dx$$
$$\Big\downarrow \text{2952}$$
$$\frac{\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$
$$\Big\downarrow \text{2733}$$

$$\frac{\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-2B\int\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-2B\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}-\frac{B(a+bx)}{c+dx}\right)}{i^2(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x)^2,x]
```

output
```
(((a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c + d*x) - 2*B*((A*(a
 + b*x))/(c + d*x) - (B*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[(e*(a + b*
x))/(c + d*x)])/(c + d*x)))/((b*c - a*d)*i^2)
```

### 3.95.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

### 3.95.4 Maple [A] (verified)

Time = 0.68 (sec) , antiderivative size = 182, normalized size of antiderivative = 1.20

| method | result |
|---|---|
| norman | $\frac{\frac{(A^2-2BA+2B^2)x}{ic}-\frac{B^2a\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{i(ad-cb)}-\frac{B^2bx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{(ad-cb)i}-\frac{2aB(-B+A)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{i(ad-cb)}-\frac{2Bb(-B+A)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{i(ad-cb)}}{i(dx+c)}$ |
| parts | $-\frac{A^2}{i^2(dx+c)d}-\frac{B^2\left(\frac{e(bx+a)\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{dx+c}-\frac{2e(bx+a)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{dx+c}+\frac{2e(bx+a)}{dx+c}\right)}{i^2e(ad-cb)}-\frac{2BA\left(\frac{e(bx+a)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{dx+c}-\frac{e(b}{d}\right.}{i^2e(ad-cb)}$ |
| parallelrisch | $-\frac{-2ABab\,d^3+2AB\,b^2c\,d^2+B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)^2b^2d^3-2B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)b^2d^3+B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2ab\,d^3-2B^2\ln\left(\frac{e(bx+a)}{dx+c}\right.}{i^2(dx+c)b\,d^3(ad-}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{d^2A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^2e^2i^2}+\frac{2d^2AB\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{(ad-cb)^2e^2i^2}+\frac{d^2B^2\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.\right.}{}\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(\frac{d^2A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^2e^2i^2}+\frac{2d^2AB\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\frac{be}{d}\right)}{(ad-cb)^2e^2i^2}+\frac{d^2B^2\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.\right.}{}\right.}{d^2}$ |
| risch | $-\frac{A^2}{i^2(dx+c)d}-\frac{B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2bx}{i^2(ad-cb)(dx+c)}-\frac{B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)^2a}{i^2(ad-cb)(dx+c)}+\frac{2B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)bx}{i^2(ad-cb)(dx+c)}+\frac{2B^2\ln\left(\frac{e(bx+a)}{dx+c}\right)a}{i^2(ad-cb)(dx+c)}-\frac{2}{i^2(ad-}$ |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x,method=_RETURNVERBOSE)
```

output
```
((A^2-2*A*B+2*B^2)/i/c*x-B^2*a/i/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))^2-B^2*b/(
a*d-b*c)/i*x*ln(e*(b*x+a)/(d*x+c))^2-2*a*B*(-B+A)/i/(a*d-b*c)*ln(e*(b*x+a)
/(d*x+c))-2*B*b*(-B+A)/i/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c)))/i/(d*x+c)
```

### 3.95.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 155, normalized size of antiderivative = 1.02

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx =$$
$$-\frac{(A^2-2\,AB+2\,B^2)bc-(A^2-2\,AB+2\,B^2)ad-(B^2bdx+B^2ad)\log\left(\frac{bex+ae}{dx+c}\right)^2-2\left((AB-B^2)bdx+\right.}{(bcd^2-ad^3)i^2x+(bc^2d-acd^2)i^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algorithm="frica
s")
```

output
```
-((A^2 - 2*A*B + 2*B^2)*b*c - (A^2 - 2*A*B + 2*B^2)*a*d - (B^2*b*d*x + B^2
*a*d)*log((b*e*x + a*e)/(d*x + c))^2 - 2*((A*B - B^2)*b*d*x + (A*B - B^2)*
a*d)*log((b*e*x + a*e)/(d*x + c)))/((b*c*d^2 - a*d^3)*i^2*x + (b*c^2*d - a
*c*d^2)*i^2)
```

### 3.95.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 432 vs. $2(128) = 256$.

Time = 1.14 (sec) , antiderivative size = 432, normalized size of antiderivative = 2.84

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \frac{2Bb(A-B)\log\left(x+\frac{2ABabd+2ABb^2c-2B^2abd-2B^2b^2c-\frac{2Ba^2bd^2(A-B)}{ad-bc}+\frac{4Bab^2cd(A-B)}{ad-bc}-\frac{2Bb^3c^2(A-B)}{ad-bc}}{4ABb^2d-4B^2b^2d}\right)}{di^2\,(ad-bc)} - \frac{2Bb(A-B)\log\left(x+\frac{2ABabd+2ABb^2c-2B^2abd-2B^2b^2c+\frac{2Ba^2bd^2(A-B)}{ad-bc}-\frac{4Bab^2cd(A-B)}{ad-bc}+\frac{2Bb^3c^2(A-B)}{ad-bc}}{4ABb^2d-4B^2b^2d}\right)}{di^2\,(ad-bc)} + \frac{(-2AB+2B^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{cdi^2+d^2i^2x} + \frac{(-B^2a-B^2bx)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{acdi^2+ad^2i^2x-bc^2i^2-bcdi^2x} + \frac{-A^2+2AB-2B^2}{cdi^2+d^2i^2x}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**2,x)
```

output
```
2*B*b*(A - B)*log(x + (2*A*B*a*b*d + 2*A*B*b**2*c - 2*B**2*a*b*d - 2*B**2*
b**2*c - 2*B*a**2*b*d**2*(A - B)/(a*d - b*c) + 4*B*a*b**2*c*d*(A - B)/(a*d
 - b*c) - 2*B*b**3*c**2*(A - B)/(a*d - b*c))/(4*A*B*b**2*d - 4*B**2*b**2*d
))/(d*i**2*(a*d - b*c)) - 2*B*b*(A - B)*log(x + (2*A*B*a*b*d + 2*A*B*b**2*
c - 2*B**2*a*b*d - 2*B**2*b**2*c + 2*B*a**2*b*d**2*(A - B)/(a*d - b*c) - 4
*B*a*b**2*c*d*(A - B)/(a*d - b*c) + 2*B*b**3*c**2*(A - B)/(a*d - b*c))/(4*
A*B*b**2*d - 4*B**2*b**2*d))/(d*i**2*(a*d - b*c)) + (-2*A*B + 2*B**2)*log(
e*(a + b*x)/(c + d*x))/(c*d*i**2 + d**2*i**2*x) + (-B**2*a - B**2*b*x)*log
(e*(a + b*x)/(c + d*x))**2/(a*c*d*i**2 + a*d**2*i**2*x - b*c**2*i**2 - b*c
*d*i**2*x) + (-A**2 + 2*A*B - 2*B**2)/(c*d*i**2 + d**2*i**2*x)
```

### 3.95.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(152) = 304$.

Time = 0.22 (sec) , antiderivative size = 416, normalized size of antiderivative = 2.74

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \left(2\left(\frac{1}{d^2i^2x+cdi^2}+\frac{b\log(bx+a)}{(bcd-ad^2)i^2}-\frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right)\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)-\frac{(bdx+bc)\log(bx+a)^2}{}\right.$$
$$-2\,AB\left(\frac{\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^2i^2x+cdi^2}-\frac{1}{d^2i^2x+cdi^2}-\frac{b\log(bx+a)}{(bcd-ad^2)i^2}+\frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right)$$
$$-\frac{B^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)^2}{d^2i^2x+cdi^2}-\frac{A^2}{d^2i^2x+cdi^2}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algorithm="maxim
a")
```

output
```
(2*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2) - b*log
(d*x + c)/((b*c*d - a*d^2)*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - ((
b*d*x + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d
 + 2*(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x +
 a))*log(d*x + c))/(b*c^2*d*i^2 - a*c*d^2*i^2 + (b*c*d^2*i^2 - a*d^3*i^2)*
x))*B^2 - 2*A*B*(log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^2*i^2*x + c*d*i^2
) - 1/(d^2*i^2*x + c*d*i^2) - b*log(b*x + a)/((b*c*d - a*d^2)*i^2) + b*log
(d*x + c)/((b*c*d - a*d^2)*i^2)) - B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c)
)^2/(d^2*i^2*x + c*d*i^2) - A^2/(d^2*i^2*x + c*d*i^2)
```

### 3.95.8 Giac [A] (verification not implemented)

Time = 0.40 (sec) , antiderivative size = 173, normalized size of antiderivative = 1.14

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \left(\frac{(bex+ae)B^2\log\left(\frac{bex+ae}{dx+c}\right)^2}{(dx+c)i^2}+\frac{2\,(bex+ae)(AB-B^2)\log\left(\frac{bex+ae}{dx+c}\right)}{(dx+c)i^2}+\frac{(bex+ae)(A^2-2\,AB+2\,B^2)}{(dx+c)i^2}\right)\left(\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^2,x, algorithm="giac"
)
```

output
```
((b*e*x + a*e)*B^2*log((b*e*x + a*e)/(d*x + c))^2/((d*x + c)*i^2) + 2*(b*e
*x + a*e)*(A*B - B^2)*log((b*e*x + a*e)/(d*x + c))/((d*x + c)*i^2) + (b*e*
x + a*e)*(A^2 - 2*A*B + 2*B^2)/((d*x + c)*i^2))*(b*c/((b*c*e - a*d*e)*(b*c
 - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.95.9 Mupad [B] (verification not implemented)

Time = 2.26 (sec) , antiderivative size = 222, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^2}\,dx = \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,\left(\frac{2\,B^2}{b\,d^2\,i^2}-\frac{2\,A\,B}{b\,d^2\,i^2}\right)}{\frac{x}{b}+\frac{c}{b\,d}} - \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B^2}{d^2\,i^2\,\left(x+\frac{c}{d}\right)}+\frac{B^2\,b}{d\,i^2\,(a\,d-b\,c)}\right) - \frac{A^2-2\,A\,B+2\,B^2}{x\,d^2\,i^2+c\,d\,i^2} + \frac{B\,b\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a\,d^2\,i^2+b\,c\,d\,i^2}{d\,i^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(A-B)\,4\mathrm{i}}{d\,i^2\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(c*i + d*i*x)^2,x)
```

output
```
(log((e*(a + b*x))/(c + d*x))*((2*B^2)/(b*d^2*i^2) - (2*A*B)/(b*d^2*i^2)))
/(x/b + c/(b*d)) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(d^2*i^2*(x + c/d))
 + (B^2*b)/(d*i^2*(a*d - b*c))) - (A^2 + 2*B^2 - 2*A*B)/(d^2*i^2*x + c*d*i
^2) + (B*b*atan(((2*b*d*x + (a*d^2*i^2 + b*c*d*i^2)/(d*i^2))*1i)/(a*d - b*
c))*(A - B)*4i)/(d*i^2*(a*d - b*c))
```

# 3.96 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$



## 3.96.1 Optimal result

Integrand size = 42, antiderivative size = 214

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \frac{2ABd(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{2B^2d(a+bx)}{(bc-ad)^2gi^2(c+dx)} + \frac{2B^2d(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^2gi^2(c+dx)} - \frac{d(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2gi^2(c+dx)} + \frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^2gi^2}$$

output

```
2*A*B*d*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*x+c)-2*B^2*d*(b*x+a)/(-a*d+b*c)^2/g/
i^2/(d*x+c)+2*B^2*d*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^2/g/i^2/(d*x+
c)-d*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^2/g/i^2/(d*x+c)+1/3*
b*(A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^2/g/i^2
```

### 3.96.2 Mathematica [A] (verified)

Time = 0.35 (sec) , antiderivative size = 187, normalized size of antiderivative = 0.87

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \frac{3b(A^2-2AB+2B^2)(c+dx)\log(a+bx)+6(A-B)B(bc-ad)\log\left(\frac{e(a+bx)}{c+dx}\right)+3B(-Bd(a+bx)+Ab}{3(bc-a}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i
*x)^2),x]
```

output
```
(3*b*(A^2 - 2*A*B + 2*B^2)*(c + d*x)*Log[a + b*x] + 6*(A - B)*B*(b*c - a*d
)*Log[(e*(a + b*x))/(c + d*x)] + 3*B*(-(B*d*(a + b*x)) + A*b*(c + d*x))*Lo
g[(e*(a + b*x))/(c + d*x)]^2 + b*B^2*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)
]^3 - 3*(A^2 - 2*A*B + 2*B^2)*(-(b*c) + a*d + b*(c + d*x)*Log[c + d*x]))/(
3*(b*c - a*d)^2*g*i^2*(c + d*x))
```

### 3.96.3 Rubi [A] (warning: unable to verify)

Time = 0.51 (sec) , antiderivative size = 140, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {2962, 2788, 2733, 2009, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{2788}$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{2733}$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-2B\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d\frac{a+bx}{c+dx}\right)}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-2B\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}-\frac{B(a+bx)}{c+dx}\right)\right)}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{2739}$$

$$\frac{\frac{b\int \frac{(a+bx)^2}{(c+dx)^2}d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}-d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-2B\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}-\frac{B(a+bx)}{c+dx}\right)\right)}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{\frac{b(a+bx)^3}{3B(c+dx)^3}-d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-2B\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}-\frac{B(a+bx)}{c+dx}\right)\right)}{gi^2(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i*x)^2)
,x]
```

output
```
((b*(a + b*x)^3)/(3*B*(c + d*x)^3) - d*(((a + b*x)*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/(c + d*x) - 2*B*((A*(a + b*x))/(c + d*x) - (B*(a + b*x))/
(c + d*x) + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(c + d*x))))/((b*c
- a*d)^2*g*i^2)
```

## 3.96. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$

### 3.96.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.96. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$

### 3.96.4 Maple [A] (verified)

Time = 0.98 (sec) , antiderivative size = 398, normalized size of antiderivative = 1.86

| method | result |
|---|---|
| norman | $\frac{\frac{\left(A^2bc-2ABad+2B^2ad\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{B(Abc-Bad)\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{d\left(A^2b-2ABb+2B^2b\right)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(A^2-2BA+2B^2\right)}{gic(ad-cb)}}{i(dx+c)}$ |
| parallelrisch | $\frac{-3A^2ab^2d^4+3A^2b^3cd^3+6ABab^2d^4-6ABb^3cd^3-6B^2ab^2d^4+6B^2b^3cd^3+B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)^3b^3d^4-3B^2x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{}$ |
| parts | $\frac{A^2\left(-\frac{1}{(ad-cb)(dx+c)}-\frac{b\ln(dx+c)}{(ad-cb)^2}+\frac{b\ln(bx+a)}{(ad-cb)^2}\right)}{g\,i^2}-\frac{B^2\left(\frac{d\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2-2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\right.\right.}{ad-cb}\right.}{g\,i^2(ad-cb)e}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{d^2A^2b\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^3g}+\frac{d^3A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^3g}-\frac{d^2ABb\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{e\,i^2(ad-cb)^3g}+\frac{2d^3AB\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\right.}{e^2i^2}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{d^2A^2b\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^3g}+\frac{d^3A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^3g}-\frac{d^2ABb\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{e\,i^2(ad-cb)^3g}+\frac{2d^3AB\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\right.}{e^2i^2}\right.}{}$ |
| risch | $-\frac{A^2}{g\,i^2(ad-cb)(dx+c)}-\frac{A^2b\ln(dx+c)}{g\,i^2(ad-cb)^2}+\frac{A^2b\ln(bx+a)}{g\,i^2(ad-cb)^2}-\frac{B^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2b}{g\,i^2(ad-cb)^2}-\frac{B^2d\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2a}{g\,i^2(ad-cb)^2(dx+c)}+$ |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x,method=_RETU
RNVERBOSE)
```

output
```
((A^2*b*c-2*A*B*a*d+2*B^2*a*d)/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a
)/(d*x+c))+B*(A*b*c-B*a*d)/g/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(d
*x+c))^2+1/g/i*d*(A^2*b-2*A*B*b+2*B^2*b)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(
e*(b*x+a)/(d*x+c))+(A^2-2*A*B+2*B^2)*d/g/i/c/(a*d-b*c)*x+b*d*B*(-B+A)/g/i/
(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^2+1/3*B^2*b*c/g/i/(a^2
*d^2-2*a*b*c*d+b^2*c^2)*ln(e*(b*x+a)/(d*x+c))^3+1/3*b*B^2*d/g/i/(a^2*d^2-2
*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^3)/i/(d*x+c)
```

### 3.96.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 236, normalized size of antiderivative = 1.10

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \frac{(B^2bdx+B^2bc)\log\left(\frac{bex+ae}{dx+c}\right)^3+3\,(A^2-2\,AB+2\,B^2)bc-3\,(A^2-2\,AB+2\,B^2)ad+3\,(ABbc-B^2ad+}{3\,((b^2c^2d-2\,abcd^2+a^2d^3)gi^2x+(}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algo
rithm="fricas")
```

output
```
1/3*((B^2*b*d*x + B^2*b*c)*log((b*e*x + a*e)/(d*x + c))^3 + 3*(A^2 - 2*A*B
 + 2*B^2)*b*c - 3*(A^2 - 2*A*B + 2*B^2)*a*d + 3*(A*B*b*c - B^2*a*d + (A*B
- B^2)*b*d*x)*log((b*e*x + a*e)/(d*x + c))^2 + 3*(A^2*b*c + (A^2 - 2*A*B +
 2*B^2)*b*d*x - 2*(A*B - B^2)*a*d)*log((b*e*x + a*e)/(d*x + c)))/((b^2*c^2
*d - 2*a*b*c*d^2 + a^2*d^3)*g*i^2*x + (b^2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*
g*i^2)
```

### 3.96.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(185) = 370$.

Time = 0.73 (sec) , antiderivative size = 539, normalized size of antiderivative = 2.52

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{B^2b\log\left(\frac{e(a+bx)}{c+dx}\right)^3}{3a^2d^2gi^2-6abcdgi^2+3b^2c^2gi^2} + \frac{(-2AB+2B^2)\log\left(\frac{e(a+bx)}{c+dx}\right)}{acdgi^2+ad^2gi^2x-bc^2gi^2-bcdgi^2x}$$
$$+ (A^2-2AB+2B^2)\left(-\frac{b\log\left(x+\frac{-\frac{a^3bd^3}{(ad-bc)^2}+\frac{3a^2b^2cd^2}{(ad-bc)^2}-\frac{3ab^3c^2d}{(ad-bc)^2}+abd+\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{gi^2\,(ad-bc)^2} + \frac{b\log\left(x+\frac{\frac{a^3bd^3}{(ad-bc)^2}-\frac{3a^2b^2cd^2}{(ad-bc)^2}+\frac{3ab^3c^2d}{(ad-bc)^2}+abd-\frac{b^4c^3}{(ad-bc)^2}+b^2c}{2b^2d}\right)}{gi^2\,(ad-bc)^2} - \frac{1}{acdgi^2-bc^2gi^2+x\,(ad^2gi^2-bcdgi^2)}\right)$$
$$+ \frac{(ABbc+ABbdx-B^2ad-B^2bdx)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{a^2cd^2gi^2+a^2d^3gi^2x-2abc^2dgi^2-2abcd^2gi^2x+b^2c^3gi^2+b^2c^2dgi^2x}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)/(d*i*x+c*i)**2,x)
```

output
```
B**2*b*log(e*(a + b*x)/(c + d*x))**3/(3*a**2*d**2*g*i**2 - 6*a*b*c*d*g*i**
2 + 3*b**2*c**2*g*i**2) + (-2*A*B + 2*B**2)*log(e*(a + b*x)/(c + d*x))/(a*
c*d*g*i**2 + a*d**2*g*i**2*x - b*c**2*g*i**2 - b*c*d*g*i**2*x) + (A**2 - 2
*A*B + 2*B**2)*(-b*log(x + (-a**3*b*d**3/(a*d - b*c)**2 + 3*a**2*b**2*c*d*
*2/(a*d - b*c)**2 - 3*a*b**3*c**2*d/(a*d - b*c)**2 + a*b*d + b**4*c**3/(a*
d - b*c)**2 + b**2*c)/(2*b**2*d))/(g*i**2*(a*d - b*c)**2) + b*log(x + (a**
3*b*d**3/(a*d - b*c)**2 - 3*a**2*b**2*c*d**2/(a*d - b*c)**2 + 3*a*b**3*c**
2*d/(a*d - b*c)**2 + a*b*d - b**4*c**3/(a*d - b*c)**2 + b**2*c)/(2*b**2*d)
)/(g*i**2*(a*d - b*c)**2) - 1/(a*c*d*g*i**2 - b*c**2*g*i**2 + x*(a*d**2*g*
i**2 - b*c*d*g*i**2))) + (A*B*b*c + A*B*b*d*x - B**2*a*d - B**2*b*d*x)*log
(e*(a + b*x)/(c + d*x))**2/(a**2*c*d**2*g*i**2 + a**2*d**3*g*i**2*x - 2*a*
b*c**2*d*g*i**2 - 2*a*b*c*d**2*g*i**2*x + b**2*c**3*g*i**2 + b**2*c**2*d*g
*i**2*x)
```

3.96. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$

### 3.96.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1004 vs. 2(212) = 424.

Time = 0.26 (sec) , antiderivative size = 1004, normalized size of antiderivative = 4.69

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$

$$= B^2\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)\log\left(\frac{b}{dx}\right.$$
$$\left.+\frac{ae}{dx+c}\right)^2$$
$$+2\,AB\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)\log$$
$$\left.+\frac{ae}{dx+c}\right)$$
$$-\frac{1}{3}B^2\left(\frac{3\left((bdx+bc)\log(bx+a)^2+(bdx+bc)\log(dx+c)^2+2\,bc-2\,ad+2\,(bdx+bc)\log(bx+a)-\right.}{b^2c^3gi^2-2\,abc^2dgi^2+a^2cd^2gi^2+(b^2c^2dgi^2-2}\right.$$
$$+A^2\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)$$
$$-\frac{((bdx+bc)\log(bx+a)^2+(bdx+bc)\log(dx+c)^2+2\,bc-2\,ad+2\,(bdx+bc)\log(bx+a)-2\,(bdx+}{b^2c^3gi^2-2\,abc^2dgi^2+a^2cd^2gi^2+(b^2c^2dgi^2-2\,abcd^2gi^2+a^2d^3}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algo
rithm="maxima")
```

output
```
B^2*(1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/
((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*g*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2 + 2*A*B*(1/
((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c
^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d +
a^2*d^2)*g*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/3*B^2*(3*((b*d*x
 + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*
(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*
log(d*x + c))*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^2*c^3*g*i^2 - 2*a*b*
c^2*d*g*i^2 + a^2*c*d^2*g*i^2 + (b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2
*d^3*g*i^2)*x) - ((b*d*x + b*c)*log(b*x + a)^3 - (b*d*x + b*c)*log(d*x + c
)^3 + 3*(b*d*x + b*c)*log(b*x + a)^2 + 3*(b*d*x + b*c + (b*d*x + b*c)*log(
b*x + a))*log(d*x + c)^2 + 6*b*c - 6*a*d + 6*(b*d*x + b*c)*log(b*x + a) -
3*(2*b*d*x + (b*d*x + b*c)*log(b*x + a)^2 + 2*b*c + 2*(b*d*x + b*c)*log(b*
x + a))*log(d*x + c))/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2*g*i^2
 + (b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x)) + A^2*(1/((b*
c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c^2 -
 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*
d^2)*g*i^2)) - ((b*d*x + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^
2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*...
```

### 3.96.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 339, normalized size of antiderivative = 1.58

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=\frac{1}{3}\left(\frac{B^2be\log\left(\frac{bex+ae}{dx+c}\right)^3}{bcgi^2-adgi^2}+\frac{3\,A^2be\log\left(\frac{bex+ae}{dx+c}\right)}{bcgi^2-adgi^2}+3\left(\frac{ABbe}{bcgi^2-adgi^2}-\frac{(bex+ae)B^2d}{(bcgi^2-adgi^2)(dx+c)}\right)\log\left(\frac{bex+}{dx+}\right.\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x, algo
rithm="giac")
```

output
```
1/3*(B^2*b*e*log((b*e*x + a*e)/(d*x + c))^3/(b*c*g*i^2 - a*d*g*i^2) + 3*A^
2*b*e*log((b*e*x + a*e)/(d*x + c))/(b*c*g*i^2 - a*d*g*i^2) + 3*(A*B*b*e/(b
*c*g*i^2 - a*d*g*i^2) - (b*e*x + a*e)*B^2*d/((b*c*g*i^2 - a*d*g*i^2)*(d*x
+ c)))*log((b*e*x + a*e)/(d*x + c))^2 - 6*(A*B*d - B^2*d)*(b*e*x + a*e)*lo
g((b*e*x + a*e)/(d*x + c))/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) - 3*(A^2*d
- 2*A*B*d + 2*B^2*d)*(b*e*x + a*e)/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)))*(b
*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.96.9 Mupad [B] (verification not implemented)

Time = 2.92 (sec) , antiderivative size = 423, normalized size of antiderivative = 1.98

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{B\,b\,(A-B)}{g\,i^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{B^2\,(a\,d-b\,c)}{b\,d\,g\,i^2\,\left(\frac{x}{b}+\frac{c}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)-\frac{A^2-2\,A\,B+2\,B^2}{(a\,d-b\,c)\,\left(c\,g\,i^2+d\,g\,i^2\,x\right)}$$
$$+\frac{B^2\,b\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g\,i^2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}-\frac{2\,B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\,(A-B)\,(a\,d-b\,c)}{b\,d\,g\,i^2\,\left(\frac{x}{b}+\frac{c}{b\,d}\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}$$
$$-\frac{b\,\mathrm{atan}\left(\frac{b\,\left(2\,b\,d\,x+\frac{a^2\,d^2\,g\,i^2-b^2\,c^2\,g\,i^2}{g\,i^2\,(a\,d-b\,c)}\right)\,\left(A^2-2\,A\,B+2\,B^2\right)\,1\mathrm{i}}{(a\,d-b\,c)\,\left(b\,A^2-2\,b\,A\,B+2\,b\,B^2\right)}\right)\,\left(A^2-2\,A\,B+2\,B^2\right)\,2\mathrm{i}}{g\,i^2\,{(a\,d-b\,c)}^2}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)*(c*i + d*i*x)^2)
,x)
```

output
```
log((e*(a + b*x))/(c + d*x))^2*((B*b*(A - B))/(g*i^2*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d)) - (B^2*(a*d - b*c))/(b*d*g*i^2*(x/b + c/(b*d))*(a^2*d^2 + b^2*
c^2 - 2*a*b*c*d))) - (A^2 + 2*B^2 - 2*A*B)/((a*d - b*c)*(c*g*i^2 + d*g*i^2
*x)) + (B^2*b*log((e*(a + b*x))/(c + d*x))^3)/(3*g*i^2*(a^2*d^2 + b^2*c^2
- 2*a*b*c*d)) - (b*atan((b*(2*b*d*x + (a^2*d^2*g*i^2 - b^2*c^2*g*i^2)/(g*i
^2*(a*d - b*c)))*(A^2 + 2*B^2 - 2*A*B)*1i)/((a*d - b*c)*(A^2*b + 2*B^2*b -
 2*A*B*b)))*(A^2 + 2*B^2 - 2*A*B)*2i)/(g*i^2*(a*d - b*c)^2) - (2*B*log((e*
(a + b*x))/(c + d*x))*(A - B)*(a*d - b*c))/(b*d*g*i^2*(x/b + c/(b*d))*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d))
```

# 3.97 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$



## 3.97.1 Optimal result

Integrand size = 42, antiderivative size = 365

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx &= -\frac{2ABd^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}+\frac{2B^2d^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}\\
&\quad-\frac{2b^2B^2(c+dx)}{(bc-ad)^3g^2i^2(a+bx)}-\frac{2B^2d^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^3g^2i^2(c+dx)}\\
&\quad-\frac{2b^2B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^3g^2i^2(a+bx)}\\
&\quad+\frac{d^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^2i^2(c+dx)}\\
&\quad-\frac{b^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3g^2i^2(a+bx)}\\
&\quad-\frac{2bd\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3g^2i^2}\end{aligned}$$

output
```
-2*A*B*d^2*(b*x+a)/(-a*d+b*c)^3/g^2/i^2/(d*x+c)+2*B^2*d^2*(b*x+a)/(-a*d+b*
c)^3/g^2/i^2/(d*x+c)-2*b^2*B^2*(d*x+c)/(-a*d+b*c)^3/g^2/i^2/(b*x+a)-2*B^2*
d^2*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-2*b^2*B*(d*
x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g^2/i^2/(b*x+a)+d^2*(b*x+a)*
(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-b^2*(d*x+c)*(A+
B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g^2/i^2/(b*x+a)-2/3*b*d*(A+B*ln(e*
(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^3/g^2/i^2
```

### 3.97.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 307, normalized size of antiderivative = 0.84

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = -\frac{-3(A^2-2AB+2B^2)\,d(-bc+ad)(a+bx)+3b(A^2+2AB+2B^2)\,(bc-ad)(c+dx)+6b(A^2+2B^2)}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d
*i*x)^2),x]
```

output
```
-1/3*(-3*(A^2 - 2*A*B + 2*B^2)*d*(-(b*c) + a*d)*(a + b*x) + 3*b*(A^2 + 2*A
*B + 2*B^2)*(b*c - a*d)*(c + d*x) + 6*b*(A^2 + 2*B^2)*d*(a + b*x)*(c + d*x
)*Log[a + b*x] + 6*B*(b*c - a*d)*(A*b*c + b*B*c + a*A*d - a*B*d + 2*A*b*d*
x)*Log[(e*(a + b*x))/(c + d*x)] + 3*B*(-(a^2*B*d^2) + 2*a*b*d*(-(B*d*x) +
A*(c + d*x)) + b^2*(2*A*d*x*(c + d*x) + B*c*(c + 2*d*x)))*Log[(e*(a + b*x)
)/(c + d*x)]^2 + 2*b*B^2*d*(a + b*x)*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)
]^3 - 6*b*(A^2 + 2*B^2)*d*(a + b*x)*(c + d*x)*Log[c + d*x])/((b*c - a*d)^3
*g^2*i^2*(a + b*x)*(c + d*x))
```

### 3.97.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 254, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$$

$$\Big\downarrow \text{ 2962}$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i^2(bc-ad)^3}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+\frac{b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}-\frac{2bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}\right)d\frac{a+bx}{c+dx}}{g^2i^2(bc-ad)^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{b^2(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}-\frac{2b^2B(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}+\frac{d^2(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-\frac{2ABd^2(a+bx)}{c+dx}-\frac{2bd\left(B\log\right.}{}}{g^2i^2(bc-ad)^3}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d*i*x)^
2),x]
```

output
```
((-2*A*B*d^2*(a + b*x))/(c + d*x) + (2*B^2*d^2*(a + b*x))/(c + d*x) - (2*b
^2*B^2*(c + d*x))/(a + b*x) - (2*B^2*d^2*(a + b*x)*Log[(e*(a + b*x))/(c +
d*x)])/(c + d*x) - (2*b^2*B*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])
)/(a + b*x) + (d^2*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c +
d*x) - (b^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x) -
(2*b*d*(A + B*Log[(e*(a + b*x))/(c + d*x)])^3)/(3*B))/((b*c - a*d)^3*g^2*i
^2)
```

3.97. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$

#### 3.97.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.97.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 765 vs. $2(363) = 726$.

Time = 1.22 (sec) , antiderivative size = 766, normalized size of antiderivative = 2.10

| method | result |
|---|---|
| parts | $\frac{A^2\left(-\frac{d}{(ad-cb)^2(dx+c)}-\frac{2db\ln(dx+c)}{(ad-cb)^3}-\frac{b}{(ad-cb)^2(bx+a)}+\frac{2db\ln(bx+a)}{(ad-cb)^3}\right)}{g^2i^2}-\frac{B^2\left(\frac{d^2\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2-2\right.}{}\right.}{}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{d^2A^2b^2}{i^2(ad-cb)^4g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{2d^3A^2b\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^4g^2}+\frac{d^4A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^4g^2}+\frac{2d^2ABb^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(a}{d}\right.}{\frac{be}{d}+\frac{(a}{d(}}\right.}{i^2(a}\right.}{}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{d^2A^2b^2}{i^2(ad-cb)^4g^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}-\frac{2d^3A^2b\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^2(ad-cb)^4g^2}+\frac{d^4A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^2(ad-cb)^4g^2}+\frac{2d^2ABb^2\left(-\frac{\ln\left(\frac{be}{d}+\frac{(a}{d}\right.}{\frac{be}{d}+\frac{(a}{d(}}\right.}{i^2(a}\right.}{}$ |
| norman | $\frac{\left(2A^2abcd-2ABa^2d^2+2ABb^2c^2+2B^2a^2d^2+2B^2b^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)}{gi\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{B\left(2Aabcd-B\,a^2d^2+B\,b^2c^2\right)\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{gi\left(a^3d^3-3a^2bc\,d^2+3a\,b^2c^2d-b^3c^3\right)}+\frac{\left(2A^2ab\,d^2+\right.}{}$ |
| parallelrisch | Expression too large to display |
| risch | Expression too large to display |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,method=_RE
TURNVERBOSE)
```

output
```
1/g^2*A^2/i^2*(-d/(a*d-b*c)^2/(d*x+c)-2*d/(a*d-b*c)^3*b*ln(d*x+c)-b/(a*d-b
*c)^2/(b*x+a)+2*d/(a*d-b*c)^3*b*ln(b*x+a))-B^2/g^2/i^2/(a*d-b*c)/e*(d^2/(a
*d-b*c)^2*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2
-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*
c)*e/d/(d*x+c)+2*b*e/d)-2/3/(a*d-b*c)^2*b*d*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))^3+1/(a*d-b*c)^2*e^2*b^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*
d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e
/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))-2*B*A/g^2/i^2/(a*d-b*c)/e*(d
^2/(a*d-b*c)^2*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+
c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-1/(a*d-b*c)^2*b*d*e*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^2+1/(a*d-b*c)^2*e^2*b^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

## 3.97. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$

### 3.97.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 515, normalized size of antiderivative = 1.41

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \frac{12\,ABabcd-3\,(A^2+2\,AB+2\,B^2)b^2c^2+3\,(A^2-2\,AB+2\,B^2)a^2d^2-2\,(B^2b^2d^2x^2+B^2abcd+(B^2b^2cd}{}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output
```
1/3*(12*A*B*a*b*c*d - 3*(A^2 + 2*A*B + 2*B^2)*b^2*c^2 + 3*(A^2 - 2*A*B + 2
*B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + B^2*a*b*c*d + (B^2*b^2*c*d + B^2*a*b*
d^2)*x)*log((b*e*x + a*e)/(d*x + c))^3 - 3*(2*A*B*b^2*d^2*x^2 + B^2*b^2*c^
2 + 2*A*B*a*b*c*d - B^2*a^2*d^2 + 2*((A*B + B^2)*b^2*c*d + (A*B - B^2)*a*b
*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 6*((A^2 + 2*B^2)*b^2*c*d - (A^2
+ 2*B^2)*a*b*d^2)*x - 6*((A^2 + 2*B^2)*b^2*d^2*x^2 + A^2*a*b*c*d + (A*B +
B^2)*b^2*c^2 - (A*B - B^2)*a^2*d^2 + ((A^2 + 2*A*B + 2*B^2)*b^2*c*d + (A^2
 - 2*A*B + 2*B^2)*a*b*d^2)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^4*c^3*d -
3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b*d^4)*g^2*i^2*x^2 + (b^4*c^4 - 2*
a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*g^2*i^2*x + (a*b^3*c^4 - 3*a^2*b^2*
c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*g^2*i^2)
```

### 3.97.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1404 vs. 2(335) = 670.

Time = 3.08 (sec) , antiderivative size = 1404, normalized size of antiderivative = 3.85

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2/(d*i*x+c*i)**2,x)
```

output
```
2*B**2*b*d*log(e*(a + b*x)/(c + d*x))**3/(3*a**3*d**3*g**2*i**2 - 9*a**2*b
*c*d**2*g**2*i**2 + 9*a*b**2*c**2*d*g**2*i**2 - 3*b**3*c**3*g**2*i**2) - 2
*b*d*(A**2 + 2*B**2)*log(x + (2*A**2*a*b*d**2 + 2*A**2*b**2*c*d + 4*B**2*a
*b*d**2 + 4*B**2*b**2*c*d - 2*a**4*b*d**5*(A**2 + 2*B**2)/(a*d - b*c)**3 +
 8*a**3*b**2*c*d**4*(A**2 + 2*B**2)/(a*d - b*c)**3 - 12*a**2*b**3*c**2*d**
3*(A**2 + 2*B**2)/(a*d - b*c)**3 + 8*a*b**4*c**3*d**2*(A**2 + 2*B**2)/(a*d
 - b*c)**3 - 2*b**5*c**4*d*(A**2 + 2*B**2)/(a*d - b*c)**3)/(4*A**2*b**2*d*
*2 + 8*B**2*b**2*d**2))/(g**2*i**2*(a*d - b*c)**3) + 2*b*d*(A**2 + 2*B**2)
*log(x + (2*A**2*a*b*d**2 + 2*A**2*b**2*c*d + 4*B**2*a*b*d**2 + 4*B**2*b**
2*c*d + 2*a**4*b*d**5*(A**2 + 2*B**2)/(a*d - b*c)**3 - 8*a**3*b**2*c*d**4*
(A**2 + 2*B**2)/(a*d - b*c)**3 + 12*a**2*b**3*c**2*d**3*(A**2 + 2*B**2)/(a
*d - b*c)**3 - 8*a*b**4*c**3*d**2*(A**2 + 2*B**2)/(a*d - b*c)**3 + 2*b**5*
c**4*d*(A**2 + 2*B**2)/(a*d - b*c)**3)/(4*A**2*b**2*d**2 + 8*B**2*b**2*d**
2))/(g**2*i**2*(a*d - b*c)**3) + (-2*A*B*a*d - 2*A*B*b*c - 4*A*B*b*d*x + 2
*B**2*a*d - 2*B**2*b*c)*log(e*(a + b*x)/(c + d*x))/(a**3*c*d**2*g**2*i**2
+ a**3*d**3*g**2*i**2*x - 2*a**2*b*c**2*d*g**2*i**2 - a**2*b*c*d**2*g**2*i
**2*x + a**2*b*d**3*g**2*i**2*x**2 + a*b**2*c**3*g**2*i**2 - a*b**2*c**2*d
*g**2*i**2*x - 2*a*b**2*c*d**2*g**2*i**2*x**2 + b**3*c**3*g**2*i**2*x + b*
*3*c**2*d*g**2*i**2*x**2) + (2*A*B*a*b*c*d + 2*A*B*a*b*d**2*x + 2*A*B*b**2
*c*d*x + 2*A*B*b**2*d**2*x**2 - B**2*a**2*d**2 - 2*B**2*a*b*d**2*x + B*...
```

### 3.97.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1995 vs. $2(363) = 726$.

Time = 0.31 (sec) , antiderivative size = 1995, normalized size of antiderivative = 5.47

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output
```
-B^2*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i
^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^
2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/
((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))*log(b*e*x/(
d*x + c) + a*e/(d*x + c))^2 - 2*A*B*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d
^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2)
 + 2*b*d*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*g^2*i^2) - 2*b*d*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*g^2*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 2/3*B^2*(3*(b^2
*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*
x)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(
b*x + a)*log(d*x + c) - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*lo
g(d*x + c)^2)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(a*b^3*c^4*g^2*i^2 - 3*
a^2*b^2*c^3*d*g^2*i^2 + 3*a^3*b*c^2*d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4
*c^3*d*g^2*i^2 - 3*a*b^3*c^2*d^2*g^2*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b
*d^4*g^2*i^2)*x^2 + (b^4*c^4*g^2*i^2 - 2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d
^3*g^2*i^2 - a^4*d^4*g^2*i^2)*x) + (3*b^2*c^2 - 3*a^2*d^2 + (b^2*d^2*x^2 +
 a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)^3 + 3*(b^2*d^2*x^2 + a*b...
```

### 3.97.8 Giac [A] (verification not implemented)

Time = 57.24 (sec) , antiderivative size = 203, normalized size of antiderivative = 0.56

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx =$$
$$-\left(\frac{(dx+c)B^2e^2\log\left(\frac{bex+ae}{dx+c}\right)^2}{(bex+ae)g^2i^2}+\frac{2\,(ABe^2+B^2e^2)(dx+c)\log\left(\frac{bex+ae}{dx+c}\right)}{(bex+ae)g^2i^2}+\frac{(A^2e^2+2\,ABe^2+2\,B^2e^2)(dx}{(bex+ae)g^2i^2}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output
```
-((d*x + c)*B^2*e^2*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x + a*e)*g^2*i^2)
 + 2*(A*B*e^2 + B^2*e^2)*(d*x + c)*log((b*e*x + a*e)/(d*x + c))/((b*e*x +
a*e)*g^2*i^2) + (A^2*e^2 + 2*A*B*e^2 + 2*B^2*e^2)*(d*x + c)/((b*e*x + a*e)
*g^2*i^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
- a*d)))^2
```

### 3.97.9 Mupad [B] (verification not implemented)

Time = 3.76 (sec) , antiderivative size = 731, normalized size of antiderivative = 2.00

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \frac{2\,B^2\,b\,d\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g^2\,i^2\,(a\,d-b\,c)^3} - \frac{\frac{A^2\,a\,d+A^2\,b\,c+2\,B^2\,a\,d+2\,B^2\,b\,c-2\,A\,B\,a\,d+2\,A\,B\,b\,c}{a\,d-b\,c}+\frac{2\,x\,(b\,d\,A^2+2\,b\,d\,B^2)}{a\,d-b\,c}}{x\,(a^2\,d^2\,g^2\,i^2-b^2\,c^2\,g^2\,i^2)+x^2\,(a\,b\,d^2\,g^2\,i^2-b^2\,c\,d\,g^2\,i^2)-a\,b\,c^2\,g^2\,i^2+a^2\,c\,d\,g^2\,i^2} - \frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{2\,(B^2\,b\,c-B^2\,a\,d+A\,B\,a\,d+A\,B\,b\,c)}{g^2\,i^2\,(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d)}+\frac{4\,A\,B\,x}{g^2\,i^2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}\right)}{x^2+\frac{x\,(a\,d+b\,c)}{b\,d}+\frac{a\,c}{b\,d}} - \ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{\frac{B^2\,(a\,d+b\,c)}{g^2\,i^2\,(a^2\,b\,d^3-2\,a\,b^2\,c\,d^2+b^3\,c^2\,d)}+\frac{2\,B^2\,x}{g^2\,i^2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}}{x^2+\frac{x\,(a\,d+b\,c)}{b\,d}+\frac{a\,c}{b\,d}} - \frac{2\,A\,B\,b\,d}{g^2\,i^2\,(a\,d-b\,c)^3}\right) - \frac{b\,d\,\mathrm{atan}\left(\frac{b\,d\,(A^2+2\,B^2)\left(\frac{a^3\,d^3\,g^2\,i^2-a^2\,b\,c\,d^2\,g^2\,i^2-a\,b^2\,c^2\,d\,g^2\,i^2+b^3\,c^3\,g^2\,i^2}{a^2\,d^2\,g^2\,i^2-2\,a\,b\,c\,d\,g^2\,i^2+b^2\,c^2\,g^2\,i^2}+2\,b\,d\,x\right)(a^2\,d^2\,g^2\,i^2-2\,a\,b\,c\,d\,g^2\,i^2+b^2\,c^2\,g^2\,i^2)\,2\mathrm{i}}{g^2\,i^2\,(a\,d-b\,c)^3\,(2\,b\,d\,A^2+4\,b\,d\,B^2)}\right)(A^2+}{g^2\,i^2\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^2*(c*i + d*i*x)^
2),x)
```

output
```
(2*B^2*b*d*log((e*(a + b*x))/(c + d*x))^3)/(3*g^2*i^2*(a*d - b*c)^3) - ((A
^2*a*d + A^2*b*c + 2*B^2*a*d + 2*B^2*b*c - 2*A*B*a*d + 2*A*B*b*c)/(a*d - b
*c) + (2*x*(A^2*b*d + 2*B^2*b*d))/(a*d - b*c))/(x*(a^2*d^2*g^2*i^2 - b^2*c
^2*g^2*i^2) + x^2*(a*b*d^2*g^2*i^2 - b^2*c*d*g^2*i^2) - a*b*c^2*g^2*i^2 +
a^2*c*d*g^2*i^2) - (log((e*(a + b*x))/(c + d*x))*((2*(B^2*b*c - B^2*a*d +
A*B*a*d + A*B*b*c))/(g^2*i^2*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (4
*A*B*x)/(g^2*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))))/(x^2 + (x*(a*d + b*c))
/(b*d) + (a*c)/(b*d)) - (b*d*atan((b*d*(A^2 + 2*B^2)*((a^3*d^3*g^2*i^2 + b
^3*c^3*g^2*i^2 - a*b^2*c^2*d*g^2*i^2 - a^2*b*c*d^2*g^2*i^2)/(a^2*d^2*g^2*i
^2 + b^2*c^2*g^2*i^2 - 2*a*b*c*d*g^2*i^2) + 2*b*d*x)*(a^2*d^2*g^2*i^2 + b^
2*c^2*g^2*i^2 - 2*a*b*c*d*g^2*i^2)*2i)/(g^2*i^2*(a*d - b*c)^3*(2*A^2*b*d +
 4*B^2*b*d)))*(A^2 + 2*B^2)*4i)/(g^2*i^2*(a*d - b*c)^3) - log((e*(a + b*x)
)/(c + d*x))^2*(((B^2*(a*d + b*c))/(g^2*i^2*(a^2*b*d^3 + b^3*c^2*d - 2*a*b
^2*c*d^2)) + (2*B^2*x)/(g^2*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x^2 + (
x*(a*d + b*c))/(b*d) + (a*c)/(b*d)) - (2*A*B*b*d)/(g^2*i^2*(a*d - b*c)^3))
```

# 3.98 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$



## 3.98.1 Optimal result

Integrand size = 42, antiderivative size = 523

$$\begin{aligned}
\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx &= \frac{2ABd^3(a+bx)}{(bc-ad)^4g^3i^2(c+dx)}-\frac{2B^2d^3(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} \\
&+\frac{6b^2B^2d(c+dx)}{(bc-ad)^4g^3i^2(a+bx)}-\frac{b^3B^2(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} \\
&+\frac{2B^2d^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^4g^3i^2(c+dx)} \\
&+\frac{6b^2Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} \\
&-\frac{b^3B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} \\
&-\frac{d^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^3i^2(c+dx)} \\
&+\frac{3b^2d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^3i^2(a+bx)} \\
&-\frac{b^3(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^3i^2(a+bx)^2} \\
&+\frac{bd^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^4g^3i^2}
\end{aligned}$$

output
```
2*A*B*d^3*(b*x+a)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)-2*B^2*d^3*(b*x+a)/(-a*d+b*c
)^4/g^3/i^2/(d*x+c)+6*b^2*B^2*d*(d*x+c)/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/4*b
^3*B^2*(d*x+c)^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2+2*B^2*d^3*(b*x+a)*ln(e*(b*
x+a)/(d*x+c))/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+6*b^2*B*d*(d*x+c)*(A+B*ln(e*(b*
x+a)/(d*x+c)))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/2*b^3*B*(d*x+c)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2-d^3*(b*x+a)*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+3*b^2*d*(d*x+c)*(A+B*ln(e*(b
*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/2*b^3*(d*x+c)^2*(A+B*ln(e
*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2+b*d^2*(A+B*ln(e*(b*x+a
)/(d*x+c)))^3/B/(-a*d+b*c)^4/g^3/i^2
```

### 3.98.2 Mathematica [A] (verified)

Time = 0.70 (sec) , antiderivative size = 466, normalized size of antiderivative = 0.89

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$$
$$=\frac{4(A^2-2AB+2B^2)\,d^2(bc-ad)(a+bx)^2-b(2A^2+2AB+B^2)\,(bc-ad)^2(c+dx)+2b(4A^2+10AB+}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d
*i*x)^2),x]
```

output
```
(4*(A^2 - 2*A*B + 2*B^2)*d^2*(b*c - a*d)*(a + b*x)^2 - b*(2*A^2 + 2*A*B +
B^2)*(b*c - a*d)^2*(c + d*x) + 2*b*(4*A^2 + 10*A*B + 11*B^2)*d*(b*c - a*d)
*(a + b*x)*(c + d*x) + 6*b*(2*A^2 + 2*A*B + 5*B^2)*d^2*(a + b*x)^2*(c + d*
x)*Log[a + b*x] + 2*B*(b*c - a*d)*(4*(A - B)*d^2*(a + b*x)^2 - b*(2*A + B)
*(b*c - a*d)*(c + d*x) + 2*b*(4*A + 5*B)*d*(a + b*x)*(c + d*x))*Log[(e*(a
+ b*x))/(c + d*x)] - 2*B*(2*a^3*B*d^3 - 6*a^2*b*d^2*(-(B*d*x) + A*(c + d*x
)) - 6*a*b^2*d*(2*A*d*x*(c + d*x) + B*c*(c + 2*d*x)) + b^3*(-6*A*d^2*x^2*(
c + d*x) + B*(c^3 - 3*c^2*d*x - 9*c*d^2*x^2 - 3*d^3*x^3)))*Log[(e*(a + b*x
))/(c + d*x)]^2 + 4*b*B^2*d^2*(a + b*x)^2*(c + d*x)*Log[(e*(a + b*x))/(c +
 d*x)]^3 - 6*b*(2*A^2 + 2*A*B + 5*B^2)*d^2*(a + b*x)^2*(c + d*x)*Log[c + d
*x])/(4*(b*c - a*d)^4*g^3*i^2*(a + b*x)^2*(c + d*x))
```

### 3.98.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3(ci+dix)^2}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i^2(bc-ad)^4}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(-\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d^3+\frac{3b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d^2}{a+bx}-\frac{3b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d}{(a+bx)^2}+\frac{b^3(c+dx)^3\left(A+B\log\right.}{(a+bx)}\right.}{g^3i^2(bc-ad)^4}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^3(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}-\frac{b^3B(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}+\frac{6b^2Bd(c+dx)\left(B\log\left(\frac{e(a-}{c+}\right.\right.}{a+bx}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d*i*x)^
2),x]
```

output
```
((2*A*B*d^3*(a + b*x))/(c + d*x) - (2*B^2*d^3*(a + b*x))/(c + d*x) + (6*b^
2*B^2*d*(c + d*x))/(a + b*x) - (b^3*B^2*(c + d*x)^2)/(4*(a + b*x)^2) + (2*
B^2*d^3*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(c + d*x) + (6*b^2*B*d*(c
+ d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - (b^3*B*(c + d*x)^
2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(a + b*x)^2) - (d^3*(a + b*x)*(
A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c + d*x) + (3*b^2*d*(c + d*x)*(A +
 B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x) - (b^3*(c + d*x)^2*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)])^2)/(2*(a + b*x)^2) + (b*d^2*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])^3)/B)/((b*c - a*d)^4*g^3*i^2)
```

#### 3.98.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.98.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(517) = 1034$.

Time = 2.71 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.07

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1080 |
| derivativedivides | Expression too large to display | 1228 |
| default | Expression too large to display | 1228 |
| risch | Expression too large to display | 1736 |
| parallelrisch | Expression too large to display | 1763 |
| norman | Expression too large to display | 1849 |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,method=_RE
TURNVERBOSE)
```

output
```
A^2/g^3/i^2*(-d^2/(a*d-b*c)^3/(d*x+c)-3*d^2/(a*d-b*c)^4*b*ln(d*x+c)-1/2*b/
(a*d-b*c)^2/(b*x+a)^2+3*d^2/(a*d-b*c)^4*b*ln(b*x+a)-2*b/(a*d-b*c)^3*d/(b*x
+a))-B^2/g^3/i^2/(a*d-b*c)/e*(d^3/(a*d-b*c)^3*((b*e/d+(a*d-b*c)*e/d/(d*x+c
))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d)-1/(a*d-b*c)^3*
b*d^2*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3+3/(a*d-b*c)^3*b^2*d*e^2*(-1/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-
b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(
d*x+c)))-1/(a*d-b*c)^3*b^3*e^3*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+
(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))-2*B*A/g^3/i^2
/(a*d-b*c)/e*(d^3/(a*d-b*c)^3*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d
-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-3/2/(a*d-b*c)^3*b*d^2*e*ln
(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+3/(a*d-b*c)^3*b^2*d*e^2*(-1/(b*e/d+(a*d-b*
c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*
x+c)))-1/(a*d-b*c)^3*b^3*e^3*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/
d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2))
```

### 3.98.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 1005, normalized size of antiderivative = 1.92

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx =$$

$$-\frac{(2A^2+2AB+B^2)b^3c^3-12(A^2+2AB+2B^2)ab^2c^2d+3(2A^2+10AB+5B^2)a^2bcd^2+4(A^2-2$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output
```
-1/4*((2*A^2 + 2*A*B + B^2)*b^3*c^3 - 12*(A^2 + 2*A*B + 2*B^2)*a*b^2*c^2*d
 + 3*(2*A^2 + 10*A*B + 5*B^2)*a^2*b*c*d^2 + 4*(A^2 - 2*A*B + 2*B^2)*a^3*d^
3 - 4*(B^2*b^3*d^3*x^3 + B^2*a^2*b*c*d^2 + (B^2*b^3*c*d^2 + 2*B^2*a*b^2*d^
3)*x^2 + (2*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c
))^3 - 6*((2*A^2 + 2*A*B + 5*B^2)*b^3*c*d^2 - (2*A^2 + 2*A*B + 5*B^2)*a*b^
2*d^3)*x^2 - 2*(3*(2*A*B + B^2)*b^3*d^3*x^3 - B^2*b^3*c^3 + 6*B^2*a*b^2*c^
2*d + 6*A*B*a^2*b*c*d^2 - 2*B^2*a^3*d^3 + 3*(4*A*B*a*b^2*d^3 + (2*A*B + 3*
B^2)*b^3*c*d^2)*x^2 + 3*(B^2*b^3*c^2*d + 4*(A*B + B^2)*a*b^2*c*d^2 + 2*(A*
B - B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x + c))^2 - 3*((2*A^2 + 6*A*B
+ 7*B^2)*b^3*c^2*d + 2*(2*A^2 - 2*A*B + 3*B^2)*a*b^2*c*d^2 - (6*A^2 + 2*A*
B + 13*B^2)*a^2*b*d^3)*x - 2*(3*(2*A^2 + 2*A*B + 5*B^2)*b^3*d^3*x^3 + 6*A^
2*a^2*b*c*d^2 - (2*A*B + B^2)*b^3*c^3 + 12*(A*B + B^2)*a*b^2*c^2*d - 4*(A*
B - B^2)*a^3*d^3 + 3*((2*A^2 + 6*A*B + 7*B^2)*b^3*c*d^2 + 4*(A^2 + 2*B^2)*
a*b^2*d^3)*x^2 + 3*((2*A*B + 3*B^2)*b^3*c^2*d + 4*(A^2 + 2*A*B + 2*B^2)*a*
b^2*c*d^2 + 2*(A^2 - 2*A*B + 2*B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x +
 c)))/((b^6*c^4*d - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4
+ a^4*b^2*d^5)*g^3*i^2*x^3 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2
+ 8*a^3*b^3*c^2*d^3 - 7*a^4*b^2*c*d^4 + 2*a^5*b*d^5)*g^3*i^2*x^2 + (2*a*b^
5*c^5 - 7*a^2*b^4*c^4*d + 8*a^3*b^3*c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*
c*d^4 + a^6*d^5)*g^3*i^2*x + (a^2*b^4*c^5 - 4*a^3*b^3*c^4*d + 6*a^4*b^2...
```

### 3.98.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

### 3.98.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4187 vs. 2(517) = 1034.

Time = 0.51 (sec) , antiderivative size = 4187, normalized size of antiderivative = 8.01

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output
```
1/2*B^2*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3
*a*b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4
)*g^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d
^3 - 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2
*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3
*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^
4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2)
- 6*b*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3 + a^4*d^4)*g^3*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2 + A
*B*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*b*
d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*g^3
*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3 -
2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^2*
d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d +
3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 - 4
*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6*b
*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*
c*d^3 + a^4*d^4)*g^3*i^2))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/4*B^2*
(2*(b^3*c^3 - 12*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3 - 6*(b^3*c*d^2 -
 a*b^2*d^3)*x^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d...
```

### 3.98.8 Giac [A] (verification not implemented)

Time = 74.13 (sec) , antiderivative size = 460, normalized size of antiderivative = 0.88

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = -\frac{1}{4}\left(\frac{2\left(B^2be^3-\frac{2\,(bex+ae)B^2de^2}{dx+c}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^2bcg^3i^2}{(dx+c)^2}-\frac{(bex+ae)^2adg^3i^2}{(dx+c)^2}}+\frac{2\left(2\,ABbe^3+B^2be^3-\frac{4\,(bex+ae)ABde^2}{dx+c}-\frac{4\,(bex+ae)B^2de^2}{dx+c}\right)\mathrm{l}}{\frac{(bex+ae)^2bcg^3i^2}{(dx+c)^2}-\frac{(bex+ae)^2adg^3i^2}{(dx+c)^2}}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output
```
-1/4*(2*(B^2*b*e^3 - 2*(b*e*x + a*e)*B^2*d*e^2/(d*x + c))*log((b*e*x + a*e
)/(d*x + c))^2/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*e*x + a*e)^2*
a*d*g^3*i^2/(d*x + c)^2) + 2*(2*A*B*b*e^3 + B^2*b*e^3 - 4*(b*e*x + a*e)*A*
B*d*e^2/(d*x + c) - 4*(b*e*x + a*e)*B^2*d*e^2/(d*x + c))*log((b*e*x + a*e)
/(d*x + c))/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*e*x + a*e)^2*a*d
*g^3*i^2/(d*x + c)^2) + (2*A^2*b*e^3 + 2*A*B*b*e^3 + B^2*b*e^3 - 4*(b*e*x
+ a*e)*A^2*d*e^2/(d*x + c) - 8*(b*e*x + a*e)*A*B*d*e^2/(d*x + c) - 8*(b*e*
x + a*e)*B^2*d*e^2/(d*x + c))/((b*e*x + a*e)^2*b*c*g^3*i^2/(d*x + c)^2 - (
b*e*x + a*e)^2*a*d*g^3*i^2/(d*x + c)^2))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)
) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

### 3.98.9 Mupad [B] (verification not implemented)

Time = 7.22 (sec) , antiderivative size = 1497, normalized size of antiderivative = 2.86

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^3*(c*i + d*i*x)^
2),x)
```

output
```
(B^2*b*d^2*log((e*(a + b*x))/(c + d*x))^3)/(g^3*i^2*(a*d - b*c)^2*(a^2*d^2
 + b^2*c^2 - 2*a*b*c*d)) - ((4*A^2*a^2*d^2 - 2*A^2*b^2*c^2 + 8*B^2*a^2*d^2
 - B^2*b^2*c^2 - 8*A*B*a^2*d^2 - 2*A*B*b^2*c^2 + 10*A^2*a*b*c*d + 23*B^2*a
*b*c*d + 22*A*B*a*b*c*d)/(2*(a*d - b*c)) + (3*x^2*(2*A^2*b^2*d^2 + 5*B^2*b
^2*d^2 + 2*A*B*b^2*d^2))/(a*d - b*c) + (3*x*(6*A^2*a*b*d^2 + 13*B^2*a*b*d^
2 + 2*A^2*b^2*c*d + 7*B^2*b^2*c*d + 2*A*B*a*b*d^2 + 6*A*B*b^2*c*d))/(2*(a*
d - b*c)))/(x*(2*a^4*d^3*g^3*i^2 + 4*a*b^3*c^3*g^3*i^2 - 6*a^2*b^2*c^2*d*g
^3*i^2) + x^2*(2*b^4*c^3*g^3*i^2 + 4*a^3*b*d^3*g^3*i^2 - 6*a^2*b^2*c*d^2*g
^3*i^2) + x^3*(2*a^2*b^2*d^3*g^3*i^2 + 2*b^4*c^2*d*g^3*i^2 - 4*a*b^3*c*d^2
*g^3*i^2) + 2*a^2*b^2*c^3*g^3*i^2 + 2*a^4*c*d^2*g^3*i^2 - 4*a^3*b*c^2*d*g^
3*i^2) - (log((e*(a + b*x))/(c + d*x))*((B^2*b*c - 4*B^2*a*d + 4*A*B*a*d +
 2*A*B*b*c)/(2*g^3*i^2*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) - x*((3*(B
^2 - 2*A*B))/(2*g^3*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (3*B*(2*A + B)*
(a*d + b*c))/(g^3*i^2*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) + (3*B
*a*c*(2*A + B))/(g^3*i^2*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (3
*B*b*d*x^2*(2*A + B))/(g^3*i^2*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)
)))/(b*x^3 + (a^2*c)/(b*d) + (x^2*(b^2*c + 2*a*b*d))/(b*d) + (x*(a^2*d + 2
*a*b*c))/(b*d)) - (b*d^2*atan((b*d^2*(2*A^2 + 5*B^2 + 2*A*B)*(2*a^4*d^4*g^
3*i^2 - 2*b^4*c^4*g^3*i^2 + 4*a*b^3*c^3*d*g^3*i^2 - 4*a^3*b*c*d^3*g^3*i^2)
*3i)/(2*g^3*i^2*(a*d - b*c)^4*(6*A^2*b*d^2 + 15*B^2*b*d^2 + 6*A*B*b*d^2...
```

3.98. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$

# 3.99 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$

### 3.99.1 Optimal result

Integrand size = 42, antiderivative size = 682

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{2ABd^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)}+\frac{2B^2d^4(a+bx)}{(bc-ad)^5g^4i^2(c+dx)} -\frac{12b^2B^2d^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)}+\frac{b^3B^2d(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2} -\frac{2b^4B^2(c+dx)^3}{27(bc-ad)^5g^4i^2(a+bx)^3}-\frac{2B^2d^4(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^5g^4i^2(c+dx)} -\frac{12b^2Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)} +\frac{2b^3Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2} -\frac{2b^4B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^5g^4i^2(a+bx)^3} +\frac{d^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(c+dx)} -\frac{6b^2d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)} +\frac{2b^3d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2} -\frac{b^4(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3} -\frac{4bd^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^5g^4i^2}$$

output
```
-2*A*B*d^4*(b*x+a)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)+2*B^2*d^4*(b*x+a)/(-a*d+b*
c)^5/g^4/i^2/(d*x+c)-12*b^2*B^2*d^2*(d*x+c)/(-a*d+b*c)^5/g^4/i^2/(b*x+a)+b
^3*B^2*d*(d*x+c)^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-2/27*b^4*B^2*(d*x+c)^3/(
-a*d+b*c)^5/g^4/i^2/(b*x+a)^3-2*B^2*d^4*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*
d+b*c)^5/g^4/i^2/(d*x+c)-12*b^2*B*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/
(-a*d+b*c)^5/g^4/i^2/(b*x+a)+2*b^3*B*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c)
))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-2/9*b^4*B*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d
*x+c)))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3+d^4*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+
c)))^2/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-6*b^2*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d
*x+c)))^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)+2*b^3*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)
/(d*x+c)))^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-1/3*b^4*(d*x+c)^3*(A+B*ln(e*(b
*x+a)/(d*x+c)))^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3-4/3*b*d^3*(A+B*ln(e*(b*x+
a)/(d*x+c)))^3/B/(-a*d+b*c)^5/g^4/i^2
```

### 3.99.2 Mathematica [A] (verified)

Time = 1.00 (sec) , antiderivative size = 613, normalized size of antiderivative = 0.90

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx =$$

$$-\frac{-27(A^2-2AB+2B^2)\,d^3(-bc+ad)(a+bx)^3+b(9A^2+6AB+2B^2)\,(bc-ad)^3(c+dx)-3b(9A^2+}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d
*i*x)^2),x]
```

output
```
-1/27*(-27*(A^2 - 2*A*B + 2*B^2)*d^3*(-(b*c) + a*d)*(a + b*x)^3 + b*(9*A^2
 + 6*A*B + 2*B^2)*(b*c - a*d)^3*(c + d*x) - 3*b*(9*A^2 + 12*A*B + 7*B^2)*d
*(b*c - a*d)^2*(a + b*x)*(c + d*x) + 3*b*(27*A^2 + 78*A*B + 92*B^2)*d^2*(b
*c - a*d)*(a + b*x)^2*(c + d*x) + 6*b*(18*A^2 + 30*A*B + 55*B^2)*d^3*(a +
b*x)^3*(c + d*x)*Log[a + b*x] + 6*B*(b*c - a*d)*(9*(A - B)*d^3*(a + b*x)^3
 + b*(3*A + B)*(b*c - a*d)^2*(c + d*x) - 3*b*(3*A + 2*B)*d*(b*c - a*d)*(a
+ b*x)*(c + d*x) + 3*b*(9*A + 13*B)*d^2*(a + b*x)^2*(c + d*x))*Log[(e*(a +
 b*x))/(c + d*x)] + 9*B*(-3*a^4*B*d^4 + 12*a^3*b*d^3*(-(B*d*x) + A*(c + d*
x)) + 18*a^2*b^2*d^2*(2*A*d*x*(c + d*x) + B*c*(c + 2*d*x)) + 6*a*b^3*d*(6*
A*d^2*x^2*(c + d*x) + B*(-c^3 + 3*c^2*d*x + 9*c*d^2*x^2 + 3*d^3*x^3)) + b^
4*(12*A*d^3*x^3*(c + d*x) + B*(c^4 - 2*c^3*d*x + 6*c^2*d^2*x^2 + 22*c*d^3*
x^3 + 10*d^4*x^4)))*Log[(e*(a + b*x))/(c + d*x)]^2 + 36*b*B^2*d^3*(a + b*x
)^3*(c + d*x)*Log[(e*(a + b*x))/(c + d*x)]^3 - 6*b*(18*A^2 + 30*A*B + 55*B
^2)*d^3*(a + b*x)^3*(c + d*x)*Log[c + d*x])/((b*c - a*d)^5*g^4*i^2*(a + b*
x)^3*(c + d*x))
```

### 3.99.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 475, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^4(ci+dix)^2}dx$$

$$\Big\downarrow \text{ 2962}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i^2(bc-ad)^5}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d^4-\frac{4b(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d^3}{a+bx}+\frac{6b^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d^2}{(a+bx)^2}-\frac{4b^3(c+dx)^3\left(A+B\log\right.}{(a+bx)}\right.}{g^4i^2(bc-ad)^5}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^4(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}-\frac{2b^4B(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9(a+bx)^3}+\frac{2b^3d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(a+bx)^2}+\frac{2b^3Bd(c+dx)^2\left(B\log\left(\right.\right.}{(a+bx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d*i*x)^
2),x]
```

output
```
((-2*A*B*d^4*(a + b*x))/(c + d*x) + (2*B^2*d^4*(a + b*x))/(c + d*x) - (12*
b^2*B^2*d^2*(c + d*x))/(a + b*x) + (b^3*B^2*d*(c + d*x)^2)/(a + b*x)^2 - (
2*b^4*B^2*(c + d*x)^3)/(27*(a + b*x)^3) - (2*B^2*d^4*(a + b*x)*Log[(e*(a +
 b*x))/(c + d*x)])/(c + d*x) - (12*b^2*B*d^2*(c + d*x)*(A + B*Log[(e*(a +
b*x))/(c + d*x)]))/(a + b*x) + (2*b^3*B*d*(c + d*x)^2*(A + B*Log[(e*(a + b
*x))/(c + d*x)]))/(a + b*x)^2 - (2*b^4*B*(c + d*x)^3*(A + B*Log[(e*(a + b*
x))/(c + d*x)]))/(9*(a + b*x)^3) + (d^4*(a + b*x)*(A + B*Log[(e*(a + b*x))
/(c + d*x)])^2)/(c + d*x) - (6*b^2*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/
(c + d*x)])^2)/(a + b*x) + (2*b^3*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(
c + d*x)])^2)/(a + b*x)^2 - (b^4*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c +
 d*x)])^2)/(3*(a + b*x)^3) - (4*b*d^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^3)/(3*B))/((b*c - a*d)^5*g^4*i^2)
```

### 3.99.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.99. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$

### 3.99.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1385 vs. $2(674) = 1348$.

Time = 2.94 (sec) , antiderivative size = 1386, normalized size of antiderivative = 2.03

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1386 |
| derivativedivides | Expression too large to display | 1588 |
| default | Expression too large to display | 1588 |
| risch | Expression too large to display | 2173 |
| parallelrisch | Expression too large to display | 2708 |
| norman | Expression too large to display | 2748 |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,method=_RE
TURNVERBOSE)
```

output
```
A^2/g^4/i^2*(-d^3/(a*d-b*c)^4/(d*x+c)-4*d^3/(a*d-b*c)^5*b*ln(d*x+c)-1/3*b/
(a*d-b*c)^2/(b*x+a)^3+4*d^3/(a*d-b*c)^5*b*ln(b*x+a)-3*b/(a*d-b*c)^4*d^2/(b
*x+a)-b/(a*d-b*c)^3*d/(b*x+a)^2)-B^2/g^4/i^2/(a*d-b*c)/e*(d^4/(a*d-b*c)^4*
((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*
x+c)+2*b*e/d)-4/3/(a*d-b*c)^4*b*d^3*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3+6/
(a*d-b*c)^4*b^2*d^2*e^2*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*
c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))-4/(a*d-b*c)^4*b^3*d*e^3*(-1/2/(b*
e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2)+1/(a*d-b*c)^4*b^4*e^4*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+
c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/27/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2
*B*A/g^4/i^2/(a*d-b*c)/e*(d^4/(a*d-b*c)^4*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)-2/(a*d-b*c)^4*
b*d^3*e*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2+6/(a*d-b*c)^4*b^2*d^2*e^2*(-1/(b
*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-
b*c)*e/d/(d*x+c)))-4/(a*d-b*c)^4*b^3*d*e^3*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c)...
```

### 3.99.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1534 vs. $2(674) = 1348$.

Time = 0.36 (sec) , antiderivative size = 1534, normalized size of antiderivative = 2.25

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output
```
-1/27*((9*A^2 + 6*A*B + 2*B^2)*b^4*c^4 - 27*(2*A^2 + 2*A*B + B^2)*a*b^3*c^
3*d + 162*(A^2 + 2*A*B + 2*B^2)*a^2*b^2*c^2*d^2 - 5*(18*A^2 + 66*A*B + 49*
B^2)*a^3*b*c*d^3 - 27*(A^2 - 2*A*B + 2*B^2)*a^4*d^4 + 6*((18*A^2 + 30*A*B
+ 55*B^2)*b^4*c*d^3 - (18*A^2 + 30*A*B + 55*B^2)*a*b^3*d^4)*x^3 + 36*(B^2*
b^4*d^4*x^4 + B^2*a^3*b*c*d^3 + (B^2*b^4*c*d^3 + 3*B^2*a*b^3*d^4)*x^3 + 3*
(B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*x^2 + (3*B^2*a^2*b^2*c*d^3 + B^2*a^3*b
*d^4)*x)*log((b*e*x + a*e)/(d*x + c))^3 + 3*((18*A^2 + 66*A*B + 85*B^2)*b^
4*c^2*d^2 + 8*(9*A^2 + 6*A*B + 20*B^2)*a*b^3*c*d^3 - (90*A^2 + 114*A*B + 2
45*B^2)*a^2*b^2*d^4)*x^2 + 9*(2*(6*A*B + 5*B^2)*b^4*d^4*x^4 + B^2*b^4*c^4
- 6*B^2*a*b^3*c^3*d + 18*B^2*a^2*b^2*c^2*d^2 + 12*A*B*a^3*b*c*d^3 - 3*B^2*
a^4*d^4 + 2*((6*A*B + 11*B^2)*b^4*c*d^3 + 9*(2*A*B + B^2)*a*b^3*d^4)*x^3 +
 6*(B^2*b^4*c^2*d^2 + 6*A*B*a^2*b^2*d^4 + 3*(2*A*B + 3*B^2)*a*b^3*c*d^3)*x
^2 - 2*(B^2*b^4*c^3*d - 9*B^2*a*b^3*c^2*d^2 - 18*(A*B + B^2)*a^2*b^2*c*d^3
 - 6*(A*B - B^2)*a^3*b*d^4)*x)*log((b*e*x + a*e)/(d*x + c))^2 - ((18*A^2 +
 30*A*B + 19*B^2)*b^4*c^3*d - 81*(2*A^2 + 6*A*B + 7*B^2)*a*b^3*c^2*d^2 - 3
*(18*A^2 - 114*A*B - 29*B^2)*a^2*b^2*c*d^3 + (198*A^2 + 114*A*B + 461*B^2)
*a^3*b*d^4)*x + 6*((18*A^2 + 30*A*B + 55*B^2)*b^4*d^4*x^4 + 18*A^2*a^3*b*c
*d^3 + (3*A*B + B^2)*b^4*c^4 - 9*(2*A*B + B^2)*a*b^3*c^3*d + 54*(A*B + B^2
)*a^2*b^2*c^2*d^2 - 9*(A*B - B^2)*a^4*d^4 + ((18*A^2 + 66*A*B + 85*B^2)*b^
4*c*d^3 + 27*(2*A^2 + 2*A*B + 5*B^2)*a*b^3*d^4)*x^3 + 3*((6*A*B + 11*B^...
```

### 3.99.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

### 3.99.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6160 vs. $2(674) = 1348$.

Time = 0.70 (sec) , antiderivative size = 6160, normalized size of antiderivative = 9.03

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output
```
-1/3*B^2*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a
^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 -
11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3
*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5
*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*
x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d
^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^
4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*
g^4*i^2*x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c
^2*d^3 + a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c
^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*
g^4*i^2) - 12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^
3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(b*e*x/
(d*x + c) + a*e/(d*x + c))^2 - 2/3*A*B*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^
2*c^2*d + 13*a^2*b*c*d^2 + 3*a^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2
*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d
^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7
*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c
*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3
*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2...
```

### 3.99.8 Giac [A] (verification not implemented)

Time = 88.80 (sec) , antiderivative size = 754, normalized size of antiderivative = 1.11

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx =$$

$$-\frac{1}{54}\left(\frac{18\left(B^2b^2e^4-\frac{3\,(bex+ae)B^2bde^3}{dx+c}+\frac{3\,(bex+ae)^2B^2d^2e^2}{(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2}{\frac{(bex+ae)^3b^2c^2g^4i^2}{(dx+c)^3}-\frac{2\,(bex+ae)^3abcdg^4i^2}{(dx+c)^3}+\frac{(bex+ae)^3a^2d^2g^4i^2}{(dx+c)^3}}+\frac{6\left(6\,ABb^2e^4+2\,B^2b^2e^4-\frac{18\,(bex+}{dx}\right.}{(bex+}\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output
```
-1/54*(18*(B^2*b^2*e^4 - 3*(b*e*x + a*e)*B^2*b*d*e^3/(d*x + c) + 3*(b*e*x
+ a*e)^2*B^2*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))^2/((b*e*x +
 a*e)^3*b^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4*i^2/(d
*x + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d*x + c)^3) + 6*(6*A*B*b^2*e^
4 + 2*B^2*b^2*e^4 - 18*(b*e*x + a*e)*A*B*b*d*e^3/(d*x + c) - 9*(b*e*x + a*
e)*B^2*b*d*e^3/(d*x + c) + 18*(b*e*x + a*e)^2*A*B*d^2*e^2/(d*x + c)^2 + 18
*(b*e*x + a*e)^2*B^2*d^2*e^2/(d*x + c)^2)*log((b*e*x + a*e)/(d*x + c))/((b
*e*x + a*e)^3*b^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*c*d*g^4*
i^2/(d*x + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d*x + c)^3) + (18*A^2*b
^2*e^4 + 12*A*B*b^2*e^4 + 4*B^2*b^2*e^4 - 54*(b*e*x + a*e)*A^2*b*d*e^3/(d*
x + c) - 54*(b*e*x + a*e)*A*B*b*d*e^3/(d*x + c) - 27*(b*e*x + a*e)*B^2*b*d
*e^3/(d*x + c) + 54*(b*e*x + a*e)^2*A^2*d^2*e^2/(d*x + c)^2 + 108*(b*e*x +
 a*e)^2*A*B*d^2*e^2/(d*x + c)^2 + 108*(b*e*x + a*e)^2*B^2*d^2*e^2/(d*x + c
)^2)/((b*e*x + a*e)^3*b^2*c^2*g^4*i^2/(d*x + c)^3 - 2*(b*e*x + a*e)^3*a*b*
c*d*g^4*i^2/(d*x + c)^3 + (b*e*x + a*e)^3*a^2*d^2*g^4*i^2/(d*x + c)^3))*(b
*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))^2
```

### 3.99.9 Mupad [B] (verification not implemented)

Time = 10.02 (sec) , antiderivative size = 2701, normalized size of antiderivative = 3.96

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^4*(c*i + d*i*x)^
2),x)
```

output
```
(log((e*(a + b*x))/(c + d*x))*(x^2*((4*B^2*b*d)/(g^4*i^2*(a*d - b*c)^3) -
(4*b*d^3*(b*d*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c
))/(2*b*d^2)) + ((a*d + b*c)*(a*d - b*c))/d^2)*(5*B^2 + 6*A*B))/(3*g^4*i^2
*(a*d - b*c)^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) + x*(
(8*(2*B^2 - 3*A*B))/(9*g^4*i^2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (4*B^2*(
a*d + b*c))/(g^4*i^2*(a*d - b*c)^3) - (4*b*d^3*(((2*a^2*d^2 + b^2*c^2 - 3*
a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^2))*(a*d + b*c) + (a*c*(a*d -
b*c))/d^2)*(5*B^2 + 6*A*B))/(3*g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^3*c^3 +
3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (2*(B^2*b*c - 9*B^2*a*d + 9*A*B*a*d + 3
*A*B*b*c))/(9*g^4*i^2*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (4*B^2*a*
c)/(g^4*i^2*(a*d - b*c)^3) - (4*b^2*d^2*x^3*(5*B^2 + 6*A*B))/(3*g^4*i^2*(a
*d - b*c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (4*a*b*c*
d^3*((2*a^2*d^2 + b^2*c^2 - 3*a*b*c*d)/(2*b*d^3) + (a*(a*d - b*c))/(2*b*d^
2))*(5*B^2 + 6*A*B))/(3*g^4*i^2*(a*d - b*c)^2*(a^3*d^3 - b^3*c^3 + 3*a*b^2
*c^2*d - 3*a^2*b*c*d^2))))/(b^2*x^4 + (a^3*c)/(b*d) + (x*(a^3*d + 3*a^2*b*
c))/(b*d) + (x^3*(b^3*c + 3*a*b^2*d))/(b*d) + (x^2*(3*a*b^2*c + 3*a^2*b*d)
)/(b*d)) - ((27*A^2*a^3*d^3 + 9*A^2*b^3*c^3 + 54*B^2*a^3*d^3 + 2*B^2*b^3*c
^3 - 54*A*B*a^3*d^3 + 6*A*B*b^3*c^3 - 45*A^2*a*b^2*c^2*d + 117*A^2*a^2*b*c
*d^2 - 25*B^2*a*b^2*c^2*d + 299*B^2*a^2*b*c*d^2 - 48*A*B*a*b^2*c^2*d + 276
*A*B*a^2*b*c*d^2)/(3*(a*d - b*c)) + (2*x^3*(18*A^2*b^3*d^3 + 55*B^2*b^3...
```

## 3.99. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$

# 3.100 $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$

### 3.100.1 Optimal result

Integrand size = 42, antiderivative size = 635

$$
\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx \\
&= \frac{B^2(bc-ad)g^3(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{4AbB(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} \\
&\quad + \frac{4bB^2(bc-ad)g^3(a+bx)}{d^3i^3(c+dx)} - \frac{4bB^2(bc-ad)g^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^3i^3(c+dx)} \\
&\quad - \frac{B(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2d^2i^3(c+dx)^2} \\
&\quad + \frac{2b^2B(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{d^4i^3} \\
&\quad + \frac{b^2g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3} + \frac{(bc-ad)g^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2d^2i^3(c+dx)^2} \\
&\quad + \frac{2b(bc-ad)g^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3(c+dx)} \\
&\quad + \frac{3b^2(bc-ad)g^3\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^4i^3} \\
&\quad + \frac{2b^2B^2(bc-ad)g^3\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} \\
&\quad + \frac{6b^2B(bc-ad)g^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3} \\
&\quad - \frac{6b^2B^2(bc-ad)g^3\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}
\end{aligned}
$$

output
```
1/4*B^2*(-a*d+b*c)*g^3*(b*x+a)^2/d^2/i^3/(d*x+c)^2-4*A*b*B*(-a*d+b*c)*g^3*
(b*x+a)/d^3/i^3/(d*x+c)+4*b*B^2*(-a*d+b*c)*g^3*(b*x+a)/d^3/i^3/(d*x+c)-4*b
*B^2*(-a*d+b*c)*g^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/d^3/i^3/(d*x+c)-1/2*B*(-
a*d+b*c)*g^3*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^2/i^3/(d*x+c)^2+2*b^2
*B*(-a*d+b*c)*g^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))/d^4
/i^3+b^2*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^3+1/2*(-a*d+b*c)*
g^3*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i^3/(d*x+c)^2+2*b*(-a*d+b*
c)*g^3*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^3/(d*x+c)+3*b^2*(-a*d+b
*c)*g^3*ln((-a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^4/i^3+2*b
^2*B^2*(-a*d+b*c)*g^3*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^3+6*b^2*B*(-a*d
+b*c)*g^3*(A+B*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i
^3-6*b^2*B^2*(-a*d+b*c)*g^3*polylog(3,d*(b*x+a)/b/(d*x+c))/d^4/i^3
```

### 3.100.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5929 vs. $2(635) = 1270$.

Time = 7.36 (sec) , antiderivative size = 5929, normalized size of antiderivative = 9.34

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i +
d*i*x)^3,x]
```

output
```
Result too large to show
```

### 3.100.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 504, normalized size of antiderivative = 0.79, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^3}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{g^3(bc-ad)\int \frac{(a+bx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{g^3(bc-ad)\int \left(\frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^3}{d^3\left(\frac{d(a+bx)}{c+dx}-b\right)^2}+\frac{3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^2}{d^3\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b}{d^3}+\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2(c+dx)}\right)}{i^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^3(bc-ad)\left(\frac{6b^2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^4}+\frac{3b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^4}+\frac{2b^2B\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)(B}{d^4}\right.}{}$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)
^3,x]
```

output
```
((b*c - a*d)*g^3*((B^2*(a + b*x)^2)/(4*d^2*(c + d*x)^2) - (4*A*b*B*(a + b*
x))/(d^3*(c + d*x)) + (4*b*B^2*(a + b*x))/(d^3*(c + d*x)) - (4*b*B^2*(a +
b*x)*Log[(e*(a + b*x))/(c + d*x)])/(d^3*(c + d*x)) - (B*(a + b*x)^2*(A + B
*Log[(e*(a + b*x))/(c + d*x)]))/(2*d^2*(c + d*x)^2) + ((a + b*x)^2*(A + B*
Log[(e*(a + b*x))/(c + d*x)])^2)/(2*d^2*(c + d*x)^2) + (2*b*(a + b*x)*(A +
 B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^3*(c + d*x)) + (b^2*(a + b*x)*(A +
B*Log[(e*(a + b*x))/(c + d*x)])^2)/(d^3*(c + d*x)*(b - (d*(a + b*x))/(c +
d*x))) + (2*b^2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Log[1 - (d*(a + b*x
))/(b*(c + d*x))])/d^4 + (3*b^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2*Log
[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 + (2*b^2*B^2*PolyLog[2, (d*(a + b*x
))/(b*(c + d*x))])/d^4 + (6*b^2*B*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Pol
yLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 - (6*b^2*B^2*PolyLog[3, (d*(a +
b*x))/(b*(c + d*x))])/d^4))/i^3
```

## 3.100. $\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$

### 3.100.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.100.4 Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix+ci)^3}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

## 3.100.5 Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3 \left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output
```
integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*
a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*b^
2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log((b*e*x + a*e)/(d*x + c)))
/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

### 3.100.6 Sympy [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{g^3 \left(\int \frac{A^2a^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{A^2b^3x^3}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2a^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2ABa^3\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right.}{c^3+3c^2dx+3cd^2x^2+d^3x}\right.}{}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**3,x)
```

output
```
g**3*(Integral(A**2*a**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(A**2*b**3*x**3/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**
3), x) + Integral(B**2*a**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*a**3*log(a*
e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x
**3), x) + Integral(3*A**2*a*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
+ d**3*x**3), x) + Integral(3*A**2*a**2*b*x/(c**3 + 3*c**2*d*x + 3*c*d**2*
x**2 + d**3*x**3), x) + Integral(B**2*b**3*x**3*log(a*e/(c + d*x) + b*e*x/
(c + d*x))**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integr
al(2*A*B*b**3*x**3*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(3*B**2*a*b**2*x**2*log(a*e/(c
 + d*x) + b*e*x/(c + d*x))**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x*
*3), x) + Integral(3*B**2*a**2*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2
/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(6*A*B*a*b*
*2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2
*x**2 + d**3*x**3), x) + Integral(6*A*B*a**2*b*x*log(a*e/(c + d*x) + b*e*x
/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

### 3.100.7 Maxima [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output
```
-3/2*A*B*a^2*b*g^3*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^
4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2
*a*d^2)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b
*c^3*d^2 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^
2 - 2*a*b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c
^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3)) - 1/2*A^2*b^3*g^3*((6*c^2*d*x + 5*c^
3)/(d^6*i^3*x^2 + 2*c*d^5*i^3*x + c^2*d^4*i^3) - 2*x/(d^3*i^3) + 6*c*log(d
*x + c)/(d^4*i^3)) + 1/2*A*B*a^3*g^3*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 -
a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3
) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c*d^2*i^3*x +
c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)
- 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 3/2*A^2*
a*b^2*g^3*((4*c*d*x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) +
 2*log(d*x + c)/(d^3*i^3)) - 3/2*(2*d*x + c)*A^2*a^2*b*g^3/(d^4*i^3*x^2 +
2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*A^2*a^3*g^3/(d^3*i^3*x^2 + 2*c*d^2*i^3*
x + c^2*d*i^3) - 1/2*(2*((b^3*c*d^2*g^3 - a*b^2*d^3*g^3)*B^2*x^2 + 2*(b^3*
c^2*d*g^3 - a*b^2*c*d^2*g^3)*B^2*x + (b^3*c^3*g^3 - a*b^2*c^2*d*g^3)*B^2)*
log(d*x + c)^3 - (2*B^2*b^3*d^3*g^3*x^3 + 4*B^2*b^3*c*d^2*g^3*x^2 - 2*(2*b
^3*c^2*d*g^3 - 6*a*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B^2*x - (5*b^3*c^3*g^3
 - 9*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 + a^3*d^3*g^3)*B^2)*log(d*x + ...
```

### 3.100.8 Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*
i)^3, x)
```

### 3.100.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3} \, dx = \int \frac{(a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^3} \, dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^3,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^3, x)
```

# 3.101 $\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$



## 3.101.1 Optimal result

Integrand size = 42, antiderivative size = 410

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = -\frac{B^2g^2(a+bx)^2}{4di^3(c+dx)^2}+\frac{2AbBg^2(a+bx)}{d^2i^3(c+dx)}-\frac{2bB^2g^2(a+bx)}{d^2i^3(c+dx)}+\frac{2bB^2g^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{d^2i^3(c+dx)}+\frac{Bg^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2di^3(c+dx)^2}-\frac{g^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2di^3(c+dx)^2}-\frac{bg^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2i^3(c+dx)}-\frac{b^2g^2\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^3i^3}-\frac{2b^2Bg^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}+\frac{2b^2B^2g^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}$$

output
```
-1/4*B^2*g^2*(b*x+a)^2/d/i^3/(d*x+c)^2+2*A*b*B*g^2*(b*x+a)/d^2/i^3/(d*x+c)
-2*b*B^2*g^2*(b*x+a)/d^2/i^3/(d*x+c)+2*b*B^2*g^2*(b*x+a)*ln(e*(b*x+a)/(d*x
+c))/d^2/i^3/(d*x+c)+1/2*B*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/d/i^3
/(d*x+c)^2-1/2*g^2*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d/i^3/(d*x+c)^2
-b*g^2*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^2/i^3/(d*x+c)-b^2*g^2*ln((-
a*d+b*c)/b/(d*x+c))*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/d^3/i^3-2*b^2*B*g^2*(A+B
*ln(e*(b*x+a)/(d*x+c)))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^3+2*b^2*B^2*g
^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^3/i^3
```

### 3.101.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3384 vs. $2(410) = 820$.

Time = 4.08 (sec) , antiderivative size = 3384, normalized size of antiderivative = 8.25

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i +
d*i*x)^3,x]
```

output
```
(g^2*((-6*A^2*(b*c - a*d)^2)/(c + d*x)^2 + (24*A^2*b*(b*c - a*d))/(c + d*x
) + 12*A^2*b^2*Log[c + d*x] - (12*a*A*b*B*d*(-(b^2*c^3) + 4*a*b*c^2*d - 3*
a^2*c*d^2 - 2*b^2*c^2*d*x + 6*a*b*c*d^2*x - 4*a^2*d^3*x - 2*b*(b*c - 2*a*d
)*(c + d*x)^2*Log[a + b*x] + 2*(b*c - a*d)^2*(c + 2*d*x)*Log[(e*(a + b*x))
/(c + d*x)] + 2*b^2*c^3*Log[c + d*x] - 4*a*b*c^2*d*Log[c + d*x] + 4*b^2*c^
2*d*x*Log[c + d*x] - 8*a*b*c*d^2*x*Log[c + d*x] + 2*b^2*c*d^2*x^2*Log[c +
d*x] - 4*a*b*d^3*x^2*Log[c + d*x]))/((b*c - a*d)^2*(c + d*x)^2) - (6*a^2*A
*B*d^2*(-(b^2*c^2) + 4*a*b*c*d - a^2*d^2 + 2*b^2*c*d*x + 2*a*b*d^2*x + 2*b
^2*d^2*x^2 - 2*b^2*(c + d*x)^2*Log[a/b + x] + 2*(b*c - a*d)^2*Log[(e*(a +
b*x))/(c + d*x)] + 2*b^2*c^2*Log[(b*(c + d*x))/(b*c - a*d)] + 4*b^2*c*d*x*
Log[(b*(c + d*x))/(b*c - a*d)] + 2*b^2*d^2*x^2*Log[(b*(c + d*x))/(b*c - a*
d)]))/((b*c - a*d)^2*(c + d*x)^2) + 6*A*b^2*B*(-2*Log[c/d + x]^2 - (8*c*(1
 + Log[c/d + x]))/(c + d*x) + (c^2*(1 + 2*Log[c/d + x]))/(c + d*x)^2 + 8*c
*(Log[a/b + x]/(c + d*x) + (b*(Log[a + b*x] - Log[c + d*x]))/(-(b*c) + a*d
)) + 2*(-Log[a/b + x] + Log[c/d + x] + Log[(e*(a + b*x))/(c + d*x)])*((c*(
3*c + 4*d*x))/(c + d*x)^2 + 2*Log[c + d*x]) + (2*c^2*(-Log[a/b + x] + (b*(
c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x])
)/(b*c - a*d)^2))/(c + d*x)^2 + 4*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a
*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + (3*a^2*B^2*d^2*(-(b*c
- a*d)^2 - 6*b*(b*c - a*d)*(c + d*x) - 6*b^2*(c + d*x)^2*Log[a + b*x] -...
```

### 3.101.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 358, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2 \left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^3}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{g^2\int \frac{(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{g^2\int \left(-\frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d(c+dx)}-\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}-\frac{b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{d^2}\right)d\frac{a+bx}{c+dx}}{i^3}$$

$$\Big\downarrow \text{2009}$$

$$g^2\left(-\frac{2b^2B\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{d^3}-\frac{b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^3}-\frac{b(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{d^2(c+dx)}+ \right.$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)
^3,x]
```

output
```
(g^2*(-1/4*(B^2*(a + b*x)^2)/(d*(c + d*x)^2) + (2*A*b*B*(a + b*x))/(d^2*(c
 + d*x)) - (2*b*B^2*(a + b*x))/(d^2*(c + d*x)) + (2*b*B^2*(a + b*x)*Log[(e
*(a + b*x))/(c + d*x)])/(d^2*(c + d*x)) + (B*(a + b*x)^2*(A + B*Log[(e*(a
+ b*x))/(c + d*x)]))/(2*d*(c + d*x)^2) - ((a + b*x)^2*(A + B*Log[(e*(a + b
*x))/(c + d*x)])^2)/(2*d*(c + d*x)^2) - (b*(a + b*x)*(A + B*Log[(e*(a + b*
x))/(c + d*x)])^2)/(d^2*(c + d*x)) - (b^2*(A + B*Log[(e*(a + b*x))/(c + d*
x)])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 - (2*b^2*B*(A + B*Log[(e*
(a + b*x))/(c + d*x)])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 + (2*b
^2*B^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^3))/i^3
```

### 3.101.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.101.4 Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(\frac{e(bx+a)}{dx+c}\right)\right)^2}{(dix+ci)^3}dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x)
```

## 3.101.5 Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output
```
integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x
^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log((b*e*x + a*e)/(d*x + c))^2 + 2*(A*
B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log((b*e*x + a*e)/(d*x + c)
))/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

### 3.101.6 Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{g^2\left(\int \frac{A^2a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{A^2b^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2a^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2ABa^2\log\left(\frac{ae}{c+dx}+\frac{bex}{c+dx}\right)}{c^3+3c^2dx+3cd^2x^2+d^3x}\right.}{}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**3,x)
```

output
```
g**2*(Integral(A**2*a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(A**2*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**
3), x) + Integral(B**2*a**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))**2/(c**3
+ 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*a**2*log(a*
e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x
**3), x) + Integral(2*A**2*a*b*x/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3
*x**3), x) + Integral(B**2*b**2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))*
*2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*b*
*2*x**2*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*c*d**2
*x**2 + d**3*x**3), x) + Integral(2*B**2*a*b*x*log(a*e/(c + d*x) + b*e*x/(
c + d*x))**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integra
l(4*A*B*a*b*x*log(a*e/(c + d*x) + b*e*x/(c + d*x))/(c**3 + 3*c**2*d*x + 3*
c*d**2*x**2 + d**3*x**3), x))/i**3
```

### 3.101.7 Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output
```
-A*B*a*b*g^2*(2*(2*d*x + c)*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^4*i^3*
x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2
)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d
^2 - a*c^2*d^3)*i^3) - 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*
a*b*c*d^3 + a^2*d^4)*i^3) + 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2
 - 2*a*b*c*d^3 + a^2*d^4)*i^3)) + 1/2*A*B*a^2*g^2*((2*b*d*x + 3*b*c - a*d)
/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a
*c^2*d^2)*i^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(d^3*i^3*x^2 + 2*c
*d^2*i^3*x + c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a
^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3
)) + 1/2*A^2*b^2*g^2*((4*c*d*x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2
*d^3*i^3) + 2*log(d*x + c)/(d^3*i^3)) - (2*d*x + c)*A^2*a*b*g^2/(d^4*i^3*x
^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*A^2*a^2*g^2/(d^3*i^3*x^2 + 2*c*d^2
*i^3*x + c^2*d*i^3) + 1/6*(2*(B^2*b^2*d^2*g^2*x^2 + 2*B^2*b^2*c*d*g^2*x +
B^2*b^2*c^2*g^2)*log(d*x + c)^3 + 3*(4*(b^2*c*d*g^2 - a*b*d^2*g^2)*B^2*x +
 (3*b^2*c^2*g^2 - 2*a*b*c*d*g^2 - a^2*d^2*g^2)*B^2)*log(d*x + c)^2)/(d^5*i
^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) - integrate(-(2*B^2*a*b*d^2*g^2*x*lo
g(e)^2 + B^2*a^2*d^2*g^2*log(e)^2 + (B^2*b^2*d^2*g^2*log(e)^2 + 2*A*B*b^2*
d^2*g^2*log(e))*x^2 + (B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d^2*g^2*x + B^2*a^2
*d^2*g^2)*log(b*x + a)^2 + 2*(2*B^2*a*b*d^2*g^2*x*log(e) + B^2*a^2*d^2*...
```

### 3.101.8 Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log((b*x + a)*e/(d*x + c)) + A)^2/(d*i*x + c*
i)^3, x)
```

### 3.101.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\right)^2}{(c\,i+d\,i\,x)^3}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^3,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)
^3, x)
```

# 3.102 $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$



## 3.102.1 Optimal result

Integrand size = 40, antiderivative size = 141

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \frac{B^2g(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} - \frac{Bg(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)i^3(c+dx)^2} + \frac{g(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)i^3(c+dx)^2}$$

output
```
1/4*B^2*g*(b*x+a)^2/(-a*d+b*c)/i^3/(d*x+c)^2-1/2*B*g*(b*x+a)^2*(A+B*ln(e*(
b*x+a)/(d*x+c)))/(-a*d+b*c)/i^3/(d*x+c)^2+1/2*g*(b*x+a)^2*(A+B*ln(e*(b*x+a
)/(d*x+c)))^2/(-a*d+b*c)/i^3/(d*x+c)^2
```

### 3.102.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 767, normalized size of antiderivative = 5.44

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{g\left(2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-4b(bc-ad)(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2+4bB(c+dx)\left(2(b\right.\right.}{}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*
i*x)^3,x]
```

output
```
(g*(2*(b*c - a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 - 4*b*(b*c - a*
d)*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + 4*b*B*(c + d*x)*(2*(
b*c - a*d)*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 2*b*(c + d*x)*Log[a + b*
x]*(A + B*Log[(e*(a + b*x))/(c + d*x)]) - 2*b*(c + d*x)*(A + B*Log[(e*(a +
 b*x))/(c + d*x)])*Log[c + d*x] - 2*B*(b*c - a*d + b*(c + d*x)*Log[a + b*x
] - b*(c + d*x)*Log[c + d*x]) - b*B*(c + d*x)*(Log[a + b*x]*(Log[a + b*x]
- 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
 a*d)]) + b*B*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*
x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - B*(2*(b*c -
 a*d)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*b*(b*c - a*d)*(c + d*x)*(
A + B*Log[(e*(a + b*x))/(c + d*x)]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A +
B*Log[(e*(a + b*x))/(c + d*x)]) - 4*b^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)])*Log[c + d*x] - 4*b*B*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log
[a + b*x] - b*(c + d*x)*Log[c + d*x]) - B*((b*c - a*d)^2 + 2*b*(b*c - a*d)
*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*
x]) - 2*b^2*B*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x)
)/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*(c
 + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*
x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))))/(4*d^2*(b*c - a*d)*i^3*(c
 + d*x)^2)
```

### 3.102.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.075, Rules used = {2962, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^3}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{c+dx}d\frac{a+bx}{c+dx}}{i^3(bc-ad)}$$

$$\Big\downarrow \text{2742}$$

$$\frac{g\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(c+dx)^2}-B\int \frac{(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{c+dx}d\frac{a+bx}{c+dx}\right)}{i^3(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{g\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(c+dx)^2}-B\left(\frac{(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2}-\frac{B(a+bx)^2}{4(c+dx)^2}\right)\right)}{i^3(bc-ad)}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c*i + d*i*x)^3
,x]
```

output
```
(g*(((a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(c + d*x)^2) -
 B*(-1/4*(B*(a + b*x)^2)/(c + d*x)^2 + ((a + b*x)^2*(A + B*Log[(e*(a + b*x
))/(c + d*x)]))/(2*(c + d*x)^2))))/((b*c - a*d)*i^3)
```

### 3.102.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.102.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(135) = 270$.

Time = 0.90 (sec) , antiderivative size = 344, normalized size of antiderivative = 2.44

| method | result |
|---|---|
| norman | $\frac{-\frac{2A^2adg+2A^2bcg-2BadgA-2BbcgA+B^2adg+B^2bcg}{4i\,d^2}-\frac{\left(2A^2bg-2BbgA+B^2bg\right)x}{2id}-\frac{B^2a^2g\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{2i(ad-cb)}-\frac{B^2b^2g\,x^2\ln\left(\frac{e(bx+c}{dx+c}\right.}{2(ad-cb)i}}{i^2(dx+}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{g\,d^2A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^2e^3i^3}+\frac{2g\,d^2AB\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}\right)}{(ad-cb)^2e^3i^3}+\frac{g\,d^2B^2\left(\left(\right.\right.}{}\right.}{d^2}$ |
| default | $-\frac{e(ad-cb)\left(\frac{g\,d^2A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^2e^3i^3}+\frac{2g\,d^2AB\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}\right)}{(ad-cb)^2e^3i^3}+\frac{g\,d^2B^2\left(\left(\right.\right.}{}\right.}{d^2}$ |
| parallelrisch | $-\frac{8ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)a\,b^2d^4g+2AB\,b^3c^2d^2g+2A^2a^2b\,d^4g-2A^2b^3c^2d^2g+B^2a^2b\,d^4g-B^2b^3c^2d^2g-2B^2x^2\ln\left(\frac{e(bx+a)}{dx+c}\right)b}{}$ |
| parts | $\frac{g\,A^2\left(-\frac{b}{d^2(dx+c)}-\frac{ad-cb}{2d^2(dx+c)^2}\right)}{i^3}-\frac{g\,B^2d\left(a\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right.}{2}\right.\right.}{}$ |
| risch | Expression too large to display |

input
```
int((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output
```
(-1/4*(2*A^2*a*d*g+2*A^2*b*c*g-2*A*B*a*d*g-2*A*B*b*c*g+B^2*a*d*g+B^2*b*c*g
)/i/d^2-1/2*(2*A^2*b*g-2*A*B*b*g+B^2*b*g)/i/d*x-1/2*B^2*a^2*g/i/(a*d-b*c)*
ln(e*(b*x+a)/(d*x+c))^2-1/2*B^2*b^2*g/(a*d-b*c)/i*x^2*ln(e*(b*x+a)/(d*x+c)
)^2-1/2*(2*A-B)*g*a^2*B/i/(a*d-b*c)*ln(e*(b*x+a)/(d*x+c))-B^2*a*b*g/i/(a*d
-b*c)*x*ln(e*(b*x+a)/(d*x+c))^2-1/2*b^2/i*g*B*(2*A-B)/(a*d-b*c)*x^2*ln(e*(
b*x+a)/(d*x+c))-g*a*B/i*b*(2*A-B)/(a*d-b*c)*x*ln(e*(b*x+a)/(d*x+c)))/i^2/(
d*x+c)^2
```

3.102. $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$

### 3.102.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 295 vs. $2(135) = 270$.

Time = 0.30 (sec) , antiderivative size = 295, normalized size of antiderivative = 2.09

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = -\frac{2\left((2A^2-2AB+B^2)b^2cd-(2A^2-2AB+B^2)abd^2\right)gx-2\left(B^2b^2d^2gx^2+2B^2abd^2gx+B^2a^2d^2g\right)\log}{4\left((bc\right.}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output
```
-1/4*(2*((2*A^2 - 2*A*B + B^2)*b^2*c*d - (2*A^2 - 2*A*B + B^2)*a*b*d^2)*g*
x - 2*(B^2*b^2*d^2*g*x^2 + 2*B^2*a*b*d^2*g*x + B^2*a^2*d^2*g)*log((b*e*x +
 a*e)/(d*x + c))^2 + ((2*A^2 - 2*A*B + B^2)*b^2*c^2 - (2*A^2 - 2*A*B + B^2
)*a^2*d^2)*g - 2*((2*A*B - B^2)*b^2*d^2*g*x^2 + 2*(2*A*B - B^2)*a*b*d^2*g*
x + (2*A*B - B^2)*a^2*d^2*g)*log((b*e*x + a*e)/(d*x + c)))/((b*c*d^4 - a*d
^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3)
```

### 3.102.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(121) = 242$.

Time = 5.09 (sec) , antiderivative size = 712, normalized size of antiderivative = 5.05

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{Bb^2g(2A-B)\log\left(x+\frac{2ABab^2dg+2ABb^3cg-B^2ab^2dg-B^2b^3cg-\frac{Ba^2b^2d^2g(2A-B)}{ad-bc}+\frac{2Bab^3cdg(2A-B)}{ad-bc}-\frac{Bb^4c^2g(2A-B)}{ad-bc}}{4ABb^3dg-2B^2b^3dg}\right)}{2d^2i^3(ad-bc)}$$
$$-\frac{Bb^2g(2A-B)\log\left(x+\frac{2ABab^2dg+2ABb^3cg-B^2ab^2dg-B^2b^3cg+\frac{Ba^2b^2d^2g(2A-B)}{ad-bc}-\frac{2Bab^3cdg(2A-B)}{ad-bc}+\frac{Bb^4c^2g(2A-B)}{ad-bc}}{4ABb^3dg-2B^2b^3dg}\right)}{2d^2i^3(ad-bc)}$$
$$+\frac{(-B^2a^2g-2B^2abgx-B^2b^2gx^2)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2ac^2di^3+4acd^2i^3x+2ad^3i^3x^2-2bc^3i^3-4bc^2di^3x-2bcd^2i^3x^2}$$
$$+\frac{-2A^2adg-2A^2bcg+2ABadg+2ABbcg-B^2adg-B^2bcg+x(-4A^2bdg+4ABbdg-2B^2bdg)}{4c^2d^2i^3+8cd^3i^3x+4d^4i^3x^2}$$
$$+\frac{(-2ABadg-2ABbcg-4ABbdgx+B^2adg+B^2bcg+2B^2bdgx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2c^2d^2i^3+4cd^3i^3x+2d^4i^3x^2}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**3,x)
```

output
```
B*b**2*g*(2*A - B)*log(x + (2*A*B*a*b**2*d*g + 2*A*B*b**3*c*g - B**2*a*b**
2*d*g - B**2*b**3*c*g - B*a**2*b**2*d**2*g*(2*A - B)/(a*d - b*c) + 2*B*a*b
**3*c*d*g*(2*A - B)/(a*d - b*c) - B*b**4*c**2*g*(2*A - B)/(a*d - b*c))/(4*
A*B*b**3*d*g - 2*B**2*b**3*d*g))/(2*d**2*i**3*(a*d - b*c)) - B*b**2*g*(2*A
 - B)*log(x + (2*A*B*a*b**2*d*g + 2*A*B*b**3*c*g - B**2*a*b**2*d*g - B**2*
b**3*c*g + B*a**2*b**2*d**2*g*(2*A - B)/(a*d - b*c) - 2*B*a*b**3*c*d*g*(2*
A - B)/(a*d - b*c) + B*b**4*c**2*g*(2*A - B)/(a*d - b*c))/(4*A*B*b**3*d*g
- 2*B**2*b**3*d*g))/(2*d**2*i**3*(a*d - b*c)) + (-B**2*a**2*g - 2*B**2*a*b
*g*x - B**2*b**2*g*x**2)*log(e*(a + b*x)/(c + d*x))**2/(2*a*c**2*d*i**3 +
4*a*c*d**2*i**3*x + 2*a*d**3*i**3*x**2 - 2*b*c**3*i**3 - 4*b*c**2*d*i**3*x
 - 2*b*c*d**2*i**3*x**2) + (-2*A**2*a*d*g - 2*A**2*b*c*g + 2*A*B*a*d*g + 2
*A*B*b*c*g - B**2*a*d*g - B**2*b*c*g + x*(-4*A**2*b*d*g + 4*A*B*b*d*g - 2*
B**2*b*d*g))/(4*c**2*d**2*i**3 + 8*c*d**3*i**3*x + 4*d**4*i**3*x**2) + (-2
*A*B*a*d*g - 2*A*B*b*c*g - 4*A*B*b*d*g*x + B**2*a*d*g + B**2*b*c*g + 2*B**
2*b*d*g*x)*log(e*(a + b*x)/(c + d*x))/(2*c**2*d**2*i**3 + 4*c*d**3*i**3*x
+ 2*d**4*i**3*x**2)
```

3.102. $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$

### 3.102.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1966 vs. $2(135) = 270$.

Time = 0.29 (sec) , antiderivative size = 1966, normalized size of antiderivative = 13.94

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output
```
-1/2*(2*d*x + c)*B^2*b*g*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2/(d^4*i^3*x
^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) + 1/4*(2*((2*b*d*x + 3*b*c - a*d)/((b*c*
d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^
2)*i^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2
*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3))*log(b*e*x/(d*
x + c) + a*e/(d*x + c)) - (7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^
2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^
2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c
^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))/(
b^2*c^4*d*i^3 - 2*a*b*c^3*d^2*i^3 + a^2*c^2*d^3*i^3 + (b^2*c^2*d^3*i^3 - 2
*a*b*c*d^4*i^3 + a^2*d^5*i^3)*x^2 + 2*(b^2*c^3*d^2*i^3 - 2*a*b*c^2*d^3*i^3
 + a^2*c*d^4*i^3)*x))*B^2*a*g + 1/4*(2*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*
d^2)*x)/((b*c*d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^
3*d^2 - a*c^2*d^3)*i^3) + 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 -
 2*a*b*c*d^3 + a^2*d^4)*i^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*
d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) -
(b^2*c^3 - 8*a*b*c^2*d + 7*a^2*c*d^2 + 2*(b^2*c^3 - 2*a*b*c^2*d + (b^2*c*d
^2 - 2*a*b*d^3)*x^2 + 2*(b^2*c^2*d - 2*a*b*c*d^2)*x)*log(b*x + a)^2 + 2*(b
^2*c^3 - 2*a*b*c^2*d + (b^2*c*d^2 - 2*a*b*d^3)*x^2 + 2*(b^2*c^2*d - 2*a...
```

### 3.102.8 Giac [A] (verification not implemented)

Time = 0.46 (sec) , antiderivative size = 198, normalized size of antiderivative = 1.40

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\,(bex+ae)^2B^2g\log\left(\frac{bex+ae}{dx+c}\right)^2}{(dx+c)^2ei^3}+\frac{2\,(2\,ABg-B^2g)(bex+ae)^2\log\left(\frac{bex+ae}{dx+c}\right)}{(dx+c)^2ei^3}+\frac{(2\,A^2g-2\,ABg+B^2g)}{(dx+c)^2ei^3}\right.$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output
```
1/4*(2*(b*e*x + a*e)^2*B^2*g*log((b*e*x + a*e)/(d*x + c))^2/((d*x + c)^2*e
*i^3) + 2*(2*A*B*g - B^2*g)*(b*e*x + a*e)^2*log((b*e*x + a*e)/(d*x + c))/(
(d*x + c)^2*e*i^3) + (2*A^2*g - 2*A*B*g + B^2*g)*(b*e*x + a*e)^2/((d*x + c
)^2*e*i^3))*(b*c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c
 - a*d)))
```

### 3.102.9 Mupad [B] (verification not implemented)

Time = 2.86 (sec) , antiderivative size = 474, normalized size of antiderivative = 3.36

$$\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx=$$
$$-\frac{x\,(2\,b\,d\,g\,A^2-2\,b\,d\,g\,A\,B+b\,d\,g\,B^2)+A^2\,a\,d\,g+A^2\,b\,c\,g+\frac{B^2\,a\,d\,g}{2}+\frac{B^2\,b\,c\,g}{2}-A\,B\,a\,d\,g-A\,B\,b\,c\,g}{2\,c^2\,d^2\,i^3+4\,c\,d^3\,i^3\,x+2\,d^4\,i^3\,x^2}$$
$$-\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{\frac{B^2\,a\,g}{2\,d^2\,i^3}+\frac{B^2\,b\,c\,g}{2\,d^3\,i^3}+\frac{B^2\,b\,g\,x}{d^2\,i^3}}{2\,c\,x+d\,x^2+\frac{c^2}{d}}+\frac{B^2\,b^2\,g}{2\,d^2\,i^3\,(a\,d-b\,c)}\right)$$
$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{A\,B\,c\,g}{d^3\,i^3}-x\left(\frac{B^2\,g}{d^2\,i^3}-\frac{2\,A\,B\,g}{d^2\,i^3}\right)+\frac{B\,g\,(A\,a\,d-B\,a\,d+B\,b\,c)}{b\,d^3\,i^3}+\frac{B^2\,b^2\,g\left(\frac{a^2\,d^2-3\,a\,b\,c\,d+2\,b^2\,c^2}{2\,b^3\,d}-\frac{c\,(a\,d-b\,c)}{2\,b^2\,d}\right)}{d^2\,i^3\,(a\,d-b\,c)}\right)}{\frac{d\,x^2}{b}+\frac{c^2}{b\,d}+\frac{2\,c\,x}{b}}$$
$$+\frac{B\,b^2\,g\,\mathrm{atan}\left(\frac{\left(\frac{2\,a\,d^3\,i^3+2\,b\,c\,d^2\,i^3}{2\,d^2\,i^3}+2\,b\,d\,x\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,(2\,A-B)\,1\mathrm{i}}{d^2\,i^3\,(a\,d-b\,c)}$$

input
```
int(((a*g + b*g*x)*(A + B*log((e*(a + b*x))/(c + d*x)))^2)/(c*i + d*i*x)^3
,x)
```

output
```
(B*b^2*g*atan((((2*a*d^3*i^3 + 2*b*c*d^2*i^3)/(2*d^2*i^3) + 2*b*d*x)*1i)/(
a*d - b*c))*(2*A - B)*1i)/(d^2*i^3*(a*d - b*c)) - log((e*(a + b*x))/(c + d
*x))^2*(((B^2*a*g)/(2*d^2*i^3) + (B^2*b*c*g)/(2*d^3*i^3) + (B^2*b*g*x)/(d^
2*i^3))/(2*c*x + d*x^2 + c^2/d) + (B^2*b^2*g)/(2*d^2*i^3*(a*d - b*c))) - (
log((e*(a + b*x))/(c + d*x))*((A*B*c*g)/(d^3*i^3) - x*((B^2*g)/(d^2*i^3) -
 (2*A*B*g)/(d^2*i^3)) + (B*g*(A*a*d - B*a*d + B*b*c))/(b*d^3*i^3) + (B^2*b
^2*g*((a^2*d^2 + 2*b^2*c^2 - 3*a*b*c*d)/(2*b^3*d) - (c*(a*d - b*c))/(2*b^2
*d)))/(d^2*i^3*(a*d - b*c))))/((d*x^2)/b + c^2/(b*d) + (2*c*x)/b) - (x*(2*
A^2*b*d*g + B^2*b*d*g - 2*A*B*b*d*g) + A^2*a*d*g + A^2*b*c*g + (B^2*a*d*g)
/2 + (B^2*b*c*g)/2 - A*B*a*d*g - A*B*b*c*g)/(2*c^2*d^2*i^3 + 2*d^4*i^3*x^2
 + 4*c*d^3*i^3*x)
```

---

3.102. $\int \frac{(ag+bgx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$

# 3.103 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$



## 3.103.1 Optimal result

Integrand size = 32, antiderivative size = 296

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = & -\frac{B^2d(a+bx)^2}{4(bc-ad)^2i^3(c+dx)^2}-\frac{2AbB(a+bx)}{(bc-ad)^2i^3(c+dx)} \\ & +\frac{2bB^2(a+bx)}{(bc-ad)^2i^3(c+dx)}-\frac{2bB^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^2i^3(c+dx)} \\ & +\frac{Bd(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^2i^3(c+dx)^2} \\ & -\frac{d(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^2i^3(c+dx)^2} \\ & +\frac{b(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^2i^3(c+dx)}\end{aligned}$$

output
```
-1/4*B^2*d*(b*x+a)^2/(-a*d+b*c)^2/i^3/(d*x+c)^2-2*A*b*B*(b*x+a)/(-a*d+b*c)
^2/i^3/(d*x+c)+2*b*B^2*(b*x+a)/(-a*d+b*c)^2/i^3/(d*x+c)-2*b*B^2*(b*x+a)*ln
(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^2/i^3/(d*x+c)+1/2*B*d*(b*x+a)^2*(A+B*ln(e*(
b*x+a)/(d*x+c)))/(-a*d+b*c)^2/i^3/(d*x+c)^2-1/2*d*(b*x+a)^2*(A+B*ln(e*(b*x
+a)/(d*x+c)))^2/(-a*d+b*c)^2/i^3/(d*x+c)^2+b*(b*x+a)*(A+B*ln(e*(b*x+a)/(d*
x+c)))^2/(-a*d+b*c)^2/i^3/(d*x+c)
```

### 3.103.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 444, normalized size of antiderivative = 1.50

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{-2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 + \frac{B\left(2(bc-ad)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4b(bc-ad)(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)+4b^2(c+dx)^2\log(a+bx)\left(\right.}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x)^3,x]
```

output
```
(-2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2 + (B*(2*(b*c - a*d)^2*(A + B*Lo
g[(e*(a + b*x))/(c + d*x)]) + 4*b*(b*c - a*d)*(c + d*x)*(A + B*Log[(e*(a +
 b*x))/(c + d*x)]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A + B*Log[(e*(a + b*x
))/(c + d*x)]) - 4*b^2*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])*Lo
g[c + d*x] - 4*b*B*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c
+ d*x)*Log[c + d*x]) - B*((b*c - a*d)^2 + 2*b*(b*c - a*d)*(c + d*x) + 2*b^
2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x]) - 2*b^2*B*(c
+ d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) -
 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*(c + d*x)^2*((2*Log
[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*d*i^3*(c + d*x)^2)
```

### 3.103.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 219, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.094$, Rules used = {2952, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ci+dix)^3}dx$$
$$\Big\downarrow \; 2952$$

$$\frac{\int \left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2 d\frac{a+bx}{c+dx}}{i^3(bc-ad)^2}$$

$$\Big\downarrow \; 2767$$

$$\frac{\int \left(b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{d(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{c+dx}\right)d\frac{a+bx}{c+dx}}{i^3(bc-ad)^2}$$

$$\Big\downarrow \; 2009$$

$$\frac{\frac{Bd(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2}+\frac{b(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-\frac{d(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(c+dx)^2}-\frac{2AbB(a+bx)}{c+dx}-\frac{2bB^2(a+bx)\log}{c+dx}}{i^3(bc-ad)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/(c*i + d*i*x)^3,x]
```

output
```
(-1/4*(B^2*d*(a + b*x)^2)/(c + d*x)^2 - (2*A*b*B*(a + b*x))/(c + d*x) + (2
*b*B^2*(a + b*x))/(c + d*x) - (2*b*B^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*
x)])/(c + d*x) + (B*d*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2
*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*
(c + d*x)^2) + (b*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c + d
*x))/((b*c - a*d)^2*i^3)
```

### 3.103.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2952
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(
m + 1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (
a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && EqQ[
n + mn, 0] && IGtQ[n, 0] && NeQ[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[d*f
 - c*g, 0] && (GtQ[p, 0] || LtQ[m, -1])
```

## 3.103.4 Maple [A] (verified)

Time = 0.97 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.63

| method | result |
|---|---|
| norman | $\frac{Bb(2Abc-Bad-2Bbc)x\ln\left(\frac{e(bx+a)}{dx+c}\right)}{i\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{B^2b^2cx\ln\left(\frac{e(bx+a)}{dx+c}\right)^2}{i\left(a^2d^2-2abcd+b^2c^2\right)}+\frac{\left(2A^2ad-2A^2bc-2ABad+4ABbc+B^2ad-4B^2bc\right)x}{2ci(ad-cb)}-\frac{Ba(2Aad-\cdots}{2\cdots}$ |
| parts | $-\frac{A^2}{2i^3(dx+c)^2d}-\frac{B^2d\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2}-\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{2}+\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{4}\cdots\right.}{i^3(ad-\cdots}$ |
| parallelrisch | $-\frac{-8ABx\ln\left(\frac{e(bx+a)}{dx+c}\right)b^3cd^4-8AB\ln\left(\frac{e(bx+a)}{dx+c}\right)ab^2cd^4-4A^2ab^2cd^4-2ABa^2bd^5-6ABb^3c^2d^3-8B^2ab^2cd^4+8ABa\cdots}{\cdots}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(-\frac{d^2A^2b\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^3e^2i^3}+\frac{d^3A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^3e^3i^3}-\frac{2d^2ABb\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\cdots\right)}{(ad-cb)^3e^2i^3}\cdots\right.}{\cdots}$ |
| default | $-\frac{e(ad-cb)\left(-\frac{d^2A^2b\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{(ad-cb)^3e^2i^3}+\frac{d^3A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2(ad-cb)^3e^3i^3}-\frac{2d^2ABb\left(\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)-\frac{(ad-cb)e}{d(dx+c)}-\cdots\right)}{(ad-cb)^3e^2i^3}\cdots\right.}{\cdots}$ |
| risch | Expression too large to display |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x,method=_RETURNVERBOSE)
```

output
```
(B/i*b*(2*A*b*c-B*a*d-2*B*b*c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/
(d*x+c))+B^2*b^2*c/i/(a^2*d^2-2*a*b*c*d+b^2*c^2)*x*ln(e*(b*x+a)/(d*x+c))^2
+1/2*(2*A^2*a*d-2*A^2*b*c-2*A*B*a*d+4*A*B*b*c+B^2*a*d-4*B^2*b*c)/c/i/(a*d-
b*c)*x-1/2*B*a*(2*A*a*d-4*A*b*c-B*a*d+4*B*b*c)/i/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(e*(b*x+a)/(d*x+c))-1/2*B^2*a*(a*d-2*b*c)/i/(a^2*d^2-2*a*b*c*d+b^2*c^
2)*ln(e*(b*x+a)/(d*x+c))^2+1/4*(2*A^2*a*d-2*A^2*b*c-2*A*B*a*d+6*A*B*b*c+B^
2*a*d-7*B^2*b*c)/c^2*d/i/(a*d-b*c)*x^2+1/2*b^2*d*B^2/i/(a^2*d^2-2*a*b*c*d+
b^2*c^2)*x^2*ln(e*(b*x+a)/(d*x+c))^2+1/2*d*B/i*b^2*(2*A-3*B)/(a^2*d^2-2*a*
b*c*d+b^2*c^2)*x^2*ln(e*(b*x+a)/(d*x+c)))/i^2/(d*x+c)^2
```

### 3.103.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 373, normalized size of antiderivative = 1.26

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx =$$

$$-\frac{(2A^2-6AB+7B^2)b^2c^2-4(A^2-2AB+2B^2)abcd+(2A^2-2AB+B^2)a^2d^2-2(B^2b^2d^2x^2+2B}{}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algorithm="frica
s")
```

output
```
-1/4*((2*A^2 - 6*A*B + 7*B^2)*b^2*c^2 - 4*(A^2 - 2*A*B + 2*B^2)*a*b*c*d +
(2*A^2 - 2*A*B + B^2)*a^2*d^2 - 2*(B^2*b^2*d^2*x^2 + 2*B^2*b^2*c*d*x + 2*B
^2*a*b*c*d - B^2*a^2*d^2)*log((b*e*x + a*e)/(d*x + c))^2 - 2*((2*A*B - 3*B
^2)*b^2*c*d - (2*A*B - 3*B^2)*a*b*d^2)*x - 2*((2*A*B - 3*B^2)*b^2*d^2*x^2
+ 4*(A*B - B^2)*a*b*c*d - (2*A*B - B^2)*a^2*d^2 - 2*(B^2*a*b*d^2 - 2*(A*B
- B^2)*b^2*c*d)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d
^4 + a^2*d^5)*i^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*i^3*x
+ (b^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*i^3)
```

### 3.103.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 892 vs. $2(269) = 538$.

Time = 2.21 (sec) , antiderivative size = 892, normalized size of antiderivative = 3.01

$$\int \frac{\left(A + B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = -\frac{Bb^2\cdot(2A-3B)\log\left(x + \frac{2ABab^2d+2ABb^3c-3B^2ab^2d-3B^2b^3c-\frac{Ba^3b^2d^3\cdot(2A-3B)}{(ad-bc)^2}+\frac{3Ba^2b^3cd^2\cdot(2A-3B)}{(ad-bc)^2}-\frac{3Bab^4c^2d(2A-3B)}{(ad-bc)^2}+\frac{Bb}{}}{4ABb^3d-6B^2b^3d}\right.}{2di^3\,(ad-bc)^2} + \frac{Bb^2\cdot(2A-3B)\log\left(x + \frac{2ABab^2d+2ABb^3c-3B^2ab^2d-3B^2b^3c+\frac{Ba^3b^2d^3\cdot(2A-3B)}{(ad-bc)^2}-\frac{3Ba^2b^3cd^2\cdot(2A-3B)}{(ad-bc)^2}+\frac{3Bab^4c^2d(2A-3B)}{(ad-bc)^2}-\frac{Bb}{}}{4ABb^3d-6B^2b^3d}\right.}{2di^3\,(ad-bc)^2} + \frac{(-B^2a^2d+2B^2abc+2B^2b^2cx+B^2b^2dx^2)\log\left(\frac{e(a+bx)}{c+dx}\right)^2}{2a^2c^2d^2i^3+4a^2cd^3i^3x+2a^2d^4i^3x^2-4abc^3di^3-8abc^2d^2i^3x-4abcd^3i^3x^2+2b^2c^4i^3+4b^2c^3di^3x+2b^2c^2} + \frac{(-2ABad+2ABbc+B^2ad-3B^2bc-2B^2bdx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{2ac^2d^2i^3+4acd^3i^3x+2ad^4i^3x^2-2bc^3di^3-4bc^2d^2i^3x-2bcd^3i^3x^2} + \frac{-2A^2ad+2A^2bc+2ABad-6ABbc-B^2ad+7B^2bc+x(-4ABbd+6B^2bd)}{4ac^2d^2i^3-4bc^3di^3+x^2\cdot(4ad^4i^3-4bcd^3i^3)+x\,(8acd^3i^3-8bc^2d^2i^3)}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(d*i*x+c*i)**3,x)
```

output
```
-B*b**2*(2*A - 3*B)*log(x + (2*A*B*a*b**2*d + 2*A*B*b**3*c - 3*B**2*a*b**2
*d - 3*B**2*b**3*c - B*a**3*b**2*d**3*(2*A - 3*B)/(a*d - b*c)**2 + 3*B*a**
2*b**3*c*d**2*(2*A - 3*B)/(a*d - b*c)**2 - 3*B*a*b**4*c**2*d*(2*A - 3*B)/(
a*d - b*c)**2 + B*b**5*c**3*(2*A - 3*B)/(a*d - b*c)**2)/(4*A*B*b**3*d - 6*
B**2*b**3*d))/(2*d*i**3*(a*d - b*c)**2) + B*b**2*(2*A - 3*B)*log(x + (2*A*
B*a*b**2*d + 2*A*B*b**3*c - 3*B**2*a*b**2*d - 3*B**2*b**3*c + B*a**3*b**2*
d**3*(2*A - 3*B)/(a*d - b*c)**2 - 3*B*a**2*b**3*c*d**2*(2*A - 3*B)/(a*d -
b*c)**2 + 3*B*a*b**4*c**2*d*(2*A - 3*B)/(a*d - b*c)**2 - B*b**5*c**3*(2*A
- 3*B)/(a*d - b*c)**2)/(4*A*B*b**3*d - 6*B**2*b**3*d))/(2*d*i**3*(a*d - b*
c)**2) + (-B**2*a**2*d + 2*B**2*a*b*c + 2*B**2*b**2*c*x + B**2*b**2*d*x**2
)*log(e*(a + b*x)/(c + d*x))**2/(2*a**2*c**2*d**2*i**3 + 4*a**2*c*d**3*i**
3*x + 2*a**2*d**4*i**3*x**2 - 4*a*b*c**3*d*i**3 - 8*a*b*c**2*d**2*i**3*x -
 4*a*b*c*d**3*i**3*x**2 + 2*b**2*c**4*i**3 + 4*b**2*c**3*d*i**3*x + 2*b**2
*c**2*d**2*i**3*x**2) + (-2*A*B*a*d + 2*A*B*b*c + B**2*a*d - 3*B**2*b*c -
2*B**2*b*d*x)*log(e*(a + b*x)/(c + d*x))/(2*a*c**2*d**2*i**3 + 4*a*c*d**3*
i**3*x + 2*a*d**4*i**3*x**2 - 2*b*c**3*d*i**3 - 4*b*c**2*d**2*i**3*x - 2*b
*c*d**3*i**3*x**2) + (-2*A**2*a*d + 2*A**2*b*c + 2*A*B*a*d - 6*A*B*b*c - B
**2*a*d + 7*B**2*b*c + x*(-4*A*B*b*d + 6*B**2*b*d))/(4*a*c**2*d**2*i**3 -
4*b*c**3*d*i**3 + x**2*(4*a*d**4*i**3 - 4*b*c*d**3*i**3) + x*(8*a*c*d**3*i
**3 - 8*b*c**2*d**2*i**3))
```

### 3.103.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 848 vs. $2(290) = 580$.

Time = 0.23 (sec) , antiderivative size = 848, normalized size of antiderivative = 2.86

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$

$$= \frac{1}{4}\left(2\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}+\frac{2\,b^2\log\left(bx+a\right)}{(b^2c^2d-2\,abcd^2+a^2d^3)i^3}-\frac{2\,b}{(b^2c^2d-}\right.\right.$$

$$+\frac{1}{2}\,AB\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}-\frac{2\,\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)}{d^3i^3x^2+2\,cd^2i^3x+c^2di^3}+\frac{2}{(b^2c^2d}\right.$$

$$-\frac{B^2\log\left(\frac{bex}{dx+c}+\frac{ae}{dx+c}\right)^2}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}-\frac{A^2}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algorithm="maxim
a")
```

output
```
1/4*(2*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2
- a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x + a)/((b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a*
b*c*d^2 + a^2*d^3)*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - (7*b^2*c^2
 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x +
 a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*
d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*
(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x +
b^2*c^2)*log(b*x + a))*log(d*x + c))/(b^2*c^4*d*i^3 - 2*a*b*c^3*d^2*i^3 +
a^2*c^2*d^3*i^3 + (b^2*c^2*d^3*i^3 - 2*a*b*c*d^4*i^3 + a^2*d^5*i^3)*x^2 +
2*(b^2*c^3*d^2*i^3 - 2*a*b*c^2*d^3*i^3 + a^2*c*d^4*i^3)*x))*B^2 + 1/2*A*B*
((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d
^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) - 2*log(b*e*x/(d*x + c) + a*e/(d*x
+ c))/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 2*b^2*log(b*x + a)/((b^2
*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*
a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*B^2*log(b*e*x/(d*x + c) + a*e/(d*x + c))^
2/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A^2/(d^3*i^3*x^2 + 2*c*d
^2*i^3*x + c^2*d*i^3)
```

### 3.103.8 Giac [A] (verification not implemented)

Time = 0.44 (sec) , antiderivative size = 371, normalized size of antiderivative = 1.25

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx$$

$$= \frac{1}{4}\left(2\left(\frac{2\,(bex+ae)B^2b}{(bci^3-adi^3)(dx+c)}-\frac{(bex+ae)^2B^2d}{(bcei^3-adei^3)(dx+c)^2}\right)\log\left(\frac{bex+ae}{dx+c}\right)^2-2\left(\frac{(2\,ABd-B^2d)(bex+}{(bcei^3-adei^3)(dx+}\right.\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(d*i*x+c*i)^3,x, algorithm="giac"
)
```

output
```
1/4*(2*(2*(b*e*x + a*e)*B^2*b/((b*c*i^3 - a*d*i^3)*(d*x + c)) - (b*e*x + a
*e)^2*B^2*d/((b*c*e*i^3 - a*d*e*i^3)*(d*x + c)^2))*log((b*e*x + a*e)/(d*x
+ c))^2 - 2*((2*A*B*d - B^2*d)*(b*e*x + a*e)^2/((b*c*e*i^3 - a*d*e*i^3)*(d
*x + c)^2) - 4*(A*B*b - B^2*b)*(b*e*x + a*e)/((b*c*i^3 - a*d*i^3)*(d*x + c
)))*log((b*e*x + a*e)/(d*x + c)) - (2*A^2*d - 2*A*B*d + B^2*d)*(b*e*x + a*
e)^2/((b*c*e*i^3 - a*d*e*i^3)*(d*x + c)^2) + 4*(A^2*b - 2*A*B*b + 2*B^2*b)
*(b*e*x + a*e)/((b*c*i^3 - a*d*i^3)*(d*x + c)))*(b*c/((b*c*e - a*d*e)*(b*c
 - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

### 3.103.9 Mupad [B] (verification not implemented)

Time = 2.88 (sec) , antiderivative size = 507, normalized size of antiderivative = 1.71

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ci+dix)^3}\,dx = -\frac{\frac{2A^2ad-2A^2bc+B^2ad-7B^2bc-2ABad+6ABbc}{2(ad-bc)}-\frac{x(3B^2bd-2ABbd)}{ad-bc}}{2c^2di^3+4cd^2i^3x+2d^3i^3x^2} - \ln\left(\frac{e(a+bx)}{c+dx}\right)^2\left(\frac{B^2}{2d^2i^3\left(2cx+dx^2+\frac{c^2}{d}\right)}-\frac{B^2b^2}{2di^3\left(a^2d^2-2abcd+b^2c^2\right)}\right) - \frac{\ln\left(\frac{e(a+bx)}{c+dx}\right)\left(\frac{AB}{bd^2i^3}+\frac{B^2x(ad-bc)}{di^3(a^2d^2-2abcd+b^2c^2)}-\frac{B^2b^2\left(\frac{a^2d^2-3abcd+2b^2c^2}{2b^3d}-\frac{c(ad-bc)}{2b^2d}\right)}{di^3(a^2d^2-2abcd+b^2c^2)}\right)}{\frac{dx^2}{b}+\frac{c^2}{bd}+\frac{2cx}{b}} + \frac{Bb^2\,\mathrm{atan}\left(\frac{Bb^2\left(2bdx+\frac{a^2d^3i^3-b^2c^2di^3}{di^3(ad-bc)}\right)(2A-3B)\,1i}{(ad-bc)(3B^2b^2-2ABb^2)}\right)(2A-3B)\,1i}{di^3(ad-bc)^2}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/(c*i + d*i*x)^3,x)
```

output
```
(B*b^2*atan((B*b^2*(2*b*d*x + (a^2*d^3*i^3 - b^2*c^2*d*i^3)/(d*i^3*(a*d -
b*c)))*(2*A - 3*B)*1i)/((a*d - b*c)*(3*B^2*b^2 - 2*A*B*b^2)))*(2*A - 3*B)*
1i)/(d*i^3*(a*d - b*c)^2) - log((e*(a + b*x))/(c + d*x))^2*(B^2/(2*d^2*i^3
*(2*c*x + d*x^2 + c^2/d)) - (B^2*b^2)/(2*d*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*
c*d))) - (log((e*(a + b*x))/(c + d*x))*((A*B)/(b*d^2*i^3) + (B^2*x*(a*d -
b*c))/(d*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (B^2*b^2*((a^2*d^2 + 2*b^2
*c^2 - 3*a*b*c*d)/(2*b^3*d) - (c*(a*d - b*c))/(2*b^2*d)))/(d*i^3*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d))))/((d*x^2)/b + c^2/(b*d) + (2*c*x)/b) - ((2*A^2*a*
d - 2*A^2*b*c + B^2*a*d - 7*B^2*b*c - 2*A*B*a*d + 6*A*B*b*c)/(2*(a*d - b*c
)) - (x*(3*B^2*b*d - 2*A*B*b*d))/(a*d - b*c))/(2*c^2*d*i^3 + 2*d^3*i^3*x^2
 + 4*c*d^2*i^3*x)
```

# 3.104 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$



## 3.104.1 Optimal result

Integrand size = 42, antiderivative size = 375

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx &= \frac{B^2d^2(a+bx)^2}{4(bc-ad)^3gi^3(c+dx)^2}+\frac{4AbBd(a+bx)}{(bc-ad)^3gi^3(c+dx)}\\
&\quad-\frac{4bB^2d(a+bx)}{(bc-ad)^3gi^3(c+dx)}+\frac{4bB^2d(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^3gi^3(c+dx)}\\
&\quad-\frac{Bd^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2}\\
&\quad+\frac{d^2(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^3gi^3(c+dx)^2}\\
&\quad-\frac{2bd(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^3gi^3(c+dx)}\\
&\quad+\frac{b^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^3gi^3}\end{aligned}$$

output
```
1/4*B^2*d^2*(b*x+a)^2/(-a*d+b*c)^3/g/i^3/(d*x+c)^2+4*A*b*B*d*(b*x+a)/(-a*d
+b*c)^3/g/i^3/(d*x+c)-4*b*B^2*d*(b*x+a)/(-a*d+b*c)^3/g/i^3/(d*x+c)+4*b*B^2
*d*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^3/g/i^3/(d*x+c)-1/2*B*d^2*(b*x
+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^3/g/i^3/(d*x+c)^2+1/2*d^2*(b*
x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g/i^3/(d*x+c)^2-2*b*d*(b
*x+a)*(A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^3/g/i^3/(d*x+c)+1/3*b^2*(A+
B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^3/g/i^3
```

## 3.104.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 290, normalized size of antiderivative = 0.77

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{\frac{3(2A^2-2AB+B^2)(bc-ad)^2}{(c+dx)^2} + \frac{6b(2A^2-6AB+7B^2)(bc-ad)}{c+dx} + 6b^2(2A^2-6AB+7B^2)\log(a+bx) + \frac{6B(bc-ad)(B(-7bc+ad}{}}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i
*x)^3),x]
```

output
```
((3*(2*A^2 - 2*A*B + B^2)*(b*c - a*d)^2)/(c + d*x)^2 + (6*b*(2*A^2 - 6*A*B
 + 7*B^2)*(b*c - a*d))/(c + d*x) + 6*b^2*(2*A^2 - 6*A*B + 7*B^2)*Log[a + b
*x] + (6*B*(b*c - a*d)*(B*(-7*b*c + a*d - 6*b*d*x) + A*(6*b*c - 2*a*d + 4*
b*d*x))*Log[(e*(a + b*x))/(c + d*x)])/(c + d*x)^2 + (6*B*(2*A*b^2*(c + d*x
)^2 + B*d*(a + b*x)*(-4*b*c + a*d - 3*b*d*x))*Log[(e*(a + b*x))/(c + d*x)]
^2)/(c + d*x)^2 + 4*b^2*B^2*Log[(e*(a + b*x))/(c + d*x)]^3 - 6*b^2*(2*A^2
- 6*A*B + 7*B^2)*Log[c + d*x])/(12*(b*c - a*d)^3*g*i^3)
```

## 3.104.3 Rubi [A] (warning: unable to verify)

Time = 0.83 (sec) , antiderivative size = 351, normalized size of antiderivative = 0.94, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}} = 0.214$, Rules used = {2962, 2788, 2767, 2009, 2788, 2733, 2009, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$

$$\Big\downarrow \; 2962$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}}{gi^3(bc-ad)^3}$$

$$\Big\downarrow \; 2788$$

$$\frac{b\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\int \left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d\frac{a+bx}{c+dx}}{gi^3(bc-ad)^3}$$

$$\Big\downarrow \; 2767$$

$$\frac{b\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\int \left(b\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2-\frac{d(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{c+dx}\right)d\frac{a+bx}{c+dx}}{gi^3(bc-ad)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{b\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{Bd(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2}+\frac{b(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-\frac{d(a+b}{}\right.}{gi^3(bc-ad)^3}$$

$$\Big\downarrow \; 2788$$

$$\frac{b\left(b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2d\frac{a+bx}{c+dx}\right)-d\left(\frac{Bd(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(c+dx)^2}+\frac{b(}{}\right.}{gi^3(}$$

$$\Big\downarrow \; 2733$$

$$b\left(b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-2B\int \left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)d\frac{a+bx}{c+dx}\right)\right)-d\Big($$

$$\Big\downarrow \; 2009$$

$$b\left(b\int \frac{(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-2B\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}-\frac{B(a+bx}{c+dx}\right.\right.\right.$$

---

3.104. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$

$$\downarrow 2739$$

$$b\left(\frac{b\int \frac{(a+bx)^2}{(c+dx)^2}d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{B}-d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-2B\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}-\frac{B(a+bx)}{c+dx}\right)\right)\right)$$

$$\downarrow 15$$

$$b\left(\frac{b(a+bx)^3}{3B(c+dx)^3}-d\left(\frac{(a+bx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{c+dx}-2B\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{c+dx}-\frac{B(a+bx)}{c+dx}\right)\right)\right)-d\left(\frac{Bd(a+bx)^2\left(B\right.}{2(c}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)*(c*i + d*i*x)^3)
,x]
```

output
```
(-(d*(-1/4*(B^2*d*(a + b*x)^2)/(c + d*x)^2 - (2*A*b*B*(a + b*x))/(c + d*x)
 + (2*b*B^2*(a + b*x))/(c + d*x) - (2*b*B^2*(a + b*x)*Log[(e*(a + b*x))/(c
 + d*x)])/(c + d*x) + (B*d*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(2*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2
)/(2*(c + d*x)^2) + (b*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(
c + d*x))) + b*((b*(a + b*x)^3)/(3*B*(c + d*x)^3) - d*(((a + b*x)*(A + B*L
og[(e*(a + b*x))/(c + d*x)])^2)/(c + d*x) - 2*B*((A*(a + b*x))/(c + d*x) -
 (B*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(c +
 d*x)))))/((b*c - a*d)^3*g*i^3)
```

### 3.104.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

3.104. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.104.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 768 vs. $2(367) = 734$.

Time = 1.37 (sec) , antiderivative size = 769, normalized size of antiderivative = 2.05

| method | result |
|---|---|
| parts | $\frac{A^2\left(-\frac{1}{2(ad-cb)(dx+c)^2}+\frac{b^2\ln(dx+c)}{(ad-cb)^3}+\frac{b}{(ad-cb)^2(dx+c)}-\frac{b^2\ln(bx+a)}{(ad-cb)^3}\right)}{g\,i^3}-\frac{B^2 d\left(d\left(\frac{\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2}\cdots\right.\right.}{\cdots}$ |
| derivativedivides | $-\frac{e(ad-cb)\left(\frac{d^2A^2b^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^4g}-\frac{2d^3A^2b\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^3(ad-cb)^4g}+\frac{d^4A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e^3i^3(ad-cb)^4g}+\frac{d^2AB\,b^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{e\,i^3(ad-cb)^4g}\cdots\right.}{\cdots}$ |
| default | $-\frac{e(ad-cb)\left(\frac{d^2A^2b^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e\,i^3(ad-cb)^4g}-\frac{2d^3A^2b\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)}{e^2i^3(ad-cb)^4g}+\frac{d^4A^2\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{2e^3i^3(ad-cb)^4g}+\frac{d^2AB\,b^2\ln\left(\frac{be}{d}+\frac{(ad-cb)e}{d(dx+c)}\right)^2}{e\,i^3(ad-cb)^4g}\cdots\right.}{\cdots}$ |
| norman | $-\frac{2A^2ad^3-6A^2bcd^2-2ABad^3+14ABbcd^2+B^2ad^3-15B^2bcd^2}{4igd^2\left(a^2d^2-2abcd+b^2c^2\right)}-\frac{\left(2A^2b^2c^2+2ABa^2d^2-8ABabcd-B^2a^2d^2+8B^2abcd\right)\ln\left(\frac{e(bx+\cdots}{dx+\cdots}\right.}{2gi\left(a^3d^3-3a^2bcd^2+3ab^2c^2d-b^3c^3\right)}\cdots$ |
| parallelrisch | Expression too large to display |
| risch | Expression too large to display |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x,method=_RETU
RNVERBOSE)
```

output
```
A^2/g/i^3*(-1/2/(a*d-b*c)/(d*x+c)^2+b^2/(a*d-b*c)^3*ln(d*x+c)+b/(a*d-b*c)^
2/(d*x+c)-b^2/(a*d-b*c)^3*ln(b*x+a))-B^2/g/i^3*d/(a*d-b*c)^2/e^2*(d/(a*d-b
*c)*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2
-1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+1/4*(
b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*b*e/(a*d-b*c)*((b*e/d+(a*d-b*c)*e/d/(d*x
+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(
b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d)+1/3/d/(a*d-b
*c)*e^2*b^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3)-2*B*A/g/i^3*d/(a*d-b*c)^2/e
^2*(d/(a*d-b*c)*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-2*b*e/(a*d-b*c)*((b*e/d+(a
*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)
-b*e/d)+1/2/d/(a*d-b*c)*e^2*b^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)
```

3.104. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$

### 3.104.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 545, normalized size of antiderivative = 1.45

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \frac{3\,(6\,A^2-14\,AB+15\,B^2)b^2c^2-24\,(A^2-2\,AB+2\,B^2)abcd+3\,(2\,A^2-2\,AB+B^2)a^2d^2+4\,(B^2b^2d^2x^2+$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algo
rithm="fricas")
```

output
```
1/12*(3*(6*A^2 - 14*A*B + 15*B^2)*b^2*c^2 - 24*(A^2 - 2*A*B + 2*B^2)*a*b*c
*d + 3*(2*A^2 - 2*A*B + B^2)*a^2*d^2 + 4*(B^2*b^2*d^2*x^2 + 2*B^2*b^2*c*d*
x + B^2*b^2*c^2)*log((b*e*x + a*e)/(d*x + c))^3 + 6*((2*A*B - 3*B^2)*b^2*d
^2*x^2 + 2*A*B*b^2*c^2 - 4*B^2*a*b*c*d + B^2*a^2*d^2 - 2*(B^2*a*b*d^2 - 2*
(A*B - B^2)*b^2*c*d)*x)*log((b*e*x + a*e)/(d*x + c))^2 + 6*((2*A^2 - 6*A*B
 + 7*B^2)*b^2*c*d - (2*A^2 - 6*A*B + 7*B^2)*a*b*d^2)*x + 6*((2*A^2 - 6*A*B
 + 7*B^2)*b^2*d^2*x^2 + 2*A^2*b^2*c^2 - 8*(A*B - B^2)*a*b*c*d + (2*A*B - B
^2)*a^2*d^2 + 2*(2*(A^2 - 2*A*B + 2*B^2)*b^2*c*d - (2*A*B - 3*B^2)*a*b*d^2
)*x)*log((b*e*x + a*e)/(d*x + c)))/((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2
*b*c*d^4 - a^3*d^5)*g*i^3*x^2 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c
^2*d^3 - a^3*c*d^4)*g*i^3*x + (b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 -
 a^3*c^2*d^3)*g*i^3)
```

### 3.104.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1488 vs. 2(333) = 666.

Time = 4.20 (sec) , antiderivative size = 1488, normalized size of antiderivative = 3.97

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)/(d*i*x+c*i)**3,x)
```

output
```
-B**2*b**2*log(e*(a + b*x)/(c + d*x))**3/(3*a**3*d**3*g*i**3 - 9*a**2*b*c*
d**2*g*i**3 + 9*a*b**2*c**2*d*g*i**3 - 3*b**3*c**3*g*i**3) + b**2*(2*A**2
- 6*A*B + 7*B**2)*log(x + (2*A**2*a*b**2*d + 2*A**2*b**3*c - 6*A*B*a*b**2*
d - 6*A*B*b**3*c + 7*B**2*a*b**2*d + 7*B**2*b**3*c - a**4*b**2*d**4*(2*A**
2 - 6*A*B + 7*B**2)/(a*d - b*c)**3 + 4*a**3*b**3*c*d**3*(2*A**2 - 6*A*B +
7*B**2)/(a*d - b*c)**3 - 6*a**2*b**4*c**2*d**2*(2*A**2 - 6*A*B + 7*B**2)/(
a*d - b*c)**3 + 4*a*b**5*c**3*d*(2*A**2 - 6*A*B + 7*B**2)/(a*d - b*c)**3 -
 b**6*c**4*(2*A**2 - 6*A*B + 7*B**2)/(a*d - b*c)**3)/(4*A**2*b**3*d - 12*A
*B*b**3*d + 14*B**2*b**3*d))/(2*g*i**3*(a*d - b*c)**3) - b**2*(2*A**2 - 6*
A*B + 7*B**2)*log(x + (2*A**2*a*b**2*d + 2*A**2*b**3*c - 6*A*B*a*b**2*d -
6*A*B*b**3*c + 7*B**2*a*b**2*d + 7*B**2*b**3*c + a**4*b**2*d**4*(2*A**2 -
6*A*B + 7*B**2)/(a*d - b*c)**3 - 4*a**3*b**3*c*d**3*(2*A**2 - 6*A*B + 7*B*
*2)/(a*d - b*c)**3 + 6*a**2*b**4*c**2*d**2*(2*A**2 - 6*A*B + 7*B**2)/(a*d
- b*c)**3 - 4*a*b**5*c**3*d*(2*A**2 - 6*A*B + 7*B**2)/(a*d - b*c)**3 + b**
6*c**4*(2*A**2 - 6*A*B + 7*B**2)/(a*d - b*c)**3)/(4*A**2*b**3*d - 12*A*B*b
**3*d + 14*B**2*b**3*d))/(2*g*i**3*(a*d - b*c)**3) + (-2*A*B*a*d + 6*A*B*b
*c + 4*A*B*b*d*x + B**2*a*d - 7*B**2*b*c - 6*B**2*b*d*x)*log(e*(a + b*x)/(
c + d*x))/(2*a**2*c**2*d**2*g*i**3 + 4*a**2*c*d**3*g*i**3*x + 2*a**2*d**4*
g*i**3*x**2 - 4*a*b*c**3*d*g*i**3 - 8*a*b*c**2*d**2*g*i**3*x - 4*a*b*c*d**
3*g*i**3*x**2 + 2*b**2*c**4*g*i**3 + 4*b**2*c**3*d*g*i**3*x + 2*b**2*c*...
```

### 3.104.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2116 vs. $2(367) = 734$.

Time = 0.34 (sec) , antiderivative size = 2116, normalized size of antiderivative = 5.64

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algo
rithm="maxima")
```

output
```
1/2*B^2*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*
i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2
*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(b*e*x/(d*x + c) + a*e/
(d*x + c))^2 + A*B*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 +
a^2*d^4)*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (
b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(b*e*x/(d*x
+ c) + a*e/(d*x + c)) - 1/12*B^2*(6*(7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(
b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b
^2*c*d*x + b^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*
x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x
 + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d
*x + c))*log(b*e*x/(d*x + c) + a*e/(d*x + c))/(b^3*c^5*g*i^3 - 3*a*b^2*c^4
*d*g*i^3 + 3*a^2*b*c^3*d^2*g*i^3 - a^3*c^2*d^3*g*i^3 + (b^3*c^3*d^2*g*i^3
- 3*a*b^2*c^2*d^3*g*i^3 + 3*a^2*b*c*d^4*g*i^3 - a^3*d^5*g*i^3)*x^2 + 2*(b^
3*c^4*d*g*i^3 - 3*a*b^2*c^3*d^2*g*i^3 + 3*a^2*b*c^2*d^3*g*i^3 - a^3*c*d^4*
g*i^3)*x) - (45*b^2*c^2 - 48*a*b*c*d + 3*a^2*d^2 + 4*(b^2*d^2*x^2 + 2*b...
```

### 3.104.8 Giac [A] (verification not implemented)

Time = 0.47 (sec) , antiderivative size = 672, normalized size of antiderivative = 1.79

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{1}{12}\left(\frac{4\,B^2b^2e\log\left(\frac{bex+ae}{dx+c}\right)^3}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}+\frac{12\,A^2b^2e\log\left(\frac{bex+ae}{dx+c}\right)}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}+6\left(\frac{2\,ABb^2e}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}-\right.\right.$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x, algo
rithm="giac")
```

output
```
1/12*(4*B^2*b^2*e*log((b*e*x + a*e)/(d*x + c))^3/(b^2*c^2*g*i^3 - 2*a*b*c*
d*g*i^3 + a^2*d^2*g*i^3) + 12*A^2*b^2*e*log((b*e*x + a*e)/(d*x + c))/(b^2*
c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3) + 6*(2*A*B*b^2*e/(b^2*c^2*g*i
^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3) - 4*(b*e*x + a*e)*B^2*b*d/((b^2*c^2*
g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)) + (b*e*x + a*e)^2*B^2*
d^2/((b^2*c^2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a^2*d^2*e*g*i^3)*(d*x + c)^2))
*log((b*e*x + a*e)/(d*x + c))^2 + 6*((2*A*B*d^2 - B^2*d^2)*(b*e*x + a*e)^2
/((b^2*c^2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a^2*d^2*e*g*i^3)*(d*x + c)^2) - 8
*(A*B*b*d - B^2*b*d)*(b*e*x + a*e)/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2
*d^2*g*i^3)*(d*x + c)))*log((b*e*x + a*e)/(d*x + c)) + 3*(2*A^2*d^2 - 2*A*
B*d^2 + B^2*d^2)*(b*e*x + a*e)^2/((b^2*c^2*e*g*i^3 - 2*a*b*c*d*e*g*i^3 + a
^2*d^2*e*g*i^3)*(d*x + c)^2) - 24*(A^2*b*d - 2*A*B*b*d + 2*B^2*b*d)*(b*e*x
 + a*e)/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)))*(b*
c/((b*c*e - a*d*e)*(b*c - a*d)) - a*d/((b*c*e - a*d*e)*(b*c - a*d)))
```

## 3.104.9 Mupad [B] (verification not implemented)

Time = 4.86 (sec) , antiderivative size = 984, normalized size of antiderivative = 2.62

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$

$$= -\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^2\left(\frac{\frac{B^2\,b^2\left(\frac{a^2\,d^2-3\,a\,b\,c\,d+2\,b^2\,c^2}{2\,b^3\,d}-\frac{c\,(a\,d-b\,c)}{2\,b^2\,d}\right)}{g\,i^3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}-\frac{B^2\,x\,(a\,d-b\,c)}{g\,i^3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}}{\frac{d\,x^2}{b}+\frac{c^2}{b\,d}+\frac{2\,c\,x}{b}} + \frac{B\,b^2\,(2\,A-3\,B)}{2\,g\,i^3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)$$

$$-\frac{\frac{2\,A^2\,a\,d-6\,A^2\,b\,c+B^2\,a\,d-15\,B^2\,b\,c-2\,A\,B\,a\,d+14\,A\,B\,b\,c}{2\,(a\,d-b\,c)}-\frac{x\,(2\,b\,d\,A^2-6\,b\,d\,A\,B+7\,b\,d\,B^2)}{a\,d-b\,c}}{x^2\,(2\,a\,d^3\,g\,i^3-2\,b\,c\,d^2\,g\,i^3)+x\,(4\,a\,c\,d^2\,g\,i^3-4\,b\,c^2\,d\,g\,i^3)-2\,b\,c^3\,g\,i^3+2\,a\,c^2\,d\,g\,i^3}$$

$$-\frac{\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)\left(\frac{B^2}{b\,d\,g\,i^3\,(a\,d-b\,c)}+\frac{B\,b^2\left(\frac{a^2\,d^2-3\,a\,b\,c\,d+2\,b^2\,c^2}{2\,b^3\,d}-\frac{c\,(a\,d-b\,c)}{2\,b^2\,d}\right)(2\,A-3\,B)}{g\,i^3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}-\frac{B\,x\,(2\,A-3\,B)\,(a\,d-b\,c)}{g\,i^3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}\right)}{\frac{d\,x^2}{b}+\frac{c^2}{b\,d}+\frac{2\,c\,x}{b}}$$

$$-\frac{B^2\,b^2\ln\left(\frac{e\,(a+b\,x)}{c+d\,x}\right)^3}{3\,g\,i^3\,(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3)}$$

$$+\frac{b^2\,\mathrm{atan}\left(\frac{b^2\left(A^2-3\,A\,B+\frac{7\,B^2}{2}\right)(2\,g\,a^3\,d^3\,i^3-2\,g\,a^2\,b\,c\,d^2\,i^3-2\,g\,a\,b^2\,c^2\,d\,i^3+2\,g\,b^3\,c^3\,i^3)\,1\mathrm{i}}{g\,i^3\,(a\,d-b\,c)^3\,(2\,A^2\,b^2-6\,A\,B\,b^2+7\,B^2\,b^2)}+\frac{b^3\,d\,x\,(g\,a^2\,d^2\,i^3-2\,g\,a\,b\,c\,d\,i^3+g\,b^2\,c^2\,i^3)\,(A}{g\,i^3\,(a\,d-b\,c)^3\,(2\,A^2\,b^2-6\,A\,B\,b^2+}\right.}{g\,i^3\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)*(c*i + d*i*x)^3)
,x)
```

output
```
(b^2*atan((b^2*(A^2 + (7*B^2)/2 - 3*A*B)*(2*a^3*d^3*g*i^3 + 2*b^3*c^3*g*i^
3 - 2*a*b^2*c^2*d*g*i^3 - 2*a^2*b*c*d^2*g*i^3)*1i)/(g*i^3*(a*d - b*c)^3*(2
*A^2*b^2 + 7*B^2*b^2 - 6*A*B*b^2)) + (b^3*d*x*(a^2*d^2*g*i^3 + b^2*c^2*g*i
^3 - 2*a*b*c*d*g*i^3)*(A^2 + (7*B^2)/2 - 3*A*B)*4i)/(g*i^3*(a*d - b*c)^3*(
2*A^2*b^2 + 7*B^2*b^2 - 6*A*B*b^2)))*(A^2 + (7*B^2)/2 - 3*A*B)*2i)/(g*i^3*
(a*d - b*c)^3) - ((2*A^2*a*d - 6*A^2*b*c + B^2*a*d - 15*B^2*b*c - 2*A*B*a*
d + 14*A*B*b*c)/(2*(a*d - b*c)) - (x*(2*A^2*b*d + 7*B^2*b*d - 6*A*B*b*d))/
(a*d - b*c))/(x^2*(2*a*d^3*g*i^3 - 2*b*c*d^2*g*i^3) + x*(4*a*c*d^2*g*i^3 -
 4*b*c^2*d*g*i^3) - 2*b*c^3*g*i^3 + 2*a*c^2*d*g*i^3) - (log((e*(a + b*x))/
(c + d*x))*(B^2/(b*d*g*i^3*(a*d - b*c)) + (B*b^2*((a^2*d^2 + 2*b^2*c^2 - 3
*a*b*c*d)/(2*b^3*d) - (c*(a*d - b*c))/(2*b^2*d))*(2*A - 3*B))/(g*i^3*(a^3*
d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B*x*(2*A - 3*B)*(a*d -
b*c))/(g*i^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))))/((d*x^
2)/b + c^2/(b*d) + (2*c*x)/b) - log((e*(a + b*x))/(c + d*x))^2*(((B^2*b^2*
((a^2*d^2 + 2*b^2*c^2 - 3*a*b*c*d)/(2*b^3*d) - (c*(a*d - b*c))/(2*b^2*d)))
/(g*i^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) - (B^2*x*(a*d
 - b*c))/(g*i^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)))/((d*
x^2)/b + c^2/(b*d) + (2*c*x)/b) + (B*b^2*(2*A - 3*B))/(2*g*i^3*(a^3*d^3 -
b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2))) - (B^2*b^2*log((e*(a + b*x))/(c
 + d*x))^3)/(3*g*i^3*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2...
```

## 3.104. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$

# 3.105 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx$



## 3.105.1 Optimal result

Integrand size = 42, antiderivative size = 525

$$\begin{aligned}\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = & -\frac{B^2d^3(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2}-\frac{6AbBd^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)} \\ & +\frac{6bB^2d^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}-\frac{2b^3B^2(c+dx)}{(bc-ad)^4g^2i^3(a+bx)} \\ & -\frac{6bB^2d^2(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^4g^2i^3(c+dx)} \\ & +\frac{Bd^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2} \\ & -\frac{2b^3B(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^4g^2i^3(a+bx)} \\ & -\frac{d^3(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^4g^2i^3(c+dx)^2} \\ & +\frac{3bd^2(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^2i^3(c+dx)} \\ & -\frac{b^3(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^4g^2i^3(a+bx)} \\ & -\frac{b^2d\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^4g^2i^3}\end{aligned}$$

output
```
-1/4*B^2*d^3*(b*x+a)^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-6*A*b*B*d^2*(b*x+a)/
(-a*d+b*c)^4/g^2/i^3/(d*x+c)+6*b*B^2*d^2*(b*x+a)/(-a*d+b*c)^4/g^2/i^3/(d*x
+c)-2*b^3*B^2*(d*x+c)/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-6*b*B^2*d^2*(b*x+a)*ln(
e*(b*x+a)/(d*x+c))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)+1/2*B*d^3*(b*x+a)^2*(A+B*l
n(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-2*b^3*B*(d*x+c)*(A+B*
ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-1/2*d^3*(b*x+a)^2*(A+B
*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2+3*b*d^2*(b*x+a)*(
A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)-b^3*(d*x+c)*(A+B
*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-b^2*d*(A+B*ln(e*(b*
x+a)/(d*x+c)))^3/B/(-a*d+b*c)^4/g^2/i^3
```

### 3.105.2 Mathematica [A] (verified)

Time = 0.67 (sec) , antiderivative size = 453, normalized size of antiderivative = 0.86

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx =$$
$$-\frac{(2A^2-2AB+B^2)\,d(bc-ad)^2(a+bx)+2b(4A^2-10AB+11B^2)\,d(bc-ad)(a+bx)(c+dx)+4b^2(A}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d
*i*x)^3),x]
```

output
```
-1/4*((2*A^2 - 2*A*B + B^2)*d*(b*c - a*d)^2*(a + b*x) + 2*b*(4*A^2 - 10*A*
B + 11*B^2)*d*(b*c - a*d)*(a + b*x)*(c + d*x) + 4*b^2*(A^2 + 2*A*B + 2*B^2
)*(b*c - a*d)*(c + d*x)^2 + 6*b^2*(2*A^2 - 2*A*B + 5*B^2)*d*(a + b*x)*(c +
 d*x)^2*Log[a + b*x] + 2*B*(b*c - a*d)*((2*A - B)*d*(b*c - a*d)*(a + b*x)
+ 2*b*(4*A - 5*B)*d*(a + b*x)*(c + d*x) + 4*b^2*(A + B)*(c + d*x)^2)*Log[(
e*(a + b*x))/(c + d*x)] + 2*B*(a^3*B*d^3 - 3*a^2*b*B*d^2*(2*c + d*x) + 3*a
*b^2*d*(2*A*(c + d*x)^2 - B*d*x*(4*c + 3*d*x)) + b^3*(6*A*d*x*(c + d*x)^2
+ B*(2*c^3 + 6*c^2*d*x - 3*d^3*x^3)))*Log[(e*(a + b*x))/(c + d*x)]^2 + 4*b
^2*B^2*d*(a + b*x)*(c + d*x)^2*Log[(e*(a + b*x))/(c + d*x)]^3 - 6*b^2*(2*A
^2 - 2*A*B + 5*B^2)*d*(a + b*x)*(c + d*x)^2*Log[c + d*x])/((b*c - a*d)^4*g
^2*i^3*(a + b*x)*(c + d*x)^2)
```

### 3.105.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^2(ci+dix)^3}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i^3(bc-ad)^4}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^3}{(a+bx)^2}-\frac{3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^2}{a+bx}+3d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b-\frac{d^3(a+bx)\left(A+B\log\left(\right.\right.}{c+dx}\right.}{g^2i^3(bc-ad)^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^3(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}-\frac{2b^3B(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{a+bx}-\frac{b^2d\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^3}{B}-\frac{d^3(a+bx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(c+dx)^2}}{}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^2*(c*i + d*i*x)^
3),x]
```

output
```
(-1/4*(B^2*d^3*(a + b*x)^2)/(c + d*x)^2 - (6*A*b*B*d^2*(a + b*x))/(c + d*x
) + (6*b*B^2*d^2*(a + b*x))/(c + d*x) - (2*b^3*B^2*(c + d*x))/(a + b*x) -
(6*b*B^2*d^2*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/(c + d*x) + (B*d^3*(a
 + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(c + d*x)^2) - (2*b^3*B
*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(a + b*x) - (d^3*(a + b*x
)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(2*(c + d*x)^2) + (3*b*d^2*(a
+ b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(c + d*x) - (b^3*(c + d*x)*
(A + B*Log[(e*(a + b*x))/(c + d*x)])^2)/(a + b*x) - (b^2*d*(A + B*Log[(e*(
a + b*x))/(c + d*x)])^3)/B)/((b*c - a*d)^4*g^2*i^3)
```

3.105. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}dx$

### 3.105.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

## 3.105.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. 2(519) = 1038.

Time = 2.78 (sec) , antiderivative size = 1080, normalized size of antiderivative = 2.06

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1080 |
| derivativedivides | Expression too large to display | 1233 |
| default | Expression too large to display | 1233 |
| parallelrisch | Expression too large to display | 1762 |
| norman | Expression too large to display | 1852 |
| risch | Expression too large to display | 2114 |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,method=_RE
TURNVERBOSE)
```

output
```
1/g^2*A^2/i^3*(-1/2*d/(a*d-b*c)^2/(d*x+c)^2+3*d/(a*d-b*c)^4*b^2*ln(d*x+c)+
2*d/(a*d-b*c)^3*b/(d*x+c)+b^2/(a*d-b*c)^3/(b*x+a)-3*d/(a*d-b*c)^4*b^2*ln(b
*x+a))-B^2/g^2/i^3*d/(a*d-b*c)^2/e^2*(d^2/(a*d-b*c)^2*(1/2*(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2*(b*e/d+(a*d-b*c)*e/
d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+1/4*(b*e/d+(a*d-b*c)*e/d/(d*x
+c))^2)-3/(a*d-b*c)^2*b*d*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/
(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d)+1/(a*d-b*c)^2*b^2*e^2*ln(b*e/d+(
a*d-b*c)*e/d/(d*x+c))^3-1/d/(a*d-b*c)^2*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(
d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*
ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))-2*B*A/g^
2/i^3*d/(a*d-b*c)^2/e^2*(d^2/(a*d-b*c)^2*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c)
)^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-3
*d/(a*d-b*c)^2*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+3/2/(a*d-b*c)^2*b^2*e^2*ln(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^2-1/d/(a*d-b*c)^2*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x
+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))))
```

### 3.105.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 1008, normalized size of antiderivative = 1.92

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = -\frac{4\,(A^2+2\,AB+2\,B^2)b^3c^3+3\,(2\,A^2-10\,AB+5\,B^2)ab^2c^2d-12\,(A^2-2\,AB+2\,B^2)a^2bcd^2+(2\,A^2-\cdots}{\cdots}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output
```
-1/4*(4*(A^2 + 2*A*B + 2*B^2)*b^3*c^3 + 3*(2*A^2 - 10*A*B + 5*B^2)*a*b^2*c
^2*d - 12*(A^2 - 2*A*B + 2*B^2)*a^2*b*c*d^2 + (2*A^2 - 2*A*B + B^2)*a^3*d^
3 + 4*(B^2*b^3*d^3*x^3 + B^2*a*b^2*c^2*d + (2*B^2*b^3*c*d^2 + B^2*a*b^2*d^
3)*x^2 + (B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2)*x)*log((b*e*x + a*e)/(d*x + c
))^3 + 6*((2*A^2 - 2*A*B + 5*B^2)*b^3*c*d^2 - (2*A^2 - 2*A*B + 5*B^2)*a*b^
2*d^3)*x^2 + 2*(3*(2*A*B - B^2)*b^3*d^3*x^3 + 2*B^2*b^3*c^3 + 6*A*B*a*b^2*
c^2*d - 6*B^2*a^2*b*c*d^2 + B^2*a^3*d^3 + 3*(4*A*B*b^3*c*d^2 + (2*A*B - 3*
B^2)*a*b^2*d^3)*x^2 - 3*(B^2*a^2*b*d^3 - 2*(A*B + B^2)*b^3*c^2*d - 4*(A*B
- B^2)*a*b^2*c*d^2)*x)*log((b*e*x + a*e)/(d*x + c))^2 + 3*((6*A^2 - 2*A*B
+ 13*B^2)*b^3*c^2*d - 2*(2*A^2 + 2*A*B + 3*B^2)*a*b^2*c*d^2 - (2*A^2 - 6*A
*B + 7*B^2)*a^2*b*d^3)*x + 2*(3*(2*A^2 - 2*A*B + 5*B^2)*b^3*d^3*x^3 + 6*A^
2*a*b^2*c^2*d + 4*(A*B + B^2)*b^3*c^3 - 12*(A*B - B^2)*a^2*b*c*d^2 + (2*A*
B - B^2)*a^3*d^3 + 3*(4*(A^2 + 2*B^2)*b^3*c*d^2 + (2*A^2 - 6*A*B + 7*B^2)*
a*b^2*d^3)*x^2 + 3*(2*(A^2 + 2*A*B + 2*B^2)*b^3*c^2*d + 4*(A^2 - 2*A*B + 2
*B^2)*a*b^2*c*d^2 - (2*A*B - 3*B^2)*a^2*b*d^3)*x)*log((b*e*x + a*e)/(d*x +
 c)))/((b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^
5 + a^4*b*d^6)*g^2*i^3*x^3 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^
3*d^3 - 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*g^2*i^3*x^2 + (b^5*c^
6 - 2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^
4 + 2*a^5*c*d^5)*g^2*i^3*x + (a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c...
```

### 3.105.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**2/(d*i*x+c*i)**3,x)
```

output
```
Timed out
```

### 3.105.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4188 vs. $2(519) = 1038$.

Time = 0.50 (sec) , antiderivative size = 4188, normalized size of antiderivative = 7.98

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output
```
-1/2*B^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d
+ a*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^
5)*g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*
b*c*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^
2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c
^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3)
 - 6*b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)*g^2*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c))^2 -
A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a*b
*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*g^
2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c*d
^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 + 5
*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d + 3
*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4 -
4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) - 6*
b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b
*c*d^3 + a^4*d^4)*g^2*i^3))*log(b*e*x/(d*x + c) + a*e/(d*x + c)) - 1/4*B^2
*(2*(4*b^3*c^3 - 15*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3 - 6*(b^3*c*d^2
- a*b^2*d^3)*x^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*...
```

### 3.105.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^2(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/((b*g*x + a*g)^2*(d*i*x + c
*i)^3), x)
```

### 3.105.9 Mupad [B] (verification not implemented)

Time = 7.47 (sec) , antiderivative size = 1505, normalized size of antiderivative = 2.87

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx=\text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^2*(c*i + d*i*x)^
3),x)
```

output
```
((4*A^2*b^2*c^2 - 2*A^2*a^2*d^2 - B^2*a^2*d^2 + 8*B^2*b^2*c^2 + 2*A*B*a^2*
d^2 + 8*A*B*b^2*c^2 + 10*A^2*a*b*c*d + 23*B^2*a*b*c*d - 22*A*B*a*b*c*d)/(2
*(a*d - b*c)) + (3*x^2*(2*A^2*b^2*d^2 + 5*B^2*b^2*d^2 - 2*A*B*b^2*d^2))/(a
*d - b*c) + (3*x*(2*A^2*a*b*d^2 + 7*B^2*a*b*d^2 + 6*A^2*b^2*c*d + 13*B^2*b
^2*c*d - 6*A*B*a*b*d^2 - 2*A*B*b^2*c*d))/(2*(a*d - b*c)))/(x*(2*b^3*c^4*g^
2*i^3 + 4*a^3*c*d^3*g^2*i^3 - 6*a^2*b*c^2*d^2*g^2*i^3) + x^2*(2*a^3*d^4*g^
2*i^3 + 4*b^3*c^3*d*g^2*i^3 - 6*a*b^2*c^2*d^2*g^2*i^3) + x^3*(2*b^3*c^2*d^
2*g^2*i^3 + 2*a^2*b*d^4*g^2*i^3 - 4*a*b^2*c*d^3*g^2*i^3) + 2*a^3*c^2*d^2*g
^2*i^3 + 2*a*b^2*c^4*g^2*i^3 - 4*a^2*b*c^3*d*g^2*i^3) - log((e*(a + b*x))/
(c + d*x))^2*((x*((3*B^2)/(2*g^2*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (3
*B^2*(a*d + b*c))/(g^2*i^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) +
 (B^2*(a*d + 2*b*c))/(2*g^2*i^3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) -
 (3*B^2*a*c)/(g^2*i^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (3*B^
2*b*d*x^2)/(g^2*i^3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(d*x^3 +
 (a*c^2)/(b*d) + (x^2*(a*d^2 + 2*b*c*d))/(b*d) + (x*(b*c^2 + 2*a*c*d))/(b*
d)) + (3*B*b^2*d*(2*A - B))/(2*g^2*i^3*(a*d - b*c)^2*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d))) - (log((e*(a + b*x))/(c + d*x))*(x*((3*(B^2 + 2*A*B))/(2*g^2*
i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (3*B*(2*A - B)*(a*d + b*c))/(g^2*i^
3*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) + (4*B^2*b*c - B^2*a*d + 2
*A*B*a*d + 4*A*B*b*c)/(2*g^2*i^3*(a^2*b*d^3 + b^3*c^2*d - 2*a*b^2*c*d^2...
```

3.105. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx$

# 3.106 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$

### 3.106.1 Optimal result

Integrand size $= 42$, antiderivative size $= 685$

$$
\begin{aligned}
\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx &= \frac{B^2d^4(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2}+\frac{8AbBd^3(a+bx)}{(bc-ad)^5g^3i^3(c+dx)} \\
&\quad-\frac{8bB^2d^3(a+bx)}{(bc-ad)^5g^3i^3(c+dx)}+\frac{8b^3B^2d(c+dx)}{(bc-ad)^5g^3i^3(a+bx)} \\
&\quad-\frac{b^4B^2(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2}+\frac{8bB^2d^3(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^5g^3i^3(c+dx)} \\
&\quad-\frac{Bd^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2} \\
&\quad+\frac{8b^3Bd(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^5g^3i^3(a+bx)} \\
&\quad-\frac{b^4B(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2} \\
&\quad+\frac{d^4(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^5g^3i^3(c+dx)^2} \\
&\quad-\frac{4bd^3(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^3i^3(c+dx)} \\
&\quad+\frac{4b^3d(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^5g^3i^3(a+bx)} \\
&\quad-\frac{b^4(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^5g^3i^3(a+bx)^2} \\
&\quad+\frac{2b^2d^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{B(bc-ad)^5g^3i^3}
\end{aligned}
$$

output
```
1/4*B^2*d^4*(b*x+a)^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+8*A*b*B*d^3*(b*x+a)/(
-a*d+b*c)^5/g^3/i^3/(d*x+c)-8*b*B^2*d^3*(b*x+a)/(-a*d+b*c)^5/g^3/i^3/(d*x+
c)+8*b^3*B^2*d*(d*x+c)/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/4*b^4*B^2*(d*x+c)^2/
(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+8*b*B^2*d^3*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(
-a*d+b*c)^5/g^3/i^3/(d*x+c)-1/2*B*d^4*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c))
)/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+8*b^3*B*d*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+
c)))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*b^4*B*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d
*x+c)))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+1/2*d^4*(b*x+a)^2*(A+B*ln(e*(b*x+a)
/(d*x+c)))^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2-4*b*d^3*(b*x+a)*(A+B*ln(e*(b*x
+a)/(d*x+c)))^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*d*(d*x+c)*(A+B*ln(e*(b*
x+a)/(d*x+c)))^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*b^4*(d*x+c)^2*(A+B*ln(e*
(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+2*b^2*d^2*(A+B*ln(e*(b*
x+a)/(d*x+c)))^3/B/(-a*d+b*c)^5/g^3/i^3
```

### 3.106.2 Mathematica [A] (verified)

Time = 0.88 (sec) , antiderivative size = 611, normalized size of antiderivative = 0.89

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$$
$$= \frac{(2A^2-2AB+B^2)\,d^2(bc-ad)^2(a+bx)^2+2b(6A^2-14AB+15B^2)\,d^2(bc-ad)(a+bx)^2(c+dx)-b^2(2}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d
*i*x)^3),x]
```

output
```
((2*A^2 - 2*A*B + B^2)*d^2*(b*c - a*d)^2*(a + b*x)^2 + 2*b*(6*A^2 - 14*A*B
 + 15*B^2)*d^2*(b*c - a*d)*(a + b*x)^2*(c + d*x) - b^2*(2*A^2 + 2*A*B + B^
2)*(b*c - a*d)^2*(c + d*x)^2 + 2*b^2*(6*A^2 + 14*A*B + 15*B^2)*d*(b*c - a*
d)*(a + b*x)*(c + d*x)^2 + 12*b^2*(2*A^2 + 5*B^2)*d^2*(a + b*x)^2*(c + d*x
)^2*Log[a + b*x] + 2*B*(b*c - a*d)*((2*A - B)*d^2*(b*c - a*d)*(a + b*x)^2
+ 2*b*(6*A - 7*B)*d^2*(a + b*x)^2*(c + d*x) - b^2*(2*A + B)*(b*c - a*d)*(c
 + d*x)^2 + 2*b^2*(6*A + 7*B)*d*(a + b*x)*(c + d*x)^2)*Log[(e*(a + b*x))/(
c + d*x)] - 2*B*(-(a^4*B*d^4) + 4*a^3*b*B*d^3*(2*c + d*x) - 6*a^2*b^2*d^2*
(2*A*(c + d*x)^2 - B*d*x*(4*c + 3*d*x)) + b^4*(-12*A*d^2*x^2*(c + d*x)^2 +
 B*c*(c^3 - 4*c^2*d*x - 18*c*d^2*x^2 - 12*d^3*x^3)) - 4*a*b^3*d*(6*A*d*x*(
c + d*x)^2 + B*(2*c^3 + 6*c^2*d*x - 3*d^3*x^3)))*Log[(e*(a + b*x))/(c + d*
x)]^2 + 8*b^2*B^2*d^2*(a + b*x)^2*(c + d*x)^2*Log[(e*(a + b*x))/(c + d*x)]
^3 - 12*b^2*(2*A^2 + 5*B^2)*d^2*(a + b*x)^2*(c + d*x)^2*Log[c + d*x])/(4*(
b*c - a*d)^5*g^3*i^3*(a + b*x)^2*(c + d*x)^2)
```

### 3.106.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 478, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^3(ci+dix)^3}dx$$

$$\Big\downarrow \text{ 2962}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i^3(bc-ad)^5}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\frac{(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^4}{(a+bx)^3}-\frac{4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^3}{(a+bx)^2}+\frac{6d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^2}{a+bx}-4d^3\left(A+B\log\left(\right.\right.\right.}{g^3i^3(bc-ad)^5}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^4(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}-\frac{b^4B(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{2(a+bx)^2}+\frac{4b^3d(c+dx)\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{a+bx}+\frac{8b^3Bd(c+dx)\left(B\log\left(\frac{e(a+}{c+}\right.\right.}{a+bx}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^3*(c*i + d*i*x)^
3),x]
```

output
```
((B^2*d^4*(a + b*x)^2)/(4*(c + d*x)^2) + (8*A*b*B*d^3*(a + b*x))/(c + d*x)
 - (8*b*B^2*d^3*(a + b*x))/(c + d*x) + (8*b^3*B^2*d*(c + d*x))/(a + b*x) -
 (b^4*B^2*(c + d*x)^2)/(4*(a + b*x)^2) + (8*b*B^2*d^3*(a + b*x)*Log[(e*(a
+ b*x))/(c + d*x)])/(c + d*x) - (B*d^4*(a + b*x)^2*(A + B*Log[(e*(a + b*x)
)/(c + d*x)]))/(2*(c + d*x)^2) + (8*b^3*B*d*(c + d*x)*(A + B*Log[(e*(a + b
*x))/(c + d*x)]))/(a + b*x) - (b^4*B*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/
(c + d*x)]))/(2*(a + b*x)^2) + (d^4*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(
c + d*x)])^2)/(2*(c + d*x)^2) - (4*b*d^3*(a + b*x)*(A + B*Log[(e*(a + b*x)
)/(c + d*x)])^2)/(c + d*x) + (4*b^3*d*(c + d*x)*(A + B*Log[(e*(a + b*x))/(
c + d*x)])^2)/(a + b*x) - (b^4*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d
*x)])^2)/(2*(a + b*x)^2) + (2*b^2*d^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^3)/B)/((b*c - a*d)^5*g^3*i^3)
```

### 3.106.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

---

3.106. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$

### 3.106.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1395 vs. 2(673) = 1346.

Time = 2.34 (sec) , antiderivative size = 1396, normalized size of antiderivative = 2.04

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1396 |
| derivativedivides | Expression too large to display | 1589 |
| default | Expression too large to display | 1589 |
| risch | Expression too large to display | 2582 |
| parallelrisch | Expression too large to display | 2645 |
| norman | Expression too large to display | 2758 |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,method=_RE
TURNVERBOSE)
```

output
```
A^2/g^3/i^3*(-1/2*d^2/(a*d-b*c)^3/(d*x+c)^2+6*d^2/(a*d-b*c)^5*b^2*ln(d*x+c
)+3*d^2/(a*d-b*c)^4*b/(d*x+c)+1/2*b^2/(a*d-b*c)^3/(b*x+a)^2-6*d^2/(a*d-b*c
)^5*b^2*ln(b*x+a)+3*b^2/(a*d-b*c)^4*d/(b*x+a))-B^2/g^3/i^3*d/(a*d-b*c)^2/e
^2*(d^3/(a*d-b*c)^3*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^2-1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/
d/(d*x+c))+1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-4/(a*d-b*c)^3*b*d^2*e*((b*
e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d
-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)
+2*b*e/d)+2*d/(a*d-b*c)^3*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3-4/(a*d
-b*c)^3*b^3*e^3*(-1/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(
d*x+c))^2-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-
2/(b*e/d+(a*d-b*c)*e/d/(d*x+c)))+1/d/(a*d-b*c)^3*b^4*e^4*(-1/2/(b*e/d+(a*d
-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c
)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/
(d*x+c))^2))-2*B*A/g^3/i^3*d/(a*d-b*c)^2/e^2*(d^3/(a*d-b*c)^3*(1/2*(b*e/d+
(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/4*(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2)-4*d^2/(a*d-b*c)^3*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*l
n(b*e/d+(a*d-b*c)*e/d/(d*x+c))-(a*d-b*c)*e/d/(d*x+c)-b*e/d)+3*d/(a*d-b*c)^
3*b^2*e^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-4/(a*d-b*c)^3*b^3*e^3*(-1/(b*e
/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-1/(b*e/d+(a*d...
```

### 3.106.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1517 vs. $2(673) = 1346$.

Time = 0.41 (sec) , antiderivative size = 1517, normalized size of antiderivative = 2.21

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output
```
-1/4*(60*A*B*a^2*b^2*c^2*d^2 + (2*A^2 + 2*A*B + B^2)*b^4*c^4 - 16*(A^2 + 2
*A*B + 2*B^2)*a*b^3*c^3*d + 16*(A^2 - 2*A*B + 2*B^2)*a^3*b*c*d^3 - (2*A^2
- 2*A*B + B^2)*a^4*d^4 - 12*((2*A^2 + 5*B^2)*b^4*c*d^3 - (2*A^2 + 5*B^2)*a
*b^3*d^4)*x^3 - 8*(B^2*b^4*d^4*x^4 + B^2*a^2*b^2*c^2*d^2 + 2*(B^2*b^4*c*d^
3 + B^2*a*b^3*d^4)*x^3 + (B^2*b^4*c^2*d^2 + 4*B^2*a*b^3*c*d^3 + B^2*a^2*b^
2*d^4)*x^2 + 2*(B^2*a*b^3*c^2*d^2 + B^2*a^2*b^2*c*d^3)*x)*log((b*e*x + a*e
)/(d*x + c))^3 + 6*(8*A*B*a*b^3*c*d^3 - (6*A^2 + 4*A*B + 15*B^2)*b^4*c^2*d
^2 + (6*A^2 - 4*A*B + 15*B^2)*a^2*b^2*d^4)*x^2 - 2*(12*A*B*b^4*d^4*x^4 - B
^2*b^4*c^4 + 8*B^2*a*b^3*c^3*d + 12*A*B*a^2*b^2*c^2*d^2 - 8*B^2*a^3*b*c*d^
3 + B^2*a^4*d^4 + 12*((2*A*B + B^2)*b^4*c*d^3 + (2*A*B - B^2)*a*b^3*d^4)*x
^3 + 6*(8*A*B*a*b^3*c*d^3 + (2*A*B + 3*B^2)*b^4*c^2*d^2 + (2*A*B - 3*B^2)*
a^2*b^2*d^4)*x^2 + 4*(B^2*b^4*c^3*d - B^2*a^3*b*d^4 + 6*(A*B + B^2)*a*b^3*
c^2*d^2 + 6*(A*B - B^2)*a^2*b^2*c*d^3)*x)*log((b*e*x + a*e)/(d*x + c))^2 -
 4*((2*A^2 + 6*A*B + 7*B^2)*b^4*c^3*d + 6*(2*A^2 - A*B + 4*B^2)*a*b^3*c^2*
d^2 - 6*(2*A^2 + A*B + 4*B^2)*a^2*b^2*c*d^3 - (2*A^2 - 6*A*B + 7*B^2)*a^3*
b*d^4)*x - 2*(6*(2*A^2 + 5*B^2)*b^4*d^4*x^4 + 12*A^2*a^2*b^2*c^2*d^2 - (2*
A*B + B^2)*b^4*c^4 + 16*(A*B + B^2)*a*b^3*c^3*d - 16*(A*B - B^2)*a^3*b*c*d
^3 + (2*A*B - B^2)*a^4*d^4 + 12*((2*A^2 + 2*A*B + 5*B^2)*b^4*c*d^3 + (2*A^
2 - 2*A*B + 5*B^2)*a*b^3*d^4)*x^3 + 6*((2*A^2 + 6*A*B + 7*B^2)*b^4*c^2*d^2
 + 8*(A^2 + 2*B^2)*a*b^3*c*d^3 + (2*A^2 - 6*A*B + 7*B^2)*a^2*b^2*d^4)*x...
```

### 3.106.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**3/(d*i*x+c*i)**3,x)
```

output
```
Timed out
```

### 3.106.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5583 vs. $2(673) = 1346$.

Time = 0.55 (sec) , antiderivative size = 5583, normalized size of antiderivative = 8.15

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output
```
1/2*B^2*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d
^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b
*d^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c
*d^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b
^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3
+ (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 +
a^6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2
+ 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^
6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g
^3*i^3) + 12*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c
^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d
^2*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^
2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(b*e*x/(d*x + c) + a*e/(
d*x + c))^2 + A*B*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d
^2 - a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d
^2 + a^2*b*d^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4
*a^3*b^3*c*d^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2
 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^
3*i^3*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*
c^2*d^4 + a^6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3...
```

### 3.106.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^3(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/((b*g*x + a*g)^3*(d*i*x + c
*i)^3), x)
```

### 3.106.9 Mupad [B] (verification not implemented)

Time = 11.14 (sec) , antiderivative size = 2155, normalized size of antiderivative = 3.15

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^3*(c*i + d*i*x)^
3),x)
```

output
```
((2*x*(2*A^2*a^2*b*d^3 + 7*B^2*a^2*b*d^3 + 2*A^2*b^3*c^2*d + 7*B^2*b^3*c^2
*d + 14*A^2*a*b^2*c*d^2 + 31*B^2*a*b^2*c*d^2 - 6*A*B*a^2*b*d^3 + 6*A*B*b^3
*c^2*d))/(a*d - b*c) - (2*A^2*a^3*d^3 + 2*A^2*b^3*c^3 + B^2*a^3*d^3 + B^2*
b^3*c^3 - 2*A*B*a^3*d^3 + 2*A*B*b^3*c^3 - 14*A^2*a*b^2*c^2*d - 14*A^2*a^2*
b*c*d^2 - 31*B^2*a*b^2*c^2*d - 31*B^2*a^2*b*c*d^2 - 30*A*B*a*b^2*c^2*d + 3
0*A*B*a^2*b*c*d^2)/(2*(a*d - b*c)) + (6*x^3*(2*A^2*b^3*d^3 + 5*B^2*b^3*d^3
))/(a*d - b*c) + (3*x^2*(6*A^2*a*b^2*d^3 + 15*B^2*a*b^2*d^3 + 6*A^2*b^3*c*
d^2 + 15*B^2*b^3*c*d^2 - 4*A*B*a*b^2*d^3 + 4*A*B*b^3*c*d^2))/(a*d - b*c))/
(x^4*(2*a^3*b^2*d^5*g^3*i^3 - 2*b^5*c^3*d^2*g^3*i^3 + 6*a*b^4*c^2*d^3*g^3*
i^3 - 6*a^2*b^3*c*d^4*g^3*i^3) - x*(4*a*b^4*c^5*g^3*i^3 - 4*a^5*c*d^4*g^3*
i^3 - 8*a^2*b^3*c^4*d*g^3*i^3 + 8*a^4*b*c^2*d^3*g^3*i^3) + x^3*(4*a^4*b*d^
5*g^3*i^3 - 4*b^5*c^4*d*g^3*i^3 + 8*a*b^4*c^3*d^2*g^3*i^3 - 8*a^3*b^2*c*d^
4*g^3*i^3) + x^2*(2*a^5*d^5*g^3*i^3 - 2*b^5*c^5*g^3*i^3 - 2*a*b^4*c^4*d*g^
3*i^3 + 2*a^4*b*c*d^4*g^3*i^3 + 16*a^2*b^3*c^3*d^2*g^3*i^3 - 16*a^3*b^2*c^
2*d^3*g^3*i^3) - 2*a^2*b^3*c^5*g^3*i^3 + 2*a^5*c^2*d^3*g^3*i^3 + 6*a^3*b^2
*c^4*d*g^3*i^3 - 6*a^4*b*c^3*d^2*g^3*i^3) + log((e*(a + b*x))/(c + d*x))^2
*((x*((3*B^2*(a*d + b*c)^2)/(g^3*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) -
B^2/(g^3*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (6*B^2*a*b*c*d)/(g^3*i^3*(
a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2)) - (B^2*(a*d + b*c))/(2*g^3*i^3*(a^2*b*d
^3 + b^3*c^2*d - 2*a*b^2*c*d^2)) + (6*B^2*b^2*d^2*x^3)/(g^3*i^3*(a^2*d^...
```

# 3.107 $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx$

### 3.107.1 Optimal result

Integrand size = 42, antiderivative size = 851

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = -\frac{B^2d^5(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2} - \frac{10AbBd^4(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} + \frac{10bB^2d^4(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} - \frac{20b^3B^2d^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4B^2d(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2} - \frac{2b^5B^2(c+dx)^3}{27(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10bB^2d^4(a+bx)\log\left(\frac{e(a+bx)}{c+dx}\right)}{(bc-ad)^6g^4i^3(c+dx)} + \frac{Bd^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} - \frac{20b^3Bd^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4Bd(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{2b^5B(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)}{9(bc-ad)^6g^4i^3(a+bx)^3} - \frac{d^5(a+bx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^6g^4i^3(c+dx)} - \frac{10b^3d^2(c+dx)\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4d(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{b^5(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10b^2d^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^3}{3B(bc-ad)^6g^4i^3}$$

output
```
-1/4*B^2*d^5*(b*x+a)^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-10*A*b*B*d^4*(b*x+a)
/(-a*d+b*c)^6/g^4/i^3/(d*x+c)+10*b*B^2*d^4*(b*x+a)/(-a*d+b*c)^6/g^4/i^3/(d
*x+c)-20*b^3*B^2*d^2*(d*x+c)/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/4*b^4*B^2*d*(d
*x+c)^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-2/27*b^5*B^2*(d*x+c)^3/(-a*d+b*c)^6
/g^4/i^3/(b*x+a)^3-10*b*B^2*d^4*(b*x+a)*ln(e*(b*x+a)/(d*x+c))/(-a*d+b*c)^6
/g^4/i^3/(d*x+c)+1/2*B*d^5*(b*x+a)^2*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a*d+b*c
)^6/g^4/i^3/(d*x+c)^2-20*b^3*B*d^2*(d*x+c)*(A+B*ln(e*(b*x+a)/(d*x+c)))/(-a
*d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*B*d*(d*x+c)^2*(A+B*ln(e*(b*x+a)/(d*x+c))
)/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-2/9*b^5*B*(d*x+c)^3*(A+B*ln(e*(b*x+a)/(d*
x+c)))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-1/2*d^5*(b*x+a)^2*(A+B*ln(e*(b*x+a)/
(d*x+c)))^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2+5*b*d^4*(b*x+a)*(A+B*ln(e*(b*x+
a)/(d*x+c)))^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)-10*b^3*d^2*(d*x+c)*(A+B*ln(e*(
b*x+a)/(d*x+c)))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*d*(d*x+c)^2*(A+B*l
n(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/3*b^5*(d*x+c)^3*(
A+B*ln(e*(b*x+a)/(d*x+c)))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-10/3*b^2*d^3*(
A+B*ln(e*(b*x+a)/(d*x+c)))^3/B/(-a*d+b*c)^6/g^4/i^3
```

### 3.107.2 Mathematica [A] (verified)

Time = 1.28 (sec) , antiderivative size = 793, normalized size of antiderivative = 0.93

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx =$$

$$-\frac{27(2A^2-2AB+B^2)\,d^3(bc-ad)^2(a+bx)^3+54b(8A^2-18AB+19B^2)\,d^3(bc-ad)(a+bx)^3(c+dx)}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d
*i*x)^3),x]
```

output
```
-1/108*(27*(2*A^2 - 2*A*B + B^2)*d^3*(b*c - a*d)^2*(a + b*x)^3 + 54*b*(8*A
^2 - 18*A*B + 19*B^2)*d^3*(b*c - a*d)*(a + b*x)^3*(c + d*x) + 4*b^2*(9*A^2
 + 6*A*B + 2*B^2)*(b*c - a*d)^3*(c + d*x)^2 - 3*b^2*(54*A^2 + 66*A*B + 37*
B^2)*d*(b*c - a*d)^2*(a + b*x)*(c + d*x)^2 + 6*b^2*(108*A^2 + 282*A*B + 31
9*B^2)*d^2*(b*c - a*d)*(a + b*x)^2*(c + d*x)^2 + 60*b^2*(18*A^2 + 12*A*B +
 49*B^2)*d^3*(a + b*x)^3*(c + d*x)^2*Log[a + b*x] + 6*B*(b*c - a*d)*(9*(2*
A - B)*d^3*(b*c - a*d)*(a + b*x)^3 + 18*b*(8*A - 9*B)*d^3*(a + b*x)^3*(c +
 d*x) + 4*b^2*(3*A + B)*(b*c - a*d)^2*(c + d*x)^2 - 3*b^2*(18*A + 11*B)*d*
(b*c - a*d)*(a + b*x)*(c + d*x)^2 + 6*b^2*(36*A + 47*B)*d^2*(a + b*x)^2*(c
 + d*x)^2)*Log[(e*(a + b*x))/(c + d*x)] + 18*B*(3*a^5*B*d^5 - 15*a^4*b*B*d
^4*(2*c + d*x) + 30*a^3*b^2*d^3*(2*A*(c + d*x)^2 - B*d*x*(4*c + 3*d*x)) +
30*a^2*b^3*d^2*(6*A*d*x*(c + d*x)^2 + B*(2*c^3 + 6*c^2*d*x - 3*d^3*x^3)) +
 15*a*b^4*d*(12*A*d^2*x^2*(c + d*x)^2 + B*c*(-c^3 + 4*c^2*d*x + 18*c*d^2*x
^2 + 12*d^3*x^3)) + b^5*(60*A*d^3*x^3*(c + d*x)^2 + B*(2*c^5 - 5*c^4*d*x +
 20*c^3*d^2*x^2 + 110*c^2*d^3*x^3 + 100*c*d^4*x^4 + 20*d^5*x^5)))*Log[(e*(
a + b*x))/(c + d*x)]^2 + 360*b^2*B^2*d^3*(a + b*x)^3*(c + d*x)^2*Log[(e*(a
 + b*x))/(c + d*x)]^3 - 60*b^2*(18*A^2 + 12*A*B + 49*B^2)*d^3*(a + b*x)^3*
(c + d*x)^2*Log[c + d*x])/((b*c - a*d)^6*g^4*i^3*(a + b*x)^3*(c + d*x)^2)
```

### 3.107.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 596, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {2962, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{(ag+bgx)^4(ci+dix)^3}dx$$

$$\Big\downarrow \text{2962}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i^3(bc-ad)^6}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{(c+dx)^4\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^5}{(a+bx)^4}-\frac{5d(c+dx)^3\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^4}{(a+bx)^3}+\frac{10d^2(c+dx)^2\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2b^3}{(a+bx)^2}-\frac{10d^3(c+dx)\left(A+B\right.}{a\ldots}\right.}{g^4i^3(bc-ad)^6}$$

$$\downarrow \text{2009}$$

$$-\frac{b^5(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{3(a+bx)^3}-\frac{2b^5B(c+dx)^3\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)}{9(a+bx)^3}+\frac{5b^4d(c+dx)^2\left(B\log\left(\frac{e(a+bx)}{c+dx}\right)+A\right)^2}{2(a+bx)^2}+\frac{5b^4Bd(c+dx)^2\left(B\log\left(\right.\right.}{2(a+bx)^2}$$

input
```
Int[(A + B*Log[(e*(a + b*x))/(c + d*x)])^2/((a*g + b*g*x)^4*(c*i + d*i*x)^
3),x]
```

output
```
(-1/4*(B^2*d^5*(a + b*x)^2)/(c + d*x)^2 - (10*A*b*B*d^4*(a + b*x))/(c + d*
x) + (10*b*B^2*d^4*(a + b*x))/(c + d*x) - (20*b^3*B^2*d^2*(c + d*x))/(a +
b*x) + (5*b^4*B^2*d*(c + d*x)^2)/(4*(a + b*x)^2) - (2*b^5*B^2*(c + d*x)^3)
/(27*(a + b*x)^3) - (10*b*B^2*d^4*(a + b*x)*Log[(e*(a + b*x))/(c + d*x)])/
(c + d*x) + (B*d^5*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/(2*(c
 + d*x)^2) - (20*b^3*B*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x)]))
/(a + b*x) + (5*b^4*B*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)]))/
(2*(a + b*x)^2) - (2*b^5*B*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x)]
))/(9*(a + b*x)^3) - (d^5*(a + b*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)])
^2)/(2*(c + d*x)^2) + (5*b*d^4*(a + b*x)*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2)/(c + d*x) - (10*b^3*d^2*(c + d*x)*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2)/(a + b*x) + (5*b^4*d*(c + d*x)^2*(A + B*Log[(e*(a + b*x))/(c + d*x)
])^2)/(2*(a + b*x)^2) - (b^5*(c + d*x)^3*(A + B*Log[(e*(a + b*x))/(c + d*x
)])^2)/(3*(a + b*x)^3) - (10*b^2*d^3*(A + B*Log[(e*(a + b*x))/(c + d*x)])^
3)/(3*B))/((b*c - a*d)^6*g^4*i^3)
```

### 3.107.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2962
```
Int[((A_.) + Log[(e_.)*((a_.) + (b_.)*(x_))^(n_.)*((c_.) + (d_.)*(x_))^(mn_
)]*(B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Sy
mbol] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A +
 B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /;
 FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && EqQ[n + mn, 0] && IGt
Q[n, 0] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && I
ntegersQ[m, q]
```

### 3.107.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1707 vs. $2(831) = 1662$.

Time = 7.75 (sec) , antiderivative size = 1708, normalized size of antiderivative = 2.01

| method | result | size |
|---|---|---|
| parts | Expression too large to display | 1708 |
| derivativedivides | Expression too large to display | 1951 |
| default | Expression too large to display | 1951 |
| risch | Expression too large to display | 3021 |
| parallelrisch | Expression too large to display | 3969 |
| norman | Expression too large to display | 4027 |

input
```
int((A+B*ln(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,method=_RE
TURNVERBOSE)
```

output
```
A^2/g^4/i^3*(-1/2*d^3/(a*d-b*c)^4/(d*x+c)^2+10*d^3/(a*d-b*c)^6*b^2*ln(d*x+
c)+4*d^3/(a*d-b*c)^5*b/(d*x+c)+1/3*b^2/(a*d-b*c)^3/(b*x+a)^3-10*d^3/(a*d-b
*c)^6*b^2*ln(b*x+a)+6*b^2/(a*d-b*c)^5*d^2/(b*x+a)+3/2*b^2/(a*d-b*c)^4*d/(b
*x+a)^2)-B^2/g^4/i^3*d/(a*d-b*c)^2/e^2*(d^4/(a*d-b*c)^4*(1/2*(b*e/d+(a*d-b
*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-1/2*(b*e/d+(a*d-b*c)*
e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))+1/4*(b*e/d+(a*d-b*c)*e/d/(d
*x+c))^2)-5*d^3/(a*d-b*c)^4*b*e*((b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))^2-2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*
e/d/(d*x+c))+2*(a*d-b*c)*e/d/(d*x+c)+2*b*e/d)+10/3*d^2/(a*d-b*c)^4*b^2*e^2
*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3-10*d/(a*d-b*c)^4*b^3*e^3*(-1/(b*e/d+(a*
d-b*c)*e/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2-2/(b*e/d+(a*d-b*c)*e
/d/(d*x+c))*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/(b*e/d+(a*d-b*c)*e/d/(d*x+c)
))+5/(a*d-b*c)^4*b^4*e^4*(-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a
*d-b*c)*e/d/(d*x+c))^2-1/2/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b
*c)*e/d/(d*x+c))-1/4/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-1/d/(a*d-b*c)^4*b^5*
e^5*(-1/3/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))^
2-2/9/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3*ln(b*e/d+(a*d-b*c)*e/d/(d*x+c))-2/27
/(b*e/d+(a*d-b*c)*e/d/(d*x+c))^3))-2*B*A/g^4/i^3*d/(a*d-b*c)^2/e^2*(d^4/(a
*d-b*c)^4*(1/2*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2*ln(b*e/d+(a*d-b*c)*e/d/(d*x
+c))-1/4*(b*e/d+(a*d-b*c)*e/d/(d*x+c))^2)-5*d^3/(a*d-b*c)^4*b*e*((b*e/d...
```

### 3.107.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2257 vs. $2(831) = 1662$.

Time = 0.44 (sec) , antiderivative size = 2257, normalized size of antiderivative = 2.65

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output
```
-1/108*(4*(9*A^2 + 6*A*B + 2*B^2)*b^5*c^5 - 135*(2*A^2 + 2*A*B + B^2)*a*b^
4*c^4*d + 1080*(A^2 + 2*A*B + 2*B^2)*a^2*b^3*c^3*d^2 - 20*(18*A^2 + 147*A*
B + 49*B^2)*a^3*b^2*c^2*d^3 - 540*(A^2 - 2*A*B + 2*B^2)*a^4*b*c*d^4 + 27*(
2*A^2 - 2*A*B + B^2)*a^5*d^5 + 60*((18*A^2 + 12*A*B + 49*B^2)*b^5*c*d^4 -
(18*A^2 + 12*A*B + 49*B^2)*a*b^4*d^5)*x^4 + 30*(3*(18*A^2 + 24*A*B + 53*B^
2)*b^5*c^2*d^3 + 2*(18*A^2 - 24*A*B + 37*B^2)*a*b^4*c*d^4 - (90*A^2 + 24*A
*B + 233*B^2)*a^2*b^3*d^5)*x^3 + 360*(B^2*b^5*d^5*x^5 + B^2*a^3*b^2*c^2*d^
3 + (2*B^2*b^5*c*d^4 + 3*B^2*a*b^4*d^5)*x^4 + (B^2*b^5*c^2*d^3 + 6*B^2*a*b
^4*c*d^4 + 3*B^2*a^2*b^3*d^5)*x^3 + (3*B^2*a*b^4*c^2*d^3 + 6*B^2*a^2*b^3*c
*d^4 + B^2*a^3*b^2*d^5)*x^2 + (3*B^2*a^2*b^3*c^2*d^3 + 2*B^2*a^3*b^2*c*d^4
)*x)*log((b*e*x + a*e)/(d*x + c))^3 + 10*(2*(18*A^2 + 66*A*B + 85*B^2)*b^5
*c^3*d^2 + 3*(126*A^2 + 84*A*B + 307*B^2)*a*b^4*c^2*d^3 - 12*(18*A^2 + 39*
A*B + 49*B^2)*a^2*b^3*c*d^4 - (198*A^2 - 84*A*B + 503*B^2)*a^3*b^2*d^5)*x^
2 + 18*(20*(3*A*B + B^2)*b^5*d^5*x^5 + 2*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*d
+ 60*B^2*a^2*b^3*c^3*d^2 + 60*A*B*a^3*b^2*c^2*d^3 - 30*B^2*a^4*b*c*d^4 + 3
*B^2*a^5*d^5 + 20*(9*A*B*a*b^4*d^5 + (6*A*B + 5*B^2)*b^5*c*d^4)*x^4 + 10*(
(6*A*B + 11*B^2)*b^5*c^2*d^3 + 18*(2*A*B + B^2)*a*b^4*c*d^4 + 9*(2*A*B - B
^2)*a^2*b^3*d^5)*x^3 + 10*(2*B^2*b^5*c^3*d^2 + 36*A*B*a^2*b^3*c*d^4 + 9*(2
*A*B + 3*B^2)*a*b^4*c^2*d^3 + 3*(2*A*B - 3*B^2)*a^3*b^2*d^5)*x^2 - 5*(B^2*
b^5*c^4*d - 12*B^2*a*b^4*c^3*d^2 + 3*B^2*a^4*b*d^5 - 36*(A*B + B^2)*a^2...
```

### 3.107.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)/(d*x+c)))**2/(b*g*x+a*g)**4/(d*i*x+c*i)**3,x)
```

output
```
Timed out
```

### 3.107.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9282 vs. 2(831) = 1662.

Time = 1.08 (sec) , antiderivative size = 9282, normalized size of antiderivative = 10.91

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output
```
-1/6*B^2*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^
2 + 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(
2*b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a
*b^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*
c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*
b^3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3
+ 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d
^7)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5
*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*
b*d^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25
*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 -
 a^8*d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5
*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*
d^6)*g^4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*
a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log
(b*x + a)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*
d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*
log(d*x + c)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c
^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(b*e*x
/(d*x + c) + a*e/(d*x + c))^2 - 1/3*A*B*((60*b^4*d^4*x^4 + 2*b^4*c^4 - ...
```

### 3.107.8 Giac [F]

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)e}{dx+c}\right)+A\right)^2}{(bgx+ag)^4(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)/(d*x+c)))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output
```
integrate((B*log((b*x + a)*e/(d*x + c)) + A)^2/((b*g*x + a*g)^4*(d*i*x + c
*i)^3), x)
```

### 3.107.9 Mupad [B] (verification not implemented)

Time = 15.42 (sec) , antiderivative size = 3550, normalized size of antiderivative = 4.17

$$\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx=\text{Too large to display}$$

input
```
int((A + B*log((e*(a + b*x))/(c + d*x)))^2/((a*g + b*g*x)^4*(c*i + d*i*x)^
3),x)
```

output
```
((36*A^2*b^4*c^4 - 54*A^2*a^4*d^4 - 27*B^2*a^4*d^4 + 8*B^2*b^4*c^4 + 54*A*
B*a^4*d^4 + 24*A*B*b^4*c^4 + 846*A^2*a^2*b^2*c^2*d^2 + 2033*B^2*a^2*b^2*c^
2*d^2 - 234*A^2*a*b^3*c^3*d + 486*A^2*a^3*b*c*d^3 - 127*B^2*a*b^3*c^3*d +
1053*B^2*a^3*b*c*d^3 - 246*A*B*a*b^3*c^3*d - 1026*A*B*a^3*b*c*d^3 + 1914*A
*B*a^2*b^2*c^2*d^2)/(6*(a*d - b*c)) + (10*x^4*(18*A^2*b^4*d^4 + 49*B^2*b^4
*d^4 + 12*A*B*b^4*d^4))/(a*d - b*c) + (5*x*(54*A^2*a^3*b*d^4 + 189*B^2*a^3
*b*d^4 - 18*A^2*b^4*c^3*d - 19*B^2*b^4*c^3*d + 198*A^2*a*b^3*c^2*d^2 + 630
*A^2*a^2*b^2*c*d^3 + 737*B^2*a*b^3*c^2*d^2 + 1445*B^2*a^2*b^2*c*d^3 - 162*
A*B*a^3*b*d^4 - 30*A*B*b^4*c^3*d + 618*A*B*a*b^3*c^2*d^2 + 150*A*B*a^2*b^2
*c*d^3))/(6*(a*d - b*c)) + (5*x^2*(198*A^2*a^2*b^2*d^4 + 503*B^2*a^2*b^2*d
^4 + 36*A^2*b^4*c^2*d^2 + 170*B^2*b^4*c^2*d^2 - 84*A*B*a^2*b^2*d^4 + 132*A
*B*b^4*c^2*d^2 + 414*A^2*a*b^3*c*d^3 + 1091*B^2*a*b^3*c*d^3 + 384*A*B*a*b^
3*c*d^3))/(3*(a*d - b*c)) + (5*x^3*(90*A^2*a*b^3*d^4 + 233*B^2*a*b^3*d^4 +
 54*A^2*b^4*c*d^3 + 159*B^2*b^4*c*d^3 + 24*A*B*a*b^3*d^4 + 72*A*B*b^4*c*d^
3))/(a*d - b*c))/(x^5*(18*a^4*b^3*d^6*g^4*i^3 + 18*b^7*c^4*d^2*g^4*i^3 - 7
2*a*b^6*c^3*d^3*g^4*i^3 - 72*a^3*b^4*c*d^5*g^4*i^3 + 108*a^2*b^5*c^2*d^4*g
^4*i^3) + x*(54*a^2*b^5*c^6*g^4*i^3 + 36*a^7*c*d^5*g^4*i^3 - 180*a^3*b^4*c
^5*d*g^4*i^3 - 90*a^6*b*c^2*d^4*g^4*i^3 + 180*a^4*b^3*c^4*d^2*g^4*i^3) + x
^2*(18*a^7*d^6*g^4*i^3 + 54*a*b^6*c^6*g^4*i^3 + 36*a^6*b*c*d^5*g^4*i^3 - 1
08*a^2*b^5*c^5*d*g^4*i^3 - 90*a^3*b^4*c^4*d^2*g^4*i^3 + 360*a^4*b^3*c^3...
```

3.107. $\int \frac{\left(A+B\log\left(\frac{e(a+bx)}{c+dx}\right)\right)^2}{(agi+bgx)^4(ci+dix)^3}\,dx$

# 3.108 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$



## 3.108.1 Optimal result

Integrand size = 41, antiderivative size = 223

$$\begin{aligned}&\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\&=-\frac{B(bc-ad)^4g^3inx}{20bd^3}+\frac{B(bc-ad)^3g^3in(a+bx)^2}{40b^2d^2}\\&\quad-\frac{B(bc-ad)^2g^3in(a+bx)^3}{60b^2d}+\frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5b}\\&\quad+\frac{(bc-ad)g^3i(a+bx)^4\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2}+\frac{B(bc-ad)^5g^3in\log(c+dx)}{20b^2d^4}\end{aligned}$$

output
```
-1/20*B*(-a*d+b*c)^4*g^3*i*n*x/b/d^3+1/40*B*(-a*d+b*c)^3*g^3*i*n*(b*x+a)^2
/b^2/d^2-1/60*B*(-a*d+b*c)^2*g^3*i*n*(b*x+a)^3/b^2/d+1/5*g^3*i*(b*x+a)^4*(
d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b+1/20*(-a*d+b*c)*g^3*i*(b*x+a)^4*(
A-B*n+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/20*B*(-a*d+b*c)^5*g^3*i*n*ln(d*x+
c)/b^2/d^4
```

### 3.108.2 Mathematica [A] (verified)

Time = 0.16 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.21

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{g^3i\Big(30(bc-ad)(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+24d(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-\frac{5B(bc-ad)^2n(6b}{}}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]),x]
```

output
```
(g^3*i*(30*(b*c - a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
+ 24*d*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (5*B*(b*c - a*
d)^2*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(
a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/d^4 + (2*B*(b*c - a*d)*n*(12*b
*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c - a*d)*(
a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]))/d^4))/(12
0*b^2)
```

### 3.108.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 197, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {2959, 27, 2947, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$
$$\Big\downarrow \text{ 2959}$$
$$\frac{i(bc-ad)\int g^3(a+bx)^3\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{5b}+\frac{g^3i(a+bx)^4(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}$$
$$\Big\downarrow \text{ 27}$$

$$\frac{g^3 i(bc-ad)\int (a+bx)^3 \left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}$$

$\big\downarrow$ 2947

$$\frac{g^3 i(bc-ad)\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{4b}-\frac{Bn(bc-ad)\int \frac{(a+bx)^3}{c+dx}dx}{4b}\right)}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}$$

$\big\downarrow$ 49

$$\frac{g^3 i(bc-ad)\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{4b}-\frac{Bn(bc-ad)\int \left(\frac{(ad-bc)^3}{d^3(c+dx)}+\frac{b(bc-ad)^2}{d^3}+\frac{b(a+bx)^2}{d}-\frac{b(bc-ad)(a+bx)}{d^2}\right)dx}{4b}\right)}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}$$

$\big\downarrow$ 2009

$$\frac{g^3 i(bc-ad)\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{4b}-\frac{Bn(bc-ad)\left(-\frac{(bc-ad)^3\log(c+dx)}{d^4}+\frac{bx(bc-ad)^2}{d^3}-\frac{(a+bx)^2(bc-ad)}{2d^2}+\frac{(a+bx)^3}{3d}\right)}{4b}\right)}{5b}+\frac{g^3 i(a+bx)^4(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b}$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x
]
```

output
```
(g^3*i*(a + b*x)^4*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b)
 + ((b*c - a*d)*g^3*i*(((a + b*x)^4*(A - B*n + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(4*b) - (B*(b*c - a*d)*n*((b*(b*c - a*d)^2*x)/d^3 - ((b*c - a*d)*(
a + b*x)^2)/(2*d^2) + (a + b*x)^3/(3*d) - ((b*c - a*d)^3*Log[c + d*x])/d^4
))/(4*b)))/(5*b)
```

3.108. $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$

### 3.108.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

rule 2959
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Simp[(f
 + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 2
))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B*n + B
*Log[e*((a + b*x)/(c + d*x))^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
i, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c
*i, 0] && IGtQ[m, -2]
```

## 3.108.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1194 vs. $2(211) = 422$.

Time = 11.44 (sec) , antiderivative size = 1195, normalized size of antiderivative = 5.36

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1195 |

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RE
TURNVERBOSE)
```

output
```
1/120*(22*B*x^3*a^2*b^3*d^5*g^3*i*n^2-2*B*x^3*b^5*c^2*d^3*g^3*i*n^2+120*A*
x^3*a^2*b^3*d^5*g^3*i*n+27*B*x^2*a^3*b^2*d^5*g^3*i*n^2+3*B*x^2*b^5*c^3*d^2
*g^3*i*n^2+60*A*x^2*a^3*b^2*d^5*g^3*i*n+6*B*x*a^4*b*d^5*g^3*i*n^2-6*B*x*b^
5*c^4*d*g^3*i*n^2+24*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g^3*i*n+6*B*x
^4*a*b^4*d^5*g^3*i*n^2-6*B*x^4*b^5*c*d^4*g^3*i*n^2+90*A*x^4*a*b^4*d^5*g^3*
i*n+30*A*x^4*b^5*c*d^4*g^3*i*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c
*d^4*g^3*i*n+180*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c*d^4*g^3*i*n+120
*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c*d^4*g^3*i*n-6*B*ln(b*x+a)*a^5*d^5
*g^3*i*n^2+24*A*x^5*b^5*d^5*g^3*i*n-6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^5*
g^3*i*n+6*B*ln(b*x+a)*b^5*c^5*g^3*i*n^2+30*B*x*a^3*b^2*c*d^4*g^3*i*n^2-60*
B*x*a^2*b^3*c^2*d^3*g^3*i*n^2+30*B*x*a*b^4*c^3*d^2*g^3*i*n^2+120*A*x*a^3*b
^2*c*d^4*g^3*i*n+60*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^2*d^3*g^3*i*n-60
*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^3*d^2*g^3*i*n+30*B*ln(e*((b*x+a)/(d
*x+c))^n)*a*b^4*c^4*d*g^3*i*n+30*B*ln(b*x+a)*a^4*b*c*d^4*g^3*i*n^2-60*B*ln
(b*x+a)*a^3*b^2*c^2*d^3*g^3*i*n^2+60*B*ln(b*x+a)*a^2*b^3*c^3*d^2*g^3*i*n^2
+90*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*d^5*g^3*i*n+30*B*x^4*ln(e*((b*x+
a)/(d*x+c))^n)*b^5*c*d^4*g^3*i*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b
^3*d^5*g^3*i*n-20*B*x^3*a*b^4*c*d^4*g^3*i*n^2-30*B*ln(b*x+a)*a*b^4*c^4*d*g
^3*i*n^2+120*A*x^3*a*b^4*c*d^4*g^3*i*n+60*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*
a^3*b^2*d^5*g^3*i*n-15*B*x^2*a^2*b^3*c*d^4*g^3*i*n^2-15*B*x^2*a*b^4*c^2...
```

### 3.108.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 720 vs. $2(213) = 426$.

Time = 0.48 (sec) , antiderivative size = 720, normalized size of antiderivative = 3.23

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{24\,Ab^5d^5g^3ix^5+6\,(5\,Ba^4bcd^4-Ba^5d^5)g^3in\log(bx+a)+6\,(Bb^5c^5-5\,Bab^4c^4d+10\,Ba^2b^3c^3d^2-10\,Ba}{}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="fricas")
```

output
```
1/120*(24*A*b^5*d^5*g^3*i*x^5 + 6*(5*B*a^4*b*c*d^4 - B*a^5*d^5)*g^3*i*n*lo
g(b*x + a) + 6*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - 10*B*
a^3*b^2*c^2*d^3)*g^3*i*n*log(d*x + c) - 6*((B*b^5*c*d^4 - B*a*b^4*d^5)*g^3
*i*n - 5*(A*b^5*c*d^4 + 3*A*a*b^4*d^5)*g^3*i)*x^4 - 2*((B*b^5*c^2*d^3 + 10
*B*a*b^4*c*d^4 - 11*B*a^2*b^3*d^5)*g^3*i*n - 60*(A*a*b^4*c*d^4 + A*a^2*b^3
*d^5)*g^3*i)*x^3 + 3*((B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 - 5*B*a^2*b^3*c*d
^4 + 9*B*a^3*b^2*d^5)*g^3*i*n + 20*(3*A*a^2*b^3*c*d^4 + A*a^3*b^2*d^5)*g^3
*i)*x^2 + 6*(20*A*a^3*b^2*c*d^4*g^3*i - (B*b^5*c^4*d - 5*B*a*b^4*c^3*d^2 +
 10*B*a^2*b^3*c^2*d^3 - 5*B*a^3*b^2*c*d^4 - B*a^4*b*d^5)*g^3*i*n)*x + 6*(4
*B*b^5*d^5*g^3*i*x^5 + 20*B*a^3*b^2*c*d^4*g^3*i*x + 5*(B*b^5*c*d^4 + 3*B*a
*b^4*d^5)*g^3*i*x^4 + 20*(B*a*b^4*c*d^4 + B*a^2*b^3*d^5)*g^3*i*x^3 + 10*(3
*B*a^2*b^3*c*d^4 + B*a^3*b^2*d^5)*g^3*i*x^2)*log(e) + 6*(4*B*b^5*d^5*g^3*i
*n*x^5 + 20*B*a^3*b^2*c*d^4*g^3*i*n*x + 5*(B*b^5*c*d^4 + 3*B*a*b^4*d^5)*g^
3*i*n*x^4 + 20*(B*a*b^4*c*d^4 + B*a^2*b^3*d^5)*g^3*i*n*x^3 + 10*(3*B*a^2*b
^3*c*d^4 + B*a^3*b^2*d^5)*g^3*i*n*x^2)*log((b*x + a)/(d*x + c)))/(b^2*d^4)
```

### 3.108.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

### 3.108.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. 2(213) = 426.

Time = 0.22 (sec) , antiderivative size = 1118, normalized size of antiderivative = 5.01

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="maxima")
```

output
```
1/5*B*b^3*d*g^3*i*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*b^3*d
*g^3*i*x^5 + 1/4*B*b^3*c*g^3*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 3/4*B*a*b^2*d*g^3*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b
^3*c*g^3*i*x^4 + 3/4*A*a*b^2*d*g^3*i*x^4 + B*a*b^2*c*g^3*i*x^3*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n) + B*a^2*b*d*g^3*i*x^3*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) + A*a*b^2*c*g^3*i*x^3 + A*a^2*b*d*g^3*i*x^3 + 3/2*B*a^2*b*c
*g^3*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*B*a^3*d*g^3*i*x^2*
log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*a^2*b*c*g^3*i*x^2 + 1/2*A*a
^3*d*g^3*i*x^2 + 1/60*B*b^3*d*g^3*i*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*lo
g(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2
*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4
*d^4)) - 1/24*B*b^3*c*g^3*i*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)
/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*
(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/8*B*a*b^2*d*g^3*i*n*(6*a^4*log(b*x +
 a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3
*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*a*b^
2*c*g^3*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d -
 a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/2*B*a^2*b*d*g^3*i*
n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*
x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3/2*B*a^2*b*c*g^3*i*n*(a^2*...
```

### 3.108.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3945 vs. $2(213) = 426$.

Time = 1.23 (sec) , antiderivative size = 3945, normalized size of antiderivative = 17.69

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="giac")
```

output
```
-1/120*(6*(B*b^9*c^6*g^3*i*n - 6*B*a*b^8*c^5*d*g^3*i*n - 5*(b*x + a)*B*b^8
*c^6*d*g^3*i*n/(d*x + c) + 15*B*a^2*b^7*c^4*d^2*g^3*i*n + 30*(b*x + a)*B*a
*b^7*c^5*d^2*g^3*i*n/(d*x + c) + 10*(b*x + a)^2*B*b^7*c^6*d^2*g^3*i*n/(d*x
 + c)^2 - 20*B*a^3*b^6*c^3*d^3*g^3*i*n - 75*(b*x + a)*B*a^2*b^6*c^4*d^3*g^
3*i*n/(d*x + c) - 60*(b*x + a)^2*B*a*b^6*c^5*d^3*g^3*i*n/(d*x + c)^2 - 10*
(b*x + a)^3*B*b^6*c^6*d^3*g^3*i*n/(d*x + c)^3 + 15*B*a^4*b^5*c^2*d^4*g^3*i
*n + 100*(b*x + a)*B*a^3*b^5*c^3*d^4*g^3*i*n/(d*x + c) + 150*(b*x + a)^2*B
*a^2*b^5*c^4*d^4*g^3*i*n/(d*x + c)^2 + 60*(b*x + a)^3*B*a*b^5*c^5*d^4*g^3*
i*n/(d*x + c)^3 - 6*B*a^5*b^4*c*d^5*g^3*i*n - 75*(b*x + a)*B*a^4*b^4*c^2*d
^5*g^3*i*n/(d*x + c) - 200*(b*x + a)^2*B*a^3*b^4*c^3*d^5*g^3*i*n/(d*x + c)
^2 - 150*(b*x + a)^3*B*a^2*b^4*c^4*d^5*g^3*i*n/(d*x + c)^3 + B*a^6*b^3*d^6
*g^3*i*n + 30*(b*x + a)*B*a^5*b^3*c*d^6*g^3*i*n/(d*x + c) + 150*(b*x + a)^
2*B*a^4*b^3*c^2*d^6*g^3*i*n/(d*x + c)^2 + 200*(b*x + a)^3*B*a^3*b^3*c^3*d^
6*g^3*i*n/(d*x + c)^3 - 5*(b*x + a)*B*a^6*b^2*d^7*g^3*i*n/(d*x + c) - 60*(
b*x + a)^2*B*a^5*b^2*c*d^7*g^3*i*n/(d*x + c)^2 - 150*(b*x + a)^3*B*a^4*b^2
*c^2*d^7*g^3*i*n/(d*x + c)^3 + 10*(b*x + a)^2*B*a^6*b*d^8*g^3*i*n/(d*x + c
)^2 + 60*(b*x + a)^3*B*a^5*b*c*d^8*g^3*i*n/(d*x + c)^3 - 10*(b*x + a)^3*B*
a^6*d^9*g^3*i*n/(d*x + c)^3)*log((b*x + a)/(d*x + c))/(b^5*d^4 - 5*(b*x +
a)*b^4*d^5/(d*x + c) + 10*(b*x + a)^2*b^3*d^6/(d*x + c)^2 - 10*(b*x + a)^3
*b^2*d^7/(d*x + c)^3 + 5*(b*x + a)^4*b*d^8/(d*x + c)^4 - (b*x + a)^5*d^...
```

### 3.108.9 Mupad [B] (verification not implemented)

Time = 2.41 (sec) , antiderivative size = 1237, normalized size of antiderivative = 5.55

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x
)
```

output
```
x*((a*c*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d*n - B
*b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d) - (b*g^3*i*(24*
A*a^2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d - 2*B
*a*b*c*d*n))/(4*d) + A*a*b^2*c*g^3*i))/(b*d) - ((20*a*d + 20*b*c)*(((20*a*
d + 20*b*c)*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d*n
 - B*b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*d) - (b*g^3*i*
(24*A*a^2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d -
 2*B*a*b*c*d*n))/(4*d) + A*a*b^2*c*g^3*i))/(20*b*d) - (a*c*((b^2*g^3*i*(20
*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c)
)/20))/(b*d) + (a*g^3*i*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c
^2*n + 12*A*a*b*c*d))/d))/(20*b*d) + (a^2*g^3*i*(2*A*a^2*d^2 + 12*A*b^2*c^
2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 16*A*a*b*c*d + 2*B*a*b*c*d*n))/(2*b*d))
+ x^2*(((20*a*d + 20*b*c)*(((20*a*d + 20*b*c)*((b^2*g^3*i*(20*A*a*d + 10*A
*b*c + B*a*d*n - B*b*c*n))/5 - (A*b^2*g^3*i*(20*a*d + 20*b*c))/20))/(20*b*
d) - (b*g^3*i*(24*A*a^2*d^2 + 4*A*b^2*c^2 + 3*B*a^2*d^2*n - B*b^2*c^2*n +
32*A*a*b*c*d - 2*B*a*b*c*d*n))/(4*d) + A*a*b^2*c*g^3*i))/(40*b*d) - (a*c*(
(b^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b^2*g^3*i*(20
*a*d + 20*b*c))/20))/(2*b*d) + (a*g^3*i*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2
*d^2*n - B*b^2*c^2*n + 12*A*a*b*c*d))/(2*d)) - x^3*(((20*a*d + 20*b*c)*((b
^2*g^3*i*(20*A*a*d + 10*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b^2*g^3*i*(2...
```

3.108. $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

# 3.109 $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$



## 3.109.1 Optimal result

Integrand size = 41, antiderivative size = 190

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$=\frac{B(bc-ad)^3g^2inx}{12bd^2}-\frac{B(bc-ad)^2g^2in(a+bx)^2}{24b^2d}$$
$$+\frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b}$$
$$+\frac{(bc-ad)g^2i(a+bx)^3\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2}-\frac{B(bc-ad)^4g^2in\log(c+dx)}{12b^2d^3}$$

output
```
1/12*B*(-a*d+b*c)^3*g^2*i*n*x/b/d^2-1/24*B*(-a*d+b*c)^2*g^2*i*n*(b*x+a)^2/
b^2/d+1/4*g^2*i*(b*x+a)^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b+1/12*(
-a*d+b*c)*g^2*i*(b*x+a)^3*(A-B*n+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2-1/12*B*(
-a*d+b*c)^4*g^2*i*n*ln(d*x+c)/b^2/d^3
```

## 3.109.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 225, normalized size of antiderivative = 1.18

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$$
$$=\frac{g^2i\Big(8(bc-ad)(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+6d(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{4B(bc-ad)^2n(2bd(b\cdots}{}}{24b^2\cdots}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]),x]
```

output
```
(g^2*i*(8*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
 6*d*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (4*B*(b*c - a*d)
^2*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]
))/d^3 - (B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a
 + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]))/d^3))/(24*b
^2)
```

### 3.109.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 173, normalized size of antiderivative = 0.91, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2959, 27, 2947, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2959}$$

$$\frac{i(bc-ad)\int g^2(a+bx)^2\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{4b}+\frac{g^2i(a+bx)^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}$$

$$\Big\downarrow \text{27}$$

$$\frac{g^2i(bc-ad)\int (a+bx)^2\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{4b}+\frac{g^2i(a+bx)^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}$$

$$\Big\downarrow \text{2947}$$

$$\frac{g^2i(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{3b}-\frac{Bn(bc-ad)\int\frac{(a+bx)^2}{c+dx}dx}{3b}\right)}{4b}+\frac{g^2i(a+bx)^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}$$

$$\Big\downarrow \; 49$$

$$\frac{g^2 i(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{3b}-\frac{Bn(bc-ad)\int\left(\frac{(ad-bc)^2}{d^2(c+dx)}-\frac{b(bc-ad)}{d^2}+\frac{b(a+bx)}{d}\right)dx}{3b}\right)}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}$$

$$\Big\downarrow \; 2009$$

$$\frac{g^2 i(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{3b}-\frac{Bn(bc-ad)\left(\frac{(bc-ad)^2\log(c+dx)}{d^3}-\frac{bx(bc-ad)}{d^2}+\frac{(a+bx)^2}{2d}\right)}{3b}\right)}{4b}+\frac{g^2 i(a+bx)^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b}$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x
]
```

output
```
(g^2*i*(a + b*x)^3*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b)
 + ((b*c - a*d)*g^2*i*(((a + b*x)^3*(A - B*n + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(3*b) - (B*(b*c - a*d)*n*(-((b*(b*c - a*d)*x)/d^2) + (a + b*x)^2/(
2*d) + ((b*c - a*d)^2*Log[c + d*x])/d^3))/(3*b)))/(4*b)
```

### 3.109.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

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rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

rule 2959
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Simp[(f
 + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 2
))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B*n + B
*Log[e*((a + b*x)/(c + d*x))^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
i, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c
*i, 0] && IGtQ[m, -2]
```

## 3.109.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(180) = 360$.

Time = 5.00 (sec) , antiderivative size = 876, normalized size of antiderivative = 4.61

| method | result |
|---|---|
| parallelrisch | $\frac{2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^4c^4g^2in+6A\,x^4b^4d^4g^2in-2B\ln(bx+a)b^4c^4g^2i\,n^2-2B\ln(bx+a)a^4d^4g^2i\,n^2+24B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a\,b^3c\,\cdots}{}$ |

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RE
TURNVERBOSE)
```

output
```
1/24*(2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^4*g^2*i*n+6*A*x^4*b^4*d^4*g^2*i*
n-2*B*ln(b*x+a)*b^4*c^4*g^2*i*n^2-2*B*ln(b*x+a)*a^4*d^4*g^2*i*n^2+24*B*x^2
*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c*d^3*g^2*i*n+24*B*x*ln(e*((b*x+a)/(d*x+c
))^n)*a^2*b^2*c*d^3*g^2*i*n+16*A*x^3*a*b^3*d^4*g^2*i*n+8*A*x^3*b^4*c*d^3*g
^2*i*n+5*B*x^2*a^2*b^2*d^4*g^2*i*n^2-B*x^2*b^4*c^2*d^2*g^2*i*n^2+12*A*x^2*
a^2*b^2*d^4*g^2*i*n+2*B*x*a^3*b*d^4*g^2*i*n^2+2*B*x*b^4*c^3*d*g^2*i*n^2+6*
B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*g^2*i*n-12*B*ln(b*x+a)*a^2*b^2*c^2
*d^2*g^2*i*n^2+8*B*ln(b*x+a)*a*b^3*c^3*d*g^2*i*n^2-8*B*x*a*b^3*c^2*d^2*g^2
*i*n^2+24*A*x*a^2*b^2*c*d^3*g^2*i*n+8*B*ln(b*x+a)*a^3*b*c*d^3*g^2*i*n^2+12
*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^2*d^2*g^2*i*n-8*B*ln(e*((b*x+a)/(d*
x+c))^n)*a*b^3*c^3*d*g^2*i*n+16*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*d^4*
g^2*i*n+8*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3*g^2*i*n+12*B*x^2*ln(e*
((b*x+a)/(d*x+c))^n)*a^2*b^2*d^4*g^2*i*n-4*B*x^2*a*b^3*c*d^3*g^2*i*n^2+24*
A*x^2*a*b^3*c*d^3*g^2*i*n+4*B*x*a^2*b^2*c*d^3*g^2*i*n^2-11*B*a^3*b*c*d^3*g
^2*i*n^2+8*B*a^2*b^2*c^2*d^2*g^2*i*n^2+7*B*a*b^3*c^3*d*g^2*i*n^2-36*A*a^3*
b*c*d^3*g^2*i*n-48*A*a^2*b^2*c^2*d^2*g^2*i*n+2*B*x^3*a*b^3*d^4*g^2*i*n^2-2
*B*x^3*b^4*c*d^3*g^2*i*n^2-2*B*b^4*c^4*g^2*i*n^2-2*B*a^4*d^4*g^2*i*n^2)/b^
2/d^3/n
```

### 3.109.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(182) = 364$.

Time = 0.43 (sec) , antiderivative size = 529, normalized size of antiderivative = 2.78

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{6\,Ab^4d^4g^2ix^4+2\,(4\,Ba^3bcd^3-Ba^4d^4)g^2in\log(bx+a)-2\,(Bb^4c^4-4\,Bab^3c^3d+6\,Ba^2b^2c^2d^2)g^2in\log(d}{}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="fricas")
```

output
```
1/24*(6*A*b^4*d^4*g^2*i*x^4 + 2*(4*B*a^3*b*c*d^3 - B*a^4*d^4)*g^2*i*n*log(
b*x + a) - 2*(B*b^4*c^4 - 4*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2)*g^2*i*n*l
og(d*x + c) - 2*((B*b^4*c*d^3 - B*a*b^3*d^4)*g^2*i*n - 4*(A*b^4*c*d^3 + 2*
A*a*b^3*d^4)*g^2*i)*x^3 - ((B*b^4*c^2*d^2 + 4*B*a*b^3*c*d^3 - 5*B*a^2*b^2*
d^4)*g^2*i*n - 12*(2*A*a*b^3*c*d^3 + A*a^2*b^2*d^4)*g^2*i)*x^2 + 2*(12*A*a
^2*b^2*c*d^3*g^2*i + (B*b^4*c^3*d - 4*B*a*b^3*c^2*d^2 + 2*B*a^2*b^2*c*d^3
+ B*a^3*b*d^4)*g^2*i*n)*x + 2*(3*B*b^4*d^4*g^2*i*x^4 + 12*B*a^2*b^2*c*d^3*
g^2*i*x + 4*(B*b^4*c*d^3 + 2*B*a*b^3*d^4)*g^2*i*x^3 + 6*(2*B*a*b^3*c*d^3 +
 B*a^2*b^2*d^4)*g^2*i*x^2)*log(e) + 2*(3*B*b^4*d^4*g^2*i*n*x^4 + 12*B*a^2*
b^2*c*d^3*g^2*i*n*x + 4*(B*b^4*c*d^3 + 2*B*a*b^3*d^4)*g^2*i*n*x^3 + 6*(2*B
*a*b^3*c*d^3 + B*a^2*b^2*d^4)*g^2*i*n*x^2)*log((b*x + a)/(d*x + c)))/(b^2*
d^3)
```

### 3.109.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1013 vs. $2(173) = 346$.

Time = 84.02 (sec) , antiderivative size = 1013, normalized size of antiderivative = 5.33

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= \begin{cases} a^2cg^2ix\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ a^2g^2\left(Acix+\frac{Adix^2}{2}+\frac{Bc^2i\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d}+\frac{Bcinx}{2}+Bcix\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{Bdinx^2}{4}+\frac{Bdix^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2}\right) \\ ci\left(Aa^2g^2x+Aabg^2x^2+\frac{Ab^2g^2x^3}{3}+\frac{Ba^3g^2\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{3b}-\frac{Ba^2g^2nx}{3}+Ba^2g^2x\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)-\frac{Babg^2nx^2}{3}\right. \\ Aa^2cg^2ix+\frac{Aa^2dg^2ix^2}{2}+Aabcg^2ix^2+\frac{2Aabdg^2ix^3}{3}+\frac{Ab^2cg^2ix^3}{3}+\frac{Ab^2dg^2ix^4}{4}-\frac{Ba^4dg^2in\log\left(\frac{c}{d}+x\right)}{12b^2}-\frac{Ba^4dg^2i\log\left(e\left(\frac{}{c}\right.\right.}{12b} \end{cases}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((a**2*c*g**2*i*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)),
(a**2*g**2*(A*c*i*x + A*d*i*x**2/2 + B*c**2*i*log(e*(a/(c + d*x))**n)/(2*d
) + B*c*i*n*x/2 + B*c*i*x*log(e*(a/(c + d*x))**n) + B*d*i*n*x**2/4 + B*d*i
*x**2*log(e*(a/(c + d*x))**n)/2), Eq(b, 0)), (c*i*(A*a**2*g**2*x + A*a*b*g
**2*x**2 + A*b**2*g**2*x**3/3 + B*a**3*g**2*log(e*(a/c + b*x/c)**n)/(3*b)
- B*a**2*g**2*n*x/3 + B*a**2*g**2*x*log(e*(a/c + b*x/c)**n) - B*a*b*g**2*n
*x**2/3 + B*a*b*g**2*x**2*log(e*(a/c + b*x/c)**n) - B*b**2*g**2*n*x**3/9 +
 B*b**2*g**2*x**3*log(e*(a/c + b*x/c)**n)/3), Eq(d, 0)), (A*a**2*c*g**2*i*
x + A*a**2*d*g**2*i*x**2/2 + A*a*b*c*g**2*i*x**2 + 2*A*a*b*d*g**2*i*x**3/3
 + A*b**2*c*g**2*i*x**3/3 + A*b**2*d*g**2*i*x**4/4 - B*a**4*d*g**2*i*n*log
(c/d + x)/(12*b**2) - B*a**4*d*g**2*i*log(e*(a/(c + d*x) + b*x/(c + d*x))*
*n)/(12*b**2) + B*a**3*c*g**2*i*n*log(c/d + x)/(3*b) + B*a**3*c*g**2*i*log
(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(3*b) + B*a**3*d*g**2*i*n*x/(12*b) -
B*a**2*c**2*g**2*i*n*log(c/d + x)/(2*d) + B*a**2*c*g**2*i*n*x/6 + B*a**2*c
*g**2*i*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + 5*B*a**2*d*g**2*i*n*x*
*2/24 + B*a**2*d*g**2*i*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/2 + B
*a*b*c**3*g**2*i*n*log(c/d + x)/(3*d**2) - B*a*b*c**2*g**2*i*n*x/(3*d) - B
*a*b*c*g**2*i*n*x**2/6 + B*a*b*c*g**2*i*x**2*log(e*(a/(c + d*x) + b*x/(c +
 d*x))**n) + B*a*b*d*g**2*i*n*x**3/12 + 2*B*a*b*d*g**2*i*x**3*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n)/3 - B*b**2*c**4*g**2*i*n*log(c/d + x)/(12*d...
```

## 3.109.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(182) = 364$.

Time = 0.21 (sec) , antiderivative size = 740, normalized size of antiderivative = 3.89

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx \\
&= \frac{1}{4}\,Bb^2dg^2ix^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{4}\,Ab^2dg^2ix^4 \\
&\quad+\frac{1}{3}\,Bb^2cg^2ix^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{2}{3}\,Babdg^2ix^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\
&\quad+\frac{1}{3}\,Ab^2cg^2ix^3+\frac{2}{3}\,Aabdg^2ix^3+Babcg^2ix^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\
&\quad+\frac{1}{2}\,Ba^2dg^2ix^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aabcg^2ix^2+\frac{1}{2}\,Aa^2dg^2ix^2 \\
&\quad-\frac{1}{24}\,Bb^2dg^2in\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c}{b^3d^3}\right. \\
&\quad+\frac{1}{6}\,Bb^2cg^2in\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right) \\
&\quad+\frac{1}{3}\,Babdg^2in\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right) \\
&\quad-Babcg^2in\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad-\frac{1}{2}\,Ba^2dg^2in\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad+Ba^2cg^2in\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) \\
&\quad+Ba^2cg^2ix\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aa^2cg^2ix
\end{aligned}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="maxima")
```

3.109. $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

output
```
1/4*B*b^2*d*g^2*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^2*d
*g^2*i*x^4 + 1/3*B*b^2*c*g^2*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 2/3*B*a*b*d*g^2*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b^2
*c*g^2*i*x^3 + 2/3*A*a*b*d*g^2*i*x^3 + B*a*b*c*g^2*i*x^2*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) + 1/2*B*a^2*d*g^2*i*x^2*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) + A*a*b*c*g^2*i*x^2 + 1/2*A*a^2*d*g^2*i*x^2 - 1/24*B*b^2*d*g^2*
i*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b
^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^
3*d^3)) + 1/6*B*b^2*c*g^2*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/
3*B*a*b*d*g^2*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*a*b*c*g^2*i*
n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) - 1/
2*B*a^2*d*g^2*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*
d)*x/(b*d)) + B*a^2*c*g^2*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a^
2*c*g^2*i*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a^2*c*g^2*i*x
```

### 3.109.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2571 vs. 2(182) = 364.

Time = 1.32 (sec) , antiderivative size = 2571, normalized size of antiderivative = 13.53

$$\int (ag + bgx)^2(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="giac")
```

output
```
1/24*(2*(B*b^7*c^5*g^2*i*n - 5*B*a*b^6*c^4*d*g^2*i*n - 4*(b*x + a)*B*b^6*c
^5*d*g^2*i*n/(d*x + c) + 10*B*a^2*b^5*c^3*d^2*g^2*i*n + 20*(b*x + a)*B*a*b
^5*c^4*d^2*g^2*i*n/(d*x + c) + 6*(b*x + a)^2*B*b^5*c^5*d^2*g^2*i*n/(d*x +
c)^2 - 10*B*a^3*b^4*c^2*d^3*g^2*i*n - 40*(b*x + a)*B*a^2*b^4*c^3*d^3*g^2*i
*n/(d*x + c) - 30*(b*x + a)^2*B*a*b^4*c^4*d^3*g^2*i*n/(d*x + c)^2 + 5*B*a^
4*b^3*c*d^4*g^2*i*n + 40*(b*x + a)*B*a^3*b^3*c^2*d^4*g^2*i*n/(d*x + c) + 6
0*(b*x + a)^2*B*a^2*b^3*c^3*d^4*g^2*i*n/(d*x + c)^2 - B*a^5*b^2*d^5*g^2*i*
n - 20*(b*x + a)*B*a^4*b^2*c*d^5*g^2*i*n/(d*x + c) - 60*(b*x + a)^2*B*a^3*
b^2*c^2*d^5*g^2*i*n/(d*x + c)^2 + 4*(b*x + a)*B*a^5*b*d^6*g^2*i*n/(d*x + c
) + 30*(b*x + a)^2*B*a^4*b*c*d^6*g^2*i*n/(d*x + c)^2 - 6*(b*x + a)^2*B*a^5
*d^7*g^2*i*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/(b^4*d^3 - 4*(b*x + a)*
b^3*d^4/(d*x + c) + 6*(b*x + a)^2*b^2*d^5/(d*x + c)^2 - 4*(b*x + a)^3*b*d^
6/(d*x + c)^3 + (b*x + a)^4*d^7/(d*x + c)^4) + (B*b^8*c^5*g^2*i*n - 5*B*a*
b^7*c^4*d*g^2*i*n - 2*(b*x + a)*B*b^7*c^5*d*g^2*i*n/(d*x + c) + 10*B*a^2*b
^6*c^3*d^2*g^2*i*n + 10*(b*x + a)*B*a*b^6*c^4*d^2*g^2*i*n/(d*x + c) - (b*x
 + a)^2*B*b^6*c^5*d^2*g^2*i*n/(d*x + c)^2 - 10*B*a^3*b^5*c^2*d^3*g^2*i*n -
 20*(b*x + a)*B*a^2*b^5*c^3*d^3*g^2*i*n/(d*x + c) + 5*(b*x + a)^2*B*a*b^5*
c^4*d^3*g^2*i*n/(d*x + c)^2 + 2*(b*x + a)^3*B*b^5*c^5*d^3*g^2*i*n/(d*x + c
)^3 + 5*B*a^4*b^4*c*d^4*g^2*i*n + 20*(b*x + a)*B*a^3*b^4*c^2*d^4*g^2*i*n/(
d*x + c) - 10*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^2*i*n/(d*x + c)^2 - 10*(b...
```

3.109. $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

## 3.109.9 Mupad [B] (verification not implemented)

Time = 1.81 (sec) , antiderivative size = 663, normalized size of antiderivative = 3.49

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,a^2\,c\,g^2\,i\,x+\frac{B\,a\,g^2\,i\,x^2\,(a\,d+2\,b\,c)}{2}+\frac{B\,b\,g^2\,i\,x^3\,(2\,a\,d+b\,c)}{3}+\frac{B\,b^2\,d\,g^2\,i\,x^4}{4}\right)$$
$$+x^3\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{12}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{36}\right)$$
$$+x\left(\frac{(12\,a\,d+12\,b\,c)\left(\frac{(12\,a\,d+12\,b\,c)\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)}{12\,b\,d}-\frac{g^2\,i\,(9\,A\,a^2\,d^2+3\,A\,b^2\,c^2+2\,B}{}\right.}{12\,b\,d}\right.$$
$$-\frac{a\,c\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)}{b\,d}$$
$$\left.+\frac{a\,g^2\,i\,(2\,A\,a^2\,d^2+6\,A\,b^2\,c^2+B\,a^2\,d^2\,n-2\,B\,b^2\,c^2\,n+12\,A\,a\,b\,c\,d+B\,a\,b\,c\,d\,n)}{2\,b\,d}\right)$$
$$-x^2\left(\frac{(12\,a\,d+12\,b\,c)\left(\frac{b\,g^2\,i\,(12\,A\,a\,d+8\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4}-\frac{A\,b\,g^2\,i\,(12\,a\,d+12\,b\,c)}{12}\right)}{24\,b\,d}\right.$$
$$-\frac{g^2\,i\,(9\,A\,a^2\,d^2+3\,A\,b^2\,c^2+2\,B\,a^2\,d^2\,n-B\,b^2\,c^2\,n+18\,A\,a\,b\,c\,d-B\,a\,b\,c\,d\,n)}{6\,d}$$
$$\left.+\frac{A\,a\,b\,c\,g^2\,i}{2}\right)-\frac{\ln(a+b\,x)\,\left(B\,a^4\,d\,g^2\,i\,n-4\,B\,a^3\,b\,c\,g^2\,i\,n\right)}{12\,b^2}$$
$$-\frac{\ln(c+d\,x)\,\left(6\,B\,i\,n\,a^2\,c^2\,d^2\,g^2-4\,B\,i\,n\,a\,b\,c^3\,d\,g^2+B\,i\,n\,b^2\,c^4\,g^2\right)}{12\,d^3}+\frac{A\,b^2\,d\,g^2\,i\,x^4}{4}$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x
)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*(B*a^2*c*g^2*i*x + (B*a*g^2*i*x^2*(a*d + 2*
b*c))/2 + (B*b*g^2*i*x^3*(2*a*d + b*c))/3 + (B*b^2*d*g^2*i*x^4)/4) + x^3*(
(b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d*n - B*b*c*n))/12 - (A*b*g^2*i*(12*a*d
 + 12*b*c))/36) + x*(((12*a*d + 12*b*c)*(((12*a*d + 12*b*c)*((b*g^2*i*(12*
A*a*d + 8*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*b*g^2*i*(12*a*d + 12*b*c))/12
))/(12*b*d) - (g^2*i*(9*A*a^2*d^2 + 3*A*b^2*c^2 + 2*B*a^2*d^2*n - B*b^2*c^
2*n + 18*A*a*b*c*d - B*a*b*c*d*n))/(3*d) + A*a*b*c*g^2*i))/(12*b*d) - (a*c
*((b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*b*g^2*i*(12*a*
d + 12*b*c))/12))/(b*d) + (a*g^2*i*(2*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*
n - 2*B*b^2*c^2*n + 12*A*a*b*c*d + B*a*b*c*d*n))/(2*b*d)) - x^2*(((12*a*d
+ 12*b*c)*((b*g^2*i*(12*A*a*d + 8*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*b*g^2
*i*(12*a*d + 12*b*c))/12))/(24*b*d) - (g^2*i*(9*A*a^2*d^2 + 3*A*b^2*c^2 +
2*B*a^2*d^2*n - B*b^2*c^2*n + 18*A*a*b*c*d - B*a*b*c*d*n))/(6*d) + (A*a*b*
c*g^2*i)/2) - (log(a + b*x)*(B*a^4*d*g^2*i*n - 4*B*a^3*b*c*g^2*i*n))/(12*b
^2) - (log(c + d*x)*(B*b^2*c^4*g^2*i*n + 6*B*a^2*c^2*d^2*g^2*i*n - 4*B*a*b
*c^3*d*g^2*i*n))/(12*d^3) + (A*b^2*d*g^2*i*x^4)/4
```

3.109. $\int (ag + bgx)^2 (ci + dix) \left(A + B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

# 3.110 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$



## 3.110.1 Optimal result

Integrand size = 39, antiderivative size = 149

$$\begin{aligned}&\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\&=-\frac{B(bc-ad)^2ginx}{6bd}+\frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b}\\&\quad+\frac{(bc-ad)gi(a+bx)^2\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2}+\frac{B(bc-ad)^3gin\log(c+dx)}{6b^2d^2}\end{aligned}$$

output
```
-1/6*B*(-a*d+b*c)^2*g*i*n*x/b/d+1/3*g*i*(b*x+a)^2*(d*x+c)*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))/b+1/6*(-a*d+b*c)*g*i*(b*x+a)^2*(A-B*n+B*ln(e*((b*x+a)/(d*x
+c))^n))/b^2+1/6*B*(-a*d+b*c)^3*g*i*n*ln(d*x+c)/b^2/d^2
```

## 3.110.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.27

$$\begin{aligned}&\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\&=\frac{gi\left(-a^2Bd^2(3bc+ad)n\log(a+bx)+b\left(dx(a^2Bd^2n-b^2Bcn(c+dx)+Ab^2dx(3c+2dx)+abd(6Ac+3A\right.\right.}{}\end{aligned}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]),x]
```

output
```
(g*i*(-(a^2*B*d^2*(3*b*c + a*d)*n*Log[a + b*x]) + b*(d*x*(a^2*B*d^2*n - b^
2*B*c*n*(c + d*x) + A*b^2*d*x*(3*c + 2*d*x) + a*b*d*(6*A*c + 3*A*d*x + B*d
*n*x)) + B*d^2*(6*a^2*c + 3*a*b*x*(2*c + d*x) + b^2*x^2*(3*c + 2*d*x))*Log
[e*((a + b*x)/(c + d*x))^n] + B*c*(b^2*c^2 - 3*a*b*c*d + 6*a^2*d^2)*n*Log[
c + d*x])))/(6*b^2*d^2)
```

### 3.110.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.97, number of steps used = 5, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.128, Rules used = {2959, 27, 2947, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2959}$$

$$\frac{i(bc-ad)\int g(a+bx)\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}$$

$$\Big\downarrow \text{27}$$

$$\frac{gi(bc-ad)\int (a+bx)\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}$$

$$\Big\downarrow \text{2947}$$

$$\frac{gi(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{2b}-\frac{Bn(bc-ad)\int \frac{a+bx}{c+dx}dx}{2b}\right)}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}$$

$$\Big\downarrow \text{49}$$

$$\frac{gi(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{2b}-\frac{Bn(bc-ad)\int\left(\frac{b}{d}+\frac{ad-bc}{d(c+dx)}\right)dx}{2b}\right)}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{gi(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{2b}-\frac{Bn(bc-ad)\left(\frac{bx}{d}-\frac{(bc-ad)\log(c+dx)}{d^2}\right)}{2b}\right)}{3b}+\frac{gi(a+bx)^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b}$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(g*i*(a + b*x)^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b) +
 ((b*c - a*d)*g*i*(((a + b*x)^2*(A - B*n + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(2*b) - (B*(b*c - a*d)*n*((b*x)/d - ((b*c - a*d)*Log[c + d*x])/d^2))/(
2*b)))/(3*b)
```

### 3.110.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

rule 2959
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Simp[(f
 + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 2
))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B*n + B
*Log[e*((a + b*x)/(c + d*x))^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
i, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c
*i, 0] && IGtQ[m, -2]
```

### 3.110.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 506 vs. $2(141) = 282$.

Time = 1.72 (sec) , antiderivative size = 507, normalized size of antiderivative = 3.40

| method | result |
|---|---|
| parallelrisch | $\frac{6Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a\,b^{2}c\,d^{2}gin+2B\,x^{3}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}d^{3}gin+3B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}c\,d^{2}gin+6Axa\,b^{2}c\,d^{2}gin+3B\,x^{2}\ln\left(e\right.}{}$ |

input
```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETU
RNVERBOSE)
```

output
```
1/6*(6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^2*g*i*n+2*B*x^3*ln(e*((b*x+
a)/(d*x+c))^n)*b^3*d^3*g*i*n+3*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^2*g
*i*n+6*A*x*a*b^2*c*d^2*g*i*n+3*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^3*g
*i*n+3*B*ln(b*x+a)*a^2*b*c*d^2*g*i*n^2-3*B*ln(b*x+a)*a*b^2*c^2*d*g*i*n^2+3
*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c^2*d*g*i*n-2*B*a^2*b*c*d^2*g*i*n^2+2*B
*a*b^2*c^2*d*g*i*n^2-9*A*a^2*b*c*d^2*g*i*n-9*A*a*b^2*c^2*d*g*i*n+B*b^3*c^3
*g*i*n^2-B*a^3*d^3*g*i*n^2+B*x^2*a*b^2*d^3*g*i*n^2-B*x^2*b^3*c*d^2*g*i*n^2
+3*A*x^2*a*b^2*d^3*g*i*n+3*A*x^2*b^3*c*d^2*g*i*n+B*x*a^2*b*d^3*g*i*n^2-B*x
*b^3*c^2*d*g*i*n^2-B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^3*g*i*n+2*A*x^3*b^3*d
^3*g*i*n-B*ln(b*x+a)*a^3*d^3*g*i*n^2+B*ln(b*x+a)*b^3*c^3*g*i*n^2)/n/b^2/d^
2
```

### 3.110.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(143) = 286$.

Time = 0.37 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.07

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{2\,Ab^3d^3gix^3+(3\,Ba^2bcd^2-Ba^3d^3)gin\log(bx+a)+(Bb^3c^3-3\,Bab^2c^2d)gin\log(dx+c)-((Bb^3cd^2-$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algo
rithm="fricas")
```

output
```
1/6*(2*A*b^3*d^3*g*i*x^3 + (3*B*a^2*b*c*d^2 - B*a^3*d^3)*g*i*n*log(b*x + a
) + (B*b^3*c^3 - 3*B*a*b^2*c^2*d)*g*i*n*log(d*x + c) - ((B*b^3*c*d^2 - B*a
*b^2*d^3)*g*i*n - 3*(A*b^3*c*d^2 + A*a*b^2*d^3)*g*i)*x^2 + (6*A*a*b^2*c*d^
2*g*i - (B*b^3*c^2*d - B*a^2*b*d^3)*g*i*n)*x + (2*B*b^3*d^3*g*i*x^3 + 6*B*
a*b^2*c*d^2*g*i*x + 3*(B*b^3*c*d^2 + B*a*b^2*d^3)*g*i*x^2)*log(e) + (2*B*b
^3*d^3*g*i*n*x^3 + 6*B*a*b^2*c*d^2*g*i*n*x + 3*(B*b^3*c*d^2 + B*a*b^2*d^3)
*g*i*n*x^2)*log((b*x + a)/(d*x + c)))/(b^2*d^2)
```

### 3.110.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(134) = 268$.

Time = 19.63 (sec) , antiderivative size = 643, normalized size of antiderivative = 4.32

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \begin{cases} acgix\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ ag\left(Acix+\frac{Adix^2}{2}+\frac{Bc^2i\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d}+\frac{Bcinx}{2}+Bcix\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{Bdinx^2}{4}+\frac{Bdix^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2}\right) \\ ci\left(Aagx+\frac{Abgx^2}{2}+\frac{Ba^2g\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2b}-\frac{Bagnx}{2}+Bagx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)-\frac{Bbgnx^2}{4}+\frac{Bbgx^2\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2}\right. \\ Aacgix+\frac{Aadgix^2}{2}+\frac{Abcgix^2}{2}+\frac{Abdgix^3}{3}-\frac{Ba^3dgin\log\left(\frac{c}{d}+x\right)}{6b^2}-\frac{Ba^3dgi\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{6b^2}+\frac{Ba^2cgin\log\left(\frac{c}{d}+x\right)}{2b}+ \end{cases}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((a*c*g*i*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (a*g*(
A*c*i*x + A*d*i*x**2/2 + B*c**2*i*log(e*(a/(c + d*x))**n)/(2*d) + B*c*i*n*
x/2 + B*c*i*x*log(e*(a/(c + d*x))**n) + B*d*i*n*x**2/4 + B*d*i*x**2*log(e*
(a/(c + d*x))**n)/2), Eq(b, 0)), (c*i*(A*a*g*x + A*b*g*x**2/2 + B*a**2*g*l
og(e*(a/c + b*x/c)**n)/(2*b) - B*a*g*n*x/2 + B*a*g*x*log(e*(a/c + b*x/c)**
n) - B*b*g*n*x**2/4 + B*b*g*x**2*log(e*(a/c + b*x/c)**n)/2), Eq(d, 0)), (A
*a*c*g*i*x + A*a*d*g*i*x**2/2 + A*b*c*g*i*x**2/2 + A*b*d*g*i*x**3/3 - B*a*
*3*d*g*i*n*log(c/d + x)/(6*b**2) - B*a**3*d*g*i*log(e*(a/(c + d*x) + b*x/(
c + d*x))**n)/(6*b**2) + B*a**2*c*g*i*n*log(c/d + x)/(2*b) + B*a**2*c*g*i*
log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(2*b) + B*a**2*d*g*i*n*x/(6*b) - B
*a*c**2*g*i*n*log(c/d + x)/(2*d) + B*a*c*g*i*x*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n) + B*a*d*g*i*n*x**2/6 + B*a*d*g*i*x**2*log(e*(a/(c + d*x) + b*
x/(c + d*x))**n)/2 + B*b*c**3*g*i*n*log(c/d + x)/(6*d**2) - B*b*c**2*g*i*n
*x/(6*d) - B*b*c*g*i*n*x**2/6 + B*b*c*g*i*x**2*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)/2 + B*b*d*g*i*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/3,
 True))
```

### 3.110.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 393 vs. $2(143) = 286$.

Time = 0.20 (sec) , antiderivative size = 393, normalized size of antiderivative = 2.64

$$
\begin{aligned}
&\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx \\
&= \frac{1}{3}\,Bbdgix^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}\,Abdgix^3 \\
&\quad+\frac{1}{2}\,Bbcgix^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\
&\quad+\frac{1}{2}\,Badgix^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}\,Abcgix^2+\frac{1}{2}\,Aadgix^2 \\
&\quad+\frac{1}{6}\,Bbdgin\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right) \\
&\quad-\frac{1}{2}\,Bbcgin\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad-\frac{1}{2}\,Badgin\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right) \\
&\quad+Bacgin\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right) \\
&\quad+Bacgix\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aacgix
\end{aligned}
$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algo
rithm="maxima")
```

output
```
1/3*B*b*d*g*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b*d*g*i*x
^3 + 1/2*B*b*c*g*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*B*a*d*
g*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*b*c*g*i*x^2 + 1/2*A
*a*d*g*i*x^2 + 1/6*B*b*d*g*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c
)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 1
/2*B*b*c*g*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*
x/(b*d)) - 1/2*B*a*d*g*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 +
(b*c - a*d)*x/(b*d)) + B*a*c*g*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) +
 B*a*c*g*i*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*c*g*i*x
```

## 3.110.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1276 vs. 2(143) = 286.

Time = 0.66 (sec) , antiderivative size = 1276, normalized size of antiderivative = 8.56

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algo
rithm="giac")
```

output
```
-1/6*((B*b^5*c^4*g*i*n - 4*B*a*b^4*c^3*d*g*i*n - 3*(b*x + a)*B*b^4*c^4*d*g
*i*n/(d*x + c) + 6*B*a^2*b^3*c^2*d^2*g*i*n + 12*(b*x + a)*B*a*b^3*c^3*d^2*
g*i*n/(d*x + c) - 4*B*a^3*b^2*c*d^3*g*i*n - 18*(b*x + a)*B*a^2*b^2*c^2*d^3
*g*i*n/(d*x + c) + B*a^4*b*d^4*g*i*n + 12*(b*x + a)*B*a^3*b*c*d^4*g*i*n/(d
*x + c) - 3*(b*x + a)*B*a^4*d^5*g*i*n/(d*x + c))*log((b*x + a)/(d*x + c))/
(b^3*d^2 - 3*(b*x + a)*b^2*d^3/(d*x + c) + 3*(b*x + a)^2*b*d^4/(d*x + c)^2
 - (b*x + a)^3*d^5/(d*x + c)^3) + ((b*x + a)*B*b^5*c^4*d*g*i*n/(d*x + c) -
 4*(b*x + a)*B*a*b^4*c^3*d^2*g*i*n/(d*x + c) - (b*x + a)^2*B*b^4*c^4*d^2*g
*i*n/(d*x + c)^2 + 6*(b*x + a)*B*a^2*b^3*c^2*d^3*g*i*n/(d*x + c) + 4*(b*x
+ a)^2*B*a*b^3*c^3*d^3*g*i*n/(d*x + c)^2 - 4*(b*x + a)*B*a^3*b^2*c*d^4*g*i
*n/(d*x + c) - 6*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g*i*n/(d*x + c)^2 + (b*x +
a)*B*a^4*b*d^5*g*i*n/(d*x + c) + 4*(b*x + a)^2*B*a^3*b*c*d^5*g*i*n/(d*x +
c)^2 - (b*x + a)^2*B*a^4*d^6*g*i*n/(d*x + c)^2 + B*b^6*c^4*g*i*log(e) - 4*
B*a*b^5*c^3*d*g*i*log(e) - 3*(b*x + a)*B*b^5*c^4*d*g*i*log(e)/(d*x + c) +
6*B*a^2*b^4*c^2*d^2*g*i*log(e) + 12*(b*x + a)*B*a*b^4*c^3*d^2*g*i*log(e)/(
d*x + c) - 4*B*a^3*b^3*c*d^3*g*i*log(e) - 18*(b*x + a)*B*a^2*b^3*c^2*d^3*g
*i*log(e)/(d*x + c) + B*a^4*b^2*d^4*g*i*log(e) + 12*(b*x + a)*B*a^3*b^2*c*
d^4*g*i*log(e)/(d*x + c) - 3*(b*x + a)*B*a^4*b*d^5*g*i*log(e)/(d*x + c) +
A*b^6*c^4*g*i - 4*A*a*b^5*c^3*d*g*i - 3*(b*x + a)*A*b^5*c^4*d*g*i/(d*x + c
) + 6*A*a^2*b^4*c^2*d^2*g*i + 12*(b*x + a)*A*a*b^4*c^3*d^2*g*i/(d*x + c...
```

### 3.110.9 Mupad [B] (verification not implemented)

Time = 1.53 (sec) , antiderivative size = 295, normalized size of antiderivative = 1.98

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,b\,d\,g\,i\,x^3}{3}+\frac{B\,g\,i\,(a\,d+b\,c)\,x^2}{2}+B\,a\,c\,g\,i\,x\right)$$
$$-x\left(\frac{\left(\frac{g\,i\,(6\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3}-\frac{A\,g\,i\,(6\,a\,d+6\,b\,c)}{6}\right)(6\,a\,d+6\,b\,c)}{6\,b\,d}+A\,a\,c\,g\,i\right.$$
$$\left.-\frac{g\,i\,(2\,A\,a^2\,d^2+2\,A\,b^2\,c^2+B\,a^2\,d^2\,n-B\,b^2\,c^2\,n+8\,A\,a\,b\,c\,d)}{2\,b\,d}\right)$$
$$+x^2\left(\frac{g\,i\,(6\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6}-\frac{A\,g\,i\,(6\,a\,d+6\,b\,c)}{12}\right)$$
$$-\frac{\ln(a+b\,x)\,(B\,a^3\,d\,g\,i\,n-3\,B\,a^2\,b\,c\,g\,i\,n)}{6\,b^2}$$
$$+\frac{\ln(c+d\,x)\,(B\,b\,c^3\,g\,i\,n-3\,B\,a\,c^2\,d\,g\,i\,n)}{6\,d^2}+\frac{A\,b\,d\,g\,i\,x^3}{3}$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((B*g*i*x^2*(a*d + b*c))/2 + (B*b*d*g*i*x^3
)/3 + B*a*c*g*i*x) - x*((((g*i*(6*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/3
- (A*g*i*(6*a*d + 6*b*c))/6)*(6*a*d + 6*b*c))/(6*b*d) + A*a*c*g*i - (g*i*(
2*A*a^2*d^2 + 2*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 8*A*a*b*c*d))/(2*b
*d)) + x^2*((g*i*(6*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/6 - (A*g*i*(6*a*
d + 6*b*c))/12) - (log(a + b*x)*(B*a^3*d*g*i*n - 3*B*a^2*b*c*g*i*n))/(6*b^
2) + (log(c + d*x)*(B*b*c^3*g*i*n - 3*B*a*c^2*d*g*i*n))/(6*d^2) + (A*b*d*g
*i*x^3)/3
```

3.110. $\int (ag + bgx)(ci + dix) \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

# 3.111 $\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$



## 3.111.1 Optimal result

Integrand size = 31, antiderivative size = 86

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = -\frac{B(bc-ad)inx}{2b}-\frac{B(bc-ad)^2in\log(a+bx)}{2b^2d}+\frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d}$$

output
```
-1/2*B*(-a*d+b*c)*i*n*x/b-1/2*B*(-a*d+b*c)^2*i*n*ln(b*x+a)/b^2/d+1/2*i*(d*
x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

## 3.111.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 74, normalized size of antiderivative = 0.86

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{i\left(-\frac{B(bc-ad)n(bdx+(bc-ad)\log(a+bx))}{b^2}+(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{2d}$$

input
```
Integrate[(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(i*(-((B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a + b*x]))/b^2) + (c + d*x
)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(2*d)
```

### 3.111.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.129, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bn(bc-ad)\int \frac{i^2(c+dx)}{a+bx}dx}{2di}$$

$$\Big\downarrow \text{27}$$

$$\frac{i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bin(bc-ad)\int \frac{c+dx}{a+bx}dx}{2d}$$

$$\Big\downarrow \text{49}$$

$$\frac{i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bin(bc-ad)\int \left(\frac{d}{b}+\frac{bc-ad}{b(a+bx)}\right)dx}{2d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d}-\frac{Bin(bc-ad)\left(\frac{(bc-ad)\log(a+bx)}{b^2}+\frac{dx}{b}\right)}{2d}$$

input
```
Int[(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/2*(B*(b*c - a*d)*i*n*((d*x)/b + ((b*c - a*d)*Log[a + b*x])/b^2))/d + (i
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d)
```

#### 3.111.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

### 3.111.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 249 vs. $2(80) = 160$.

Time = 0.68 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.91

| method | result |
|---|---|
| parallelrisch | $\frac{B\,x^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^2d^2in+A\,x^2b^2d^2in-B\ln(bx+a)a^2d^2i\,n^2+2B\ln(bx+a)abcdi\,n^2-B\ln(bx+a)b^2c^2i\,n^2+2Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)\right.}{2b^2}$ |

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/2*(B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*i*n+A*x^2*b^2*d^2*i*n-B*ln(b*
x+a)*a^2*d^2*i*n^2+2*B*ln(b*x+a)*a*b*c*d*i*n^2-B*ln(b*x+a)*b^2*c^2*i*n^2+2
*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*i*n+B*x*a*b*d^2*i*n^2-B*x*b^2*c*d*i
*n^2+2*A*x*b^2*c*d*i*n+B*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c^2*i*n-B*a^2*d^2*i
*n^2+B*b^2*c^2*i*n^2-3*A*a*b*c*d*i*n-2*A*b^2*c^2*i*n)/b^2/n/d
```

### 3.111.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 162 vs. $2(80) = 160$.

Time = 0.35 (sec) , antiderivative size = 162, normalized size of antiderivative = 1.88

$$\int (ci + dix) \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \frac{Ab^2d^2ix^2 - Bb^2c^2in\log(dx+c) + (2\,Babcd - Ba^2d^2)in\log(bx+a) + (2\,Ab^2cdi - (Bb^2cd - Babd^2)in)x}{2\,b^2d}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="frica
s")
```

output
```
1/2*(A*b^2*d^2*i*x^2 - B*b^2*c^2*i*n*log(d*x + c) + (2*B*a*b*c*d - B*a^2*d
^2)*i*n*log(b*x + a) + (2*A*b^2*c*d*i - (B*b^2*c*d - B*a*b*d^2)*i*n)*x + (
B*b^2*d^2*i*x^2 + 2*B*b^2*c*d*i*x)*log(e) + (B*b^2*d^2*i*n*x^2 + 2*B*b^2*c
*d*i*n*x)*log((b*x + a)/(d*x + c)))/(b^2*d)
```

### 3.111.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(73) = 146$.

Time = 5.09 (sec) , antiderivative size = 382, normalized size of antiderivative = 4.44

$$\int (ci + dix) \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = \begin{cases} cix\left(A + B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ Acix + \frac{Adix^2}{2} + \frac{Bc^2i\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d} + \frac{Bcinx}{2} + Bcix\log\left(e\left(\frac{a}{c+dx}\right)^n\right) + \frac{Bdinx^2}{4} + \frac{Bdix^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2} \\ ci\left(Ax + \frac{Ba\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b} - Bnx + Bx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)\right) \\ Acix + \frac{Adix^2}{2} - \frac{Ba^2din\log\left(\frac{c}{d}+x\right)}{2b^2} - \frac{Ba^2di\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{2b^2} + \frac{Bacin\log\left(\frac{c}{d}+x\right)}{b} + \frac{Baci\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{b} + \frac{Badi}{2b} \end{cases}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((c*i*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c*i*x +
 A*d*i*x**2/2 + B*c**2*i*log(e*(a/(c + d*x))**n)/(2*d) + B*c*i*n*x/2 + B*c
*i*x*log(e*(a/(c + d*x))**n) + B*d*i*n*x**2/4 + B*d*i*x**2*log(e*(a/(c + d
*x))**n)/2, Eq(b, 0)), (c*i*(A*x + B*a*log(e*(a/c + b*x/c)**n)/b - B*n*x +
 B*x*log(e*(a/c + b*x/c)**n)), Eq(d, 0)), (A*c*i*x + A*d*i*x**2/2 - B*a**2
*d*i*n*log(c/d + x)/(2*b**2) - B*a**2*d*i*log(e*(a/(c + d*x) + b*x/(c + d*
x))**n)/(2*b**2) + B*a*c*i*n*log(c/d + x)/b + B*a*c*i*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n)/b + B*a*d*i*n*x/(2*b) - B*c**2*i*n*log(c/d + x)/(2*d)
- B*c*i*n*x/2 + B*c*i*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d*i*x*
*2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/2, True))
```

### 3.111.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 156, normalized size of antiderivative = 1.81

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{1}{2}\,Bdix^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}\,Adix^2$$
$$-\frac{1}{2}\,Bdin\left(\frac{a^2\log\left(bx+a\right)}{b^2}-\frac{c^2\log\left(dx+c\right)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bcin\left(\frac{a\log\left(bx+a\right)}{b}-\frac{c\log\left(dx+c\right)}{d}\right)+Bcix\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acix$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="maxim
a")
```

output
```
1/2*B*d*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*d*i*x^2 - 1/2
*B*d*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d
)) + B*c*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c*i*x*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) + A*c*i*x
```

### 3.111.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(80) = 160.

Time = 0.46 (sec) , antiderivative size = 580, normalized size of antiderivative = 6.74

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=\frac{1}{2}\left(\frac{(Bb^3c^3in-3\,Bab^2c^2din+3\,Ba^2bcd^2in-Ba^3d^3in)\log\left(\frac{bx+a}{dx+c}\right)}{b^2d-\frac{2\,(bx+a)bd^2}{dx+c}+\frac{(bx+a)^2d^3}{(dx+c)^2}}-\frac{Bb^4c^3in-3\,Bab^3c^2din-\frac{(bx+a)Bb^3c^3}{dx+c}}{}\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="giac"
)
```

output
```
1/2*((B*b^3*c^3*i*n - 3*B*a*b^2*c^2*d*i*n + 3*B*a^2*b*c*d^2*i*n - B*a^3*d^
3*i*n)*log((b*x + a)/(d*x + c))/(b^2*d - 2*(b*x + a)*b*d^2/(d*x + c) + (b*
x + a)^2*d^3/(d*x + c)^2) - (B*b^4*c^3*i*n - 3*B*a*b^3*c^2*d*i*n - (b*x +
a)*B*b^3*c^3*d*i*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*i*n + 3*(b*x + a)*B*a*b^2
*c^2*d^2*i*n/(d*x + c) - B*a^3*b*d^3*i*n - 3*(b*x + a)*B*a^2*b*c*d^3*i*n/(
d*x + c) + (b*x + a)*B*a^3*d^4*i*n/(d*x + c) - B*b^4*c^3*i*log(e) + 3*B*a*
b^3*c^2*d*i*log(e) - 3*B*a^2*b^2*c*d^2*i*log(e) + B*a^3*b*d^3*i*log(e) - A
*b^4*c^3*i + 3*A*a*b^3*c^2*d*i - 3*A*a^2*b^2*c*d^2*i + A*a^3*b*d^3*i)/(b^3
*d - 2*(b*x + a)*b^2*d^2/(d*x + c) + (b*x + a)^2*b*d^3/(d*x + c)^2) + (B*b
^3*c^3*i*n - 3*B*a*b^2*c^2*d*i*n + 3*B*a^2*b*c*d^2*i*n - B*a^3*d^3*i*n)*lo
g(-b + (b*x + a)*d/(d*x + c))/(b^2*d) - (B*b^3*c^3*i*n - 3*B*a*b^2*c^2*d*i
*n + 3*B*a^2*b*c*d^2*i*n - B*a^3*d^3*i*n)*log((b*x + a)/(d*x + c))/(b^2*d)
)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.111.9 Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 134, normalized size of antiderivative = 1.56

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$=x\left(\frac{i\,(2\,A\,a\,d+4\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,b}-\frac{A\,i\,(2\,a\,d+2\,b\,c)}{2\,b}\right)$$
$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,d\,i\,x^2}{2}+B\,c\,i\,x\right)$$
$$-\frac{\ln(a+b\,x)\,(B\,a^2\,d\,i\,n-2\,B\,a\,b\,c\,i\,n)}{2\,b^2}+\frac{A\,d\,i\,x^2}{2}-\frac{B\,c^2\,i\,n\,\ln(c+d\,x)}{2\,d}$$

input
```
int((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x*((i*(2*A*a*d + 4*A*b*c + B*a*d*n - B*b*c*n))/(2*b) - (A*i*(2*a*d + 2*b*c
))/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d*i*x^2)/2 + B*c*i*x) - (lo
g(a + b*x)*(B*a^2*d*i*n - 2*B*a*b*c*i*n))/(2*b^2) + (A*d*i*x^2)/2 - (B*c^2
*i*n*log(c + d*x))/(2*d)
```

## 3.111. $\int (ci + dix) \left(A + B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

# 3.112 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$



## 3.112.1 Optimal result

Integrand size = 41, antiderivative size = 141

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx &= \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg} \\ &- \frac{(bc-ad)i\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g} \\ &+ \frac{B(bc-ad)in\operatorname{PolyLog}\left(2,1+\frac{bc-ad}{d(a+bx)}\right)}{b^2g}\end{aligned}$$

output
```
i*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g-(-a*d+b*c)*i*ln((a*d-b*c)/d/
(b*x+a))*(A-B*n+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g+B*(-a*d+b*c)*i*n*polylo
g(2,1+(-a*d+b*c)/d/(b*x+a))/b^2/g
```

### 3.112.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 172, normalized size of antiderivative = 1.22

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$=\frac{i\left(B(-bc+ad)n\log^2(a+bx)+2\left(Abdx+Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B(-bc+ad)n\log(c+dx)\right)+2(b\right.}{2b^2}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*
g*x),x]
```

output
```
(i*(B*(-(b*c) + a*d)*n*Log[a + b*x]^2 + 2*(A*b*d*x + B*d*(a + b*x)*Log[e*(
(a + b*x)/(c + d*x))^n] + B*(-(b*c) + a*d)*n*Log[c + d*x]) + 2*(b*c - a*d)
*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(b*(c + d*x)
)/(b*c - a*d)]) + 2*B*(b*c - a*d)*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)]))/(2*b^2*g)
```

### 3.112.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.95, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.195, Rules used = {2959, 27, 2941, 2858, 27, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{ag+bgx}\,dx$$
$$\Big\downarrow \; 2959$$
$$\frac{i(bc-ad)\int \frac{A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{g(a+bx)}dx}{b}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$
$$\Big\downarrow \; 27$$
$$\frac{i(bc-ad)\int \frac{A-Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}dx}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$
$$\Big\downarrow \; 2941$$

$$\frac{i(bc-ad)\left(\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(c+dx)}dx}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{ 2858}$$

$$\frac{i(bc-ad)\left(\frac{Bn(bc-ad)\int \frac{b\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)\left(b\left(c-\frac{ad}{b}\right)+d(a+bx)\right)}d(a+bx)}{b^2}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{i(bc-ad)\left(\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{(a+bx)(bc-ad+d(a+bx))}d(a+bx)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{ 2778}$$

$$\frac{i(bc-ad)\left(-\frac{Bn(bc-ad)\int \frac{(a+bx)\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{bc-ad+d(a+bx)}d\frac{1}{a+bx}}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{ 2005}$$

$$\frac{i(bc-ad)\left(-\frac{Bn(bc-ad)\int \frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)}{d+\frac{bc-ad}{a+bx}}d\frac{1}{a+bx}}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

$$\Big\downarrow \text{ 2752}$$

---

3.112. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

$$\frac{i(bc-ad)\left(\frac{Bn\,\text{PolyLog}\left(2,\frac{bc-ad}{d(a+bx)}+1\right)}{b}-\frac{\log\left(-\frac{bc-ad}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A-Bn\right)}{b}\right)}{bg}+\frac{i(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bg}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x),x
]
```

output
```
(i*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b*g) + ((b*c - a*d)*
i*(-((Log[-((b*c - a*d)/(d*(a + b*x)))]*(A - B*n + B*Log[e*((a + b*x)/(c +
 d*x))^n]))/b) + (B*n*PolyLog[2, 1 + (b*c - a*d)/(d*(a + b*x))])/b))/(b*g)
```

### 3.112.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

---

## 3.112. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

rule 2941
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[-(b*c - a*d)/(d*(a + b*
x))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Simp[B*n*((b*c - a*d
)/g)   Int[Log[-(b*c - a*d)/(d*(a + b*x))]/((a + b*x)*(c + d*x)), x], x] /;
 FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f -
 a*g, 0]
```

rule 2959
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_)), x_Symbol] :> Simp[(f
 + g*x)^(m + 1)*(h + i*x)*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 2
))), x] + Simp[i*((b*c - a*d)/(b*d*(m + 2)))   Int[(f + g*x)^m*(A - B*n + B
*Log[e*((a + b*x)/(c + d*x))^n]), x], x] /; FreeQ[{a, b, c, d, e, f, g, h,
i, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c
*i, 0] && IGtQ[m, -2]
```

## 3.112.4 Maple [F]

$$\int \frac{(dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{bgx+ag}dx$$

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

## 3.112.5 Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algo
rithm="fricas")
```

output
```
integral((A*d*i*x + A*c*i + (B*d*i*x + B*c*i)*log(e*((b*x + a)/(d*x + c))^
n))/(b*g*x + a*g), x)
```

### 3.112.6 Sympy [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = \frac{i\left(\int \frac{Ac}{a+bx}\,dx+\int \frac{Adx}{a+bx}\,dx+\int \frac{Bc\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx+\int \frac{Bdx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx\right)}{g}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
```

output
```
i*(Integral(A*c/(a + b*x), x) + Integral(A*d*x/(a + b*x), x) + Integral(B*
c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + Integral(B*d*x*l
og(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x))/g
```

### 3.112.7 Maxima [A] (verification not implemented)

Time = 0.48 (sec) , antiderivative size = 276, normalized size of antiderivative = 1.96

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = Adi\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)-\frac{Bcin\log(dx+c)}{bg} + \frac{Aci\log(bgx+ag)}{bg}+\frac{(bcin-adin)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\text{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^2g} + \frac{2\,Bbdix\log(e)-(bcin-adin)B\log(bx+a)^2+2\,(bci\log(e)+(in-i\log(e))ad)B\log(bx+a)+2\,(B}{2}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algo
rithm="maxima")
```

output
```
A*d*i*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - B*c*i*n*log(d*x + c)/(b*g) + A*
c*i*log(b*g*x + a*g)/(b*g) + (b*c*i*n - a*d*i*n)*(log(b*x + a)*log((b*d*x
+ a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^2*g) + 1
/2*(2*B*b*d*i*x*log(e) - (b*c*i*n - a*d*i*n)*B*log(b*x + a)^2 + 2*(b*c*i*l
og(e) + (i*n - i*log(e))*a*d)*B*log(b*x + a) + 2*(B*b*d*i*x + (b*c*i - a*d
*i)*B*log(b*x + a))*log((b*x + a)^n) - 2*(B*b*d*i*x + (b*c*i - a*d*i)*B*lo
g(b*x + a))*log((d*x + c)^n))/(b^2*g)
```

### 3.112.8 Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, algo
rithm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a*
g), x)
```

### 3.112.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx=\int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x),x
)
```

output
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x),
x)
```

# 3.113 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$



## 3.113.1 Optimal result

Integrand size = 41, antiderivative size = 150

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = -\frac{Bin(c+dx)}{bg^2(a+bx)} - \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{Bdin\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}$$

output
```
-B*i*n*(d*x+c)/b/g^2/(b*x+a)-i*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g
^2/(b*x+a)-d*i*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b
^2/g^2+B*d*i*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g^2
```

### 3.113.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.26

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{i\left(-\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2(a+bx)}+\frac{d\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2}-\frac{Bn\left(\frac{bc-ad}{a+bx}+d\log(a+bx)-d\log(c+dx)\right)}{b^2}-\frac{Bdn\left(\log^2(a+\right.}{}\right.}{g^2}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*
g*x)^2,x]
```

output
```
(i*(-(((b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2*(a + b*x))
) + (d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/b^2 - (B*n*((b
*c - a*d)/(a + b*x) + d*Log[a + b*x] - d*Log[c + d*x]))/b^2 - (B*d*n*(Log[
a + b*x]^2 - 2*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2*PolyLog[2,
-((d*(a + b*x))/(b*c - a*d))]))/(2*b^2)))/g^2
```

### 3.113.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.95, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {2961, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^2}\,dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{i\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{ 2780}$$

$$\frac{i\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^2}$$

$$\Big\downarrow \text{2741}$$

$$\frac{i\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{g^2}$$

$$\Big\downarrow \text{2779}$$

$$\frac{i\left(\frac{d\left(\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{g^2}$$

$$\Big\downarrow \text{2838}$$

$$\frac{i\left(\frac{d\left(\frac{Bn\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{g^2}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^2
,x]
```

output
```
(i*((-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n]))/(a + b*x))/b + (d*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*
Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x))/(
d*(a + b*x))])/b))/b))/g^2
```

---

3.113. $\displaystyle\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$

### 3.113.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.113.4 Maple [F]

$$\int \frac{(dix + ci)\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{(bgx + ag)^{2}}dx$$

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

### 3.113.5 Fricas [F]

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^2}\,dx = \int \frac{(dix + ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)}{(bgx + ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, al
gorithm="fricas")
```

output
```
integral((A*d*i*x + A*c*i + (B*d*i*x + B*c*i)*log(e*((b*x + a)/(d*x + c))^
n))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

### 3.113.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x)
```

output
```
Timed out
```

### 3.113.7 Maxima [F]

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^2}\,dx = \int \frac{(dix + ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)}{(bgx + ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, al
gorithm="maxima")
```

output
```
-B*c*i*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^2) -
 d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) + B*d*i*((((b*x + a)*log(b*x + a) +
 a)*log((b*x + a)^n) - ((b*x + a)*log(b*x + a) + a)*log((d*x + c)^n))/(b^3
*g^2*x + a*b^2*g^2) + integrate((b^2*d*x^2*log(e) + b^2*c*x*log(e) - a*b*c
*n + a^2*d*n - (a*b*c*n - a^2*d*n + (b^2*c*n - a*b*d*n)*x)*log(b*x + a))/(
b^4*d*g^2*x^3 + a^2*b^2*c*g^2 + (b^4*c*g^2 + 2*a*b^3*d*g^2)*x^2 + (2*a*b^3
*c*g^2 + a^2*b^2*d*g^2)*x), x)) + A*d*i*(a/(b^3*g^2*x + a*b^2*g^2) + log(b
*x + a)/(b^2*g^2)) - B*c*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^2*g^2
*x + a*b*g^2) - A*c*i/(b^2*g^2*x + a*b*g^2)
```

## 3.113.8 Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x, al
gorithm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x + a*
g)^2, x)
```

## 3.113.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{{\left(a\,g+b\,g\,x\right)}^2}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^2
,x)
```

output
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^2
, x)
```

# 3.114 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$



## 3.114.1 Optimal result

Integrand size = 41, antiderivative size = 89

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = -\frac{Bin(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)g^3(a+bx)^2}$$

output
```
-1/4*B*i*n*(d*x+c)^2/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*i*(d*x+c)^2*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^3/(b*x+a)^2
```

## 3.114.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 216 vs. 2(89) = 178.

Time = 0.10 (sec) , antiderivative size = 216, normalized size of antiderivative = 2.43

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \frac{i\left(-\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2(a+bx)^2} - \frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2(a+bx)} - \frac{Bdn\left(\frac{1}{a+bx}+\frac{d\log(a+bx)}{bc-ad}-\frac{d\log(c+dx)}{bc-ad}\right)}{b^2} - \frac{Bn\left(\frac{bc-ad}{(a+bx)^2}-\frac{2d}{a+bx}-\frac{2d^2}{}\right.}{4}\right.}{g^3}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*
g*x)^3,x]
```

output
```
(i*(-1/2*((b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2*(a + b*
x)^2) - (d*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2*(a + b*x)) - (B*d*
n*((a + b*x)^(-1) + (d*Log[a + b*x])/(b*c - a*d) - (d*Log[c + d*x])/(b*c -
 a*d)))/b^2 - (B*n*((b*c - a*d)/(a + b*x)^2 - (2*d)/(a + b*x) - (2*d^2*Log
[a + b*x])/(b*c - a*d) + (2*d^2*Log[c + d*x])/(b*c - a*d)))/(4*b^2)))/g^3
```

### 3.114.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.049$, Rules used = $\{2961, 2741\}$

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^3}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{i\left(-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}\right)}{g^3(bc-ad)}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^3
,x]
```

output
```
(i*(-1/4*(B*n*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(2*(a + b*x)^2)))/((b*c - a*d)*g^3)
```

### 3.114.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.114.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(85) = 170$.

Time = 5.16 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.66

| method | result |
|---|---|
| parallelrisch | $-\frac{-4Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}c\,d^{2}in+B\,a^{2}b^{2}d^{3}i\,n^{2}-B\,b^{4}c^{2}di\,n^{2}+2A\,a^{2}b^{2}d^{3}in-2A\,b^{4}c^{2}din-2B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}d^{3}in+2Bx}{4g^{3}(bx+a)^{2}b^{4}dn(ad-cb)}$ |

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,method=_RE
TURNVERBOSE)
```

output
```
-1/4*(-4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^2*i*n+B*a^2*b^2*d^3*i*n^2-B
*b^4*c^2*d*i*n^2+2*A*a^2*b^2*d^3*i*n-2*A*b^4*c^2*d*i*n-2*B*x^2*ln(e*((b*x+
a)/(d*x+c))^n)*b^4*d^3*i*n+2*B*x*a*b^3*d^3*i*n^2-2*B*x*b^4*c*d^2*i*n^2+4*A
*x*a*b^3*d^3*i*n-4*A*x*b^4*c*d^2*i*n-2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2
*d*i*n)/g^3/(b*x+a)^2/b^4/d/n/(a*d-b*c)
```

### 3.114.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(85) = 170$.

Time = 0.37 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.81

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = -\frac{(Bb^2c^2-Ba^2d^2)in+2\,(Ab^2c^2-Aa^2d^2)i+2\,((Bb^2cd-Babd^2)in+2\,(Ab^2cd-Aabd^2)i)x+2\,(2\,(Bb^2c}{4\,((b^5c-ab^4d)g^3x^2+2\,(ab^4c-a^2b^3}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, al
gorithm="fricas")
```

output
```
-1/4*((B*b^2*c^2 - B*a^2*d^2)*i*n + 2*(A*b^2*c^2 - A*a^2*d^2)*i + 2*((B*b^
2*c*d - B*a*b*d^2)*i*n + 2*(A*b^2*c*d - A*a*b*d^2)*i)*x + 2*(2*(B*b^2*c*d
- B*a*b*d^2)*i*x + (B*b^2*c^2 - B*a^2*d^2)*i)*log(e) + 2*(B*b^2*d^2*i*n*x^
2 + 2*B*b^2*c*d*i*n*x + B*b^2*c^2*i*n)*log((b*x + a)/(d*x + c)))/((b^5*c -
 a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d
)*g^3)
```

### 3.114.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1360 vs. $2(76) = 152$.

Time = 80.89 (sec) , antiderivative size = 1360, normalized size of antiderivative = 15.28

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \begin{cases} \frac{\tilde{\infty}(A+B\log(0^ne))\left(-\frac{ci}{2x^2}-\frac{di}{x}\right)}{g^3} \\ -\frac{Ad^2i}{b^3cg^3+b^3dg^3x}-\frac{Bd^2i\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)}{b^3cg^3+b^3dg^3x} \\ \frac{Acix+\frac{Adix^2}{2}+\frac{Bc^2i\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2d}+\frac{Bcinx}{2}+Bcix\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{Bdinx^2}{4}+\frac{Bdix^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{2}}{a^3g^3} \\ -\frac{2Aa^2d^2i}{4a^3b^2dg^3-4a^2b^3cg^3+8a^2b^3dg^3x-8ab^4cg^3x+4ab^4dg^3x^2-4b^5cg^3x^2}-\frac{4Aabd^2ix}{4a^3b^2dg^3-4a^2b^3cg^3+8a^2b^3dg^3x-8ab^4cg^3x+4ab^4dg^3x^2-4b^5cg^3x^2} \end{cases}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x)
```

output
```
Piecewise((zoo*(A + B*log(0**n*e))*(-c*i/(2*x**2) - d*i/x)/g**3, Eq(a, 0)
& Eq(b, 0)), (-A*d**2*i/(b**3*c*g**3 + b**3*d*g**3*x) - B*d**2*i*log(e*(b*
c/(c*d + d**2*x) + b*x/(c + d*x))**n)/(b**3*c*g**3 + b**3*d*g**3*x), Eq(a,
 b*c/d)), ((A*c*i*x + A*d*i*x**2/2 + B*c**2*i*log(e*(a/(c + d*x))**n)/(2*d
) + B*c*i*n*x/2 + B*c*i*x*log(e*(a/(c + d*x))**n) + B*d*i*n*x**2/4 + B*d*i
*x**2*log(e*(a/(c + d*x))**n)/2)/(a**3*g**3), Eq(b, 0)), (-2*A*a**2*d**2*i
/(4*a**3*b**2*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**
4*c*g**3*x + 4*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) - 4*A*a*b*d**2*i*x
/(4*a**3*b**2*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**
4*c*g**3*x + 4*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) + 2*A*b**2*c**2*i/
(4*a**3*b**2*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**4
*c*g**3*x + 4*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) + 4*A*b**2*c*d*i*x/
(4*a**3*b**2*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**4
*c*g**3*x + 4*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) - B*a**2*d**2*i*n/(
4*a**3*b**2*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**4*
c*g**3*x + 4*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) - 2*B*a*b*d**2*i*n*x
/(4*a**3*b**2*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**
4*c*g**3*x + 4*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) + B*b**2*c**2*i*n/
(4*a**3*b**2*d*g**3 - 4*a**2*b**3*c*g**3 + 8*a**2*b**3*d*g**3*x - 8*a*b**4
*c*g**3*x + 4*a*b**4*d*g**3*x**2 - 4*b**5*c*g**3*x**2) + 2*B*b**2*c**2*...
```

### 3.114.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(85) = 170$.

Time = 0.20 (sec) , antiderivative size = 582, normalized size of antiderivative = 6.54

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{4}\,Bdin\left(\frac{3\,abc-a^2d+2\,(2\,b^2c-abd)x}{(b^5c-ab^4d)g^3x^2+2\,(ab^4c-a^2b^3d)g^3x+(a^2b^3c-a^3b^2d)g^3}+\frac{2\,(2\,bcd-ad^2)\log(bx+a)}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)g^3}-\right.$$

$$+\frac{1}{4}\,Bcin\left(\frac{2\,bdx-bc+3\,ad}{(b^4c-ab^3d)g^3x^2+2\,(ab^3c-a^2b^2d)g^3x+(a^2b^2c-a^3bd)g^3}+\frac{2\,d^2\log(bx+a)}{(b^3c^2-2\,ab^2cd+a^2bd^2)g^3}-\frac{}{(b^3}\right.$$

$$-\frac{(2\,bx+a)Bdi\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(b^4g^3x^2+2\,ab^3g^3x+a^2b^2g^3)}-\frac{(2\,bx+a)Adi}{2\,(b^4g^3x^2+2\,ab^3g^3x+a^2b^2g^3)}$$

$$-\frac{Bci\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}-\frac{Aci}{2\,(b^3g^3x^2+2\,ab^2g^3x+a^2bg^3)}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, al
gorithm="maxima")
```

output
```
-1/4*B*d*i*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^4*d)
*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3) +
2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^
3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^
2)*g^3)) + 1/4*B*c*i*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2
 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(
b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((
b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*(2*b*x + a)*B*d*i*log(e*(b*
x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3)
- 1/2*(2*b*x + a)*A*d*i/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*
B*c*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x
+ a^2*b*g^3) - 1/2*A*c*i/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3)
```

### 3.114.8 Giac [A] (verification not implemented)

Time = 0.78 (sec) , antiderivative size = 100, normalized size of antiderivative = 1.12

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = -\frac{1}{4}\left(\frac{2\,(dx+c)^2Bin\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^2g^3}+\frac{(Bin+2\,Bi\log(e)+2\,Ai)(dx+c)^2}{(bx+a)^2g^3}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x, al
gorithm="giac")
```

output
```
-1/4*(2*(d*x + c)^2*B*i*n*log((b*x + a)/(d*x + c))/((b*x + a)^2*g^3) + (B*
i*n + 2*B*i*log(e) + 2*A*i)*(d*x + c)^2/((b*x + a)^2*g^3))*(b*c/(b*c - a*d
)^2 - a*d/(b*c - a*d)^2)
```

### 3.114.9 Mupad [B] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 204, normalized size of antiderivative = 2.29

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = -\frac{x\,(2\,A\,b\,d\,i+B\,b\,d\,i\,n)+A\,a\,d\,i+A\,b\,c\,i+\frac{B\,a\,d\,i\,n}{2}+\frac{B\,b\,c\,i\,n}{2}}{2\,a^2\,b^2\,g^3+4\,a\,b^3\,g^3\,x+2\,b^4\,g^3\,x^2} - \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(\frac{B\,c\,i}{2\,b}+\frac{B\,a\,d\,i}{2\,b^2}+\frac{B\,d\,i\,x}{b}\right)}{a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2} - \frac{B\,d^2\,i\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,1\mathrm{i}}{b^2\,g^3\,(a\,d-b\,c)}$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^3
,x)
```

output
```
- (x*(2*A*b*d*i + B*b*d*i*n) + A*a*d*i + A*b*c*i + (B*a*d*i*n)/2 + (B*b*c*
i*n)/2)/(2*a^2*b^2*g^3 + 2*b^4*g^3*x^2 + 4*a*b^3*g^3*x) - (log(e*((a + b*x
)/(c + d*x))^n)*((B*c*i)/(2*b) + (B*a*d*i)/(2*b^2) + (B*d*i*x)/b))/(a^2*g^
3 + b^2*g^3*x^2 + 2*a*b*g^3*x) - (B*d^2*i*n*atan((b*c*2i + b*d*x*2i)/(a*d
- b*c) + 1i)*1i)/(b^2*g^3*(a*d - b*c))
```

## 3.114. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$

# 3.115 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$



## 3.115.1 Optimal result

Integrand size = 41, antiderivative size = 181

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \frac{Bdin(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2} - \frac{bBin(c+dx)^3}{9(bc-ad)^2g^4(a+bx)^3} + \frac{di(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2g^4(a+bx)^3}$$

output
```
1/4*B*d*i*n*(d*x+c)^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-1/9*b*B*i*n*(d*x+c)^3/(-a
*d+b*c)^2/g^4/(b*x+a)^3+1/2*d*i*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/
(-a*d+b*c)^2/g^4/(b*x+a)^2-1/3*b*i*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))/(-a*d+b*c)^2/g^4/(b*x+a)^3
```

## 3.115.2 Mathematica [A] (verified)

Time = 0.28 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.08

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{i\left(\frac{12Abc}{(a+bx)^3} - \frac{12aAd}{(a+bx)^3} + \frac{4bBcn}{(a+bx)^3} - \frac{4aBdn}{(a+bx)^3} + \frac{18Ad}{(a+bx)^2} + \frac{3Bdn}{(a+bx)^2} - \frac{6Bd^2n}{(bc-ad)(a+bx)} - \frac{6Bd^3n\log(a+bx)}{(bc-ad)^2} + \frac{6B(2bc+ad+3bd\ldots}{(a\ldots}\right.}{36b^2g^4}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*
g*x)^4,x]
```

output
```
-1/36*(i*((12*A*b*c)/(a + b*x)^3 - (12*a*A*d)/(a + b*x)^3 + (4*b*B*c*n)/(a
 + b*x)^3 - (4*a*B*d*n)/(a + b*x)^3 + (18*A*d)/(a + b*x)^2 + (3*B*d*n)/(a
+ b*x)^2 - (6*B*d^2*n)/((b*c - a*d)*(a + b*x)) - (6*B*d^3*n*Log[a + b*x])/
(b*c - a*d)^2 + (6*B*(2*b*c + a*d + 3*b*d*x)*Log[e*((a + b*x)/(c + d*x))^n
])/(a + b*x)^3 + (6*B*d^3*n*Log[c + d*x])/(b*c - a*d)^2))/(b^2*g^4)
```

## 3.115.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 143, normalized size of antiderivative = 0.79, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2961, 2772, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^4}dx$$

$$\Big\downarrow \ 2961$$

$$\frac{i\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)^2}$$

$$\Big\downarrow \ 2772$$

$$\frac{i\left(-Bn\int -\frac{(c+dx)^4\left(2b-\frac{3d(a+bx)}{c+dx}\right)}{6(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}\right)}{g^4(bc-ad)^2}$$

$$\Big\downarrow \ 27$$

$$\frac{i\left(\frac{1}{6}Bn\int \frac{(c+dx)^4\left(2b-\frac{3d(a+bx)}{c+dx}\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}\right)}{g^4(bc-ad)^2}$$

$$\Big\downarrow \ 53$$

$$\frac{i\left(\frac{1}{6}Bn\int\left(\frac{2b(c+dx)^4}{(a+bx)^4}-\frac{3d(c+dx)^3}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}\right)}{g^4(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{b(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{1}{6}Bn\left(\frac{3d(c+dx)^2}{2(a+bx)^2}-\frac{2b(c+dx)^3}{3(a+bx)^3}\right)\right)}{g^4(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^4
,x]
```

output
```
(i*((B*n*((3*d*(c + d*x)^2)/(2*(a + b*x)^2) - (2*b*(c + d*x)^3)/(3*(a + b*
x)^3)))/6 + (d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a +
 b*x)^2) - (b*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a +
b*x)^3)))/((b*c - a*d)^2*g^4)
```

### 3.115.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.115.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 454 vs. $2(173) = 346$.

Time = 10.20 (sec) , antiderivative size = 455, normalized size of antiderivative = 2.51

| method | result |
|---|---|
| parallelrisch | $-\frac{18Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{6}c^{2}d^{2}in-18Bxab^{5}cd^{3}in^{2}-36Axab^{5}cd^{3}in-18B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)ab^{5}c^{2}d^{2}in-18Bx^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right.}{}$ |

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,method=_RE
TURNVERBOSE)
```

output
```
-1/36*(18*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^2*d^2*i*n-18*B*x*a*b^5*c*d^3
*i*n^2-36*A*x*a*b^5*c*d^3*i*n-18*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^2*d^2
*i*n-18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^4*i*n-36*B*x*ln(e*((b*x+a)
/(d*x+c))^n)*a*b^5*c*d^3*i*n+5*B*a^3*b^3*d^4*i*n^2+4*B*b^6*c^3*d*i*n^2+6*A
*a^3*b^3*d^4*i*n+12*A*b^6*c^3*d*i*n-6*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*
d^4*i*n+6*B*x^2*a*b^5*d^4*i*n^2-6*B*x^2*b^6*c*d^3*i*n^2+15*B*x*a^2*b^4*d^4
*i*n^2+3*B*x*b^6*c^2*d^2*i*n^2+18*A*x*a^2*b^4*d^4*i*n+18*A*x*b^6*c^2*d^2*i
*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^3*d*i*n-9*B*a*b^5*c^2*d^2*i*n^2-18
*A*a*b^5*c^2*d^2*i*n)/g^4/(b*x+a)^3/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^5/d
```

### 3.115.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(173) = 346$.

Time = 0.38 (sec) , antiderivative size = 478, normalized size of antiderivative = 2.64

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{(ag+bgx)^{4}}\,dx$$
$$=\frac{6\left(Bb^{3}cd^{2}-Bab^{2}d^{3}\right)inx^{2}-\left(4\,Bb^{3}c^{3}-9\,Bab^{2}c^{2}d+5\,Ba^{3}d^{3}\right)in-6\left(2\,Ab^{3}c^{3}-3\,Aab^{2}c^{2}d+Aa^{3}d^{3}\right)i-3\left(\right.}{}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, al
gorithm="fricas")
```

output
```
1/36*(6*(B*b^3*c*d^2 - B*a*b^2*d^3)*i*n*x^2 - (4*B*b^3*c^3 - 9*B*a*b^2*c^2
*d + 5*B*a^3*d^3)*i*n - 6*(2*A*b^3*c^3 - 3*A*a*b^2*c^2*d + A*a^3*d^3)*i -
3*((B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b*d^3)*i*n + 6*(A*b^3*c^2*d -
2*A*a*b^2*c*d^2 + A*a^2*b*d^3)*i)*x - 6*(3*(B*b^3*c^2*d - 2*B*a*b^2*c*d^2
+ B*a^2*b*d^3)*i*x + (2*B*b^3*c^3 - 3*B*a*b^2*c^2*d + B*a^3*d^3)*i)*log(e)
 + 6*(B*b^3*d^3*i*n*x^3 + 3*B*a*b^2*d^3*i*n*x^2 - 3*(B*b^3*c^2*d - 2*B*a*b
^2*c*d^2)*i*n*x - (2*B*b^3*c^3 - 3*B*a*b^2*c^2*d)*i*n)*log((b*x + a)/(d*x
+ c)))/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a
^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b
^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4)
```

## 3.115.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x)
```

output
```
Timed out
```

### 3.115.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(173) = 346$.

Time = 0.22 (sec) , antiderivative size = 945, normalized size of antiderivative = 5.22

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{1}{18}\,Bcin\left(\frac{6\,b^2d^2x^2+2\,b^2c^2-7\,abcd+11\,a^2d^2-3\,(b^2cd-5\,abd^2\cdots}{(b^6c^2-2\,ab^5cd+a^2b^4d^2)g^4x^3+3\,(ab^5c^2-2\,a^2b^4cd+a^3b^3d^2)g^4x^2+3\,(a^2b^4c^2-2\,a^3b^3cd+a\cdots}\right.$$
$$-\frac{1}{36}\,Bdin\left(\frac{5\,ab^2c^2-22\,a^2bcd+5\,a^3d^2-6\,(3\,b^3cd-ab^2d^2)x^2+3\,(3\,b^3c^2-16\,a\cdots}{(b^7c^2-2\,ab^6cd+a^2b^5d^2)g^4x^3+3\,(ab^6c^2-2\,a^2b^5cd+a^3b^4d^2)g^4x^2+3\,(a^2b^5c^2-2\,a^3b^4cd+a\cdots}\right.$$
$$-\frac{(3\,bx+a)Bdi\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{6\,(b^5g^4x^3+3\,ab^4g^4x^2+3\,a^2b^3g^4x+a^3b^2g^4)}$$
$$-\frac{(3\,bx+a)Adi}{6\,(b^5g^4x^3+3\,ab^4g^4x^2+3\,a^2b^3g^4x+a^3b^2g^4)}$$
$$-\frac{Bci\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{3\,(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4)}-\frac{Aci}{3\,(b^4g^4x^3+3\,ab^3g^4x^2+3\,a^2b^2g^4x+a^3bg^4)}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, al
gorithm="maxima")
```

output
```
-1/18*B*c*i*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^
2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(
a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*
b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g
^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^3
*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*
d^2 - a^3*b*d^3)*g^4)) - 1/36*B*d*i*n*((5*a*b^2*c^2 - 22*a^2*b*c*d + 5*a^3
*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*d + 5*a^2
*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^2 -
 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d + a
^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4) - 6*(
3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^
2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 - 3*a
*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/6*(3*b*x + a)*B*d*i*
log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^5*g^4*x^3 + 3*a*b^4*g^4*x^2 + 3*
a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/6*(3*b*x + a)*A*d*i/(b^5*g^4*x^3 + 3*a*b^
4*g^4*x^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/3*B*c*i*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n)/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3
*b*g^4) - 1/3*A*c*i/(b^4*g^4*x^3 + 3*a*b^3*g^4*x^2 + 3*a^2*b^2*g^4*x + a^3
*b*g^4)
```

### 3.115.8 Giac [A] (verification not implemented)

Time = 1.08 (sec) , antiderivative size = 234, normalized size of antiderivative = 1.29

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{1}{36}\left(\frac{6\left(2\,Bbin-\frac{3\,(bx+a)Bdin}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3bcg^4}{(dx+c)^3}-\frac{(bx+a)^3adg^4}{(dx+c)^3}}+\frac{4\,Bbin-\frac{9\,(bx+a)Bdin}{dx+c}+12\,Bbi\log\left(e\right)-\frac{18\,(bx+a)Bdi\log(e)}{dx+c}+1}{\frac{(bx+a)^3bcg^4}{(dx+c)^3}-\frac{(bx+a)^3adg^4}{(dx+c)^3}}\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x, al
gorithm="giac")
```

output
```
-1/36*(6*(2*B*b*i*n - 3*(b*x + a)*B*d*i*n/(d*x + c))*log((b*x + a)/(d*x +
c))/((b*x + a)^3*b*c*g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^4/(d*x + c)^3) +
(4*B*b*i*n - 9*(b*x + a)*B*d*i*n/(d*x + c) + 12*B*b*i*log(e) - 18*(b*x + a
)*B*d*i*log(e)/(d*x + c) + 12*A*b*i - 18*(b*x + a)*A*d*i/(d*x + c))/((b*x
+ a)^3*b*c*g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^4/(d*x + c)^3))*(b*c/(b*c -
 a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.115.9 Mupad [B] (verification not implemented)

Time = 2.12 (sec) , antiderivative size = 374, normalized size of antiderivative = 2.07

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{\frac{6\,A\,a^2\,d^2\,i-12\,A\,b^2\,c^2\,i+5\,B\,a^2\,d^2\,i\,n-4\,B\,b^2\,c^2\,i\,n+6\,A\,a\,b\,c\,d\,i+5\,B\,a\,b\,c\,d\,i\,n}{6\,(a\,d-b\,c)}+\frac{x\,(6\,A\,a\,b\,d^2\,i-6\,A\,b^2\,c\,d\,i-B\,b^2\,c\,d\,i\,n+5\,B\,a\,b\,d^2\,i\,n)}{2\,(a\,d-b\,c)}+B}{6\,a^3\,b^2\,g^4+18\,a^2\,b^3\,g^4\,x+18\,a\,b^4\,g^4\,x^2+6\,b^5\,g^4\,x^3}$$

$$-\frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,c\,i}{3\,b}+\frac{B\,a\,d\,i}{6\,b^2}+\frac{B\,d\,i\,x}{2\,b}\right)}{a^3\,g^4+3\,a^2\,b\,g^4\,x+3\,a\,b^2\,g^4\,x^2+b^3\,g^4\,x^3}-\frac{B\,d^3\,i\,n\,\mathrm{atanh}\left(\frac{6\,b^4\,c^2\,g^4-6\,a^2\,b^2\,d^2\,g^4}{6\,b^2\,g^4\,(a\,d-b\,c)^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{3\,b^2\,g^4\,(a\,d-b\,c)^2}$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^4
,x)
```

output
```
- ((6*A*a^2*d^2*i - 12*A*b^2*c^2*i + 5*B*a^2*d^2*i*n - 4*B*b^2*c^2*i*n + 6
*A*a*b*c*d*i + 5*B*a*b*c*d*i*n)/(6*(a*d - b*c)) + (x*(6*A*a*b*d^2*i - 6*A*
b^2*c*d*i - B*b^2*c*d*i*n + 5*B*a*b*d^2*i*n))/(2*(a*d - b*c)) + (B*b^2*d^2
*i*n*x^2)/(a*d - b*c))/(6*a^3*b^2*g^4 + 6*b^5*g^4*x^3 + 18*a^2*b^3*g^4*x +
 18*a*b^4*g^4*x^2) - (log(e*((a + b*x)/(c + d*x))^n)*((B*c*i)/(3*b) + (B*a
*d*i)/(6*b^2) + (B*d*i*x)/(2*b)))/(a^3*g^4 + b^3*g^4*x^3 + 3*a*b^2*g^4*x^2
 + 3*a^2*b*g^4*x) - (B*d^3*i*n*atanh((6*b^4*c^2*g^4 - 6*a^2*b^2*d^2*g^4)/(
6*b^2*g^4*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(3*b^2*g^4*(a*d - b*c)^
2)
```

# 3.116 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$



## 3.116.1 Optimal result

Integrand size = 41, antiderivative size = 281

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = -\frac{Bd^2in(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2} + \frac{2bBdin(c+dx)^3}{9(bc-ad)^3g^5(a+bx)^3} - \frac{b^2Bin(c+dx)^4}{16(bc-ad)^3g^5(a+bx)^4} - \frac{d^2i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^5(a+bx)^2} + \frac{2bdi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^5(a+bx)^3} - \frac{b^2i(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^3g^5(a+bx)^4}$$

output
```
-1/4*B*d^2*i*n*(d*x+c)^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+2/9*b*B*d*i*n*(d*x+c)^
3/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/16*b^2*B*i*n*(d*x+c)^4/(-a*d+b*c)^3/g^5/(b*
x+a)^4-1/2*d^2*i*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^
5/(b*x+a)^2+2/3*b*d*i*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)
^3/g^5/(b*x+a)^3-1/4*b^2*i*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d
+b*c)^3/g^5/(b*x+a)^4
```

## 3.116.2 Mathematica [A] (verified)

Time = 0.32 (sec) , antiderivative size = 220, normalized size of antiderivative = 0.78

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{i\left(\frac{36Abc}{(a+bx)^4}-\frac{36aAd}{(a+bx)^4}+\frac{9bBcn}{(a+bx)^4}-\frac{9aBdn}{(a+bx)^4}+\frac{48Ad}{(a+bx)^3}+\frac{4Bdn}{(a+bx)^3}-\frac{6Bd^2n}{(bc-ad)(a+bx)^2}+\frac{12Bd^3n}{(bc-ad)^2(a+bx)}+\frac{12Bd^4n\log(a+b}{(bc-ad)^3}\right.}{144b^2g^5}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*
g*x)^5,x]
```

output
```
-1/144*(i*((36*A*b*c)/(a + b*x)^4 - (36*a*A*d)/(a + b*x)^4 + (9*b*B*c*n)/(
a + b*x)^4 - (9*a*B*d*n)/(a + b*x)^4 + (48*A*d)/(a + b*x)^3 + (4*B*d*n)/(a
 + b*x)^3 - (6*B*d^2*n)/((b*c - a*d)*(a + b*x)^2) + (12*B*d^3*n)/((b*c - a
*d)^2*(a + b*x)) + (12*B*d^4*n*Log[a + b*x])/(b*c - a*d)^3 + (12*B*(3*b*c
+ a*d + 4*b*d*x)*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^4 - (12*B*d^4*n
*Log[c + d*x])/(b*c - a*d)^3))/(b^2*g^5)
```

## 3.116.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 211, normalized size of antiderivative = 0.75, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2961, 2772, 27, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^5}\,dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^3}$$

$$\Big\downarrow \text{2772}$$

$$\frac{i\left(-Bn\int -\frac{(c+dx)^5\left(3b^2-\frac{8d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{12(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}\right.}{g^5(bc-ad)^3}$$

$\downarrow$ 27

$$\frac{i\left(\frac{1}{12}Bn\int \frac{(c+dx)^5\left(3b^2-\frac{8d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\right.}{g^5(bc-ad)^3}$$

$\downarrow$ 1140

$$\frac{i\left(\frac{1}{12}Bn\int \left(\frac{3b^2(c+dx)^5}{(a+bx)^5}-\frac{8bd(c+dx)^4}{(a+bx)^4}+\frac{6d^2(c+dx)^3}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{2(a+bx)^2}\right.}{g^5(bc-ad)^3}$$

$\downarrow$ 2009

$$\frac{i\left(-\frac{b^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{2bd(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{1}{12}Bn\left(-\frac{3b^2(c+}{4(a+}\right.\right.}{g^5(bc-ad)^3}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)^5
,x]
```

output
```
(i*((B*n*((-3*d^2*(c + d*x)^2)/(a + b*x)^2 + (8*b*d*(c + d*x)^3)/(3*(a + b
*x)^3) - (3*b^2*(c + d*x)^4)/(4*(a + b*x)^4)))/12 - (d^2*(c + d*x)^2*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + (2*b*d*(c + d*x)^3*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3) - (b^2*(c + d*x)^4*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*(a + b*x)^4)))/((b*c - a*d)^3*g^
5)
```

## 3.116. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$

#### 3.116.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.116.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 985 vs. 2(269) = 538.

Time = 27.71 (sec) , antiderivative size = 986, normalized size of antiderivative = 3.51

| method | result |
|---|---|
| parallelrisch | $\frac{48A\,x^{3}a^{7}bc\,d^{4}in-288A\,x^{3}a^{5}b^{3}c^{3}d^{2}in-144Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{7}b\,c^{3}d^{2}in+48Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{6}b^{2}c^{4}din+384A\,x^{3}a^{4}b^{4}c^{4}din}{}$ |

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,method=_RE
TURNVERBOSE)
```

---

3.116. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{(ag+bgx)^{5}}\,dx$

output
```
1/144*(48*A*x^3*a^7*b*c*d^4*i*n-288*A*x^3*a^5*b^3*c^3*d^2*i*n-144*B*x*ln(e
*((b*x+a)/(d*x+c))^n)*a^7*b*c^3*d^2*i*n+48*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a
^6*b^2*c^4*d*i*n+384*A*x^3*a^4*b^4*c^4*d*i*n+72*B*x^2*ln(e*((b*x+a)/(d*x+c
))^n)*a^8*c*d^4*i*n+48*B*x^2*a^7*b*c^2*d^3*i*n^2-222*B*x^2*a^6*b^2*c^3*d^2
*i*n^2+192*B*x^2*a^5*b^3*c^4*d*i*n^2-432*A*x^2*a^6*b^2*c^3*d^2*i*n+576*A*x
^2*a^5*b^3*c^4*d*i*n+144*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^8*c^2*d^3*i*n-168
*B*x*a^7*b*c^3*d^2*i*n^2+132*B*x*a^6*b^2*c^4*d*i*n^2-432*A*x*a^7*b*c^3*d^2
*i*n+432*A*x*a^6*b^2*c^4*d*i*n-96*B*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^4*d*
i*n+12*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c*d^4*i*n+48*B*x^3*ln(e*((b
*x+a)/(d*x+c))^n)*a^7*b*c*d^4*i*n+13*B*x^4*a^6*b^2*c*d^4*i*n^2-36*B*x^4*a^
4*b^4*c^3*d^2*i*n^2+32*B*x^4*a^3*b^5*c^4*d*i*n^2+12*A*x^4*a^6*b^2*c*d^4*i*
n-72*A*x^4*a^4*b^4*c^3*d^2*i*n+96*A*x^4*a^3*b^5*c^4*d*i*n+40*B*x^3*a^7*b*c
*d^4*i*n^2+12*B*x^3*a^6*b^2*c^2*d^3*i*n^2-144*B*x^3*a^5*b^3*c^3*d^2*i*n^2+
128*B*x^3*a^4*b^4*c^4*d*i*n^2-9*B*x^4*a^2*b^6*c^5*i*n^2-36*A*x^4*a^2*b^6*c
^5*i*n-36*B*x^3*a^3*b^5*c^5*i*n^2-144*A*x^3*a^3*b^5*c^5*i*n+36*B*x^2*a^8*c
*d^4*i*n^2-54*B*x^2*a^4*b^4*c^5*i*n^2+72*A*x^2*a^8*c*d^4*i*n-216*A*x^2*a^4
*b^4*c^5*i*n+72*B*x*a^8*c^2*d^3*i*n^2-36*B*x*a^5*b^3*c^5*i*n^2+144*A*x*a^8
*c^2*d^3*i*n-144*A*x*a^5*b^3*c^5*i*n+72*B*ln(e*((b*x+a)/(d*x+c))^n)*a^8*c^
3*d^2*i*n+36*B*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^5*i*n)/g^5/(b*x+a)^4/n/
(a^3*d^3-3*a^2*b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/a^6/c
```

### 3.116.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 773 vs. $2(269) = 538$.

Time = 0.35 (sec) , antiderivative size = 773, normalized size of antiderivative = 2.75

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{12\,(Bb^4cd^3 - Bab^3d^4)inx^3 - 6\,(Bb^4c^2d^2 - 8\,Bab^3cd^3 + 7\,Ba^2b^2d^4)inx^2 + (9\,Bb^4c^4 - 32\,Bab^3c^3d + 36\,B}{}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, al
gorithm="fricas")
```

output
```
-1/144*(12*(B*b^4*c*d^3 - B*a*b^3*d^4)*i*n*x^3 - 6*(B*b^4*c^2*d^2 - 8*B*a*
b^3*c*d^3 + 7*B*a^2*b^2*d^4)*i*n*x^2 + (9*B*b^4*c^4 - 32*B*a*b^3*c^3*d + 3
6*B*a^2*b^2*c^2*d^2 - 13*B*a^4*d^4)*i*n + 12*(3*A*b^4*c^4 - 8*A*a*b^3*c^3*
d + 6*A*a^2*b^2*c^2*d^2 - A*a^4*d^4)*i + 4*((B*b^4*c^3*d - 6*B*a*b^3*c^2*d
^2 + 18*B*a^2*b^2*c*d^3 - 13*B*a^3*b*d^4)*i*n + 12*(A*b^4*c^3*d - 3*A*a*b^
3*c^2*d^2 + 3*A*a^2*b^2*c*d^3 - A*a^3*b*d^4)*i)*x + 12*(4*(B*b^4*c^3*d - 3
*B*a*b^3*c^2*d^2 + 3*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*i*x + (3*B*b^4*c^4 - 8
*B*a*b^3*c^3*d + 6*B*a^2*b^2*c^2*d^2 - B*a^4*d^4)*i)*log(e) + 12*(B*b^4*d^
4*i*n*x^4 + 4*B*a*b^3*d^4*i*n*x^3 + 6*B*a^2*b^2*d^4*i*n*x^2 + 4*(B*b^4*c^3
*d - 3*B*a*b^3*c^2*d^2 + 3*B*a^2*b^2*c*d^3)*i*n*x + (3*B*b^4*c^4 - 8*B*a*b
^3*c^3*d + 6*B*a^2*b^2*c^2*d^2)*i*n)*log((b*x + a)/(d*x + c)))/((b^9*c^3 -
 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3
*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 -
 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3
 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 -
 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2*d^3)*g^5)
```

### 3.116.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**5,x)
```

output
```
Timed out
```

### 3.116.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1398 vs. $2(269) = 538$.

Time = 0.25 (sec) , antiderivative size = 1398, normalized size of antiderivative = 4.98

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx=\text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, al
gorithm="maxima")
```

output
```
1/48*B*c*i*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*d^
2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^2*
c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a^3
*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a^4
*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 - a
^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2 -
 a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 - a
^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3
*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5*c
^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5)
) - 1/144*B*d*i*n*((7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 13*a
^4*d^3 + 12*(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c*d^
2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d^2
- 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^6*d
^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^5*d
^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*b^4
*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^6*b
^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*b^2
*d^3)*g^5) + 12*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^3*d
 + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b*c*...
```

### 3.116.8 Giac [A] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.40

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{144}\left(\frac{12\left(3\,Bb^2in-\frac{8\,(bx+a)Bbdin}{dx+c}+\frac{6\,(bx+a)^2Bd^2in}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^4b^2c^2g^5}{(dx+c)^4}-\frac{2\,(bx+a)^4abcdg^5}{(dx+c)^4}+\frac{(bx+a)^4a^2d^2g^5}{(dx+c)^4}}+\frac{9\,Bb^2in-\frac{32\,(bx+a)Bbdin}{dx+c}+\frac{36\,(bx+a)^2Bd^2in}{(dx+c)^2}}{}\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x, al
gorithm="giac")
```

output
```
-1/144*(12*(3*B*b^2*i*n - 8*(b*x + a)*B*b*d*i*n/(d*x + c) + 6*(b*x + a)^2*
B*d^2*i*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^4*b^2*c^2*g^5/(
d*x + c)^4 - 2*(b*x + a)^4*a*b*c*d*g^5/(d*x + c)^4 + (b*x + a)^4*a^2*d^2*g
^5/(d*x + c)^4) + (9*B*b^2*i*n - 32*(b*x + a)*B*b*d*i*n/(d*x + c) + 36*(b*
x + a)^2*B*d^2*i*n/(d*x + c)^2 + 36*B*b^2*i*log(e) - 96*(b*x + a)*B*b*d*i*
log(e)/(d*x + c) + 72*(b*x + a)^2*B*d^2*i*log(e)/(d*x + c)^2 + 36*A*b^2*i
- 96*(b*x + a)*A*b*d*i/(d*x + c) + 72*(b*x + a)^2*A*d^2*i/(d*x + c)^2)/((b
*x + a)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*x + a)^4*a*b*c*d*g^5/(d*x + c)^4
+ (b*x + a)^4*a^2*d^2*g^5/(d*x + c)^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*
d)^2)
```

### 3.116.9 Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 610, normalized size of antiderivative = 2.17

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{B\,d^4\,i\,n\,\mathrm{atanh}\left(\frac{12\,a^3\,b^2\,d^3\,g^5-12\,a^2\,b^3\,c\,d^2\,g^5-12\,a\,b^4\,c^2\,d\,g^5+12\,b^5\,c^3\,g^5}{12\,b^2\,g^5\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{6\,b^2\,g^5\,(a\,d-b\,c)^3}$$
$$-\frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B\,c\,i}{4\,b}+\frac{B\,a\,d\,i}{12\,b^2}+\frac{B\,d\,i\,x}{3\,b}\right)}{a^4\,g^5+4\,a^3\,b\,g^5\,x+6\,a^2\,b^2\,g^5\,x^2+4\,a\,b^3\,g^5\,x^3+b^4\,g^5\,x^4}$$
$$-\frac{\frac{12\,A\,a^3\,d^3\,i+36\,A\,b^3\,c^3\,i+13\,B\,a^3\,d^3\,i\,n+9\,B\,b^3\,c^3\,i\,n-60\,A\,a\,b^2\,c^2\,d\,i+12\,A\,a^2\,b\,c\,d^2\,i-23\,B\,a\,b^2\,c^2\,d\,i\,n+13\,B\,a^2\,b\,c\,d^2\,i\,n}{12\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{x\,(12\,A\,a^2\,b\,d^3\,i+}{}}{12\,a^4\,b^2\,g^5+48\,a^3\,b^3\,g^5\,x+72\,a^2\,b^4}$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)^5
,x)
```

output
```
(B*d^4*i*n*atanh((12*b^5*c^3*g^5 + 12*a^3*b^2*d^3*g^5 - 12*a*b^4*c^2*d*g^5
 - 12*a^2*b^3*c*d^2*g^5)/(12*b^2*g^5*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(6*b^2*g^5*(a*d - b*c)^3) - (log(e*(
(a + b*x)/(c + d*x))^n)*((B*c*i)/(4*b) + (B*a*d*i)/(12*b^2) + (B*d*i*x)/(3
*b)))/(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2*g^5*x^2 + 4*a^3
*b*g^5*x) - ((12*A*a^3*d^3*i + 36*A*b^3*c^3*i + 13*B*a^3*d^3*i*n + 9*B*b^3
*c^3*i*n - 60*A*a*b^2*c^2*d*i + 12*A*a^2*b*c*d^2*i - 23*B*a*b^2*c^2*d*i*n
+ 13*B*a^2*b*c*d^2*i*n)/(12*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(12*A*a^
2*b*d^3*i + 12*A*b^3*c^2*d*i - 24*A*a*b^2*c*d^2*i + 13*B*a^2*b*d^3*i*n + B
*b^3*c^2*d*i*n - 5*B*a*b^2*c*d^2*i*n))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))
 - (d*x^2*(B*b^3*c*d*i*n - 7*B*a*b^2*d^2*i*n))/(2*(a^2*d^2 + b^2*c^2 - 2*a
*b*c*d)) + (B*b^3*d^3*i*n*x^3)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(12*a^4*b^
2*g^5 + 12*b^6*g^5*x^4 + 48*a^3*b^3*g^5*x + 48*a*b^5*g^5*x^3 + 72*a^2*b^4*
g^5*x^2)
```

## 3.116. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$

# 3.117 $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$



## 3.117.1 Optimal result

Integrand size = 43, antiderivative size = 442

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= \frac{B(bc-ad)^5g^3i^2nx}{60b^2d^3}+\frac{B(bc-ad)^4g^3i^2n(c+dx)^2}{120bd^4}-\frac{19B(bc-ad)^3g^3i^2n(c+dx)^3}{180d^4} \\
&\quad+\frac{13bB(bc-ad)^2g^3i^2n(c+dx)^4}{120d^4}-\frac{b^2B(bc-ad)g^3i^2n(c+dx)^5}{30d^4} \\
&\quad-\frac{(bc-ad)^3g^3i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^4} \\
&\quad+\frac{3b(bc-ad)^2g^3i^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^4} \\
&\quad-\frac{3b^2(bc-ad)g^3i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^4} \\
&\quad+\frac{b^3g^3i^2(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^4} \\
&\quad+\frac{B(bc-ad)^6g^3i^2n\log\left(\frac{a+bx}{c+dx}\right)}{60b^3d^4}+\frac{B(bc-ad)^6g^3i^2n\log(c+dx)}{60b^3d^4}
\end{aligned}$$

output
```
1/60*B*(-a*d+b*c)^5*g^3*i^2*n*x/b^2/d^3+1/120*B*(-a*d+b*c)^4*g^3*i^2*n*(d*
x+c)^2/b/d^4-19/180*B*(-a*d+b*c)^3*g^3*i^2*n*(d*x+c)^3/d^4+13/120*b*B*(-a*
d+b*c)^2*g^3*i^2*n*(d*x+c)^4/d^4-1/30*b^2*B*(-a*d+b*c)*g^3*i^2*n*(d*x+c)^5
/d^4-1/3*(-a*d+b*c)^3*g^3*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^
4+3/4*b*(-a*d+b*c)^2*g^3*i^2*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4
-3/5*b^2*(-a*d+b*c)*g^3*i^2*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+
1/6*b^3*g^3*i^2*(d*x+c)^6*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+1/60*B*(-a*d
+b*c)^6*g^3*i^2*n*ln((b*x+a)/(d*x+c))/b^3/d^4+1/60*B*(-a*d+b*c)^6*g^3*i^2*
n*ln(d*x+c)/b^3/d^4
```

### 3.117.2 Mathematica [A] (verified)

Time = 0.26 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.00

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{g^3i^2\left(90d^4(bc-ad)^2(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+144d^5(bc-ad)(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]),x]
```

output
```
(g^3*i^2*(90*d^4*(b*c - a*d)^2*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 144*d^5*(b*c - a*d)*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]) + 60*d^6*(a + b*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 15*B*
(b*c - a*d)^3*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2
+ 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 12*B*(b*c - a*d)^2*n
*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c -
a*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]) - B*
(b*c - a*d)*n*(60*b*d*(b*c - a*d)^4*x + 30*d^2*(-(b*c) + a*d)^3*(a + b*x)^
2 + 20*d^3*(b*c - a*d)^2*(a + b*x)^3 + 15*d^4*(-(b*c) + a*d)*(a + b*x)^4 +
 12*d^5*(a + b*x)^5 - 60*(b*c - a*d)^5*Log[c + d*x]))/(360*b^3*d^4)
```

### 3.117.3 Rubi [A] (verified)

Time = 0.63 (sec) , antiderivative size = 388, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2961, 2782, 27, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2961}$$

$$g^3i^2(bc-ad)^6\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2782}$$

$$g^3i^2(bc-ad)^6\left(-Bn\int -\frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{60d^4(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}+\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\right.$$

$$\Big\downarrow \text{27}$$

$$g^3i^2(bc-ad)^6\left(\frac{Bn\int \frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{60d^4}+\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3b^2\left(B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{5d^4\left(b-\frac{d(a}{c}\right.}\right.$$

$$\Big\downarrow \text{2123}$$

$$g^3i^2(bc-ad)^6\left(\frac{Bn\int \left(-\frac{10db^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}+\frac{26db}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{19d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3b}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2b^2}+\frac{c+dx}{(a+bx)b^3}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{60d^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$g^3i^2(bc-ad)^6\left(\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^4\left(b-\right.}\right.$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
,x]
```

output
```
(b*c - a*d)^6*g^3*i^2*((b^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*d^4
*(b - (d*(a + b*x))/(c + d*x))^6) - (3*b^2*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(5*d^4*(b - (d*(a + b*x))/(c + d*x))^5) + (3*b*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(4*d^4*(b - (d*(a + b*x))/(c + d*x))^4) - (A + B*Log
[e*((a + b*x)/(c + d*x))^n])/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) + (B*
n*((-2*b^2)/(b - (d*(a + b*x))/(c + d*x))^5 + (13*b)/(2*(b - (d*(a + b*x))
/(c + d*x))^4) - 19/(3*(b - (d*(a + b*x))/(c + d*x))^3) + 1/(2*b*(b - (d*(
a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a +
 b*x)/(c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(60*d^4))
```

### 3.117.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.117.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1760 vs. $2(420) = 840$.

Time = 24.52 (sec) , antiderivative size = 1761, normalized size of antiderivative = 3.98

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1761 |

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_
RETURNVERBOSE)
```

output
```
1/360*(540*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c^2*d^4*g^3*i^2*n+360*B
*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^3*c^2*d^4*g^3*i^2*n+540*B*x^4*ln(e*((b*
x+a)/(d*x+c))^n)*a*b^5*c*d^5*g^3*i^2*n+6*B*a^6*d^6*g^3*i^2*n^2+6*B*b^6*c^6
*g^3*i^2*n^2+120*A*x^3*a^3*b^3*d^6*g^3*i^2*n+720*B*x^3*ln(e*((b*x+a)/(d*x+
c))^n)*a^2*b^4*c*d^5*g^3*i^2*n+360*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c
^2*d^4*g^3*i^2*n+360*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^3*c*d^5*g^3*i^2
*n+36*B*x*a*b^5*c^4*d^2*g^3*i^2*n^2+360*A*x*a^3*b^3*c^2*d^4*g^3*i^2*n+120*
B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^3*c^3*d^3*g^3*i^2*n-90*B*ln(e*((b*x+a)/(
d*x+c))^n)*a^2*b^4*c^4*d^2*g^3*i^2*n+36*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*
c^5*d*g^3*i^2*n-36*B*ln(b*x+a)*a^5*b*c*d^5*g^3*i^2*n^2+90*B*ln(b*x+a)*a^4*
b^2*c^2*d^4*g^3*i^2*n^2-120*B*ln(b*x+a)*a^3*b^3*c^3*d^3*g^3*i^2*n^2+90*B*l
n(b*x+a)*a^2*b^4*c^4*d^2*g^3*i^2*n^2-36*B*ln(b*x+a)*a*b^5*c^5*d*g^3*i^2*n^
2+360*A*x^3*a*b^5*c^2*d^4*g^3*i^2*n+102*B*x^2*a^3*b^3*c*d^5*g^3*i^2*n^2-90
*B*x^2*a^2*b^4*c^2*d^4*g^3*i^2*n^2-18*B*x^2*a*b^5*c^3*d^3*g^3*i^2*n^2+360*
A*x^2*a^3*b^3*c*d^5*g^3*i^2*n+540*A*x^2*a^2*b^4*c^2*d^4*g^3*i^2*n+36*B*x*a
^4*b^2*c*d^5*g^3*i^2*n^2+30*B*x*a^3*b^3*c^2*d^4*g^3*i^2*n^2-90*B*x*a^2*b^4
*c^3*d^3*g^3*i^2*n^2+216*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^6*g^3*i^2
*n+144*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c*d^5*g^3*i^2*n+270*B*x^4*ln(e*
((b*x+a)/(d*x+c))^n)*a^2*b^4*d^6*g^3*i^2*n+90*B*x^4*ln(e*((b*x+a)/(d*x+c))
^n)*b^6*c^2*d^4*g^3*i^2*n-18*B*x^4*a*b^5*c*d^5*g^3*i^2*n^2+540*A*x^4*a*...
```

### 3.117.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(420) = 840$.

Time = 0.60 (sec) , antiderivative size = 1074, normalized size of antiderivative = 2.43

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{60\,Ab^6d^6g^3i^2x^6+6\,(15\,Ba^4b^2c^2d^4-6\,Ba^5bcd^5+Ba^6d^6)g^3i^2n\log(bx+a)+6\,(Bb^6c^6-6\,Bab^5c^5d+15\,B}{}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="fricas")
```

output
```
1/360*(60*A*b^6*d^6*g^3*i^2*x^6 + 6*(15*B*a^4*b^2*c^2*d^4 - 6*B*a^5*b*c*d^
5 + B*a^6*d^6)*g^3*i^2*n*log(b*x + a) + 6*(B*b^6*c^6 - 6*B*a*b^5*c^5*d + 1
5*B*a^2*b^4*c^4*d^2 - 20*B*a^3*b^3*c^3*d^3)*g^3*i^2*n*log(d*x + c) - 12*((
B*b^6*c*d^5 - B*a*b^5*d^6)*g^3*i^2*n - 6*(2*A*b^6*c*d^5 + 3*A*a*b^5*d^6)*g
^3*i^2)*x^5 - 3*((7*B*b^6*c^2*d^4 + 6*B*a*b^5*c*d^5 - 13*B*a^2*b^4*d^6)*g^
3*i^2*n - 30*(A*b^6*c^2*d^4 + 6*A*a*b^5*c*d^5 + 3*A*a^2*b^4*d^6)*g^3*i^2)*
x^4 - 2*((B*b^6*c^3*d^3 + 39*B*a*b^5*c^2*d^4 - 21*B*a^2*b^4*c*d^5 - 19*B*a
^3*b^3*d^6)*g^3*i^2*n - 60*(3*A*a*b^5*c^2*d^4 + 6*A*a^2*b^4*c*d^5 + A*a^3*
b^3*d^6)*g^3*i^2)*x^3 + 3*((B*b^6*c^4*d^2 - 6*B*a*b^5*c^3*d^3 - 30*B*a^2*b
^4*c^2*d^4 + 34*B*a^3*b^3*c*d^5 + B*a^4*b^2*d^6)*g^3*i^2*n + 60*(3*A*a^2*b
^4*c^2*d^4 + 2*A*a^3*b^3*c*d^5)*g^3*i^2)*x^2 + 6*(60*A*a^3*b^3*c^2*d^4*g^3
*i^2 - (B*b^6*c^5*d - 6*B*a*b^5*c^4*d^2 + 15*B*a^2*b^4*c^3*d^3 - 5*B*a^3*b
^3*c^2*d^4 - 6*B*a^4*b^2*c*d^5 + B*a^5*b*d^6)*g^3*i^2*n)*x + 6*(10*B*b^6*d
^6*g^3*i^2*x^6 + 60*B*a^3*b^3*c^2*d^4*g^3*i^2*x + 12*(2*B*b^6*c*d^5 + 3*B*
a*b^5*d^6)*g^3*i^2*x^5 + 15*(B*b^6*c^2*d^4 + 6*B*a*b^5*c*d^5 + 3*B*a^2*b^4
*d^6)*g^3*i^2*x^4 + 20*(3*B*a*b^5*c^2*d^4 + 6*B*a^2*b^4*c*d^5 + B*a^3*b^3*
d^6)*g^3*i^2*x^3 + 30*(3*B*a^2*b^4*c^2*d^4 + 2*B*a^3*b^3*c*d^5)*g^3*i^2*x^
2)*log(e) + 6*(10*B*b^6*d^6*g^3*i^2*n*x^6 + 60*B*a^3*b^3*c^2*d^4*g^3*i^2*n
*x + 12*(2*B*b^6*c*d^5 + 3*B*a*b^5*d^6)*g^3*i^2*n*x^5 + 15*(B*b^6*c^2*d^4
+ 6*B*a*b^5*c*d^5 + 3*B*a^2*b^4*d^6)*g^3*i^2*n*x^4 + 20*(3*B*a*b^5*c^2*...
```

### 3.117.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x
)
```

output
```
Timed out
```

### 3.117.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1978 vs. 2(420) = 840.

Time = 0.24 (sec) , antiderivative size = 1978, normalized size of antiderivative = 4.48

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="maxima")
```

output
```
1/6*B*b^3*d^2*g^3*i^2*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/6*A*b
^3*d^2*g^3*i^2*x^6 + 2/5*B*b^3*c*d*g^3*i^2*x^5*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n) + 3/5*B*a*b^2*d^2*g^3*i^2*x^5*log(e*(b*x/(d*x + c) + a/(d*x +
c))^n) + 2/5*A*b^3*c*d*g^3*i^2*x^5 + 3/5*A*a*b^2*d^2*g^3*i^2*x^5 + 1/4*B*b
^3*c^2*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*B*a*b^2*c*
d*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/4*B*a^2*b*d^2*g^3
*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^3*c^2*g^3*i^2*x^
4 + 3/2*A*a*b^2*c*d*g^3*i^2*x^4 + 3/4*A*a^2*b*d^2*g^3*i^2*x^4 + B*a*b^2*c^
2*g^3*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*B*a^2*b*c*d*g^3*i
^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*B*a^3*d^2*g^3*i^2*x^3*
log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*b^2*c^2*g^3*i^2*x^3 + 2*A*a^2
*b*c*d*g^3*i^2*x^3 + 1/3*A*a^3*d^2*g^3*i^2*x^3 + 3/2*B*a^2*b*c^2*g^3*i^2*x
^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + B*a^3*c*d*g^3*i^2*x^2*log(e*(b
*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*a^2*b*c^2*g^3*i^2*x^2 + A*a^3*c*d*g
^3*i^2*x^2 - 1/360*B*b^3*d^2*g^3*i^2*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*l
og(d*x + c)/d^6 + (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*
b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*
d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5)) + 1/30*B*b^3*c*d*g^3*i^2*n
*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^
3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d...
```

### 3.117.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5133 vs. $2(420) = 840$.

Time = 1.80 (sec) , antiderivative size = 5133, normalized size of antiderivative = 11.61

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="giac")
```

output
```
-1/360*(6*(B*b^10*c^7*g^3*i^2*n - 7*B*a*b^9*c^6*d*g^3*i^2*n - 6*(b*x + a)*
B*b^9*c^7*d*g^3*i^2*n/(d*x + c) + 21*B*a^2*b^8*c^5*d^2*g^3*i^2*n + 42*(b*x
 + a)*B*a*b^8*c^6*d^2*g^3*i^2*n/(d*x + c) + 15*(b*x + a)^2*B*b^8*c^7*d^2*g
^3*i^2*n/(d*x + c)^2 - 35*B*a^3*b^7*c^4*d^3*g^3*i^2*n - 126*(b*x + a)*B*a^
2*b^7*c^5*d^3*g^3*i^2*n/(d*x + c) - 105*(b*x + a)^2*B*a*b^7*c^6*d^3*g^3*i^
2*n/(d*x + c)^2 - 20*(b*x + a)^3*B*b^7*c^7*d^3*g^3*i^2*n/(d*x + c)^3 + 35*
B*a^4*b^6*c^3*d^4*g^3*i^2*n + 210*(b*x + a)*B*a^3*b^6*c^4*d^4*g^3*i^2*n/(d
*x + c) + 315*(b*x + a)^2*B*a^2*b^6*c^5*d^4*g^3*i^2*n/(d*x + c)^2 + 140*(b
*x + a)^3*B*a*b^6*c^6*d^4*g^3*i^2*n/(d*x + c)^3 - 21*B*a^5*b^5*c^2*d^5*g^3
*i^2*n - 210*(b*x + a)*B*a^4*b^5*c^3*d^5*g^3*i^2*n/(d*x + c) - 525*(b*x +
a)^2*B*a^3*b^5*c^4*d^5*g^3*i^2*n/(d*x + c)^2 - 420*(b*x + a)^3*B*a^2*b^5*c
^5*d^5*g^3*i^2*n/(d*x + c)^3 + 7*B*a^6*b^4*c*d^6*g^3*i^2*n + 126*(b*x + a)
*B*a^5*b^4*c^2*d^6*g^3*i^2*n/(d*x + c) + 525*(b*x + a)^2*B*a^4*b^4*c^3*d^6
*g^3*i^2*n/(d*x + c)^2 + 700*(b*x + a)^3*B*a^3*b^4*c^4*d^6*g^3*i^2*n/(d*x
+ c)^3 - B*a^7*b^3*d^7*g^3*i^2*n - 42*(b*x + a)*B*a^6*b^3*c*d^7*g^3*i^2*n/
(d*x + c) - 315*(b*x + a)^2*B*a^5*b^3*c^2*d^7*g^3*i^2*n/(d*x + c)^2 - 700*
(b*x + a)^3*B*a^4*b^3*c^3*d^7*g^3*i^2*n/(d*x + c)^3 + 6*(b*x + a)*B*a^7*b^
2*d^8*g^3*i^2*n/(d*x + c) + 105*(b*x + a)^2*B*a^6*b^2*c*d^8*g^3*i^2*n/(d*x
 + c)^2 + 420*(b*x + a)^3*B*a^5*b^2*c^2*d^8*g^3*i^2*n/(d*x + c)^3 - 15*(b*
x + a)^2*B*a^7*b*d^9*g^3*i^2*n/(d*x + c)^2 - 140*(b*x + a)^3*B*a^6*b*c*...
```

### 3.117.9 Mupad [B] (verification not implemented)

Time = 2.81 (sec) , antiderivative size = 2555, normalized size of antiderivative = 5.78

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))
,x)
```

output
```
x^2*((a*c*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n - B*b*c*n))/6 -
 (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (b*
g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2*n - 2*B*b^2*c^2*n + 60*
A*a*b*c*d - B*a*b*c*d*n))/5 + A*a*b^2*c*d*g^3*i^2))/(2*b*d) - ((60*a*d + 6
0*b*c)*((g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3*n - B*b^3*c^3*n
 + 48*A*a*b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d
^2*n))/(4*d) + ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c +
 B*a*d*n - B*b*c*n))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a*d +
 60*b*c))/(60*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2*d^2
*n - 2*B*b^2*c^2*n + 60*A*a*b*c*d - B*a*b*c*d*n))/5 + A*a*b^2*c*d*g^3*i^2)
)/(60*b*d) - (a*c*((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b*c + B*a*d*n - B*b*c*n
))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60))/(b*d)))/(120*b*d) + (a*g^3
*i^2*(3*A*a^3*d^3 + 12*A*b^3*c^3 + B*a^3*d^3*n - 3*B*b^3*c^3*n + 54*A*a*b^
2*c^2*d + 36*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 5*B*a^2*b*c*d^2*n))/(6*b*
d)) + x^3*((g^3*i^2*(16*A*a^3*d^3 + 4*A*b^3*c^3 + 3*B*a^3*d^3*n - B*b^3*c^
3*n + 48*A*a*b^2*c^2*d + 72*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d*n + 3*B*a^2*b*
c*d^2*n))/(12*d) + ((60*a*d + 60*b*c)*((((b^2*d*g^3*i^2*(24*A*a*d + 18*A*b
*c + B*a*d*n - B*b*c*n))/6 - (A*b^2*d*g^3*i^2*(60*a*d + 60*b*c))/60)*(60*a
*d + 60*b*c))/(60*b*d) - (b*g^3*i^2*(30*A*a^2*d^2 + 15*A*b^2*c^2 + 3*B*a^2
*d^2*n - 2*B*b^2*c^2*n + 60*A*a*b*c*d - B*a*b*c*d*n))/5 + A*a*b^2*c*d*g...
```

3.117. $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

# 3.118 $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$



## 3.118.1 Optimal result

Integrand size = 43, antiderivative size = 352

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx\\
&=-\frac{B(bc-ad)^4g^2i^2nx}{30b^2d^2}-\frac{B(bc-ad)^3g^2i^2n(c+dx)^2}{60bd^3}+\frac{B(bc-ad)^2g^2i^2n(c+dx)^3}{10d^3}\\
&\quad-\frac{bB(bc-ad)g^2i^2n(c+dx)^4}{20d^3}+\frac{(bc-ad)^2g^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^3}\\
&\quad-\frac{b(bc-ad)g^2i^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^3}\\
&\quad+\frac{b^2g^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3}\\
&\quad-\frac{B(bc-ad)^5g^2i^2n\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3}-\frac{B(bc-ad)^5g^2i^2n\log(c+dx)}{30b^3d^3}
\end{aligned}$$

output
```
-1/30*B*(-a*d+b*c)^4*g^2*i^2*n*x/b^2/d^2-1/60*B*(-a*d+b*c)^3*g^2*i^2*n*(d*
x+c)^2/b/d^3+1/10*B*(-a*d+b*c)^2*g^2*i^2*n*(d*x+c)^3/d^3-1/20*b*B*(-a*d+b*
c)*g^2*i^2*n*(d*x+c)^4/d^3+1/3*(-a*d+b*c)^2*g^2*i^2*(d*x+c)^3*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/d^3-1/2*b*(-a*d+b*c)*g^2*i^2*(d*x+c)^4*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))/d^3+1/5*b^2*g^2*i^2*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))/d^3-1/30*B*(-a*d+b*c)^5*g^2*i^2*n*ln((b*x+a)/(d*x+c))/b^3/d^3-1/30*B*
(-a*d+b*c)^5*g^2*i^2*n*ln(d*x+c)/b^3/d^3
```

### 3.118.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 374, normalized size of antiderivative = 1.06

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{g^2i^2\left(20d^3(bc-ad)^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+30d^4(bc-ad)(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]),x]
```

output
```
(g^2*i^2*(20*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 30*d^4*(b*c - a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]) + 12*d^5*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 10*B*(
b*c - a*d)^3*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Lo
g[c + d*x]) - 5*B*(b*c - a*d)^2*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) +
 a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + B*
(b*c - a*d)*n*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 +
4*d^3*(b*c - a*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c
 + d*x])))/(60*b^3*d^3)
```

### 3.118.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 311, normalized size of antiderivative = 0.88, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2961, 2782, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2961}$$

$$g^2i^2(bc-ad)^5\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2782}$$

$$g^2i^2(bc-ad)^5\left(-Bn\int\frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{30d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{2d^3\left(b-\frac{d(a\right.}{c+}}\right.$$

$$\Big\downarrow \text{27}$$

$$g^2i^2(bc-ad)^5\left(-\frac{Bn\int\frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{30d^3}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right.$$

$$\Big\downarrow \text{1195}$$

$$g^2i^2(bc-ad)^5\left(-\frac{Bn\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{9d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{6bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{30d^3}+\right.$$

$$\Big\downarrow \text{2009}$$

$$g^2i^2(bc-ad)^5\left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\text{lo}}{}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
,x]
```

output
```
(b*c - a*d)^5*g^2*i^2*((b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d^3
*(b - (d*(a + b*x))/(c + d*x))^5) - (b*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]))/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^4) + (A + B*Log[e*((a + b*x)/(c
+ d*x))^n])/(3*d^3*(b - (d*(a + b*x))/(c + d*x))^3) - (B*n*((3*b)/(2*(b -
(d*(a + b*x))/(c + d*x))^4) - 3/(b - (d*(a + b*x))/(c + d*x))^3 + 1/(2*b*(
b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) +
Log[(a + b*x)/(c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(30*
d^3))
```

3.118. $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

### 3.118.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.118.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1273 vs. $2(334) = 668$.

Time = 11.13 (sec) , antiderivative size = 1274, normalized size of antiderivative = 3.62

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1274 |

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_
RETURNVERBOSE)
```

output
```
1/60*(60*A*x^2*a^2*b^3*c*d^4*g^2*i^2*n+60*A*x^2*a*b^4*c^2*d^3*g^2*i^2*n+2*
B*ln(b*x+a)*a^5*d^5*g^2*i^2*n^2+12*A*x^5*b^5*d^5*g^2*i^2*n+2*B*ln(e*((b*x+
a)/(d*x+c))^n)*b^5*c^5*g^2*i^2*n-2*B*ln(b*x+a)*b^5*c^5*g^2*i^2*n^2+12*B*x^
5*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g^2*i^2*n+80*B*x^3*ln(e*((b*x+a)/(d*x+
c))^n)*a*b^4*c*d^4*g^2*i^2*n+60*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c*
d^4*g^2*i^2*n+60*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^2*d^3*g^2*i^2*n+6
0*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*c^2*d^3*g^2*i^2*n+2*B*a^5*d^5*g^2*
i^2*n^2-2*B*b^5*c^5*g^2*i^2*n^2+10*B*x*a^3*b^2*c*d^4*g^2*i^2*n^2-10*B*x*a*
b^4*c^3*d^2*g^2*i^2*n^2+60*A*x*a^2*b^3*c^2*d^3*g^2*i^2*n+20*B*ln(e*((b*x+a
)/(d*x+c))^n)*a^2*b^3*c^3*d^2*g^2*i^2*n-10*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b
^4*c^4*d*g^2*i^2*n-10*B*ln(b*x+a)*a^4*b*c*d^4*g^2*i^2*n^2+20*B*ln(b*x+a)*a
^3*b^2*c^2*d^3*g^2*i^2*n^2-20*B*ln(b*x+a)*a^2*b^3*c^3*d^2*g^2*i^2*n^2+10*B
*ln(b*x+a)*a*b^4*c^4*d*g^2*i^2*n^2+30*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^
4*d^5*g^2*i^2*n+30*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^4*g^2*i^2*n+20*
B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^3*d^5*g^2*i^2*n+20*B*x^3*ln(e*((b*x+
a)/(d*x+c))^n)*b^5*c^2*d^3*g^2*i^2*n+80*A*x^3*a*b^4*c*d^4*g^2*i^2*n+15*B*x
^2*a^2*b^3*c*d^4*g^2*i^2*n^2-15*B*x^2*a*b^4*c^2*d^3*g^2*i^2*n^2-9*B*a^4*b*
c*d^4*g^2*i^2*n^2-25*B*a^3*b^2*c^2*d^3*g^2*i^2*n^2+25*B*a^2*b^3*c^3*d^2*g^
2*i^2*n^2+9*B*a*b^4*c^4*d*g^2*i^2*n^2-120*A*a^3*b^2*c^2*d^3*g^2*i^2*n-120*
A*a^2*b^3*c^3*d^2*g^2*i^2*n+3*B*x^4*a*b^4*d^5*g^2*i^2*n^2-3*B*x^4*b^5*c...
```

### 3.118.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 774 vs. $2(334) = 668$.

Time = 0.44 (sec) , antiderivative size = 774, normalized size of antiderivative = 2.20

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{12\,Ab^5d^5g^2i^2x^5+2\,(10\,Ba^3b^2c^2d^3-5\,Ba^4bcd^4+Ba^5d^5)g^2i^2n\log{(bx+a)}-2\,(Bb^5c^5-5\,Bab^4c^4d+10\,B}{}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="fricas")
```

output
```
1/60*(12*A*b^5*d^5*g^2*i^2*x^5 + 2*(10*B*a^3*b^2*c^2*d^3 - 5*B*a^4*b*c*d^4
 + B*a^5*d^5)*g^2*i^2*n*log(b*x + a) - 2*(B*b^5*c^5 - 5*B*a*b^4*c^4*d + 10
*B*a^2*b^3*c^3*d^2)*g^2*i^2*n*log(d*x + c) - 3*((B*b^5*c*d^4 - B*a*b^4*d^5
)*g^2*i^2*n - 10*(A*b^5*c*d^4 + A*a*b^4*d^5)*g^2*i^2)*x^4 - 2*(3*(B*b^5*c^
2*d^3 - B*a^2*b^3*d^5)*g^2*i^2*n - 10*(A*b^5*c^2*d^3 + 4*A*a*b^4*c*d^4 + A
*a^2*b^3*d^5)*g^2*i^2)*x^3 - ((B*b^5*c^3*d^2 + 15*B*a*b^4*c^2*d^3 - 15*B*a
^2*b^3*c*d^4 - B*a^3*b^2*d^5)*g^2*i^2*n - 60*(A*a*b^4*c^2*d^3 + A*a^2*b^3*
c*d^4)*g^2*i^2)*x^2 + 2*(30*A*a^2*b^3*c^2*d^3*g^2*i^2 + (B*b^5*c^4*d - 5*B
*a*b^4*c^3*d^2 + 5*B*a^3*b^2*c*d^4 - B*a^4*b*d^5)*g^2*i^2*n)*x + 2*(6*B*b^
5*d^5*g^2*i^2*x^5 + 30*B*a^2*b^3*c^2*d^3*g^2*i^2*x + 15*(B*b^5*c*d^4 + B*a
*b^4*d^5)*g^2*i^2*x^4 + 10*(B*b^5*c^2*d^3 + 4*B*a*b^4*c*d^4 + B*a^2*b^3*d^
5)*g^2*i^2*x^3 + 30*(B*a*b^4*c^2*d^3 + B*a^2*b^3*c*d^4)*g^2*i^2*x^2)*log(e
) + 2*(6*B*b^5*d^5*g^2*i^2*n*x^5 + 30*B*a^2*b^3*c^2*d^3*g^2*i^2*n*x + 15*(
B*b^5*c*d^4 + B*a*b^4*d^5)*g^2*i^2*n*x^4 + 10*(B*b^5*c^2*d^3 + 4*B*a*b^4*c
*d^4 + B*a^2*b^3*d^5)*g^2*i^2*n*x^3 + 30*(B*a*b^4*c^2*d^3 + B*a^2*b^3*c*d^
4)*g^2*i^2*n*x^2)*log((b*x + a)/(d*x + c)))/(b^3*d^3)
```

### 3.118.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x
)
```

output
```
Timed out
```

### 3.118.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. $2(334) = 668$.

Time = 0.23 (sec) , antiderivative size = 1336, normalized size of antiderivative = 3.80

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="maxima")
```

output
```
1/5*B*b^2*d^2*g^2*i^2*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*b
^2*d^2*g^2*i^2*x^5 + 1/2*B*b^2*c*d*g^2*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n) + 1/2*B*a*b*d^2*g^2*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + 1/2*A*b^2*c*d*g^2*i^2*x^4 + 1/2*A*a*b*d^2*g^2*i^2*x^4 + 1/3*B*b^2*c
^2*g^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 4/3*B*a*b*c*d*g^2*
i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*B*a^2*d^2*g^2*i^2*x^3
*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b^2*c^2*g^2*i^2*x^3 + 4/3*
A*a*b*c*d*g^2*i^2*x^3 + 1/3*A*a^2*d^2*g^2*i^2*x^3 + B*a*b*c^2*g^2*i^2*x^2*
log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + B*a^2*c*d*g^2*i^2*x^2*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n) + A*a*b*c^2*g^2*i^2*x^2 + A*a^2*c*d*g^2*i^2*x^
2 + 1/60*B*b^2*d^2*g^2*i^2*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c
)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3
 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) -
1/12*B*b^2*c*d*g^2*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4
+ (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*
c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/12*B*a*b*d^2*g^2*i^2*n*(6*a^4*log(b*x + a
)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c
^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/6*B*b^2*c^
2*g^2*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d -
 a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 2/3*B*a*b*c*d*g^2...
```

### 3.118.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3352 vs. $2(334) = 668$.

Time = 1.27 (sec) , antiderivative size = 3352, normalized size of antiderivative = 9.52

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="giac")
```

output
```
1/60*(2*(B*b^8*c^6*g^2*i^2*n - 6*B*a*b^7*c^5*d*g^2*i^2*n - 5*(b*x + a)*B*b
^7*c^6*d*g^2*i^2*n/(d*x + c) + 15*B*a^2*b^6*c^4*d^2*g^2*i^2*n + 30*(b*x +
a)*B*a*b^6*c^5*d^2*g^2*i^2*n/(d*x + c) + 10*(b*x + a)^2*B*b^6*c^6*d^2*g^2*
i^2*n/(d*x + c)^2 - 20*B*a^3*b^5*c^3*d^3*g^2*i^2*n - 75*(b*x + a)*B*a^2*b^
5*c^4*d^3*g^2*i^2*n/(d*x + c) - 60*(b*x + a)^2*B*a*b^5*c^5*d^3*g^2*i^2*n/(
d*x + c)^2 + 15*B*a^4*b^4*c^2*d^4*g^2*i^2*n + 100*(b*x + a)*B*a^3*b^4*c^3*
d^4*g^2*i^2*n/(d*x + c) + 150*(b*x + a)^2*B*a^2*b^4*c^4*d^4*g^2*i^2*n/(d*x
 + c)^2 - 6*B*a^5*b^3*c*d^5*g^2*i^2*n - 75*(b*x + a)*B*a^4*b^3*c^2*d^5*g^2
*i^2*n/(d*x + c) - 200*(b*x + a)^2*B*a^3*b^3*c^3*d^5*g^2*i^2*n/(d*x + c)^2
 + B*a^6*b^2*d^6*g^2*i^2*n + 30*(b*x + a)*B*a^5*b^2*c*d^6*g^2*i^2*n/(d*x +
 c) + 150*(b*x + a)^2*B*a^4*b^2*c^2*d^6*g^2*i^2*n/(d*x + c)^2 - 5*(b*x + a
)*B*a^6*b*d^7*g^2*i^2*n/(d*x + c) - 60*(b*x + a)^2*B*a^5*b*c*d^7*g^2*i^2*n
/(d*x + c)^2 + 10*(b*x + a)^2*B*a^6*d^8*g^2*i^2*n/(d*x + c)^2)*log((b*x +
a)/(d*x + c))/(b^5*d^3 - 5*(b*x + a)*b^4*d^4/(d*x + c) + 10*(b*x + a)^2*b^
3*d^5/(d*x + c)^2 - 10*(b*x + a)^3*b^2*d^6/(d*x + c)^3 + 5*(b*x + a)^4*b*d
^7/(d*x + c)^4 - (b*x + a)^5*d^8/(d*x + c)^5) + (2*(b*x + a)*B*b^9*c^6*d*g
^2*i^2*n/(d*x + c) - 12*(b*x + a)*B*a*b^8*c^5*d^2*g^2*i^2*n/(d*x + c) - 9*
(b*x + a)^2*B*b^8*c^6*d^2*g^2*i^2*n/(d*x + c)^2 + 30*(b*x + a)*B*a^2*b^7*c
^4*d^3*g^2*i^2*n/(d*x + c) + 54*(b*x + a)^2*B*a*b^7*c^5*d^3*g^2*i^2*n/(d*x
 + c)^2 + 9*(b*x + a)^3*B*b^7*c^6*d^3*g^2*i^2*n/(d*x + c)^3 - 40*(b*x +...
```

### 3.118.9 Mupad [B] (verification not implemented)

Time = 2.11 (sec) , antiderivative size = 1328, normalized size of antiderivative = 3.77

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))
,x)
```

output
```
x^2*(((30*a*d + 30*b*c)*((((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d*n - B
*b*c*n))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30)*(30*a*d + 30*b*c))/(30*
b*d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 1
8*A*a*b*c*d))/2 + A*a*b*c*d*g^2*i^2))/(60*b*d) - (a*c*((b*d*g^2*i^2*(15*A*
a*d + 15*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))
/30))/(2*b*d) + (g^2*i^2*(3*A*a^3*d^3 + 3*A*b^3*c^3 + B*a^3*d^3*n - B*b^3*
c^3*n + 27*A*a*b^2*c^2*d + 27*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 3*B*a^2*
b*c*d^2*n))/(6*b*d)) + log(e*((a + b*x)/(c + d*x))^n)*((B*g^2*i^2*x^3*(a^2
*d^2 + b^2*c^2 + 4*a*b*c*d))/3 + B*a^2*c^2*g^2*i^2*x + (B*b^2*d^2*g^2*i^2*
x^5)/5 + B*a*c*g^2*i^2*x^2*(a*d + b*c) + (B*b*d*g^2*i^2*x^4*(a*d + b*c))/2
) - x^3*((((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*
b*d*g^2*i^2*(30*a*d + 30*b*c))/30)*(30*a*d + 30*b*c))/(90*b*d) - (g^2*i^2*
(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 18*A*a*b*c*d))/6
+ (A*a*b*c*d*g^2*i^2)/3) + x*((a*c*((((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c +
B*a*d*n - B*b*c*n))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30)*(30*a*d + 30
*b*c))/(30*b*d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*n - B*b^
2*c^2*n + 18*A*a*b*c*d))/2 + A*a*b*c*d*g^2*i^2))/(b*d) - ((30*a*d + 30*b*c
)*(((30*a*d + 30*b*c)*((((b*d*g^2*i^2*(15*A*a*d + 15*A*b*c + B*a*d*n - B*b
*c*n))/5 - (A*b*d*g^2*i^2*(30*a*d + 30*b*c))/30)*(30*a*d + 30*b*c))/(30*b*
d) - (g^2*i^2*(6*A*a^2*d^2 + 6*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + ...
```

3.118. $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) \, dx$

# 3.119 $\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$



## 3.119.1 Optimal result

Integrand size = 41, antiderivative size = 250

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{B(bc-ad)^3gi^2nx}{12b^2d}+\frac{B(bc-ad)^2gi^2n(c+dx)^2}{24bd^2}-\frac{B(bc-ad)gi^2n(c+dx)^3}{12d^2}-\frac{(bc-ad)gi^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^2}+\frac{bgi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2}+\frac{B(bc-ad)^4gi^2n\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2}+\frac{B(bc-ad)^4gi^2n\log(c+dx)}{12b^3d^2}$$

output
```
1/12*B*(-a*d+b*c)^3*g*i^2*n*x/b^2/d+1/24*B*(-a*d+b*c)^2*g*i^2*n*(d*x+c)^2/
b/d^2-1/12*B*(-a*d+b*c)*g*i^2*n*(d*x+c)^3/d^2-1/3*(-a*d+b*c)*g*i^2*(d*x+c)
^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^2+1/4*b*g*i^2*(d*x+c)^4*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/d^2+1/12*B*(-a*d+b*c)^4*g*i^2*n*ln((b*x+a)/(d*x+c))/b^3
/d^2+1/12*B*(-a*d+b*c)^4*g*i^2*n*ln(d*x+c)/b^3/d^2
```

### 3.119.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 224, normalized size of antiderivative = 0.90

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{gi^2\left(\frac{4B(bc-ad)^2n(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx))}{b^3}-\frac{B(bc-ad)n(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-\dots}{b^3}\right.}{24d^2}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]),x]
```

output
```
(g*i^2*((4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b
*c - a*d)^2*Log[a + b*x]))/b^3 - (B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x +
 3*b^2*(b*c - a*d)*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a
 + b*x]))/b^3 - 8*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]) + 6*b*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(24*d^2)
```

### 3.119.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.94, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {2961, 2782, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$
$$\Big\downarrow \text{2961}$$
$$gi^2(bc-ad)^4\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$
$$\Big\downarrow \text{2782}$$
$$gi^2(bc-ad)^4\left(-Bn\int -\frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{12d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)$$

$$\Big\downarrow \ 27$$

$$gi^2(bc-ad)^4\left(\frac{Bn\int \frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)$$

$$\Big\downarrow \ 86$$

$$gi^2(bc-ad)^4\left(\frac{Bn\int \left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.$$

$$\Big\downarrow \ 2009$$

$$gi^2(bc-ad)^4\left(-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^3}+\frac{1}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\right.}{12d^2}\right.$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x
]
```

output
```
(b*c - a*d)^4*g*i^2*((b*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d^2*(b
- (d*(a + b*x))/(c + d*x))^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*
d^2*(b - (d*(a + b*x))/(c + d*x))^3) + (B*n*(-(b - (d*(a + b*x))/(c + d*x)
)^(-3) + 1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x
))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c +
 d*x)]/b^3))/(12*d^2))
```

3.119. $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

### 3.119.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.119.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 836 vs. 2(236) = 472.

Time = 4.76 (sec) , antiderivative size = 837, normalized size of antiderivative = 3.35

| method | result |
|---|---|
| parallelrisch | $\frac{24B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^3c\,d^3g\,i^2n+24Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a\,b^3c^2d^2g\,i^2n+6B\,x^4\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4d^4g\,i^2n-36Aa\,b^3c^3dg\,i^2n+}{}$ |

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RE
TURNVERBOSE)
```

output
```
1/24*(24*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^3*c*d^3*g*i^2*n+24*B*x*ln(e*(
(b*x+a)/(d*x+c))^n)*a*b^3*c^2*d^2*g*i^2*n+6*B*x^4*ln(e*((b*x+a)/(d*x+c))^n
)*b^4*d^4*g*i^2*n-36*A*a*b^3*c^3*d*g*i^2*n+2*B*x^3*a*b^3*d^4*g*i^2*n^2-2*B
*x^3*b^4*c*d^3*g*i^2*n^2+8*A*x^3*a*b^3*d^4*g*i^2*n+16*A*x^3*b^4*c*d^3*g*i^
2*n+B*x^2*a^2*b^2*d^4*g*i^2*n^2-5*B*x^2*b^4*c^2*d^2*g*i^2*n^2+12*A*x^2*b^4
*c^2*d^2*g*i^2*n-2*B*x*a^3*b*d^4*g*i^2*n^2-2*B*x*b^4*c^3*d*g*i^2*n^2+8*B*l
n(e*((b*x+a)/(d*x+c))^n)*a*b^3*c^3*d*g*i^2*n+8*B*x^3*ln(e*((b*x+a)/(d*x+c)
)^n)*a*b^3*d^4*g*i^2*n+16*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3*g*i^2*
n+12*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d^2*g*i^2*n+24*A*x*a*b^3*c^2*
d^2*g*i^2*n-8*B*ln(b*x+a)*a^3*b*c*d^3*g*i^2*n^2+12*B*ln(b*x+a)*a^2*b^2*c^2
*d^2*g*i^2*n^2-8*B*ln(b*x+a)*a*b^3*c^3*d*g*i^2*n^2+4*B*x^2*a*b^3*c*d^3*g*i
^2*n^2+24*A*x^2*a*b^3*c*d^3*g*i^2*n+8*B*x*a^2*b^2*c*d^3*g*i^2*n^2-4*B*x*a*
b^3*c^2*d^2*g*i^2*n^2-7*B*a^3*b*c*d^3*g*i^2*n^2-8*B*a^2*b^2*c^2*d^2*g*i^2*
n^2+11*B*a*b^3*c^3*d*g*i^2*n^2-2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^4*g*i^2
*n+6*A*x^4*b^4*d^4*g*i^2*n+2*B*ln(b*x+a)*b^4*c^4*g*i^2*n^2+2*B*ln(b*x+a)*a
^4*d^4*g*i^2*n^2+2*B*b^4*c^4*g*i^2*n^2+2*B*a^4*d^4*g*i^2*n^2-48*A*a^2*b^2*
c^2*d^2*g*i^2*n)/b^3/d^2/n
```

### 3.119.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(236) = 472$.

Time = 0.40 (sec) , antiderivative size = 530, normalized size of antiderivative = 2.12

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{6\,Ab^4d^4gi^2x^4+2\,(6\,Ba^2b^2c^2d^2-4\,Ba^3bcd^3+Ba^4d^4)gi^2n\log\left(bx+a\right)+2\,(Bb^4c^4-4\,Bab^3c^3d)gi^2n\log\left(d\right.}{}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="fricas")
```

output
```
1/24*(6*A*b^4*d^4*g*i^2*x^4 + 2*(6*B*a^2*b^2*c^2*d^2 - 4*B*a^3*b*c*d^3 + B
*a^4*d^4)*g*i^2*n*log(b*x + a) + 2*(B*b^4*c^4 - 4*B*a*b^3*c^3*d)*g*i^2*n*l
og(d*x + c) - 2*((B*b^4*c*d^3 - B*a*b^3*d^4)*g*i^2*n - 4*(2*A*b^4*c*d^3 +
A*a*b^3*d^4)*g*i^2)*x^3 - ((5*B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 - B*a^2*b^2*
d^4)*g*i^2*n - 12*(A*b^4*c^2*d^2 + 2*A*a*b^3*c*d^3)*g*i^2)*x^2 + 2*(12*A*a
*b^3*c^2*d^2*g*i^2 - (B*b^4*c^3*d + 2*B*a*b^3*c^2*d^2 - 4*B*a^2*b^2*c*d^3
+ B*a^3*b*d^4)*g*i^2*n)*x + 2*(3*B*b^4*d^4*g*i^2*x^4 + 12*B*a*b^3*c^2*d^2*
g*i^2*x + 4*(2*B*b^4*c*d^3 + B*a*b^3*d^4)*g*i^2*x^3 + 6*(B*b^4*c^2*d^2 + 2
*B*a*b^3*c*d^3)*g*i^2*x^2)*log(e) + 2*(3*B*b^4*d^4*g*i^2*n*x^4 + 12*B*a*b^
3*c^2*d^2*g*i^2*n*x + 4*(2*B*b^4*c*d^3 + B*a*b^3*d^4)*g*i^2*n*x^3 + 6*(B*b
^4*c^2*d^2 + 2*B*a*b^3*c*d^3)*g*i^2*n*x^2)*log((b*x + a)/(d*x + c)))/(b^3*
d^2)
```

### 3.119.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1054 vs. $2(233) = 466$.

Time = 83.82 (sec) , antiderivative size = 1054, normalized size of antiderivative = 4.22

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((a*c**2*g*i**2*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)),
(a*g*(A*c**2*i**2*x + A*c*d*i**2*x**2 + A*d**2*i**2*x**3/3 + B*c**3*i**2*l
og(e*(a/(c + d*x))**n)/(3*d) + B*c**2*i**2*n*x/3 + B*c**2*i**2*x*log(e*(a/
(c + d*x))**n) + B*c*d*i**2*n*x**2/3 + B*c*d*i**2*x**2*log(e*(a/(c + d*x))
**n) + B*d**2*i**2*n*x**3/9 + B*d**2*i**2*x**3*log(e*(a/(c + d*x))**n)/3),
 Eq(b, 0)), (c**2*i**2*(A*a*g*x + A*b*g*x**2/2 + B*a**2*g*log(e*(a/c + b*x
/c)**n)/(2*b) - B*a*g*n*x/2 + B*a*g*x*log(e*(a/c + b*x/c)**n) - B*b*g*n*x*
*2/4 + B*b*g*x**2*log(e*(a/c + b*x/c)**n)/2), Eq(d, 0)), (A*a*c**2*g*i**2*
x + A*a*c*d*g*i**2*x**2 + A*a*d**2*g*i**2*x**3/3 + A*b*c**2*g*i**2*x**2/2
+ 2*A*b*c*d*g*i**2*x**3/3 + A*b*d**2*g*i**2*x**4/4 + B*a**4*d**2*g*i**2*n*
log(c/d + x)/(12*b**3) + B*a**4*d**2*g*i**2*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)/(12*b**3) - B*a**3*c*d*g*i**2*n*log(c/d + x)/(3*b**2) - B*a**3*c
*d*g*i**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(3*b**2) - B*a**3*d**2*g
*i**2*n*x/(12*b**2) + B*a**2*c**2*g*i**2*n*log(c/d + x)/(2*b) + B*a**2*c**
2*g*i**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(2*b) + B*a**2*c*d*g*i**2
*n*x/(3*b) + B*a**2*d**2*g*i**2*n*x**2/(24*b) - B*a*c**3*g*i**2*n*log(c/d
+ x)/(3*d) - B*a*c**2*g*i**2*n*x/6 + B*a*c**2*g*i**2*x*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n) + B*a*c*d*g*i**2*n*x**2/6 + B*a*c*d*g*i**2*x**2*log(e
*(a/(c + d*x) + b*x/(c + d*x))**n) + B*a*d**2*g*i**2*n*x**3/12 + B*a*d**2*
g*i**2*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/3 + B*b*c**4*g*i**2...
```

### 3.119.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 740 vs. $2(236) = 472$.

Time = 0.20 (sec) , antiderivative size = 740, normalized size of antiderivative = 2.96

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{1}{4}Bbd^2gi^2x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{4}Abd^2gi^2x^4$$
$$+\frac{2}{3}Bbcdgi^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}Bad^2gi^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+\frac{2}{3}Abcdgi^2x^3+\frac{1}{3}Aad^2gi^2x^3+\frac{1}{2}Bbc^2gi^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+Bacdgi^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{2}Abc^2gi^2x^2+Aacdgi^2x^2$$
$$-\frac{1}{24}Bbd^2gi^2n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c}{b^3d^3}\right.$$
$$+\frac{1}{3}Bbcdgi^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$+\frac{1}{6}Bad^2gi^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-\frac{1}{2}Bbc^2gi^2n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$-Bacdgi^2n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bac^2gi^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bac^2gi^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Aac^2gi^2x$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="maxima")
```

3.119. $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

output
```
1/4*B*b*d^2*g*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b*d^2
*g*i^2*x^4 + 2/3*B*b*c*d*g*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 1/3*B*a*d^2*g*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2/3*A*b*c
*d*g*i^2*x^3 + 1/3*A*a*d^2*g*i^2*x^3 + 1/2*B*b*c^2*g*i^2*x^2*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) + B*a*c*d*g*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) + 1/2*A*b*c^2*g*i^2*x^2 + A*a*c*d*g*i^2*x^2 - 1/24*B*b*d^2*g*i^
2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b
^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^
3*d^3)) + 1/3*B*b*c*d*g*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/
6*B*a*d^2*g*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 1/2*B*b*c^2*g*
i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
- B*a*c*d*g*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*
d)*x/(b*d)) + B*a*c^2*g*i^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*a*
c^2*g*i^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*a*c^2*g*i^2*x
```

### 3.119.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1997 vs. $2(236) = 472$.

Time = 1.00 (sec) , antiderivative size = 1997, normalized size of antiderivative = 7.99

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="giac")
```

output
```
-1/24*(2*(B*b^6*c^5*g*i^2*n - 5*B*a*b^5*c^4*d*g*i^2*n - 4*(b*x + a)*B*b^5*
c^5*d*g*i^2*n/(d*x + c) + 10*B*a^2*b^4*c^3*d^2*g*i^2*n + 20*(b*x + a)*B*a*
b^4*c^4*d^2*g*i^2*n/(d*x + c) - 10*B*a^3*b^3*c^2*d^3*g*i^2*n - 40*(b*x + a
)*B*a^2*b^3*c^3*d^3*g*i^2*n/(d*x + c) + 5*B*a^4*b^2*c*d^4*g*i^2*n + 40*(b*
x + a)*B*a^3*b^2*c^2*d^4*g*i^2*n/(d*x + c) - B*a^5*b*d^5*g*i^2*n - 20*(b*x
 + a)*B*a^4*b*c*d^5*g*i^2*n/(d*x + c) + 4*(b*x + a)*B*a^5*d^6*g*i^2*n/(d*x
 + c))*log((b*x + a)/(d*x + c))/(b^4*d^2 - 4*(b*x + a)*b^3*d^3/(d*x + c) +
 6*(b*x + a)^2*b^2*d^4/(d*x + c)^2 - 4*(b*x + a)^3*b*d^5/(d*x + c)^3 + (b*
x + a)^4*d^6/(d*x + c)^4) - (B*b^8*c^5*g*i^2*n - 5*B*a*b^7*c^4*d*g*i^2*n -
 6*(b*x + a)*B*b^7*c^5*d*g*i^2*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g*i^2*n
+ 30*(b*x + a)*B*a*b^6*c^4*d^2*g*i^2*n/(d*x + c) + 7*(b*x + a)^2*B*b^6*c^5
*d^2*g*i^2*n/(d*x + c)^2 - 10*B*a^3*b^5*c^2*d^3*g*i^2*n - 60*(b*x + a)*B*a
^2*b^5*c^3*d^3*g*i^2*n/(d*x + c) - 35*(b*x + a)^2*B*a*b^5*c^4*d^3*g*i^2*n/
(d*x + c)^2 - 2*(b*x + a)^3*B*b^5*c^5*d^3*g*i^2*n/(d*x + c)^3 + 5*B*a^4*b^
4*c*d^4*g*i^2*n + 60*(b*x + a)*B*a^3*b^4*c^2*d^4*g*i^2*n/(d*x + c) + 70*(b
*x + a)^2*B*a^2*b^4*c^3*d^4*g*i^2*n/(d*x + c)^2 + 10*(b*x + a)^3*B*a*b^4*c
^4*d^4*g*i^2*n/(d*x + c)^3 - B*a^5*b^3*d^5*g*i^2*n - 30*(b*x + a)*B*a^4*b^
3*c*d^5*g*i^2*n/(d*x + c) - 70*(b*x + a)^2*B*a^3*b^3*c^2*d^5*g*i^2*n/(d*x
+ c)^2 - 20*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g*i^2*n/(d*x + c)^3 + 6*(b*x + a
)*B*a^5*b^2*d^6*g*i^2*n/(d*x + c) + 35*(b*x + a)^2*B*a^4*b^2*c*d^6*g*i^...
```

3.119. $\int (ag+bgx)(ci+dix)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\, dx$

### 3.119.9 Mupad [B] (verification not implemented)

Time = 1.78 (sec) , antiderivative size = 661, normalized size of antiderivative = 2.64

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(B\,a\,c^2\,g\,i^2\,x+\frac{B\,c\,g\,i^2\,x^2\,(2\,a\,d+b\,c)}{2}+\frac{B\,d\,g\,i^2\,x^3\,(a\,d+2\,b\,c)}{3}+\frac{B\,b\,d^2\,g\,i^2\,x^4}{4}\right)$$

$$+x^3\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{12}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{36}\right)$$

$$+x\left(\frac{(12\,a\,d+12\,b\,c)\,\left(\frac{\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)(12\,a\,d+12\,b\,c)}{12\,b\,d}-\frac{g\,i^2\,(3\,A\,a^2\,d^2+9\,A\,b^2\,c^2+B\,a\cdots}{}\right.}{12\,b\,d}\right.$$

$$\left.-\frac{a\,c\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)}{b\,d}+\frac{c\,g\,i^2\,(6\,A\,a^2\,d^2+2\,A\,b^2\,c^2+2\,B\,a^2\,d^2\,n-B\,b^2\,c^2\,n+12\,A\,a\,b\,c\,d-B\,a\,b\,c\,d\,n)}{2\,b\,d}\right)$$

$$-x^2\left(\frac{\left(\frac{d\,g\,i^2\,(8\,A\,a\,d+12\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4}-\frac{A\,d\,g\,i^2\,(12\,a\,d+12\,b\,c)}{12}\right)\,(12\,a\,d+12\,b\,c)}{24\,b\,d}-\frac{g\,i^2\,(3\,A\,a^2\,d^2+9\,A\,b^2\,c^2+B\,a^2\,d^2\,n-2\,B\,b^2\,c^2\,n+18\,A\,a\,b\,c\,d+B\,a\,b\,c\,d\,n)}{6\,b}+\frac{A\,a\,c\,d\,g\,i^2}{2}\right)+\frac{\ln(c+d\,x)\,(B\,b\,c^4\,g\,i^2\,n-4\,B\,a\,c^3\,d\,g\,i^2\,n)}{12\,d^2}$$

$$+\frac{\ln(a+b\,x)\,(B\,g\,n\,a^4\,d^2\,i^2-4\,B\,g\,n\,a^3\,b\,c\,d\,i^2+6\,B\,g\,n\,a^2\,b^2\,c^2\,i^2)}{12\,b^3}+\frac{A\,b\,d^2\,g\,i^2\,x^4}{4}$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x
)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*(B*a*c^2*g*i^2*x + (B*c*g*i^2*x^2*(2*a*d +
b*c))/2 + (B*d*g*i^2*x^3*(a*d + 2*b*c))/3 + (B*b*d^2*g*i^2*x^4)/4) + x^3*(
(d*g*i^2*(8*A*a*d + 12*A*b*c + B*a*d*n - B*b*c*n))/12 - (A*d*g*i^2*(12*a*d
 + 12*b*c))/36) + x*(((12*a*d + 12*b*c)*((((d*g*i^2*(8*A*a*d + 12*A*b*c +
B*a*d*n - B*b*c*n))/4 - (A*d*g*i^2*(12*a*d + 12*b*c))/12)*(12*a*d + 12*b*c
))/(12*b*d) - (g*i^2*(3*A*a^2*d^2 + 9*A*b^2*c^2 + B*a^2*d^2*n - 2*B*b^2*c^
2*n + 18*A*a*b*c*d + B*a*b*c*d*n))/(3*b) + A*a*c*d*g*i^2))/(12*b*d) - (a*c
*((d*g*i^2*(8*A*a*d + 12*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*d*g*i^2*(12*a*
d + 12*b*c))/12))/(b*d) + (c*g*i^2*(6*A*a^2*d^2 + 2*A*b^2*c^2 + 2*B*a^2*d^
2*n - B*b^2*c^2*n + 12*A*a*b*c*d - B*a*b*c*d*n))/(2*b*d)) - x^2*((((d*g*i^
2*(8*A*a*d + 12*A*b*c + B*a*d*n - B*b*c*n))/4 - (A*d*g*i^2*(12*a*d + 12*b*
c))/12)*(12*a*d + 12*b*c))/(24*b*d) - (g*i^2*(3*A*a^2*d^2 + 9*A*b^2*c^2 +
B*a^2*d^2*n - 2*B*b^2*c^2*n + 18*A*a*b*c*d + B*a*b*c*d*n))/(6*b) + (A*a*c*
d*g*i^2)/2) + (log(c + d*x)*(B*b*c^4*g*i^2*n - 4*B*a*c^3*d*g*i^2*n))/(12*d
^2) + (log(a + b*x)*(B*a^4*d^2*g*i^2*n + 6*B*a^2*b^2*c^2*g*i^2*n - 4*B*a^3
*b*c*d*g*i^2*n))/(12*b^3) + (A*b*d^2*g*i^2*x^4)/4
```

3.119. $\int (ag + bgx)(ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$

# 3.120 $\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$



## 3.120.1 Optimal result

Integrand size = 33, antiderivative size = 124

$$\begin{aligned}\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\ =-\frac{B(bc-ad)^2i^2nx}{3b^2}-\frac{B(bc-ad)i^2n(c+dx)^2}{6bd}\\ -\frac{B(bc-ad)^3i^2n\log(a+bx)}{3b^3d}+\frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d}\end{aligned}$$

output
```
-1/3*B*(-a*d+b*c)^2*i^2*n*x/b^2-1/6*B*(-a*d+b*c)*i^2*n*(d*x+c)^2/b/d-1/3*B
*(-a*d+b*c)^3*i^2*n*ln(b*x+a)/b^3/d+1/3*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(
d*x+c))^n))/d
```

## 3.120.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 101, normalized size of antiderivative = 0.81

$$\begin{aligned}\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx\\ =\frac{i^2\left(-\frac{B(bc-ad)n(2bd(bc-ad)x+b^2(c+dx)^2+2(bc-ad)^2\log(a+bx))}{2b^3}+(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{3d}\end{aligned}$$

input
```
Integrate[(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(i^2*(-1/2*(B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*
c - a*d)^2*Log[a + b*x]))/b^3 + (c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])))/(3*d)
```

### 3.120.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 109, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^2 \left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) \, dx$$

$$\Big\downarrow \text{2947}$$

$$\frac{i^2 (c+dx)^3 \left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bn(bc-ad)\int \frac{i^3 (c+dx)^2}{a+bx}dx}{3di}$$

$$\Big\downarrow \text{27}$$

$$\frac{i^2 (c+dx)^3 \left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bi^2 n(bc-ad)\int \frac{(c+dx)^2}{a+bx}dx}{3d}$$

$$\Big\downarrow \text{49}$$

$$\frac{i^2 (c+dx)^3 \left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bi^2 n(bc-ad)\int \left(\frac{(bc-ad)^2}{b^2 (a+bx)}+\frac{d(bc-ad)}{b^2}+\frac{d(c+dx)}{b}\right)dx}{3d}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i^2 (c+dx)^3 \left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d}-\frac{Bi^2 n(bc-ad) \left(\frac{(bc-ad)^2 \log (a+bx)}{b^3}+\frac{dx(bc-ad)}{b^2}+\frac{(c+dx)^2}{2b}\right)}{3d}$$

input
```
Int[(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/3*(B*(b*c - a*d)*i^2*n*((d*(b*c - a*d)*x)/b^2 + (c + d*x)^2/(2*b) + ((b
*c - a*d)^2*Log[a + b*x])/b^3))/d + (i^2*(c + d*x)^3*(A + B*Log[e*((a + b*
x)/(c + d*x))^n]))/(3*d)
```

### 3.120.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

## 3.120.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 462 vs. $2(116) = 232$.

Time = 1.96 (sec) , antiderivative size = 463, normalized size of antiderivative = 3.73

| method | result |
|---|---|
| parallelrisch | $\frac{2A\,x^3b^3d^3i^2n+2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3c^3i^2n+2B\ln(bx+a)a^3d^3i^2n^2-2B\ln(bx+a)b^3c^3i^2n^2+6B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3c\,d^2i^2n+2}{}$ |

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/6*(2*A*x^3*b^3*d^3*i^2*n+2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^3*i^2*n+2*B
*ln(b*x+a)*a^3*d^3*i^2*n^2-2*B*ln(b*x+a)*b^3*c^3*i^2*n^2+6*B*x^2*ln(e*((b*
x+a)/(d*x+c))^n)*b^3*c*d^2*i^2*n+2*B*a^3*d^3*i^2*n^2+4*B*b^3*c^3*i^2*n^2-6
*A*b^3*c^3*i^2*n-6*B*ln(b*x+a)*a^2*b*c*d^2*i^2*n^2+6*B*ln(b*x+a)*a*b^2*c^2
*d*i^2*n^2+6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c^2*d*i^2*n+6*B*x*a*b^2*c*d
^2*i^2*n^2-5*B*a^2*b*c*d^2*i^2*n^2-B*a*b^2*c^2*d*i^2*n^2-12*A*a*b^2*c^2*d*
i^2*n+2*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^3*i^2*n+B*x^2*a*b^2*d^3*i^2*
n^2-B*x^2*b^3*c*d^2*i^2*n^2+6*A*x^2*b^3*c*d^2*i^2*n-2*B*x*a^2*b*d^3*i^2*n^
2-4*B*x*b^3*c^2*d*i^2*n^2+6*A*x*b^3*c^2*d*i^2*n)/b^3/d/n
```

## 3.120.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(116) = 232$.

Time = 0.34 (sec) , antiderivative size = 297, normalized size of antiderivative = 2.40

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{2\,Ab^3d^3i^2x^3 - 2\,Bb^3c^3i^2n\log(dx+c) + 2\,(3\,Bab^2c^2d - 3\,Ba^2bcd^2 + Ba^3d^3)i^2n\log(bx+a) + (6\,Ab^3cd^2i^2}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/6*(2*A*b^3*d^3*i^2*x^3 - 2*B*b^3*c^3*i^2*n*log(d*x + c) + 2*(3*B*a*b^2*c
^2*d - 3*B*a^2*b*c*d^2 + B*a^3*d^3)*i^2*n*log(b*x + a) + (6*A*b^3*c*d^2*i^
2 - (B*b^3*c*d^2 - B*a*b^2*d^3)*i^2*n)*x^2 + 2*(3*A*b^3*c^2*d*i^2 - (2*B*b
^3*c^2*d - 3*B*a*b^2*c*d^2 + B*a^2*b*d^3)*i^2*n)*x + 2*(B*b^3*d^3*i^2*x^3
+ 3*B*b^3*c*d^2*i^2*x^2 + 3*B*b^3*c^2*d*i^2*x)*log(e) + 2*(B*b^3*d^3*i^2*n
*x^3 + 3*B*b^3*c*d^2*i^2*n*x^2 + 3*B*b^3*c^2*d*i^2*n*x)*log((b*x + a)/(d*x
 + c)))/(b^3*d)
```

### 3.120.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 656 vs. $2(107) = 214$.

Time = 18.64 (sec) , antiderivative size = 656, normalized size of antiderivative = 5.29

$$\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \begin{cases} c^2i^2x\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ Ac^2i^2x+Acdi^2x^2+\frac{Ad^2i^2x^3}{3}+\frac{Bc^3i^2\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{3d}+\frac{Bc^2i^2nx}{3}+Bc^2i^2x\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{Bcdi^2nx^2}{3}+Bcdi^2x \\ c^2i^2\left(Ax+\frac{Ba\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}-Bnx+Bx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)\right) \\ Ac^2i^2x+Acdi^2x^2+\frac{Ad^2i^2x^3}{3}+\frac{Ba^3d^2i^2n\log\left(\frac{c}{d}+x\right)}{3b^3}+\frac{Ba^3d^2i^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{3b^3}-\frac{Ba^2cdi^2n\log\left(\frac{c}{d}+x\right)}{b^2}-\frac{Ba^2cdi^2\log}{} \end{cases}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise((c**2*i**2*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c
**2*i**2*x + A*c*d*i**2*x**2 + A*d**2*i**2*x**3/3 + B*c**3*i**2*log(e*(a/(
c + d*x))**n)/(3*d) + B*c**2*i**2*n*x/3 + B*c**2*i**2*x*log(e*(a/(c + d*x)
)**n) + B*c*d*i**2*n*x**2/3 + B*c*d*i**2*x**2*log(e*(a/(c + d*x))**n) + B*
d**2*i**2*n*x**3/9 + B*d**2*i**2*x**3*log(e*(a/(c + d*x))**n)/3, Eq(b, 0))
, (c**2*i**2*(A*x + B*a*log(e*(a/c + b*x/c)**n)/b - B*n*x + B*x*log(e*(a/c
 + b*x/c)**n)), Eq(d, 0)), (A*c**2*i**2*x + A*c*d*i**2*x**2 + A*d**2*i**2*
x**3/3 + B*a**3*d**2*i**2*n*log(c/d + x)/(3*b**3) + B*a**3*d**2*i**2*log(e
*(a/(c + d*x) + b*x/(c + d*x))**n)/(3*b**3) - B*a**2*c*d*i**2*n*log(c/d +
x)/b**2 - B*a**2*c*d*i**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b**2 - B
*a**2*d**2*i**2*n*x/(3*b**2) + B*a*c**2*i**2*n*log(c/d + x)/b + B*a*c**2*i
**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b + B*a*c*d*i**2*n*x/b + B*a*d
**2*i**2*n*x**2/(6*b) - B*c**3*i**2*n*log(c/d + x)/(3*d) - 2*B*c**2*i**2*n
*x/3 + B*c**2*i**2*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) - B*c*d*i**2*
n*x**2/6 + B*c*d*i**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) + B*d**
2*i**2*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/3, True))
```

### 3.120.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 309 vs. $2(116) = 232$.

Time = 0.20 (sec) , antiderivative size = 309, normalized size of antiderivative = 2.49

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{1}{3}\,Bd^2i^2x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{1}{3}\,Ad^2i^2x^3$$
$$+Bcdi^2x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acdi^2x^2$$
$$+\frac{1}{6}\,Bd^2i^2n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-Bcdi^2n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bc^2i^2n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bc^2i^2x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^2i^2x$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/3*B*d^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*d^2*i^2*x
^3 + B*c*d*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d*i^2*x^2
+ 1/6*B*d^2*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2
*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - B*c*d*i^2*n*(a
^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^2*
i^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + B*c^2*i^2*x*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n) + A*c^2*i^2*x
```

### 3.120.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 990 vs. $2(116) = 232$.

Time = 0.65 (sec) , antiderivative size = 990, normalized size of antiderivative = 7.98

$$\int (ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{1}{6}\left(\frac{2\,(Bb^4c^4i^2n - 4\,Bab^3c^3di^2n + 6\,Ba^2b^2c^2d^2i^2n - 4\,Ba^3bcd^3i^2n + Ba^4d^4i^2n)\log\left(\frac{bx+a}{dx+c}\right)}{b^3d - \frac{3\,(bx+a)b^2d^2}{dx+c} + \frac{3\,(bx+a)^2bd^3}{(dx+c)^2} - \frac{(bx+a)^3d^4}{(dx+c)^3}} - \frac{3\,Bb^6c^4i^2n -}{}\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/6*(2*(B*b^4*c^4*i^2*n - 4*B*a*b^3*c^3*d*i^2*n + 6*B*a^2*b^2*c^2*d^2*i^2*
n - 4*B*a^3*b*c*d^3*i^2*n + B*a^4*d^4*i^2*n)*log((b*x + a)/(d*x + c))/(b^3
*d - 3*(b*x + a)*b^2*d^2/(d*x + c) + 3*(b*x + a)^2*b*d^3/(d*x + c)^2 - (b*
x + a)^3*d^4/(d*x + c)^3) - (3*B*b^6*c^4*i^2*n - 12*B*a*b^5*c^3*d*i^2*n -
5*(b*x + a)*B*b^5*c^4*d*i^2*n/(d*x + c) + 18*B*a^2*b^4*c^2*d^2*i^2*n + 20*
(b*x + a)*B*a*b^4*c^3*d^2*i^2*n/(d*x + c) + 2*(b*x + a)^2*B*b^4*c^4*d^2*i^
2*n/(d*x + c)^2 - 12*B*a^3*b^3*c*d^3*i^2*n - 30*(b*x + a)*B*a^2*b^3*c^2*d^
3*i^2*n/(d*x + c) - 8*(b*x + a)^2*B*a*b^3*c^3*d^3*i^2*n/(d*x + c)^2 + 3*B*
a^4*b^2*d^4*i^2*n + 20*(b*x + a)*B*a^3*b^2*c*d^4*i^2*n/(d*x + c) + 12*(b*x
 + a)^2*B*a^2*b^2*c^2*d^4*i^2*n/(d*x + c)^2 - 5*(b*x + a)*B*a^4*b*d^5*i^2*
n/(d*x + c) - 8*(b*x + a)^2*B*a^3*b*c*d^5*i^2*n/(d*x + c)^2 + 2*(b*x + a)^
2*B*a^4*d^6*i^2*n/(d*x + c)^2 - 2*B*b^6*c^4*i^2*log(e) + 8*B*a*b^5*c^3*d*i
^2*log(e) - 12*B*a^2*b^4*c^2*d^2*i^2*log(e) + 8*B*a^3*b^3*c*d^3*i^2*log(e)
 - 2*B*a^4*b^2*d^4*i^2*log(e) - 2*A*b^6*c^4*i^2 + 8*A*a*b^5*c^3*d*i^2 - 12
*A*a^2*b^4*c^2*d^2*i^2 + 8*A*a^3*b^3*c*d^3*i^2 - 2*A*a^4*b^2*d^4*i^2)/(b^5
*d - 3*(b*x + a)*b^4*d^2/(d*x + c) + 3*(b*x + a)^2*b^3*d^3/(d*x + c)^2 - (
b*x + a)^3*b^2*d^4/(d*x + c)^3) + 2*(B*b^4*c^4*i^2*n - 4*B*a*b^3*c^3*d*i^2
*n + 6*B*a^2*b^2*c^2*d^2*i^2*n - 4*B*a^3*b*c*d^3*i^2*n + B*a^4*d^4*i^2*n)*
log(b - (b*x + a)*d/(d*x + c))/(b^3*d) - 2*(B*b^4*c^4*i^2*n - 4*B*a*b^3*c^
3*d*i^2*n + 6*B*a^2*b^2*c^2*d^2*i^2*n - 4*B*a^3*b*c*d^3*i^2*n + B*a^4*d...
```

### 3.120.9 Mupad [B] (verification not implemented)

Time = 1.40 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.44

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,c^2\,i^2\,x+B\,c\,d\,i^2\,x^2+\frac{B\,d^2\,i^2\,x^3}{3}\right) - x\left(\frac{(3\,a\,d+3\,b\,c)\left(\frac{d\,i^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{3\,b}-\frac{A\,d\,i^2\,(3\,a\,d+3\,b\,c)}{3\,b}\right)}{3\,b\,d} - \frac{c\,i^2\,(3\,A\,a\,d+3\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{b}+\frac{A\,a\,c\,d\,i^2}{b}\right) + x^2\left(\frac{d\,i^2\,(3\,A\,a\,d+9\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{6\,b}-\frac{A\,d\,i^2\,(3\,a\,d+3\,b\,c)}{6\,b}\right) + \frac{\ln(a+b\,x)\,\left(B\,n\,a^3\,d^2\,i^2-3\,B\,n\,a^2\,b\,c\,d\,i^2+3\,B\,n\,a\,b^2\,c^2\,i^2\right)}{3\,b^3} + \frac{A\,d^2\,i^2\,x^3}{3}-\frac{B\,c^3\,i^2\,n\,\ln(c+d\,x)}{3\,d}$$

input
```
int((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((B*d^2*i^2*x^3)/3 + B*c^2*i^2*x + B*c*d*i^
2*x^2) - x*(((3*a*d + 3*b*c)*((d*i^2*(3*A*a*d + 9*A*b*c + B*a*d*n - B*b*c*
n))/(3*b) - (A*d*i^2*(3*a*d + 3*b*c))/(3*b)))/(3*b*d) - (c*i^2*(3*A*a*d +
3*A*b*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d*i^2)/b) + x^2*((d*i^2*(3*A*a*d
+ 9*A*b*c + B*a*d*n - B*b*c*n))/(6*b) - (A*d*i^2*(3*a*d + 3*b*c))/(6*b)) +
 (log(a + b*x)*(B*a^3*d^2*i^2*n + 3*B*a*b^2*c^2*i^2*n - 3*B*a^2*b*c*d*i^2*
n))/(3*b^3) + (A*d^2*i^2*x^3)/3 - (B*c^3*i^2*n*log(c + d*x))/(3*d)
```

# 3.121 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$



## 3.121.1 Optimal result

Integrand size = 43, antiderivative size = 289

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx = -\frac{Bd(bc-ad)i^2nx}{2b^2g}+\frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g} + \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg}-\frac{B(bc-ad)^2i^2n\log\left(\frac{a+bx}{c+dx}\right)}{2b^3g} - \frac{3B(bc-ad)^2i^2n\log(c+dx)}{2b^3g}-\frac{(bc-ad)^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} + \frac{B(bc-ad)^2i^2n\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}$$

output
```
-1/2*B*d*(-a*d+b*c)*i^2*n*x/b^2/g+d*(-a*d+b*c)*i^2*(b*x+a)*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))/b^3/g+1/2*i^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b
/g-1/2*B*(-a*d+b*c)^2*i^2*n*ln((b*x+a)/(d*x+c))/b^3/g-3/2*B*(-a*d+b*c)^2*i
^2*n*ln(d*x+c)/b^3/g-(-a*d+b*c)^2*i^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1
-b*(d*x+c)/d/(b*x+a))/b^3/g+B*(-a*d+b*c)^2*i^2*n*polylog(2,b*(d*x+c)/d/(b*
x+a))/b^3/g
```

### 3.121.2 Mathematica [A] (verified)

Time = 0.13 (sec) , antiderivative size = 264, normalized size of antiderivative = 0.91

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^2\Big(2Abd(bc-ad)x - B(bc-ad)n(bdx+(bc-ad)\log(a+bx)) + 2Bd(bc-ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) -}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x),x]
```

output
```
(i^2*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a +
 b*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*
(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*Log[g
*(a + b*x)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*L
og[c + d*x] + B*(b*c - a*d)^2*n*(-(Log[g*(a + b*x)]*(Log[g*(a + b*x)] - 2*
Log[(b*(c + d*x))/(b*c - a*d)])) + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)])))/(2*b^3*g)
```

### 3.121.3 Rubi [A] (verified)

Time = 0.89 (sec) , antiderivative size = 319, normalized size of antiderivative = 1.10, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2961, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{ag+bgx}\,dx$$

$$\Big\downarrow \ 2961$$

$$\frac{i^2(bc-ad)^2\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \ 2789$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$\Big\downarrow$ 2756

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int \frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$\Big\downarrow$ 54

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int \left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+}{c+}}{b}}{g}$$

$\Big\downarrow$ 2009

$$\frac{i^2(bc-ad)^2\left(\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}\right)}{g}$$

$\Big\downarrow$ 2789

---

3.121. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

$$\frac{i^2(bc-ad)^2\left(\frac{\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d\cdots}{\cdots}\right)}{b^{\cdots}}\cdots\right)}{2\cdots}\right)}{b}\cdots\right)}{g}$$

$$\Big\downarrow \text{2751}$$

$$\frac{i^2(bc-ad)^2\left(\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\cdots}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\cdots\right)}{\cdots}\cdots\right)}{g}$$

$$\Big\downarrow \text{16}$$

$$\frac{i^2(bc-ad)^2\left(\frac{\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\cdots\right)}{\cdots}\cdots\right)}{g}$$

$$\Big\downarrow \text{2779}$$

---

3.121. $\displaystyle\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

$$i^2(bc-ad)^2\left(\frac{\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}\right.$$

$$\overline{\qquad\qquad\qquad\qquad g\qquad\qquad\qquad}$$

$\downarrow$ 2838

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{\frac{Bn\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\ldots}{b}}{\ldots}\right.$$

$$\overline{\qquad\qquad\qquad\qquad g\qquad\qquad\qquad}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
,x]
```

output
```
((b*c - a*d)^2*i^2*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(2*d*(b - (
d*(a + b*x))/(c + d*x))^2) - (B*n*(1/(b*(b - (d*(a + b*x))/(c + d*x))) + L
og[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(2*d)
))/b + ((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b*(c + d*x
)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b - (d*(a + b*x))/(c + d*x)])/
(b*d))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x
))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b
)/b))/g
```

## 3.121. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

### 3.121.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a + 
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

---

## 3.121. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.121.4 Maple [F]

$$\int \frac{(dix+ci)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{bgx+ag}dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

### 3.121.5 Fricas [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, al
gorithm="fricas")
```

output
```
integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B
*c*d*i^2*x + B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n))/(b*g*x + a*g), x)
```

### 3.121.6 Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$=\frac{i^2\left(\int\frac{Ac^2}{a+bx}\,dx+\int\frac{Ad^2x^2}{a+bx}\,dx+\int\frac{Bc^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx+\int\frac{2Acdx}{a+bx}\,dx+\int\frac{Bd^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx+\int 2\right.}{g}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
```

output
```
i**2*(Integral(A*c**2/(a + b*x), x) + Integral(A*d**2*x**2/(a + b*x), x) +
 Integral(B*c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + I
ntegral(2*A*c*d*x/(a + b*x), x) + Integral(B*d**2*x**2*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n)/(a + b*x), x) + Integral(2*B*c*d*x*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n)/(a + b*x), x))/g
```

## 3.121.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 580 vs. 2(280) = 560.

Time = 0.49 (sec) , antiderivative size = 580, normalized size of antiderivative = 2.01

$$
\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx \\
&= 2\,Acdi^2\left(\frac{x}{bg}-\frac{a\log(bx+a)}{b^2g}\right)+\frac{1}{2}\,Ad^2i^2\left(\frac{2\,a^2\log(bx+a)}{b^3g}+\frac{bx^2-2\,ax}{b^2g}\right) \\
&\quad+\frac{Ac^2i^2\log(bgx+ag)}{bg}-\frac{(3\,bc^2i^2n-2\,acdi^2n)B\log(dx+c)}{2\,b^2g} \\
&\quad+\frac{(b^2c^2i^2n-2\,abcdi^2n+a^2d^2i^2n)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^3g} \\
&\quad+\frac{Bb^2d^2i^2x^2\log(e)-(b^2c^2i^2n-2\,abcdi^2n+a^2d^2i^2n)B\log(bx+a)^2-((i^2n-4\,i^2\log(e))b^2cd-(i^2n-
\end{aligned}
$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, al
gorithm="maxima")
```

output
```
2*A*c*d*i^2*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + 1/2*A*d^2*i^2*(2*a^2*log(
b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A*c^2*i^2*log(b*g*x + a*g)/(
b*g) - 1/2*(3*b*c^2*i^2*n - 2*a*c*d*i^2*n)*B*log(d*x + c)/(b^2*g) + (b^2*c
^2*i^2*n - 2*a*b*c*d*i^2*n + a^2*d^2*i^2*n)*(log(b*x + a)*log((b*d*x + a*d
)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^3*g) + 1/2*(B
*b^2*d^2*i^2*x^2*log(e) - (b^2*c^2*i^2*n - 2*a*b*c*d*i^2*n + a^2*d^2*i^2*n
)*B*log(b*x + a)^2 - ((i^2*n - 4*i^2*log(e))*b^2*c*d - (i^2*n - 2*i^2*log(
e))*a*b*d^2)*B*x + (2*b^2*c^2*i^2*log(e) + 4*(i^2*n - i^2*log(e))*a*b*c*d
- (3*i^2*n - 2*i^2*log(e))*a^2*d^2)*B*log(b*x + a) + (B*b^2*d^2*i^2*x^2 +
2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B*x + 2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2
*d^2*i^2)*B*log(b*x + a))*log((b*x + a)^n) - (B*b^2*d^2*i^2*x^2 + 2*(2*b^2
*c*d*i^2 - a*b*d^2*i^2)*B*x + 2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2
)*B*log(b*x + a))*log((d*x + c)^n))/(b^3*g)
```

### 3.121.8 Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, al
gorithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g), x)
```

### 3.121.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx=\int \frac{(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
, x)
```

# 3.122 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$



## 3.122.1 Optimal result

Integrand size = 43, antiderivative size = 259

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx &= -\frac{B(bc-ad)i^2n(c+dx)}{b^2g^2(a+bx)}+\frac{d^2i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2}\\ &\quad-\frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^2(a+bx)}-\frac{Bd(bc-ad)i^2n\log(c+dx)}{b^3g^2}\\ &\quad-\frac{2d(bc-ad)i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}\\ &\quad+\frac{2Bd(bc-ad)i^2n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}\end{aligned}$$

output
```
-B*(-a*d+b*c)*i^2*n*(d*x+c)/b^2/g^2/(b*x+a)+d^2*i^2*(b*x+a)*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))/b^3/g^2-(-a*d+b*c)*i^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/b^2/g^2/(b*x+a)-B*d*(-a*d+b*c)*i^2*n*ln(d*x+c)/b^3/g^2-2*d*(-a*d+b*
c)*i^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^2+2
*B*d*(-a*d+b*c)*i^2*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^2
```

### 3.122.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 233, normalized size of antiderivative = 0.90

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{i^2\left(Abd^2x-\frac{B(bc-ad)^2n}{a+bx}+Bd(-bc+ad)n\log(a+bx)+Bd^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\frac{(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x)^2,x]
```

output
```
(i^2*(A*b*d^2*x - (B*(b*c - a*d)^2*n)/(a + b*x) + B*d*(-(b*c) + a*d)*n*Log
[a + b*x] + B*d^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - ((b*c - a*d)^
2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + 2*d*(b*c - a*d)*Log[
a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + B*d*(-(b*c) + a*d)*n*(Lo
g[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2
, (d*(a + b*x))/(-(b*c) + a*d)])))/(b^3*g^2)
```

### 3.122.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 235, normalized size of antiderivative = 0.91, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {2961, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^2}\,dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i^2(bc-ad)\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{2793}$$

$$\frac{i^2(bc-ad)\int\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d^2}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d}{b^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2(a+bx)^2}\right)d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i^2(bc-ad)\left(\frac{d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2d\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^2(a+bx)}+\frac{2B\cdots}{\cdots}\right.}{g^2}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^2,x]
```

output
```
((b*c - a*d)*i^2*(-((B*n*(c + d*x))/(b^2*(a + b*x))) - ((c + d*x)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]))/(b^2*(a + b*x)) + (d^2*(a + b*x)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(b^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)
)) + (B*d*n*Log[b - (d*(a + b*x))/(c + d*x)])/b^3 - (2*d*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b^3 + (2*B*d*n
*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b^3))/g^2
```

### 3.122.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.122.4 Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^2}dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

### 3.122.5 Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,
algorithm="fricas")
```

output
```
integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B
*c*d*i^2*x + B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n))/(b^2*g^2*x^2 + 2*a
*b*g^2*x + a^2*g^2), x)
```

### 3.122.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x
)
```

output
```
Timed out
```

### 3.122.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1190 vs. 2(258) = 516.

Time = 0.47 (sec) , antiderivative size = 1190, normalized size of antiderivative = 4.59

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,
algorithm="maxima")
```

output
```
-B*c^2*i^2*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^
2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - A*(a^2/(b^4*g^2*x + a*b^3*g^2
) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2))*d^2*i^2 + 2*A*c*d*i^2*(a/(b^
3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - B*c^2*i^2*log(e*(b*x/(d*x
 + c) + a/(d*x + c))^n)/(b^2*g^2*x + a*b*g^2) - A*c^2*i^2/(b^2*g^2*x + a*b
*g^2) - (b^2*c^2*d*i^2*n + a*b*c*d^2*i^2*n - a^2*d^3*i^2*n)*B*log(d*x + c)
/(b^4*c*g^2 - a*b^3*d*g^2) + ((b^3*c*d^2*i^2*log(e) - a*b^2*d^3*i^2*log(e)
)*B*x^2 + (a*b^2*c*d^2*i^2*log(e) - a^2*b*d^3*i^2*log(e))*B*x - ((b^3*c^2*
d*i^2*n - 2*a*b^2*c*d^2*i^2*n + a^2*b*d^3*i^2*n)*B*x + (a*b^2*c^2*d*i^2*n
- 2*a^2*b*c*d^2*i^2*n + a^3*d^3*i^2*n)*B)*log(b*x + a)^2 + (2*(i^2*n + i^2
*log(e))*a*b^2*c^2*d - 3*(i^2*n + i^2*log(e))*a^2*b*c*d^2 + (i^2*n + i^2*l
og(e))*a^3*d^3)*B + ((2*b^3*c^2*d*i^2*log(e) + (3*i^2*n - 4*i^2*log(e))*a*
b^2*c*d^2 - 2*(i^2*n - i^2*log(e))*a^2*b*d^3)*B*x + (2*a*b^2*c^2*d*i^2*log
(e) + (3*i^2*n - 4*i^2*log(e))*a^2*b*c*d^2 - 2*(i^2*n - i^2*log(e))*a^3*d^
3)*B)*log(b*x + a) + ((b^3*c*d^2*i^2 - a*b^2*d^3*i^2)*B*x^2 + (a*b^2*c*d^2
*i^2 - a^2*b*d^3*i^2)*B*x + (2*a*b^2*c^2*d*i^2 - 3*a^2*b*c*d^2*i^2 + a^3*d
^3*i^2)*B + 2*((b^3*c^2*d*i^2 - 2*a*b^2*c*d^2*i^2 + a^2*b*d^3*i^2)*B*x + (
a*b^2*c^2*d*i^2 - 2*a^2*b*c*d^2*i^2 + a^3*d^3*i^2)*B)*log(b*x + a))*log((b
*x + a)^n) - ((b^3*c*d^2*i^2 - a*b^2*d^3*i^2)*B*x^2 + (a*b^2*c*d^2*i^2 - a
^2*b*d^3*i^2)*B*x + (2*a*b^2*c^2*d*i^2 - 3*a^2*b*c*d^2*i^2 + a^3*d^3*i^...
```

### 3.122.8 Giac [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,
algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g)^2, x)
```

### 3.122.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{{\left(c\,i+d\,i\,x\right)}^2\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{{\left(a\,g+b\,g\,x\right)}^2}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^2,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^2, x)
```

# 3.123 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$



## 3.123.1 Optimal result

Integrand size = 43, antiderivative size = 242

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = -\frac{Bdi^2n(c+dx)}{b^2g^3(a+bx)}-\frac{Bi^2n(c+dx)^2}{4bg^3(a+bx)^2} - \frac{di^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^3(a+bx)} - \frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2} - \frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} + \frac{Bd^2i^2n\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}$$

output
```
-B*d*i^2*n*(d*x+c)/b^2/g^3/(b*x+a)-1/4*B*i^2*n*(d*x+c)^2/b/g^3/(b*x+a)^2-d
*i^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^3/(b*x+a)-1/2*i^2*(d*x+
c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^3/(b*x+a)^2-d^2*i^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^3+B*d^2*i^2*n*polylog(2
,b*(d*x+c)/d/(b*x+a))/b^3/g^3
```

### 3.123.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 258, normalized size of antiderivative = 1.07

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{i^2\left(-\frac{B(bc-ad)^2n}{(a+bx)^2}+\frac{6Bd(-bc+ad)n}{a+bx}-6Bd^2n\log(a+bx)-\frac{2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}+\frac{8d(-bc+ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x)^3,x]
```

output
```
(i^2*(-((B*(b*c - a*d)^2*n)/(a + b*x)^2) + (6*B*d*(-(b*c) + a*d)*n)/(a + b
*x) - 6*B*d^2*n*Log[a + b*x] - (2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(a + b*x)^2 + (8*d*(-(b*c) + a*d)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(a + b*x) + 4*d^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]) + 6*B*d^2*n*Log[c + d*x] - 2*B*d^2*n*(Log[a + b*x]*(Log[a + b*x]
 - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c)
+ a*d)])))/(4*b^3*g^3)
```

### 3.123.3 Rubi [A] (verified)

Time = 0.76 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.89, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.163, Rules used = {2961, 2780, 2741, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^3}\,dx$$
$$\Big\downarrow \; 2961$$
$$\frac{i^2\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^3}$$
$$\Big\downarrow \; 2780$$

$$\frac{i^2\left(\frac{\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^3}$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{i^2\left(\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}}{b}\right)}{g^3}$$

$$\Big\downarrow \text{ 2780}$$

$$\frac{i^2\left(\frac{d\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}}{b}\right)}{g^3}$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{i^2\left(\frac{d\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}}{b}\right)}{g^3}$$

$$\Big\downarrow \text{ 2779}$$

---

3.123. $\displaystyle\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$

$$\frac{i^2\left(\frac{d\left(\frac{d\left(\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{b}+\frac{-\frac{(c+}{}}{}\right.}{g^3}$$

$\Big\downarrow$ 2838

$$\frac{i^2\left(\frac{d\left(\frac{d\left(\frac{Bn\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\right.\right.}{2(a+}}{}\right.}{g^3}$$

input

```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^3,x]
```

output

```
(i^2*((-1/4*(B*n*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]))/(2*(a + b*x)^2))/b + (d*((-((B*n*(c + d*x))/(a + b*
x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x))/b + (d
*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b))/b))/b))/g^3
```

## 3.123. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$

#### 3.123.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.123.4 Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^3}dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

### 3.123.5 Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,
algorithm="fricas")
```

output
```
integral((A*d^2*i^2*x^2 + 2*A*c*d*i^2*x + A*c^2*i^2 + (B*d^2*i^2*x^2 + 2*B
*c*d*i^2*x + B*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n))/(b^3*g^3*x^3 + 3*a
*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

### 3.123.6 Sympy [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \frac{i^2\left(\int \frac{Ac^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{Ad^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{Bc^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx+\int \frac{2Acdx}{a^3+3a^2bx+3ab^2x^2+b^3}\right.}{g^3}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x
)
```

output
```
i**2*(Integral(A*c**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
+ Integral(A*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x)
 + Integral(B*c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*
b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*c*d*x/(a**3 + 3*a**2*b
*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(B*d**2*x**2*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
x) + Integral(2*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*
a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

### 3.123.7 Maxima [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,
algorithm="maxima")
```

output
```
-1/2*B*c*d*i^2*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^
4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3
) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2
)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^
2*d^2)*g^3)) + 1/4*B*c^2*i^2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)
*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*
d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x
 + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) + 1/2*A*d^2*i^2*((4*a*b*x
 + 3*a^2)/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^
3*g^3)) + 1/2*B*d^2*i^2*(((4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x + a^2)*l
og(b*x + a))*log((b*x + a)^n) - (4*a*b*x + 3*a^2 + 2*(b^2*x^2 + 2*a*b*x +
a^2)*log(b*x + a))*log((d*x + c)^n))/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^
3*g^3) + 2*integrate(1/2*(2*b^3*d*x^3*log(e) + 2*b^3*c*x^2*log(e) - 3*a^2*
b*c*n + 3*a^3*d*n - 4*(a*b^2*c*n - a^2*b*d*n)*x - 2*(a^2*b*c*n - a^3*d*n +
 (b^3*c*n - a*b^2*d*n)*x^2 + 2*(a*b^2*c*n - a^2*b*d*n)*x)*log(b*x + a))/(b
^6*d*g^3*x^4 + a^3*b^3*c*g^3 + (b^6*c*g^3 + 3*a*b^5*d*g^3)*x^3 + 3*(a*b^5*
c*g^3 + a^2*b^4*d*g^3)*x^2 + (3*a^2*b^4*c*g^3 + a^3*b^3*d*g^3)*x), x)) - (
2*b*x + a)*B*c*d*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^3*x^2 +
 2*a*b^3*g^3*x + a^2*b^2*g^3) - (2*b*x + a)*A*c*d*i^2/(b^4*g^3*x^2 + 2*a*b
^3*g^3*x + a^2*b^2*g^3) - 1/2*B*c^2*i^2*log(e*(b*x/(d*x + c) + a/(d*x +...
```

### 3.123.8 Giac [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,
algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g)^3, x)
```

### 3.123.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^3}\,dx = \int \frac{\left(c\,i + d\,i\,x\right)^2 \left(A + B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{\left(a\,g + b\,g\,x\right)^3}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^3,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^3, x)
```

# 3.124 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$



## 3.124.1 Optimal result

Integrand size = 43, antiderivative size = 93

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx=-\frac{Bi^2n(c+dx)^3}{9(bc-ad)g^4(a+bx)^3}-\frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)g^4(a+bx)^3}$$

output
```
-1/9*B*i^2*n*(d*x+c)^3/(-a*d+b*c)/g^4/(b*x+a)^3-1/3*i^2*(d*x+c)^3*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^4/(b*x+a)^3
```

## 3.124.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(93) = 186.

Time = 0.19 (sec) , antiderivative size = 329, normalized size of antiderivative = 3.54

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx=-\frac{i^2\left(3Ab^3c^3-3a^3Ad^3+b^3Bc^3n-a^3Bd^3n+9Ab^3c^2dx-9a^2Abd^3x+3b^3Bc^2dnx-3a^2bBd^3nx+9Ab^3c\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x)^4,x]
```

output
```
-1/9*(i^2*(3*A*b^3*c^3 - 3*a^3*A*d^3 + b^3*B*c^3*n - a^3*B*d^3*n + 9*A*b^3
*c^2*d*x - 9*a^2*A*b*d^3*x + 3*b^3*B*c^2*d*n*x - 3*a^2*b*B*d^3*n*x + 9*A*b
^3*c*d^2*x^2 - 9*a*A*b^2*d^3*x^2 + 3*b^3*B*c*d^2*n*x^2 - 3*a*b^2*B*d^3*n*x
^2 + 3*B*d^3*n*(a + b*x)^3*Log[a + b*x] + 3*B*(b*c - a*d)*(a^2*d^2 + a*b*d
*(c + 3*d*x) + b^2*(c^2 + 3*c*d*x + 3*d^2*x^2))*Log[e*((a + b*x)/(c + d*x)
)^n] - 3*a^3*B*d^3*n*Log[c + d*x] - 9*a^2*b*B*d^3*n*x*Log[c + d*x] - 9*a*b
^2*B*d^3*n*x^2*Log[c + d*x] - 3*b^3*B*d^3*n*x^3*Log[c + d*x]))/(b^3*(b*c -
 a*d)*g^4*(a + b*x)^3)
```

### 3.124.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.84, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2961, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{i^2 \int \frac{(c+dx)^4 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)}$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{i^2 \left(-\frac{(c+dx)^3 \left(B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}-\frac{Bn(c+dx)^3}{9(a+bx)^3}\right)}{g^4(bc-ad)}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^4,x]
```

output
```
(i^2*(-1/9*(B*n*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]))/(3*(a + b*x)^3)))/((b*c - a*d)*g^4)
```

### 3.124.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.124.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(89) = 178.

Time = 10.00 (sec) , antiderivative size = 376, normalized size of antiderivative = 4.04

| method | result |
|---|---|
| parallelrisch | $-\frac{B a^3 b^2 d^4 i^2 n^2 - B b^5 c^3 d i^2 n^2 + 3A a^3 b^2 d^4 i^2 n - 3A b^5 c^3 d i^2 n - 3B x^3 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) b^5 d^4 i^2 n + 3B x^2 a b^4 d^4 i^2 n^2 - 3B x^2 b^5 c d^3 i}{}$ |

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,method=_
RETURNVERBOSE)
```

output
```
-1/9*(B*a^3*b^2*d^4*i^2*n^2-B*b^5*c^3*d*i^2*n^2+3*A*a^3*b^2*d^4*i^2*n-3*A*
b^5*c^3*d*i^2*n-3*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^4*i^2*n+3*B*x^2*a*
b^4*d^4*i^2*n^2-3*B*x^2*b^5*c*d^3*i^2*n^2+9*A*x^2*a*b^4*d^4*i^2*n-9*A*x^2*
b^5*c*d^3*i^2*n+3*B*x*a^2*b^3*d^4*i^2*n^2-3*B*x*b^5*c^2*d^2*i^2*n^2+9*A*x*
a^2*b^3*d^4*i^2*n-9*A*x*b^5*c^2*d^2*i^2*n-3*B*ln(e*((b*x+a)/(d*x+c))^n)*b^
5*c^3*d*i^2*n-9*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^3*i^2*n-9*B*x*ln(e
*((b*x+a)/(d*x+c))^n)*b^5*c^2*d^2*i^2*n)/g^4/(b*x+a)^3/b^5/d/n/(a*d-b*c)
```

### 3.124.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(89) = 178$.

Time = 0.33 (sec) , antiderivative size = 409, normalized size of antiderivative = 4.40

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = -\frac{(Bb^3c^3-Ba^3d^3)i^2n+3\,(Ab^3c^3-Aa^3d^3)i^2+3\,((Bb^3cd^2-Bab^2d^3)i^2n+3\,(Ab^3cd^2-Aab^2d^3)i^2)x^2+3}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,
algorithm="fricas")
```

output
```
-1/9*((B*b^3*c^3 - B*a^3*d^3)*i^2*n + 3*(A*b^3*c^3 - A*a^3*d^3)*i^2 + 3*((
B*b^3*c*d^2 - B*a*b^2*d^3)*i^2*n + 3*(A*b^3*c*d^2 - A*a*b^2*d^3)*i^2)*x^2
+ 3*((B*b^3*c^2*d - B*a^2*b*d^3)*i^2*n + 3*(A*b^3*c^2*d - A*a^2*b*d^3)*i^2
)*x + 3*(3*(B*b^3*c*d^2 - B*a*b^2*d^3)*i^2*x^2 + 3*(B*b^3*c^2*d - B*a^2*b*
d^3)*i^2*x + (B*b^3*c^3 - B*a^3*d^3)*i^2)*log(e) + 3*(B*b^3*d^3*i^2*n*x^3
+ 3*B*b^3*c*d^2*i^2*n*x^2 + 3*B*b^3*c^2*d*i^2*n*x + B*b^3*c^3*i^2*n)*log((
b*x + a)/(d*x + c)))/((b^7*c - a*b^6*d)*g^4*x^3 + 3*(a*b^6*c - a^2*b^5*d)*
g^4*x^2 + 3*(a^2*b^5*c - a^3*b^4*d)*g^4*x + (a^3*b^4*c - a^4*b^3*d)*g^4)
```

### 3.124.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x
)
```

output
```
Timed out
```

### 3.124.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1544 vs. 2(89) = 178.

Time = 0.25 (sec) , antiderivative size = 1544, normalized size of antiderivative = 16.60

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,
algorithm="maxima")
```

output
```
-1/18*B*d^2*i^2*n*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c^
2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a^
3*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2
- 2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d +
a^4*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6*
(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^2
*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 +
a^2*d^3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^
3*d^3)*g^4)) - 1/18*B*c^2*i^2*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d +
11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*
d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^
2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*
c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a
^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^
2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/18*B*c*d*i^2*n*((5*a*b^2*c^2
- 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2
- 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^
4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c
^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a
^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b...
```

### 3.124.8 Giac [A] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 108, normalized size of antiderivative = 1.16

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$
$$-\frac{1}{9}\left(\frac{3\,(dx+c)^3 Bi^2 n\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^3 g^4}+\frac{(Bi^2n+3\,Bi^2\log(e)+3\,Ai^2)(dx+c)^3}{(bx+a)^3 g^4}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,
algorithm="giac")
```

output
```
-1/9*(3*(d*x + c)^3*B*i^2*n*log((b*x + a)/(d*x + c))/((b*x + a)^3*g^4) + (
B*i^2*n + 3*B*i^2*log(e) + 3*A*i^2)*(d*x + c)^3/((b*x + a)^3*g^4))*(b*c/(b
*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.124.9 Mupad [B] (verification not implemented)

Time = 2.23 (sec) , antiderivative size = 421, normalized size of antiderivative = 4.53

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx =$$

$$-\frac{x\,(3\,A\,a\,b\,d^2\,i^2+3\,A\,b^2\,c\,d\,i^2+B\,a\,b\,d^2\,i^2\,n+B\,b^2\,c\,d\,i^2\,n)+x^2\,(3\,A\,b^2\,d^2\,i^2+B\,b^2\,d^2\,i^2\,n)+A\,a^2\,d^2\,i^2+}{3\,a^3\,b^3\,g^4+9\,a^2\,b^4\,g^4\,x+9\,a\,b^5\,g^4\,x^2+3\,b^6\,g^4\,x}$$

$$-\frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(a\left(\frac{B\,a\,d^2\,i^2}{3\,b^3}+\frac{B\,c\,d\,i^2}{3\,b^2}\right)+x\left(b\left(\frac{B\,a\,d^2\,i^2}{3\,b^3}+\frac{B\,c\,d\,i^2}{3\,b^2}\right)+\frac{2\,B\,a\,d^2\,i^2}{3\,b^2}+\frac{2\,B\,c\,d\,i^2}{3\,b}\right)+\frac{B\,c^2\,i^2}{3\,b}+\frac{B\,d^2\,i^2\,x^2}{b}\right.}{a^3\,g^4+3\,a^2\,b\,g^4\,x+3\,a\,b^2\,g^4\,x^2+b^3\,g^4\,x^3}$$

$$-\frac{B\,d^3\,i^2\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{3\,b^3\,g^4\,(a\,d-b\,c)}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^4,x)
```

output
```
- (x*(3*A*a*b*d^2*i^2 + 3*A*b^2*c*d*i^2 + B*a*b*d^2*i^2*n + B*b^2*c*d*i^2*
n) + x^2*(3*A*b^2*d^2*i^2 + B*b^2*d^2*i^2*n) + A*a^2*d^2*i^2 + A*b^2*c^2*i
^2 + (B*a^2*d^2*i^2*n)/3 + (B*b^2*c^2*i^2*n)/3 + A*a*b*c*d*i^2 + (B*a*b*c*
d*i^2*n)/3)/(3*a^3*b^3*g^4 + 3*b^6*g^4*x^3 + 9*a^2*b^4*g^4*x + 9*a*b^5*g^4
*x^2) - (log(e*((a + b*x)/(c + d*x))^n)*(a*((B*a*d^2*i^2)/(3*b^3) + (B*c*d
*i^2)/(3*b^2)) + x*(b*((B*a*d^2*i^2)/(3*b^3) + (B*c*d*i^2)/(3*b^2)) + (2*B
*a*d^2*i^2)/(3*b^2) + (2*B*c*d*i^2)/(3*b)) + (B*c^2*i^2)/(3*b) + (B*d^2*i^
2*x^2)/b))/(a^3*g^4 + b^3*g^4*x^3 + 3*a*b^2*g^4*x^2 + 3*a^2*b*g^4*x) - (B*
d^3*i^2*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(3*b^3*g^4*(a*d -
 b*c))
```

# 3.125 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$



## 3.125.1 Optimal result

Integrand size = 43, antiderivative size = 189

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx = \frac{Bdi^2n(c+dx)^3}{9(bc-ad)^2g^5(a+bx)^3} - \frac{bBi^2n(c+dx)^4}{16(bc-ad)^2g^5(a+bx)^4} + \frac{di^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^2g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^2g^5(a+bx)^4}$$

output
```
1/9*B*d*i^2*n*(d*x+c)^3/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/16*b*B*i^2*n*(d*x+c)^
4/(-a*d+b*c)^2/g^5/(b*x+a)^4+1/3*d*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/4*b*i^2*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(
d*x+c))^n))/(-a*d+b*c)^2/g^5/(b*x+a)^4
```

### 3.125.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 474 vs. $2(189) = 378$.

Time = 0.26 (sec) , antiderivative size = 474, normalized size of antiderivative = 2.51

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$$
$$= -\frac{(bc-ad)^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^3g^5(a+bx)^4} - \frac{2d(bc-ad)i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3g^5(a+bx)^3} - \frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^3g^5(a+bx)^2} - \frac{Bd^2i^2n\left(\frac{1}{(a+bx)^2}-\frac{2d}{(bc-ad)(a+bx)}-\frac{2d^2\log(a+bx)}{(bc-ad)^2}+\frac{2d^2\log(c+dx)}{(bc-ad)^2}\right)}{4b^3g^5} - \frac{Bdi^2n\left(\frac{2(bc-ad)}{(a+bx)^3}-\frac{3d}{(a+bx)^2}+\frac{6d^2}{(bc-ad)(a+bx)}+\frac{6d^3\log(a+bx)}{(bc-ad)^2}-\frac{6d^3\log(c+dx)}{(bc-ad)^2}\right)}{9b^3g^5} - \frac{Bi^2n\left(\frac{3(bc-ad)^2}{(a+bx)^4}-\frac{4d(bc-ad)}{(a+bx)^3}+\frac{6d^2}{(a+bx)^2}-\frac{12d^3}{(bc-ad)(a+bx)}-\frac{12d^4\log(a+bx)}{(bc-ad)^2}+\frac{12d^4\log(c+dx)}{(bc-ad)^2}\right)}{48b^3g^5}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x)^5,x]
```

output
```
-1/4*((b*c - a*d)^2*i^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*g^5*(
a + b*x)^4) - (2*d*(b*c - a*d)*i^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
/(3*b^3*g^5*(a + b*x)^3) - (d^2*i^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(2*b^3*g^5*(a + b*x)^2) - (B*d^2*i^2*n*((a + b*x)^(-2) - (2*d)/((b*c - a
*d)*(a + b*x)) - (2*d^2*Log[a + b*x])/(b*c - a*d)^2 + (2*d^2*Log[c + d*x])
/(b*c - a*d)^2))/(4*b^3*g^5) - (B*d*i^2*n*((2*(b*c - a*d))/(a + b*x)^3 - (
3*d)/(a + b*x)^2 + (6*d^2)/((b*c - a*d)*(a + b*x)) + (6*d^3*Log[a + b*x])/
(b*c - a*d)^2 - (6*d^3*Log[c + d*x])/(b*c - a*d)^2))/(9*b^3*g^5) - (B*i^2*
n*((3*(b*c - a*d)^2)/(a + b*x)^4 - (4*d*(b*c - a*d))/(a + b*x)^3 + (6*d^2)
/(a + b*x)^2 - (12*d^3)/((b*c - a*d)*(a + b*x)) - (12*d^4*Log[a + b*x])/(b
*c - a*d)^2 + (12*d^4*Log[c + d*x])/(b*c - a*d)^2))/(48*b^3*g^5)
```

### 3.125.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 145, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2961, 2772, 27, 53, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i^2\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^2}$$

$$\Big\downarrow \text{2772}$$

$$\frac{i^2\left(-Bn\int -\frac{(c+dx)^5\left(3b-\frac{4d(a+bx)}{c+dx}\right)}{12(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}\right)}{g^5(bc-ad)^2}$$

$$\Big\downarrow \text{27}$$

$$\frac{i^2\left(\frac{1}{12}Bn\int \frac{(c+dx)^5\left(3b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)^5}d\frac{a+bx}{c+dx}-\frac{b(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}\right)}{g^5(bc-ad)^2}$$

$$\Big\downarrow \text{53}$$

$$\frac{i^2\left(\frac{1}{12}Bn\int \left(\frac{3b(c+dx)^5}{(a+bx)^5}-\frac{4d(c+dx)^4}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}\right)}{g^5(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i^2\left(-\frac{b(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{1}{12}Bn\left(\frac{4d(c+dx)^3}{3(a+bx)^3}-\frac{3b(c+dx)^4}{4(a+bx)^4}\right)\right)}{g^5(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^5,x]
```

3.125. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}dx$

output
```
(i^2*((B*n*((4*d*(c + d*x)^3)/(3*(a + b*x)^3) - (3*b*(c + d*x)^4)/(4*(a +
b*x)^4)))/12 + (d*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(
a + b*x)^3) - (b*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*(a
 + b*x)^4)))/((b*c - a*d)^2*g^5)
```

### 3.125.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 53
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && LeQ[7*m + 4*n + 4, 0])
|| LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.125. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$

### 3.125.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 854 vs. 2(181) = 362.

Time = 27.07 (sec) , antiderivative size = 855, normalized size of antiderivative = 4.52

| method | result |
|---|---|
| parallelrisch | $\frac{9B x^4 a^2 b^5 c^5 i^2 n^2+36A x^4 a^2 b^5 c^5 i^2 n+16B x^3 a^7 c d^4 i^2 n^2+36B x^3 a^3 b^4 c^5 i^2 n^2+48A x^3 a^7 c d^4 i^2 n+144A x^3 a^3 b^4 c^5 i^2 n+48B x^2 a^7}{}$ |

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,method=_
RETURNVERBOSE)
```

output
```
1/144*(9*B*x^4*a^2*b^5*c^5*i^2*n^2+36*A*x^4*a^2*b^5*c^5*i^2*n+16*B*x^3*a^7
*c*d^4*i^2*n^2+36*B*x^3*a^3*b^4*c^5*i^2*n^2+48*A*x^3*a^7*c*d^4*i^2*n+144*A
*x^3*a^3*b^4*c^5*i^2*n+48*B*x^2*a^7*c^2*d^3*i^2*n^2+54*B*x^2*a^4*b^3*c^5*i
^2*n^2+144*A*x^2*a^7*c^2*d^3*i^2*n+216*A*x^2*a^4*b^3*c^5*i^2*n+48*B*x*a^7*
c^3*d^2*i^2*n^2+36*B*x*a^5*b^2*c^5*i^2*n^2+144*A*x*a^7*c^3*d^2*i^2*n+144*A
*x*a^5*b^2*c^5*i^2*n+48*B*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^4*d*i^2*n-36*B*l
n(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^5*i^2*n+48*B*x^3*ln(e*((b*x+a)/(d*x+c))^n
)*a^7*c*d^4*i^2*n+7*B*x^4*a^6*b*c*d^4*i^2*n^2-16*B*x^4*a^3*b^4*c^4*d*i^2*n
^2+12*A*x^4*a^6*b*c*d^4*i^2*n-48*A*x^4*a^3*b^4*c^4*d*i^2*n+144*B*x^2*ln(e*
((b*x+a)/(d*x+c))^n)*a^7*c^2*d^3*i^2*n-6*B*x^2*a^6*b*c^3*d^2*i^2*n^2-96*B*
x^2*a^5*b^2*c^4*d*i^2*n^2-72*A*x^2*a^6*b*c^3*d^2*i^2*n-288*A*x^2*a^5*b^2*c
^4*d*i^2*n+144*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^3*d^2*i^2*n-84*B*x*a^6*
b*c^4*d*i^2*n^2-288*A*x*a^6*b*c^4*d*i^2*n+12*B*x^3*a^6*b*c^2*d^3*i^2*n^2-6
4*B*x^3*a^4*b^3*c^4*d*i^2*n^2-192*A*x^3*a^4*b^3*c^4*d*i^2*n+12*B*x^4*ln(e*
((b*x+a)/(d*x+c))^n)*a^6*b*c*d^4*i^2*n-72*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*
a^6*b*c^3*d^2*i^2*n-96*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^4*d*i^2*n)/g^
5/(b*x+a)^4/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/a^6/c
```

### 3.125.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 710 vs. 2(181) = 362.

Time = 0.37 (sec) , antiderivative size = 710, normalized size of antiderivative = 3.76

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$$
$$= \frac{12\,(Bb^4cd^3-Bab^3d^4)i^2nx^3-(9\,Bb^4c^4-16\,Bab^3c^3d+7\,Ba^4d^4)i^2n-12\,(3\,Ab^4c^4-4\,Aab^3c^3d+Aa^4d^4)i}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,
algorithm="fricas")
```

output
```
1/144*(12*(B*b^4*c*d^3 - B*a*b^3*d^4)*i^2*n*x^3 - (9*B*b^4*c^4 - 16*B*a*b^
3*c^3*d + 7*B*a^4*d^4)*i^2*n - 12*(3*A*b^4*c^4 - 4*A*a*b^3*c^3*d + A*a^4*d
^4)*i^2 - 6*((B*b^4*c^2*d^2 - 8*B*a*b^3*c*d^3 + 7*B*a^2*b^2*d^4)*i^2*n + 1
2*(A*b^4*c^2*d^2 - 2*A*a*b^3*c*d^3 + A*a^2*b^2*d^4)*i^2)*x^2 - 4*((5*B*b^4
*c^3*d - 12*B*a*b^3*c^2*d^2 + 7*B*a^3*b*d^4)*i^2*n + 12*(2*A*b^4*c^3*d - 3
*A*a*b^3*c^2*d^2 + A*a^3*b*d^4)*i^2)*x - 12*(6*(B*b^4*c^2*d^2 - 2*B*a*b^3*
c*d^3 + B*a^2*b^2*d^4)*i^2*x^2 + 4*(2*B*b^4*c^3*d - 3*B*a*b^3*c^2*d^2 + B*
a^3*b*d^4)*i^2*x + (3*B*b^4*c^4 - 4*B*a*b^3*c^3*d + B*a^4*d^4)*i^2)*log(e)
 + 12*(B*b^4*d^4*i^2*n*x^4 + 4*B*a*b^3*d^4*i^2*n*x^3 - 6*(B*b^4*c^2*d^2 -
2*B*a*b^3*c*d^3)*i^2*n*x^2 - 4*(2*B*b^4*c^3*d - 3*B*a*b^3*c^2*d^2)*i^2*n*x
 - (3*B*b^4*c^4 - 4*B*a*b^3*c^3*d)*i^2*n)*log((b*x + a)/(d*x + c)))/((b^9*
c^2 - 2*a*b^8*c*d + a^2*b^7*d^2)*g^5*x^4 + 4*(a*b^8*c^2 - 2*a^2*b^7*c*d +
a^3*b^6*d^2)*g^5*x^3 + 6*(a^2*b^7*c^2 - 2*a^3*b^6*c*d + a^4*b^5*d^2)*g^5*x
^2 + 4*(a^3*b^6*c^2 - 2*a^4*b^5*c*d + a^5*b^4*d^2)*g^5*x + (a^4*b^5*c^2 -
2*a^5*b^4*c*d + a^6*b^3*d^2)*g^5)
```

### 3.125.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**5,x
)
```

output
```
Timed out
```

### 3.125.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2247 vs. $2(181) = 362$.

Time = 0.30 (sec) , antiderivative size = 2247, normalized size of antiderivative = 11.89

$$\int \frac{(ci + dix)^2 \left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^5} \, dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,
algorithm="maxima")
```

output
```
1/48*B*c^2*i^2*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*
c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*
b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 -
 a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 -
 a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2
 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d
^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2
 - a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2
*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b
^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*
g^5)) - 1/144*B*d^2*i^2*n*((13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*b*c
*d^2 - 7*a^5*d^3 - 12*(6*b^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 + 6*
(6*b^5*c^3 - 46*a*b^4*c^2*d + 29*a^2*b^3*c*d^2 - 7*a^3*b^2*d^3)*x^2 + 4*(1
0*a*b^4*c^3 - 63*a^2*b^3*c^2*d + 33*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*x)/((b^10
*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*g^5*x^4 + 4*(a*b^9*c
^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*g^5*x^3 + 6*(a^2*b^8
*c^3 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*g^5*x^2 + 4*(a^3*b
^7*c^3 - 3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*g^5*x + (a^4*b^6
*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3)*g^5) - 12*(6*b^2*c
^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(b*x + a)/((b^7*c^4 - 4*a*b^6*c^3*d ...
```

### 3.125.8 Giac [A] (verification not implemented)

Time = 1.98 (sec) , antiderivative size = 250, normalized size of antiderivative = 1.32

$$\int \frac{(ci + dix)^2 \left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^5} \, dx =$$

$$-\frac{1}{144}\left(\frac{12\left(3\,Bbi^2n - \frac{4\,(bx+a)Bdi^2n}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^4bcg^5}{(dx+c)^4} - \frac{(bx+a)^4adg^5}{(dx+c)^4}} + \frac{9\,Bbi^2n - \frac{16\,(bx+a)Bdi^2n}{dx+c} + 36\,Bbi^2\log(e) - \frac{48\,(bx+a)Bdi^2}{dx+c}}{\frac{(bx+a)^4bcg^5}{(dx+c)^4} - \frac{(bx+a)^4adg^5}{(dx+c)^4}}\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^5,x,
algorithm="giac")
```

output
```
-1/144*(12*(3*B*b*i^2*n - 4*(b*x + a)*B*d*i^2*n/(d*x + c))*log((b*x + a)/(
d*x + c))/((b*x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5/(d*x + c)
^4) + (9*B*b*i^2*n - 16*(b*x + a)*B*d*i^2*n/(d*x + c) + 36*B*b*i^2*log(e)
- 48*(b*x + a)*B*d*i^2*log(e)/(d*x + c) + 36*A*b*i^2 - 48*(b*x + a)*A*d*i^
2/(d*x + c))/((b*x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5/(d*x +
 c)^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.125.9 Mupad [B] (verification not implemented)

Time = 2.68 (sec) , antiderivative size = 652, normalized size of antiderivative = 3.45

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx =$$

$$-\frac{\frac{12\,A\,a^3\,d^3\,i^2-36\,A\,b^3\,c^3\,i^2+7\,B\,a^3\,d^3\,i^2\,n-9\,B\,b^3\,c^3\,i^2\,n+12\,A\,a\,b^2\,c^2\,d\,i^2+12\,A\,a^2\,b\,c\,d^2\,i^2+7\,B\,a\,b^2\,c^2\,d\,i^2\,n+7\,B\,a^2\,b\,c\,d^2\,i^2\,n}{12\,(a\,d-b\,c)}+\frac{x\,(12\,A\,a^2\,\cdots}{\cdots}}{12\,a^4\,b^3\,g^5+48\,a^3\,b^4\,\cdots}$$

$$-\frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(a\,\left(\frac{B\,a\,d^2\,i^2}{12\,b^3}+\frac{B\,c\,d\,i^2}{6\,b^2}\right)+x\,\left(b\,\left(\frac{B\,a\,d^2\,i^2}{12\,b^3}+\frac{B\,c\,d\,i^2}{6\,b^2}\right)+\frac{B\,a\,d^2\,i^2}{4\,b^2}+\frac{B\,c\,d\,i^2}{2\,b}\right)+\frac{B\,c^2\,i^2}{4\,b}+\frac{B\,d^2\,i^2\,x^2}{2\,b}\right)}{a^4\,g^5+4\,a^3\,b\,g^5\,x+6\,a^2\,b^2\,g^5\,x^2+4\,a\,b^3\,g^5\,x^3+b^4\,g^5\,x^4}$$

$$-\frac{B\,d^4\,i^2\,n\,\mathrm{atanh}\left(\frac{12\,b^5\,c^2\,g^5-12\,a^2\,b^3\,d^2\,g^5}{12\,b^3\,g^5\,{(a\,d-b\,c)}^2}-\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{6\,b^3\,g^5\,{(a\,d-b\,c)}^2}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^5,x)
```

output
```
- ((12*A*a^3*d^3*i^2 - 36*A*b^3*c^3*i^2 + 7*B*a^3*d^3*i^2*n - 9*B*b^3*c^3*
i^2*n + 12*A*a*b^2*c^2*d*i^2 + 12*A*a^2*b*c*d^2*i^2 + 7*B*a*b^2*c^2*d*i^2*
n + 7*B*a^2*b*c*d^2*i^2*n)/(12*(a*d - b*c)) + (x*(12*A*a^2*b*d^3*i^2 - 24*
A*b^3*c^2*d*i^2 + 12*A*a*b^2*c*d^2*i^2 + 7*B*a^2*b*d^3*i^2*n - 5*B*b^3*c^2
*d*i^2*n + 7*B*a*b^2*c*d^2*i^2*n))/(3*(a*d - b*c)) + (x^2*(12*A*a*b^2*d^3*
i^2 - 12*A*b^3*c*d^2*i^2 + 7*B*a*b^2*d^3*i^2*n - B*b^3*c*d^2*i^2*n))/(2*(a
*d - b*c)) + (B*b^3*d^3*i^2*n*x^3)/(a*d - b*c))/(12*a^4*b^3*g^5 + 12*b^7*g
^5*x^4 + 48*a^3*b^4*g^5*x + 48*a*b^6*g^5*x^3 + 72*a^2*b^5*g^5*x^2) - (log(
e*((a + b*x)/(c + d*x))^n)*(a*((B*a*d^2*i^2)/(12*b^3) + (B*c*d*i^2)/(6*b^2
)) + x*(b*((B*a*d^2*i^2)/(12*b^3) + (B*c*d*i^2)/(6*b^2)) + (B*a*d^2*i^2)/(
4*b^2) + (B*c*d*i^2)/(2*b)) + (B*c^2*i^2)/(4*b) + (B*d^2*i^2*x^2)/(2*b)))/
(a^4*g^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2*g^5*x^2 + 4*a^3*b*g^5
*x) - (B*d^4*i^2*n*atanh((12*b^5*c^2*g^5 - 12*a^2*b^3*d^2*g^5)/(12*b^3*g^5
*(a*d - b*c)^2) - (2*b*d*x)/(a*d - b*c)))/(6*b^3*g^5*(a*d - b*c)^2)
```

3.125. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^5}\,dx$

# 3.126 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx$



## 3.126.1 Optimal result

Integrand size = 43, antiderivative size = 293

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx = -\frac{Bd^2i^2n(c+dx)^3}{9(bc-ad)^3g^6(a+bx)^3} + \frac{bBdi^2n(c+dx)^4}{8(bc-ad)^3g^6(a+bx)^4} - \frac{b^2Bi^2n(c+dx)^5}{25(bc-ad)^3g^6(a+bx)^5} - \frac{d^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^3g^6(a+bx)^3} + \frac{bdi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^6(a+bx)^4} - \frac{b^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5(bc-ad)^3g^6(a+bx)^5}$$

output
```
-1/9*B*d^2*i^2*n*(d*x+c)^3/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/8*b*B*d*i^2*n*(d*x
+c)^4/(-a*d+b*c)^3/g^6/(b*x+a)^4-1/25*b^2*B*i^2*n*(d*x+c)^5/(-a*d+b*c)^3/g
^6/(b*x+a)^5-1/3*d^2*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b
*c)^3/g^6/(b*x+a)^3+1/2*b*d*i^2*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/
(-a*d+b*c)^3/g^6/(b*x+a)^4-1/5*b^2*i^2*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/(-a*d+b*c)^3/g^6/(b*x+a)^5
```

### 3.126.2 Mathematica [A] (verified)

Time = 0.56 (sec) , antiderivative size = 357, normalized size of antiderivative = 1.22

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx$$
$$= \frac{i^2\left(-\frac{360Ab^2c^2}{(a+bx)^5}+\frac{720aAbcd}{(a+bx)^5}-\frac{360a^2Ad^2}{(a+bx)^5}-\frac{72b^2Bc^2n}{(a+bx)^5}+\frac{144abBcdn}{(a+bx)^5}-\frac{72a^2Bd^2n}{(a+bx)^5}-\frac{900Abcd}{(a+bx)^4}+\frac{900aAd^2}{(a+bx)^4}-\frac{135bBcdn}{(a+bx)^4}+\frac{135aB}{(a+b}\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x)^6,x]
```

output
```
(i^2*((-360*A*b^2*c^2)/(a + b*x)^5 + (720*a*A*b*c*d)/(a + b*x)^5 - (360*a^
2*A*d^2)/(a + b*x)^5 - (72*b^2*B*c^2*n)/(a + b*x)^5 + (144*a*b*B*c*d*n)/(a
 + b*x)^5 - (72*a^2*B*d^2*n)/(a + b*x)^5 - (900*A*b*c*d)/(a + b*x)^4 + (90
0*a*A*d^2)/(a + b*x)^4 - (135*b*B*c*d*n)/(a + b*x)^4 + (135*a*B*d^2*n)/(a
+ b*x)^4 - (600*A*d^2)/(a + b*x)^3 - (20*B*d^2*n)/(a + b*x)^3 + (30*B*d^3*
n)/((b*c - a*d)*(a + b*x)^2) - (60*B*d^4*n)/((b*c - a*d)^2*(a + b*x)) - (6
0*B*d^5*n*Log[a + b*x])/(b*c - a*d)^3 - (60*B*(a^2*d^2 + a*b*d*(3*c + 5*d*
x) + b^2*(6*c^2 + 15*c*d*x + 10*d^2*x^2))*Log[e*((a + b*x)/(c + d*x))^n])/
(a + b*x)^5 + (60*B*d^5*n*Log[c + d*x])/(b*c - a*d)^3))/(1800*b^3*g^6)
```

### 3.126.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 215, normalized size of antiderivative = 0.73, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2772, 27, 1140, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^6}\,dx$$
$$\Big\downarrow \ 2961$$
$$\frac{i^2\int \frac{(c+dx)^6\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^6}d\frac{a+bx}{c+dx}}{g^6(bc-ad)^3}$$
$$\Big\downarrow \ 2772$$

$$\frac{i^2\left(-Bn\int -\frac{(c+dx)^6\left(6b^2-\frac{15d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{30(a+bx)^6}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{3(a+bx)^3}}{g^6(bc-ad)^3}$$

$$\Big\downarrow \; 27$$

$$\frac{i^2\left(\frac{1}{30}Bn\int \frac{(c+dx)^6\left(6b^2-\frac{15d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)^6}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}}{g^6(bc-ad)^3}$$

$$\Big\downarrow \; 1140$$

$$\frac{i^2\left(\frac{1}{30}Bn\int \left(\frac{6b^2(c+dx)^6}{(a+bx)^6}-\frac{15bd(c+dx)^5}{(a+bx)^5}+\frac{10d^2(c+dx)^4}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{3(a+bx)^3}}{g^6(bc-ad)^3}$$

$$\Big\downarrow \; 2009$$

$$\frac{i^2\left(-\frac{b^2(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{bd(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^4}+\frac{1}{30}Bn\left(-\frac{6b^2(c+}{5(a+}\right.\right.}{g^6(bc-ad)^3}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^6,x]
```

output
```
(i^2*((B*n*((-10*d^2*(c + d*x)^3)/(3*(a + b*x)^3) + (15*b*d*(c + d*x)^4)/(
4*(a + b*x)^4) - (6*b^2*(c + d*x)^5)/(5*(a + b*x)^5)))/30 - (d^2*(c + d*x)
^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3) + (b*d*(c + d*x
)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^4) - (b^2*(c + d*
x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*(a + b*x)^5)))/((b*c - a*d
)^3*g^6)
```

### 3.126. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx$

### 3.126.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1140
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(p_.), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(a + b*x + c*x^2)^p, x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.126.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1048 vs. $2(281) = 562$.

Time = 57.56 (sec) , antiderivative size = 1049, normalized size of antiderivative = 3.58

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1049 |

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^6,x,method=_
RETURNVERBOSE)
```

output
```
-1/1800*(-1800*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c*d^5*i^2*n+1800*B*
x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^2*d^4*i^2*n-1800*B*x*ln(e*((b*x+a)/(
d*x+c))^n)*a^2*b^7*c^2*d^4*i^2*n+2400*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*
c^3*d^3*i^2*n-300*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*d^6*i^2*n-600*B*x^
3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*d^6*i^2*n-300*B*x^3*a*b^8*c*d^5*i^2*n^
2-600*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^3*d^3*i^2*n-600*B*x^2*a^2*b^7*
c*d^5*i^2*n^2+150*B*x^2*a*b^8*c^2*d^4*i^2*n^2-1800*A*x^2*a^2*b^7*c*d^5*i^2
*n+1800*A*x^2*a*b^8*c^2*d^4*i^2*n-900*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^
4*d^2*i^2*n-600*B*x*a^2*b^7*c^2*d^4*i^2*n^2+500*B*x*a*b^8*c^3*d^3*i^2*n^2-
1800*A*x*a^2*b^7*c^2*d^4*i^2*n+2400*A*x*a*b^8*c^3*d^3*i^2*n-600*B*ln(e*((b
*x+a)/(d*x+c))^n)*a^2*b^7*c^3*d^3*i^2*n+900*B*ln(e*((b*x+a)/(d*x+c))^n)*a*
b^8*c^4*d^2*i^2*n+47*B*a^5*b^4*d^6*i^2*n^2-72*B*b^9*c^5*d*i^2*n^2+60*A*a^5
*b^4*d^6*i^2*n-360*A*b^9*c^5*d*i^2*n-60*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^
9*d^6*i^2*n+60*B*x^4*a*b^8*d^6*i^2*n^2-60*B*x^4*b^9*c*d^5*i^2*n^2+270*B*x^
3*a^2*b^7*d^6*i^2*n^2+30*B*x^3*b^9*c^2*d^4*i^2*n^2+470*B*x^2*a^3*b^6*d^6*i
^2*n^2-20*B*x^2*b^9*c^3*d^3*i^2*n^2+600*A*x^2*a^3*b^6*d^6*i^2*n-600*A*x^2*
b^9*c^3*d^3*i^2*n+235*B*x*a^4*b^5*d^6*i^2*n^2-135*B*x*b^9*c^4*d^2*i^2*n^2+
300*A*x*a^4*b^5*d^6*i^2*n-900*A*x*b^9*c^4*d^2*i^2*n-360*B*ln(e*((b*x+a)/(d
*x+c))^n)*b^9*c^5*d*i^2*n-200*B*a^2*b^7*c^3*d^3*i^2*n^2+225*B*a*b^8*c^4*d^
2*i^2*n^2-600*A*a^2*b^7*c^3*d^3*i^2*n+900*A*a*b^8*c^4*d^2*i^2*n)/g^6/(b...
```

### 3.126.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1087 vs. $2(281) = 562$.

Time = 0.37 (sec) , antiderivative size = 1087, normalized size of antiderivative = 3.71

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx = -\frac{60\,(Bb^5cd^4-Bab^4d^5)i^2nx^4-30\,(Bb^5c^2d^3-10\,Bab^4cd^4+9\,Ba^2b^3d^5)i^2nx^3+(72\,Bb^5c^5-225\,Bab^4c^4d}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^6,x,
algorithm="fricas")
```

output
```
-1/1800*(60*(B*b^5*c*d^4 - B*a*b^4*d^5)*i^2*n*x^4 - 30*(B*b^5*c^2*d^3 - 10
*B*a*b^4*c*d^4 + 9*B*a^2*b^3*d^5)*i^2*n*x^3 + (72*B*b^5*c^5 - 225*B*a*b^4*
c^4*d + 200*B*a^2*b^3*c^3*d^2 - 47*B*a^5*d^5)*i^2*n + 60*(6*A*b^5*c^5 - 15
*A*a*b^4*c^4*d + 10*A*a^2*b^3*c^3*d^2 - A*a^5*d^5)*i^2 + 10*((2*B*b^5*c^3*
d^2 - 15*B*a*b^4*c^2*d^3 + 60*B*a^2*b^3*c*d^4 - 47*B*a^3*b^2*d^5)*i^2*n +
60*(A*b^5*c^3*d^2 - 3*A*a*b^4*c^2*d^3 + 3*A*a^2*b^3*c*d^4 - A*a^3*b^2*d^5)
*i^2)*x^2 + 5*((27*B*b^5*c^4*d - 100*B*a*b^4*c^3*d^2 + 120*B*a^2*b^3*c^2*d
^3 - 47*B*a^4*b*d^5)*i^2*n + 60*(3*A*b^5*c^4*d - 8*A*a*b^4*c^3*d^2 + 6*A*a
^2*b^3*c^2*d^3 - A*a^4*b*d^5)*i^2)*x + 60*(10*(B*b^5*c^3*d^2 - 3*B*a*b^4*c
^2*d^3 + 3*B*a^2*b^3*c*d^4 - B*a^3*b^2*d^5)*i^2*x^2 + 5*(3*B*b^5*c^4*d - 8
*B*a*b^4*c^3*d^2 + 6*B*a^2*b^3*c^2*d^3 - B*a^4*b*d^5)*i^2*x + (6*B*b^5*c^5
 - 15*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2 - B*a^5*d^5)*i^2)*log(e) + 60*(
B*b^5*d^5*i^2*n*x^5 + 5*B*a*b^4*d^5*i^2*n*x^4 + 10*B*a^2*b^3*d^5*i^2*n*x^3
 + 10*(B*b^5*c^3*d^2 - 3*B*a*b^4*c^2*d^3 + 3*B*a^2*b^3*c*d^4)*i^2*n*x^2 +
5*(3*B*b^5*c^4*d - 8*B*a*b^4*c^3*d^2 + 6*B*a^2*b^3*c^2*d^3)*i^2*n*x + (6*B
*b^5*c^5 - 15*B*a*b^4*c^4*d + 10*B*a^2*b^3*c^3*d^2)*i^2*n)*log((b*x + a)/(
d*x + c)))/((b^11*c^3 - 3*a*b^10*c^2*d + 3*a^2*b^9*c*d^2 - a^3*b^8*d^3)*g^
6*x^5 + 5*(a*b^10*c^3 - 3*a^2*b^9*c^2*d + 3*a^3*b^8*c*d^2 - a^4*b^7*d^3)*g
^6*x^4 + 10*(a^2*b^9*c^3 - 3*a^3*b^8*c^2*d + 3*a^4*b^7*c*d^2 - a^5*b^6*d^3
)*g^6*x^3 + 10*(a^3*b^8*c^3 - 3*a^4*b^7*c^2*d + 3*a^5*b^6*c*d^2 - a^6*b...
```

### 3.126.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**6,x
)
```

output
```
Timed out
```

### 3.126.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3058 vs. $2(281) = 562$.

Time = 0.35 (sec) , antiderivative size = 3058, normalized size of antiderivative = 10.44

$$\int \frac{(ci + dix)^2 \left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^6} \, dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^6,x,
algorithm="maxima")
```

output
```
-1/300*B*c^2*i^2*n*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*a^
2*b^2*c^2*d^2 - 163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*d^
4)*x^3 + 10*(2*b^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3*b
^4*c^3*d - 17*a*b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10*c
^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*g^
6*x^5 + 5*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c*d
^3 + a^5*b^5*d^4)*g^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^6*
c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a^4
*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2 +
5*(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 + a
^8*b^2*d^4)*g^6*x + (a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 - 4
*a^8*b^2*c*d^3 + a^9*b*d^4)*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b^5
*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b
*d^5)*g^6) - 60*d^5*log(d*x + c)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*c^
3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6)) - 1/1800*B
*d^2*i^2*n*((47*a^2*b^4*c^4 - 278*a^3*b^3*c^3*d + 822*a^4*b^2*c^2*d^2 - 27
8*a^5*b*c*d^3 + 47*a^6*d^4 + 60*(10*b^6*c^2*d^2 - 5*a*b^5*c*d^3 + a^2*b^4*
d^4)*x^4 - 30*(10*b^6*c^3*d - 95*a*b^5*c^2*d^2 + 46*a^2*b^4*c*d^3 - 9*a^3*
b^3*d^4)*x^3 + 10*(20*b^6*c^4 - 140*a*b^5*c^3*d + 537*a^2*b^4*c^2*d^2 - 24
8*a^3*b^3*c*d^3 + 47*a^4*b^2*d^4)*x^2 + 5*(35*a*b^5*c^4 - 218*a^2*b^4*c...
```

### 3.126.8 Giac [A] (verification not implemented)

Time = 2.58 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.43

$$\int \frac{(ci + dix)^2 \left(A + B \log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^6} \, dx =$$

$$-\frac{1}{1800} \left( \frac{60 \left(6 Bb^2 i^2 n - \frac{15 (bx+a)Bbdi^2 n}{dx+c} + \frac{10 (bx+a)^2 Bd^2 i^2 n}{(dx+c)^2}\right) \log \left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^5 b^2 c^2 g^6}{(dx+c)^5} - \frac{2 (bx+a)^5 abcdg^6}{(dx+c)^5} + \frac{(bx+a)^5 a^2 d^2 g^6}{(dx+c)^5}} + \frac{72 Bb^2 i^2 n - \frac{225 (bx+a)Bbdi^2 n}{dx+c} + \frac{200 (}{}}{} \right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^6,x,
algorithm="giac")
```

output
```
-1/1800*(60*(6*B*b^2*i^2*n - 15*(b*x + a)*B*b*d*i^2*n/(d*x + c) + 10*(b*x
+ a)^2*B*d^2*i^2*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^5*b^2*
c^2*g^6/(d*x + c)^5 - 2*(b*x + a)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*x + a)^5*
a^2*d^2*g^6/(d*x + c)^5) + (72*B*b^2*i^2*n - 225*(b*x + a)*B*b*d*i^2*n/(d*
x + c) + 200*(b*x + a)^2*B*d^2*i^2*n/(d*x + c)^2 + 360*B*b^2*i^2*log(e) -
900*(b*x + a)*B*b*d*i^2*log(e)/(d*x + c) + 600*(b*x + a)^2*B*d^2*i^2*log(e
)/(d*x + c)^2 + 360*A*b^2*i^2 - 900*(b*x + a)*A*b*d*i^2/(d*x + c) + 600*(b
*x + a)^2*A*d^2*i^2/(d*x + c)^2)/((b*x + a)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*
(b*x + a)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*x + a)^5*a^2*d^2*g^6/(d*x + c)^5)
)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.126.9 Mupad [B] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 954, normalized size of antiderivative = 3.26

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx$$
$$= \frac{B\,d^5\,i^2\,n\,\mathrm{atanh}\left(\frac{30\,a^3\,b^3\,d^3\,g^6-30\,a^2\,b^4\,c\,d^2\,g^6-30\,a\,b^5\,c^2\,d\,g^6+30\,b^6\,c^3\,g^6}{30\,b^3\,g^6\,(a\,d-b\,c)^3}+\frac{2\,b\,d\,x\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}{(a\,d-b\,c)^3}\right)}{15\,b^3\,g^6\,(a\,d-b\,c)^3}$$
$$-\frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(a\left(\frac{B\,a\,d^2\,i^2}{30\,b^3}+\frac{B\,c\,d\,i^2}{10\,b^2}\right)+x\left(b\left(\frac{B\,a\,d^2\,i^2}{30\,b^3}+\frac{B\,c\,d\,i^2}{10\,b^2}\right)+\frac{2\,B\,a\,d^2\,i^2}{15\,b^2}+\frac{2\,B\,c\,d\,i^2}{5\,b}\right)+\frac{B\,c^2\,i^2}{5\,b}+\frac{B\,d^2\,i^2\,x^2}{3\,b}\right.}{a^5\,g^6+5\,a^4\,b\,g^6\,x+10\,a^3\,b^2\,g^6\,x^2+10\,a^2\,b^3\,g^6\,x^3+5\,a\,b^4\,g^6\,x^4+b^5\,g^6\,x^5}$$
$$-\frac{\frac{60\,A\,a^4\,d^4\,i^2+360\,A\,b^4\,c^4\,i^2+47\,B\,a^4\,d^4\,i^2\,n+72\,B\,b^4\,c^4\,i^2\,n+60\,A\,a^2\,b^2\,c^2\,d^2\,i^2-540\,A\,a\,b^3\,c^3\,d\,i^2+60\,A\,a^3\,b\,c\,d^3\,i^2-153\,B\,a\,b^3\,c^3\,d\,i^2\,n+47\,B}{60\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}}{}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^6,x)
```

output
```
(B*d^5*i^2*n*atanh((30*b^6*c^3*g^6 + 30*a^3*b^3*d^3*g^6 - 30*a*b^5*c^2*d*g
^6 - 30*a^2*b^4*c*d^2*g^6)/(30*b^3*g^6*(a*d - b*c)^3) + (2*b*d*x*(a^2*d^2
+ b^2*c^2 - 2*a*b*c*d))/(a*d - b*c)^3))/(15*b^3*g^6*(a*d - b*c)^3) - (log(
e*((a + b*x)/(c + d*x))^n)*(a*((B*a*d^2*i^2)/(30*b^3) + (B*c*d*i^2)/(10*b^
2)) + x*(b*((B*a*d^2*i^2)/(30*b^3) + (B*c*d*i^2)/(10*b^2)) + (2*B*a*d^2*i^
2)/(15*b^2) + (2*B*c*d*i^2)/(5*b)) + (B*c^2*i^2)/(5*b) + (B*d^2*i^2*x^2)/(
3*b)))/(a^5*g^6 + b^5*g^6*x^5 + 5*a*b^4*g^6*x^4 + 10*a^3*b^2*g^6*x^2 + 10*
a^2*b^3*g^6*x^3 + 5*a^4*b*g^6*x) - ((60*A*a^4*d^4*i^2 + 360*A*b^4*c^4*i^2
+ 47*B*a^4*d^4*i^2*n + 72*B*b^4*c^4*i^2*n + 60*A*a^2*b^2*c^2*d^2*i^2 - 540
*A*a*b^3*c^3*d*i^2 + 60*A*a^3*b*c*d^3*i^2 - 153*B*a*b^3*c^3*d*i^2*n + 47*B
*a^3*b*c*d^3*i^2*n + 47*B*a^2*b^2*c^2*d^2*i^2*n)/(60*(a^2*d^2 + b^2*c^2 -
2*a*b*c*d)) + (x^2*(60*A*a^2*b^2*d^4*i^2 + 60*A*b^4*c^2*d^2*i^2 + 47*B*a^2
*b^2*d^4*i^2*n + 2*B*b^4*c^2*d^2*i^2*n - 120*A*a*b^3*c*d^3*i^2 - 13*B*a*b^
3*c*d^3*i^2*n))/(6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (x*(60*A*a^3*b*d^4*i
^2 + 180*A*b^4*c^3*d*i^2 - 300*A*a*b^3*c^2*d^2*i^2 + 60*A*a^2*b^2*c*d^3*i^
2 + 47*B*a^3*b*d^4*i^2*n + 27*B*b^4*c^3*d*i^2*n - 73*B*a*b^3*c^2*d^2*i^2*n
 + 47*B*a^2*b^2*c*d^3*i^2*n))/(12*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (d*x^
3*(9*B*a*b^3*d^3*i^2*n - B*b^4*c*d^2*i^2*n))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b
*c*d)) + (B*b^4*d^4*i^2*n*x^4)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))/(30*a^5*b^
3*g^6 + 30*b^8*g^6*x^5 + 150*a^4*b^4*g^6*x + 150*a*b^7*g^6*x^4 + 300*a^...
```

3.126. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^6}\,dx$

# 3.127 $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$



## 3.127.1 Optimal result

Integrand size = 43, antiderivative size = 477

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= \frac{B(bc-ad)^6g^3i^3nx}{140b^3d^3} + \frac{B(bc-ad)^5g^3i^3n(c+dx)^2}{280b^2d^4} + \frac{B(bc-ad)^4g^3i^3n(c+dx)^3}{420bd^4} \\
&\quad - \frac{17B(bc-ad)^3g^3i^3n(c+dx)^4}{280d^4} + \frac{bB(bc-ad)^2g^3i^3n(c+dx)^5}{14d^4} \\
&\quad - \frac{b^2B(bc-ad)g^3i^3n(c+dx)^6}{42d^4} - \frac{(bc-ad)^3g^3i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^4} \\
&\quad + \frac{3b(bc-ad)^2g^3i^3(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^4} \\
&\quad - \frac{b^2(bc-ad)g^3i^3(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^4} \\
&\quad + \frac{b^3g^3i^3(c+dx)^7\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{7d^4} \\
&\quad + \frac{B(bc-ad)^7g^3i^3n\log\left(\frac{a+bx}{c+dx}\right)}{140b^4d^4} + \frac{B(bc-ad)^7g^3i^3n\log(c+dx)}{140b^4d^4}
\end{aligned}$$

output
```
1/140*B*(-a*d+b*c)^6*g^3*i^3*n*x/b^3/d^3+1/280*B*(-a*d+b*c)^5*g^3*i^3*n*(d
*x+c)^2/b^2/d^4+1/420*B*(-a*d+b*c)^4*g^3*i^3*n*(d*x+c)^3/b/d^4-17/280*B*(-
a*d+b*c)^3*g^3*i^3*n*(d*x+c)^4/d^4+1/14*b*B*(-a*d+b*c)^2*g^3*i^3*n*(d*x+c)
^5/d^4-1/42*b^2*B*(-a*d+b*c)*g^3*i^3*n*(d*x+c)^6/d^4-1/4*(-a*d+b*c)^3*g^3*
i^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+3/5*b*(-a*d+b*c)^2*g^3*i
^3*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4-1/2*b^2*(-a*d+b*c)*g^3*i^
3*(d*x+c)^6*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+1/7*b^3*g^3*i^3*(d*x+c)^7*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+1/140*B*(-a*d+b*c)^7*g^3*i^3*n*ln((b*x
+a)/(d*x+c))/b^4/d^4+1/140*B*(-a*d+b*c)^7*g^3*i^3*n*ln(d*x+c)/b^4/d^4
```

### 3.127.2 Mathematica [A] (verified)

Time = 0.37 (sec) , antiderivative size = 631, normalized size of antiderivative = 1.32

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{g^3i^3\left(210d^4(bc-ad)^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+504d^5(bc-ad)^2(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]),x]
```

output
```
(g^3*i^3*(210*d^4*(b*c - a*d)^3*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]) + 504*d^5*(b*c - a*d)^2*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]) + 420*d^6*(b*c - a*d)*(a + b*x)^6*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]) + 120*d^7*(a + b*x)^7*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 35
*B*(b*c - a*d)^4*n*(6*b*d*(b*c - a*d)^2*x + 3*d^2*(-(b*c) + a*d)*(a + b*x)
^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c + d*x]) + 42*B*(b*c - a*d)^
3*n*(12*b*d*(b*c - a*d)^3*x - 6*d^2*(b*c - a*d)^2*(a + b*x)^2 + 4*d^3*(b*c
 - a*d)*(a + b*x)^3 - 3*d^4*(a + b*x)^4 - 12*(b*c - a*d)^4*Log[c + d*x]) -
 7*B*(b*c - a*d)^2*n*(60*b*d*(b*c - a*d)^4*x + 30*d^2*(-(b*c) + a*d)^3*(a
+ b*x)^2 + 20*d^3*(b*c - a*d)^2*(a + b*x)^3 + 15*d^4*(-(b*c) + a*d)*(a + b
*x)^4 + 12*d^5*(a + b*x)^5 - 60*(b*c - a*d)^5*Log[c + d*x]) + 2*B*(b*c - a
*d)*n*(60*b*d*(b*c - a*d)^5*x - 30*d^2*(b*c - a*d)^4*(a + b*x)^2 + 20*d^3*
(b*c - a*d)^3*(a + b*x)^3 - 15*d^4*(b*c - a*d)^2*(a + b*x)^4 + 12*d^5*(b*c
 - a*d)*(a + b*x)^5 - 10*d^6*(a + b*x)^6 - 60*(b*c - a*d)^6*Log[c + d*x]))
)/(840*b^4*d^4)
```

### 3.127.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 414, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2961, 2782, 27, 2123, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2961}$$

$$g^3i^3(bc-ad)^7\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^8}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2782}$$

$$g^3i^3(bc-ad)^7\left(-Bn\int -\frac{(c+dx)\left(b^3-\frac{7d(a+bx)b^2}{c+dx}+\frac{21d^2(a+bx)^2b}{(c+dx)^2}-\frac{35d^3(a+bx)^3}{(c+dx)^3}\right)}{140d^4(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}+\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{7d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\right.$$

$$\Big\downarrow \text{27}$$

$$g^3i^3(bc-ad)^7\left(\frac{Bn\int \frac{(c+dx)\left(b^3-\frac{7d(a+bx)b^2}{c+dx}+\frac{21d^2(a+bx)^2b}{(c+dx)^2}-\frac{35d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}}{140d^4}+\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{7d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.$$

$$\Big\downarrow \text{2123}$$

$$g^3i^3(bc-ad)^7\left(\frac{Bn\int\left(-\frac{20db^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^7}+\frac{50db}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{34d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4b}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3b^2}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2b^3}+\frac{c+}{(a+}\right.}{140d^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$g^3 i^3 (bc - ad)^7 \left( \frac{b^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{7d^4\left(b - \frac{d(a+bx)}{c+dx}\right)^7} - \frac{b^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{2d^4\left(b - \frac{d(a+bx)}{c+dx}\right)^6} + \frac{3b \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)}{5d^4\left(b - \frac{d(a+bx)}{c+dx}\right)^5} - \frac{B\log\left(e\left(\frac{a+b}{c+d}\right.\right.}{4d^4\left(b - \frac{d}{}\right.}\right.$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
,x]
```

output
```
(b*c - a*d)^7*g^3*i^3*((b^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(7*d^4
*(b - (d*(a + b*x))/(c + d*x))^7) - (b^2*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^6) + (3*b*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(5*d^4*(b - (d*(a + b*x))/(c + d*x))^5) - (A + B*Log[e
*((a + b*x)/(c + d*x))^n])/(4*d^4*(b - (d*(a + b*x))/(c + d*x))^4) + (B*n*
((-10*b^2)/(3*(b - (d*(a + b*x))/(c + d*x))^6) + (10*b)/(b - (d*(a + b*x))
/(c + d*x))^5 - 17/(2*(b - (d*(a + b*x))/(c + d*x))^4) + 1/(3*b*(b - (d*(a
 + b*x))/(c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^
3*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^4 - Log[b -
(d*(a + b*x))/(c + d*x)]/b^4))/(140*d^4))
```

### 3.127.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.127.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2178 vs. $2(453) = 906$.

Time = 50.10 (sec) , antiderivative size = 2179, normalized size of antiderivative = 4.57

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 2179 |

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_
RETURNVERBOSE)
```

output
```
1/840*(-6*B*x*b^7*c^6*d*g^3*i^3*n^2+120*B*x^7*ln(e*((b*x+a)/(d*x+c))^n)*b^
7*d^7*g^3*i^3*n+20*B*x^6*a*b^6*d^7*g^3*i^3*n^2-20*B*x^6*b^7*c*d^6*g^3*i^3*
n^2+420*A*x^6*a*b^6*d^7*g^3*i^3*n+420*A*x^6*b^7*c*d^6*g^3*i^3*n+60*B*x^5*a
^2*b^5*d^7*g^3*i^3*n^2-60*B*x^5*b^7*c^2*d^5*g^3*i^3*n^2+504*A*x^5*a^2*b^5*
d^7*g^3*i^3*n+504*A*x^5*b^7*c^2*d^5*g^3*i^3*n+51*B*x^4*a^3*b^4*d^7*g^3*i^3
*n^2-51*B*x^4*b^7*c^3*d^4*g^3*i^3*n^2+1512*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)
*a*b^6*c*d^6*g^3*i^3*n+1890*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*c*d^6*
g^3*i^3*n+1890*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c^2*d^5*g^3*i^3*n+840
*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*c*d^6*g^3*i^3*n+2520*B*x^3*ln(e*(
(b*x+a)/(d*x+c))^n)*a^2*b^5*c^2*d^5*g^3*i^3*n+840*B*x^3*ln(e*((b*x+a)/(d*x
+c))^n)*a*b^6*c^3*d^4*g^3*i^3*n+1260*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b
^4*c^2*d^5*g^3*i^3*n+1260*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*c^3*d^4*
g^3*i^3*n+840*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*c^3*d^4*g^3*i^3*n-21*B
*x^2*a*b^6*c^4*d^3*g^3*i^3*n^2+1260*A*x^2*a^3*b^4*c^2*d^5*g^3*i^3*n+1260*A
*x^2*a^2*b^5*c^3*d^4*g^3*i^3*n+420*B*x^6*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*d
^7*g^3*i^3*n+420*B*x^6*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c*d^6*g^3*i^3*n+504*B
*x^5*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*d^7*g^3*i^3*n+39*B*a^6*b*c*d^6*g^3*
i^3*n^2-105*B*a^5*b^2*c^2*d^5*g^3*i^3*n^2-378*B*a^4*b^3*c^3*d^4*g^3*i^3*n^
2+378*B*a^3*b^4*c^4*d^3*g^3*i^3*n^2+504*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^
7*c^2*d^5*g^3*i^3*n+1512*A*x^5*a*b^6*c*d^6*g^3*i^3*n+210*B*x^4*ln(e*((b...
```

### 3.127.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1336 vs. 2(453) = 906.

Time = 0.93 (sec) , antiderivative size = 1336, normalized size of antiderivative = 2.80

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="fricas")
```

output
```
1/840*(120*A*b^7*d^7*g^3*i^3*x^7 + 6*(35*B*a^4*b^3*c^3*d^4 - 21*B*a^5*b^2*
c^2*d^5 + 7*B*a^6*b*c*d^6 - B*a^7*d^7)*g^3*i^3*n*log(b*x + a) + 6*(B*b^7*c
^7 - 7*B*a*b^6*c^6*d + 21*B*a^2*b^5*c^5*d^2 - 35*B*a^3*b^4*c^4*d^3)*g^3*i^
3*n*log(d*x + c) - 20*((B*b^7*c*d^6 - B*a*b^6*d^7)*g^3*i^3*n - 21*(A*b^7*c
*d^6 + A*a*b^6*d^7)*g^3*i^3)*x^6 - 12*(5*(B*b^7*c^2*d^5 - B*a^2*b^5*d^7)*g
^3*i^3*n - 42*(A*b^7*c^2*d^5 + 3*A*a*b^6*c*d^6 + A*a^2*b^5*d^7)*g^3*i^3)*x
^5 - 3*((17*B*b^7*c^3*d^4 + 49*B*a*b^6*c^2*d^5 - 49*B*a^2*b^5*c*d^6 - 17*B
*a^3*b^4*d^7)*g^3*i^3*n - 70*(A*b^7*c^3*d^4 + 9*A*a*b^6*c^2*d^5 + 9*A*a^2*
b^5*c*d^6 + A*a^3*b^4*d^7)*g^3*i^3)*x^4 - 2*((B*b^7*c^4*d^3 + 98*B*a*b^6*c
^3*d^4 - 98*B*a^3*b^4*c*d^6 - B*a^4*b^3*d^7)*g^3*i^3*n - 420*(A*a*b^6*c^3*
d^4 + 3*A*a^2*b^5*c^2*d^5 + A*a^3*b^4*c*d^6)*g^3*i^3)*x^3 + 3*((B*b^7*c^5*
d^2 - 7*B*a*b^6*c^4*d^3 - 84*B*a^2*b^5*c^3*d^4 + 84*B*a^3*b^4*c^2*d^5 + 7*
B*a^4*b^3*c*d^6 - B*a^5*b^2*d^7)*g^3*i^3*n + 420*(A*a^2*b^5*c^3*d^4 + A*a^
3*b^4*c^2*d^5)*g^3*i^3)*x^2 + 6*(140*A*a^3*b^4*c^3*d^4*g^3*i^3 - (B*b^7*c^
6*d - 7*B*a*b^6*c^5*d^2 + 21*B*a^2*b^5*c^4*d^3 - 21*B*a^4*b^3*c^2*d^5 + 7*
B*a^5*b^2*c*d^6 - B*a^6*b*d^7)*g^3*i^3*n)*x + 6*(20*B*b^7*d^7*g^3*i^3*x^7
+ 140*B*a^3*b^4*c^3*d^4*g^3*i^3*x + 70*(B*b^7*c*d^6 + B*a*b^6*d^7)*g^3*i^3
*x^6 + 84*(B*b^7*c^2*d^5 + 3*B*a*b^6*c*d^6 + B*a^2*b^5*d^7)*g^3*i^3*x^5 +
35*(B*b^7*c^3*d^4 + 9*B*a*b^6*c^2*d^5 + 9*B*a^2*b^5*c*d^6 + B*a^3*b^4*d^7)
*g^3*i^3*x^4 + 140*(B*a*b^6*c^3*d^4 + 3*B*a^2*b^5*c^2*d^5 + B*a^3*b^4*c...
```

### 3.127.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x
)
```

output
```
Timed out
```

### 3.127.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2901 vs. 2(453) = 906.

Time = 0.28 (sec) , antiderivative size = 2901, normalized size of antiderivative = 6.08

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="maxima")
```

output
```
1/7*B*b^3*d^3*g^3*i^3*x^7*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/7*A*b
^3*d^3*g^3*i^3*x^7 + 1/2*B*b^3*c*d^2*g^3*i^3*x^6*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + 1/2*B*a*b^2*d^3*g^3*i^3*x^6*log(e*(b*x/(d*x + c) + a/(d*x
+ c))^n) + 1/2*A*b^3*c*d^2*g^3*i^3*x^6 + 1/2*A*a*b^2*d^3*g^3*i^3*x^6 + 3/5
*B*b^3*c^2*d*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 9/5*B*a*
b^2*c*d^2*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/5*B*a^2*b
*d^3*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/5*A*b^3*c^2*d*
g^3*i^3*x^5 + 9/5*A*a*b^2*c*d^2*g^3*i^3*x^5 + 3/5*A*a^2*b*d^3*g^3*i^3*x^5
+ 1/4*B*b^3*c^3*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 9/4*B
*a*b^2*c^2*d*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 9/4*B*a^
2*b*c*d^2*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*B*a^3*d
^3*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*b^3*c^3*g^3*
i^3*x^4 + 9/4*A*a*b^2*c^2*d*g^3*i^3*x^4 + 9/4*A*a^2*b*c*d^2*g^3*i^3*x^4 +
1/4*A*a^3*d^3*g^3*i^3*x^4 + B*a*b^2*c^3*g^3*i^3*x^3*log(e*(b*x/(d*x + c) +
 a/(d*x + c))^n) + 3*B*a^2*b*c^2*d*g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n) + B*a^3*c*d^2*g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n) + A*a*b^2*c^3*g^3*i^3*x^3 + 3*A*a^2*b*c^2*d*g^3*i^3*x^3 + A*a^3*c*d^2*g
^3*i^3*x^3 + 3/2*B*a^2*b*c^3*g^3*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + 3/2*B*a^3*c^2*d*g^3*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
 + 3/2*A*a^2*b*c^3*g^3*i^3*x^2 + 3/2*A*a^3*c^2*d*g^3*i^3*x^2 + 1/420*B*...
```

### 3.127.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5934 vs. 2(453) = 906.

Time = 2.50 (sec) , antiderivative size = 5934, normalized size of antiderivative = 12.44

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="giac")
```

output
```
-1/840*(6*(B*b^11*c^8*g^3*i^3*n - 8*B*a*b^10*c^7*d*g^3*i^3*n - 7*(b*x + a)
*B*b^10*c^8*d*g^3*i^3*n/(d*x + c) + 28*B*a^2*b^9*c^6*d^2*g^3*i^3*n + 56*(b
*x + a)*B*a*b^9*c^7*d^2*g^3*i^3*n/(d*x + c) + 21*(b*x + a)^2*B*b^9*c^8*d^2
*g^3*i^3*n/(d*x + c)^2 - 56*B*a^3*b^8*c^5*d^3*g^3*i^3*n - 196*(b*x + a)*B*
a^2*b^8*c^6*d^3*g^3*i^3*n/(d*x + c) - 168*(b*x + a)^2*B*a*b^8*c^7*d^3*g^3*
i^3*n/(d*x + c)^2 - 35*(b*x + a)^3*B*b^8*c^8*d^3*g^3*i^3*n/(d*x + c)^3 + 7
0*B*a^4*b^7*c^4*d^4*g^3*i^3*n + 392*(b*x + a)*B*a^3*b^7*c^5*d^4*g^3*i^3*n/
(d*x + c) + 588*(b*x + a)^2*B*a^2*b^7*c^6*d^4*g^3*i^3*n/(d*x + c)^2 + 280*
(b*x + a)^3*B*a*b^7*c^7*d^4*g^3*i^3*n/(d*x + c)^3 - 56*B*a^5*b^6*c^3*d^5*g
^3*i^3*n - 490*(b*x + a)*B*a^4*b^6*c^4*d^5*g^3*i^3*n/(d*x + c) - 1176*(b*x
 + a)^2*B*a^3*b^6*c^5*d^5*g^3*i^3*n/(d*x + c)^2 - 980*(b*x + a)^3*B*a^2*b^
6*c^6*d^5*g^3*i^3*n/(d*x + c)^3 + 28*B*a^6*b^5*c^2*d^6*g^3*i^3*n + 392*(b*
x + a)*B*a^5*b^5*c^3*d^6*g^3*i^3*n/(d*x + c) + 1470*(b*x + a)^2*B*a^4*b^5*
c^4*d^6*g^3*i^3*n/(d*x + c)^2 + 1960*(b*x + a)^3*B*a^3*b^5*c^5*d^6*g^3*i^3
*n/(d*x + c)^3 - 8*B*a^7*b^4*c*d^7*g^3*i^3*n - 196*(b*x + a)*B*a^6*b^4*c^2
*d^7*g^3*i^3*n/(d*x + c) - 1176*(b*x + a)^2*B*a^5*b^4*c^3*d^7*g^3*i^3*n/(d
*x + c)^2 - 2450*(b*x + a)^3*B*a^4*b^4*c^4*d^7*g^3*i^3*n/(d*x + c)^3 + B*a
^8*b^3*d^8*g^3*i^3*n + 56*(b*x + a)*B*a^7*b^3*c*d^8*g^3*i^3*n/(d*x + c) +
588*(b*x + a)^2*B*a^6*b^3*c^2*d^8*g^3*i^3*n/(d*x + c)^2 + 1960*(b*x + a)^3
*B*a^5*b^3*c^3*d^8*g^3*i^3*n/(d*x + c)^3 - 7*(b*x + a)*B*a^8*b^2*d^9*g^...
```

### 3.127.9 Mupad [B] (verification not implemented)

Time = 3.06 (sec) , antiderivative size = 4476, normalized size of antiderivative = 9.38

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))
,x)
```

output
```
x^4*((g^3*i^3*(20*A*a^3*d^3 + 20*A*b^3*c^3 + 3*B*a^3*d^3*n - 3*B*b^3*c^3*n
 + 120*A*a*b^2*c^2*d + 120*A*a^2*b*c*d^2 - 6*B*a*b^2*c^2*d*n + 6*B*a^2*b*c
*d^2*n))/20 + ((140*a*d + 140*b*c)*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*
c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(
140*a*d + 140*b*c))/(140*b*d) - (b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2
+ B*a^2*d^2*n - B*b^2*c^2*n + 32*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i^3))/(
560*b*d) - (a*c*((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n
))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140))/(4*b*d)) + x^3*((g^3*
i^3*(4*A*a^4*d^4 + 4*A*b^4*c^4 + B*a^4*d^4*n - B*b^4*c^4*n + 144*A*a^2*b^2
*c^2*d^2 + 64*A*a*b^3*c^3*d + 64*A*a^3*b*c*d^3 - 8*B*a*b^3*c^3*d*n + 8*B*a
^3*b*c*d^3*n))/(12*b*d) - ((140*a*d + 140*b*c)*((g^3*i^3*(20*A*a^3*d^3 + 2
0*A*b^3*c^3 + 3*B*a^3*d^3*n - 3*B*b^3*c^3*n + 120*A*a*b^2*c^2*d + 120*A*a^
2*b*c*d^2 - 6*B*a*b^2*c^2*d*n + 6*B*a^2*b*c*d^2*n))/5 + ((140*a*d + 140*b*
c)*((((b^2*d^2*g^3*i^3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b
^2*d^2*g^3*i^3*(140*a*d + 140*b*c))/140)*(140*a*d + 140*b*c))/(140*b*d) -
(b*d*g^3*i^3*(12*A*a^2*d^2 + 12*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c^2*n + 32
*A*a*b*c*d))/2 + A*a*b^2*c*d^2*g^3*i^3))/(140*b*d) - (a*c*((b^2*d^2*g^3*i^
3*(28*A*a*d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a
*d + 140*b*c))/140))/(b*d)))/(420*b*d) + (a*c*((((b^2*d^2*g^3*i^3*(28*A*a*
d + 28*A*b*c + B*a*d*n - B*b*c*n))/7 - (A*b^2*d^2*g^3*i^3*(140*a*d + 14...
```

3.127. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

# 3.128 $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$



## 3.128.1 Optimal result

Integrand size = 43, antiderivative size = 387

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx \\
&= -\frac{B(bc-ad)^5g^2i^3nx}{60b^3d^2} - \frac{B(bc-ad)^4g^2i^3n(c+dx)^2}{120b^2d^3} \\
&\quad - \frac{B(bc-ad)^3g^2i^3n(c+dx)^3}{180bd^3} + \frac{7B(bc-ad)^2g^2i^3n(c+dx)^4}{120d^3} \\
&\quad - \frac{bB(bc-ad)g^2i^3n(c+dx)^5}{30d^3} + \frac{(bc-ad)^2g^2i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^3} \\
&\quad - \frac{2b(bc-ad)g^2i^3(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^3} \\
&\quad + \frac{b^2g^2i^3(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3} \\
&\quad - \frac{B(bc-ad)^6g^2i^3n\log\left(\frac{a+bx}{c+dx}\right)}{60b^4d^3} - \frac{B(bc-ad)^6g^2i^3n\log(c+dx)}{60b^4d^3}
\end{aligned}$$

output
```
-1/60*B*(-a*d+b*c)^5*g^2*i^3*n*x/b^3/d^2-1/120*B*(-a*d+b*c)^4*g^2*i^3*n*(d
*x+c)^2/b^2/d^3-1/180*B*(-a*d+b*c)^3*g^2*i^3*n*(d*x+c)^3/b/d^3+7/120*B*(-a
*d+b*c)^2*g^2*i^3*n*(d*x+c)^4/d^3-1/30*b*B*(-a*d+b*c)*g^2*i^3*n*(d*x+c)^5/
d^3+1/4*(-a*d+b*c)^2*g^2*i^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3
-2/5*b*(-a*d+b*c)*g^2*i^3*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3+1/
6*b^2*g^2*i^3*(d*x+c)^6*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3-1/60*B*(-a*d+b
*c)^6*g^2*i^3*n*ln((b*x+a)/(d*x+c))/b^4/d^3-1/60*B*(-a*d+b*c)^6*g^2*i^3*n*
ln(d*x+c)/b^4/d^3
```

### 3.128.2 Mathematica [A] (verified)

Time = 0.22 (sec) , antiderivative size = 441, normalized size of antiderivative = 1.14

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{g^2i^3\left(-15B(bc-ad)^3n(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx)\right)}{}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]),x]
```

output
```
(g^2*i^3*(-15*B*(b*c - a*d)^3*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)
*(c + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) + 12*B*(b
*c - a*d)^2*n*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 +
4*b^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)^4*Log[a
 + b*x]) - B*(b*c - a*d)*n*(60*b*d*(b*c - a*d)^4*x + 30*b^2*(b*c - a*d)^3*
(c + d*x)^2 + 20*b^3*(b*c - a*d)^2*(c + d*x)^3 + 15*b^4*(b*c - a*d)*(c + d
*x)^4 + 12*b^5*(c + d*x)^5 + 60*(b*c - a*d)^5*Log[a + b*x]) + 90*b^4*(b*c
- a*d)^2*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 144*b^5*(b*c
 - a*d)*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 60*b^6*(c + d
*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(360*b^4*d^3)
```

### 3.128.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 337, normalized size of antiderivative = 0.87, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2961, 2782, 27, 1195, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$
$$\Big\downarrow \text{2961}$$
$$g^2i^3(bc-ad)^6\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$
$$\Big\downarrow \text{2782}$$

$$g^2 i^3 (bc-ad)^6 \left(-Bn\int \frac{(c+dx)\left(b^2-\frac{6d(a+bx)b}{c+dx}+\frac{15d^2(a+bx)^2}{(c+dx)^2}\right)}{60d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{2b\left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{5d^3\left(b-\frac{d(a}{c-}\right.}\right.$$

$$\downarrow 27$$

$$g^2 i^3 (bc-ad)^6 \left(-\frac{Bn\int \frac{(c+dx)\left(b^2-\frac{6d(a+bx)b}{c+dx}+\frac{15d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{60d^3}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{2b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right.$$

$$\downarrow 1195$$

$$g^2 i^3 (bc-ad)^6 \left(-\frac{Bn\int \left(\frac{d}{b^4\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{14d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{10bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}+\frac{c}{b^4(}\right.}{60d^3}\right.$$

$$\downarrow 2009$$

$$g^2 i^3 (bc-ad)^6 \left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{2b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^4}-\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
,x]
```

output
```
(b*c - a*d)^6*g^2*i^3*((b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*d^3
*(b - (d*(a + b*x))/(c + d*x))^6) - (2*b*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(5*d^3*(b - (d*(a + b*x))/(c + d*x))^5) + (A + B*Log[e*((a + b*x)/(
c + d*x))^n])/(4*d^3*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*((2*b)/(b - (
d*(a + b*x))/(c + d*x))^5 - 7/(2*(b - (d*(a + b*x))/(c + d*x))^4) + 1/(3*b
*(b - (d*(a + b*x))/(c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b*x))/(c + d*x))
^2) + 1/(b^3*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^4
 - Log[b - (d*(a + b*x))/(c + d*x)]/b^4))/(60*d^3))
```

## 3.128. $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

### 3.128.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.128.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1720 vs. $2(367) = 734$.

Time = 25.41 (sec) , antiderivative size = 1721, normalized size of antiderivative = 4.45

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1721 |

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_
RETURNVERBOSE)
```

output
```
1/360*(12*B*x^5*a*b^5*d^6*g^2*i^3*n^2+540*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*
a*b^5*c*d^5*g^2*i^3*n+360*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c*d^5*g^
2*i^3*n+720*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^2*d^4*g^2*i^3*n+540*B*
x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c^2*d^4*g^2*i^3*n+360*B*x^2*ln(e*((b
*x+a)/(d*x+c))^n)*a*b^5*c^3*d^3*g^2*i^3*n+360*B*x*ln(e*((b*x+a)/(d*x+c))^n
)*a^2*b^4*c^3*d^3*g^2*i^3*n+33*B*a^5*b*c*d^5*g^2*i^3*n^2-72*B*a^4*b^2*c^2*
d^4*g^2*i^3*n^2-150*B*a^3*b^3*c^3*d^3*g^2*i^3*n^2+168*B*a^2*b^4*c^4*d^2*g^
2*i^3*n^2+33*B*a*b^5*c^5*d*g^2*i^3*n^2-900*A*a^3*b^3*c^3*d^3*g^2*i^3*n-720
*A*a^2*b^4*c^4*d^2*g^2*i^3*n+90*B*x^2*a^2*b^4*c^2*d^4*g^2*i^3*n^2-102*B*x^
2*a*b^5*c^3*d^3*g^2*i^3*n^2+540*A*x^2*a^2*b^4*c^2*d^4*g^2*i^3*n+360*A*x^2*
a*b^5*c^3*d^3*g^2*i^3*n-36*B*x*a^4*b^2*c*d^5*g^2*i^3*n^2+90*B*x*a^3*b^3*c^
2*d^4*g^2*i^3*n^2-30*B*x*a^2*b^4*c^3*d^3*g^2*i^3*n^2-36*B*x*a*b^5*c^4*d^2*
g^2*i^3*n^2+360*A*x*a^2*b^4*c^3*d^3*g^2*i^3*n+90*B*ln(e*((b*x+a)/(d*x+c))^
n)*a^2*b^4*c^4*d^2*g^2*i^3*n+144*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^6
*g^2*i^3*n+216*B*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c*d^5*g^2*i^3*n+90*B*x^
4*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*d^6*g^2*i^3*n+270*B*x^4*ln(e*((b*x+a)/
(d*x+c))^n)*b^6*c^2*d^4*g^2*i^3*n+18*B*x^4*a*b^5*c*d^5*g^2*i^3*n^2+540*A*x
^4*a*b^5*c*d^5*g^2*i^3*n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^3*d^3*g
^2*i^3*n+78*B*x^3*a^2*b^4*c*d^5*g^2*i^3*n^2-42*B*x^3*a*b^5*c^2*d^4*g^2*i^3
*n^2+360*A*x^3*a^2*b^4*c*d^5*g^2*i^3*n+720*A*x^3*a*b^5*c^2*d^4*g^2*i^3*...
```

### 3.128.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1075 vs. $2(367) = 734$.

Time = 0.66 (sec) , antiderivative size = 1075, normalized size of antiderivative = 2.78

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{60\,Ab^6d^6g^2i^3x^6+6\,(20\,Ba^3b^3c^3d^3-15\,Ba^4b^2c^2d^4+6\,Ba^5bcd^5-Ba^6d^6)g^2i^3n\log{(bx+a)}-6\,(Bb^6c^6-6\cdots}{\cdots}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="fricas")
```

output
```
1/360*(60*A*b^6*d^6*g^2*i^3*x^6 + 6*(20*B*a^3*b^3*c^3*d^3 - 15*B*a^4*b^2*c
^2*d^4 + 6*B*a^5*b*c*d^5 - B*a^6*d^6)*g^2*i^3*n*log(b*x + a) - 6*(B*b^6*c^
6 - 6*B*a*b^5*c^5*d + 15*B*a^2*b^4*c^4*d^2)*g^2*i^3*n*log(d*x + c) - 12*((
B*b^6*c*d^5 - B*a*b^5*d^6)*g^2*i^3*n - 6*(3*A*b^6*c*d^5 + 2*A*a*b^5*d^6)*g
^2*i^3)*x^5 - 3*((13*B*b^6*c^2*d^4 - 6*B*a*b^5*c*d^5 - 7*B*a^2*b^4*d^6)*g^
2*i^3*n - 30*(3*A*b^6*c^2*d^4 + 6*A*a*b^5*c*d^5 + A*a^2*b^4*d^6)*g^2*i^3)*
x^4 - 2*((19*B*b^6*c^3*d^3 + 21*B*a*b^5*c^2*d^4 - 39*B*a^2*b^4*c*d^5 - B*a
^3*b^3*d^6)*g^2*i^3*n - 60*(A*b^6*c^3*d^3 + 6*A*a*b^5*c^2*d^4 + 3*A*a^2*b^
4*c*d^5)*g^2*i^3)*x^3 - 3*((B*b^6*c^4*d^2 + 34*B*a*b^5*c^3*d^3 - 30*B*a^2*
b^4*c^2*d^4 - 6*B*a^3*b^3*c*d^5 + B*a^4*b^2*d^6)*g^2*i^3*n - 60*(2*A*a*b^5
*c^3*d^3 + 3*A*a^2*b^4*c^2*d^4)*g^2*i^3)*x^2 + 6*(60*A*a^2*b^4*c^3*d^3*g^2
*i^3 + (B*b^6*c^5*d - 6*B*a*b^5*c^4*d^2 - 5*B*a^2*b^4*c^3*d^3 + 15*B*a^3*b
^3*c^2*d^4 - 6*B*a^4*b^2*c*d^5 + B*a^5*b*d^6)*g^2*i^3*n)*x + 6*(10*B*b^6*d
^6*g^2*i^3*x^6 + 60*B*a^2*b^4*c^3*d^3*g^2*i^3*x + 12*(3*B*b^6*c*d^5 + 2*B*
a*b^5*d^6)*g^2*i^3*x^5 + 15*(3*B*b^6*c^2*d^4 + 6*B*a*b^5*c*d^5 + B*a^2*b^4
*d^6)*g^2*i^3*x^4 + 20*(B*b^6*c^3*d^3 + 6*B*a*b^5*c^2*d^4 + 3*B*a^2*b^4*c*
d^5)*g^2*i^3*x^3 + 30*(2*B*a*b^5*c^3*d^3 + 3*B*a^2*b^4*c^2*d^4)*g^2*i^3*x^
2)*log(e) + 6*(10*B*b^6*d^6*g^2*i^3*n*x^6 + 60*B*a^2*b^4*c^3*d^3*g^2*i^3*n
*x + 12*(3*B*b^6*c*d^5 + 2*B*a*b^5*d^6)*g^2*i^3*n*x^5 + 15*(3*B*b^6*c^2*d^
4 + 6*B*a*b^5*c*d^5 + B*a^2*b^4*d^6)*g^2*i^3*n*x^4 + 20*(B*b^6*c^3*d^3 ...
```

### 3.128.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x
)
```

output
```
Timed out
```

### 3.128.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1978 vs. $2(367) = 734$.

Time = 0.25 (sec) , antiderivative size = 1978, normalized size of antiderivative = 5.11

$$\int (ag + bgx)^2(ci + dix)^3 \left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="maxima")
```

output
```
1/6*B*b^2*d^3*g^2*i^3*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/6*A*b
^2*d^3*g^2*i^3*x^6 + 3/5*B*b^2*c*d^2*g^2*i^3*x^5*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + 2/5*B*a*b*d^3*g^2*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x +
c))^n) + 3/5*A*b^2*c*d^2*g^2*i^3*x^5 + 2/5*A*a*b*d^3*g^2*i^3*x^5 + 3/4*B*b
^2*c^2*d*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*B*a*b*c*
d^2*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*B*a^2*d^3*g^2
*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/4*A*b^2*c^2*d*g^2*i^3*
x^4 + 3/2*A*a*b*c*d^2*g^2*i^3*x^4 + 1/4*A*a^2*d^3*g^2*i^3*x^4 + 1/3*B*b^2*
c^3*g^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*B*a*b*c^2*d*g^2
*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + B*a^2*c*d^2*g^2*i^3*x^3*
log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A*b^2*c^3*g^2*i^3*x^3 + 2*A*a
*b*c^2*d*g^2*i^3*x^3 + A*a^2*c*d^2*g^2*i^3*x^3 + B*a*b*c^3*g^2*i^3*x^2*log
(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*B*a^2*c^2*d*g^2*i^3*x^2*log(e*(b
*x/(d*x + c) + a/(d*x + c))^n) + A*a*b*c^3*g^2*i^3*x^2 + 3/2*A*a^2*c^2*d*g
^2*i^3*x^2 - 1/360*B*b^2*d^3*g^2*i^3*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*l
og(d*x + c)/d^6 + (12*(b^5*c*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*
b^3*d^5)*x^4 + 20*(b^5*c^3*d^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*
d^5)*x^2 + 60*(b^5*c^5 - a^5*d^5)*x)/(b^5*d^5)) + 1/20*B*b^2*c*d^2*g^2*i^3
*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*
b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b...
```

### 3.128.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4300 vs. $2(367) = 734$.

Time = 1.78 (sec) , antiderivative size = 4300, normalized size of antiderivative = 11.11

$$\int (ag + bgx)^2(ci + dix)^3 \left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right) dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x,
algorithm="giac")
```

output
```
1/360*(6*(B*b^9*c^7*g^2*i^3*n - 7*B*a*b^8*c^6*d*g^2*i^3*n - 6*(b*x + a)*B*
b^8*c^7*d*g^2*i^3*n/(d*x + c) + 21*B*a^2*b^7*c^5*d^2*g^2*i^3*n + 42*(b*x +
 a)*B*a*b^7*c^6*d^2*g^2*i^3*n/(d*x + c) + 15*(b*x + a)^2*B*b^7*c^7*d^2*g^2
*i^3*n/(d*x + c)^2 - 35*B*a^3*b^6*c^4*d^3*g^2*i^3*n - 126*(b*x + a)*B*a^2*
b^6*c^5*d^3*g^2*i^3*n/(d*x + c) - 105*(b*x + a)^2*B*a*b^6*c^6*d^3*g^2*i^3*
n/(d*x + c)^2 + 35*B*a^4*b^5*c^3*d^4*g^2*i^3*n + 210*(b*x + a)*B*a^3*b^5*c
^4*d^4*g^2*i^3*n/(d*x + c) + 315*(b*x + a)^2*B*a^2*b^5*c^5*d^4*g^2*i^3*n/(
d*x + c)^2 - 21*B*a^5*b^4*c^2*d^5*g^2*i^3*n - 210*(b*x + a)*B*a^4*b^4*c^3*
d^5*g^2*i^3*n/(d*x + c) - 525*(b*x + a)^2*B*a^3*b^4*c^4*d^5*g^2*i^3*n/(d*x
 + c)^2 + 7*B*a^6*b^3*c*d^6*g^2*i^3*n + 126*(b*x + a)*B*a^5*b^3*c^2*d^6*g^
2*i^3*n/(d*x + c) + 525*(b*x + a)^2*B*a^4*b^3*c^3*d^6*g^2*i^3*n/(d*x + c)^
2 - B*a^7*b^2*d^7*g^2*i^3*n - 42*(b*x + a)*B*a^6*b^2*c*d^7*g^2*i^3*n/(d*x
+ c) - 315*(b*x + a)^2*B*a^5*b^2*c^2*d^7*g^2*i^3*n/(d*x + c)^2 + 6*(b*x +
a)*B*a^7*b*d^8*g^2*i^3*n/(d*x + c) + 105*(b*x + a)^2*B*a^6*b*c*d^8*g^2*i^3
*n/(d*x + c)^2 - 15*(b*x + a)^2*B*a^7*d^9*g^2*i^3*n/(d*x + c)^2)*log((b*x
+ a)/(d*x + c))/(b^6*d^3 - 6*(b*x + a)*b^5*d^4/(d*x + c) + 15*(b*x + a)^2*
b^4*d^5/(d*x + c)^2 - 20*(b*x + a)^3*b^3*d^6/(d*x + c)^3 + 15*(b*x + a)^4*
b^2*d^7/(d*x + c)^4 - 6*(b*x + a)^5*b*d^8/(d*x + c)^5 + (b*x + a)^6*d^9/(d
*x + c)^6) - (2*B*b^12*c^7*g^2*i^3*n - 14*B*a*b^11*c^6*d*g^2*i^3*n - 18*(b
*x + a)*B*b^11*c^7*d*g^2*i^3*n/(d*x + c) + 42*B*a^2*b^10*c^5*d^2*g^2*i^...
```

### 3.128.9 Mupad [B] (verification not implemented)

Time = 2.83 (sec) , antiderivative size = 2547, normalized size of antiderivative = 6.58

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))
,x)
```

output
```
x^2*((a*c*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n - B*b*c*n))/6 -
 (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d + 60*b*c))/(60*b*d) - (d*
g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2*n - 3*B*b^2*c^2*n + 60*
A*a*b*c*d + B*a*b*c*d*n))/5 + A*a*b*c*d^2*g^2*i^3))/(2*b*d) - ((60*a*d + 6
0*b*c)*((g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3*n - 3*B*b^3*c^3*n
 + 72*A*a*b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 5*B*a^2*b*c*d
^2*n))/(4*b) + ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c +
 B*a*d*n - B*b*c*n))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a*d +
 60*b*c))/(60*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2*d^2
*n - 3*B*b^2*c^2*n + 60*A*a*b*c*d + B*a*b*c*d*n))/5 + A*a*b*c*d^2*g^2*i^3)
)/(60*b*d) - (a*c*((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b*c + B*a*d*n - B*b*c*n
))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60))/(b*d)))/(120*b*d) + (c*g^2
*i^3*(12*A*a^3*d^3 + 3*A*b^3*c^3 + 3*B*a^3*d^3*n - B*b^3*c^3*n + 36*A*a*b^
2*c^2*d + 54*A*a^2*b*c*d^2 - 5*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n))/(6*b*
d)) + x^3*((g^2*i^3*(4*A*a^3*d^3 + 16*A*b^3*c^3 + B*a^3*d^3*n - 3*B*b^3*c^
3*n + 72*A*a*b^2*c^2*d + 48*A*a^2*b*c*d^2 - 3*B*a*b^2*c^2*d*n + 5*B*a^2*b*
c*d^2*n))/(12*b) + ((60*a*d + 60*b*c)*((((b*d^2*g^2*i^3*(18*A*a*d + 24*A*b
*c + B*a*d*n - B*b*c*n))/6 - (A*b*d^2*g^2*i^3*(60*a*d + 60*b*c))/60)*(60*a
*d + 60*b*c))/(60*b*d) - (d*g^2*i^3*(15*A*a^2*d^2 + 30*A*b^2*c^2 + 2*B*a^2
*d^2*n - 3*B*b^2*c^2*n + 60*A*a*b*c*d + B*a*b*c*d*n))/5 + A*a*b*c*d^2*g...
```

3.128. $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

# 3.129 $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx$



## 3.129.1 Optimal result

Integrand size = 41, antiderivative size = 283

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx \\
&= \frac{B(bc-ad)^4gi^3nx}{20b^3d}+\frac{B(bc-ad)^3gi^3n(c+dx)^2}{40b^2d^2} \\
&\quad+\frac{B(bc-ad)^2gi^3n(c+dx)^3}{60bd^2}-\frac{B(bc-ad)gi^3n(c+dx)^4}{20d^2} \\
&\quad-\frac{(bc-ad)gi^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2}+\frac{bgi^3(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5d^2} \\
&\quad+\frac{B(bc-ad)^5gi^3n\log\left(\frac{a+bx}{c+dx}\right)}{20b^4d^2}+\frac{B(bc-ad)^5gi^3n\log(c+dx)}{20b^4d^2}
\end{aligned}$$

output
```
1/20*B*(-a*d+b*c)^4*g*i^3*n*x/b^3/d+1/40*B*(-a*d+b*c)^3*g*i^3*n*(d*x+c)^2/
b^2/d^2+1/60*B*(-a*d+b*c)^2*g*i^3*n*(d*x+c)^3/b/d^2-1/20*B*(-a*d+b*c)*g*i^
3*n*(d*x+c)^4/d^2-1/4*(-a*d+b*c)*g*i^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/d^2+1/5*b*g*i^3*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^2+1/20*
B*(-a*d+b*c)^5*g*i^3*n*ln((b*x+a)/(d*x+c))/b^4/d^2+1/20*B*(-a*d+b*c)^5*g*i
^3*n*ln(d*x+c)/b^4/d^2
```

### 3.129.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 269, normalized size of antiderivative = 0.95

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{gi^3\left(\frac{5B(bc-ad)^2n(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx))}{b^4}-\frac{2B(bc-ad)n(12bd(bc-ad)^3x+6b^2(bc-ad)^2(c}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]),x]
```

output
```
(g*i^3*((5*B*(b*c - a*d)^2*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c
 + d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 - (2*B*
(b*c - a*d)*n*(12*b*d*(b*c - a*d)^3*x + 6*b^2*(b*c - a*d)^2*(c + d*x)^2 +
4*b^3*(b*c - a*d)*(c + d*x)^3 + 3*b^4*(c + d*x)^4 + 12*(b*c - a*d)^4*Log[a
 + b*x]))/b^4 - 30*(b*c - a*d)*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 24*b*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(120*d
^2)
```

### 3.129.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.92, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.122, Rules used = {2961, 2782, 27, 86, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2961}$$

$$gi^3(bc-ad)^5\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2782}$$

$$gi^3(bc-ad)^5\left(-Bn\int -\frac{(c+dx)\left(b-\frac{5d(a+bx)}{c+dx}\right)}{20d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)$$

$\downarrow$ 27

$$gi^3(bc-ad)^5\left(\frac{Bn\int \frac{(c+dx)\left(b-\frac{5d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{20d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)$$

$\downarrow$ 86

$$gi^3(bc-ad)^5\left(\frac{Bn\int \left(\frac{d}{b^4\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{4d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{c+dx}{b^4(a+bx)}\right)d\frac{a+bx}{c+dx}}{20d^2}-\right.$$

$\downarrow$ 2009

$$gi^3(bc-ad)^5\left(-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^4}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^4}+\frac{1}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\right.}{20}\right.$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x
]
```

output
```
(b*c - a*d)^5*g*i^3*((b*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d^2*(b
- (d*(a + b*x))/(c + d*x))^5) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(4*
d^2*(b - (d*(a + b*x))/(c + d*x))^4) + (B*n*(-(b - (d*(a + b*x))/(c + d*x)
)^(-4) + 1/(3*b*(b - (d*(a + b*x))/(c + d*x))^3) + 1/(2*b^2*(b - (d*(a + b
*x))/(c + d*x))^2) + 1/(b^3*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)
/(c + d*x)]/b^4 - Log[b - (d*(a + b*x))/(c + d*x)]/b^4))/(20*d^2))
```

3.129. $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

### 3.129.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.129.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. 2(267) = 534.

Time = 11.62 (sec) , antiderivative size = 1119, normalized size of antiderivative = 3.95

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1119 |

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RE
TURNVERBOSE)
```

output
```
1/120*(6*B*ln(b*x+a)*b^5*c^5*g*i^3*n^2-6*B*ln(b*x+a)*a^5*d^5*g*i^3*n^2-6*B
*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^5*g*i^3*n+24*A*x^5*b^5*d^5*g*i^3*n+24*B*x
^5*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^5*g*i^3*n+120*B*x^3*ln(e*((b*x+a)/(d*x+
c))^n)*a*b^4*c*d^4*g*i^3*n+180*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^2*d
^3*g*i^3*n+120*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^3*d^2*g*i^3*n+30*B*ln
(e*((b*x+a)/(d*x+c))^n)*a*b^4*c^4*d*g*i^3*n+30*B*x^4*ln(e*((b*x+a)/(d*x+c)
)^n)*a*b^4*d^5*g*i^3*n+90*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^4*g*i^3*
n+120*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^2*d^3*g*i^3*n-27*B*x^2*b^5*c^3
*d^2*g*i^3*n^2+60*A*x^2*b^5*c^3*d^2*g*i^3*n+6*B*x*a^4*b*d^5*g*i^3*n^2-6*B*
x*b^5*c^4*d*g*i^3*n^2+6*B*b^5*c^5*g*i^3*n^2-6*B*a^5*d^5*g*i^3*n^2+20*B*x^3
*a*b^4*c*d^4*g*i^3*n^2+120*A*x^3*a*b^4*c*d^4*g*i^3*n+60*B*x^2*ln(e*((b*x+a
)/(d*x+c))^n)*b^5*c^3*d^2*g*i^3*n+15*B*x^2*a^2*b^3*c*d^4*g*i^3*n^2+15*B*x^
2*a*b^4*c^2*d^3*g*i^3*n^2+180*A*x^2*a*b^4*c^2*d^3*g*i^3*n-30*B*x*a^3*b^2*c
*d^4*g*i^3*n^2+60*B*x*a^2*b^3*c^2*d^3*g*i^3*n^2-30*B*x*a*b^4*c^3*d^2*g*i^3
*n^2+120*A*x*a*b^4*c^3*d^2*g*i^3*n+30*B*ln(b*x+a)*a^4*b*c*d^4*g*i^3*n^2-60
*B*ln(b*x+a)*a^3*b^2*c^2*d^3*g*i^3*n^2+60*B*ln(b*x+a)*a^2*b^3*c^3*d^2*g*i^
3*n^2-30*B*ln(b*x+a)*a*b^4*c^4*d*g*i^3*n^2+27*B*a^4*b*c*d^4*g*i^3*n^2-45*B
*a^3*b^2*c^2*d^3*g*i^3*n^2-45*B*a^2*b^3*c^3*d^2*g*i^3*n^2+63*B*a*b^4*c^4*d
*g*i^3*n^2-300*A*a^2*b^3*c^3*d^2*g*i^3*n-180*A*a*b^4*c^4*d*g*i^3*n+6*B*x^4
*a*b^4*d^5*g*i^3*n^2-6*B*x^4*b^5*c*d^4*g*i^3*n^2+30*A*x^4*a*b^4*d^5*g*i...
```

### 3.129.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 721 vs. $2(267) = 534$.

Time = 0.45 (sec) , antiderivative size = 721, normalized size of antiderivative = 2.55

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\frac{24\,Ab^5d^5gi^3x^5+6\,(10\,Ba^2b^3c^3d^2-10\,Ba^3b^2c^2d^3+5\,Ba^4bcd^4-Ba^5d^5)gi^3n\log\left(bx+a\right)+6\,(Bb^5c^5-5\,B}{}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="fricas")
```

output
```
1/120*(24*A*b^5*d^5*g*i^3*x^5 + 6*(10*B*a^2*b^3*c^3*d^2 - 10*B*a^3*b^2*c^2
*d^3 + 5*B*a^4*b*c*d^4 - B*a^5*d^5)*g*i^3*n*log(b*x + a) + 6*(B*b^5*c^5 -
5*B*a*b^4*c^4*d)*g*i^3*n*log(d*x + c) - 6*((B*b^5*c*d^4 - B*a*b^4*d^5)*g*i
^3*n - 5*(3*A*b^5*c*d^4 + A*a*b^4*d^5)*g*i^3)*x^4 - 2*((11*B*b^5*c^2*d^3 -
 10*B*a*b^4*c*d^4 - B*a^2*b^3*d^5)*g*i^3*n - 60*(A*b^5*c^2*d^3 + A*a*b^4*c
*d^4)*g*i^3)*x^3 - 3*((9*B*b^5*c^3*d^2 - 5*B*a*b^4*c^2*d^3 - 5*B*a^2*b^3*c
*d^4 + B*a^3*b^2*d^5)*g*i^3*n - 20*(A*b^5*c^3*d^2 + 3*A*a*b^4*c^2*d^3)*g*i
^3)*x^2 + 6*(20*A*a*b^4*c^3*d^2*g*i^3 - (B*b^5*c^4*d + 5*B*a*b^4*c^3*d^2 -
 10*B*a^2*b^3*c^2*d^3 + 5*B*a^3*b^2*c*d^4 - B*a^4*b*d^5)*g*i^3*n)*x + 6*(4
*B*b^5*d^5*g*i^3*x^5 + 20*B*a*b^4*c^3*d^2*g*i^3*x + 5*(3*B*b^5*c*d^4 + B*a
*b^4*d^5)*g*i^3*x^4 + 20*(B*b^5*c^2*d^3 + B*a*b^4*c*d^4)*g*i^3*x^3 + 10*(B
*b^5*c^3*d^2 + 3*B*a*b^4*c^2*d^3)*g*i^3*x^2)*log(e) + 6*(4*B*b^5*d^5*g*i^3
*n*x^5 + 20*B*a*b^4*c^3*d^2*g*i^3*n*x + 5*(3*B*b^5*c*d^4 + B*a*b^4*d^5)*g*
i^3*n*x^4 + 20*(B*b^5*c^2*d^3 + B*a*b^4*c*d^4)*g*i^3*n*x^3 + 10*(B*b^5*c^3
*d^2 + 3*B*a*b^4*c^2*d^3)*g*i^3*n*x^2)*log((b*x + a)/(d*x + c)))/(b^4*d^2)
```

### 3.129.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Timed out
```

### 3.129.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1118 vs. 2(267) = 534.

Time = 0.23 (sec) , antiderivative size = 1118, normalized size of antiderivative = 3.95

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="maxima")
```

output
```
1/5*B*b*d^3*g*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A*b*d^3
*g*i^3*x^5 + 3/4*B*b*c*d^2*g*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
) + 1/4*B*a*d^3*g*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/4*A*b
*c*d^2*g*i^3*x^4 + 1/4*A*a*d^3*g*i^3*x^4 + B*b*c^2*d*g*i^3*x^3*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n) + B*a*c*d^2*g*i^3*x^3*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) + A*b*c^2*d*g*i^3*x^3 + A*a*c*d^2*g*i^3*x^3 + 1/2*B*b*c^3*g
*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*B*a*c^2*d*g*i^3*x^2*
log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A*b*c^3*g*i^3*x^2 + 3/2*A*a*c
^2*d*g*i^3*x^2 + 1/60*B*b*d^3*g*i^3*n*(12*a^5*log(b*x + a)/b^5 - 12*c^5*lo
g(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2*d^2 - a^2*b^2
*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c^4 - a^4*d^4)*x)/(b^4
*d^4)) - 1/8*B*b*c*d^2*g*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c
)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6
*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/24*B*a*d^3*g*i^3*n*(6*a^4*log(b*x +
 a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3
*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*b*c^
2*d*g*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d -
 a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) + 1/2*B*a*c*d^2*g*i^3*
n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*
x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 1/2*B*b*c^3*g*i^3*n*(a^2*lo...
```

### 3.129.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2584 vs. $2(267) = 534$.

Time = 1.18 (sec) , antiderivative size = 2584, normalized size of antiderivative = 9.13

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, al
gorithm="giac")
```

output
```
-1/120*(6*(B*b^7*c^6*g*i^3*n - 6*B*a*b^6*c^5*d*g*i^3*n - 5*(b*x + a)*B*b^6
*c^6*d*g*i^3*n/(d*x + c) + 15*B*a^2*b^5*c^4*d^2*g*i^3*n + 30*(b*x + a)*B*a
*b^5*c^5*d^2*g*i^3*n/(d*x + c) - 20*B*a^3*b^4*c^3*d^3*g*i^3*n - 75*(b*x +
a)*B*a^2*b^4*c^4*d^3*g*i^3*n/(d*x + c) + 15*B*a^4*b^3*c^2*d^4*g*i^3*n + 10
0*(b*x + a)*B*a^3*b^3*c^3*d^4*g*i^3*n/(d*x + c) - 6*B*a^5*b^2*c*d^5*g*i^3*
n - 75*(b*x + a)*B*a^4*b^2*c^2*d^5*g*i^3*n/(d*x + c) + B*a^6*b*d^6*g*i^3*n
 + 30*(b*x + a)*B*a^5*b*c*d^6*g*i^3*n/(d*x + c) - 5*(b*x + a)*B*a^6*d^7*g*
i^3*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^5*d^2 - 5*(b*x + a)*b^4*d^3/(
d*x + c) + 10*(b*x + a)^2*b^3*d^4/(d*x + c)^2 - 10*(b*x + a)^3*b^2*d^5/(d*
x + c)^3 + 5*(b*x + a)^4*b*d^6/(d*x + c)^4 - (b*x + a)^5*d^7/(d*x + c)^5)
- (5*B*b^10*c^6*g*i^3*n - 30*B*a*b^9*c^5*d*g*i^3*n - 31*(b*x + a)*B*b^9*c^
6*d*g*i^3*n/(d*x + c) + 75*B*a^2*b^8*c^4*d^2*g*i^3*n + 186*(b*x + a)*B*a*b
^8*c^5*d^2*g*i^3*n/(d*x + c) + 47*(b*x + a)^2*B*b^8*c^6*d^2*g*i^3*n/(d*x +
 c)^2 - 100*B*a^3*b^7*c^3*d^3*g*i^3*n - 465*(b*x + a)*B*a^2*b^7*c^4*d^3*g*
i^3*n/(d*x + c) - 282*(b*x + a)^2*B*a*b^7*c^5*d^3*g*i^3*n/(d*x + c)^2 - 27
*(b*x + a)^3*B*b^7*c^6*d^3*g*i^3*n/(d*x + c)^3 + 75*B*a^4*b^6*c^2*d^4*g*i^
3*n + 620*(b*x + a)*B*a^3*b^6*c^3*d^4*g*i^3*n/(d*x + c) + 705*(b*x + a)^2*
B*a^2*b^6*c^4*d^4*g*i^3*n/(d*x + c)^2 + 162*(b*x + a)^3*B*a*b^6*c^5*d^4*g*
i^3*n/(d*x + c)^3 + 6*(b*x + a)^4*B*b^6*c^6*d^4*g*i^3*n/(d*x + c)^4 - 30*B
*a^5*b^5*c*d^5*g*i^3*n - 465*(b*x + a)*B*a^4*b^5*c^2*d^5*g*i^3*n/(d*x +...
```

### 3.129.9 Mupad [B] (verification not implemented)

Time = 2.33 (sec) , antiderivative size = 1234, normalized size of antiderivative = 4.36

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx=\text{Too large to display}$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x
)
```

output
```
x*((a*c*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n - B
*b*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d) - (d*g*i^3*(4*A
*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 32*A*a*b*c*d + 2*B
*a*b*c*d*n))/(4*b) + A*a*c*d^2*g*i^3))/(b*d) - ((20*a*d + 20*b*c)*(((20*a*
d + 20*b*c)*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n
 - B*b*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d) - (d*g*i^3*
(4*A*a^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 32*A*a*b*c*d +
 2*B*a*b*c*d*n))/(4*b) + A*a*c*d^2*g*i^3))/(20*b*d) - (a*c*((d^2*g*i^3*(10
*A*a*d + 20*A*b*c + B*a*d*n - B*b*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c)
)/20))/(b*d) + (c*g*i^3*(4*A*a^2*d^2 + 4*A*b^2*c^2 + B*a^2*d^2*n - B*b^2*c
^2*n + 12*A*a*b*c*d))/b))/(20*b*d) + (c^2*g*i^3*(12*A*a^2*d^2 + 2*A*b^2*c^
2 + 3*B*a^2*d^2*n - B*b^2*c^2*n + 16*A*a*b*c*d - 2*B*a*b*c*d*n))/(2*b*d))
- x^3*(((20*a*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n - B*b
*c*n))/5 - (A*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(60*b*d) - (d*g*i^3*(4*A*a
^2*d^2 + 24*A*b^2*c^2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 32*A*a*b*c*d + 2*B*a
*b*c*d*n))/(12*b) + (A*a*c*d^2*g*i^3)/3) + x^2*(((20*a*d + 20*b*c)*(((20*a
*d + 20*b*c)*((d^2*g*i^3*(10*A*a*d + 20*A*b*c + B*a*d*n - B*b*c*n))/5 - (A
*d^2*g*i^3*(20*a*d + 20*b*c))/20))/(20*b*d) - (d*g*i^3*(4*A*a^2*d^2 + 24*A
*b^2*c^2 + B*a^2*d^2*n - 3*B*b^2*c^2*n + 32*A*a*b*c*d + 2*B*a*b*c*d*n))/(4
*b) + A*a*c*d^2*g*i^3))/(40*b*d) - (a*c*((d^2*g*i^3*(10*A*a*d + 20*A*b*...
```

3.129. $\int (ag + bgx)(ci + dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

# 3.130 $\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$



## 3.130.1 Optimal result

Integrand size = 33, antiderivative size = 156

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = -\frac{B(bc-ad)^3i^3nx}{4b^3}-\frac{B(bc-ad)^2i^3n(c+dx)^2}{8b^2d}-\frac{B(bc-ad)i^3n(c+dx)^3}{12bd} - \frac{B(bc-ad)^4i^3n\log(a+bx)}{4b^4d}+\frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d}$$

output
```
-1/4*B*(-a*d+b*c)^3*i^3*n*x/b^3-1/8*B*(-a*d+b*c)^2*i^3*n*(d*x+c)^2/b^2/d-1
/12*B*(-a*d+b*c)*i^3*n*(d*x+c)^3/b/d-1/4*B*(-a*d+b*c)^4*i^3*n*ln(b*x+a)/b^
4/d+1/4*i^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
```

## 3.130.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 124, normalized size of antiderivative = 0.79

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{i^3\left(-\frac{B(bc-ad)n(6bd(bc-ad)^2x+3b^2(bc-ad)(c+dx)^2+2b^3(c+dx)^3+6(bc-ad)^3\log(a+bx))}{6b^4}+(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right)}{4d}$$

input
```
Integrate[(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
(i^3*(-1/6*(B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c
+ d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]))/b^4 + (c + d
*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])))/(4*d)
```

### 3.130.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 133, normalized size of antiderivative = 0.85, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {2947, 27, 49, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^3 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right) \, dx$$

$$\Big\downarrow \text{ 2947}$$

$$\frac{i^3 (c+dx)^4 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bn(bc-ad) \int \frac{i^4 (c+dx)^3}{a+bx}dx}{4di}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{i^3 (c+dx)^4 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bi^3 n(bc-ad) \int \frac{(c+dx)^3}{a+bx}dx}{4d}$$

$$\Big\downarrow \text{ 49}$$

$$\frac{i^3 (c+dx)^4 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bi^3 n(bc-ad) \int \left(\frac{(bc-ad)^3}{b^3 (a+bx)}+\frac{d(bc-ad)^2}{b^3}+\frac{d(c+dx)(bc-ad)}{b^2}+\frac{d(c+dx)^2}{b}\right)dx}{4d}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i^3 (c+dx)^4 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d}-\frac{Bi^3 n(bc-ad) \left(\frac{(bc-ad)^3 \log(a+bx)}{b^4}+\frac{dx(bc-ad)^2}{b^3}+\frac{(c+dx)^2 (bc-ad)}{2b^2}+\frac{(c+dx)^3}{3b}\right)}{4d}$$

input
```
Int[(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-1/4*(B*(b*c - a*d)*i^3*n*((d*(b*c - a*d)^2*x)/b^3 + ((b*c - a*d)*(c + d*x
)^2)/(2*b^2) + (c + d*x)^3/(3*b) + ((b*c - a*d)^3*Log[a + b*x])/b^4))/d +
(i^3*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d)
```

### 3.130.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

## 3.130.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 651 vs. $2(146) = 292$.

Time = 4.81 (sec) , antiderivative size = 652, normalized size of antiderivative = 4.18

| method | result |
|---|---|
| parallelrisch | $\frac{24B\,x^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c\,d^3i^3n+6A\,x^4b^4d^4i^3n+6B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4c^4i^3n-6B\ln(bx+a)a^4d^4i^3n^2-6B\ln(bx+a)b^4c^4i^3n^2+}{}$ |

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,method=_RETURNVERBOSE)
```

output
```
1/24*(24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^3*i^3*n+6*A*x^4*b^4*d^4*i
^3*n+6*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^4*i^3*n-6*B*ln(b*x+a)*a^4*d^4*i^3
*n^2-6*B*ln(b*x+a)*b^4*c^4*i^3*n^2+36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^4*
c^2*d^2*i^3*n+12*B*x^2*a*b^3*c*d^3*i^3*n^2+24*B*x*ln(e*((b*x+a)/(d*x+c))^n
)*b^4*c^3*d*i^3*n-24*B*x*a^2*b^2*c*d^3*i^3*n^2+36*B*x*a*b^3*c^2*d^2*i^3*n^
2+24*B*ln(b*x+a)*a^3*b*c*d^3*i^3*n^2-36*B*ln(b*x+a)*a^2*b^2*c^2*d^2*i^3*n^
2+24*B*ln(b*x+a)*a*b^3*c^3*d*i^3*n^2+21*B*a^3*b*c*d^3*i^3*n^2-24*B*a^2*b^2
*c^2*d^2*i^3*n^2-9*B*a*b^3*c^3*d*i^3*n^2-60*A*a*b^3*c^3*d*i^3*n+6*B*x^4*ln
(e*((b*x+a)/(d*x+c))^n)*b^4*d^4*i^3*n+2*B*x^3*a*b^3*d^4*i^3*n^2-2*B*x^3*b^
4*c*d^3*i^3*n^2+24*A*x^3*b^4*c*d^3*i^3*n-3*B*x^2*a^2*b^2*d^4*i^3*n^2-9*B*x
^2*b^4*c^2*d^2*i^3*n^2+36*A*x^2*b^4*c^2*d^2*i^3*n+6*B*x*a^3*b*d^4*i^3*n^2-
18*B*x*b^4*c^3*d*i^3*n^2+24*A*x*b^4*c^3*d*i^3*n-6*B*a^4*d^4*i^3*n^2+18*B*b
^4*c^4*i^3*n^2-24*A*b^4*c^4*i^3*n)/b^4/d/n
```

### 3.130.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 429 vs. $2(146) = 292$.

Time = 0.38 (sec) , antiderivative size = 429, normalized size of antiderivative = 2.75

$$\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \frac{6\,Ab^4d^4i^3x^4 - 6\,Bb^4c^4i^3n\log\left(dx+c\right) + 6\left(4\,Bab^3c^3d - 6\,Ba^2b^2c^2d^2 + 4\,Ba^3bcd^3 - Ba^4d^4\right)i^3n\log\left(bx+a\right)}{}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="fri
cas")
```

output
```
1/24*(6*A*b^4*d^4*i^3*x^4 - 6*B*b^4*c^4*i^3*n*log(d*x + c) + 6*(4*B*a*b^3*
c^3*d - 6*B*a^2*b^2*c^2*d^2 + 4*B*a^3*b*c*d^3 - B*a^4*d^4)*i^3*n*log(b*x +
 a) + 2*(12*A*b^4*c*d^3*i^3 - (B*b^4*c*d^3 - B*a*b^3*d^4)*i^3*n)*x^3 + 3*(
12*A*b^4*c^2*d^2*i^3 - (3*B*b^4*c^2*d^2 - 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)
*i^3*n)*x^2 + 6*(4*A*b^4*c^3*d*i^3 - (3*B*b^4*c^3*d - 6*B*a*b^3*c^2*d^2 +
4*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*i^3*n)*x + 6*(B*b^4*d^4*i^3*x^4 + 4*B*b^4
*c*d^3*i^3*x^3 + 6*B*b^4*c^2*d^2*i^3*x^2 + 4*B*b^4*c^3*d*i^3*x)*log(e) + 6
*(B*b^4*d^4*i^3*n*x^4 + 4*B*b^4*c*d^3*i^3*n*x^3 + 6*B*b^4*c^2*d^2*i^3*n*x^
2 + 4*B*b^4*c^3*d*i^3*n*x)*log((b*x + a)/(d*x + c)))/(b^4*d)
```

### 3.130.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 945 vs. $2(136) = 272$.

Time = 78.68 (sec) , antiderivative size = 945, normalized size of antiderivative = 6.06

$$\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\begin{cases} c^3i^3x\left(A+B\log\left(e\left(\frac{a}{c}\right)^n\right)\right) \\ Ac^3i^3x+\frac{3Ac^2di^3x^2}{2}+Acd^2i^3x^3+\frac{Ad^3i^3x^4}{4}+\frac{Bc^4i^3\log\left(e\left(\frac{a}{c+dx}\right)^n\right)}{4d}+\frac{Bc^3i^3nx}{4}+Bc^3i^3x\log\left(e\left(\frac{a}{c+dx}\right)^n\right)+\frac{3Bc^2di^3}{8} \\ c^3i^3\left(Ax+\frac{Ba\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}-Bnx+Bx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)\right) \\ Ac^3i^3x+\frac{3Ac^2di^3x^2}{2}+Acd^2i^3x^3+\frac{Ad^3i^3x^4}{4}-\frac{Ba^4d^3i^3n\log\left(\frac{c}{d}+x\right)}{4b^4}-\frac{Ba^4d^3i^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{4b^4}+\frac{Ba^3cd^2i^3n\log\left(\frac{c}{d}+\right.}{b^3} \end{cases}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n)),x)
```

output
```
Piecewise(((c**3*i**3*x*(A + B*log(e*(a/c)**n)), Eq(b, 0) & Eq(d, 0)), (A*c
**3*i**3*x + 3*A*c**2*d*i**3*x**2/2 + A*c*d**2*i**3*x**3 + A*d**3*i**3*x**
4/4 + B*c**4*i**3*log(e*(a/(c + d*x))**n)/(4*d) + B*c**3*i**3*n*x/4 + B*c*
*3*i**3*x*log(e*(a/(c + d*x))**n) + 3*B*c**2*d*i**3*n*x**2/8 + 3*B*c**2*d*
i**3*x**2*log(e*(a/(c + d*x))**n)/2 + B*c*d**2*i**3*n*x**3/4 + B*c*d**2*i*
*3*x**3*log(e*(a/(c + d*x))**n) + B*d**3*i**3*n*x**4/16 + B*d**3*i**3*x**4
*log(e*(a/(c + d*x))**n)/4, Eq(b, 0)), (c**3*i**3*(A*x + B*a*log(e*(a/c +
b*x/c)**n)/b - B*n*x + B*x*log(e*(a/c + b*x/c)**n)), Eq(d, 0)), (A*c**3*i*
*3*x + 3*A*c**2*d*i**3*x**2/2 + A*c*d**2*i**3*x**3 + A*d**3*i**3*x**4/4 -
B*a**4*d**3*i**3*n*log(c/d + x)/(4*b**4) - B*a**4*d**3*i**3*log(e*(a/(c +
d*x) + b*x/(c + d*x))**n)/(4*b**4) + B*a**3*c*d**2*i**3*n*log(c/d + x)/b**
3 + B*a**3*c*d**2*i**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/b**3 + B*a*
*3*d**3*i**3*n*x/(4*b**3) - 3*B*a**2*c**2*d*i**3*n*log(c/d + x)/(2*b**2) -
 3*B*a**2*c**2*d*i**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(2*b**2) - B
*a**2*c*d**2*i**3*n*x/b**2 - B*a**2*d**3*i**3*n*x**2/(8*b**2) + B*a*c**3*i
**3*n*log(c/d + x)/b + B*a*c**3*i**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**
n)/b + 3*B*a*c**2*d*i**3*n*x/(2*b) + B*a*c*d**2*i**3*n*x**2/(2*b) + B*a*d*
*3*i**3*n*x**3/(12*b) - B*c**4*i**3*n*log(c/d + x)/(4*d) - 3*B*c**3*i**3*n
*x/4 + B*c**3*i**3*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n) - 3*B*c**2*d*
i**3*n*x**2/8 + 3*B*c**2*d*i**3*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x)...
```

### 3.130.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 479 vs. $2(146) = 292$.

Time = 0.21 (sec) , antiderivative size = 479, normalized size of antiderivative = 3.07

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \frac{1}{4}Bd^3i^3x^4\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+\frac{1}{4}Ad^3i^3x^4+Bcd^2i^3x^3\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Acd^2i^3x^3$$
$$+\frac{3}{2}Bc^2di^3x^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+\frac{3}{2}Ac^2di^3x^2$$
$$-\frac{1}{24}Bd^3i^3n\left(\frac{6\,a^4\log(bx+a)}{b^4}-\frac{6\,c^4\log(dx+c)}{d^4}+\frac{2\,(b^3cd^2-ab^2d^3)x^3-3\,(b^3c^2d-a^2bd^3)x^2+6\,(b^3c^3-}{b^3d^3}\right.$$
$$+\frac{1}{2}Bcd^2i^3n\left(\frac{2\,a^3\log(bx+a)}{b^3}-\frac{2\,c^3\log(dx+c)}{d^3}-\frac{(b^2cd-abd^2)x^2-2\,(b^2c^2-a^2d^2)x}{b^2d^2}\right)$$
$$-\frac{3}{2}Bc^2di^3n\left(\frac{a^2\log(bx+a)}{b^2}-\frac{c^2\log(dx+c)}{d^2}+\frac{(bc-ad)x}{bd}\right)$$
$$+Bc^3i^3n\left(\frac{a\log(bx+a)}{b}-\frac{c\log(dx+c)}{d}\right)$$
$$+Bc^3i^3x\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)+Ac^3i^3x$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="max
ima")
```

output
```
1/4*B*d^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A*d^3*i^3*x
^4 + B*c*d^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c*d^2*i^3*
x^3 + 3/2*B*c^2*d*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A*c
^2*d*i^3*x^2 - 1/24*B*d^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x +
c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 +
6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/2*B*c*d^2*i^3*n*(2*a^3*log(b*x + a
)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a
^2*d^2)*x)/(b^2*d^2)) - 3/2*B*c^2*d*i^3*n*(a^2*log(b*x + a)/b^2 - c^2*log(
d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + B*c^3*i^3*n*(a*log(b*x + a)/b - c*lo
g(d*x + c)/d) + B*c^3*i^3*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*c^3
*i^3*x
```

### 3.130.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1402 vs. $2(146) = 292$.

Time = 0.80 (sec) , antiderivative size = 1402, normalized size of antiderivative = 8.99

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n)),x, algorithm="gia
c")
```

output
```
1/24*(6*(B*b^5*c^5*i^3*n - 5*B*a*b^4*c^4*d*i^3*n + 10*B*a^2*b^3*c^3*d^2*i^
3*n - 10*B*a^3*b^2*c^2*d^3*i^3*n + 5*B*a^4*b*c*d^4*i^3*n - B*a^5*d^5*i^3*n
)*log((b*x + a)/(d*x + c))/(b^4*d - 4*(b*x + a)*b^3*d^2/(d*x + c) + 6*(b*x
 + a)^2*b^2*d^3/(d*x + c)^2 - 4*(b*x + a)^3*b*d^4/(d*x + c)^3 + (b*x + a)^
4*d^5/(d*x + c)^4) - (11*B*b^8*c^5*i^3*n - 55*B*a*b^7*c^4*d*i^3*n - 26*(b*
x + a)*B*b^7*c^5*d*i^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*d^2*i^3*n + 130*(b*
x + a)*B*a*b^6*c^4*d^2*i^3*n/(d*x + c) + 21*(b*x + a)^2*B*b^6*c^5*d^2*i^3*
n/(d*x + c)^2 - 110*B*a^3*b^5*c^2*d^3*i^3*n - 260*(b*x + a)*B*a^2*b^5*c^3*
d^3*i^3*n/(d*x + c) - 105*(b*x + a)^2*B*a*b^5*c^4*d^3*i^3*n/(d*x + c)^2 -
6*(b*x + a)^3*B*b^5*c^5*d^3*i^3*n/(d*x + c)^3 + 55*B*a^4*b^4*c*d^4*i^3*n +
 260*(b*x + a)*B*a^3*b^4*c^2*d^4*i^3*n/(d*x + c) + 210*(b*x + a)^2*B*a^2*b
^4*c^3*d^4*i^3*n/(d*x + c)^2 + 30*(b*x + a)^3*B*a*b^4*c^4*d^4*i^3*n/(d*x +
 c)^3 - 11*B*a^5*b^3*d^5*i^3*n - 130*(b*x + a)*B*a^4*b^3*c*d^5*i^3*n/(d*x
+ c) - 210*(b*x + a)^2*B*a^3*b^3*c^2*d^5*i^3*n/(d*x + c)^2 - 60*(b*x + a)^
3*B*a^2*b^3*c^3*d^5*i^3*n/(d*x + c)^3 + 26*(b*x + a)*B*a^5*b^2*d^6*i^3*n/(
d*x + c) + 105*(b*x + a)^2*B*a^4*b^2*c*d^6*i^3*n/(d*x + c)^2 + 60*(b*x + a
)^3*B*a^3*b^2*c^2*d^6*i^3*n/(d*x + c)^3 - 21*(b*x + a)^2*B*a^5*b*d^7*i^3*n
/(d*x + c)^2 - 30*(b*x + a)^3*B*a^4*b*c*d^7*i^3*n/(d*x + c)^3 + 6*(b*x + a
)^3*B*a^5*d^8*i^3*n/(d*x + c)^3 - 6*B*b^8*c^5*i^3*log(e) + 30*B*a*b^7*c^4*
d*i^3*log(e) - 60*B*a^2*b^6*c^3*d^2*i^3*log(e) + 60*B*a^3*b^5*c^2*d^3*i...
```

### 3.130.9 Mupad [B] (verification not implemented)

Time = 1.55 (sec) , antiderivative size = 588, normalized size of antiderivative = 3.77

$$\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= x^3\left(\frac{d^2\,i^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{12\,b}-\frac{A\,d^2\,i^3\,(4\,a\,d+4\,b\,c)}{12\,b}\right)$$
$$-x^2\left(\frac{\left(\frac{d^2\,i^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,b}-\frac{A\,d^2\,i^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)(4\,a\,d+4\,b\,c)}{8\,b\,d}-\frac{c\,d\,i^3\,(4\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{2\,b}+\frac{A\,a\,c\,d^2\,i^3}{2\,b}\right)$$
$$+\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(B\,c^3\,i^3\,x+\frac{3\,B\,c^2\,d\,i^3\,x^2}{2}+B\,c\,d^2\,i^3\,x^3+\frac{B\,d^3\,i^3\,x^4}{4}\right)$$
$$+x\left(\frac{(4\,a\,d+4\,b\,c)\left(\frac{\left(\frac{d^2\,i^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,b}-\frac{A\,d^2\,i^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)(4\,a\,d+4\,b\,c)}{4\,b\,d}-\frac{c\,d\,i^3\,(4\,A\,a\,d+6\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n}{b}\right.}{4\,b\,d}+\frac{c^2\,i^3\,(12\,A\,a\,d+8\,A\,b\,c+3\,B\,a\,d\,n-3\,B\,b\,c\,n)}{2\,b}-\frac{a\,c\left(\frac{d^2\,i^3\,(4\,A\,a\,d+16\,A\,b\,c+B\,a\,d\,n-B\,b\,c\,n)}{4\,b}-\frac{A\,d^2\,i^3\,(4\,a\,d+4\,b\,c)}{4\,b}\right)}{b\,d}\right)$$
$$-\frac{\ln(a+b\,x)\,(B\,n\,a^4\,d^3\,i^3-4\,B\,n\,a^3\,b\,c\,d^2\,i^3+6\,B\,n\,a^2\,b^2\,c^2\,d\,i^3-4\,B\,n\,a\,b^3\,c^3\,i^3)}{4\,b^4}$$
$$+\frac{A\,d^3\,i^3\,x^4}{4}-\frac{B\,c^4\,i^3\,n\,\ln(c+d\,x)}{4\,d}$$

input
```
int((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)),x)
```

output
```
x^3*((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(12*b) - (A*d^2*i^
3*(4*a*d + 4*b*c))/(12*b)) - x^2*((((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d*n
 - B*b*c*n))/(4*b) - (A*d^2*i^3*(4*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(
8*b*d) - (c*d*i^3*(4*A*a*d + 6*A*b*c + B*a*d*n - B*b*c*n))/(2*b) + (A*a*c*
d^2*i^3)/(2*b)) + log(e*((a + b*x)/(c + d*x))^n)*((B*d^3*i^3*x^4)/4 + B*c^
3*i^3*x + (3*B*c^2*d*i^3*x^2)/2 + B*c*d^2*i^3*x^3) + x*(((4*a*d + 4*b*c)*(
(((d^2*i^3*(4*A*a*d + 16*A*b*c + B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*i^3*(4
*a*d + 4*b*c))/(4*b))*(4*a*d + 4*b*c))/(4*b*d) - (c*d*i^3*(4*A*a*d + 6*A*b
*c + B*a*d*n - B*b*c*n))/b + (A*a*c*d^2*i^3)/b))/(4*b*d) + (c^2*i^3*(12*A*
a*d + 8*A*b*c + 3*B*a*d*n - 3*B*b*c*n))/(2*b) - (a*c*((d^2*i^3*(4*A*a*d +
16*A*b*c + B*a*d*n - B*b*c*n))/(4*b) - (A*d^2*i^3*(4*a*d + 4*b*c))/(4*b)))
/(b*d)) - (log(a + b*x)*(B*a^4*d^3*i^3*n - 4*B*a*b^3*c^3*i^3*n - 4*B*a^3*b
*c*d^2*i^3*n + 6*B*a^2*b^2*c^2*d*i^3*n))/(4*b^4) + (A*d^3*i^3*x^4)/4 - (B*
c^4*i^3*n*log(c + d*x))/(4*d)
```

3.130. $\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$

# 3.131 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$



## 3.131.1 Optimal result

Integrand size = 43, antiderivative size = 373

$$\begin{aligned}\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx &= -\frac{5Bd(bc-ad)^2i^3nx}{6b^3g}-\frac{B(bc-ad)i^3n(c+dx)^2}{6b^2g}\\ &+\frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g}\\ &+\frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g}+\frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg}\\ &-\frac{5B(bc-ad)^3i^3n\log\left(\frac{a+bx}{c+dx}\right)}{6b^4g}-\frac{11B(bc-ad)^3i^3n\log(c+dx)}{6b^4g}\\ &-\frac{(bc-ad)^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}\\ &+\frac{B(bc-ad)^3i^3n\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}\end{aligned}$$

output
```
-5/6*B*d*(-a*d+b*c)^2*i^3*n*x/b^3/g-1/6*B*(-a*d+b*c)*i^3*n*(d*x+c)^2/b^2/g
+d*(-a*d+b*c)^2*i^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/g+1/2*(-a*
d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g+1/3*i^3*(d*x+c)
^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g-5/6*B*(-a*d+b*c)^3*i^3*n*ln((b*x+a)
/(d*x+c))/b^4/g-11/6*B*(-a*d+b*c)^3*i^3*n*ln(d*x+c)/b^4/g-(-a*d+b*c)^3*i^3
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g+B*(-a*d+b
*c)^3*i^3*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g
```

## 3.131.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 368, normalized size of antiderivative = 0.99

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$=\frac{i^3\Big(6Abd(bc-ad)^2x-3B(bc-ad)^2n(bdx+(bc-ad)\log(a+bx))-B(bc-ad)n(2bd(bc-ad)x+b^2(c\cdots}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x),x]
```

output
```
(i^3*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a*d)*L
og[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*
(b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*
x)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
 6*(b*c - a*d)^3*Log[g*(a + b*x)]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) -
 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[g*(a + b*x)]*
(Log[g*(a + b*x)] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)])))/(6*b^4*g)
```

### 3.131.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.28, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.326, Rules used = {2961, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{ag+bgx} \, dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i^3 (bc-ad)^3 \int \frac{(c+dx) \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx) \left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \text{2789}$$

$$\frac{i^3 (bc-ad)^3 \left(\frac{d \int \frac{A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx) \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx) \left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \text{2756}$$

$$\frac{i^3 (bc-ad)^3 \left(\frac{d \left(\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3 d \left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{B n \int \frac{c+dx}{(a+bx) \left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3 d}\right)}{b}+\frac{\int \frac{(c+dx) \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx) \left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \text{54}$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\ldots\right.}{(a+bx)\left(b-\ldots\right.}}{}\right.}{g}$$

$$\Big\downarrow \textcolor{blue}{2009}$$

$$\frac{i^3(bc-ad)^3\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^3}+\frac{1}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\ldots}{2b\left(b-\ldots\right.}\right.}{3d}\right.}{b}\right.}{g}$$

$$\Big\downarrow \textcolor{blue}{2789}$$

$$\frac{i^3(bc-ad)^3\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b-\frac{d\ldots}{\ldots}\right.}{b^3}\right.}{}\right.}{b}\right.}{g}$$

$$\Big\downarrow \textcolor{blue}{2756}$$

3.131. $\displaystyle\int\frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

$$\frac{i^3(bc-ad)^3\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right.}{}\right.}{g}$$

$$\Big\downarrow \; 54$$

$$\frac{i^3(bc-ad)^3\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b}\right.}{g}$$

$$\Big\downarrow \; 2009$$

$$\frac{i^3(bc-ad)^3\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}}{b}+\frac{d\left(\right.}{}\right.}{g}$$

$$\Big\downarrow \; 2789$$

---

3.131. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

$$i^3(bc-ad)^3\left(\frac{\frac{\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+\cdots}{c+\cdots}\right.}{b^2}\cdots\right.}{2d\cdots}\cdots\right.}{b\cdots}}{b}\cdots\right.$$

$$\cdots g$$

↓ 2751

$$i^3(bc-ad)^3\left(\frac{\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int \frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\cdots\right.}{\cdots}}{b}\cdots\right.$$

↓ 16

$$i^3(bc-ad)^3\left(\frac{\frac{\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\cdots}{\cdots}\right.}{\cdots}}{b}\cdots\right.$$

↓ 2779

3.131. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

$$i^3(bc-ad)^3\left(\frac{\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+\cdots}{c+\cdots}\right.}{bd}\right.}{b}\right.$$

$$\downarrow \text{2838}$$

$$i^3(bc-ad)^3\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{\frac{\frac{Bn\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\cdots\right.}{b}}{\cdots}}{b}\right.$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
,x]
```

output
```
((b*c - a*d)^3*i^3*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*d*(b - (
d*(a + b*x))/(c + d*x))^3) - (B*n*(1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2)
 + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 -
Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(3*d)))/b + ((d*((A + B*Log[e*((a +
 b*x)/(c + d*x))^n])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(1/(b*(b
 - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(
a + b*x))/(c + d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Lo
g[b - (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b
*(c + d*x))/(d*(a + b*x))])/b)/b)/b)/b))/g
```

## 3.131. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

### 3.131.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

## 3.131. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.131.4 Maple [F]

$$\int \frac{(dix + ci)^3 \left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{bgx + ag}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x)
```

### 3.131.5 Fricas [F]

$$\int \frac{(ci + dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag + bgx}\,dx = \int \frac{(dix + ci)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)}{bgx + ag}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, al
gorithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log(e*
((b*x + a)/(d*x + c))^n))/(b*g*x + a*g), x)
```

### 3.131.6 Sympy [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= \frac{i^3\left(\int \frac{Ac^3}{a+bx}\,dx + \int \frac{Ad^3x^3}{a+bx}\,dx + \int \frac{Bc^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a+bx}\,dx + \int \frac{3Acd^2x^2}{a+bx}\,dx + \int \frac{3Ac^2dx}{a+bx}\,dx + \int \frac{Bd^3x^3\log\left(e\left(\frac{a}{c+dx}+\right.\right.}{a+bx}\right.}{g}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g),x)
```

output
```
i**3*(Integral(A*c**3/(a + b*x), x) + Integral(A*d**3*x**3/(a + b*x), x) +
 Integral(B*c**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + I
ntegral(3*A*c*d**2*x**2/(a + b*x), x) + Integral(3*A*c**2*d*x/(a + b*x), x
) + Integral(B*d**3*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x)
, x) + Integral(3*B*c*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a
 + b*x), x) + Integral(3*B*c**2*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n
)/(a + b*x), x))/g
```

### 3.131.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 935 vs. $2(360) = 720$.

Time = 0.50 (sec) , antiderivative size = 935, normalized size of antiderivative = 2.51

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx$$
$$= 3\,Ac^2di^3\left(\frac{x}{bg} - \frac{a\log(bx+a)}{b^2g}\right) - \frac{1}{6}\,Ad^3i^3\left(\frac{6\,a^3\log(bx+a)}{b^4g} - \frac{2\,b^2x^3 - 3\,abx^2 + 6\,a^2x}{b^3g}\right)$$
$$+ \frac{3}{2}\,Acd^2i^3\left(\frac{2\,a^2\log(bx+a)}{b^3g} + \frac{bx^2 - 2\,ax}{b^2g}\right) + \frac{Ac^3i^3\log(bgx+ag)}{bg}$$
$$- \frac{(11\,b^2c^3i^3n - 15\,abc^2di^3n + 6\,a^2cd^2i^3n)B\log(dx+c)}{6\,b^3g}$$
$$+ \frac{(b^3c^3i^3n - 3\,ab^2c^2di^3n + 3\,a^2bcd^2i^3n - a^3d^3i^3n)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{b^4g}$$
$$+ \frac{2\,Bb^3d^3i^3x^3\log(e) - ((i^3n - 9\,i^3\log(e))b^3cd^2 - (i^3n - 3\,i^3\log(e))ab^2d^3)Bx^2 - 3\,(b^3c^3i^3n - 3\,ab^2c^2di^3}{}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, al
gorithm="maxima")
```

output
```
3*A*c^2*d*i^3*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - 1/6*A*d^3*i^3*(6*a^3*lo
g(b*x + a)/(b^4*g) - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/(b^3*g)) + 3/2*A*c*
d^2*i^3*(2*a^2*log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A*c^3*i^3
*log(b*g*x + a*g)/(b*g) - 1/6*(11*b^2*c^3*i^3*n - 15*a*b*c^2*d*i^3*n + 6*a
^2*c*d^2*i^3*n)*B*log(d*x + c)/(b^3*g) + (b^3*c^3*i^3*n - 3*a*b^2*c^2*d*i^
3*n + 3*a^2*b*c*d^2*i^3*n - a^3*d^3*i^3*n)*(log(b*x + a)*log((b*d*x + a*d)
/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(b^4*g) + 1/6*(2*
B*b^3*d^3*i^3*x^3*log(e) - ((i^3*n - 9*i^3*log(e))*b^3*c*d^2 - (i^3*n - 3*
i^3*log(e))*a*b^2*d^3)*B*x^2 - 3*(b^3*c^3*i^3*n - 3*a*b^2*c^2*d*i^3*n + 3*
a^2*b*c*d^2*i^3*n - a^3*d^3*i^3*n)*B*log(b*x + a)^2 - ((7*i^3*n - 18*i^3*l
og(e))*b^3*c^2*d - 6*(2*i^3*n - 3*i^3*log(e))*a*b^2*c*d^2 + (5*i^3*n - 6*i
^3*log(e))*a^2*b*d^3)*B*x + (6*b^3*c^3*i^3*log(e) + 18*(i^3*n - i^3*log(e)
)*a*b^2*c^2*d - 9*(3*i^3*n - 2*i^3*log(e))*a^2*b*c*d^2 + (11*i^3*n - 6*i^3
*log(e))*a^3*d^3)*B*log(b*x + a) + (2*B*b^3*d^3*i^3*x^3 + 3*(3*b^3*c*d^2*i
^3 - a*b^2*d^3*i^3)*B*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2*c*d^2*i^3 + a^2*b
*d^3*i^3)*B*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^2*i^3 - a
^3*d^3*i^3)*B*log(b*x + a))*log((b*x + a)^n) - (2*B*b^3*d^3*i^3*x^3 + 3*(3
*b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2*c*d^2
*i^3 + a^2*b*d^3*i^3)*B*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c
*d^2*i^3 - a^3*d^3*i^3)*B*log(b*x + a))*log((d*x + c)^n))/(b^4*g)
```

### 3.131.8 Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g),x, al
gorithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g), x)
```

### 3.131.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ag+bgx}\,dx=\int \frac{\left(c\,i+d\,i\,x\right)^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
, x)
```

# 3.132 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$



## 3.132.1 Optimal result

Integrand size = 43, antiderivative size = 390

$$\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx\\
&= -\frac{Bd^2(bc-ad)i^3nx}{2b^3g^2}-\frac{B(bc-ad)^2i^3n(c+dx)}{b^3g^2(a+bx)}\\
&\quad+\frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^2}\\
&\quad-\frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2(a+bx)}+\frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^2}\\
&\quad-\frac{Bd(bc-ad)^2i^3n\log\left(\frac{a+bx}{c+dx}\right)}{2b^4g^2}-\frac{5Bd(bc-ad)^2i^3n\log(c+dx)}{2b^4g^2}\\
&\quad-\frac{3d(bc-ad)^2i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}\\
&\quad+\frac{3Bd(bc-ad)^2i^3n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}
\end{aligned}$$

output
```
-1/2*B*d^2*(-a*d+b*c)*i^3*n*x/b^3/g^2-B*(-a*d+b*c)^2*i^3*n*(d*x+c)/b^3/g^2
/(b*x+a)+2*d^2*(-a*d+b*c)*i^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/
g^2-(-a*d+b*c)^2*i^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g^2/(b*x+
a)+1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^2-1/2*B*d*(-a
*d+b*c)^2*i^3*n*ln((b*x+a)/(d*x+c))/b^4/g^2-5/2*B*d*(-a*d+b*c)^2*i^3*n*ln(
d*x+c)/b^4/g^2-3*d*(-a*d+b*c)^2*i^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b
*(d*x+c)/d/(b*x+a))/b^4/g^2+3*B*d*(-a*d+b*c)^2*i^3*n*polylog(2,b*(d*x+c)/d
/(b*x+a))/b^4/g^2
```

### 3.132.2 Mathematica [A] (verified)

Time = 0.24 (sec) , antiderivative size = 394, normalized size of antiderivative = 1.01

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx$$
$$= \frac{i^3\left(2Abd^2(3bc-2ad)x - bBd^2(bc-ad)nx - \frac{2B(bc-ad)^3n}{a+bx} - a^2Bd^3n\log(a+bx) - 2Bd(bc-ad)^2n\log(a+\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x)^2,x]
```

output
```
(i^3*(2*A*b*d^2*(3*b*c - 2*a*d)*x - b*B*d^2*(b*c - a*d)*n*x - (2*B*(b*c -
a*d)^3*n)/(a + b*x) - a^2*B*d^3*n*Log[a + b*x] - 2*B*d*(b*c - a*d)^2*n*Log
[a + b*x] + 2*B*d^2*(3*b*c - 2*a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^
n] + b^2*d^3*x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - (2*(b*c - a*d)^3
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + 6*d*(b*c - a*d)^2*Log
[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + b^2*B*c^2*d*n*Log[c + d
*x] + 2*B*d*(b*c - a*d)^2*n*Log[c + d*x] - 2*B*d*(-(b*c) + a*d)*(-3*b*c +
2*a*d)*n*Log[c + d*x] - 3*B*d*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x]
- 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) +
 a*d)]))/(2*b^4*g^2)
```

### 3.132.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 343, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {2961, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i^3(bc-ad)^2\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{2793}$$

$$\frac{i^3(bc-ad)^2\int \left(\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d^2}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d^2}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{(c+dx)^2\left(A+B\log\left(\right.\right.}{b^3(a+bx)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i^3(bc-ad)^2\left(\frac{2d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{3d\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3(a+bx)}+\right.}{g^2}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^2,x]
```

output
```
((b*c - a*d)^2*i^3*(-((B*n*(c + d*x))/(b^3*(a + b*x))) - (B*d*n)/(2*b^3*(b
 - (d*(a + b*x))/(c + d*x))) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(b^3*(a + b*x)) + (d*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b^
2*(b - (d*(a + b*x))/(c + d*x))^2) + (2*d^2*(a + b*x)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(b^4*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (B*d*n
*Log[(a + b*x)/(c + d*x)])/(2*b^4) + (5*B*d*n*Log[b - (d*(a + b*x))/(c + d
*x)])/(2*b^4) - (3*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c
+ d*x))/(d*(a + b*x))])/b^4 + (3*B*d*n*PolyLog[2, (b*(c + d*x))/(d*(a + b*
x))])/b^4))/g^2
```

### 3.132.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.132.4 Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^2}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x)
```

## 3.132.5 Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^3\left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,
algorithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log(e*
((b*x + a)/(d*x + c))^n))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

### 3.132.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2,x
)
```

output
```
Timed out
```

### 3.132.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1785 vs. 2(381) = 762.

Time = 0.50 (sec) , antiderivative size = 1785, normalized size of antiderivative = 4.58

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx=\text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,
algorithm="maxima")
```

output
```
-B*c^3*i^3*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^
2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - 3*A*(a^2/(b^4*g^2*x + a*b^3*g
^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2))*c*d^2*i^3 + 1/2*(2*a^3/(b^
5*g^2*x + a*b^4*g^2) + 6*a^2*log(b*x + a)/(b^4*g^2) + (b*x^2 - 4*a*x)/(b^3
*g^2))*A*d^3*i^3 + 3*A*c^2*d*i^3*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)
/(b^2*g^2)) - B*c^3*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^2*g^2*x
+ a*b*g^2) - A*c^3*i^3/(b^2*g^2*x + a*b*g^2) - 1/2*(5*b^3*c^3*d*i^3*n - 3*
a*b^2*c^2*d^2*i^3*n - 2*a^2*b*c*d^3*i^3*n + 2*a^3*d^4*i^3*n)*B*log(d*x + c
)/(b^5*c*g^2 - a*b^4*d*g^2) + 1/2*((b^4*c*d^3*i^3*log(e) - a*b^3*d^4*i^3*l
og(e))*B*x^3 - ((i^3*n - 6*i^3*log(e))*b^4*c^2*d^2 - (2*i^3*n - 9*i^3*log(
e))*a*b^3*c*d^3 + (i^3*n - 3*i^3*log(e))*a^2*b^2*d^4)*B*x^2 - ((i^3*n - 6*
i^3*log(e))*a*b^3*c^2*d^2 - 2*(i^3*n - 5*i^3*log(e))*a^2*b^2*c*d^3 + (i^3*
n - 4*i^3*log(e))*a^3*b*d^4)*B*x - 3*((b^4*c^3*d*i^3*n - 3*a*b^3*c^2*d^2*i
^3*n + 3*a^2*b^2*c*d^3*i^3*n - a^3*b*d^4*i^3*n)*B*x + (a*b^3*c^3*d*i^3*n -
 3*a^2*b^2*c^2*d^2*i^3*n + 3*a^3*b*c*d^3*i^3*n - a^4*d^4*i^3*n)*B)*log(b*x
 + a)^2 + 2*(3*(i^3*n + i^3*log(e))*a*b^3*c^3*d - 6*(i^3*n + i^3*log(e))*a
^2*b^2*c^2*d^2 + 4*(i^3*n + i^3*log(e))*a^3*b*c*d^3 - (i^3*n + i^3*log(e))
*a^4*d^4)*B + ((6*b^4*c^3*d*i^3*log(e) + 6*(2*i^3*n - 3*i^3*log(e))*a*b^3*
c^2*d^2 - (17*i^3*n - 18*i^3*log(e))*a^2*b^2*c*d^3 + (7*i^3*n - 6*i^3*log(
e))*a^3*b*d^4)*B*x + (6*a*b^3*c^3*d*i^3*log(e) + 6*(2*i^3*n - 3*i^3*log...
```

## 3.132.8 Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2,x,
algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g)^2, x)
```

### 3.132.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^2,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^2, x)
```

# 3.133 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$



## 3.133.1 Optimal result

Integrand size = 43, antiderivative size = 361

$$\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx \\
&= -\frac{2Bd(bc-ad)i^3n(c+dx)}{b^3g^3(a+bx)}-\frac{B(bc-ad)i^3n(c+dx)^2}{4b^2g^3(a+bx)^2} \\
&\quad+\frac{d^3i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^3}-\frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^3(a+bx)} \\
&\quad-\frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^3(a+bx)^2}-\frac{Bd^2(bc-ad)i^3n\log(c+dx)}{b^4g^3} \\
&\quad-\frac{3d^2(bc-ad)i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3} \\
&\quad+\frac{3Bd^2(bc-ad)i^3n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}
\end{aligned}$$

output
```
-2*B*d*(-a*d+b*c)*i^3*n*(d*x+c)/b^3/g^3/(b*x+a)-1/4*B*(-a*d+b*c)*i^3*n*(d*
x+c)^2/b^2/g^3/(b*x+a)^2+d^3*i^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b
^4/g^3-2*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g^3/
(b*x+a)-1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g
^3/(b*x+a)^2-B*d^2*(-a*d+b*c)*i^3*n*ln(d*x+c)/b^4/g^3-3*d^2*(-a*d+b*c)*i^3
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^3+3*B*d^2
*(-a*d+b*c)*i^3*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^3
```

### 3.133.2 Mathematica [A] (verified)

Time = 0.23 (sec) , antiderivative size = 331, normalized size of antiderivative = 0.92

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx$$
$$= \frac{i^3\left(4Abd^3x-\frac{B(bc-ad)^3n}{(a+bx)^2}-\frac{10Bd(bc-ad)^2n}{a+bx}+10Bd^2(-bc+ad)n\log(a+bx)+4Bd^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\cdots\right.}{\cdots}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x)^3,x]
```

output
```
(i^3*(4*A*b*d^3*x - (B*(b*c - a*d)^3*n)/(a + b*x)^2 - (10*B*d*(b*c - a*d)^
2*n)/(a + b*x) + 10*B*d^2*(-(b*c) + a*d)*n*Log[a + b*x] + 4*B*d^3*(a + b*x
)*Log[e*((a + b*x)/(c + d*x))^n] - (2*(b*c - a*d)^3*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(a + b*x)^2 - (12*d*(b*c - a*d)^2*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(a + b*x) + 12*d^2*(b*c - a*d)*Log[a + b*x]*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]) + 6*B*d^2*(b*c - a*d)*n*Log[c + d*x] + 6*B*d^2*
(-(b*c) + a*d)*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c -
a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(4*b^4*g^3)
```

### 3.133.3 Rubi [A] (verified)

Time = 0.56 (sec) , antiderivative size = 309, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^3}\,dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{i^3(bc-ad)\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^3}$$

$$\Big\downarrow \text{ 2793}$$

$$\frac{i^3(bc-ad)\int\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d^3}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d^2}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d}{b^3(a+bx)^2}+\frac{(c+dx)^3\left(A+\right.}{b^2}\right.}{g^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i^3(bc-ad)\left(\frac{d^3(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{3d^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4}-\frac{2d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3(a+bx)}-\right.}{g^3}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^3,x]
```

output
```
((b*c - a*d)*i^3*((-2*B*d*n*(c + d*x))/(b^3*(a + b*x)) - (B*n*(c + d*x)^2)
/(4*b^2*(a + b*x)^2) - (2*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(b^3*(a + b*x)) - ((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
/(2*b^2*(a + b*x)^2) + (d^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
]))/(b^4*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*d^2*n*Log[b - (d*(a
 + b*x))/(c + d*x)])/b^4 - (3*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*L
og[1 - (b*(c + d*x))/(d*(a + b*x))])/b^4 + (3*B*d^2*n*PolyLog[2, (b*(c + d
*x))/(d*(a + b*x))])/b^4))/g^3
```

### 3.133.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.133.4 Maple [F]

$$\int \frac{(dix+ci)^{3} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{(bgx+ag)^{3}}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x)
```

## 3.133.5 Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^{3}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}{(bgx+ag)^{3}}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,
algorithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log(e*
((b*x + a)/(d*x + c))^n))/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x +
 a^3*g^3), x)
```

### 3.133.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci + dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3,x
)
```

output
```
Timed out
```

### 3.133.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2746 vs. $2(356) = 712$.

Time = 0.57 (sec) , antiderivative size = 2746, normalized size of antiderivative = 7.61

$$\int \frac{(ci + dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag + bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,
algorithm="maxima")
```

output
```
-3/4*B*c^2*d*i^3*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*
b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g
^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d
^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*
b^2*d^2)*g^3)) + 1/4*B*c^3*i^3*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*
d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) +
2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d
*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A*d^3*i^3*((6*a^2
*b*x + 5*a^3)/(b^6*g^3*x^2 + 2*a*b^5*g^3*x + a^2*b^4*g^3) - 2*x/(b^3*g^3)
+ 6*a*log(b*x + a)/(b^4*g^3)) + 3/2*A*c*d^2*i^3*((4*a*b*x + 3*a^2)/(b^5*g^
3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) - 3/2*(2*
b*x + a)*B*c^2*d*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g^3*x^2 +
 2*a*b^3*g^3*x + a^2*b^2*g^3) - 3/2*(2*b*x + a)*A*c^2*d*i^3/(b^4*g^3*x^2 +
 2*a*b^3*g^3*x + a^2*b^2*g^3) - 1/2*B*c^3*i^3*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A*c^3*i^3/(b^3*
g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*(2*b^3*c^3*d^2*i^3*n + 8*a*b^2*
c^2*d^3*i^3*n - 13*a^2*b*c*d^4*i^3*n + 5*a^3*d^5*i^3*n)*B*log(d*x + c)/(b^
6*c^2*g^3 - 2*a*b^5*c*d*g^3 + a^2*b^4*d^2*g^3) + 1/4*(4*(b^5*c^2*d^3*i^3*l
og(e) - 2*a*b^4*c*d^4*i^3*log(e) + a^2*b^3*d^5*i^3*log(e))*B*x^3 + 8*(a*b^
4*c^2*d^3*i^3*log(e) - 2*a^2*b^3*c*d^4*i^3*log(e) + a^3*b^2*d^5*i^3*log...
```

## 3.133.8 Giac [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^3\left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3,x,
algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g)^3, x)
```

### 3.133.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^3}\,dx=\int \frac{(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(a\,g+b\,g\,x)^3}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^3,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^3, x)
```

# 3.134 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$



## 3.134.1 Optimal result

Integrand size = 43, antiderivative size = 326

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = -\frac{Bd^2i^3n(c+dx)}{b^3g^4(a+bx)} - \frac{Bdi^3n(c+dx)^2}{4b^2g^4(a+bx)^2} - \frac{Bi^3n(c+dx)^3}{9bg^4(a+bx)^3} - \frac{d^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^4(a+bx)} - \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^4(a+bx)^2} - \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bg^4(a+bx)^3} - \frac{d^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} + \frac{Bd^3i^3n\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}$$

output
```
-B*d^2*i^3*n*(d*x+c)/b^3/g^4/(b*x+a)-1/4*B*d*i^3*n*(d*x+c)^2/b^2/g^4/(b*x+
a)^2-1/9*B*i^3*n*(d*x+c)^3/b/g^4/(b*x+a)^3-d^2*i^3*(d*x+c)*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))/b^3/g^4/(b*x+a)-1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))/b^2/g^4/(b*x+a)^2-1/3*i^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))/b/g^4/(b*x+a)^3-d^3*i^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c
)/d/(b*x+a))/b^4/g^4+B*d^3*i^3*n*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^4
```

### 3.134.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 326, normalized size of antiderivative = 1.00

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{i^3\left(-\frac{4B(bc-ad)^3n}{(a+bx)^3}-\frac{21Bd(bc-ad)^2n}{(a+bx)^2}+\frac{66Bd^2(-bc+ad)n}{a+bx}-66Bd^3n\log(a+bx)-\frac{12(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}-\frac{54}{}\right.}{}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g +
b*g*x)^4,x]
```

output
```
(i^3*((-4*B*(b*c - a*d)^3*n)/(a + b*x)^3 - (21*B*d*(b*c - a*d)^2*n)/(a + b
*x)^2 + (66*B*d^2*(-(b*c) + a*d)*n)/(a + b*x) - 66*B*d^3*n*Log[a + b*x] -
(12*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^3 - (5
4*d*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 + (1
08*d^2*(-(b*c) + a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) +
36*d^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 66*B*d^3*n*Lo
g[c + d*x] - 18*B*d^3*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/
(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/(36*b^4*g^4)
```

### 3.134.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 286, normalized size of antiderivative = 0.88, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.209, Rules used = {2961, 2780, 2741, 2780, 2741, 2780, 2741, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ag+bgx)^4}\,dx$$
$$\Big\downarrow \text{2961}$$
$$\frac{i^3\int \frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^4}$$
$$\Big\downarrow \text{2780}$$

$$\frac{i^3\left(\frac{\int \frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^4}$$

$$\Big\downarrow \; 2741$$

$$\frac{i^3\left(\frac{d\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}-\frac{Bn(c+dx)^3}{9(a+bx)^3}}{b}\right)}{g^4}$$

$$\Big\downarrow \; 2780$$

$$\frac{i^3\left(\frac{d\left(\frac{\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}-\frac{Bn(c+dx)^3}{9(a+bx)^3}}{b}\right)}{g^4}$$

$$\Big\downarrow \; 2741$$

$$\frac{i^3\left(\frac{d\left(\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}}{b}\right)}{b}+\frac{-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}-\frac{Bn(c+dx)^3}{9(a+bx)^3}}{b}\right)}{g^4}$$

$$\Big\downarrow \; 2780$$

---

**3.134.** $\displaystyle\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$

$$\frac{i^3\left(\frac{d\left(\frac{d\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}}{b}\right)}{b}+\right.}{g^4}$$

↓ 2741

$$\frac{i^3\left(\frac{d\left(\frac{d\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}}{b}\right)}{b}\right.}{g^4}$$

↓ 2779

3.134. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$

$$\frac{i^3\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{b}+\frac{-\frac{(c\cdots}{\cdots}}{\cdots}\cdots}{b}\cdots\right)}{g^4}$$

$\downarrow$ 2838

$$\frac{i^3\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{Bn\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}\right)}{b}+\frac{-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}}{b}\right)}{b}+\frac{-\frac{(c+dx)^2\left(B\log\left(\cdots\right.\right.}{2(a\cdots}}{\cdots}\cdots}{b}\cdots\right)}{g^4}$$

---

## 3.134. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a*g + b*g*x)
^4,x]
```

output
```
(i^3*((-1/9*(B*n*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]))/(3*(a + b*x)^3))/b + (d*((-1/4*(B*n*(c + d*x)^2)/(a
 + b*x)^2 - ((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b
*x)^2))/b + (d*((-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]))/(a + b*x))/b + (d*(-(((A + B*Log[e*((a + b*x)/(c
+ d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*
(c + d*x))/(d*(a + b*x))])/b))/b))/b))/b))/g^4
```

### 3.134.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.134.4 Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(bgx+ag)^4}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x)
```

### 3.134.5 Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^4}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,
algorithm="fricas")
```

output
```
integral((A*d^3*i^3*x^3 + 3*A*c*d^2*i^3*x^2 + 3*A*c^2*d*i^3*x + A*c^3*i^3
+ (B*d^3*i^3*x^3 + 3*B*c*d^2*i^3*x^2 + 3*B*c^2*d*i^3*x + B*c^3*i^3)*log(e*
((b*x + a)/(d*x + c))^n))/(b^4*g^4*x^4 + 4*a*b^3*g^4*x^3 + 6*a^2*b^2*g^4*x
^2 + 4*a^3*b*g^4*x + a^4*g^4), x)
```

### 3.134.6 Sympy [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx$$
$$= \frac{i^3\left(\int \frac{Ac^3}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{Ad^3x^3}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{Bc^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \right.}{}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4,x
)
```

output
```
i**3*(Integral(A*c**3/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**
3 + b**4*x**4), x) + Integral(A*d**3*x**3/(a**4 + 4*a**3*b*x + 6*a**2*b**2
*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B*c**3*log(e*(a/(c + d*x
) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x*
*3 + b**4*x**4), x) + Integral(3*A*c*d**2*x**2/(a**4 + 4*a**3*b*x + 6*a**2
*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*A*c**2*d*x/(a**4
+ 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integra
l(B*d**3*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x +
 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*B*c*d**2*x
**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b*
*2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(3*B*c**2*d*x*log(e*(a/
(c + d*x) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a
*b**3*x**3 + b**4*x**4), x))/g**4
```

## 3.134.7 Maxima [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^4}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,
algorithm="maxima")
```

output
```
-1/6*B*c*d^2*i^3*n*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c
^2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a
^3*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2
 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d +
 a^4*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6
*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^
2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 +
 a^2*d^3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b
^3*d^3)*g^4)) - 1/18*B*c^3*i^3*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d +
 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4
*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a
^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2
*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*
a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c
^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/12*B*c^2*d*i^3*n*((5*a*b^2*c
^2 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c
^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)
*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^
5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d
+ a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*...
```

## 3.134.8 Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(bgx+ag)^4}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4,x,
algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(b*g*x +
a*g)^4, x)
```

### 3.134.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ag+bgx)^4}\,dx=\int \frac{(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(a\,g+b\,g\,x)^4}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^4,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^4, x)
```

# 3.135 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$



## 3.135.1 Optimal result

Integrand size = 43, antiderivative size = 269

$$\begin{aligned}\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx &= \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3di}\\ &\quad-\frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2i}\\ &\quad+\frac{(bc-ad)^2g^3(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^3i}\\ &\quad+\frac{(bc-ad)^3g^3\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{6d^4i}\\ &\quad+\frac{B(bc-ad)^3g^3n\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}\end{aligned}$$

output
```
1/3*g^3*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i-1/6*(-a*d+b*c)*g^3*(
b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/d^2/i+1/6*(-a*d+b*c)^2*g^
3*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/d^3/i+1/6*(-a*d+b*c)^3
*g^3*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d
^4/i+B*(-a*d+b*c)^3*g^3*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i
```

### 3.135.2 Mathematica [A] (verified)

Time = 0.18 (sec) , antiderivative size = 370, normalized size of antiderivative = 1.38

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^3\left(6Abd(bc-ad)^2x+6Bd(bc-ad)^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i +
d*i*x),x]
```

output
```
(g^3*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a +
b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d
)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*
n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) - 6*(b*c - a*d)^3*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])*Log[i*(c + d*x)] + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(
a + b*x))/(-(b*c) + a*d)] - Log[i*(c + d*x)])*Log[i*(c + d*x)] + 2*PolyLog
[2, (b*(c + d*x))/(b*c - a*d)])))/(6*d^4*i)
```

### 3.135.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 315, normalized size of antiderivative = 1.17, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.140, Rules used = {2961, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{ci+dix}\,dx$$
$$\Big\downarrow \ 2961$$
$$\frac{g^3(bc-ad)^3\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{i}$$
$$\Big\downarrow \ 2784$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\int\frac{(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{i}$$

$\downarrow$ 2784

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}}{3d}\right)}{i}$$

$\downarrow$ 2784

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{6A+11Bn\cdots}{\cdots}}{\cdots}}{2d}}{3d}\cdots}{i}$$

$\downarrow$ 2754

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{6Bn\int\frac{(c+\cdots}{\cdots}}{\cdots}}{\cdots}}{3d}\cdots}{i}$$

$\downarrow$ 2838

3.135. $$\int\frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$

$$\frac{g^3(bc-ad)^3\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\log\left(1-\frac{d(}{b(}\right.}{}}{}}{3d}\right.}{i}$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)
,x]
```

output
```
((b*c - a*d)^3*g^3*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a + b*x)^2*(3*A + B*
n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x
))/(c + d*x))^2) - (((a + b*x)*(6*A + 5*B*n + 6*B*Log[e*((a + b*x)/(c + d*
x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((6*A + 11*B*n +
 6*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])
/d) - (6*B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d))/(3*d)))
/i
```

### 3.135.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.135.4 Maple [F]

$$\int \frac{(bgx+ag)^{3} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{dix+ci}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

## 3.135.5 Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \int \frac{(bgx+ag)^{3} \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, al
gorithm="fricas")
```

output
```
integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
+ (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log(e*
((b*x + a)/(d*x + c))^n))/(d*i*x + c*i), x)
```

### 3.135.6 Sympy [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^3\left(\int \frac{Aa^3}{c+dx}\,dx+\int \frac{Ab^3x^3}{c+dx}\,dx+\int \frac{Ba^3\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx+\int \frac{3Aab^2x^2}{c+dx}\,dx+\int \frac{3Aa^2bx}{c+dx}\,dx+\int \frac{Bb^3x^3\log\left(e\left(\frac{a}{c+dx}+\right.\right.}{c+dx}\right.}{i}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i),x)
```

output
```
g**3*(Integral(A*a**3/(c + d*x), x) + Integral(A*b**3*x**3/(c + d*x), x) +
 Integral(B*a**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + I
ntegral(3*A*a*b**2*x**2/(c + d*x), x) + Integral(3*A*a**2*b*x/(c + d*x), x
) + Integral(B*b**3*x**3*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x)
, x) + Integral(3*B*a*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c
 + d*x), x) + Integral(3*B*a**2*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n
)/(c + d*x), x))/i
```

### 3.135.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1003 vs. 2(260) = 520.

Time = 0.49 (sec) , antiderivative size = 1003, normalized size of antiderivative = 3.73

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= 3\,Aa^2bg^3\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right)-\frac{1}{6}\,Ab^3g^3\left(\frac{6\,c^3\log(dx+c)}{d^4i}-\frac{2\,d^2x^3-3\,cdx^2+6\,c^2x}{d^3i}\right)$$
$$+\frac{3}{2}\,Aab^2g^3\left(\frac{2\,c^2\log(dx+c)}{d^3i}+\frac{dx^2-2\,cx}{d^2i}\right)+\frac{Aa^3g^3\log(dix+ci)}{di}$$
$$-\frac{\left(b^3c^3g^3n-3\,ab^2c^2dg^3n+3\,a^2bcd^2g^3n-a^3d^3g^3n\right)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^4i}$$
$$+\frac{\left(6\,a^3d^3g^3\log(e)-(11\,g^3n+6\,g^3\log(e))b^3c^3+9\,(3\,g^3n+2\,g^3\log(e))ab^2c^2d-18\,(g^3n+g^3\log(e))a^2bc\right.}{6\,d^4i}$$
$$+\frac{2\,Bb^3d^3g^3x^3\log(e)-\left((g^3n+3\,g^3\log(e))b^3cd^2-(g^3n+9\,g^3\log(e))ab^2d^3\right)Bx^2+6\,(b^3c^3g^3n-3\,ab^2c^2d}{}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, al
gorithm="maxima")
```

output
```
3*A*a^2*b*g^3*(x/(d*i) - c*log(d*x + c)/(d^2*i)) - 1/6*A*b^3*g^3*(6*c^3*lo
g(d*x + c)/(d^4*i) - (2*d^2*x^3 - 3*c*d*x^2 + 6*c^2*x)/(d^3*i)) + 3/2*A*a*
b^2*g^3*(2*c^2*log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A*a^3*g^3
*log(d*i*x + c*i)/(d*i) - (b^3*c^3*g^3*n - 3*a*b^2*c^2*d*g^3*n + 3*a^2*b*c
*d^2*g^3*n - a^3*d^3*g^3*n)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) +
1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^4*i) + 1/6*(6*a^3*d^3*g^3*log
(e) - (11*g^3*n + 6*g^3*log(e))*b^3*c^3 + 9*(3*g^3*n + 2*g^3*log(e))*a*b^2
*c^2*d - 18*(g^3*n + g^3*log(e))*a^2*b*c*d^2)*B*log(d*x + c)/(d^4*i) + 1/6
*(2*B*b^3*d^3*g^3*x^3*log(e) - ((g^3*n + 3*g^3*log(e))*b^3*c*d^2 - (g^3*n
+ 9*g^3*log(e))*a*b^2*d^3)*B*x^2 + 6*(b^3*c^3*g^3*n - 3*a*b^2*c^2*d*g^3*n
+ 3*a^2*b*c*d^2*g^3*n - a^3*d^3*g^3*n)*B*log(b*x + a)*log(d*x + c) - 3*(b^
3*c^3*g^3*n - 3*a*b^2*c^2*d*g^3*n + 3*a^2*b*c*d^2*g^3*n - a^3*d^3*g^3*n)*B
*log(d*x + c)^2 + ((5*g^3*n + 6*g^3*log(e))*b^3*c^2*d - 6*(2*g^3*n + 3*g^3
*log(e))*a*b^2*c*d^2 + (7*g^3*n + 18*g^3*log(e))*a^2*b*d^3)*B*x + (6*a*b^2
*c^2*d*g^3*n - 15*a^2*b*c*d^2*g^3*n + 11*a^3*d^3*g^3*n)*B*log(b*x + a) + (
2*B*b^3*d^3*g^3*x^3 - 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B*x^2 + 6*(b^3*c
^2*d*g^3 - 3*a*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B*x - 6*(b^3*c^3*g^3 - 3*a
*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*B*log(d*x + c))*log((b*x
 + a)^n) - (2*B*b^3*d^3*g^3*x^3 - 3*(b^3*c*d^2*g^3 - 3*a*b^2*d^3*g^3)*B*x^
2 + 6*(b^3*c^2*d*g^3 - 3*a*b^2*c*d^2*g^3 + 3*a^2*b*d^3*g^3)*B*x - 6*(b^...
```

### 3.135.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3832 vs. $2(260) = 520$.

Time = 189.49 (sec) , antiderivative size = 3832, normalized size of antiderivative = 14.25

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, al
gorithm="giac")
```

output
```
-1/120*(6*(B*b^9*c^6*g^3*n - 6*B*a*b^8*c^5*d*g^3*n - 5*(b*x + a)*B*b^8*c^6
*d*g^3*n/(d*x + c) + 15*B*a^2*b^7*c^4*d^2*g^3*n + 30*(b*x + a)*B*a*b^7*c^5
*d^2*g^3*n/(d*x + c) + 10*(b*x + a)^2*B*b^7*c^6*d^2*g^3*n/(d*x + c)^2 - 20
*B*a^3*b^6*c^3*d^3*g^3*n - 75*(b*x + a)*B*a^2*b^6*c^4*d^3*g^3*n/(d*x + c)
- 60*(b*x + a)^2*B*a*b^6*c^5*d^3*g^3*n/(d*x + c)^2 - 10*(b*x + a)^3*B*b^6*
c^6*d^3*g^3*n/(d*x + c)^3 + 15*B*a^4*b^5*c^2*d^4*g^3*n + 100*(b*x + a)*B*a
^3*b^5*c^3*d^4*g^3*n/(d*x + c) + 150*(b*x + a)^2*B*a^2*b^5*c^4*d^4*g^3*n/(
d*x + c)^2 + 60*(b*x + a)^3*B*a*b^5*c^5*d^4*g^3*n/(d*x + c)^3 - 6*B*a^5*b^
4*c*d^5*g^3*n - 75*(b*x + a)*B*a^4*b^4*c^2*d^5*g^3*n/(d*x + c) - 200*(b*x
+ a)^2*B*a^3*b^4*c^3*d^5*g^3*n/(d*x + c)^2 - 150*(b*x + a)^3*B*a^2*b^4*c^4
*d^5*g^3*n/(d*x + c)^3 + B*a^6*b^3*d^6*g^3*n + 30*(b*x + a)*B*a^5*b^3*c*d^
6*g^3*n/(d*x + c) + 150*(b*x + a)^2*B*a^4*b^3*c^2*d^6*g^3*n/(d*x + c)^2 +
200*(b*x + a)^3*B*a^3*b^3*c^3*d^6*g^3*n/(d*x + c)^3 - 5*(b*x + a)*B*a^6*b^
2*d^7*g^3*n/(d*x + c) - 60*(b*x + a)^2*B*a^5*b^2*c*d^7*g^3*n/(d*x + c)^2 -
 150*(b*x + a)^3*B*a^4*b^2*c^2*d^7*g^3*n/(d*x + c)^3 + 10*(b*x + a)^2*B*a^
6*b*d^8*g^3*n/(d*x + c)^2 + 60*(b*x + a)^3*B*a^5*b*c*d^8*g^3*n/(d*x + c)^3
 - 10*(b*x + a)^3*B*a^6*d^9*g^3*n/(d*x + c)^3)*log((b*x + a)/(d*x + c))/(b
^5*d^4*i - 5*(b*x + a)*b^4*d^5*i/(d*x + c) + 10*(b*x + a)^2*b^3*d^6*i/(d*x
 + c)^2 - 10*(b*x + a)^3*b^2*d^7*i/(d*x + c)^3 + 5*(b*x + a)^4*b*d^8*i/(d*
x + c)^4 - (b*x + a)^5*d^9*i/(d*x + c)^5) + (5*B*b^10*c^6*g^3*n - 30*B*...
```

### 3.135.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)^3\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
, x)
```

# 3.136 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$



## 3.136.1 Optimal result

Integrand size = 43, antiderivative size = 211

$$\begin{aligned}\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx &= \frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di}\\ &\quad-\frac{(bc-ad)g^2(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i}\\ &\quad-\frac{(bc-ad)^2g^2\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2d^3i}\\ &\quad-\frac{B(bc-ad)^2g^2n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}\end{aligned}$$

output

```
1/2*g^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i-1/2*(-a*d+b*c)*g^2*(
b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))/d^2/i-1/2*(-a*d+b*c)^2*g^2*
(2*A+3*B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^3/i-B
*(-a*d+b*c)^2*g^2*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i
```

## 3.136.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.26

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= \frac{g^2\left(-2Abd(bc-ad)x+2Bd(-bc+ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i +
d*i*x),x]
```

output
```
(g^2*(-2*A*b*d*(b*c - a*d)*x + 2*B*d*(-(b*c) + a*d)*(a + b*x)*Log[e*((a +
b*x)/(c + d*x))^n] + d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
) + 2*B*(b*c - a*d)^2*n*Log[c + d*x] - B*(b*c - a*d)*n*(b*d*x + (-(b*c) +
a*d)*Log[c + d*x]) + 2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*Log[i*(c + d*x)] - B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[i*(c + d*x)])*Log[i*(c + d*x)] + 2*PolyLog[2, (b*(c + d*x))/(b*c
- a*d)])))/(2*d^3*i)
```

## 3.136.3 Rubi [A] (verified)

Time = 0.57 (sec) , antiderivative size = 238, normalized size of antiderivative = 1.13, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{ci+dix}\,dx$$

$$\Big\downarrow \; 2961$$

$$\frac{g^2(bc-ad)^2 \int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \; 2784$$

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{i}$$

$$\Big\downarrow \text{2784}$$

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}}{2d}\right)}{i}$$

$$\Big\downarrow \text{2754}$$

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{2Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+b}{b(c+d}\right.}{d}}{}}{2d}\right.}{i}$$

$$\Big\downarrow \text{2838}$$

$$\frac{g^2(bc-ad)^2\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+3Bn\right)}{d}-}{d}}{2d}\right.}{i}$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)
,x]
```

output
```
((b*c - a*d)^2*g^2*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(2*A + B*n
+ 2*B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c
+ d*x))) - (-(((2*A + 3*B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 -
(d*(a + b*x))/(b*(c + d*x))])/d) - (2*B*n*PolyLog[2, (d*(a + b*x))/(b*(c +
 d*x))])/d)/d)/(2*d)))/i
```

---

3.136. $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$

### 3.136.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.136.4 Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{dix+ci}\,dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

### 3.136.5 Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, al
gorithm="fricas")
```

output
```
integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B
*a*b*g^2*x + B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n))/(d*i*x + c*i), x)
```

### 3.136.6 Sympy [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$=\frac{g^2\left(\int \frac{Aa^2}{c+dx}\,dx+\int \frac{Ab^2x^2}{c+dx}\,dx+\int \frac{Ba^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx+\int \frac{2Aabx}{c+dx}\,dx+\int \frac{Bb^2x^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx+\int \right.}{i}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i),x)
```

output
```
g**2*(Integral(A*a**2/(c + d*x), x) + Integral(A*b**2*x**2/(c + d*x), x) +
 Integral(B*a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + I
ntegral(2*A*a*b*x/(c + d*x), x) + Integral(B*b**2*x**2*log(e*(a/(c + d*x)
+ b*x/(c + d*x))**n)/(c + d*x), x) + Integral(2*B*a*b*x*log(e*(a/(c + d*x)
 + b*x/(c + d*x))**n)/(c + d*x), x))/i
```

### 3.136.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(204) = 408$.

Time = 0.50 (sec) , antiderivative size = 627, normalized size of antiderivative = 2.97

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = 2\,Aabg^2\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right) + \frac{1}{2}\,Ab^2g^2\left(\frac{2\,c^2\log(dx+c)}{d^3i}+\frac{dx^2-2\,cx}{d^2i}\right)+\frac{Aa^2g^2\log(dix+ci)}{di} + \frac{(b^2c^2g^2n-2\,abcdg^2n+a^2d^2g^2n)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^3i} + \frac{(2\,a^2d^2g^2\log(e)+(3\,g^2n+2\,g^2\log(e))b^2c^2-4\,(g^2n+g^2\log(e))abcd)B\log(dx+c)}{2\,d^3i} + \frac{Bb^2d^2g^2x^2\log(e)-2\,(b^2c^2g^2n-2\,abcdg^2n+a^2d^2g^2n)B\log(bx+a)\log(dx+c)+(b^2c^2g^2n-2\,abcdg}{}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, al
gorithm="maxima")
```

output
```
2*A*a*b*g^2*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + 1/2*A*b^2*g^2*(2*c^2*log(
d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A*a^2*g^2*log(d*i*x + c*i)/(
d*i) + (b^2*c^2*g^2*n - 2*a*b*c*d*g^2*n + a^2*d^2*g^2*n)*(log(b*x + a)*log
((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^
3*i) + 1/2*(2*a^2*d^2*g^2*log(e) + (3*g^2*n + 2*g^2*log(e))*b^2*c^2 - 4*(g
^2*n + g^2*log(e))*a*b*c*d)*B*log(d*x + c)/(d^3*i) + 1/2*(B*b^2*d^2*g^2*x^
2*log(e) - 2*(b^2*c^2*g^2*n - 2*a*b*c*d*g^2*n + a^2*d^2*g^2*n)*B*log(b*x +
 a)*log(d*x + c) + (b^2*c^2*g^2*n - 2*a*b*c*d*g^2*n + a^2*d^2*g^2*n)*B*log
(d*x + c)^2 - ((g^2*n + 2*g^2*log(e))*b^2*c*d - (g^2*n + 4*g^2*log(e))*a*b
*d^2)*B*x - (2*a*b*c*d*g^2*n - 3*a^2*d^2*g^2*n)*B*log(b*x + a) + (B*b^2*d^
2*g^2*x^2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B*x + 2*(b^2*c^2*g^2 - 2*a*b*c
*d*g^2 + a^2*d^2*g^2)*B*log(d*x + c))*log((b*x + a)^n) - (B*b^2*d^2*g^2*x^
2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B*x + 2*(b^2*c^2*g^2 - 2*a*b*c*d*g^2 +
 a^2*d^2*g^2)*B*log(d*x + c))*log((d*x + c)^n))/(d^3*i)
```

### 3.136.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2499 vs. $2(204) = 408$.

Time = 130.46 (sec) , antiderivative size = 2499, normalized size of antiderivative = 11.84

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, al
gorithm="giac")
```

output
```
1/24*(2*(B*b^7*c^5*g^2*n - 5*B*a*b^6*c^4*d*g^2*n - 4*(b*x + a)*B*b^6*c^5*d
*g^2*n/(d*x + c) + 10*B*a^2*b^5*c^3*d^2*g^2*n + 20*(b*x + a)*B*a*b^5*c^4*d
^2*g^2*n/(d*x + c) + 6*(b*x + a)^2*B*b^5*c^5*d^2*g^2*n/(d*x + c)^2 - 10*B*
a^3*b^4*c^2*d^3*g^2*n - 40*(b*x + a)*B*a^2*b^4*c^3*d^3*g^2*n/(d*x + c) - 3
0*(b*x + a)^2*B*a*b^4*c^4*d^3*g^2*n/(d*x + c)^2 + 5*B*a^4*b^3*c*d^4*g^2*n
 + 40*(b*x + a)*B*a^3*b^3*c^2*d^4*g^2*n/(d*x + c) + 60*(b*x + a)^2*B*a^2*b^
3*c^3*d^4*g^2*n/(d*x + c)^2 - B*a^5*b^2*d^5*g^2*n - 20*(b*x + a)*B*a^4*b^2
*c*d^5*g^2*n/(d*x + c) - 60*(b*x + a)^2*B*a^3*b^2*c^2*d^5*g^2*n/(d*x + c)^
2 + 4*(b*x + a)*B*a^5*b*d^6*g^2*n/(d*x + c) + 30*(b*x + a)^2*B*a^4*b*c*d^6
*g^2*n/(d*x + c)^2 - 6*(b*x + a)^2*B*a^5*d^7*g^2*n/(d*x + c)^2)*log((b*x +
 a)/(d*x + c))/(b^4*d^3*i - 4*(b*x + a)*b^3*d^4*i/(d*x + c) + 6*(b*x + a)^
2*b^2*d^5*i/(d*x + c)^2 - 4*(b*x + a)^3*b*d^6*i/(d*x + c)^3 + (b*x + a)^4*
d^7*i/(d*x + c)^4) + (B*b^8*c^5*g^2*n - 5*B*a*b^7*c^4*d*g^2*n - 2*(b*x + a
)*B*b^7*c^5*d*g^2*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g^2*n + 10*(b*x + a)*
B*a*b^6*c^4*d^2*g^2*n/(d*x + c) - (b*x + a)^2*B*b^6*c^5*d^2*g^2*n/(d*x + c
)^2 - 10*B*a^3*b^5*c^2*d^3*g^2*n - 20*(b*x + a)*B*a^2*b^5*c^3*d^3*g^2*n/(d
*x + c) + 5*(b*x + a)^2*B*a*b^5*c^4*d^3*g^2*n/(d*x + c)^2 + 2*(b*x + a)^3*
B*b^5*c^5*d^3*g^2*n/(d*x + c)^3 + 5*B*a^4*b^4*c*d^4*g^2*n + 20*(b*x + a)*B
*a^3*b^4*c^2*d^4*g^2*n/(d*x + c) - 10*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^2*n/
(d*x + c)^2 - 10*(b*x + a)^3*B*a*b^4*c^4*d^4*g^2*n/(d*x + c)^3 - B*a^5*...
```

### 3.136.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
, x)
```

# 3.137 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$



## 3.137.1 Optimal result

Integrand size = 41, antiderivative size = 134

$$\begin{aligned}\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx &= \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di} \\ &\quad+\frac{(bc-ad)g\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i} \\ &\quad+\frac{B(bc-ad)gn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}\end{aligned}$$

output
```
g*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i+(-a*d+b*c)*g*(A+B*n+B*ln(e*(
(b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^2/i+B*(-a*d+b*c)*g*n*polyl
og(2,d*(b*x+a)/b/(d*x+c))/d^2/i
```

### 3.137.2 Mathematica [A] (verified)

Time = 0.08 (sec) , antiderivative size = 170, normalized size of antiderivative = 1.27

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$$
$$= \frac{g\left(2Abdx+2Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-2B(bc-ad)n\log(c+dx)-2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\right.}{2d^2i}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*
i*x),x]
```

output
```
(g*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*(b*c
- a*d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(2
*d^2*i)
```

### 3.137.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 154, normalized size of antiderivative = 1.15, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.098, Rules used = {2961, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{ci+dix}\,dx$$

$$\Big\downarrow \ 2961$$

$$\frac{g(bc-ad)\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \ 2784$$

$$\frac{g(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int \frac{A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{i}$$

$$\Big\downarrow \text{2754}$$

$$\frac{g(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A+Bn\right)}{d}}{d}\right)}{i}$$

$$\Big\downarrow \text{2838}$$

$$\frac{g(bc-ad)\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A+Bn\right)}{d}-\frac{Bn\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d}}{d}\right)}{i}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x),x
]
```

output
```
((b*c - a*d)*g*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c +
 d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((A + B*n + B*Log[e*((a + b*x)/(
c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*n*PolyLog[2, (
d*(a + b*x))/(b*(c + d*x))])/d)/d))/i
```

### 3.137.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

3.137. $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.137.4 Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{dix+ci}dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

## 3.137.5 Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{ci+dix}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algo
rithm="fricas")
```

output
```
integral((A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log(e*((b*x + a)/(d*x + c))^
n))/(d*i*x + c*i), x)
```

## 3.137.6 Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{ci+dix}\,dx = \frac{g\left(\int \frac{Aa}{c+dx}\,dx+\int \frac{Abx}{c+dx}\,dx+\int \frac{Ba\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}{c+dx}\,dx+\int \frac{Bbx\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}{c+dx}\,dx\right)}{i}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i),x)
```

output
```
g*(Integral(A*a/(c + d*x), x) + Integral(A*b*x/(c + d*x), x) + Integral(B*
a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + Integral(B*b*x*l
og(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x))/i
```

### 3.137.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 306 vs. $2(133) = 266$.

Time = 0.51 (sec) , antiderivative size = 306, normalized size of antiderivative = 2.28

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = Abg\left(\frac{x}{di}-\frac{c\log(dx+c)}{d^2i}\right) + \frac{Aag\log(dix+ci)}{di} - \frac{(bcgn-adgn)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B}{d^2i} + \frac{(adg\log(e)-(gn+g\log(e))bc)B\log(dx+c)}{d^2i} + \frac{2\,Badgn\log(bx+a)+2\,Bbdgx\log(e)+2\,(bcgn-adgn)B\log(bx+a)\log(dx+c)-(bcgn-adgn)B}{}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algo
rithm="maxima")
```

output
```
A*b*g*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + A*a*g*log(d*i*x + c*i)/(d*i) -
(b*c*g*n - a*d*g*n)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dil
og(-(b*d*x + a*d)/(b*c - a*d)))*B/(d^2*i) + (a*d*g*log(e) - (g*n + g*log(e
))*b*c)*B*log(d*x + c)/(d^2*i) + 1/2*(2*B*a*d*g*n*log(b*x + a) + 2*B*b*d*g
*x*log(e) + 2*(b*c*g*n - a*d*g*n)*B*log(b*x + a)*log(d*x + c) - (b*c*g*n -
 a*d*g*n)*B*log(d*x + c)^2 + 2*(B*b*d*g*x - (b*c*g - a*d*g)*B*log(d*x + c)
)*log((b*x + a)^n) - 2*(B*b*d*g*x - (b*c*g - a*d*g)*B*log(d*x + c))*log((d
*x + c)^n))/(d^2*i)
```

### 3.137.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1242 vs. 2(133) = 266.

Time = 78.29 (sec) , antiderivative size = 1242, normalized size of antiderivative = 9.27

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algo
rithm="giac")
```

output
```
-1/6*((B*b^5*c^4*g*n - 4*B*a*b^4*c^3*d*g*n - 3*(b*x + a)*B*b^4*c^4*d*g*n/(
d*x + c) + 6*B*a^2*b^3*c^2*d^2*g*n + 12*(b*x + a)*B*a*b^3*c^3*d^2*g*n/(d*x
 + c) - 4*B*a^3*b^2*c*d^3*g*n - 18*(b*x + a)*B*a^2*b^2*c^2*d^3*g*n/(d*x +
c) + B*a^4*b*d^4*g*n + 12*(b*x + a)*B*a^3*b*c*d^4*g*n/(d*x + c) - 3*(b*x +
 a)*B*a^4*d^5*g*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^3*d^2*i - 3*(b*x
+ a)*b^2*d^3*i/(d*x + c) + 3*(b*x + a)^2*b*d^4*i/(d*x + c)^2 - (b*x + a)^3
*d^5*i/(d*x + c)^3) + ((b*x + a)*B*b^5*c^4*d*g*n/(d*x + c) - 4*(b*x + a)*B
*a*b^4*c^3*d^2*g*n/(d*x + c) - (b*x + a)^2*B*b^4*c^4*d^2*g*n/(d*x + c)^2 +
 6*(b*x + a)*B*a^2*b^3*c^2*d^3*g*n/(d*x + c) + 4*(b*x + a)^2*B*a*b^3*c^3*d
^3*g*n/(d*x + c)^2 - 4*(b*x + a)*B*a^3*b^2*c*d^4*g*n/(d*x + c) - 6*(b*x +
a)^2*B*a^2*b^2*c^2*d^4*g*n/(d*x + c)^2 + (b*x + a)*B*a^4*b*d^5*g*n/(d*x +
c) + 4*(b*x + a)^2*B*a^3*b*c*d^5*g*n/(d*x + c)^2 - (b*x + a)^2*B*a^4*d^6*g
*n/(d*x + c)^2 + B*b^6*c^4*g*log(e) - 4*B*a*b^5*c^3*d*g*log(e) - 3*(b*x +
a)*B*b^5*c^4*d*g*log(e)/(d*x + c) + 6*B*a^2*b^4*c^2*d^2*g*log(e) + 12*(b*x
 + a)*B*a*b^4*c^3*d^2*g*log(e)/(d*x + c) - 4*B*a^3*b^3*c*d^3*g*log(e) - 18
*(b*x + a)*B*a^2*b^3*c^2*d^3*g*log(e)/(d*x + c) + B*a^4*b^2*d^4*g*log(e) +
 12*(b*x + a)*B*a^3*b^2*c*d^4*g*log(e)/(d*x + c) - 3*(b*x + a)*B*a^4*b*d^5
*g*log(e)/(d*x + c) + A*b^6*c^4*g - 4*A*a*b^5*c^3*d*g - 3*(b*x + a)*A*b^5*
c^4*d*g/(d*x + c) + 6*A*a^2*b^4*c^2*d^2*g + 12*(b*x + a)*A*a*b^4*c^3*d^2*g
/(d*x + c) - 4*A*a^3*b^3*c*d^3*g - 18*(b*x + a)*A*a^2*b^3*c^2*d^3*g/(d*...
```

### 3.137.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x),x
)
```

output
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x),
x)
```

3.137. $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{ci+dix}\,dx$

# 3.138 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx$



## 3.138.1 Optimal result

Integrand size = 33, antiderivative size = 80

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx=-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di}-\frac{Bn\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

output
```
-(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d/i-B*n*polylog(
2,d*(b*x+a)/b/(d*x+c))/d/i
```

## 3.138.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.26

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx$$
$$=\frac{\log(i(c+dx))\left(2A-2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+Bn\log(i(c+dx))\right)-2Bn\operatorname{PolyLog}\left(2,\frac{b(c}{bc}\right.}{2di}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x),x]
```

output
```
(Log[i*(c + d*x)]*(2*A - 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*B*Log
[e*((a + b*x)/(c + d*x))^n] + B*n*Log[i*(c + d*x)]) - 2*B*n*PolyLog[2, (b*
(c + d*x))/(b*c - a*d)])/(2*d*i)
```

### 3.138.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.06, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2943, 2858, 27, 25, 2778, 2005, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{ci+dix} \, dx$$

$$\Big\downarrow \text{2943}$$

$$\frac{Bn(bc-ad) \int \frac{\log \left(\frac{bc-ad}{b(c+dx)}\right)}{(a+bx)(c+dx)}dx}{di}-\frac{\log \left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}$$

$$\Big\downarrow \text{2858}$$

$$\frac{Bn(bc-ad) \int \frac{d \log \left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx) \left(\left(a-\frac{bc}{d}\right) d+b(c+dx)\right)}d(c+dx)}{d^2 i}-\frac{\log \left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}$$

$$\Big\downarrow \text{27}$$

$$\frac{Bn(bc-ad) \int -\frac{\log \left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{di}-\frac{\log \left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}$$

$$\Big\downarrow \text{25}$$

$$-\frac{Bn(bc-ad) \int \frac{\log \left(\frac{bc-ad}{b(c+dx)}\right)}{(c+dx)(bc-ad-b(c+dx))}d(c+dx)}{di}-\frac{\log \left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}$$

$$\Big\downarrow \text{2778}$$

$$\frac{Bn(bc-ad) \int \frac{(c+dx) \log \left(\frac{bc-ad}{b(c+dx)}\right)}{bc-ad-b(c+dx)}d\frac{1}{c+dx}}{di}-\frac{\log \left(\frac{bc-ad}{b(c+dx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}$$

$$\Big\downarrow \text{2005}$$

$$\frac{Bn(bc-ad)\int \frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)}{\frac{bc-ad}{c+dx}-b}d\frac{1}{c+dx}}{di}-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}$$

$$\Big\downarrow \text{ 2752}$$

$$-\frac{\log\left(\frac{bc-ad}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{di}-\frac{Bn\operatorname{PolyLog}\left(2,1-\frac{bc-ad}{b(c+dx)}\right)}{di}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x),x]
```

output
```
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(b*c - a*d)/(b*(c + d*x))])/
(d*i)) - (B*n*PolyLog[2, 1 - (b*c - a*d)/(b*(c + d*x))])/(d*i)
```

### 3.138.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2005
```
Int[(Fx_)*(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[x^(m
+ n*p)*(b + a/x^n)^p*Fx, x] /; FreeQ[{a, b, m, n}, x] && IntegerQ[p] && Neg
Q[n]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2778
```
Int[((a_.) + Log[(c_.)*(x_)^(n_)]*(b_.))/((x_)*((d_) + (e_.)*(x_)^(r_.))),
x_Symbol] :> Simp[1/n   Subst[Int[(a + b*Log[c*x])/(x*(d + e*x^(r/n))), x],
 x, x^n], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IntegerQ[r/n]
```

rule 2858
```
Int[((a_.) + Log[(c_.)*((d_) + (e_.)*(x_))^(n_.)]*(b_.))^(p_.)*((f_.) + (g_
.)*(x_))^(q_.)*((h_.) + (i_.)*(x_))^(r_.), x_Symbol] :> Simp[1/e   Subst[In
t[(g*(x/e))^q*((e*h - d*i)/e + i*(x/e))^r*(a + b*Log[c*x^n])^p, x], x, d +
e*x], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, n, p, q, r}, x] && EqQ[e*f -
d*g, 0] && (IGtQ[p, 0] || IGtQ[r, 0]) && IntegerQ[2*r]
```

rule 2943
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))/((f_.) + (g_.)*(x_)), x_Symbol] :> Simp[(-Log[(b*c - a*d)/(b*(c + d*x
))])*((A + B*Log[e*((a + b*x)/(c + d*x))^n])/g), x] + Simp[B*n*((b*c - a*d)
/g)   Int[Log[(b*c - a*d)/(b*(c + d*x))]/((a + b*x)*(c + d*x)), x], x] /; F
reeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c - a*d, 0] && EqQ[d*f - c
*g, 0]
```

### 3.138.4 Maple [F]

$$\int \frac{A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{dix + ci}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x)
```

### 3.138.5 Fricas [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{ci + dix}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right) + A}{dix + ci}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="frica
s")
```

output
```
integral((B*log(e*((b*x + a)/(d*x + c))^n) + A)/(d*i*x + c*i), x)
```

### 3.138.6 Sympy [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx=\frac{\int \frac{A}{c+dx}\,dx+\int \frac{B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx}{i}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i),x)
```

output
```
(Integral(A/(c + d*x), x) + Integral(B*log(e*(a/(c + d*x) + b*x/(c + d*x))
**n)/(c + d*x), x))/i
```

### 3.138.7 Maxima [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{dix+ci}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="maxim
a")
```

output
```
-1/2*B*((2*n*log(b*x + a)*log(d*x + c) - n*log(d*x + c)^2 - 2*log(d*x + c)
*log((b*x + a)^n) + 2*log(d*x + c)*log((d*x + c)^n))/(d*i) - 2*integrate((
n*log(b*x + a) + log(e))/(d*i*x + c*i), x)) + A*log(d*i*x + c*i)/(d*i)
```

### 3.138.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 566 vs. $2(79) = 158$.

Time = 54.29 (sec) , antiderivative size = 566, normalized size of antiderivative = 7.08

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx$$
$$=\frac{1}{2}\left(\frac{\left(Bb^3c^3n-3\,Bab^2c^2dn+3\,Ba^2bcd^2n-Ba^3d^3n\right)\log\left(\frac{bx+a}{dx+c}\right)}{b^2di-\frac{2\,(bx+a)bd^2i}{dx+c}+\frac{(bx+a)^2d^3i}{(dx+c)^2}}-\frac{Bb^4c^3n-3\,Bab^3c^2dn-\frac{(bx+a)Bb^3c^3dn}{dx+c}+$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i),x, algorithm="giac"
)
```

output
```
1/2*((B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*l
og((b*x + a)/(d*x + c))/(b^2*d*i - 2*(b*x + a)*b*d^2*i/(d*x + c) + (b*x +
a)^2*d^3*i/(d*x + c)^2) - (B*b^4*c^3*n - 3*B*a*b^3*c^2*d*n - (b*x + a)*B*b
^3*c^3*d*n/(d*x + c) + 3*B*a^2*b^2*c*d^2*n + 3*(b*x + a)*B*a*b^2*c^2*d^2*n
/(d*x + c) - B*a^3*b*d^3*n - 3*(b*x + a)*B*a^2*b*c*d^3*n/(d*x + c) + (b*x
+ a)*B*a^3*d^4*n/(d*x + c) - B*b^4*c^3*log(e) + 3*B*a*b^3*c^2*d*log(e) - 3
*B*a^2*b^2*c*d^2*log(e) + B*a^3*b*d^3*log(e) - A*b^4*c^3 + 3*A*a*b^3*c^2*d
 - 3*A*a^2*b^2*c*d^2 + A*a^3*b*d^3)/(b^3*d*i - 2*(b*x + a)*b^2*d^2*i/(d*x
+ c) + (b*x + a)^2*b*d^3*i/(d*x + c)^2) + (B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n
 + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*log(-b + (b*x + a)*d/(d*x + c))/(b^2*d
*i) - (B*b^3*c^3*n - 3*B*a*b^2*c^2*d*n + 3*B*a^2*b*c*d^2*n - B*a^3*d^3*n)*
log((b*x + a)/(d*x + c))/(b^2*d*i))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2
)^2
```

### 3.138.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ci+dix}\,dx=\int \frac{A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{c\,i+d\,i\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*i + d*i*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*i + d*i*x), x)
```

# 3.139 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx$



## 3.139.1 Optimal result

Integrand size = 43, antiderivative size = 50

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx=\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2B(bc-ad)gin}$$

output
```
1/2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/B/(-a*d+b*c)/g/i/n
```

## 3.139.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.07 (sec) , antiderivative size = 219, normalized size of antiderivative = 4.38

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx$$
$$=\frac{2A\log(a+bx)-Bn\log^2(a+bx)+2B\log(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-2A\log(c+dx)+2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i
*x)),x]
```

output
```
(2*A*Log[a + b*x] - B*n*Log[a + b*x]^2 + 2*B*Log[a + b*x]*Log[e*((a + b*x)
/(c + d*x))^n] - 2*A*Log[c + d*x] + 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)
]*Log[c + d*x] - 2*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] - B*n*Log
[c + d*x]^2 + 2*B*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 2*B*n*Po
lyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 2*B*n*PolyLog[2, (b*(c + d*x))/(b
*c - a*d)])/(2*(b*c - a*d)*g*i)
```

### 3.139.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.047$, Rules used = {2961, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)(ci+dix)} dx$$
$$\Big\downarrow \text{2961}$$
$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}d\frac{a+bx}{c+dx}}{gi(bc-ad)}$$
$$\Big\downarrow \text{2738}$$
$$\frac{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2Bgin(bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i*x)),x
]
```

output
```
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*B*(b*c - a*d)*g*i*n)
```

### 3.139.3.1 Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.139.4 Maple [A] (verified)

Time = 1.46 (sec) , antiderivative size = 84, normalized size of antiderivative = 1.68

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{B\,a^{2}c^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}+2A\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{2}c^{2}}{2ig\,c^{2}a^{2}n(ad-cb)}$ | 84 |
| default | $\frac{A\left(\frac{\ln(dx+c)}{ad-cb}-\frac{\ln(bx+a)}{ad-cb}\right)}{gi}-\frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}{2gin(ad-cb)}$ | 88 |
| parts | $\frac{A\left(\frac{\ln(dx+c)}{ad-cb}-\frac{\ln(bx+a)}{ad-cb}\right)}{gi}-\frac{B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}}{2gin(ad-cb)}$ | 88 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i),x,method=_RETU
RNVERBOSE)
```

output
```
-1/2*(B*a^2*c^2*ln(e*((b*x+a)/(d*x+c))^n)^2+2*A*ln(e*((b*x+a)/(d*x+c))^n)*
a^2*c^2)/i/g/c^2/a^2/n/(a*d-b*c)
```

### 3.139.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 74, normalized size of antiderivative = 1.48

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx = \frac{Bn\log\left(\frac{bx+a}{dx+c}\right)^2+2\,B\log\left(e\right)\log\left(\frac{bx+a}{dx+c}\right)+2\,A\log\left(\frac{bx+a}{dx+c}\right)}{2\,(bc-ad)gi}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i),x, algo
rithm="fricas")
```

output
```
1/2*(B*n*log((b*x + a)/(d*x + c))^2 + 2*B*log(e)*log((b*x + a)/(d*x + c))
+ 2*A*log((b*x + a)/(d*x + c)))/((b*c - a*d)*g*i)
```

### 3.139.6 Sympy [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx = \frac{\int \frac{A}{ac+adx+bcx+bdx^2}\,dx+\int \frac{B\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{ac+adx+bcx+bdx^2}\,dx}{gi}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)/(d*i*x+c*i),x)
```

output
```
(Integral(A/(a*c + a*d*x + b*c*x + b*d*x**2), x) + Integral(B*log(e*(a/(c
+ d*x) + b*x/(c + d*x))**n)/(a*c + a*d*x + b*c*x + b*d*x**2), x))/(g*i)
```

### 3.139.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 175 vs. 2(48) = 96.

Time = 0.20 (sec) , antiderivative size = 175, normalized size of antiderivative = 3.50

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx &= B\left(\frac{\log\left(bx+a\right)}{(bc-ad)gi}-\frac{\log\left(dx+c\right)}{(bc-ad)gi}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) \\ &\quad-\frac{\left(\log\left(bx+a\right)^2-2\,\log\left(bx+a\right)\log\left(dx+c\right)+\log\left(dx+c\right)^2\right)Bn}{2\,(bcgi-adgi)} \\ &\quad+A\left(\frac{\log\left(bx+a\right)}{(bc-ad)gi}-\frac{\log\left(dx+c\right)}{(bc-ad)gi}\right)\end{aligned}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i),x, algo
rithm="maxima")
```

output
```
B*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))*log(e*
(b*x/(d*x + c) + a/(d*x + c))^n) - 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*lo
g(d*x + c) + log(d*x + c)^2)*B*n/(b*c*g*i - a*d*g*i) + A*(log(b*x + a)/((b
*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))
```

### 3.139.8 Giac [A] (verification not implemented)

Time = 0.43 (sec) , antiderivative size = 92, normalized size of antiderivative = 1.84

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx = \frac{\left(Bn\log\left(\frac{bx+a}{dx+c}\right)^2+2\,B\log\left(e\right)\log\left(\frac{bx+a}{dx+c}\right)+2\,A\log\left(\frac{bx+a}{dx+c}\right)\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)}{2\,gi}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i),x, algo
rithm="giac")
```

output
```
1/2*(B*n*log((b*x + a)/(d*x + c))^2 + 2*B*log(e)*log((b*x + a)/(d*x + c))
+ 2*A*log((b*x + a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(g
*i)
```

### 3.139.9 Mupad [B] (verification not implemented)

Time = 2.43 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.52

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)}\,dx = -\frac{B\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2-A\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{2\,g\,i\,n\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)*(c*i + d*i*x)),x
)
```

output
```
-(B*log(e*((a + b*x)/(c + d*x))^n)^2 - A*n*atan((b*c*2i + b*d*x*2i)/(a*d -
 b*c) + 1i)*4i)/(2*g*i*n*(a*d - b*c))
```

# 3.140 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx$



## 3.140.1 Optimal result

Integrand size = 43, antiderivative size = 181

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{bBn(c+dx)}{(bc-ad)^2g^2i(a+bx)} - \frac{b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^2i(a+bx)} - \frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^2g^2i} + \frac{Bdn\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^2g^2i}$$

output
```
-b*B*n*(d*x+c)/(-a*d+b*c)^2/g^2/i/(b*x+a)-b*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))/(-a*d+b*c)^2/g^2/i/(b*x+a)-d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(
(b*x+a)/(d*x+c))/(-a*d+b*c)^2/g^2/i+1/2*B*d*n*ln((b*x+a)/(d*x+c))^2/(-a*d+
b*c)^2/g^2/i
```

## 3.140.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.17 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.68

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx =$$

$$-\frac{2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2d(a+bx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2d(a+bx)\left(A+\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d
*i*x)),x]
```

output
```
-1/2*(2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d*(a + b*x)
*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 2*B*n*(b*c - a*d + d*(a +
 b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*n*(a + b*x)*(Log[a +
b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(
a + b*x))/(-(b*c) + a*d)]) + B*d*n*(a + b*x)*((2*Log[(d*(a + b*x))/(-(b*c)
 + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)]))/((b*c - a*d)^2*g^2*i*(a + b*x))
```

## 3.140.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.74, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.116$, Rules used = {2961, 2772, 25, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^2(ci+dix)}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i(bc-ad)^2}$$

$$\Big\downarrow \text{ 2772}$$

$$\frac{-Bn\int -\frac{(c+dx)^2\left(b+\frac{d(a+bx)\log\left(\frac{a+bx}{c+dx}\right)}{c+dx}\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{b(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i(bc-ad)^2}$$

$$\Big\downarrow \text{ 25}$$

$$\frac{Bn\int \frac{(c+dx)^2\left(b+\frac{d(a+bx)\log\left(\frac{a+bx}{c+dx}\right)}{c+dx}\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{b(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i(bc-ad)^2}$$

$$\Big\downarrow \text{ 2010}$$

$$\frac{Bn\int\left(\frac{b(c+dx)^2}{(a+bx)^2}+\frac{d\log\left(\frac{a+bx}{c+dx}\right)(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{g^2i(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{b(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+Bn\left(\frac{1}{2}d\log^2\left(\frac{a+bx}{c+dx}\right)-\frac{b(c+dx)}{a+bx}\right)}{g^2i(bc-ad)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d*i*x))
,x]
```

output
```
(-((b*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)) - d*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] + B*n*(-((b*(
c + d*x))/(a + b*x)) + (d*Log[(a + b*x)/(c + d*x)]^2)/2))/((b*c - a*d)^2*g
^2*i)
```

### 3.140.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

3.140. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.140.4 Maple [A] (verified)

Time = 4.79 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.49

| method | result |
|---|---|
| parallelrisch | $-\frac{-2Bab^3d^3n^2+2Bb^4cd^2n^2-2Aab^3d^3n+2Ab^4cd^2n+Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^4d^3+2Ax\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^4d^3+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right.}{2i\,g^2(bx+a)n(a^2d^2-2abcd+b^2c^2)b}$ |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i),x,method=_RE
TURNVERBOSE)
```

output
```
-1/2*(-2*B*a*b^3*d^3*n^2+2*B*b^4*c*d^2*n^2-2*A*a*b^3*d^3*n+2*A*b^4*c*d^2*n
+B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*d^3+2*A*x*ln(e*((b*x+a)/(d*x+c))^n)*b
^4*d^3+B*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^3*d^3+2*A*ln(e*((b*x+a)/(d*x+c))^
n)*a*b^3*d^3+2*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^3*n+2*B*ln(e*((b*x+a)/(
d*x+c))^n)*b^4*c*d^2*n)/i/g^2/(b*x+a)/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/d^
2
```

## 3.140.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 194, normalized size of antiderivative = 1.07

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-\frac{2\,Abc-2\,Aad+(Bbdnx+Badn)\log\left(\frac{bx+a}{dx+c}\right)^2+2\,(Bbc-Bad)n+2\,\left(Bbc-Bad+(Bbdx+Bad)\log\right.}{2\,((b^3c^2-2\,ab^2cd+a^2bd^2)g^2ix+(ab^2c^2-2\,a^2bcd}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i),x, al
gorithm="fricas")
```

output
```
-1/2*(2*A*b*c - 2*A*a*d + (B*b*d*n*x + B*a*d*n)*log((b*x + a)/(d*x + c))^2
 + 2*(B*b*c - B*a*d)*n + 2*(B*b*c - B*a*d + (B*b*d*x + B*a*d)*log((b*x + a
)/(d*x + c)))*log(e) + 2*(B*b*c*n + A*a*d + (B*b*d*n + A*b*d)*x)*log((b*x
+ a)/(d*x + c)))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^2*i*x + (a*b^2*c^2
 - 2*a^2*b*c*d + a^3*d^2)*g^2*i)
```

## 3.140.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2/(d*i*x+c*i),x)
```

output
```
Timed out
```

## 3.140.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 427 vs. $2(179) = 358$.

Time = 0.22 (sec) , antiderivative size = 427, normalized size of antiderivative = 2.36

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx = -B\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log\Big(e$$
$$+\frac{\left((bdx+ad)\log(bx+a)^2+(bdx+ad)\log(dx+c)^2-2\,bc+2\,ad-2\,(bdx+ad)\log(bx+a)+2\,(bdx\right.}{2\,(ab^2c^2g^2i-2\,a^2bcdg^2i+a^3d^2g^2i+(b^3c^2g^2i-2\,ab^2cdg^2i+a^2b}$$
$$-A\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i),x, al
gorithm="maxima")
```

output
```
-B*(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/(
(b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*g^2*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*((b*d*
x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2
*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))
*log(d*x + c))*B*n/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^2*g^2*i +
(b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x) - A*(1/((b^2*c -
a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g
^2*i))
```

### 3.140.8 Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx=\int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{(bgx+ag)^2(dix+ci)}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i),x, al
gorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/((b*g*x + a*g)^2*(d*i*x +
 c*i)), x)
```

### 3.140.9 Mupad [B] (verification not implemented)

Time = 2.80 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.32

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx=\frac{A}{g^2\,i\,(a\,d-b\,c)\,(a+b\,x)}+\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g^2\,i\,(a\,d-b\,c)\,(a+b\,x)}+\frac{B\,n}{g^2\,i\,(a\,d-b\,c)\,(a+b\,x)}-\frac{B\,d\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{2\,g^2\,i\,n\,(a\,d-b\,c)^2}+\frac{A\,d\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g^2\,i\,(a\,d-b\,c)^2}+\frac{B\,d\,n\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g^2\,i\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^2*(c*i + d*i*x))
,x)
```

output
```
A/(g^2*i*(a*d - b*c)*(a + b*x)) + (B*log(e*((a + b*x)/(c + d*x))^n))/(g^2*
i*(a*d - b*c)*(a + b*x)) + (A*d*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b
*c))*2i)/(g^2*i*(a*d - b*c)^2) + (B*n)/(g^2*i*(a*d - b*c)*(a + b*x)) + (B*
d*n*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g^2*i*(a*d - b*c)^
2) - (B*d*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g^2*i*n*(a*d - b*c)^2)
```

## 3.140. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)}\,dx$

# 3.141 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx$



## 3.141.1 Optimal result

Integrand size = 43, antiderivative size = 266

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx &= -\frac{Bn(c+dx)^2\left(b-\frac{4d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3g^3i(a+bx)^2} \\ &+ \frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^3i(a+bx)} \\ &- \frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2} \\ &+ \frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^3i} - \frac{Bd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3g^3i}\end{aligned}$$

output
```
-1/4*B*n*(d*x+c)^2*(b-4*d*(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+
2*b*d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^3/i/(b*x+a)-1
/2*b^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^3/i/(b*x+a
)^2+d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^3/g
^3/i-1/2*B*d^2*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^3/i
```

### 3.141.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.63

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \frac{-2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4d(bc-ad)(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4d^2(a+bx)^2\log(a}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d
*i*x)),x]
```

output
```
(-2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d*(b*c - a*d)
*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)^2*Log[
a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 4*B*d*n*(a + b*x)*(b*c - a
*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*n*((b*c - a*
d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)^2*Log[a + b*x] + 2*d
^2*(a + b*x)^2*Log[c + d*x]) - 2*B*d^2*n*(a + b*x)^2*(Log[a + b*x]*(Log[a
+ b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-
(b*c) + a*d)]) + 2*B*d^2*n*(a + b*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
)/(4*(b*c - a*d)^3*g^3*i*(a + b*x)^2)
```

### 3.141.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 204, normalized size of antiderivative = 0.77, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2961, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^3(ci+dix)}\,dx$$
$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i(bc-ad)^3}$$

$\Big\downarrow$ 2772

$$\frac{-Bn\int -\frac{(c+dx)^3\left(b^2-\frac{4d(a+bx)b}{c+dx}-\frac{2d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^2}\right)}{2(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\right.\right.\right.}{g^3i(bc-ad)^3}$$

$\Big\downarrow$ 27

$$\frac{\frac{1}{2}Bn\int \frac{(c+dx)^3\left(b^2-\frac{4d(a+bx)b}{c+dx}-\frac{2d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^2}\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{g^3i(bc-ad)^3}$$

$\Big\downarrow$ 2010

$$\frac{\frac{1}{2}Bn\int \left(\frac{b(c+dx)^3\left(b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)^3}-\frac{2d^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(\right.\right.}{g^3i(bc-ad)^3}$$

$\Big\downarrow$ 2009

$$\frac{-\frac{b^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{2bd(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}+\frac{1}{2}Bn\left(\right.}{g^3i(bc-ad)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d*i*x))
,x]
```

output
```
((2*b*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (b^2
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + d^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] + (B*n*(-
1/2*((c + d*x)^2*(b - (4*d*(a + b*x))/(c + d*x))^2)/(a + b*x)^2 - d^2*Log[
(a + b*x)/(c + d*x)]^2))/2)/((b*c - a*d)^3*g^3*i)
```

## 3.141. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx$

### 3.141.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.141.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(260) = 520$.

Time = 11.92 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.36

| method | result |
|---|---|
| parallelrisch | $-\frac{6B\,x^2 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^4b^2c^2d^2n+8Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^5b\,c^2d^2n+4Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^4b^2c^3dn-2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^4b^2c^4}{}$ |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x,method=_RE
TURNVERBOSE)
```

output
```
-1/4*(6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^2*d^2*n+8*B*x*ln(e*((b*x
+a)/(d*x+c))^n)*a^5*b*c^2*d^2*n+4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^
3*d*n-2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^4*n+B*x^2*a^2*b^4*c^4*n^2+2*
A*x^2*a^2*b^4*c^4*n+2*B*x*a^3*b^3*c^4*n^2+4*A*x*a^3*b^3*c^4*n+2*B*ln(e*((b
*x+a)/(d*x+c))^n)^2*a^6*c^2*d^2+4*A*ln(e*((b*x+a)/(d*x+c))^n)*a^6*c^2*d^2+
2*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^2*c^2*d^2+7*B*x^2*a^4*b^2*c^2*d^
2*n^2-8*B*x^2*a^3*b^3*c^3*d*n^2+4*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*
c^2*d^2+6*A*x^2*a^4*b^2*c^2*d^2*n-8*A*x^2*a^3*b^3*c^3*d*n+4*B*x*ln(e*((b*x
+a)/(d*x+c))^n)^2*a^5*b*c^2*d^2+8*B*x*a^5*b*c^2*d^2*n^2-10*B*x*a^4*b^2*c^3
*d*n^2+8*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b*c^2*d^2+8*A*x*a^5*b*c^2*d^2*n
-12*A*x*a^4*b^2*c^3*d*n+8*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b*c^3*d*n)/i/g^3
/(b*x+a)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/a^4/c^2/n
```

### 3.141.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 483, normalized size of antiderivative = 1.82

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx =$$
$$-\frac{2\,Ab^2c^2-8\,Aabcd+6\,Aa^2d^2-2\left(Bb^2d^2nx^2+2\,Babd^2nx+Ba^2d^2n\right)\log\left(\frac{bx+a}{dx+c}\right)^2+\left(Bb^2c^2-8\,Babcd+\right.}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x, al
gorithm="fricas")
```

output
```
-1/4*(2*A*b^2*c^2 - 8*A*a*b*c*d + 6*A*a^2*d^2 - 2*(B*b^2*d^2*n*x^2 + 2*B*a
*b*d^2*n*x + B*a^2*d^2*n)*log((b*x + a)/(d*x + c))^2 + (B*b^2*c^2 - 8*B*a*
b*c*d + 7*B*a^2*d^2)*n - 2*(2*A*b^2*c*d - 2*A*a*b*d^2 + 3*(B*b^2*c*d - B*a
*b*d^2)*n)*x + 2*(B*b^2*c^2 - 4*B*a*b*c*d + 3*B*a^2*d^2 - 2*(B*b^2*c*d - B
*a*b*d^2)*x - 2*(B*b^2*d^2*x^2 + 2*B*a*b*d^2*x + B*a^2*d^2)*log((b*x + a)/
(d*x + c)))*log(e) - 2*(2*A*a^2*d^2 + (3*B*b^2*d^2*n + 2*A*b^2*d^2)*x^2 -
(B*b^2*c^2 - 4*B*a*b*c*d)*n + 2*(2*A*a*b*d^2 + (B*b^2*c*d + 2*B*a*b*d^2)*n
)*x)*log((b*x + a)/(d*x + c)))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2
 - a^3*b^2*d^3)*g^3*i*x^2 + 2*(a*b^4*c^3 - 3*a^2*b^3*c^2*d + 3*a^3*b^2*c*d
^2 - a^4*b*d^3)*g^3*i*x + (a^2*b^3*c^3 - 3*a^3*b^2*c^2*d + 3*a^4*b*c*d^2 -
 a^5*d^3)*g^3*i)
```

### 3.141.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3/(d*i*x+c*i),x)
```

output
```
Timed out
```

### 3.141.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 888 vs. $2(260) = 520$.

Time = 0.25 (sec) , antiderivative size = 888, normalized size of antiderivative = 3.34

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx$$
$$=\frac{1}{2}B\left(\frac{2\,bdx-bc+3\,ad}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)g^3ix^2+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)g^3ix+(a^2b^2c^2-2\,a^3bcd+a^4d^2)g^3i}+\right.$$
$$-\frac{\left(b^2c^2-8\,abcd+7\,a^2d^2+2\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\log(bx+a)^2+2\,(b^2d^2x^2+2\,abd^2x+a^2d^2)\log(d\right.}{4\,(a^2b^3c^3g^3i-3\,a^3b^2c^2dg^3i+3\,a^4bcd^2g^3i-a^5d^3g^3i+}$$
$$+\frac{1}{2}A\left(\frac{2\,bdx-bc+3\,ad}{(b^4c^2-2\,ab^3cd+a^2b^2d^2)g^3ix^2+2\,(ab^3c^2-2\,a^2b^2cd+a^3bd^2)g^3ix+(a^2b^2c^2-2\,a^3bcd+a^4d^2)g^3i}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x, al
gorithm="maxima")
```

output
```
1/2*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3*
i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2 -
 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) - 1/4*(b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b
*d^2*x + a^2*d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)
*log(d*x + c)^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x +
 a^2*d^2)*log(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b
^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))*B*n/(a^2*b
^3*c^3*g^3*i - 3*a^3*b^2*c^2*d*g^3*i + 3*a^4*b*c*d^2*g^3*i - a^5*d^3*g^3*i
 + (b^5*c^3*g^3*i - 3*a*b^4*c^2*d*g^3*i + 3*a^2*b^3*c*d^2*g^3*i - a^3*b^2*
d^3*g^3*i)*x^2 + 2*(a*b^4*c^3*g^3*i - 3*a^2*b^3*c^2*d*g^3*i + 3*a^3*b^2*c*
d^2*g^3*i - a^4*b*d^3*g^3*i)*x) + 1/2*A*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2
 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a
^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*l
og(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) -
2*d^2*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^
3*i))
```

## 3.141.8 Giac [A] (verification not implemented)

Time = 104.07 (sec) , antiderivative size = 104, normalized size of antiderivative = 0.39

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx = -\frac{1}{4}\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)^2\left(\frac{2\,(dx+c)^2Bn\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^2g^3i}+\frac{(Bn+2\,B\log\left(e\right)+2\,A)(dx+c)^2}{(bx+a)^2g^3i}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i),x, al
gorithm="giac")
```

output
```
-1/4*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2*(2*(d*x + c)^2*B*n*log((b*x
 + a)/(d*x + c))/((b*x + a)^2*g^3*i) + (B*n + 2*B*log(e) + 2*A)*(d*x + c)^
2/((b*x + a)^2*g^3*i))
```

### 3.141.9 Mupad [B] (verification not implemented)

Time = 3.08 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.15

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \frac{B\,d^2\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{g^3\,i\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{2\,d^2}+\frac{a\,g^3\,i\,n\,(a\,d-b\,c)}{2\,d}+\frac{b\,g^3\,i\,n\,x\,(a\,d-b\,c)}{d}\right)}{g^3\,i\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)\,(i\,a^2\,g^3+2\,i\,a\,b\,g^3\,x+i\,b^2\,g^3\,x^2)}$$
$$-\frac{B\,d^2\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{2\,g^3\,i\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}$$
$$-\frac{\frac{6\,A\,a\,d-2\,A\,b\,c+7\,B\,a\,d\,n-B\,b\,c\,n}{2\,(a\,d-b\,c)}+\frac{d\,x\,(2\,A\,b+3\,B\,b\,n)}{a\,d-b\,c}}{x^2\,(2\,b^3\,c\,g^3\,i-2\,a\,b^2\,d\,g^3\,i)+x\,(4\,a\,b^2\,c\,g^3\,i-4\,a^2\,b\,d\,g^3\,i)-2\,a^3\,d\,g^3\,i+2\,a^2\,b\,c\,g^3\,i}$$
$$+\frac{d^2\,\mathrm{atan}\left(\frac{d^2\left(A+\frac{3\,B\,n}{2}\right)(2\,i\,a^3\,d^3\,g^3-2\,i\,a^2\,b\,c\,d^2\,g^3-2\,i\,a\,b^2\,c^2\,d\,g^3+2\,i\,b^3\,c^3\,g^3)\,1\mathrm{i}}{g^3\,i\,(2\,A\,d^2+3\,B\,d^2\,n)\,(a\,d-b\,c)^3}+\frac{b\,d^3\,x\left(A+\frac{3\,B\,n}{2}\right)(i\,a^2\,d^2\,g^3-2\,i\,a\,b\,c\,d\,g^3+i\,b^2\,c^2\,g^3}{g^3\,i\,(2\,A\,d^2+3\,B\,d^2\,n)\,(a\,d-b\,c)^3}\right.}{g^3\,i\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^3*(c*i + d*i*x))
,x)
```

output
```
(d^2*atan((d^2*(A + (3*B*n)/2)*(2*a^3*d^3*g^3*i + 2*b^3*c^3*g^3*i - 2*a*b^
2*c^2*d*g^3*i - 2*a^2*b*c*d^2*g^3*i)*1i)/(g^3*i*(2*A*d^2 + 3*B*d^2*n)*(a*d
 - b*c)^3) + (b*d^3*x*(A + (3*B*n)/2)*(a^2*d^2*g^3*i + b^2*c^2*g^3*i - 2*a
*b*c*d*g^3*i)*4i)/(g^3*i*(2*A*d^2 + 3*B*d^2*n)*(a*d - b*c)^3))*(A + (3*B*n
)/2)*2i)/(g^3*i*(a*d - b*c)^3) - ((6*A*a*d - 2*A*b*c + 7*B*a*d*n - B*b*c*n
)/(2*(a*d - b*c)) + (d*x*(2*A*b + 3*B*b*n))/(a*d - b*c))/(x^2*(2*b^3*c*g^3
*i - 2*a*b^2*d*g^3*i) + x*(4*a*b^2*c*g^3*i - 4*a^2*b*d*g^3*i) - 2*a^3*d*g^
3*i + 2*a^2*b*c*g^3*i) - (B*d^2*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g^3*i
*n*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*d^2*log(e*((a + b*x)/
(c + d*x))^n)*((g^3*i*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2) + (a*g^3*i*n*(a
*d - b*c))/(2*d) + (b*g^3*i*n*x*(a*d - b*c))/d))/(g^3*i*n*(a*d - b*c)*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2*g^3*i + b^2*g^3*i*x^2 + 2*a*b*g^3*i*x))
```

# 3.142 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx$



## 3.142.1 Optimal result

Integrand size = 43, antiderivative size = 389

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx = &-\frac{3bBd^2n(c+dx)}{(bc-ad)^4g^4i(a+bx)}+\frac{3b^2Bdn(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2}\\ &-\frac{b^3Bn(c+dx)^3}{9(bc-ad)^4g^4i(a+bx)^3}\\ &-\frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^4i(a+bx)}\\ &+\frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2}\\ &-\frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^4g^4i(a+bx)^3}\\ &-\frac{d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^4i}+\frac{Bd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^4i}\end{aligned}$$

output
```
-3*b*B*d^2*n*(d*x+c)/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/4*b^2*B*d*n*(d*x+c)^2/(-
a*d+b*c)^4/g^4/i/(b*x+a)^2-1/9*b^3*B*n*(d*x+c)^3/(-a*d+b*c)^4/g^4/i/(b*x+a
)^3-3*b*d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^4/i/(b*
x+a)+3/2*b^2*d*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^4/
i/(b*x+a)^2-1/3*b^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4
/g^4/i/(b*x+a)^3-d^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(
-a*d+b*c)^4/g^4/i+1/2*B*d^3*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^4/i
```

### 3.142.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 518, normalized size of antiderivative = 1.33

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx$$
$$= \frac{-\frac{12A(bc-ad)^3}{(a+bx)^3}-\frac{4B(bc-ad)^3n}{(a+bx)^3}+\frac{18Ad(bc-ad)^2}{(a+bx)^2}+\frac{15Bd(bc-ad)^2n}{(a+bx)^2}+\frac{36Ad^2(-bc+ad)}{a+bx}+\frac{66Bd^2(-bc+ad)n}{a+bx}-36Ad^3\log(a+b}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d
*i*x)),x]
```

output
```
((-12*A*(b*c - a*d)^3)/(a + b*x)^3 - (4*B*(b*c - a*d)^3*n)/(a + b*x)^3 + (
18*A*d*(b*c - a*d)^2)/(a + b*x)^2 + (15*B*d*(b*c - a*d)^2*n)/(a + b*x)^2 +
 (36*A*d^2*(-(b*c) + a*d))/(a + b*x) + (66*B*d^2*(-(b*c) + a*d)*n)/(a + b*
x) - 36*A*d^3*Log[a + b*x] - 66*B*d^3*n*Log[a + b*x] + 18*B*d^3*n*Log[a +
b*x]^2 - (12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^3 +
 (18*B*d*(b*c - a*d)^2*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x)^2 + (36*B
*d^2*(-(b*c) + a*d)*Log[e*((a + b*x)/(c + d*x))^n])/(a + b*x) - 36*B*d^3*L
og[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n] + 36*A*d^3*Log[c + d*x] + 66*B*
d^3*n*Log[c + d*x] - 36*B*d^3*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c +
d*x] + 36*B*d^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] + 18*B*d^3*n*L
og[c + d*x]^2 - 36*B*d^3*n*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] - 3
6*B*d^3*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] - 36*B*d^3*n*PolyLog[2,
 (b*(c + d*x))/(b*c - a*d)])/(36*(b*c - a*d)^4*g^4*i)
```

### 3.142.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 275, normalized size of antiderivative = 0.71, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2961, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^4(ci+dix)}\,dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i(bc-ad)^4}$$

$$\Big\downarrow \text{2772}$$

$$\frac{-Bn\int -\frac{(c+dx)^4\left(2b^3-\frac{9d(a+bx)b^2}{c+dx}+\frac{18d^2(a+bx)^2b}{(c+dx)^2}+\frac{6d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^3}\right)}{6(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(\right.}{}}{g^4i(bc-ad)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{6}Bn\int \frac{(c+dx)^4\left(2b^3-\frac{9d(a+bx)b^2}{c+dx}+\frac{18d^2(a+bx)^2b}{(c+dx)^2}+\frac{6d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)}{(c+dx)^3}\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\right.}{2(}}{g^4i(bc-ad)^4}$$

$$\Big\downarrow \text{2010}$$

$$\frac{\frac{1}{6}Bn\int \left(\frac{b\left(2b^2-\frac{9d(a+bx)b}{c+dx}+\frac{18d^2(a+bx)^2}{(c+dx)^2}\right)(c+dx)^4}{(a+bx)^4}+\frac{6d^3\log\left(\frac{a+bx}{c+dx}\right)(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+d}{}}{g^4i(bc-ad)^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{3b^2d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\left(d^3\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)-\frac{3b}{}}{g^4i(bc-ad)^4}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d*i*x))
,x]
```

output
```
((-3*b*d^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (
3*b^2*d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2
) - (b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^
3) - d^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] +
 (B*n*((-18*b*d^2*(c + d*x))/(a + b*x) + (9*b^2*d*(c + d*x)^2)/(2*(a + b*x
)^2) - (2*b^3*(c + d*x)^3)/(3*(a + b*x)^3) + 3*d^3*Log[(a + b*x)/(c + d*x)
]^2))/6)/((b*c - a*d)^4*g^4*i)
```

3.142. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx$

### 3.142.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.142.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1071 vs. $2(379) = 758$.

Time = 25.47 (sec) , antiderivative size = 1072, normalized size of antiderivative = 2.76

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1072 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x,method=_RE
TURNVERBOSE)
```

output
```
-1/36*(18*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^8*c^2*d^3+36*A*ln(e*((b*x+a)/(d*
x+c))^n)*a^8*c^2*d^3-4*B*x^3*a^2*b^6*c^5*n^2-12*A*x^3*a^2*b^6*c^5*n-12*B*x
^2*a^3*b^5*c^5*n^2-36*A*x^2*a^3*b^5*c^5*n-12*B*x*a^4*b^4*c^5*n^2-36*A*x*a^
4*b^4*c^5*n+12*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^5*n+18*B*x^3*ln(e*((b
*x+a)/(d*x+c))^n)^2*a^5*b^3*c^2*d^3+85*B*x^3*a^5*b^3*c^2*d^3*n^2-108*B*x^3
*a^4*b^4*c^3*d^2*n^2+27*B*x^3*a^3*b^5*c^4*d*n^2+36*A*x^3*ln(e*((b*x+a)/(d*
x+c))^n)*a^5*b^3*c^2*d^3+66*A*x^3*a^5*b^3*c^2*d^3*n-108*A*x^3*a^4*b^4*c^3*
d^2*n+54*A*x^3*a^3*b^5*c^4*d*n+54*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^
2*c^2*d^3+189*B*x^2*a^6*b^2*c^2*d^3*n^2-258*B*x^2*a^5*b^3*c^3*d^2*n^2+81*B
*x^2*a^4*b^4*c^4*d*n^2+108*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^2*d^3
+162*A*x^2*a^6*b^2*c^2*d^3*n-288*A*x^2*a^5*b^3*c^3*d^2*n+162*A*x^2*a^4*b^4
*c^4*d*n+54*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*b*c^2*d^3+108*B*x*a^7*b*c^
2*d^3*n^2-162*B*x*a^6*b^2*c^3*d^2*n^2+66*B*x*a^5*b^3*c^4*d*n^2+108*A*x*ln(
e*((b*x+a)/(d*x+c))^n)*a^7*b*c^2*d^3+108*A*x*a^7*b*c^2*d^3*n-216*A*x*a^6*b
^2*c^3*d^2*n+144*A*x*a^5*b^3*c^4*d*n+108*B*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b
*c^3*d^2*n-54*B*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^4*d*n+66*B*x^3*ln(e*((
b*x+a)/(d*x+c))^n)*a^5*b^3*c^2*d^3*n+162*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a
^6*b^2*c^2*d^3*n+36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^3*d^2*n+108*
B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^2*d^3*n+108*B*x*ln(e*((b*x+a)/(d*x+c
))^n)*a^6*b^2*c^3*d^2*n-18*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^4*d*...
```

### 3.142.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 859 vs. $2(379) = 758$.

Time = 0.39 (sec) , antiderivative size = 859, normalized size of antiderivative = 2.21

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx =$$
$$-\frac{12\,Ab^3c^3 - 54\,Aab^2c^2d + 108\,Aa^2bcd^2 - 66\,Aa^3d^3 + 6\left(6\,Ab^3cd^2 - 6\,Aab^2d^3 + 11\,(Bb^3cd^2 - Bab^2d^3)n\right)x}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x, al
gorithm="fricas")
```

output
```
-1/36*(12*A*b^3*c^3 - 54*A*a*b^2*c^2*d + 108*A*a^2*b*c*d^2 - 66*A*a^3*d^3
+ 6*(6*A*b^3*c*d^2 - 6*A*a*b^2*d^3 + 11*(B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2
 + 18*(B*b^3*d^3*n*x^3 + 3*B*a*b^2*d^3*n*x^2 + 3*B*a^2*b*d^3*n*x + B*a^3*d
^3*n)*log((b*x + a)/(d*x + c))^2 + (4*B*b^3*c^3 - 27*B*a*b^2*c^2*d + 108*B
*a^2*b*c*d^2 - 85*B*a^3*d^3)*n - 3*(6*A*b^3*c^2*d - 36*A*a*b^2*c*d^2 + 30*
A*a^2*b*d^3 + (5*B*b^3*c^2*d - 54*B*a*b^2*c*d^2 + 49*B*a^2*b*d^3)*n)*x + 6
*(2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2 - 11*B*a^3*d^3 + 6*(B*b
^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*d - 6*B*a*b^2*c*d^2 + 5*B*a^2*b
*d^3)*x + 6*(B*b^3*d^3*x^3 + 3*B*a*b^2*d^3*x^2 + 3*B*a^2*b*d^3*x + B*a^3*d
^3)*log((b*x + a)/(d*x + c)))*log(e) + 6*(6*A*a^3*d^3 + (11*B*b^3*d^3*n +
6*A*b^3*d^3)*x^3 + 3*(6*A*a*b^2*d^3 + (2*B*b^3*c*d^2 + 9*B*a*b^2*d^3)*n)*x
^2 + (2*B*b^3*c^3 - 9*B*a*b^2*c^2*d + 18*B*a^2*b*c*d^2)*n + 3*(6*A*a^2*b*d
^3 - (B*b^3*c^2*d - 6*B*a*b^2*c*d^2 - 6*B*a^2*b*d^3)*n)*x)*log((b*x + a)/(
d*x + c)))/((b^7*c^4 - 4*a*b^6*c^3*d + 6*a^2*b^5*c^2*d^2 - 4*a^3*b^4*c*d^3
 + a^4*b^3*d^4)*g^4*i*x^3 + 3*(a*b^6*c^4 - 4*a^2*b^5*c^3*d + 6*a^3*b^4*c^2
*d^2 - 4*a^4*b^3*c*d^3 + a^5*b^2*d^4)*g^4*i*x^2 + 3*(a^2*b^5*c^4 - 4*a^3*b
^4*c^3*d + 6*a^4*b^3*c^2*d^2 - 4*a^5*b^2*c*d^3 + a^6*b*d^4)*g^4*i*x + (a^3
*b^4*c^4 - 4*a^4*b^3*c^3*d + 6*a^5*b^2*c^2*d^2 - 4*a^6*b*c*d^3 + a^7*d^4)*
g^4*i)
```

## 3.142.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4/(d*i*x+c*i),x)
```

output
```
Timed out
```

## 3.142.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1472 vs. 2(379) = 758.

Time = 0.29 (sec) , antiderivative size = 1472, normalized size of antiderivative = 3.78

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x, al
gorithm="maxima")
```

output
```
-1/6*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d -
 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*
g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3
)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d
^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*g^
4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 -
4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c
^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))*log(e*(b*x/(d*
x + c) + a/(d*x + c))^n) - 1/36*(4*b^3*c^3 - 27*a*b^2*c^2*d + 108*a^2*b*c*
d^2 - 85*a^3*d^3 + 66*(b^3*c*d^2 - a*b^2*d^3)*x^2 - 18*(b^3*d^3*x^3 + 3*a*
b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^2 - 18*(b^3*d^3*x^3 +
3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^2 - 3*(5*b^3*c^2*d
 - 54*a*b^2*c*d^2 + 49*a^2*b*d^3)*x + 66*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 +
3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a) - 6*(11*b^3*d^3*x^3 + 33*a*b^2*d^3*x
^2 + 33*a^2*b*d^3*x + 11*a^3*d^3 - 6*(b^3*d^3*x^3 + 3*a*b^2*d^3*x^2 + 3*a^
2*b*d^3*x + a^3*d^3)*log(b*x + a))*log(d*x + c))*B*n/(a^3*b^4*c^4*g^4*i -
4*a^4*b^3*c^3*d*g^4*i + 6*a^5*b^2*c^2*d^2*g^4*i - 4*a^6*b*c*d^3*g^4*i + a^
7*d^4*g^4*i + (b^7*c^4*g^4*i - 4*a*b^6*c^3*d*g^4*i + 6*a^2*b^5*c^2*d^2*g^4
*i - 4*a^3*b^4*c*d^3*g^4*i + a^4*b^3*d^4*g^4*i)*x^3 + 3*(a*b^6*c^4*g^4*i -
 4*a^2*b^5*c^3*d*g^4*i + 6*a^3*b^4*c^2*d^2*g^4*i - 4*a^4*b^3*c*d^3*g^4*...
```

### 3.142.8 Giac [A] (verification not implemented)

Time = 139.85 (sec) , antiderivative size = 232, normalized size of antiderivative = 0.60

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx =$$
$$-\frac{1}{36}\left(\frac{6\left(2\,Bbn-\frac{3\,(bx+a)Bdn}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3bcg^4i}{(dx+c)^3}-\frac{(bx+a)^3adg^4i}{(dx+c)^3}}+\frac{4\,Bbn-\frac{9\,(bx+a)Bdn}{dx+c}+12\,Bb\log(e)-\frac{18\,(bx+a)Bd\log(e)}{dx+c}+12\,Ab}{\frac{(bx+a)^3bcg^4i}{(dx+c)^3}-\frac{(bx+a)^3adg^4i}{(dx+c)^3}}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i),x, al
gorithm="giac")
```

output
```
-1/36*(6*(2*B*b*n - 3*(b*x + a)*B*d*n/(d*x + c))*log((b*x + a)/(d*x + c))/
((b*x + a)^3*b*c*g^4*i/(d*x + c)^3 - (b*x + a)^3*a*d*g^4*i/(d*x + c)^3) +
(4*B*b*n - 9*(b*x + a)*B*d*n/(d*x + c) + 12*B*b*log(e) - 18*(b*x + a)*B*d*
log(e)/(d*x + c) + 12*A*b - 18*(b*x + a)*A*d/(d*x + c))/((b*x + a)^3*b*c*g
^4*i/(d*x + c)^3 - (b*x + a)^3*a*d*g^4*i/(d*x + c)^3))*(b*c/(b*c - a*d)^2
- a*d/(b*c - a*d)^2)^2
```

## 3.142.9 Mupad [B] (verification not implemented)

Time = 3.89 (sec) , antiderivative size = 986, normalized size of antiderivative = 2.53

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx$$
$$= \frac{\frac{66\,A\,a^2\,d^2+12\,A\,b^2\,c^2+85\,B\,a^2\,d^2\,n+4\,B\,b^2\,c^2\,n-42\,A\,a\,b\,c\,d-23\,B\,a\,b\,c\,d\,n}{6\,(a\,d-b\,c)}+\frac{x\,(30\,A\,a\,b}{}}{x\,\left(18\,i\,a^4\,b\,d^2\,g^4-36\,i\,a^3\,b^2\,c\,d\,g^4+18\,i\,a^2\,b^3\,c^2\,g^4\right)+x^2\,\left(18\,i\,a^3\,b^2\,d^2\,g^4-36\,i\,a^2\,b^3\,c\,d\,g^4+18\,i\,a\,b^4\,c^2\,g^4\right)}$$
$$-\frac{B\,d^3\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{2\,g^4\,i\,n\,(a\,d-b\,c)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}$$
$$+\frac{B\,d^3\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(x\,\left(b\,\left(\frac{g^4\,i\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{6\,d^2}+\frac{a\,g^4\,i\,n\,(a\,d-b\,c)}{3\,d}\right)+\frac{2\,a\,b\,g^4\,i\,n\,(a\,d-b\,c)}{3\,d}+\frac{b\,g^4\,i\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c}{3\,d^2}\right.\right.}{g^4\,i\,n\,(a\,d-b\,c)\,\left(a^3\,d^3-3\,a^2\,b\,c\,d^2+3\,a\,b^2\,c^2\,d-b^3\,c^3\right)}$$
$$+\frac{d^3\,\mathrm{atan}\left(\frac{d^3\,\left(\frac{i\,a^4\,d^4\,g^4-2\,i\,a^3\,b\,c\,d^3\,g^4+2\,i\,a\,b^3\,c^3\,d\,g^4-i\,b^4\,c^4\,g^4}{i\,a^3\,d^3\,g^4-3\,i\,a^2\,b\,c\,d^2\,g^4+3\,i\,a\,b^2\,c^2\,d\,g^4-i\,b^3\,c^3\,g^4}+2\,b\,d\,x\right)\,\left(A+\frac{11\,B\,n}{6}\right)\,\left(i\,a^3\,d^3\,g^4-3\,i\,a^2\,b\,c\,d^2\,g^4+3\,i\,a\,b^2\,c^2\,d\,g^4-i\,b^3\,c^3\,g^4\right)\,6i}{g^4\,i\,(6\,A\,d^3+11\,B\,d^3\,n)\,(a\,d-b\,c)^4}\right.}{g^4\,i\,(a\,d-b\,c)^4}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^4*(c*i + d*i*x))
,x)
```

output
```
((66*A*a^2*d^2 + 12*A*b^2*c^2 + 85*B*a^2*d^2*n + 4*B*b^2*c^2*n - 42*A*a*b*
c*d - 23*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x*(30*A*a*b*d^2 - 6*A*b^2*c*d + 4
9*B*a*b*d^2*n - 5*B*b^2*c*d*n))/(2*(a*d - b*c)) + (d*x^2*(6*A*b^2*d + 11*B
*b^2*d*n))/(a*d - b*c))/(x*(18*a^4*b*d^2*g^4*i + 18*a^2*b^3*c^2*g^4*i - 36
*a^3*b^2*c*d*g^4*i) + x^2*(18*a*b^4*c^2*g^4*i + 18*a^3*b^2*d^2*g^4*i - 36*
a^2*b^3*c*d*g^4*i) + x^3*(6*b^5*c^2*g^4*i + 6*a^2*b^3*d^2*g^4*i - 12*a*b^4
*c*d*g^4*i) + 6*a^5*d^2*g^4*i + 6*a^3*b^2*c^2*g^4*i - 12*a^4*b*c*d*g^4*i)
+ (d^3*atan((d^3*((a^4*d^4*g^4*i - b^4*c^4*g^4*i + 2*a*b^3*c^3*d*g^4*i - 2
*a^3*b*c*d^3*g^4*i)/(a^3*d^3*g^4*i - b^3*c^3*g^4*i + 3*a*b^2*c^2*d*g^4*i -
 3*a^2*b*c*d^2*g^4*i) + 2*b*d*x)*(A + (11*B*n)/6)*(a^3*d^3*g^4*i - b^3*c^3
*g^4*i + 3*a*b^2*c^2*d*g^4*i - 3*a^2*b*c*d^2*g^4*i)*6i)/(g^4*i*(6*A*d^3 +
11*B*d^3*n)*(a*d - b*c)^4))*(A + (11*B*n)/6)*2i)/(g^4*i*(a*d - b*c)^4) - (
B*d^3*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g^4*i*n*(a*d - b*c)*(a^3*d^3 -
b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)) + (B*d^3*log(e*((a + b*x)/(c + d
*x))^n)*(x*(b*((g^4*i*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (a*g^4*i*n*(a
*d - b*c))/(3*d)) + (2*a*b*g^4*i*n*(a*d - b*c))/(3*d) + (b*g^4*i*n*(a*d -
b*c)*(3*a*d - b*c))/(3*d^2)) + a*((g^4*i*n*(a*d - b*c)*(3*a*d - b*c))/(6*d
^2) + (a*g^4*i*n*(a*d - b*c))/(3*d)) + (g^4*i*n*(a*d - b*c)*(3*a^2*d^2 + b
^2*c^2 - 3*a*b*c*d))/(3*d^3) + (b^2*g^4*i*n*x^2*(a*d - b*c))/d))/(g^4*i*n*
(a*d - b*c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2)*(a^3*g^...
```

3.142. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)}\,dx$

# 3.143 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$



## 3.143.1 Optimal result

Integrand size = 43, antiderivative size = 359

$$\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx \\
&= \frac{3B(bc-ad)^2g^3n(a+bx)}{d^3i^2(c+dx)} - \frac{(bc-ad)^2g^3(6A+5Bn)(a+bx)}{2d^3i^2(c+dx)} \\
&\quad - \frac{3B(bc-ad)^2g^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^2(c+dx)} + \frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^2(c+dx)} \\
&\quad - \frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^2(c+dx)} \\
&\quad - \frac{b(bc-ad)^2g^3\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{2d^4i^2} \\
&\quad - \frac{3bB(bc-ad)^2g^3n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}
\end{aligned}$$

output
```
3*B*(-a*d+b*c)^2*g^3*n*(b*x+a)/d^3/i^2/(d*x+c)-1/2*(-a*d+b*c)^2*g^3*(5*B*n
+6*A)*(b*x+a)/d^3/i^2/(d*x+c)-3*B*(-a*d+b*c)^2*g^3*(b*x+a)*ln(e*((b*x+a)/(
d*x+c))^n)/d^3/i^2/(d*x+c)+1/2*g^3*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))/d/i^2/(d*x+c)-1/2*(-a*d+b*c)*g^3*(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(
d*x+c))^n))/d^2/i^2/(d*x+c)-1/2*b*(-a*d+b*c)^2*g^3*(6*A+5*B*n+6*B*ln(e*((b
*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^4/i^2-3*b*B*(-a*d+b*c)^2*g^3
*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^2
```

### 3.143.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 375, normalized size of antiderivative = 1.04

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$$
$$= \frac{g^3\left(-2Ab^2d(2bc-3ad)x-2bBd(2bc-3ad)(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+b^3d^2x^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{2}{\cdot}\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i +
d*i*x)^2,x]
```

output
```
(g^3*(-2*A*b^2*d*(2*b*c - 3*a*d)*x - 2*b*B*d*(2*b*c - 3*a*d)*(a + b*x)*Log
[e*((a + b*x)/(c + d*x))^n] + b^3*d^2*x^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]) + (2*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x
) + 2*b*B*(2*b*c - 3*a*d)*(b*c - a*d)*n*Log[c + d*x] + 6*b*(b*c - a*d)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*B*(b*c - a*d)^2*n*(
(b*c - a*d)/(c + d*x) + b*Log[a + b*x] - b*Log[c + d*x]) + b*B*n*(-(a^2*d^
2*Log[a + b*x]) + b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 3*b*B*(b*
c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(2*d^4*i^2)
```

### 3.143.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 322, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2961, 2784, 2784, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ci+dix)^2}\,dx$$
$$\Big\downarrow \; 2961$$
$$\frac{g^3(bc-ad)^2 \int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{2784}$$

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{i^2}$$

$$\Big\downarrow \text{2784}$$

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{d}}{2d}\right)}{i^2}$$

$$\Big\downarrow \text{2793}$$

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\left(-\frac{6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d}-\frac{b(6A+5Bn+6B}{d\left(\frac{d(a-}{c+}\right.}\right.}{d}}{2d}\right.}{i^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^3(bc-ad)^2\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+6A+5Bn\right)}{d^2}}{}}{2d}\right.}{i^2}$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)
^2,x]
```

3.143. $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$

output
```
((b*c - a*d)^2*g^3*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(
2*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)^2*(3*A + B*
n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)^2*(b - (d*(a + b*x))
/(c + d*x))) - ((6*B*n*(a + b*x))/(d*(c + d*x)) - ((6*A + 5*B*n)*(a + b*x)
)/(d*(c + d*x)) - (6*B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d*(c + d
*x)) - (b*(6*A + 5*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a
 + b*x))/(b*(c + d*x))])/d^2 - (6*b*B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d
*x))])/d^2)/d)/(2*d)))/i^2
```

### 3.143.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.143. $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$

### 3.143.4 Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(dix+ci)^2}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

### 3.143.5 Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output
```
integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
+ (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log(e*
((b*x + a)/(d*x + c))^n))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

### 3.143.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**2,x
)
```

output
```
Timed out
```

### 3.143.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1892 vs. $2(350) = 700$.

Time = 0.49 (sec) , antiderivative size = 1892, normalized size of antiderivative = 5.27

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output
```
B*a^3*g^3*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2
) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) + 1/2*(2*c^3/(d^5*i^2*x + c*d^4*
i^2) + 6*c^2*log(d*x + c)/(d^4*i^2) + (d*x^2 - 4*c*x)/(d^3*i^2))*A*b^3*g^3
 - 3*A*a*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)
/(d^3*i^2))*g^3 + 3*A*a^2*b*g^3*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/
(d^2*i^2)) - B*a^3*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^2*i^2*x +
 c*d*i^2) - A*a^3*g^3/(d^2*i^2*x + c*d*i^2) - 1/2*(6*a^3*b*d^3*g^3*log(e)
- (7*g^3*n + 6*g^3*log(e))*b^4*c^3 + (17*g^3*n + 18*g^3*log(e))*a*b^3*c^2*
d - 6*(2*g^3*n + 3*g^3*log(e))*a^2*b^2*c*d^2)*B*log(d*x + c)/(b*c*d^4*i^2
- a*d^5*i^2) + 1/2*((b^4*c*d^3*g^3*log(e) - a*b^3*d^4*g^3*log(e))*B*x^3 -
((g^3*n + 3*g^3*log(e))*b^4*c^2*d^2 - (2*g^3*n + 9*g^3*log(e))*a*b^3*c*d^3
 + (g^3*n + 6*g^3*log(e))*a^2*b^2*d^4)*B*x^2 - ((g^3*n + 4*g^3*log(e))*b^4
*c^3*d - 2*(g^3*n + 5*g^3*log(e))*a*b^3*c^2*d^2 + (g^3*n + 6*g^3*log(e))*a
^2*b^2*c*d^3)*B*x - 6*((b^4*c^3*d*g^3*n - 3*a*b^3*c^2*d^2*g^3*n + 3*a^2*b^
2*c*d^3*g^3*n - a^3*b*d^4*g^3*n)*B*x + (b^4*c^4*g^3*n - 3*a*b^3*c^3*d*g^3*
n + 3*a^2*b^2*c^2*d^2*g^3*n - a^3*b*c*d^3*g^3*n)*B)*log(b*x + a)*log(d*x +
 c) + 3*((b^4*c^3*d*g^3*n - 3*a*b^3*c^2*d^2*g^3*n + 3*a^2*b^2*c*d^3*g^3*n
- a^3*b*d^4*g^3*n)*B*x + (b^4*c^4*g^3*n - 3*a*b^3*c^3*d*g^3*n + 3*a^2*b^2*
c^2*d^2*g^3*n - a^3*b*c*d^3*g^3*n)*B)*log(d*x + c)^2 - 2*((g^3*n - g^3*log
(e))*b^4*c^4 - 4*(g^3*n - g^3*log(e))*a*b^3*c^3*d + 6*(g^3*n - g^3*log(...
```

### 3.143.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3072 vs. $2(350) = 700$.

Time = 295.16 (sec) , antiderivative size = 3072, normalized size of antiderivative = 8.56

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output
```
-1/24*(6*(B*b^8*c^5*g^3*n - 5*B*a*b^7*c^4*d*g^3*n - 4*(b*x + a)*B*b^7*c^5*
d*g^3*n/(d*x + c) + 10*B*a^2*b^6*c^3*d^2*g^3*n + 20*(b*x + a)*B*a*b^6*c^4*
d^2*g^3*n/(d*x + c) + 6*(b*x + a)^2*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 10*B
*a^3*b^5*c^2*d^3*g^3*n - 40*(b*x + a)*B*a^2*b^5*c^3*d^3*g^3*n/(d*x + c) -
30*(b*x + a)^2*B*a*b^5*c^4*d^3*g^3*n/(d*x + c)^2 - 4*(b*x + a)^3*B*b^5*c^5
*d^3*g^3*n/(d*x + c)^3 + 5*B*a^4*b^4*c*d^4*g^3*n + 40*(b*x + a)*B*a^3*b^4*
c^2*d^4*g^3*n/(d*x + c) + 60*(b*x + a)^2*B*a^2*b^4*c^3*d^4*g^3*n/(d*x + c)
^2 + 20*(b*x + a)^3*B*a*b^4*c^4*d^4*g^3*n/(d*x + c)^3 - B*a^5*b^3*d^5*g^3*
n - 20*(b*x + a)*B*a^4*b^3*c*d^5*g^3*n/(d*x + c) - 60*(b*x + a)^2*B*a^3*b^
3*c^2*d^5*g^3*n/(d*x + c)^2 - 40*(b*x + a)^3*B*a^2*b^3*c^3*d^5*g^3*n/(d*x
+ c)^3 + 4*(b*x + a)*B*a^5*b^2*d^6*g^3*n/(d*x + c) + 30*(b*x + a)^2*B*a^4*
b^2*c*d^6*g^3*n/(d*x + c)^2 + 40*(b*x + a)^3*B*a^3*b^2*c^2*d^6*g^3*n/(d*x
+ c)^3 - 6*(b*x + a)^2*B*a^5*b*d^7*g^3*n/(d*x + c)^2 - 20*(b*x + a)^3*B*a^
4*b*c*d^7*g^3*n/(d*x + c)^3 + 4*(b*x + a)^3*B*a^5*d^8*g^3*n/(d*x + c)^3)*l
og((b*x + a)/(d*x + c))/(b^4*d^4*i^2 - 4*(b*x + a)*b^3*d^5*i^2/(d*x + c) +
 6*(b*x + a)^2*b^2*d^6*i^2/(d*x + c)^2 - 4*(b*x + a)^3*b*d^7*i^2/(d*x + c)
^3 + (b*x + a)^4*d^8*i^2/(d*x + c)^4) + (11*B*b^8*c^5*g^3*n - 55*B*a*b^7*c
^4*d*g^3*n - 38*(b*x + a)*B*b^7*c^5*d*g^3*n/(d*x + c) + 110*B*a^2*b^6*c^3*
d^2*g^3*n + 190*(b*x + a)*B*a*b^6*c^4*d^2*g^3*n/(d*x + c) + 45*(b*x + a)^2
*B*b^6*c^5*d^2*g^3*n/(d*x + c)^2 - 110*B*a^3*b^5*c^2*d^3*g^3*n - 380*(b...
```

### 3.143.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^2,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^2, x)
```

# 3.144 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$



## 3.144.1 Optimal result

Integrand size = 43, antiderivative size = 275

$$\begin{aligned}
\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx
&= -\frac{2B(bc-ad)g^2n(a+bx)}{d^2i^2(c+dx)}+\frac{(bc-ad)g^2(2A+Bn)(a+bx)}{d^2i^2(c+dx)} \\
&\quad+\frac{2B(bc-ad)g^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^2(c+dx)}+\frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^2(c+dx)} \\
&\quad+\frac{b(bc-ad)g^2\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} \\
&\quad+\frac{2bB(bc-ad)g^2n\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}
\end{aligned}$$

output
```
-2*B*(-a*d+b*c)*g^2*n*(b*x+a)/d^2/i^2/(d*x+c)+(-a*d+b*c)*g^2*(B*n+2*A)*(b*
x+a)/d^2/i^2/(d*x+c)+2*B*(-a*d+b*c)*g^2*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/
d^2/i^2/(d*x+c)+g^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i^2/(d*x+c
)+b*(-a*d+b*c)*g^2*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b
/(d*x+c))/d^3/i^2+2*b*B*(-a*d+b*c)*g^2*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^
3/i^2
```

### 3.144.2 Mathematica [A] (verified)

Time = 0.14 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.92

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$$
$$= \frac{g^2\left(Ab^2dx+\frac{B(bc-ad)^2n}{c+dx}+bB(bc-ad)n\log(a+bx)+bBd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\frac{(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{c+dx}\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i +
d*i*x)^2,x]
```

output
```
(g^2*(A*b^2*d*x + (B*(b*c - a*d)^2*n)/(c + d*x) + b*B*(b*c - a*d)*n*Log[a
+ b*x] + b*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - ((b*c - a*d)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) - 2*b*B*(b*c - a*d)*n*Log
[c + d*x] - 2*b*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c +
 d*x] + b*B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c +
d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(d^3*i^2)
```

### 3.144.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 242, normalized size of antiderivative = 0.88, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {2961, 2784, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ci+dix)^2}\,dx$$
$$\Big\downarrow \text{2961}$$
$$\frac{g^2(bc-ad)\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^2}$$
$$\Big\downarrow \text{2784}$$

$$\frac{g^2(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{d}\right)}{i^2}$$

$$\Big\downarrow \text{ 2793}$$

$$\frac{g^2(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\left(-\frac{2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d}-\frac{b\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{d}\right)}{i^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^2(bc-ad)\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d^2}-\frac{(a+bx)(2A+Bn)}{d(c+dx)}-\frac{2bBn\,\text{PolyLog}\left(2,\frac{d(a+b}{b(c+d}\right.}{d^2}}{d}\right.}{i^2}$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)
^2,x]
```

output
```
((b*c - a*d)*g^2*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*
(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))) - ((2*B*n*(a + b*x))/(d*(c + d*
x)) - ((2*A + B*n)*(a + b*x))/(d*(c + d*x)) - (2*B*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n])/(d*(c + d*x)) - (b*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c
 + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 - (2*b*B*n*PolyLog[
2, (d*(a + b*x))/(b*(c + d*x))])/d^2)/d))/i^2
```

### 3.144.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.144.4 Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(dix+ci)^2}dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

### 3.144.5 Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output
```
integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B
*a*b*g^2*x + B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n))/(d^2*i^2*x^2 + 2*c
*d*i^2*x + c^2*i^2), x)
```

### 3.144.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**2,x
)
```

output
```
Timed out
```

### 3.144.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1273 vs. 2(274) = 548.

Time = 0.48 (sec) , antiderivative size = 1273, normalized size of antiderivative = 4.63

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output
```
B*a^2*g^2*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2
) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A*b^2*(c^2/(d^4*i^2*x + c*d^3*
i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i^2))*g^2 + 2*A*a*b*g^2*(c/(d^3
*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - B*a^2*g^2*log(e*(b*x/(d*x
+ c) + a/(d*x + c))^n)/(d^2*i^2*x + c*d*i^2) - A*a^2*g^2/(d^2*i^2*x + c*d*
i^2) - (2*a^2*b*d^2*g^2*log(e) + 2*(g^2*n + g^2*log(e))*b^3*c^2 - (3*g^2*n
 + 4*g^2*log(e))*a*b^2*c*d)*B*log(d*x + c)/(b*c*d^3*i^2 - a*d^4*i^2) + ((b
^3*c*d^2*g^2*log(e) - a*b^2*d^3*g^2*log(e))*B*x^2 + (b^3*c^2*d*g^2*log(e)
- a*b^2*c*d^2*g^2*log(e))*B*x + 2*((b^3*c^2*d*g^2*n - 2*a*b^2*c*d^2*g^2*n
+ a^2*b*d^3*g^2*n)*B*x + (b^3*c^3*g^2*n - 2*a*b^2*c^2*d*g^2*n + a^2*b*c*d^
2*g^2*n)*B)*log(b*x + a)*log(d*x + c) - ((b^3*c^2*d*g^2*n - 2*a*b^2*c*d^2*
g^2*n + a^2*b*d^3*g^2*n)*B*x + (b^3*c^3*g^2*n - 2*a*b^2*c^2*d*g^2*n + a^2*
b*c*d^2*g^2*n)*B)*log(d*x + c)^2 + ((g^2*n - g^2*log(e))*b^3*c^3 - 3*(g^2*
n - g^2*log(e))*a*b^2*c^2*d + 2*(g^2*n - g^2*log(e))*a^2*b*c*d^2)*B + ((b^
3*c^2*d*g^2*n - a*b^2*c*d^2*g^2*n - a^2*b*d^3*g^2*n)*B*x + (b^3*c^3*g^2*n
- a*b^2*c^2*d*g^2*n - a^2*b*c*d^2*g^2*n)*B)*log(b*x + a) + ((b^3*c*d^2*g^2
 - a*b^2*d^3*g^2)*B*x^2 + (b^3*c^2*d*g^2 - a*b^2*c*d^2*g^2)*B*x - (b^3*c^3
*g^2 - 3*a*b^2*c^2*d*g^2 + 2*a^2*b*c*d^2*g^2)*B - 2*((b^3*c^2*d*g^2 - 2*a*
b^2*c*d^2*g^2 + a^2*b*d^3*g^2)*B*x + (b^3*c^3*g^2 - 2*a*b^2*c^2*d*g^2 + a^
2*b*c*d^2*g^2)*B)*log(d*x + c))*log((b*x + a)^n) - ((b^3*c*d^2*g^2 - a*...
```

### 3.144.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1898 vs. $2(274) = 548$.

Time = 221.56 (sec) , antiderivative size = 1898, normalized size of antiderivative = 6.90

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output
```
1/6*(2*(B*b^6*c^4*g^2*n - 4*B*a*b^5*c^3*d*g^2*n - 3*(b*x + a)*B*b^5*c^4*d*
g^2*n/(d*x + c) + 6*B*a^2*b^4*c^2*d^2*g^2*n + 12*(b*x + a)*B*a*b^4*c^3*d^2
*g^2*n/(d*x + c) + 3*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 4*B*a^3
*b^3*c*d^3*g^2*n - 18*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 12*(b*
x + a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + B*a^4*b^2*d^4*g^2*n + 12*(b*x
 + a)*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) + 18*(b*x + a)^2*B*a^2*b^2*c^2*d^4*g
^2*n/(d*x + c)^2 - 3*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 12*(b*x + a)^
2*B*a^3*b*c*d^5*g^2*n/(d*x + c)^2 + 3*(b*x + a)^2*B*a^4*d^6*g^2*n/(d*x + c
)^2)*log((b*x + a)/(d*x + c))/(b^3*d^3*i^2 - 3*(b*x + a)*b^2*d^4*i^2/(d*x
+ c) + 3*(b*x + a)^2*b*d^5*i^2/(d*x + c)^2 - (b*x + a)^3*d^6*i^2/(d*x + c)
^3) + (3*B*b^6*c^4*g^2*n - 12*B*a*b^5*c^3*d*g^2*n - 7*(b*x + a)*B*b^5*c^4*
d*g^2*n/(d*x + c) + 18*B*a^2*b^4*c^2*d^2*g^2*n + 28*(b*x + a)*B*a*b^4*c^3*
d^2*g^2*n/(d*x + c) + 4*(b*x + a)^2*B*b^4*c^4*d^2*g^2*n/(d*x + c)^2 - 12*B
*a^3*b^3*c*d^3*g^2*n - 42*(b*x + a)*B*a^2*b^3*c^2*d^3*g^2*n/(d*x + c) - 16
*(b*x + a)^2*B*a*b^3*c^3*d^3*g^2*n/(d*x + c)^2 + 3*B*a^4*b^2*d^4*g^2*n + 2
8*(b*x + a)*B*a^3*b^2*c*d^4*g^2*n/(d*x + c) + 24*(b*x + a)^2*B*a^2*b^2*c^2
*d^4*g^2*n/(d*x + c)^2 - 7*(b*x + a)*B*a^4*b*d^5*g^2*n/(d*x + c) - 16*(b*x
 + a)^2*B*a^3*b*c*d^5*g^2*n/(d*x + c)^2 + 4*(b*x + a)^2*B*a^4*d^6*g^2*n/(d
*x + c)^2 + 2*B*b^6*c^4*g^2*log(e) - 8*B*a*b^5*c^3*d*g^2*log(e) - 6*(b*x +
 a)*B*b^5*c^4*d*g^2*log(e)/(d*x + c) + 12*B*a^2*b^4*c^2*d^2*g^2*log(e) ...
```

## 3.144.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^2,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^2, x)
```

# 3.145 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$



## 3.145.1 Optimal result

Integrand size = 41, antiderivative size = 168

$$\begin{aligned}\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = &-\frac{Ag(a+bx)}{di^2(c+dx)}+\frac{Bgn(a+bx)}{di^2(c+dx)}\\ &-\frac{Bg(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di^2(c+dx)}\\ &-\frac{bg\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i^2}\\ &-\frac{bBgn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}\end{aligned}$$

output
```
-A*g*(b*x+a)/d/i^2/(d*x+c)+B*g*n*(b*x+a)/d/i^2/(d*x+c)-B*g*(b*x+a)*ln(e*((
b*x+a)/(d*x+c))^n)/d/i^2/(d*x+c)-b*g*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-
a*d+b*c)/b/(d*x+c))/d^2/i^2-b*B*g*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i^2
```

### 3.145.2 Mathematica [A] (verified)

Time = 0.11 (sec) , antiderivative size = 183, normalized size of antiderivative = 1.09

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \frac{g\left(\frac{2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx}+2b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log(c+dx)-2Bn\left(\frac{bc-ad}{c+dx}+b\log(a+bx)-b\log\right.\right.}{2d^2i^2}$$

input

```
Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*
i*x)^2,x]
```

output

```
(g*((2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + 2*b
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*B*n*((b*c - a*d)/
(c + d*x) + b*Log[a + b*x] - b*Log[c + d*x]) - b*B*n*((2*Log[(d*(a + b*x))
/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))
/(b*c - a*d)])))/(2*d^2*i^2)
```

### 3.145.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 154, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.073, Rules used = {2961, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ci+dix)^2}\,dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{2793}$$

$$\frac{g\int \left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d}-\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g\left(-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2}-\frac{A(a+bx)}{d(c+dx)}-\frac{bBn\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2}-\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d(c+dx)}+\frac{Bn(a+bx)}{d(c+dx)}\right)}{i^2}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)^2
,x]
```

output
```
(g*(-((A*(a + b*x))/(d*(c + d*x))) + (B*n*(a + b*x))/(d*(c + d*x)) - (B*(a
 + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d*(c + d*x)) - (b*(A + B*Log[e*((
a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 - (b*B*n
*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^2))/i^2
```

### 3.145.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

3.145. $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx$

### 3.145.4 Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{(dix+ci)^{2}}dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x)
```

### 3.145.5 Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{(ci+dix)^{2}}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)}{(dix+ci)^{2}}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output
```
integral((A*b*g*x + A*a*g + (B*b*g*x + B*a*g)*log(e*((b*x + a)/(d*x + c))^
n))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

### 3.145.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)}{(ci+dix)^{2}}\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

### 3.145.7 Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output
```
B*a*g*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2) -
b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - 1/2*B*b*g*((2*(d*n*x + c*n)*log(b*
x + a)*log(d*x + c) - (d*n*x + c*n)*log(d*x + c)^2 - 2*((d*x + c)*log(d*x
+ c) + c)*log((b*x + a)^n) + 2*((d*x + c)*log(d*x + c) + c)*log((d*x + c)^
n))/(d^3*i^2*x + c*d^2*i^2) - 2*integrate((b*d^2*x^2*log(e) + a*d^2*x*log(
e) - b*c^2*n + a*c*d*n + (b*d^2*n*x^2 + a*c*d*n + (b*c*d*n + a*d^2*n)*x)*l
og(b*x + a))/(b*d^4*i^2*x^3 + a*c^2*d^2*i^2 + (2*b*c*d^3*i^2 + a*d^4*i^2)*
x^2 + (b*c^2*d^2*i^2 + 2*a*c*d^3*i^2)*x), x)) + A*b*g*(c/(d^3*i^2*x + c*d^
2*i^2) + log(d*x + c)/(d^2*i^2)) - B*a*g*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n)/(d^2*i^2*x + c*d*i^2) - A*a*g/(d^2*i^2*x + c*d*i^2)
```

### 3.145.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 906 vs. $2(167) = 334$.

Time = 142.03 (sec) , antiderivative size = 906, normalized size of antiderivative = 5.39

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx =$$
$$-\frac{1}{2}\left(\frac{\left(Bb^4c^3gn - 3\,Bab^3c^2dgn - \frac{2\,(bx+a)Bb^3c^3dgn}{dx+c} + 3\,Ba^2b^2cd^2gn + \frac{6\,(bx+a)Bab^2c^2d^2gn}{dx+c} - Ba^3bd^3gn - \frac{6\,(bx+\cdots}{\cdots}\right.}{b^2d^2i^2 - \frac{2\,(bx+a)bd^3i^2}{dx+c} + \frac{(bx+a)^2d^4i^2}{(dx+c)^2}}\right.$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output
```
-1/2*((B*b^4*c^3*g*n - 3*B*a*b^3*c^2*d*g*n - 2*(b*x + a)*B*b^3*c^3*d*g*n/(
d*x + c) + 3*B*a^2*b^2*c*d^2*g*n + 6*(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x +
c) - B*a^3*b*d^3*g*n - 6*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x + c) + 2*(b*x +
a)*B*a^3*d^4*g*n/(d*x + c))*log((b*x + a)/(d*x + c))/(b^2*d^2*i^2 - 2*(b*x
 + a)*b*d^3*i^2/(d*x + c) + (b*x + a)^2*d^4*i^2/(d*x + c)^2) + (B*b^4*c^3*
g*n - 3*B*a*b^3*c^2*d*g*n - (b*x + a)*B*b^3*c^3*d*g*n/(d*x + c) + 3*B*a^2*
b^2*c*d^2*g*n + 3*(b*x + a)*B*a*b^2*c^2*d^2*g*n/(d*x + c) - B*a^3*b*d^3*g*
n - 3*(b*x + a)*B*a^2*b*c*d^3*g*n/(d*x + c) + (b*x + a)*B*a^3*d^4*g*n/(d*x
 + c) + B*b^4*c^3*g*log(e) - 3*B*a*b^3*c^2*d*g*log(e) - 2*(b*x + a)*B*b^3*
c^3*d*g*log(e)/(d*x + c) + 3*B*a^2*b^2*c*d^2*g*log(e) + 6*(b*x + a)*B*a*b^
2*c^2*d^2*g*log(e)/(d*x + c) - B*a^3*b*d^3*g*log(e) - 6*(b*x + a)*B*a^2*b*
c*d^3*g*log(e)/(d*x + c) + 2*(b*x + a)*B*a^3*d^4*g*log(e)/(d*x + c) + A*b^
4*c^3*g - 3*A*a*b^3*c^2*d*g - 2*(b*x + a)*A*b^3*c^3*d*g/(d*x + c) + 3*A*a^
2*b^2*c*d^2*g + 6*(b*x + a)*A*a*b^2*c^2*d^2*g/(d*x + c) - A*a^3*b*d^3*g -
6*(b*x + a)*A*a^2*b*c*d^3*g/(d*x + c) + 2*(b*x + a)*A*a^3*d^4*g/(d*x + c))
/(b^2*d^2*i^2 - 2*(b*x + a)*b*d^3*i^2/(d*x + c) + (b*x + a)^2*d^4*i^2/(d*x
 + c)^2) + (B*b^3*c^3*g*n - 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*
a^3*d^3*g*n)*log(-b + (b*x + a)*d/(d*x + c))/(b*d^2*i^2) - (B*b^3*c^3*g*n
- 3*B*a*b^2*c^2*d*g*n + 3*B*a^2*b*c*d^2*g*n - B*a^3*d^3*g*n)*log((b*x + a)
/(d*x + c))/(b*d^2*i^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2
```

### 3.145.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{{(c\,i+d\,i\,x)}^2}\,dx$$

input
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^2
,x)
```

output
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^2
, x)
```

# 3.146 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx$



## 3.146.1 Optimal result

Integrand size = 33, antiderivative size = 102

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx = \frac{A(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{Bn(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)i^2(c+dx)}$$

output
```
A*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)-B*n*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)+B*(b*x
+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/i^2/(d*x+c)
```

## 3.146.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 114, normalized size of antiderivative = 1.12

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx = -\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di(ci+dix)} + \frac{B(bc-ad)n\left(\frac{1}{(bc-ad)(c+dx)}+\frac{b\log(a+bx)}{(bc-ad)^2}-\frac{b\log(c+dx)}{(bc-ad)^2}\right)}{di^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x)^2,x]
```

output
```
-((A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*i*(c*i + d*i*x))) + (B*(b*c -
a*d)*n*(1/((b*c - a*d)*(c + d*x)) + (b*Log[a + b*x])/(b*c - a*d)^2 - (b*Lo
g[c + d*x])/(b*c - a*d)^2))/(d*i^2)
```

## 3.146.3 Rubi [A] (verified)

Time = 0.22 (sec) , antiderivative size = 77, normalized size of antiderivative = 0.75, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2951, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ci+dix)^2} \, dx$$

$$\Big\downarrow \text{2951}$$

$$\frac{\int \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) d\frac{a+bx}{c+dx}}{i^2 (bc-ad)}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{A(a+bx)}{c+dx}+\frac{B(a+bx) \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{Bn(a+bx)}{c+dx}}{i^2 (bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x)^2,x]
```

output
```
((A*(a + b*x))/(c + d*x) - (B*n*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[e*
((a + b*x)/(c + d*x))^n])/(c + d*x))/((b*c - a*d)*i^2)
```

### 3.146.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

### 3.146.4 Maple [A] (verified)

Time = 2.08 (sec) , antiderivative size = 81, normalized size of antiderivative = 0.79

| method | result | size |
|---|---|---|
| default | $-\frac{A}{i^{2}\left(dx+c\right)d}-\frac{B\left(\frac{\left(bx+a\right)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{dx+c}-\frac{n\left(bx+a\right)}{dx+c}\right)}{i^{2}\left(ad-cb\right)}$ | 81 |
| parts | $-\frac{A}{i^{2}\left(dx+c\right)d}-\frac{B\left(\frac{\left(bx+a\right)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{dx+c}-\frac{n\left(bx+a\right)}{dx+c}\right)}{i^{2}\left(ad-cb\right)}$ | 81 |
| parallelrisch | $-\frac{-Bab\,d^{3}n^{2}+B\,b^{2}c\,d^{2}n^{2}+Aab\,d^{3}n-A\,b^{2}c\,d^{2}n+Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{2}d^{3}n+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)ab\,d^{3}n}{i^{2}\left(dx+c\right)b\,d^{3}n\left(ad-cb\right)}$ | 129 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x,method=_RETURNVERBOSE)
```

output
```
-A/i^2/(d*x+c)/d-1/i^2*B/(a*d-b*c)*((b*x+a)/(d*x+c)*ln(e*((b*x+a)/(d*x+c))
^n)-n*(b*x+a)/(d*x+c))
```

### 3.146.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 105, normalized size of antiderivative = 1.03

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{(ci+dix)^{2}}\,dx = -\frac{Abc-Aad-(Bbc-Bad)n+(Bbc-Bad)\log(e)-(Bbdnx+Badn)\log\left(\frac{bx+a}{dx+c}\right)}{(bcd^{2}-ad^{3})i^{2}x+(bc^{2}d-acd^{2})i^{2}}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, algorithm="fri
cas")
```

output
```
-(A*b*c - A*a*d - (B*b*c - B*a*d)*n + (B*b*c - B*a*d)*log(e) - (B*b*d*n*x
+ B*a*d*n)*log((b*x + a)/(d*x + c)))/((b*c*d^2 - a*d^3)*i^2*x + (b*c^2*d -
 a*c*d^2)*i^2)
```

### 3.146.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 444 vs. $2(82) = 164$.

Time = 11.44 (sec) , antiderivative size = 444, normalized size of antiderivative = 4.35

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci + dix)^2}\,dx = \begin{cases} -\frac{A}{cdi^2+d^2i^2x} - \frac{B\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)}{cdi^2+d^2i^2x} \\ \frac{Ax+\frac{Ba\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{b}-Bnx+Bx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{c^2i^2} \\ -\frac{Aad}{acd^2i^2+ad^3i^2x-bc^2di^2-bcd^2i^2x} + \frac{Abc}{acd^2i^2+ad^3i^2x-bc^2di^2-bcd^2i^2x} + \frac{Badn}{acd^2i^2+ad^3i^2x-bc^2di^2-bcd^2i^2x} - \frac{Bad\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right.\right.}{acd^2i^2+ad^3i^2x-bc^2di^2-b} \end{cases}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**2,x)
```

output
```
Piecewise((-A/(c*d*i**2 + d**2*i**2*x) - B*log(e*(b*c/(c*d + d**2*x) + b*x
/(c + d*x))**n)/(c*d*i**2 + d**2*i**2*x), Eq(a, b*c/d)), ((A*x + B*a*log(e
*(a/c + b*x/c)**n)/b - B*n*x + B*x*log(e*(a/c + b*x/c)**n))/(c**2*i**2), E
q(d, 0)), (-A*a*d/(a*c*d**2*i**2 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d**
2*i**2*x) + A*b*c/(a*c*d**2*i**2 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d**
2*i**2*x) + B*a*d*n/(a*c*d**2*i**2 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d
**2*i**2*x) - B*a*d*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a*c*d**2*i**2
 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d**2*i**2*x) - B*b*c*n/(a*c*d**2*i*
*2 + a*d**3*i**2*x - b*c**2*d*i**2 - b*c*d**2*i**2*x) - B*b*d*x*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)/(a*c*d**2*i**2 + a*d**3*i**2*x - b*c**2*d*i*
*2 - b*c*d**2*i**2*x), True))
```

### 3.146.7 Maxima [A] (verification not implemented)

Time = 0.19 (sec) , antiderivative size = 136, normalized size of antiderivative = 1.33

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx = Bn\left(\frac{1}{d^2i^2x+cdi^2}+\frac{b\log(bx+a)}{(bcd-ad^2)i^2}-\frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right) - \frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^2i^2x+cdi^2} - \frac{A}{d^2i^2x+cdi^2}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, algorithm="max
ima")
```

output
```
B*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2) - b*lo
g(d*x + c)/((b*c*d - a*d^2)*i^2)) - B*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)/(d^2*i^2*x + c*d*i^2) - A/(d^2*i^2*x + c*d*i^2)
```

### 3.146.8 Giac [A] (verification not implemented)

Time = 0.56 (sec) , antiderivative size = 91, normalized size of antiderivative = 0.89

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx = \left(\frac{(bx+a)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)i^2}-\frac{(Bn-B\log(e)-A)(bx+a)}{(dx+c)i^2}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^2,x, algorithm="gia
c")
```

output
```
((b*x + a)*B*n*log((b*x + a)/(d*x + c))/((d*x + c)*i^2) - (B*n - B*log(e)
- A)*(b*x + a)/((d*x + c)*i^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.146.9 Mupad [B] (verification not implemented)

Time = 1.44 (sec) , antiderivative size = 113, normalized size of antiderivative = 1.11

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^2}\,dx = -\frac{A-B\,n}{x\,d^2\,i^2+c\,d\,i^2}-\frac{B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{d\,\left(c\,i^2+d\,i^2\,x\right)} + \frac{B\,b\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,2\mathrm{i}}{d\,i^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*i + d*i*x)^2,x)
```

output
```
(B*b*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*2i)/(d*i^2*(a*d - b*c))
- (B*log(e*((a + b*x)/(c + d*x))^n))/(d*(c*i^2 + d*i^2*x)) - (A - B*n)/(d^
2*i^2*x + c*d*i^2)
```

# 3.147 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$



## 3.147.1 Optimal result

Integrand size = 43, antiderivative size = 166

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx = -\frac{Ad(a+bx)}{(bc-ad)^2gi^2(c+dx)}+\frac{Bdn(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2gi^2(c+dx)}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2B(bc-ad)^2gi^2n}$$

output
```
-A*d*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*x+c)+B*d*n*(b*x+a)/(-a*d+b*c)^2/g/i^2/(
d*x+c)-B*d*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2/g/i^2/(d*x+c)+1/
2*b*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/B/(-a*d+b*c)^2/g/i^2/n
```

## 3.147.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.16 (sec) , antiderivative size = 304, normalized size of antiderivative = 1.83

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2b(c+dx)\left(A+B\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i
*x)^2),x]
```

output
```
(2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Log[
a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*x)
*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x]*
(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b
*x))/(-(b*c) + a*d)]) + b*B*n*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*
d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]
))/(2*(b*c - a*d)^2*g*i^2*(c + d*x))
```

### 3.147.3 Rubi [A] (verified)

Time = 0.38 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.72, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {2961, 2788, 2009, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)(ci+dix)^2} \, dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{ 2788}$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}d\frac{a+bx}{c+dx}-d\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{Bn(a+bx)}{c+dx}\right)}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{ 2738}$$

$$\frac{\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2Bn} - d\left(\frac{A(a+bx)}{c+dx} + \frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx} - \frac{Bn(a+bx)}{c+dx}\right)}{gi^2(bc-ad)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i*x)^2)
,x]
```

output
```
((b*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*B*n) - d*((A*(a + b*x))/(
c + d*x) - (B*n*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[e*((a + b*x)/(c +
d*x))^n])/(c + d*x)))/((b*c - a*d)^2*g*i^2)
```

### 3.147.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.147.4 Maple [A] (verified)

Time = 4.76 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.62

| method | result |
|---|---|
| parallelrisch | $\frac{2Ba\,b^2d^4n^2-2B\,b^3c\,d^3n^2-2Aa\,b^2d^4n+2A\,b^3c\,d^3n+Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^3d^4+2Ax\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^3d^4+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b}{2i^2g(dx+c)(a^2d^2-2abcd+b^2c^2)b^2d^3n}$ |

input

```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^2,x,method=_RE
TURNVERBOSE)
```

output

```
1/2*(2*B*a*b^2*d^4*n^2-2*B*b^3*c*d^3*n^2-2*A*a*b^2*d^4*n+2*A*b^3*c*d^3*n+B
*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^4+2*A*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3
*d^4+B*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*c*d^3+2*A*ln(e*((b*x+a)/(d*x+c))^n)
*b^3*c*d^3-2*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^4*n-2*B*ln(e*((b*x+a)/(d*
x+c))^n)*a*b^2*d^4*n)/i^2/g/(d*x+c)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/d^3/n
```

### 3.147.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 196, normalized size of antiderivative = 1.18

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=\frac{2\,Abc-2\,Aad+(Bbdnx+Bbcn)\log\left(\frac{bx+a}{dx+c}\right)^2-2\,(Bbc-Bad)n+2\,\left(Bbc-Bad+(Bbdx+Bbc)\log\left(\frac{bx}{dx}\right.\right.}{2\,((b^2c^2d-2\,abcd^2+a^2d^3)gi^2x+(b^2c^3-2\,abc^2d+a^2}$$

input

```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, al
gorithm="fricas")
```

output

```
1/2*(2*A*b*c - 2*A*a*d + (B*b*d*n*x + B*b*c*n)*log((b*x + a)/(d*x + c))^2
- 2*(B*b*c - B*a*d)*n + 2*(B*b*c - B*a*d + (B*b*d*x + B*b*c)*log((b*x + a)
/(d*x + c)))*log(e) - 2*(B*a*d*n - A*b*c + (B*b*d*n - A*b*d)*x)*log((b*x +
 a)/(d*x + c)))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g*i^2*x + (b^2*c^3 -
2*a*b*c^2*d + a^2*c*d^2)*g*i^2)
```

### 3.147.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)/(d*i*x+c*i)**2,x)
```

output
```
Timed out
```

### 3.147.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 424 vs. $2(164) = 328$.

Time = 0.21 (sec) , antiderivative size = 424, normalized size of antiderivative = 2.55

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=B\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)\log\left(e\left(\frac{}{}\right.\right.$$
$$-\frac{\left((bdx+bc)\log(bx+a)^2+(bdx+bc)\log(dx+c)^2+2\,bc-2\,ad+2\,(bdx+bc)\log(bx+a)-2\,(bdx+\right.}{2\,(b^2c^3gi^2-2\,abc^2dgi^2+a^2cd^2gi^2+(b^2c^2dgi^2-2\,abcd^2gi^2+a^2}$$
$$+A\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, al
gorithm="maxima")
```

output
```
B*(1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((
b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c
*d + a^2*d^2)*g*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/2*((b*d*x
 + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*
(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*
log(d*x + c))*B*n/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2*g*i^2 + (
b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x) + A*(1/((b*c*d - a
*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2*c^2 - 2*a*b*
c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*
i^2))
```

### 3.147.8 Giac [A] (verification not implemented)

Time = 1.05 (sec) , antiderivative size = 208, normalized size of antiderivative = 1.25

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=\frac{1}{2}\left(\frac{Bbn\log\left(\frac{bx+a}{dx+c}\right)^2}{bcgi^2-adgi^2}-\frac{2\,(bx+a)Bdn\log\left(\frac{bx+a}{dx+c}\right)}{(bcgi^2-adgi^2)(dx+c)}+\frac{2\,(Bb\log(e)+Ab)\log\left(\frac{bx+a}{dx+c}\right)}{bcgi^2-adgi^2}+\frac{2\,(Bdn-Bd\log(e)\cdots}{(bcgi^2-adgi^2)\cdots}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^2,x, al
gorithm="giac")
```

output
```
1/2*(B*b*n*log((b*x + a)/(d*x + c))^2/(b*c*g*i^2 - a*d*g*i^2) - 2*(b*x + a
)*B*d*n*log((b*x + a)/(d*x + c))/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) + 2*(
B*b*log(e) + A*b)*log((b*x + a)/(d*x + c))/(b*c*g*i^2 - a*d*g*i^2) + 2*(B*
d*n - B*d*log(e) - A*d)*(b*x + a)/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)))*(b*
c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.147.9 Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 241, normalized size of antiderivative = 1.45

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx=\frac{B\,n}{g\,i^2\,\left(a\,d-b\,c\right)\,\left(c+d\,x\right)}-\frac{A}{g\,i^2\,\left(a\,d-b\,c\right)\,\left(c+d\,x\right)}$$
$$-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{g\,i^2\,\left(a\,d-b\,c\right)\,\left(c+d\,x\right)}+\frac{B\,b\,{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}^2}{2\,g\,i^2\,n\,{\left(a\,d-b\,c\right)}^2}$$
$$-\frac{A\,b\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g\,i^2\,{\left(a\,d-b\,c\right)}^2}$$
$$+\frac{B\,b\,n\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g\,i^2\,{\left(a\,d-b\,c\right)}^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)*(c*i + d*i*x)^2)
,x)
```

output
```
(B*n)/(g*i^2*(a*d - b*c)*(c + d*x)) - A/(g*i^2*(a*d - b*c)*(c + d*x)) - (B
*log(e*((a + b*x)/(c + d*x))^n))/(g*i^2*(a*d - b*c)*(c + d*x)) - (A*b*atan
((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g*i^2*(a*d - b*c)^2) + (B*
b*n*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g*i^2*(a*d - b*c)^
2) + (B*b*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g*i^2*n*(a*d - b*c)^2)
```

---

3.147. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^2}\,dx$

# 3.148 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$



## 3.148.1 Optimal result

Integrand size = 43, antiderivative size = 273

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = -\frac{Bd^2n(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}-\frac{b^2Bn(c+dx)}{(bc-ad)^3g^2i^2(a+bx)}+\frac{d^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(c+dx)}-\frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(a+bx)}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}+\frac{bBdn\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3g^2i^2}$$

output
```
-B*d^2*n*(b*x+a)/(-a*d+b*c)^3/g^2/i^2/(d*x+c)-b^2*B*n*(d*x+c)/(-a*d+b*c)^3
/g^2/i^2/(b*x+a)+d^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/
g^2/i^2/(d*x+c)-b^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g
^2/i^2/(b*x+a)-2*b*d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(
-a*d+b*c)^3/g^2/i^2+b*B*d*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g^2/i^2
```

### 3.148.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.25 (sec) , antiderivative size = 342, normalized size of antiderivative = 1.25

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$$
$$= \frac{-\frac{b^2Bcn}{a+bx}+\frac{abBdn}{a+bx}+\frac{bBcdn}{c+dx}-\frac{aBd^2n}{c+dx}-\frac{b(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}+\frac{d(-bc+ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx}-2bd\log(a+}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d
*i*x)^2),x]
```

output
```
(-((b^2*B*c*n)/(a + b*x)) + (a*b*B*d*n)/(a + b*x) + (b*B*c*d*n)/(c + d*x)
- (a*B*d^2*n)/(c + d*x) - (b*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(a + b*x) + (d*(-(b*c) + a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(c + d*x) - 2*b*d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
 2*b*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + b*B*d*n*(Log[
a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2,
(d*(a + b*x))/(-(b*c) + a*d)]) - b*B*d*n*((2*Log[(d*(a + b*x))/(-(b*c) + a
*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)
]))/((b*c - a*d)^3*g^2*i^2)
```

### 3.148.3 Rubi [A] (verified)

Time = 0.40 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {2961, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^2(ci+dix)^2}\,dx$$
$$\Big\downarrow \text{2961}$$
$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i^2(bc-ad)^3}$$

$\big\downarrow$ 2772

$$\frac{-Bn\int\left(d^2-\frac{2b(c+dx)\log\left(\frac{a+bx}{c+dx}\right)d}{a+bx}-\frac{b^2(c+dx)^2}{(a+bx)^2}\right)d\frac{a+bx}{c+dx}-\frac{b^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}+\frac{d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{c+dx}}{g^2i^2(bc-ad)^3}$$

$\big\downarrow$ 2009

$$\frac{-\frac{b^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}+\frac{d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{c+dx}-2bd\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\Big(\frac{b^2(}{c}}{g^2i^2(bc-ad)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d*i*x)^
2),x]
```

output
```
((d^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) - (b^2*(
c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - 2*b*d*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] - B*n*((d^2*(a +
b*x))/(c + d*x) + (b^2*(c + d*x))/(a + b*x) - b*d*Log[(a + b*x)/(c + d*x)]
^2))/((b*c - a*d)^3*g^2*i^2)
```

### 3.148.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

3.148. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$

### 3.148.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 665 vs. $2(273) = 546$.

Time = 9.42 (sec) , antiderivative size = 666, normalized size of antiderivative = 2.44

| method | result |
|---|---|
| parallelrisch | $\frac{-2Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{4}b\,c^{3}d^{2}n+2Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{3}b^{2}c^{4}dn+B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}a^{3}b^{2}c^{3}d^{2}-B\,x^{2}a^{4}b\,c^{2}d^{3}n^{2}+2B\,x^{2}a^{3}b^{2}}{}$ |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,method=_
RETURNVERBOSE)
```

output
```
(-2*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^3*d^2*n+2*B*x*ln(e*((b*x+a)/(d*x
+c))^n)*a^3*b^2*c^4*d*n+B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^2*c^3*d^2-
B*x^2*a^4*b*c^2*d^3*n^2+2*B*x^2*a^3*b^2*c^3*d^2*n^2-B*x^2*a^2*b^3*c^4*d*n^
2+2*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^3*d^2+A*x^2*a^4*b*c^2*d^3*n-
A*x^2*a^2*b^3*c^4*d*n+B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b*c^3*d^2+B*x*ln
(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^2*c^4*d+B*x*a^4*b*c^3*d^2*n^2+B*x*a^3*b^2*
c^4*d*n^2+2*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^3*d^2+2*A*x*ln(e*((b*x+a
)/(d*x+c))^n)*a^3*b^2*c^4*d-A*x*a^4*b*c^3*d^2*n+A*x*a^3*b^2*c^4*d*n-B*x*a^
5*c^2*d^3*n^2-B*x*a^2*b^3*c^5*n^2+A*x*a^5*c^2*d^3*n-A*x*a^2*b^3*c^5*n+B*ln
(e*((b*x+a)/(d*x+c))^n)^2*a^4*b*c^4*d-B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*c^3*
d^2*n+B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^5*n+2*A*ln(e*((b*x+a)/(d*x+c))
^n)*a^4*b*c^4*d)/i^2/g^2/(d*x+c)/(b*x+a)/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-
b*c)/c^3/a^3/n
```

### 3.148.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 450, normalized size of antiderivative = 1.65

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)}{(ag+bgx)^{2}(ci+dix)^{2}}\,dx =$$
$$-\frac{Ab^{2}c^{2}-Aa^{2}d^{2}+(Bb^{2}d^{2}nx^{2}+Babcdn+(Bb^{2}cd+Babd^{2})nx)\log\left(\frac{bx+a}{dx+c}\right)^{2}+(Bb^{2}c^{2}-2\,Babcd+Ba^{2}d}{(b}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output
```
-(A*b^2*c^2 - A*a^2*d^2 + (B*b^2*d^2*n*x^2 + B*a*b*c*d*n + (B*b^2*c*d + B*
a*b*d^2)*n*x)*log((b*x + a)/(d*x + c))^2 + (B*b^2*c^2 - 2*B*a*b*c*d + B*a^
2*d^2)*n + 2*(A*b^2*c*d - A*a*b*d^2)*x + (B*b^2*c^2 - B*a^2*d^2 + 2*(B*b^2
*c*d - B*a*b*d^2)*x + 2*(B*b^2*d^2*x^2 + B*a*b*c*d + (B*b^2*c*d + B*a*b*d^
2)*x)*log((b*x + a)/(d*x + c)))*log(e) + (2*A*b^2*d^2*x^2 + 2*A*a*b*c*d +
(B*b^2*c^2 - B*a^2*d^2)*n + 2*(A*b^2*c*d + A*a*b*d^2 + (B*b^2*c*d - B*a*b*
d^2)*n)*x)*log((b*x + a)/(d*x + c)))/((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2
*b^2*c*d^3 - a^3*b*d^4)*g^2*i^2*x^2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c
*d^3 - a^4*d^4)*g^2*i^2*x + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2
 - a^4*c*d^3)*g^2*i^2)
```

## 3.148.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2/(d*i*x+c*i)**2,x
)
```

output
```
Timed out
```

## 3.148.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 862 vs. 2(273) = 546.

Time = 0.22 (sec) , antiderivative size = 862, normalized size of antiderivative = 3.16

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx =$$
$$-B\left(\frac{2\,bdx+bc+ad}{(b^3c^2d-2\,ab^2cd^2+a^2bd^3)g^2i^2x^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)g^2i^2x+(ab^2c^3-2\,a^2bc^2d+a^3cd\right.$$
$$-\frac{\left(b^2c^2-2\,abcd+a^2d^2-(b^2d^2x^2+abcd+(b^2cd+abd^2)x)\log(bx+a)^2+2\,(b^2d^2x^2+abcd+(b^2cd+ab\right.}{ab^3c^4g^2i^2-3\,a^2b^2c^3dg^2i^2+3\,a^3bc^2d^2g^2i^2-a^4cd^3g^2i^2+(b^4c^3dg^2i^2-3\,ab^3c^2d^2g^2i^2+3\,a^2b^2cd^3}$$
$$-A\left(\frac{2\,bdx+bc+ad}{(b^3c^2d-2\,ab^2cd^2+a^2bd^3)g^2i^2x^2+(b^3c^3-ab^2c^2d-a^2bcd^2+a^3d^3)g^2i^2x+(ab^2c^3-2\,a^2bc^2d+a^3cd\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output
```
-B*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2
*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*
c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3 -
 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/((
b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) - (b^2*c^2 - 2*a*b*c*d + a^2*d^2 - (b^2*d^2*x^2
+ a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + a*b*c
*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)*log(d*x + c) - (b^2*d^2*x^2 + a*b
*c*d + (b^2*c*d + a*b*d^2)*x)*log(d*x + c)^2)*B*n/(a*b^3*c^4*g^2*i^2 - 3*a
^2*b^2*c^3*d*g^2*i^2 + 3*a^3*b*c^2*d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*
c^3*d*g^2*i^2 - 3*a*b^3*c^2*d^2*g^2*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*
d^4*g^2*i^2)*x^2 + (b^4*c^4*g^2*i^2 - 2*a*b^3*c^3*d*g^2*i^2 + 2*a^3*b*c*d^
3*g^2*i^2 - a^4*d^4*g^2*i^2)*x) - A*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2
*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d
^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2)
 + 2*b*d*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)
*g^2*i^2) - 2*b*d*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 -
 a^3*d^3)*g^2*i^2))
```

### 3.148.8 Giac [A] (verification not implemented)

Time = 106.59 (sec) , antiderivative size = 96, normalized size of antiderivative = 0.35

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx$$
$$= -\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)^2\left(\frac{(dx+c)Bn\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2i^2}+\frac{(Bn+B\log(e)+A)(dx+c)}{(bx+a)g^2i^2}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output
```
-(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2*((d*x + c)*B*n*log((b*x + a)/(d
*x + c))/((b*x + a)*g^2*i^2) + (B*n + B*log(e) + A)*(d*x + c)/((b*x + a)*g
^2*i^2))
```

### 3.148.9 Mupad [B] (verification not implemented)

Time = 2.06 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.58

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^2}\,dx = \frac{B\,b\,d\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{g^2\,i^2\,n\,(a\,d-b\,c)^3} - \frac{A\,b\,c}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{A\,a\,d}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} + \frac{B\,a\,d\,n}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,b\,c\,n}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{2\,A\,b\,d\,x}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,a\,d\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{B\,b\,c\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{2\,B\,b\,d\,x\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)}{g^2\,i^2\,(a\,d-b\,c)^2\,(a+b\,x)\,(c+d\,x)} - \frac{A\,b\,d\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,4\mathrm{i}}{g^2\,i^2\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^2*(c*i + d*i*x)^
2),x)
```

output
```
(B*b*d*log(e*((a + b*x)/(c + d*x))^n)^2)/(g^2*i^2*n*(a*d - b*c)^3) - (A*a*
d)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (A*b*c)/(g^2*i^2*(a*d - b
*c)^2*(a + b*x)*(c + d*x)) - (A*b*d*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d
 - b*c))*4i)/(g^2*i^2*(a*d - b*c)^3) + (B*a*d*n)/(g^2*i^2*(a*d - b*c)^2*(a
 + b*x)*(c + d*x)) - (B*b*c*n)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x))
 - (2*A*b*d*x)/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (B*a*d*log(e*
((a + b*x)/(c + d*x))^n))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c + d*x)) - (B
*b*c*log(e*((a + b*x)/(c + d*x))^n))/(g^2*i^2*(a*d - b*c)^2*(a + b*x)*(c +
 d*x)) - (2*B*b*d*x*log(e*((a + b*x)/(c + d*x))^n))/(g^2*i^2*(a*d - b*c)^2
*(a + b*x)*(c + d*x))
```

# 3.149 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$



## 3.149.1 Optimal result

Integrand size = 43, antiderivative size = 380

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx &= \frac{Bd^3n(a+bx)}{(bc-ad)^4g^3i^2(c+dx)}+\frac{3b^2Bdn(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} \\ &\quad-\frac{b^3Bn(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} \\ &\quad-\frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(c+dx)} \\ &\quad+\frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} \\ &\quad-\frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} \\ &\quad+\frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^3i^2} \\ &\quad-\frac{3bBd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^3i^2}\end{aligned}$$

output
```
B*d^3*n*(b*x+a)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+3*b^2*B*d*n*(d*x+c)/(-a*d+b*c
)^4/g^3/i^2/(b*x+a)-1/4*b^3*B*n*(d*x+c)^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2-d
^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+3*
b^2*d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)
-1/2*b^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(b
*x+a)^2+3*b*d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+
b*c)^4/g^3/i^2-3/2*b*B*d^2*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^3/i^2
```

## 3.149.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.38 (sec) , antiderivative size = 478, normalized size of antiderivative = 1.26

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$$

$$= \frac{-\frac{bB(bc-ad)^2n}{(a+bx)^2}+\frac{8b^2Bcdn}{a+bx}-\frac{8abBd^2n}{a+bx}+\frac{2bBd(bc-ad)n}{a+bx}-\frac{4bBcd^2n}{c+dx}+\frac{4aBd^3n}{c+dx}+6bBd^2n\log(a+bx)-\frac{2b(bc-ad)^2\left(A+B\log\ldots\right.}{(a+b\ldots}}{\ldots}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d
*i*x)^2),x]
```

output
```
(-((b*B*(b*c - a*d)^2*n)/(a + b*x)^2) + (8*b^2*B*c*d*n)/(a + b*x) - (8*a*b
*B*d^2*n)/(a + b*x) + (2*b*B*d*(b*c - a*d)*n)/(a + b*x) - (4*b*B*c*d^2*n)/
(c + d*x) + (4*a*B*d^3*n)/(c + d*x) + 6*b*B*d^2*n*Log[a + b*x] - (2*b*(b*c
 - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 + (8*b*d*(b*
c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (4*d^2*(b*c -
 a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + 12*b*d^2*Log[a +
 b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*b*B*d^2*n*Log[c + d*x] -
12*b*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 6*b*B*d^2*n
*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyL
og[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 6*b*B*d^2*n*((2*Log[(d*(a + b*x))/(
-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(
b*c - a*d)]))/(4*(b*c - a*d)^4*g^3*i^2)
```

### 3.149.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 267, normalized size of antiderivative = 0.70, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2961, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^3(ci+dix)^2}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i^2(bc-ad)^4}$$

$$\Big\downarrow \text{2772}$$

$$\frac{-Bn\int -\frac{(c+dx)^3\left(b^3-\frac{6d(a+bx)b^2}{c+dx}-\frac{6d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)b}{(c+dx)^2}+\frac{2d^3(a+bx)^3}{(c+dx)^3}\right)}{2(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\right.}{c}}{g^3i^2(bc-ad)^4}$$

$$\Big\downarrow \text{27}$$

$$\frac{\frac{1}{2}Bn\int \frac{(c+dx)^3\left(b^3-\frac{6d(a+bx)b^2}{c+dx}-\frac{6d^2(a+bx)^2\log\left(\frac{a+bx}{c+dx}\right)b}{(c+dx)^2}+\frac{2d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\right.}{a+}}{g^3i^2(bc-ad)^4}$$

$$\Big\downarrow \text{2010}$$

$$\frac{\frac{1}{2}Bn\int \left(\frac{(c+dx)^3\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{2d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)^3}-\frac{6bd^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx}{}}{g^3i^2(bc-ad)^4}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{d^3(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{c+dx}+3bd^2\log\left(\frac{a+bx}{c+dx}\right)\left(\right.}{g^3i^2(bc-ad)^4}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d*i*x)^
2),x]
```

output
```
(-((d^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)) + (3*
b^2*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (b^3*(
c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + 3*b*d
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] + (B*n*
((2*d^3*(a + b*x))/(c + d*x) + (6*b^2*d*(c + d*x))/(a + b*x) - (b^3*(c + d
*x)^2)/(2*(a + b*x)^2) - 3*b*d^2*Log[(a + b*x)/(c + d*x)]^2))/2)/((b*c - a
*d)^4*g^3*i^2)
```

### 3.149.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.149.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 976 vs. $2(374) = 748$.

Time = 28.77 (sec) , antiderivative size = 977, normalized size of antiderivative = 2.57

| method | result |
|---|---|
| parallelrisch | $\frac{24Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a\,b^{6}c\,d^{5}n+12A\,x^{2}b^{7}c\,d^{5}n+6Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}a^{2}b^{5}d^{6}-3Bx\,a^{2}b^{5}d^{6}n^{2}+9Bx\,b^{7}c^{2}d^{4}n^{2}+12Ax\ln\left(e\left(\frac{bx-}{dx-}\right.\right.}{}$ |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,method=_
RETURNVERBOSE)
```

output
```
1/4*(24*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c*d^5*n+12*A*x^2*b^7*c*d^5*n+6
*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*d^6-3*B*x*a^2*b^5*d^6*n^2+9*B*x*b
^7*c^2*d^4*n^2+12*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*d^6-18*A*x*a^2*b^5
*d^6*n+6*A*x*b^7*c^2*d^4*n+6*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*c*d^5-4
*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*d^6*n-2*B*ln(e*((b*x+a)/(d*x+c))^n)*b
^7*c^3*d^3*n+12*A*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*c*d^5-15*B*a^2*b^5*c*d
^5*n^2+12*B*a*b^6*c^2*d^4*n^2-6*A*a^2*b^5*c*d^5*n+12*A*a*b^6*c^2*d^4*n+4*B
*a^3*b^4*d^6*n^2-B*b^7*c^3*d^3*n^2-4*A*a^3*b^4*d^6*n-2*A*b^7*c^3*d^3*n+6*B
*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^7*d^6+12*A*x^3*ln(e*((b*x+a)/(d*x+c))^n
)*b^7*d^6+6*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^6*n+12*B*x^2*ln(e*((b*x+
a)/(d*x+c))^n)^2*a*b^6*d^6+6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^7*c*d^5-6
*B*x^2*a*b^6*d^6*n^2+6*B*x^2*b^7*c*d^5*n^2+24*A*x^2*ln(e*((b*x+a)/(d*x+c))
^n)*a*b^6*d^6+12*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c*d^5-12*A*x^2*a*b^6*
d^6*n+18*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c*d^5*n+12*B*x*ln(e*((b*x+a)/
(d*x+c))^n)^2*a*b^6*c*d^5-12*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^5*d^6*n+6
*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^2*d^4*n-6*B*x*a*b^6*c*d^5*n^2+24*A*x*
ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*c*d^5+12*A*x*a*b^6*c*d^5*n+12*B*ln(e*((b*x
+a)/(d*x+c))^n)*a*b^6*c^2*d^4*n)/i^2/g^3/(d*x+c)/(b*x+a)^2/(a^3*d^3-3*a^2*
b*c*d^2+3*a*b^2*c^2*d-b^3*c^3)/(a*d-b*c)/b^4/d^3/n
```

### 3.149.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 946 vs. $2(374) = 748$.

Time = 0.37 (sec) , antiderivative size = 946, normalized size of antiderivative = 2.49

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = -\frac{2\,Ab^3c^3-12\,Aab^2c^2d+6\,Aa^2bcd^2+4\,Aa^3d^3-6\,(2\,Ab^3cd^2-2\,Aab^2d^3+(Bb^3cd^2-Bab^2d^3)n)x^2-6\,(\ldots}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output
```
-1/4*(2*A*b^3*c^3 - 12*A*a*b^2*c^2*d + 6*A*a^2*b*c*d^2 + 4*A*a^3*d^3 - 6*(
2*A*b^3*c*d^2 - 2*A*a*b^2*d^3 + (B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2 - 6*(B*
b^3*d^3*n*x^3 + B*a^2*b*c*d^2*n + (B*b^3*c*d^2 + 2*B*a*b^2*d^3)*n*x^2 + (2
*B*a*b^2*c*d^2 + B*a^2*b*d^3)*n*x)*log((b*x + a)/(d*x + c))^2 + (B*b^3*c^3
 - 12*B*a*b^2*c^2*d + 15*B*a^2*b*c*d^2 - 4*B*a^3*d^3)*n - 3*(2*A*b^3*c^2*d
 + 4*A*a*b^2*c*d^2 - 6*A*a^2*b*d^3 + (3*B*b^3*c^2*d - 2*B*a*b^2*c*d^2 - B*
a^2*b*d^3)*n)*x + 2*(B*b^3*c^3 - 6*B*a*b^2*c^2*d + 3*B*a^2*b*c*d^2 + 2*B*a
^3*d^3 - 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 - 3*(B*b^3*c^2*d + 2*B*a*b^2*c*
d^2 - 3*B*a^2*b*d^3)*x - 6*(B*b^3*d^3*x^3 + B*a^2*b*c*d^2 + (B*b^3*c*d^2 +
 2*B*a*b^2*d^3)*x^2 + (2*B*a*b^2*c*d^2 + B*a^2*b*d^3)*x)*log((b*x + a)/(d*
x + c)))*log(e) - 2*(6*A*a^2*b*c*d^2 + 3*(B*b^3*d^3*n + 2*A*b^3*d^3)*x^3 +
 3*(3*B*b^3*c*d^2*n + 2*A*b^3*c*d^2 + 4*A*a*b^2*d^3)*x^2 - (B*b^3*c^3 - 6*
B*a*b^2*c^2*d + 2*B*a^3*d^3)*n + 3*(4*A*a*b^2*c*d^2 + 2*A*a^2*b*d^3 + (B*b
^3*c^2*d + 4*B*a*b^2*c*d^2 - 2*B*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c))
)/((b^6*c^4*d - 4*a*b^5*c^3*d^2 + 6*a^2*b^4*c^2*d^3 - 4*a^3*b^3*c*d^4 + a^
4*b^2*d^5)*g^3*i^2*x^3 + (b^6*c^5 - 2*a*b^5*c^4*d - 2*a^2*b^4*c^3*d^2 + 8*
a^3*b^3*c^2*d^3 - 7*a^4*b^2*c*d^4 + 2*a^5*b*d^5)*g^3*i^2*x^2 + (2*a*b^5*c^
5 - 7*a^2*b^4*c^4*d + 8*a^3*b^3*c^3*d^2 - 2*a^4*b^2*c^2*d^3 - 2*a^5*b*c*d^
4 + a^6*d^5)*g^3*i^2*x + (a^2*b^4*c^5 - 4*a^3*b^3*c^4*d + 6*a^4*b^2*c^3*d^
2 - 4*a^5*b*c^2*d^3 + a^6*c*d^4)*g^3*i^2)
```

### 3.149.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3/(d*i*x+c*i)**2,x
)
```

output
```
Timed out
```

### 3.149.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1724 vs. 2(374) = 748.

Time = 0.29 (sec) , antiderivative size = 1724, normalized size of antiderivative = 4.54

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx=\text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output
```
1/2*B*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a
*b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*
g^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3
 - 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c
^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d
 + 3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) -
6*b*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)*g^3*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/4
*(b^3*c^3 - 12*a*b^2*c^2*d + 15*a^2*b*c*d^2 - 4*a^3*d^3 - 6*(b^3*c*d^2 - a
*b^2*d^3)*x^2 + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x
^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x + a)^2 + 6*(b^3*d^3*x^3 + a^2*
b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*l
og(d*x + c)^2 - 3*(3*b^3*c^2*d - 2*a*b^2*c*d^2 - a^2*b*d^3)*x - 6*(b^3*d^3
*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*
b*d^3)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b
^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x - 2*(b^3*d^3*x^3 + a^2*b*c*d^2
 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d^3)*x)*log(b*x
+ a))*log(d*x + c))*B*n/(a^2*b^4*c^5*g^3*i^2 - 4*a^3*b^3*c^4*d*g^3*i^2 + 6
*a^4*b^2*c^3*d^2*g^3*i^2 - 4*a^5*b*c^2*d^3*g^3*i^2 + a^6*c*d^4*g^3*i^2 ...
```

### 3.149.8 Giac [A] (verification not implemented)

Time = 137.13 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.63

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx =$$

$$-\frac{1}{4}\left(\frac{2\left(Bbn-\frac{2(bx+a)Bdn}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^2bcg^3i^2}{(dx+c)^2}-\frac{(bx+a)^2adg^3i^2}{(dx+c)^2}}+\frac{Bbn-\frac{4(bx+a)Bdn}{dx+c}+2Bb\log(e)-\frac{4(bx+a)Bd\log(e)}{dx+c}+2Ab-\frac{4(bx+}{dx}}{\frac{(bx+a)^2bcg^3i^2}{(dx+c)^2}-\frac{(bx+a)^2adg^3i^2}{(dx+c)^2}}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output
```
-1/4*(2*(B*b*n - 2*(b*x + a)*B*d*n/(d*x + c))*log((b*x + a)/(d*x + c))/((b
*x + a)^2*b*c*g^3*i^2/(d*x + c)^2 - (b*x + a)^2*a*d*g^3*i^2/(d*x + c)^2) +
 (B*b*n - 4*(b*x + a)*B*d*n/(d*x + c) + 2*B*b*log(e) - 4*(b*x + a)*B*d*log
(e)/(d*x + c) + 2*A*b - 4*(b*x + a)*A*d/(d*x + c))/((b*x + a)^2*b*c*g^3*i^
2/(d*x + c)^2 - (b*x + a)^2*a*d*g^3*i^2/(d*x + c)^2))*(b*c/(b*c - a*d)^2 -
 a*d/(b*c - a*d)^2)^2
```

## 3.149.9 Mupad [B] (verification not implemented)

Time = 3.97 (sec) , antiderivative size = 1016, normalized size of antiderivative = 2.67

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx = \frac{3\,B\,b\,d^2\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{2\,g^3\,i^2\,n\,(a\,d-b\,c)^4}$$

$$-\frac{\frac{4\,A\,a^2\,d^2-2\,A\,b^2\,c^2-4\,B\,a^2\,d^2\,n-B\,b^2\,c^2\,n+10\,A\,a\,b\,c\,d+11\,B\,a\,b\,c\,d\,n}{2\,(a\,d-b\,c)}+\frac{3\,x^2\,(}{}}{x\,\left(2\,a^4\,d^3\,g^3\,i^2-6\,a^2\,b^2\,c^2\,d\,g^3\,i^2+4\,a\,b^3\,c^3\,g^3\,i^2\right)+x^2\,\left(4\,a^3\,b\,d^3\,g^3\,i^2-6\,a^2\,b^2\,c\,d^2\,g^3\,i^2+2\,b^4\,c^3\,g^3\,i^2\right)+}$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{\frac{B\,(2\,a\,d+b\,c)}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{3\,B\,b\,d\,x}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}}{x\,\left(d\,a^2\,g^3\,i^2+2\,b\,c\,a\,g^3\,i^2\right)+x^2\,\left(c\,b^2\,g^3\,i^2+2\,a\,d\,b\,g^3\,i^2\right)+a^2\,c\,g^3\,i^2+b^2\,d\,g^3\,i^2\,x^3}\right.$$

$$\left.+\frac{3\,B\,b\,d^2\left(b\,g^3\,i^2\,n\,x^2\,(a\,d-b\,c)+\frac{a\,c\,g^3\,i^2\,n\,(a\,d-b\,c)}{d}+\frac{g^3\,i^2\,n\,x\,(a\,d+b\,c)\,(a\,d-b\,c)}{d}\right)}{g^3\,i^2\,n\,(a\,d-b\,c)^4\,\left(x\,\left(d\,a^2\,g^3\,i^2+2\,b\,c\,a\,g^3\,i^2\right)+x^2\,\left(c\,b^2\,g^3\,i^2+2\,a\,d\,b\,g^3\,i^2\right)+a^2\,c\,g^3\,i^2+b^2\,d\,g^3\,i^2\,x^3\right)}\right)$$

$$-\frac{b\,d^2\,\mathrm{atan}\left(\frac{b\,d^2\,(2\,A+B\,n)\left(\frac{a^4\,d^4\,g^3\,i^2-2\,a^3\,b\,c\,d^3\,g^3\,i^2+2\,a\,b^3\,c^3\,d\,g^3\,i^2-b^4\,c^4\,g^3\,i^2}{a^3\,d^3\,g^3\,i^2-3\,a^2\,b\,c\,d^2\,g^3\,i^2+3\,a\,b^2\,c^2\,d\,g^3\,i^2-b^3\,c^3\,g^3\,i^2}+2\,b\,d\,x\right)\left(a^3\,d^3\,g^3\,i^2-3\,a^2\,b\,c\,d^2\,g^3\,i^2+3\,a\,b^2\,c^2\,d\,g^3\,i^2-b^3\right.}{g^3\,i^2\,(6\,A\,b\,d^2+3\,B\,b\,d^2\,n)\,(a\,d-b\,c)^4}}{g^3\,i^2\,(a\,d-b\,c)^4}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^3*(c*i + d*i*x)^
2),x)
```

output
```
(3*B*b*d^2*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g^3*i^2*n*(a*d - b*c)^4) -
 ((4*A*a^2*d^2 - 2*A*b^2*c^2 - 4*B*a^2*d^2*n - B*b^2*c^2*n + 10*A*a*b*c*d
+ 11*B*a*b*c*d*n)/(2*(a*d - b*c)) + (3*x^2*(2*A*b^2*d^2 + B*b^2*d^2*n))/(a
*d - b*c) + (3*x*(6*A*a*b*d^2 + 2*A*b^2*c*d + B*a*b*d^2*n + 3*B*b^2*c*d*n)
)/(2*(a*d - b*c)))/(x*(2*a^4*d^3*g^3*i^2 + 4*a*b^3*c^3*g^3*i^2 - 6*a^2*b^2
*c^2*d*g^3*i^2) + x^2*(2*b^4*c^3*g^3*i^2 + 4*a^3*b*d^3*g^3*i^2 - 6*a^2*b^2
*c*d^2*g^3*i^2) + x^3*(2*a^2*b^2*d^3*g^3*i^2 + 2*b^4*c^2*d*g^3*i^2 - 4*a*b
^3*c*d^2*g^3*i^2) + 2*a^2*b^2*c^3*g^3*i^2 + 2*a^4*c*d^2*g^3*i^2 - 4*a^3*b*
c^2*d*g^3*i^2) - (b*d^2*atan((b*d^2*(2*A + B*n)*((a^4*d^4*g^3*i^2 - b^4*c^
4*g^3*i^2 + 2*a*b^3*c^3*d*g^3*i^2 - 2*a^3*b*c*d^3*g^3*i^2)/(a^3*d^3*g^3*i^
2 - b^3*c^3*g^3*i^2 + 3*a*b^2*c^2*d*g^3*i^2 - 3*a^2*b*c*d^2*g^3*i^2) + 2*b
*d*x)*(a^3*d^3*g^3*i^2 - b^3*c^3*g^3*i^2 + 3*a*b^2*c^2*d*g^3*i^2 - 3*a^2*b
*c*d^2*g^3*i^2)*3i)/(g^3*i^2*(6*A*b*d^2 + 3*B*b*d^2*n)*(a*d - b*c)^4))*(2*
A + B*n)*3i)/(g^3*i^2*(a*d - b*c)^4) - log(e*((a + b*x)/(c + d*x))^n)*(((B
*(2*a*d + b*c))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (3*B*b*d*x)/(2*(a^2*
d^2 + b^2*c^2 - 2*a*b*c*d)))/(x*(a^2*d*g^3*i^2 + 2*a*b*c*g^3*i^2) + x^2*(b
^2*c*g^3*i^2 + 2*a*b*d*g^3*i^2) + a^2*c*g^3*i^2 + b^2*d*g^3*i^2*x^3) + (3*
B*b*d^2*(b*g^3*i^2*n*x^2*(a*d - b*c) + (a*c*g^3*i^2*n*(a*d - b*c))/d + (g^
3*i^2*n*x*(a*d + b*c)*(a*d - b*c))/d))/(g^3*i^2*n*(a*d - b*c)^4*(x*(a^2*d*
g^3*i^2 + 2*a*b*c*g^3*i^2) + x^2*(b^2*c*g^3*i^2 + 2*a*b*d*g^3*i^2) + a^...
```

3.149. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^2}\,dx$

# 3.150 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx$



## 3.150.1 Optimal result

Integrand size = 43, antiderivative size = 477

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = &-\frac{Bd^4n(a+bx)}{(bc-ad)^5g^4i^2(c+dx)}-\frac{6b^2Bd^2n(c+dx)}{(bc-ad)^5g^4i^2(a+bx)}\\ &+\frac{b^3Bdn(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2}-\frac{b^4Bn(c+dx)^3}{9(bc-ad)^5g^4i^2(a+bx)^3}\\ &+\frac{d^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(c+dx)}\\ &-\frac{6b^2d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)}\\ &+\frac{2b^3d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2}\\ &-\frac{b^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^5g^4i^2(a+bx)^3}\\ &-\frac{4bd^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}\\ &+\frac{2bBd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^4i^2}\end{aligned}$$

output
```
-B*d^4*n*(b*x+a)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-6*b^2*B*d^2*n*(d*x+c)/(-a*d+
b*c)^5/g^4/i^2/(b*x+a)+b^3*B*d*n*(d*x+c)^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-
1/9*b^4*B*n*(d*x+c)^3/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3+d^4*(b*x+a)*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-6*b^2*d^2*(d*x+c)*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)+2*b^3*d*(d*x+c)^2
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-1/3*b^4*(d
*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3-4*b
*d^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^5/g^4/
i^2+2*b*B*d^3*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^5/g^4/i^2
```

## 3.150.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.72 (sec) , antiderivative size = 549, normalized size of antiderivative = 1.15

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx =$$

$$-\frac{\frac{bB(bc-ad)^3n}{(a+bx)^3}-\frac{6bBd(bc-ad)^2n}{(a+bx)^2}+\frac{27b^2Bcd^2n}{a+bx}-\frac{27abBd^3n}{a+bx}+\frac{12bBd^2(bc-ad)n}{a+bx}-\frac{9bBcd^3n}{c+dx}+\frac{9aBd^4n}{c+dx}+30bBd^3n\log(a+}{\phantom{x}}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d
*i*x)^2),x]
```

output
```
-1/9*((b*B*(b*c - a*d)^3*n)/(a + b*x)^3 - (6*b*B*d*(b*c - a*d)^2*n)/(a + b
*x)^2 + (27*b^2*B*c*d^2*n)/(a + b*x) - (27*a*b*B*d^3*n)/(a + b*x) + (12*b*
B*d^2*(b*c - a*d)*n)/(a + b*x) - (9*b*B*c*d^3*n)/(c + d*x) + (9*a*B*d^4*n)
/(c + d*x) + 30*b*B*d^3*n*Log[a + b*x] + (3*b*(b*c - a*d)^3*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(a + b*x)^3 - (9*b*d*(b*c - a*d)^2*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 + (27*b*d^2*(b*c - a*d)*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (9*d^3*(-(b*c) + a*d)*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + 36*b*d^3*Log[a + b*x]*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]) - 30*b*B*d^3*n*Log[c + d*x] - 36*b*d^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 18*b*B*d^3*n*(Log[a + b*x]*
(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b
*x))/(-(b*c) + a*d)]) + 18*b*B*d^3*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)]
 - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/
((b*c - a*d)^5*g^4*i^2)
```

### 3.150.3 Rubi [A] (verified)

Time = 0.47 (sec) , antiderivative size = 329, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {2961, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^4(ci+dix)^2}dx$$

$$\Big\downarrow \ 2961$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i^2(bc-ad)^5}$$

$$\Big\downarrow \ 2772$$

$$\frac{-Bn\int \left(d^4-\frac{4b(c+dx)\log\left(\frac{a+bx}{c+dx}\right)d^3}{a+bx}-\frac{6b^2(c+dx)^2d^2}{(a+bx)^2}+\frac{2b^3(c+dx)^3d}{(a+bx)^3}-\frac{b^4(c+dx)^4}{3(a+bx)^4}\right)d\frac{a+bx}{c+dx}-\frac{b^4(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+}{}$$

$$\Big\downarrow \ 2009$$

$$\frac{-\frac{b^4(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{2b^3d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(a+bx)^2}-\frac{6b^2d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}+\frac{d^4(a+bx)\left(B\log\left(e\right.\right.}{c+}}{}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d*i*x)^
2),x]
```

output
```
((d^4*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) - (6*b^2
*d^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (2*b^3*
d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 - (b^4*(
c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3) - 4*b*d
^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] - B*n*(
(d^4*(a + b*x))/(c + d*x) + (6*b^2*d^2*(c + d*x))/(a + b*x) - (b^3*d*(c +
d*x)^2)/(a + b*x)^2 + (b^4*(c + d*x)^3)/(9*(a + b*x)^3) - 2*b*d^3*Log[(a +
 b*x)/(c + d*x)]^2))/((b*c - a*d)^5*g^4*i^2)
```

#### 3.150.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.150.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1909 vs. $2(473) = 946$.

Time = 44.86 (sec) , antiderivative size = 1910, normalized size of antiderivative = 4.00

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1910 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,method=_
RETURNVERBOSE)
```

output
```
1/9*(-90*A*x*a^6*b^3*c^5*d^2*n+45*A*x*a^5*b^4*c^6*d*n+54*B*ln(e*((b*x+a)/(
d*x+c))^n)*a^7*b^2*c^5*d^2*n-18*B*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^6*d*
n+18*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b^4*c^4*d^3-27*B*x^3*a^7*b^2*c^
2*d^5*n^2+126*B*x^3*a^6*b^3*c^3*d^4*n^2-77*B*x^3*a^5*b^4*c^4*d^3*n^2-27*B*
x^3*a^4*b^5*c^5*d^2*n^2+6*B*x^3*a^3*b^6*c^6*d*n^2+108*A*x^3*ln(e*((b*x+a)/
(d*x+c))^n)*a^6*b^3*c^3*d^4+36*A*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^4*c^4
*d^3+27*A*x^3*a^7*b^2*c^2*d^5*n+63*A*x^3*a^6*b^3*c^3*d^4*n-96*A*x^3*a^5*b^
4*c^4*d^3*n+9*A*x^3*a^3*b^6*c^6*d*n+9*A*x*a^9*c^2*d^5*n-9*A*x*a^4*b^5*c^7*
n+18*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^8*b*c^4*d^3-9*B*ln(e*((b*x+a)/(d*x+c)
)^n)*a^9*c^3*d^4*n+3*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^4*c^7*n+36*A*ln(e*(
(b*x+a)/(d*x+c))^n)*a^8*b*c^4*d^3-B*x^3*a^2*b^7*c^7*n^2-3*A*x^3*a^2*b^7*c^
7*n-3*B*x^2*a^3*b^6*c^7*n^2-9*A*x^2*a^3*b^6*c^7*n-9*B*x*a^9*c^2*d^5*n^2-3*
B*x*a^4*b^5*c^7*n^2+54*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*b^2*c^3*d^4+5
4*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^3*c^4*d^3-27*B*x^2*a^8*b*c^2*d^5
*n^2+81*B*x^2*a^7*b^2*c^3*d^4*n^2+27*B*x^2*a^6*b^3*c^4*d^3*n^2-102*B*x^2*a
^5*b^4*c^5*d^2*n^2+24*B*x^2*a^4*b^5*c^6*d*n^2+108*A*x^2*ln(e*((b*x+a)/(d*x
+c))^n)*a^7*b^2*c^3*d^4+108*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^3*c^4*d^
3+27*A*x^2*a^8*b*c^2*d^5*n+27*A*x^2*a^7*b^2*c^3*d^4*n-90*A*x^2*a^5*b^4*c^5
*d^2*n+45*A*x^2*a^4*b^5*c^6*d*n+18*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^8*b*c
^3*d^4+54*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*b^2*c^4*d^3+9*B*x*a^8*b*c...
```

### 3.150.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1458 vs. $2(473) = 946$.

Time = 0.43 (sec) , antiderivative size = 1458, normalized size of antiderivative = 3.06

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output
```
-1/9*(3*A*b^4*c^4 - 18*A*a*b^3*c^3*d + 54*A*a^2*b^2*c^2*d^2 - 30*A*a^3*b*c
*d^3 - 9*A*a^4*d^4 + 6*(6*A*b^4*c*d^3 - 6*A*a*b^3*d^4 + 5*(B*b^4*c*d^3 - B
*a*b^3*d^4)*n)*x^3 + 3*(6*A*b^4*c^2*d^2 + 24*A*a*b^3*c*d^3 - 30*A*a^2*b^2*
d^4 + (11*B*b^4*c^2*d^2 + 8*B*a*b^3*c*d^3 - 19*B*a^2*b^2*d^4)*n)*x^2 + 18*
(B*b^4*d^4*n*x^4 + B*a^3*b*c*d^3*n + (B*b^4*c*d^3 + 3*B*a*b^3*d^4)*n*x^3 +
 3*(B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n*x^2 + (3*B*a^2*b^2*c*d^3 + B*a^3*b*d^
4)*n*x)*log((b*x + a)/(d*x + c))^2 + (B*b^4*c^4 - 9*B*a*b^3*c^3*d + 54*B*a
^2*b^2*c^2*d^2 - 55*B*a^3*b*c*d^3 + 9*B*a^4*d^4)*n - (6*A*b^4*c^3*d - 54*A
*a*b^3*c^2*d^2 - 18*A*a^2*b^2*c*d^3 + 66*A*a^3*b*d^4 + (5*B*b^4*c^3*d - 81
*B*a*b^3*c^2*d^2 + 57*B*a^2*b^2*c*d^3 + 19*B*a^3*b*d^4)*n)*x + 3*(B*b^4*c^
4 - 6*B*a*b^3*c^3*d + 18*B*a^2*b^2*c^2*d^2 - 10*B*a^3*b*c*d^3 - 3*B*a^4*d^
4 + 12*(B*b^4*c*d^3 - B*a*b^3*d^4)*x^3 + 6*(B*b^4*c^2*d^2 + 4*B*a*b^3*c*d^
3 - 5*B*a^2*b^2*d^4)*x^2 - 2*(B*b^4*c^3*d - 9*B*a*b^3*c^2*d^2 - 3*B*a^2*b^
2*c*d^3 + 11*B*a^3*b*d^4)*x + 12*(B*b^4*d^4*x^4 + B*a^3*b*c*d^3 + (B*b^4*c
*d^3 + 3*B*a*b^3*d^4)*x^3 + 3*(B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*x^2 + (3*B*a
^2*b^2*c*d^3 + B*a^3*b*d^4)*x)*log((b*x + a)/(d*x + c)))*log(e) + 3*(12*A*
a^3*b*c*d^3 + 2*(5*B*b^4*d^4*n + 6*A*b^4*d^4)*x^4 + 2*(6*A*b^4*c*d^3 + 18*
A*a*b^3*d^4 + (11*B*b^4*c*d^3 + 9*B*a*b^3*d^4)*n)*x^3 + 6*(6*A*a*b^3*c*d^3
 + 6*A*a^2*b^2*d^4 + (B*b^4*c^2*d^2 + 9*B*a*b^3*c*d^3)*n)*x^2 + (B*b^4*c^4
 - 6*B*a*b^3*c^3*d + 18*B*a^2*b^2*c^2*d^2 - 3*B*a^4*d^4)*n + 2*(18*A*a^...
```

## 3.150.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4/(d*i*x+c*i)**2,x
)
```

output
```
Timed out
```

### 3.150.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2563 vs. $2(473) = 946$.

Time = 0.36 (sec) , antiderivative size = 2563, normalized size of antiderivative = 5.37

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^4(ci + dix)^2}\, dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output
```
-1/3*B*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a^3
*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11
*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b
^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c
^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^
3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3
 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^4*
c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*g^
4*i^2*x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c^2
*d^3 + a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4
*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^
4*i^2) - 12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*
d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(e*(b*x/(
d*x + c) + a/(d*x + c))^n) - 1/9*(b^4*c^4 - 9*a*b^3*c^3*d + 54*a^2*b^2*c^2
*d^2 - 55*a^3*b*c*d^3 + 9*a^4*d^4 + 30*(b^4*c*d^3 - a*b^3*d^4)*x^3 + 3*(11
*b^4*c^2*d^2 + 8*a*b^3*c*d^3 - 19*a^2*b^2*d^4)*x^2 - 18*(b^4*d^4*x^4 + a^3
*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)*x
^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(b*x + a)^2 - 18*(b^4*d^4*x^4 + a
^3*b*c*d^3 + (b^4*c*d^3 + 3*a*b^3*d^4)*x^3 + 3*(a*b^3*c*d^3 + a^2*b^2*d^4)
*x^2 + (3*a^2*b^2*c*d^3 + a^3*b*d^4)*x)*log(d*x + c)^2 - (5*b^4*c^3*d -...
```

### 3.150.8 Giac [A] (verification not implemented)

Time = 180.85 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.84

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag + bgx)^4(ci + dix)^2}\, dx =$$

$$-\frac{1}{18}\left(\frac{6\left(Bb^2n - \frac{3(bx+a)Bbdn}{dx+c} + \frac{3(bx+a)^2Bd^2n}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)}{\frac{(bx+a)^3b^2c^2g^4i^2}{(dx+c)^3} - \frac{2(bx+a)^3abcdg^4i^2}{(dx+c)^3} + \frac{(bx+a)^3a^2d^2g^4i^2}{(dx+c)^3}} + \frac{2Bb^2n - \frac{9(bx+a)Bbdn}{dx+c} + \frac{18(bx+a)^2Bd^2n}{(dx+c)^2} + 6Bb^2 \cdots}{\frac{(bx+a)\cdots}{(d\cdots}}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output
```
-1/18*(6*(B*b^2*n - 3*(b*x + a)*B*b*d*n/(d*x + c) + 3*(b*x + a)^2*B*d^2*n/
(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^3*b^2*c^2*g^4*i^2/(d*x +
c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4*i^2/(d*x + c)^3 + (b*x + a)^3*a^2*d^2*g^4
*i^2/(d*x + c)^3) + (2*B*b^2*n - 9*(b*x + a)*B*b*d*n/(d*x + c) + 18*(b*x +
 a)^2*B*d^2*n/(d*x + c)^2 + 6*B*b^2*log(e) - 18*(b*x + a)*B*b*d*log(e)/(d*
x + c) + 18*(b*x + a)^2*B*d^2*log(e)/(d*x + c)^2 + 6*A*b^2 - 18*(b*x + a)*
A*b*d/(d*x + c) + 18*(b*x + a)^2*A*d^2/(d*x + c)^2)/((b*x + a)^3*b^2*c^2*g
^4*i^2/(d*x + c)^3 - 2*(b*x + a)^3*a*b*c*d*g^4*i^2/(d*x + c)^3 + (b*x + a)
^3*a^2*d^2*g^4*i^2/(d*x + c)^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)^2
```

### 3.150.9 Mupad [B] (verification not implemented)

Time = 6.58 (sec) , antiderivative size = 1665, normalized size of antiderivative = 3.49

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^4*(c*i + d*i*x)^
2),x)
```

output
```
(2*B*b*d^3*log(e*((a + b*x)/(c + d*x))^n)^2)/(g^4*i^2*n*(a*d - b*c)^3*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d)) - log(e*((a + b*x)/(c + d*x))^n)*(((B*(3*a*d
+ b*c))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (4*B*b*d*x)/(3*(a^2*d^2 + b^
2*c^2 - 2*a*b*c*d)))/(x^3*(b^3*c*g^4*i^2 + 3*a*b^2*d*g^4*i^2) + x^2*(3*a*b
^2*c*g^4*i^2 + 3*a^2*b*d*g^4*i^2) + x*(a^3*d*g^4*i^2 + 3*a^2*b*c*g^4*i^2)
+ a^3*c*g^4*i^2 + b^3*d*g^4*i^2*x^4) + (4*B*b*d^3*(x*((a*d + b*c)*((a*g^4*
i^2*n*(a*d - b*c))/(2*d) + (g^4*i^2*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2))
+ (a*b*c*g^4*i^2*n*(a*d - b*c))/d) + x^2*(b*d*((a*g^4*i^2*n*(a*d - b*c))/(
2*d) + (g^4*i^2*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2)) + (b*g^4*i^2*n*(a*d
+ b*c)*(a*d - b*c))/d) + a*c*((a*g^4*i^2*n*(a*d - b*c))/(2*d) + (g^4*i^2*n
*(a*d - b*c)*(2*a*d - b*c))/(2*d^2)) + b^2*g^4*i^2*n*x^3*(a*d - b*c)))/(g^
4*i^2*n*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(x^3*(b^3*c*g^4*i^2
+ 3*a*b^2*d*g^4*i^2) + x^2*(3*a*b^2*c*g^4*i^2 + 3*a^2*b*d*g^4*i^2) + x*(a^
3*d*g^4*i^2 + 3*a^2*b*c*g^4*i^2) + a^3*c*g^4*i^2 + b^3*d*g^4*i^2*x^4))) -
(b*d^3*atan((b*d^3*((a^5*d^5*g^4*i^2 + b^5*c^5*g^4*i^2 - 3*a*b^4*c^4*d*g^4
*i^2 - 3*a^4*b*c*d^4*g^4*i^2 + 2*a^2*b^3*c^3*d^2*g^4*i^2 + 2*a^3*b^2*c^2*d
^3*g^4*i^2)/(a^4*d^4*g^4*i^2 + b^4*c^4*g^4*i^2 - 4*a*b^3*c^3*d*g^4*i^2 - 4
*a^3*b*c*d^3*g^4*i^2 + 6*a^2*b^2*c^2*d^2*g^4*i^2) + 2*b*d*x)*(6*A + 5*B*n)
*(a^4*d^4*g^4*i^2 + b^4*c^4*g^4*i^2 - 4*a*b^3*c^3*d*g^4*i^2 - 4*a^3*b*c*d^
3*g^4*i^2 + 6*a^2*b^2*c^2*d^2*g^4*i^2)*2i)/(g^4*i^2*(12*A*b*d^3 + 10*B*...
```

3.150. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^2}\,dx$

# 3.151 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$



## 3.151.1 Optimal result

Integrand size = 43, antiderivative size = 382

$$\begin{aligned}
&\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx\\
&= -\frac{3B(bc-ad)g^3n(a+bx)^2}{4d^2i^3(c+dx)^2}-\frac{3bB(bc-ad)g^3n(a+bx)}{d^3i^3(c+dx)}\\
&\quad+\frac{b(bc-ad)g^3(3A+Bn)(a+bx)}{d^3i^3(c+dx)}+\frac{3bB(bc-ad)g^3(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^3(c+dx)}\\
&\quad+\frac{g^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{di^3(c+dx)^2}\\
&\quad+\frac{(bc-ad)g^3(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^3(c+dx)^2}\\
&\quad+\frac{b^2(bc-ad)g^3\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3}\\
&\quad+\frac{3b^2B(bc-ad)g^3n\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}
\end{aligned}$$

output
```
-3/4*B*(-a*d+b*c)*g^3*n*(b*x+a)^2/d^2/i^3/(d*x+c)^2-3*b*B*(-a*d+b*c)*g^3*n
*(b*x+a)/d^3/i^3/(d*x+c)+b*(-a*d+b*c)*g^3*(B*n+3*A)*(b*x+a)/d^3/i^3/(d*x+c
)+3*b*B*(-a*d+b*c)*g^3*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/d^3/i^3/(d*x+c)+g
^3*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i^3/(d*x+c)^2+1/2*(-a*d+b*c
)*g^3*(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/d^2/i^3/(d*x+c)^2+
b^2*(-a*d+b*c)*g^3*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b
/(d*x+c))/d^4/i^3+3*b^2*B*(-a*d+b*c)*g^3*n*polylog(2,d*(b*x+a)/b/(d*x+c))/
d^4/i^3
```

### 3.151.2 Mathematica [A] (verified)

Time = 0.25 (sec) , antiderivative size = 334, normalized size of antiderivative = 0.87

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$=\frac{g^3\left(4Ab^3dx-\frac{B(bc-ad)^3n}{(c+dx)^2}+\frac{10bB(bc-ad)^2n}{c+dx}+10b^2B(bc-ad)n\log(a+bx)+4b^2Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i +
d*i*x)^3,x]
```

output
```
(g^3*(4*A*b^3*d*x - (B*(b*c - a*d)^3*n)/(c + d*x)^2 + (10*b*B*(b*c - a*d)^
2*n)/(c + d*x) + 10*b^2*B*(b*c - a*d)*n*Log[a + b*x] + 4*b^2*B*d*(a + b*x)
*Log[e*((a + b*x)/(c + d*x))^n] + (2*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(c + d*x)^2 - (12*b*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(c + d*x) - 14*b^2*B*(b*c - a*d)*n*Log[c + d*x] - 12*b^2*
(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 6*b^2*B*
(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c
+ d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(4*d^4*i^3)
```

### 3.151.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 321, normalized size of antiderivative = 0.84, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.093, Rules used = {2961, 2784, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ci+dix)^3}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{g^3(bc-ad)\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^3}$$

$$\Big\downarrow \text{ 2784}$$

$$\frac{g^3(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int \frac{(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{d}\right)}{i^3}$$

$$\Big\downarrow \text{ 2793}$$

$$\frac{g^3(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int \left(-\frac{\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}-\frac{\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{d^2}-\frac{(a+bx)\left(3A+Bn+3B\right.}{d(c+}\right.}{d}\right.}{i^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^3(bc-ad)\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{-\frac{b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{d^3}-\frac{b(a+bx)(3A+Bn)}{d^2(c+dx)}-\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a-}{c-}\right.\right.\right.}{2d(c+}\right.}{i^3}$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)
^3,x]
```

---

3.151. $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}dx$

output
```
((b*c - a*d)*g^3*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*
(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))) - ((3*B*n*(a + b*x)^2)/(4*d*(c
+ d*x)^2) + (3*b*B*n*(a + b*x))/(d^2*(c + d*x)) - (b*(3*A + B*n)*(a + b*x)
)/(d^2*(c + d*x)) - (3*b*B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d^2*
(c + d*x)) - ((a + b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n])
)/(2*d*(c + d*x)^2) - (b^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]
)*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 - (3*b^2*B*n*PolyLog[2, (d*(a
+ b*x))/(b*(c + d*x))])/d^3)/d))/i^3
```

### 3.151.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.151. $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$

### 3.151.4 Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(dix+ci)^3}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

### 3.151.5 Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output
```
integral((A*b^3*g^3*x^3 + 3*A*a*b^2*g^3*x^2 + 3*A*a^2*b*g^3*x + A*a^3*g^3
+ (B*b^3*g^3*x^3 + 3*B*a*b^2*g^3*x^2 + 3*B*a^2*b*g^3*x + B*a^3*g^3)*log(e*
((b*x + a)/(d*x + c))^n))/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x +
 c^3*i^3), x)
```

### 3.151.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**3,x
)
```

output
```
Timed out
```

### 3.151.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2894 vs. 2(377) = 754.

Time = 0.55 (sec) , antiderivative size = 2894, normalized size of antiderivative = 7.58

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output
```
3/4*B*a^2*b*g^3*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a
*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^
3) + 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^
4)*i^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a
^2*d^4)*i^3)) + 1/4*B*a^3*g^3*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4
)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2
*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*
x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*A*b^3*g^3*((6*c^2*
d*x + 5*c^3)/(d^6*i^3*x^2 + 2*c*d^5*i^3*x + c^2*d^4*i^3) - 2*x/(d^3*i^3) +
 6*c*log(d*x + c)/(d^4*i^3)) + 3/2*A*a*b^2*g^3*((4*c*d*x + 3*c^2)/(d^5*i^3
*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x + c)/(d^3*i^3)) - 3/2*(2*d
*x + c)*B*a^2*b*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*i^3*x^2 +
2*c*d^3*i^3*x + c^2*d^2*i^3) - 3/2*(2*d*x + c)*A*a^2*b*g^3/(d^4*i^3*x^2 +
2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*B*a^3*g^3*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A*a^3*g^3/(d^3*i
^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 1/2*(6*a^3*b^2*d^3*g^3*log(e) - (7*g
^3*n + 6*g^3*log(e))*b^5*c^3 + (19*g^3*n + 18*g^3*log(e))*a*b^4*c^2*d - 2*
(7*g^3*n + 9*g^3*log(e))*a^2*b^3*c*d^2)*B*log(d*x + c)/(b^2*c^2*d^4*i^3 -
2*a*b*c*d^5*i^3 + a^2*d^6*i^3) + 1/4*(4*(b^5*c^2*d^3*g^3*log(e) - 2*a*b^4*
c*d^4*g^3*log(e) + a^2*b^3*d^5*g^3*log(e))*B*x^3 + 8*(b^5*c^3*d^2*g^3*l...
```

### 3.151.8 Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(d*i*x +
c*i)^3, x)
```

### 3.151.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx=\int \frac{\left(a\,g+b\,g\,x\right)^3\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{{\left(c\,i+d\,i\,x\right)}^3}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^3,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^3, x)
```

# 3.152 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$



## 3.152.1 Optimal result

Integrand size = 43, antiderivative size = 263

$$\begin{aligned}\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx &= \frac{Bg^2n(a+bx)^2}{4di^3(c+dx)^2}-\frac{Abg^2(a+bx)}{d^2i^3(c+dx)}\\ &\quad+\frac{bBg^2n(a+bx)}{d^2i^3(c+dx)}\\ &\quad-\frac{bBg^2(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^3(c+dx)}\\ &\quad-\frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^3(c+dx)^2}\\ &\quad-\frac{b^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^3}\\ &\quad-\frac{b^2Bg^2n\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}\end{aligned}$$

output
```
1/4*B*g^2*n*(b*x+a)^2/d/i^3/(d*x+c)^2-A*b*g^2*(b*x+a)/d^2/i^3/(d*x+c)+b*B*
g^2*n*(b*x+a)/d^2/i^3/(d*x+c)-b*B*g^2*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/d^
2/i^3/(d*x+c)-1/2*g^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d/i^3/(d*x
+c)^2-b^2*g^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^3
/i^3-b^2*B*g^2*n*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^3
```

### 3.152.2 Mathematica [A] (verified)

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 0.98

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \frac{g^2\left(\frac{B(bc-ad)^2n}{(c+dx)^2}-\frac{6bB(bc-ad)n}{c+dx}-6b^2Bn\log(a+bx)-\frac{2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2}+\frac{8b(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx}\right.}{}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i +
d*i*x)^3,x]
```

output
```
(g^2*((B*(b*c - a*d)^2*n)/(c + d*x)^2 - (6*b*B*(b*c - a*d)*n)/(c + d*x) -
6*b^2*B*n*Log[a + b*x] - (2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(c + d*x)^2 + (8*b*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]))/(c + d*x) + 6*b^2*B*n*Log[c + d*x] + 4*b^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*Log[c + d*x] - 2*b^2*B*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)
] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))
)/(4*d^3*i^3)
```

### 3.152.3 Rubi [A] (verified)

Time = 0.43 (sec) , antiderivative size = 229, normalized size of antiderivative = 0.87, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {2961, 2793, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ci+dix)^3}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{g^2\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^3}$$

$$\Big\downarrow \text{2793}$$

$$\frac{g^2\int\left(-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{d^2}-\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d(c+dx)}\right)d\frac{a+bx}{c+dx}}{i^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^2\left(-\frac{b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3}-\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2}-\frac{Ab(a+bx)}{d^2(c+dx)}-\frac{b^2Bn\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3}-\frac{bB(a}{}\right.}{i^3}$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)
^3,x]
```

output
```
(g^2*((B*n*(a + b*x)^2)/(4*d*(c + d*x)^2) - (A*b*(a + b*x))/(d^2*(c + d*x)
) + (b*B*n*(a + b*x))/(d^2*(c + d*x)) - (b*B*(a + b*x)*Log[e*((a + b*x)/(c
 + d*x))^n])/(d^2*(c + d*x)) - ((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(2*d*(c + d*x)^2) - (b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*
Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 - (b^2*B*n*PolyLog[2, (d*(a + b*
x))/(b*(c + d*x))])/d^3))/i^3
```

### 3.152.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2793
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[a + b*Log[c*x^n],
 (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e,
 f, m, n, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IntegerQ[m] && Integer
Q[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.152. $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$

### 3.152.4 Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)}{(dix+ci)^3}dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x)
```

### 3.152.5 Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output
```
integral((A*b^2*g^2*x^2 + 2*A*a*b*g^2*x + A*a^2*g^2 + (B*b^2*g^2*x^2 + 2*B
*a*b*g^2*x + B*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n))/(d^3*i^3*x^3 + 3*c
*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

### 3.152.6 Sympy [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{g^2\left(\int \frac{Aa^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{Ab^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{Ba^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2Aabx}{c^3+3c^2dx+3cd^2x^2+d}\right.}{i^3}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**3,x
)
```

output
```
g**2*(Integral(A*a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
+ Integral(A*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x)
 + Integral(B*a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*
d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*a*b*x/(c**3 + 3*c**2*d
*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(B*b**2*x**2*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(2*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*
c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

### 3.152.7 Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output
```
1/2*B*a*b*g^2*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d
^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3)
 + 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)
*i^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2
*d^4)*i^3)) + 1/4*B*a^2*g^2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*
i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b
^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x
+ c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 1/2*A*b^2*g^2*((4*c*d*x
+ 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x + c)/(d^3
*i^3)) - 1/2*B*b^2*g^2*((2*(d^2*n*x^2 + 2*c*d*n*x + c^2*n)*log(b*x + a)*lo
g(d*x + c) - (d^2*n*x^2 + 2*c*d*n*x + c^2*n)*log(d*x + c)^2 - (4*c*d*x + 3
*c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c))*log((b*x + a)^n) + (4*c*d
*x + 3*c^2 + 2*(d^2*x^2 + 2*c*d*x + c^2)*log(d*x + c))*log((d*x + c)^n))/(
d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) - 2*integrate(1/2*(2*b*d^3*x^3*
log(e) + 2*a*d^3*x^2*log(e) - 3*b*c^3*n + 3*a*c^2*d*n - 4*(b*c^2*d*n - a*c
*d^2*n)*x + 2*(b*d^3*n*x^3 + a*c^2*d*n + (2*b*c*d^2*n + a*d^3*n)*x^2 + (b*
c^2*d*n + 2*a*c*d^2*n)*x)*log(b*x + a))/(b*d^6*i^3*x^4 + a*c^3*d^3*i^3 + (
3*b*c*d^5*i^3 + a*d^6*i^3)*x^3 + 3*(b*c^2*d^4*i^3 + a*c*d^5*i^3)*x^2 + (b*
c^3*d^3*i^3 + 3*a*c^2*d^4*i^3)*x), x)) - (2*d*x + c)*B*a*b*g^2*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n)/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3)...
```

### 3.152.8 Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)/(d*i*x +
c*i)^3, x)
```

### 3.152.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx=\int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(c\,i+d\,i\,x)^3}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^3,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^3, x)
```

# 3.153 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$



## 3.153.1 Optimal result

Integrand size = 41, antiderivative size = 89

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx = -\frac{Bgn(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} + \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)i^3(c+dx)^2}$$

output
```
-1/4*B*g*n*(b*x+a)^2/(-a*d+b*c)/i^3/(d*x+c)^2+1/2*g*(b*x+a)^2*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^3/(d*x+c)^2
```

## 3.153.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 215 vs. 2(89) = 178.

Time = 0.10 (sec) , antiderivative size = 215, normalized size of antiderivative = 2.42

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{g\left(\frac{(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2(c+dx)^2} - \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^2(c+dx)} + \frac{bBn\left(\frac{1}{c+dx}+\frac{b\log(a+bx)}{bc-ad}-\frac{b\log(c+dx)}{bc-ad}\right)}{d^2} - \frac{Bn\left(\frac{bc-ad}{(c+dx)^2}+\frac{2b}{c+dx}+\frac{2b^2\log}{bc-}\right.}{4d^2}\right.}{i^3}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*
i*x)^3,x]
```

output
```
(g*(((b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d^2*(c + d*x)^
2) - (b*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d^2*(c + d*x)) + (b*B*n*(
(c + d*x)^(-1) + (b*Log[a + b*x])/(b*c - a*d) - (b*Log[c + d*x])/(b*c - a*
d)))/d^2 - (B*n*((b*c - a*d)/(c + d*x)^2 + (2*b)/(c + d*x) + (2*b^2*Log[a
+ b*x])/(b*c - a*d) - (2*b^2*Log[c + d*x])/(b*c - a*d)))/(4*d^2)))/i^3
```

### 3.153.3 Rubi [A] (verified)

Time = 0.25 (sec) , antiderivative size = 76, normalized size of antiderivative = 0.85, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.049, Rules used = {2961, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{(ci+dix)^3}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx}d\frac{a+bx}{c+dx}}{i^3(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{g\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}-\frac{Bn(a+bx)^2}{4(c+dx)^2}\right)}{i^3(bc-ad)}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c*i + d*i*x)^3
,x]
```

output
```
(g*(-1/4*(B*n*(a + b*x)^2)/(c + d*x)^2 + ((a + b*x)^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(2*(c + d*x)^2)))/((b*c - a*d)*i^3)
```

### 3.153.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.153.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(85) = 170$.

Time = 4.64 (sec) , antiderivative size = 237, normalized size of antiderivative = 2.66

| method | result |
|---|---|
| parallelrisch | $-\frac{2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{2}b\,d^{4}gn+2B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}d^{4}gn-2Bxa\,b^{2}d^{4}g\,n^{2}+2Bx\,b^{3}c\,d^{3}g\,n^{2}+4Axa\,b^{2}d^{4}gn-4Ax\,b^{3}c\,d^{3}gn+}{4i^{3}(dx+c)^{2}b\,d^{4}n(ad-cb)}$ |

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,method=_RE
TURNVERBOSE)
```

output
```
-1/4*(2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^4*g*n+2*B*x^2*ln(e*((b*x+a)/(d
*x+c))^n)*b^3*d^4*g*n-2*B*x*a*b^2*d^4*g*n^2+2*B*x*b^3*c*d^3*g*n^2+4*A*x*a*
b^2*d^4*g*n-4*A*x*b^3*c*d^3*g*n+4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^4*
g*n-B*a^2*b*d^4*g*n^2+B*b^3*c^2*d^2*g*n^2+2*A*a^2*b*d^4*g*n-2*A*b^3*c^2*d^
2*g*n)/i^3/(d*x+c)^2/b/d^4/n/(a*d-b*c)
```

### 3.153.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 250 vs. 2(85) = 170.

Time = 0.32 (sec) , antiderivative size = 250, normalized size of antiderivative = 2.81

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{(Bb^2c^2-Ba^2d^2)gn-2\,(Ab^2c^2-Aa^2d^2)g+2\,((Bb^2cd-Babd^2)gn-2\,(Ab^2cd-Aabd^2)g)x-2\,(2\,(Bb^2c}{4\,((bcd^4-ad^5)i^3x^2+2\,(bc^2d^3-ac}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output
```
1/4*((B*b^2*c^2 - B*a^2*d^2)*g*n - 2*(A*b^2*c^2 - A*a^2*d^2)*g + 2*((B*b^2
*c*d - B*a*b*d^2)*g*n - 2*(A*b^2*c*d - A*a*b*d^2)*g)*x - 2*(2*(B*b^2*c*d -
 B*a*b*d^2)*g*x + (B*b^2*c^2 - B*a^2*d^2)*g)*log(e) + 2*(B*b^2*d^2*g*n*x^2
 + 2*B*a*b*d^2*g*n*x + B*a^2*d^2*g*n)*log((b*x + a)/(d*x + c)))/((b*c*d^4
- a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)
*i^3)
```

### 3.153.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1328 vs. 2(75) = 150.

Time = 111.17 (sec) , antiderivative size = 1328, normalized size of antiderivative = 14.92

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \begin{cases} -\frac{Abg}{cd^2i^3+d^3i^3x}-\frac{Bbg\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)}{cd^2i^3+d^3i^3x} \\ \frac{Aagx+\frac{Abgx^2}{2}+\frac{Ba^2g\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2b}-\frac{Bagnx}{2}+Bagx\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)-\frac{Bbgnx^2}{4}+\frac{Bbgx^2\log\left(e\left(\frac{a}{c}+\frac{bx}{c}\right)^n\right)}{2}}{c^3i^3} \\ -\frac{2Aa^2d^2g}{4ac^2d^3i^3+8acd^4i^3x+4ad^5i^3x^2-4bc^3d^2i^3-8bc^2d^3i^3x-4bcd^4i^3x^2}-\frac{4Aabd^2gx}{4ac^2d^3i^3+8acd^4i^3x+4ad^5i^3x^2-4bc^3d^2i^3-8bc^2d^3i^3x-4bcd^4i^3x^2}+ \end{cases}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**3,x)
```

output
```
Piecewise((-A*b*g/(c*d**2*i**3 + d**3*i**3*x) - B*b*g*log(e*(b*c/(c*d + d*
*2*x) + b*x/(c + d*x))**n)/(c*d**2*i**3 + d**3*i**3*x), Eq(a, b*c/d)), ((A
*a*g*x + A*b*g*x**2/2 + B*a**2*g*log(e*(a/c + b*x/c)**n)/(2*b) - B*a*g*n*x
/2 + B*a*g*x*log(e*(a/c + b*x/c)**n) - B*b*g*n*x**2/4 + B*b*g*x**2*log(e*(
a/c + b*x/c)**n)/2)/(c**3*i**3), Eq(d, 0)), (-2*A*a**2*d**2*g/(4*a*c**2*d*
*3*i**3 + 8*a*c*d**4*i**3*x + 4*a*d**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*
b*c**2*d**3*i**3*x - 4*b*c*d**4*i**3*x**2) - 4*A*a*b*d**2*g*x/(4*a*c**2*d*
*3*i**3 + 8*a*c*d**4*i**3*x + 4*a*d**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*
b*c**2*d**3*i**3*x - 4*b*c*d**4*i**3*x**2) + 2*A*b**2*c**2*g/(4*a*c**2*d**
3*i**3 + 8*a*c*d**4*i**3*x + 4*a*d**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*b
*c**2*d**3*i**3*x - 4*b*c*d**4*i**3*x**2) + 4*A*b**2*c*d*g*x/(4*a*c**2*d**
3*i**3 + 8*a*c*d**4*i**3*x + 4*a*d**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*b
*c**2*d**3*i**3*x - 4*b*c*d**4*i**3*x**2) + B*a**2*d**2*g*n/(4*a*c**2*d**3
*i**3 + 8*a*c*d**4*i**3*x + 4*a*d**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*b*
c**2*d**3*i**3*x - 4*b*c*d**4*i**3*x**2) - 2*B*a**2*d**2*g*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)/(4*a*c**2*d**3*i**3 + 8*a*c*d**4*i**3*x + 4*a*d**
5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*b*c**2*d**3*i**3*x - 4*b*c*d**4*i**3*
x**2) + 2*B*a*b*d**2*g*n*x/(4*a*c**2*d**3*i**3 + 8*a*c*d**4*i**3*x + 4*a*d
**5*i**3*x**2 - 4*b*c**3*d**2*i**3 - 8*b*c**2*d**3*i**3*x - 4*b*c*d**4*i**
3*x**2) - 4*B*a*b*d**2*g*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(4*a...
```

### 3.153.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 578 vs. $2(85) = 170$.

Time = 0.20 (sec) , antiderivative size = 578, normalized size of antiderivative = 6.49

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$= \frac{1}{4}Bbgn\left(\frac{bc^2-3\,acd+2\,(bcd-2\,ad^2)x}{(bcd^4-ad^5)i^3x^2+2\,(bc^2d^3-acd^4)i^3x+(bc^3d^2-ac^2d^3)i^3}+\frac{2\,(b^2c-2\,abd)\log(bx+a)}{(b^2c^2d^2-2\,abcd^3+a^2d^4)i^3}-\frac{2\,(b\right.$$
$$+\frac{1}{4}Bagn\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}+\frac{2\,b^2\log(bx+a)}{(b^2c^2d-2\,abcd^2+a^2d^3)i^3}-\frac{}{(b^2\right.$$
$$-\frac{(2\,dx+c)Bbg\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(d^4i^3x^2+2\,cd^3i^3x+c^2d^2i^3)}-\frac{(2\,dx+c)Abg}{2\,(d^4i^3x^2+2\,cd^3i^3x+c^2d^2i^3)}$$
$$-\frac{Bag\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}-\frac{Aag}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output
```
1/4*B*b*g*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d^5)*
i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3) + 2
*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i^3
) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4
)*i^3)) + 1/4*B*a*g*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2
+ 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b
*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b
^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*(2*d*x + c)*B*b*g*log(e*(b*x
/(d*x + c) + a/(d*x + c))^n)/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) -
 1/2*(2*d*x + c)*A*b*g/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - 1/2*B
*a*g*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x +
 c^2*d*i^3) - 1/2*A*a*g/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
```

### 3.153.8 Giac [A] (verification not implemented)

Time = 0.80 (sec) , antiderivative size = 101, normalized size of antiderivative = 1.13

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\,(bx+a)^2Bgn\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)^2i^3}-\frac{(Bgn-2\,Bg\log(e)-2\,Ag)(bx+a)^2}{(dx+c)^2i^3}\right)\left(\frac{bc}{(bc-ad)^2}-\frac{ad}{(bc-ad)^2}\right)$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output
```
1/4*(2*(b*x + a)^2*B*g*n*log((b*x + a)/(d*x + c))/((d*x + c)^2*i^3) - (B*g
*n - 2*B*g*log(e) - 2*A*g)*(b*x + a)^2/((d*x + c)^2*i^3))*(b*c/(b*c - a*d)
^2 - a*d/(b*c - a*d)^2)
```

### 3.153.9 Mupad [B] (verification not implemented)

Time = 1.93 (sec) , antiderivative size = 205, normalized size of antiderivative = 2.30

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$$
$$=-\frac{x\,(2\,A\,b\,d\,g-B\,b\,d\,g\,n)+A\,a\,d\,g+A\,b\,c\,g-\frac{B\,a\,d\,g\,n}{2}-\frac{B\,b\,c\,g\,n}{2}}{2\,c^2\,d^2\,i^3+4\,c\,d^3\,i^3\,x+2\,d^4\,i^3\,x^2}$$
$$-\frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(\frac{B\,a\,g}{2\,d}+\frac{B\,b\,c\,g}{2\,d^2}+\frac{B\,b\,g\,x}{d}\right)}{c^2\,i^3+2\,c\,d\,i^3\,x+d^2\,i^3\,x^2}+\frac{B\,b^2\,g\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,1\mathrm{i}}{d^2\,i^3\,(a\,d-b\,c)}$$

input
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)^3
,x)
```

output
```
(B*b^2*g*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*1i)/(d^2*i^3*(a*d -
b*c)) - (log(e*((a + b*x)/(c + d*x))^n)*((B*a*g)/(2*d) + (B*b*c*g)/(2*d^2)
 + (B*b*g*x)/d))/(c^2*i^3 + d^2*i^3*x^2 + 2*c*d*i^3*x) - (x*(2*A*b*d*g - B
*b*d*g*n) + A*a*d*g + A*b*c*g - (B*a*d*g*n)/2 - (B*b*c*g*n)/2)/(2*c^2*d^2*
i^3 + 2*d^4*i^3*x^2 + 4*c*d^3*i^3*x)
```

## 3.153. $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(ci+dix)^3}\,dx$

# 3.154 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx$



## 3.154.1 Optimal result

Integrand size = 33, antiderivative size = 151

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx = \frac{Bn}{4di^3(c+dx)^2}+\frac{bBn}{2d(bc-ad)i^3(c+dx)}+\frac{b^2Bn\log(a+bx)}{2d(bc-ad)^2i^3}-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2di^3(c+dx)^2}-\frac{b^2Bn\log(c+dx)}{2d(bc-ad)^2i^3}$$

output
```
1/4*B*n/d/i^3/(d*x+c)^2+1/2*b*B*n/d/(-a*d+b*c)/i^3/(d*x+c)+1/2*b^2*B*n*ln(
b*x+a)/d/(-a*d+b*c)^2/i^3+1/2*(-A-B*ln(e*((b*x+a)/(d*x+c))^n))/d/i^3/(d*x+
c)^2-1/2*b^2*B*n*ln(d*x+c)/d/(-a*d+b*c)^2/i^3
```

## 3.154.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 115, normalized size of antiderivative = 0.76

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx = \frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+\frac{Bn\left((bc-ad)(3bc-ad+2bdx)+2b^2(c+dx)^2\log(a+bx)-2b^2(c+dx)^2\log(c+dx)\right)}{(bc-ad)^2}}{4di^3(c+dx)^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x)^3,x]
```

output
```
(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + (B*n*((b*c - a*d)*(3*b*c - a*
d + 2*b*d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c +
d*x]))/(b*c - a*d)^2)/(4*d*i^3*(c + d*x)^2)
```

### 3.154.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 142, normalized size of antiderivative = 0.94, number of steps used = 4, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {2947, 27, 54, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ci+dix)^3} \, dx$$

$$\Big\downarrow \text{ 2947}$$

$$\frac{Bn(bc-ad) \int \frac{1}{i^2(a+bx)(c+dx)^3}dx}{2di}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2di^3(c+dx)^2}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{Bn(bc-ad) \int \frac{1}{(a+bx)(c+dx)^3}dx}{2di^3}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2di^3(c+dx)^2}$$

$$\Big\downarrow \text{ 54}$$

$$\frac{Bn(bc-ad) \int \left(\frac{b^3}{(bc-ad)^3(a+bx)}-\frac{db^2}{(bc-ad)^3(c+dx)}-\frac{db}{(bc-ad)^2(c+dx)^2}-\frac{d}{(bc-ad)(c+dx)^3}\right)dx}{2di^3}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2di^3(c+dx)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{Bn(bc-ad) \left(\frac{b^2 \log(a+bx)}{(bc-ad)^3}-\frac{b^2 \log(c+dx)}{(bc-ad)^3}+\frac{b}{(c+dx)(bc-ad)^2}+\frac{1}{2(c+dx)^2(bc-ad)}\right)}{2di^3}-\frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2di^3(c+dx)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(c*i + d*i*x)^3,x]
```

output
```
-1/2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])/(d*i^3*(c + d*x)^2) + (B*(b*c
- a*d)*n*(1/(2*(b*c - a*d)*(c + d*x)^2) + b/((b*c - a*d)^2*(c + d*x)) + (b
^2*Log[a + b*x])/(b*c - a*d)^3 - (b^2*Log[c + d*x])/(b*c - a*d)^3))/(2*d*i
^3)
```

### 3.154.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2947
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(f + g*x)^(m + 1)*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])/(g*(m + 1))), x] - Simp[B*n*((b*c - a*d)
/(g*(m + 1)))   Int[(f + g*x)^(m + 1)/((a + b*x)*(c + d*x)), x], x] /; Free
Q[{a, b, c, d, e, f, g, A, B, m, n}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1]
&& NeQ[m, -2]
```

## 3.154.4 Maple [A] (verified)

Time = 4.59 (sec) , antiderivative size = 272, normalized size of antiderivative = 1.80

| method | result |
|---|---|
| parallelrisch | $-\frac{2A\,a^{2}b\,d^{5}n+2A\,b^{3}c^{2}d^{3}n-B\,a^{2}b\,d^{5}n^{2}-3B\,b^{3}c^{2}d^{3}n^{2}+4Ba\,b^{2}c\,d^{4}n^{2}-4Aa\,b^{2}c\,d^{4}n-2B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}d^{5}n+2Bxa\,b^{2}d^{5}}{4i^{3}(dx+c)^{2}n(a^{2}d^{2}-2abcd+}$ |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x,method=_RETURNVERBOSE)
```

output
```
-1/4*(2*A*a^2*b*d^5*n+2*A*b^3*c^2*d^3*n-B*a^2*b*d^5*n^2-3*B*b^3*c^2*d^3*n^
2+4*B*a*b^2*c*d^4*n^2-4*A*a*b^2*c*d^4*n-2*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*
b^3*d^5*n+2*B*x*a*b^2*d^5*n^2-2*B*x*b^3*c*d^4*n^2+2*B*ln(e*((b*x+a)/(d*x+c
))^n)*a^2*b*d^5*n-4*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^4*n-4*B*ln(e*((b
*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n)/i^3/(d*x+c)^2/n/(a^2*d^2-2*a*b*c*d+b^2*c^
2)/b/d^4
```

### 3.154.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 266, normalized size of antiderivative = 1.76

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx = -\frac{2\,Ab^2c^2-4\,Aabcd+2\,Aa^2d^2-2\,(Bb^2cd-Babd^2)nx-(3\,Bb^2c^2-4\,Babcd+Ba^2d^2)n+2\,(Bb^2c^2-2}{4\,((b^2c^2d^3-2\,abcd^4+a^2d^5)i^3x^2+2\,(b^2c^3d^2-2\,abc^2d^3+}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, algorithm="fri
cas")
```

output
```
-1/4*(2*A*b^2*c^2 - 4*A*a*b*c*d + 2*A*a^2*d^2 - 2*(B*b^2*c*d - B*a*b*d^2)*
n*x - (3*B*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2)*n + 2*(B*b^2*c^2 - 2*B*a*b*c
*d + B*a^2*d^2)*log(e) - 2*(B*b^2*d^2*n*x^2 + 2*B*b^2*c*d*n*x + (2*B*a*b*c
*d - B*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 +
 a^2*d^5)*i^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*i^3*x + (b
^2*c^4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*i^3)
```

### 3.154.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2103 vs. 2(133) = 266.

Time = 108.22 (sec) , antiderivative size = 2103, normalized size of antiderivative = 13.93

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(d*i*x+c*i)**3,x)
```

output
```
Piecewise((-A/(2*c**2*d*i**3 + 4*c*d**2*i**3*x + 2*d**3*i**3*x**2) - B*log
(e*(b*c/(c*d + d**2*x) + b*x/(c + d*x))**n)/(2*c**2*d*i**3 + 4*c*d**2*i**3
*x + 2*d**3*i**3*x**2), Eq(a, b*c/d)), ((A*x + B*a*log(e*(a/c + b*x/c)**n)
/b - B*n*x + B*x*log(e*(a/c + b*x/c)**n))/(c**3*i**3), Eq(d, 0)), (-2*A*a*
*2*d**2/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4*i**3*x + 4*a**2*d**5*i**3*x
**2 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*i**3*x - 8*a*b*c*d**4*i**3*x
**2 + 4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**3*x + 4*b**2*c**2*d**3*i**3
*x**2) + 4*A*a*b*c*d/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4*i**3*x + 4*a**
2*d**5*i**3*x**2 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*i**3*x - 8*a*b*
c*d**4*i**3*x**2 + 4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**3*x + 4*b**2*c
**2*d**3*i**3*x**2) - 2*A*b**2*c**2/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4
*i**3*x + 4*a**2*d**5*i**3*x**2 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*
i**3*x - 8*a*b*c*d**4*i**3*x**2 + 4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i*
*3*x + 4*b**2*c**2*d**3*i**3*x**2) + B*a**2*d**2*n/(4*a**2*c**2*d**3*i**3
+ 8*a**2*c*d**4*i**3*x + 4*a**2*d**5*i**3*x**2 - 8*a*b*c**3*d**2*i**3 - 16
*a*b*c**2*d**3*i**3*x - 8*a*b*c*d**4*i**3*x**2 + 4*b**2*c**4*d*i**3 + 8*b*
*2*c**3*d**2*i**3*x + 4*b**2*c**2*d**3*i**3*x**2) - 2*B*a**2*d**2*log(e*(a
/(c + d*x) + b*x/(c + d*x))**n)/(4*a**2*c**2*d**3*i**3 + 8*a**2*c*d**4*i**
3*x + 4*a**2*d**5*i**3*x**2 - 8*a*b*c**3*d**2*i**3 - 16*a*b*c**2*d**3*i**3
*x - 8*a*b*c*d**4*i**3*x**2 + 4*b**2*c**4*d*i**3 + 8*b**2*c**3*d**2*i**...
```

### 3.154.7 Maxima [A] (verification not implemented)

Time = 0.20 (sec) , antiderivative size = 259, normalized size of antiderivative = 1.72

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx$$
$$= \frac{1}{4}Bn\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}+\frac{2\,b^2\log{(bx+a)}}{(b^2c^2d-2\,abcd^2+a^2d^3)i^3}-\frac{2\,b}{(b^2c^2d-\cdots}\right.$$
$$-\frac{B\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}-\frac{A}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, algorithm="max
ima")
```

output
```
1/4*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d^2
 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x + a)/((b^2*
c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2*a
*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*B*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(
d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A/(d^3*i^3*x^2 + 2*c*d^2*i^
3*x + c^2*d*i^3)
```

### 3.154.8 Giac [A] (verification not implemented)

Time = 0.77 (sec) , antiderivative size = 207, normalized size of antiderivative = 1.37

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bx+a)Bbn}{(bci^3-adi^3)(dx+c)}-\frac{(bx+a)^2Bdn}{(bci^3-adi^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)+\frac{(Bdn-2\,Bd\log{(e)}-2\,Ad)(bx}{(bci^3-adi^3)(dx+c)^2}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(d*i*x+c*i)^3,x, algorithm="gia
c")
```

output
```
1/4*(2*(2*(b*x + a)*B*b*n/((b*c*i^3 - a*d*i^3)*(d*x + c)) - (b*x + a)^2*B*
d*n/((b*c*i^3 - a*d*i^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c)) + (B*d*n -
 2*B*d*log(e) - 2*A*d)*(b*x + a)^2/((b*c*i^3 - a*d*i^3)*(d*x + c)^2) - 4*(
B*b*n - B*b*log(e) - A*b)*(b*x + a)/((b*c*i^3 - a*d*i^3)*(d*x + c)))*(b*c/
(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.154.9 Mupad [B] (verification not implemented)

Time = 1.57 (sec) , antiderivative size = 221, normalized size of antiderivative = 1.46

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx=\frac{B\,b^2\,n\,\mathrm{atanh}\left(\frac{2\,a^2\,d^3\,i^3-2\,b^2\,c^2\,d\,i^3}{2\,d\,i^3\,{\left(a\,d-b\,c\right)}^2}+\frac{2\,b\,d\,x}{a\,d-b\,c}\right)}{d\,i^3\,{\left(a\,d-b\,c\right)}^2}-\frac{B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}{2\,d\,\left(c^2\,i^3+2\,c\,d\,i^3\,x+d^2\,i^3\,x^2\right)}-\frac{\frac{2\,A\,a\,d-2\,A\,b\,c-B\,a\,d\,n+3\,B\,b\,c\,n}{2\,\left(a\,d-b\,c\right)}+\frac{B\,b\,d\,n\,x}{a\,d-b\,c}}{2\,c^2\,d\,i^3+4\,c\,d^2\,i^3\,x+2\,d^3\,i^3\,x^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/(c*i + d*i*x)^3,x)
```

output
```
(B*b^2*n*atanh((2*a^2*d^3*i^3 - 2*b^2*c^2*d*i^3)/(2*d*i^3*(a*d - b*c)^2) +
 (2*b*d*x)/(a*d - b*c)))/(d*i^3*(a*d - b*c)^2) - (B*log(e*((a + b*x)/(c +
d*x))^n))/(2*d*(c^2*i^3 + d^2*i^3*x^2 + 2*c*d*i^3*x)) - ((2*A*a*d - 2*A*b*
c - B*a*d*n + 3*B*b*c*n)/(2*(a*d - b*c)) + (B*b*d*n*x)/(a*d - b*c))/(2*c^2
*d*i^3 + 2*d^3*i^3*x^2 + 4*c*d^2*i^3*x)
```

3.154. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ci+dix)^3}\,dx$

# 3.155 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$



## 3.155.1 Optimal result

Integrand size = 43, antiderivative size = 254

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx = -\frac{Bn\left(4b-\frac{d(a+bx)}{c+dx}\right)^2}{4(bc-ad)^3gi^3}+\frac{d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2}-\frac{2bd(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3gi^3(c+dx)}+\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^3gi^3}-\frac{b^2Bn\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^3gi^3}$$

output
```
-1/4*B*n*(4*b-d*(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g/i^3+1/2*d^2*(b*x+a)^2*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g/i^3/(d*x+c)^2-2*b*d*(b*x+a)*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g/i^3/(d*x+c)+b^2*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^3/g/i^3-1/2*b^2*B*n*ln(
(b*x+a)/(d*x+c))^2/(-a*d+b*c)^3/g/i^3
```

### 3.155.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.21 (sec) , antiderivative size = 434, normalized size of antiderivative = 1.71

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx = \frac{2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b(bc-ad)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b^2(c+dx)^2\log(a+}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i
*x)^3),x]
```

output
```
(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(b*c - a*d)*
(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d*x)^2*Log[a
 + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 4*b*B*n*(c + d*x)*(b*c - a*
d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - B*n*((b*c - a*d
)^2 + 2*b*(b*c - a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(
c + d*x)^2*Log[c + d*x]) - 2*b^2*B*n*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*
x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c
) + a*d)]) + 2*b^2*B*n*(c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
 Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/(4
*(b*c - a*d)^3*g*i^3*(c + d*x)^2)
```

### 3.155.3 Rubi [A] (verified)

Time = 0.41 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {2961, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)(ci+dix)^3}\,dx$$

$$\Big\downarrow \; 2961$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}d\frac{a+bx}{c+dx}}{gi^3(bc-ad)^3}$$

$\downarrow$ 2772

$$\frac{-Bn\int\left(\frac{b^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)}{a+bx}-\frac{1}{2}d\left(4b-\frac{d(a+bx)}{c+dx}\right)\right)d\frac{a+bx}{c+dx}+b^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{d^2(a+bx)^2\left(B\log\right.}{2(c}}{gi^3(bc-ad)^3}$$

$\downarrow$ 2009

$$\frac{b^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{d^2(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}-\frac{2bd(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{c+dx}-Bn\left(\frac{1}{2}b^2\right.}{gi^3(bc-ad)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)*(c*i + d*i*x)^3)
,x]
```

output
```
((d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(c + d*x)^2)
- (2*b*d*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + b^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] - B*n*((4
*b - (d*(a + b*x))/(c + d*x))^2/4 + (b^2*Log[(a + b*x)/(c + d*x)]^2)/2))/(
(b*c - a*d)^3*g*i^3)
```

### 3.155.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

3.155. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.155.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 627 vs. $2(248) = 496$.

Time = 11.28 (sec) , antiderivative size = 628, normalized size of antiderivative = 2.47

| method | result |
|---|---|
| parallelrisch | $-\frac{2Bx\,a^4c^3d^3n^2-4Ax\,a^4c^3d^3n+2B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^4c^4d^2n-6B\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^2b^2c^4d^2n-4Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)a^3b\,c^4d^2}{}$ |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^3,x,method=_RE
TURNVERBOSE)
```

output
```
-1/4*(2*B*x*a^4*c^3*d^3*n^2-4*A*x*a^4*c^3*d^3*n+2*B*ln(e*((b*x+a)/(d*x+c))
^n)*a^4*c^4*d^2*n-6*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^4*d^2*n-4*B*
x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^4*d^2*n-8*B*x*ln(e*((b*x+a)/(d*x+c))^n
)*a^2*b^2*c^5*d*n+B*x^2*a^4*c^2*d^4*n^2-2*A*x^2*a^4*c^2*d^4*n+2*B*ln(e*((b
*x+a)/(d*x+c))^n)^2*a^2*b^2*c^6+4*A*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^6+
2*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^4*d^2-8*B*x^2*a^3*b*c^3*d^3*
n^2+7*B*x^2*a^2*b^2*c^4*d^2*n^2+4*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*
c^4*d^2+8*A*x^2*a^3*b*c^3*d^3*n-6*A*x^2*a^2*b^2*c^4*d^2*n+4*B*x*ln(e*((b*x
+a)/(d*x+c))^n)^2*a^2*b^2*c^5*d-10*B*x*a^3*b*c^4*d^2*n^2+8*B*x*a^2*b^2*c^5
*d*n^2+8*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^5*d+12*A*x*a^3*b*c^4*d^2*
n-8*A*x*a^2*b^2*c^5*d*n-8*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^5*d*n)/i^3/g
/(d*x+c)^2/(a^2*d^2-2*a*b*c*d+b^2*c^2)/(a*d-b*c)/a^2/c^4/n
```

### 3.155.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 487, normalized size of antiderivative = 1.92

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{6\,Ab^2c^2-8\,Aabcd+2\,Aa^2d^2+2\,(Bb^2d^2nx^2+2\,Bb^2cdnx+Bb^2c^2n)\log\left(\frac{bx+a}{dx+c}\right)^2-(7\,Bb^2c^2-8\,Babcd+}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, al
gorithm="fricas")
```

output
```
1/4*(6*A*b^2*c^2 - 8*A*a*b*c*d + 2*A*a^2*d^2 + 2*(B*b^2*d^2*n*x^2 + 2*B*b^
2*c*d*n*x + B*b^2*c^2*n)*log((b*x + a)/(d*x + c))^2 - (7*B*b^2*c^2 - 8*B*a
*b*c*d + B*a^2*d^2)*n + 2*(2*A*b^2*c*d - 2*A*a*b*d^2 - 3*(B*b^2*c*d - B*a*
b*d^2)*n)*x + 2*(3*B*b^2*c^2 - 4*B*a*b*c*d + B*a^2*d^2 + 2*(B*b^2*c*d - B*
a*b*d^2)*x + 2*(B*b^2*d^2*x^2 + 2*B*b^2*c*d*x + B*b^2*c^2)*log((b*x + a)/(
d*x + c)))*log(e) + 2*(2*A*b^2*c^2 - (3*B*b^2*d^2*n - 2*A*b^2*d^2)*x^2 - (
4*B*a*b*c*d - B*a^2*d^2)*n + 2*(2*A*b^2*c*d - (2*B*b^2*c*d + B*a*b*d^2)*n)
*x)*log((b*x + a)/(d*x + c)))/((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*a^2*b*c*
d^4 - a^3*d^5)*g*i^3*x^2 + 2*(b^3*c^4*d - 3*a*b^2*c^3*d^2 + 3*a^2*b*c^2*d^
3 - a^3*c*d^4)*g*i^3*x + (b^3*c^5 - 3*a*b^2*c^4*d + 3*a^2*b*c^3*d^2 - a^3*
c^2*d^3)*g*i^3)
```

### 3.155.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)/(d*i*x+c*i)**3,x)
```

output
```
Timed out
```

### 3.155.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 888 vs. $2(248) = 496$.

Time = 0.23 (sec) , antiderivative size = 888, normalized size of antiderivative = 3.50

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{1}{2}B\left(\frac{2\,bdx+3\,bc-ad}{(b^2c^2d^2-2\,abcd^3+a^2d^4)gi^3x^2+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)gi^3x+(b^2c^4-2\,abc^3d+a^2c^2d^2)gi^3}+\right.$$
$$-\frac{\left(7\,b^2c^2-8\,abcd+a^2d^2+2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log(bx+a)^2+2\,(b^2d^2x^2+2\,b^2cdx+b^2c^2)\log(dx\right.}{4\,(b^3c^5gi^3-3\,ab^2c^4dgi^3+3\,a^2bc^3d^2gi^3-a^3c^2d^3gi^3+(}$$
$$+\frac{1}{2}A\left(\frac{2\,bdx+3\,bc-ad}{(b^2c^2d^2-2\,abcd^3+a^2d^4)gi^3x^2+2\,(b^2c^3d-2\,abc^2d^2+a^2cd^3)gi^3x+(b^2c^4-2\,abc^3d+a^2c^2d^2)gi^3}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, al
gorithm="maxima")
```

output
```
1/2*B*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*i^
3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2*a
*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^
2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 - 3*a
*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) - 1/4*(7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2
*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)
*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x +
 b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b
^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*B*n/(b^3*c
^5*g*i^3 - 3*a*b^2*c^4*d*g*i^3 + 3*a^2*b*c^3*d^2*g*i^3 - a^3*c^2*d^3*g*i^3
 + (b^3*c^3*d^2*g*i^3 - 3*a*b^2*c^2*d^3*g*i^3 + 3*a^2*b*c*d^4*g*i^3 - a^3*
d^5*g*i^3)*x^2 + 2*(b^3*c^4*d*g*i^3 - 3*a*b^2*c^3*d^2*g*i^3 + 3*a^2*b*c^2*
d^3*g*i^3 - a^3*c*d^4*g*i^3)*x) + 1/2*A*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2
*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a
^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*l
og(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) -
2*b^2*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*
i^3))
```

### 3.155.8 Giac [A] (verification not implemented)

Time = 1.52 (sec) , antiderivative size = 418, normalized size of antiderivative = 1.65

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(\frac{2\,Bb^2n\log\left(\frac{bx+a}{dx+c}\right)^2}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}-2\left(\frac{4\,(bx+a)Bbdn}{(b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3)(dx+c)}-\frac{(bx+a)^2}{(b^2c^2gi^3-2\,abcdgi^3+}\right.\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)/(d*i*x+c*i)^3,x, al
gorithm="giac")
```

output
```
1/4*(2*B*b^2*n*log((b*x + a)/(d*x + c))^2/(b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3
 + a^2*d^2*g*i^3) - 2*(4*(b*x + a)*B*b*d*n/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i
^3 + a^2*d^2*g*i^3)*(d*x + c)) - (b*x + a)^2*B*d^2*n/((b^2*c^2*g*i^3 - 2*a
*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c)) + 4*(
B*b^2*log(e) + A*b^2)*log((b*x + a)/(d*x + c))/(b^2*c^2*g*i^3 - 2*a*b*c*d*
g*i^3 + a^2*d^2*g*i^3) - (B*d^2*n - 2*B*d^2*log(e) - 2*A*d^2)*(b*x + a)^2/
((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)^2) + 8*(B*b*d
*n - B*b*d*log(e) - A*b*d)*(b*x + a)/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a
^2*d^2*g*i^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.155.9 Mupad [B] (verification not implemented)

Time = 3.37 (sec) , antiderivative size = 573, normalized size of antiderivative = 2.26

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$$
$$=\frac{B\,b^2\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(\frac{c\,g\,i^3\,n\,(a\,d-b\,c)}{2\,b}-\frac{g\,i^3\,n\,(a\,d-b\,c)\,(a\,d-2\,b\,c)}{2\,b^2}+\frac{d\,g\,i^3\,n\,x\,(a\,d-b\,c)}{b}\right)}{g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)\,(g\,c^2\,i^3+2\,g\,c\,d\,i^3\,x+g\,d^2\,i^3\,x^2)}$$
$$-\frac{B\,b^2\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{2\,g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}$$
$$-\frac{\frac{2\,A\,a\,d-6\,A\,b\,c-B\,a\,d\,n+7\,B\,b\,c\,n}{2\,(a\,d-b\,c)}-\frac{b\,x\,(2\,A\,d-3\,B\,d\,n)}{a\,d-b\,c}}{x^2\,(2\,a\,d^3\,g\,i^3-2\,b\,c\,d^2\,g\,i^3)+x\,(4\,a\,c\,d^2\,g\,i^3-4\,b\,c^2\,d\,g\,i^3)-2\,b\,c^3\,g\,i^3+2\,a\,c^2\,d\,g\,i^3}$$
$$+\frac{b^2\,\mathrm{atan}\left(\frac{b^2\,\left(A-\frac{3\,B\,n}{2}\right)\,(2\,g\,a^3\,d^3\,i^3-2\,g\,a^2\,b\,c\,d^2\,i^3-2\,g\,a\,b^2\,c^2\,d\,i^3+2\,g\,b^3\,c^3\,i^3)\,1\mathrm{i}}{g\,i^3\,(2\,A\,b^2-3\,B\,b^2\,n)\,(a\,d-b\,c)^3}+\frac{b^3\,d\,x\,\left(A-\frac{3\,B\,n}{2}\right)\,(g\,a^2\,d^2\,i^3-2\,g\,a\,b\,c\,d\,i^3+g\,b^2\,c^2\,i^3)}{g\,i^3\,(2\,A\,b^2-3\,B\,b^2\,n)\,(a\,d-b\,c)^3}\right.}{g\,i^3\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)*(c*i + d*i*x)^3)
,x)
```

output
```
(b^2*atan((b^2*(A - (3*B*n)/2)*(2*a^3*d^3*g*i^3 + 2*b^3*c^3*g*i^3 - 2*a*b^
2*c^2*d*g*i^3 - 2*a^2*b*c*d^2*g*i^3)*1i)/(g*i^3*(2*A*b^2 - 3*B*b^2*n)*(a*d
 - b*c)^3) + (b^3*d*x*(A - (3*B*n)/2)*(a^2*d^2*g*i^3 + b^2*c^2*g*i^3 - 2*a
*b*c*d*g*i^3)*4i)/(g*i^3*(2*A*b^2 - 3*B*b^2*n)*(a*d - b*c)^3))*(A - (3*B*n
)/2)*2i)/(g*i^3*(a*d - b*c)^3) - ((2*A*a*d - 6*A*b*c - B*a*d*n + 7*B*b*c*n
)/(2*(a*d - b*c)) - (b*x*(2*A*d - 3*B*d*n))/(a*d - b*c))/(x^2*(2*a*d^3*g*i
^3 - 2*b*c*d^2*g*i^3) + x*(4*a*c*d^2*g*i^3 - 4*b*c^2*d*g*i^3) - 2*b*c^3*g*
i^3 + 2*a*c^2*d*g*i^3) - (B*b^2*log(e*((a + b*x)/(c + d*x))^n)^2)/(2*g*i^3
*n*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B*b^2*log(e*((a + b*x)/
(c + d*x))^n)*((c*g*i^3*n*(a*d - b*c))/(2*b) - (g*i^3*n*(a*d - b*c)*(a*d -
 2*b*c))/(2*b^2) + (d*g*i^3*n*x*(a*d - b*c))/b))/(g*i^3*n*(a*d - b*c)*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^2*g*i^3 + d^2*g*i^3*x^2 + 2*c*d*g*i^3*x))
```

3.155. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)(ci+dix)^3}\,dx$

# 3.156 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$



## 3.156.1 Optimal result

Integrand size = 43, antiderivative size = 381

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx &= \frac{Bd^3n(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2} - \frac{3bBd^2n(a+bx)}{(bc-ad)^4g^2i^3(c+dx)} \\ &\quad - \frac{b^3Bn(c+dx)}{(bc-ad)^4g^2i^3(a+bx)} \\ &\quad - \frac{d^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2} \\ &\quad + \frac{3bd^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(c+dx)} \\ &\quad - \frac{b^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(a+bx)} \\ &\quad - \frac{3b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^4g^2i^3} \\ &\quad + \frac{3b^2Bdn\log^2\left(\frac{a+bx}{c+dx}\right)}{2(bc-ad)^4g^2i^3}\end{aligned}$$

output
```
1/4*B*d^3*n*(b*x+a)^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-3*b*B*d^2*n*(b*x+a)/(
-a*d+b*c)^4/g^2/i^3/(d*x+c)-b^3*B*n*(d*x+c)/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-1
/2*d^3*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^3/(d*x
+c)^2+3*b*d^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^3
/(d*x+c)-b^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^3/
(b*x+a)-3*b^2*d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+
b*c)^4/g^2/i^3+3/2*b^2*B*d*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^4/g^2/i^3
```

## 3.156.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.39 (sec) , antiderivative size = 477, normalized size of antiderivative = 1.25

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$$

$$= \frac{-\frac{4b^3Bcn}{a+bx} + \frac{4ab^2Bdn}{a+bx} + \frac{Bd(bc-ad)^2n}{(c+dx)^2} + \frac{8b^2Bcdn}{c+dx} - \frac{8abBd^2n}{c+dx} + \frac{2bBd(bc-ad)n}{c+dx} + 6b^2Bdn\log(a+bx) - \frac{4b^2(bc-ad)\left(A+B\,\mathrm{l}\right.}{a+b}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d
*i*x)^3),x]
```

output
```
((-4*b^3*B*c*n)/(a + b*x) + (4*a*b^2*B*d*n)/(a + b*x) + (B*d*(b*c - a*d)^2
*n)/(c + d*x)^2 + (8*b^2*B*c*d*n)/(c + d*x) - (8*a*b*B*d^2*n)/(c + d*x) +
(2*b*B*d*(b*c - a*d)*n)/(c + d*x) + 6*b^2*B*d*n*Log[a + b*x] - (4*b^2*(b*c
 - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (2*d*(b*c - a*
d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)^2 - (8*b*d*(b*c - a
*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) - 12*b^2*d*Log[a + b
*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*b^2*B*d*n*Log[c + d*x] + 12
*b^2*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 6*b^2*B*d*n*(
Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog
[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 6*b^2*B*d*n*((2*Log[(d*(a + b*x))/(-(
b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*
c - a*d)]))/(4*(b*c - a*d)^4*g^2*i^3)
```

### 3.156.3 Rubi [A] (verified)

Time = 0.44 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.70, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^2(ci+dix)^3}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i^3(bc-ad)^4}$$

$$\Big\downarrow \text{ 2772}$$

$$\frac{-Bn\int\left(-\frac{(c+dx)^2b^3}{(a+bx)^2}-\frac{3d(c+dx)\log\left(\frac{a+bx}{c+dx}\right)b^2}{a+bx}+3d^2b-\frac{d^3(a+bx)}{2(c+dx)}\right)d\frac{a+bx}{c+dx}-\frac{b^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-3b^2d\log\left(\frac{a+}{c+}\right.}{g^2i^3(bc-ad)^4}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{b^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-3b^2d\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-\frac{d^3(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}+\frac{3bd^2(c}{}}{g^2i^3(bc-ad)^4}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^2*(c*i + d*i*x)^
3),x]
```

output
```
(-1/2*(d^3*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)^2
 + (3*b*d^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) -
(b^3*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - 3*b^2*d
*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] - B*n*(-1
/4*(d^3*(a + b*x)^2)/(c + d*x)^2 + (3*b*d^2*(a + b*x))/(c + d*x) + (b^3*(c
 + d*x))/(a + b*x) - (3*b^2*d*Log[(a + b*x)/(c + d*x)]^2)/2))/((b*c - a*d)
^4*g^2*i^3)
```

### 3.156.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.156.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 961 vs. $2(375) = 750$.

Time = 28.55 (sec) , antiderivative size = 962, normalized size of antiderivative = 2.52

| method | result |
|---|---|
| parallelrisch | $-\frac{-24Bx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a\,b^{5}c\,d^{6}n+6B\,x^{3}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{2}b^{6}d^{7}+12A\,x^{3}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{6}d^{7}-18B\,x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a\,b^{5}d^{7}n}{}$ |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,method=_
RETURNVERBOSE)
```

output
```
-1/4*(-24*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^6*n+6*B*x^3*ln(e*((b*x+a
)/(d*x+c))^n)^2*b^6*d^7+12*A*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^7-18*B*x^
2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^7*n+12*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2
*a*b^5*c*d^6-6*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*d^7*n+12*B*x*ln(e*((b
*x+a)/(d*x+c))^n)*b^6*c^2*d^5*n-6*B*x*a*b^5*c*d^6*n^2+24*A*x*ln(e*((b*x+a)
/(d*x+c))^n)*a*b^5*c*d^6-12*A*x*a*b^5*c*d^6*n-12*B*ln(e*((b*x+a)/(d*x+c))^
n)*a^2*b^4*c*d^6*n-6*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^7*n+6*B*x^2*ln(
e*((b*x+a)/(d*x+c))^n)^2*a*b^5*d^7+12*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^
6*c*d^6+6*B*x^2*a*b^5*d^7*n^2-6*B*x^2*b^6*c*d^6*n^2+12*A*x^2*ln(e*((b*x+a)
/(d*x+c))^n)*a*b^5*d^7+24*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c*d^6-12*A*x
^2*a*b^5*d^7*n+12*A*x^2*b^6*c*d^6*n+6*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*
c^2*d^5+9*B*x*a^2*b^4*d^7*n^2-3*B*x*b^6*c^2*d^5*n^2+12*A*x*ln(e*((b*x+a)/(
d*x+c))^n)*b^6*c^2*d^5-6*A*x*a^2*b^4*d^7*n+18*A*x*b^6*c^2*d^5*n+6*B*ln(e*(
(b*x+a)/(d*x+c))^n)^2*a*b^5*c^2*d^5+2*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^3*
d^7*n+4*B*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^3*d^4*n+12*A*ln(e*((b*x+a)/(d*x+
c))^n)*a*b^5*c^2*d^5-B*a^3*b^3*d^7*n^2+4*B*b^6*c^3*d^4*n^2+2*A*a^3*b^3*d^7
*n+4*A*b^6*c^3*d^4*n+12*B*a^2*b^4*c*d^6*n^2-15*B*a*b^5*c^2*d^5*n^2-12*A*a^
2*b^4*c*d^6*n+6*A*a*b^5*c^2*d^5*n)/i^3/g^2/(d*x+c)^2/(b*x+a)/(a*d-b*c)^2/(
a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/d^4/n
```

### 3.156.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 949 vs. $2(375) = 750$.

Time = 0.38 (sec) , antiderivative size = 949, normalized size of antiderivative = 2.49

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx =$$

$$-\frac{4\,Ab^3c^3 + 6\,Aab^2c^2d - 12\,Aa^2bcd^2 + 2\,Aa^3d^3 + 6\,(2\,Ab^3cd^2 - 2\,Aab^2d^3 - (Bb^3cd^2 - Bab^2d^3)n)x^2 + 6\,(\ldots}{\ldots}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output
```
-1/4*(4*A*b^3*c^3 + 6*A*a*b^2*c^2*d - 12*A*a^2*b*c*d^2 + 2*A*a^3*d^3 + 6*(
2*A*b^3*c*d^2 - 2*A*a*b^2*d^3 - (B*b^3*c*d^2 - B*a*b^2*d^3)*n)*x^2 + 6*(B*
b^3*d^3*n*x^3 + B*a*b^2*c^2*d*n + (2*B*b^3*c*d^2 + B*a*b^2*d^3)*n*x^2 + (B
*b^3*c^2*d + 2*B*a*b^2*c*d^2)*n*x)*log((b*x + a)/(d*x + c))^2 + (4*B*b^3*c
^3 - 15*B*a*b^2*c^2*d + 12*B*a^2*b*c*d^2 - B*a^3*d^3)*n + 3*(6*A*b^3*c^2*d
 - 4*A*a*b^2*c*d^2 - 2*A*a^2*b*d^3 - (B*b^3*c^2*d + 2*B*a*b^2*c*d^2 - 3*B*
a^2*b*d^3)*n)*x + 2*(2*B*b^3*c^3 + 3*B*a*b^2*c^2*d - 6*B*a^2*b*c*d^2 + B*a
^3*d^3 + 6*(B*b^3*c*d^2 - B*a*b^2*d^3)*x^2 + 3*(3*B*b^3*c^2*d - 2*B*a*b^2*
c*d^2 - B*a^2*b*d^3)*x + 6*(B*b^3*d^3*x^3 + B*a*b^2*c^2*d + (2*B*b^3*c*d^2
 + B*a*b^2*d^3)*x^2 + (B*b^3*c^2*d + 2*B*a*b^2*c*d^2)*x)*log((b*x + a)/(d*
x + c)))*log(e) + 2*(6*A*a*b^2*c^2*d - 3*(B*b^3*d^3*n - 2*A*b^3*d^3)*x^3 -
 3*(3*B*a*b^2*d^3*n - 4*A*b^3*c*d^2 - 2*A*a*b^2*d^3)*x^2 + (2*B*b^3*c^3 -
6*B*a^2*b*c*d^2 + B*a^3*d^3)*n + 3*(2*A*b^3*c^2*d + 4*A*a*b^2*c*d^2 + (2*B
*b^3*c^2*d - 4*B*a*b^2*c*d^2 - B*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c))
)/((b^5*c^4*d^2 - 4*a*b^4*c^3*d^3 + 6*a^2*b^3*c^2*d^4 - 4*a^3*b^2*c*d^5 +
a^4*b*d^6)*g^2*i^3*x^3 + (2*b^5*c^5*d - 7*a*b^4*c^4*d^2 + 8*a^2*b^3*c^3*d^
3 - 2*a^3*b^2*c^2*d^4 - 2*a^4*b*c*d^5 + a^5*d^6)*g^2*i^3*x^2 + (b^5*c^6 -
2*a*b^4*c^5*d - 2*a^2*b^3*c^4*d^2 + 8*a^3*b^2*c^3*d^3 - 7*a^4*b*c^2*d^4 +
2*a^5*c*d^5)*g^2*i^3*x + (a*b^4*c^6 - 4*a^2*b^3*c^5*d + 6*a^3*b^2*c^4*d^2
- 4*a^4*b*c^3*d^3 + a^5*c^2*d^4)*g^2*i^3)
```

### 3.156.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**2/(d*i*x+c*i)**3,x
)
```

output
```
Timed out
```

### 3.156.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1724 vs. $2(375) = 750$.

Time = 0.30 (sec) , antiderivative size = 1724, normalized size of antiderivative = 4.52

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output
```
-1/2*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d +
a*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)
*g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*
c*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2
+ 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d
+ 3*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4
 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) -
 6*b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^
3*b*c*d^3 + a^4*d^4)*g^2*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/
4*(4*b^3*c^3 - 15*a*b^2*c^2*d + 12*a^2*b*c*d^2 - a^3*d^3 - 6*(b^3*c*d^2 -
a*b^2*d^3)*x^2 - 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*
x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x + a)^2 - 6*(b^3*d^3*x^3 + a*b
^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*
log(d*x + c)^2 - 3*(b^3*c^2*d + 2*a*b^2*c*d^2 - 3*a^2*b*d^3)*x - 6*(b^3*d^
3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2
*c*d^2)*x)*log(b*x + a) + 6*(b^3*d^3*x^3 + a*b^2*c^2*d + (2*b^3*c*d^2 + a*
b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x + 2*(b^3*d^3*x^3 + a*b^2*c^2*
d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*d^2)*x)*log(b*x
 + a))*log(d*x + c))*B*n/(a*b^4*c^6*g^2*i^3 - 4*a^2*b^3*c^5*d*g^2*i^3 + 6*
a^3*b^2*c^4*d^2*g^2*i^3 - 4*a^4*b*c^3*d^3*g^2*i^3 + a^5*c^2*d^4*g^2*i^3...
```

### 3.156.8 Giac [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{(bgx+ag)^2(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/((b*g*x + a*g)^2*(d*i*x +
 c*i)^3), x)
```

### 3.156.9 Mupad [B] (verification not implemented)

Time = 4.07 (sec) , antiderivative size = 1018, normalized size of antiderivative = 2.67

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$$
$$= \frac{\frac{4\,A\,b^2\,c^2-2\,A\,a^2\,d^2+B\,a^2\,d^2\,n+4\,B\,b^2\,c^2\,n+10\,A\,a\,b\,c\,d-11\,B\,a\,b\,c\,d\,n}{2\,(a\,d-b\,c)}+\frac{3\,x^2\,(2\,A}{}}{x\,\left(4\,a^3\,c\,d^3\,g^2\,i^3-6\,a^2\,b\,c^2\,d^2\,g^2\,i^3+2\,b^3\,c^4\,g^2\,i^3\right)+x^2\,\left(2\,a^3\,d^4\,g^2\,i^3-6\,a\,b^2\,c^2\,d^2\,g^2\,i^3+4\,b^3\,c^3\,d\,g^2\,i^3\right)+x^3}$$
$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{\frac{B\,(a\,d+2\,b\,c)}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}+\frac{3\,B\,b\,d\,x}{2\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}}{x\,\left(b\,c^2\,g^2\,i^3+2\,a\,d\,c\,g^2\,i^3\right)+x^2\,\left(a\,d^2\,g^2\,i^3+2\,b\,c\,d\,g^2\,i^3\right)+a\,c^2\,g^2\,i^3+b\,d^2\,g^2\,i^3\,x^3}\right.$$
$$\left.-\frac{3\,B\,b^2\,d\left(d\,g^2\,i^3\,n\,x^2\,(a\,d-b\,c)+\frac{a\,c\,g^2\,i^3\,n\,(a\,d-b\,c)}{b}+\frac{g^2\,i^3\,n\,x\,(a\,d+b\,c)\,(a\,d-b\,c)}{b}\right)}{g^2\,i^3\,n\,(a\,d-b\,c)^4\,\left(x\,\left(b\,c^2\,g^2\,i^3+2\,a\,d\,c\,g^2\,i^3\right)+x^2\,\left(a\,d^2\,g^2\,i^3+2\,b\,c\,d\,g^2\,i^3\right)+a\,c^2\,g^2\,i^3+b\,d^2\,g^2\,i^3\,x^3\right)}\right)$$
$$-\frac{3\,B\,b^2\,d\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{2\,g^2\,i^3\,n\,(a\,d-b\,c)^4}$$
$$+\frac{b^2\,d\,\mathrm{atan}\left(\frac{b^2\,d\,(2\,A-B\,n)\left(\frac{a^4\,d^4\,g^2\,i^3-2\,a^3\,b\,c\,d^3\,g^2\,i^3+2\,a\,b^3\,c^3\,d\,g^2\,i^3-b^4\,c^4\,g^2\,i^3}{a^3\,d^3\,g^2\,i^3-3\,a^2\,b\,c\,d^2\,g^2\,i^3+3\,a\,b^2\,c^2\,d\,g^2\,i^3-b^3\,c^3\,g^2\,i^3}+2\,b\,d\,x\right)\left(a^3\,d^3\,g^2\,i^3-3\,a^2\,b\,c\,d^2\,g^2\,i^3+3\,a\,b^2\,c^2\,d\,g^2\,i^3-b^3\right.}{g^2\,i^3\,(6\,A\,b^2\,d-3\,B\,b^2\,d\,n)\,(a\,d-b\,c)^4}\right.}{g^2\,i^3\,(a\,d-b\,c)^4}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^2*(c*i + d*i*x)^
3),x)
```

output
```
((4*A*b^2*c^2 - 2*A*a^2*d^2 + B*a^2*d^2*n + 4*B*b^2*c^2*n + 10*A*a*b*c*d -
 11*B*a*b*c*d*n)/(2*(a*d - b*c)) + (3*x^2*(2*A*b^2*d^2 - B*b^2*d^2*n))/(a*
d - b*c) + (3*x*(2*A*a*b*d^2 + 6*A*b^2*c*d - 3*B*a*b*d^2*n - B*b^2*c*d*n))
/(2*(a*d - b*c)))/(x*(2*b^3*c^4*g^2*i^3 + 4*a^3*c*d^3*g^2*i^3 - 6*a^2*b*c^
2*d^2*g^2*i^3) + x^2*(2*a^3*d^4*g^2*i^3 + 4*b^3*c^3*d*g^2*i^3 - 6*a*b^2*c^
2*d^2*g^2*i^3) + x^3*(2*b^3*c^2*d^2*g^2*i^3 + 2*a^2*b*d^4*g^2*i^3 - 4*a*b^
2*c*d^3*g^2*i^3) + 2*a^3*c^2*d^2*g^2*i^3 + 2*a*b^2*c^4*g^2*i^3 - 4*a^2*b*c
^3*d*g^2*i^3) - log(e*((a + b*x)/(c + d*x))^n)*(((B*(a*d + 2*b*c))/(2*(a^2
*d^2 + b^2*c^2 - 2*a*b*c*d)) + (3*B*b*d*x)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c
*d)))/(x*(b*c^2*g^2*i^3 + 2*a*c*d*g^2*i^3) + x^2*(a*d^2*g^2*i^3 + 2*b*c*d*
g^2*i^3) + a*c^2*g^2*i^3 + b*d^2*g^2*i^3*x^3) - (3*B*b^2*d*(d*g^2*i^3*n*x^
2*(a*d - b*c) + (a*c*g^2*i^3*n*(a*d - b*c))/b + (g^2*i^3*n*x*(a*d + b*c)*(
a*d - b*c))/b))/(g^2*i^3*n*(a*d - b*c)^4*(x*(b*c^2*g^2*i^3 + 2*a*c*d*g^2*i
^3) + x^2*(a*d^2*g^2*i^3 + 2*b*c*d*g^2*i^3) + a*c^2*g^2*i^3 + b*d^2*g^2*i^
3*x^3))) + (b^2*d*atan((b^2*d*(2*A - B*n)*((a^4*d^4*g^2*i^3 - b^4*c^4*g^2*
i^3 + 2*a*b^3*c^3*d*g^2*i^3 - 2*a^3*b*c*d^3*g^2*i^3)/(a^3*d^3*g^2*i^3 - b^
3*c^3*g^2*i^3 + 3*a*b^2*c^2*d*g^2*i^3 - 3*a^2*b*c*d^2*g^2*i^3) + 2*b*d*x)*
(a^3*d^3*g^2*i^3 - b^3*c^3*g^2*i^3 + 3*a*b^2*c^2*d*g^2*i^3 - 3*a^2*b*c*d^2
*g^2*i^3)*3i)/(g^2*i^3*(6*A*b^2*d - 3*B*b^2*d*n)*(a*d - b*c)^4))*(2*A - B*
n)*3i)/(g^2*i^3*(a*d - b*c)^4) - (3*B*b^2*d*log(e*((a + b*x)/(c + d*x))...
```

3.156. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^2(ci+dix)^3}\,dx$

# 3.157 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx$



## 3.157.1 Optimal result

Integrand size = 43, antiderivative size = 483

$$\begin{aligned}\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx &= -\frac{Bd^4n(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2}+\frac{4bBd^3n(a+bx)}{(bc-ad)^5g^3i^3(c+dx)} \\ &\quad+\frac{4b^3Bdn(c+dx)}{(bc-ad)^5g^3i^3(a+bx)}-\frac{b^4Bn(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2} \\ &\quad+\frac{d^4(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2} \\ &\quad-\frac{4bd^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(c+dx)} \\ &\quad+\frac{4b^3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(a+bx)} \\ &\quad-\frac{b^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2} \\ &\quad+\frac{6b^2d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3} \\ &\quad-\frac{3b^2Bd^2n\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^5g^3i^3}\end{aligned}$$

output
```
-1/4*B*d^4*n*(b*x+a)^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+4*b*B*d^3*n*(b*x+a)/
(-a*d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*B*d*n*(d*x+c)/(-a*d+b*c)^5/g^3/i^3/(b*x
+a)-1/4*b^4*B*n*(d*x+c)^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+1/2*d^4*(b*x+a)^2
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2-4*b*d^3*(b
*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(d*x+c)+4*b^3*d
*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*
b^4*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)
^2+6*b^2*d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c
)^5/g^3/i^3-3*b^2*B*d^2*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^5/g^3/i^3
```

### 3.157.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.68 (sec) , antiderivative size = 561, normalized size of antiderivative = 1.16

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx =$$

$$-\frac{\frac{b^2B(bc-ad)^2n}{(a+bx)^2}-\frac{12b^3Bcdn}{a+bx}+\frac{12ab^2Bd^2n}{a+bx}-\frac{2b^2Bd(bc-ad)n}{a+bx}+\frac{Bd^2(bc-ad)^2n}{(c+dx)^2}+\frac{12b^2Bcd^2n}{c+dx}-\frac{12abBd^3n}{c+dx}+\frac{2bBd^2(bc-ad)n}{c+dx}+\cdots}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d
*i*x)^3),x]
```

output
```
-1/4*((b^2*B*(b*c - a*d)^2*n)/(a + b*x)^2 - (12*b^3*B*c*d*n)/(a + b*x) + (
12*a*b^2*B*d^2*n)/(a + b*x) - (2*b^2*B*d*(b*c - a*d)*n)/(a + b*x) + (B*d^2
*(b*c - a*d)^2*n)/(c + d*x)^2 + (12*b^2*B*c*d^2*n)/(c + d*x) - (12*a*b*B*d
^3*n)/(c + d*x) + (2*b*B*d^2*(b*c - a*d)*n)/(c + d*x) + (2*b^2*(b*c - a*d)
^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 - (12*b^2*d*(b*c -
a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (2*d^2*(b*c - a*d
)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)^2 - (12*b*d^2*(b*c -
 a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) - 24*b^2*d^2*Log[a
 + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 24*b^2*d^2*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*Log[c + d*x] + 12*b^2*B*d^2*n*(Log[a + b*x]*(Log[
a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/
(-(b*c) + a*d)]) - 12*b^2*B*d^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b
*c - a*d)^5*g^3*i^3)
```

### 3.157.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 335, normalized size of antiderivative = 0.69, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {2961, 2772, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^3(ci+dix)^3} \, dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i^3(bc-ad)^5}$$

$$\Big\downarrow \text{ 2772}$$

$$\frac{-Bn\int \left(\frac{6b^2(c+dx)\log\left(\frac{a+bx}{c+dx}\right)d^2}{a+bx}+\frac{1}{2}\left(-\frac{(c+dx)^3b^4}{(a+bx)^3}+\frac{8d(c+dx)^2b^3}{(a+bx)^2}-8d^3b+\frac{d^4(a+bx)}{c+dx}\right)\right)d\frac{a+bx}{c+dx}-\frac{b^4(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{2(a+bx)^2}}{}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{b^4(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{4b^3d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}+6b^2d^2\log\left(\frac{a+bx}{c+dx}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+\frac{d^4(}{}}{}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^3*(c*i + d*i*x)^
3),x]
```

output
```
((d^4*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(c + d*x)^2)
- (4*b*d^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) + (
4*b^3*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (b^4
*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + 6*b
^2*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c + d*x)] - B
*n*((d^4*(a + b*x)^2)/(4*(c + d*x)^2) - (4*b*d^3*(a + b*x))/(c + d*x) - (4
*b^3*d*(c + d*x))/(a + b*x) + (b^4*(c + d*x)^2)/(4*(a + b*x)^2) + 3*b^2*d^
2*Log[(a + b*x)/(c + d*x)]^2))/((b*c - a*d)^5*g^3*i^3)
```

### 3.157.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.157.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1449 vs. $2(475) = 950$.

Time = 54.00 (sec) , antiderivative size = 1450, normalized size of antiderivative = 3.00

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1450 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,method=_
RETURNVERBOSE)
```

output
```
-1/4*(12*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*b^8*d^8+24*A*x^4*ln(e*((b*x+a)/
(d*x+c))^n)*b^8*d^8-24*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*d^8*n+24*B*x^
3*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c*d^7*n+48*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)
^2*a*b^7*c*d^7-36*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6*d^8*n+36*B*x^2*l
n(e*((b*x+a)/(d*x+c))^n)*b^8*c^2*d^6*n-24*B*x^2*a*b^7*c*d^7*n^2+96*A*x^2*l
n(e*((b*x+a)/(d*x+c))^n)*a*b^7*c*d^7+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^
2*b^6*c*d^7+24*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^7*c^2*d^6-8*B*x*ln(e*((
b*x+a)/(d*x+c))^n)*a^3*b^5*d^8*n+8*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c^3*d
^5*n-12*B*x*a^2*b^6*c*d^7*n^2-12*B*x*a*b^7*c^2*d^6*n^2+48*A*x*ln(e*((b*x+a
)/(d*x+c))^n)*a^2*b^6*c*d^7+48*A*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*c^2*d^6
-48*A*x*a^2*b^6*c*d^7*n+48*A*x*a*b^7*c^2*d^6*n-16*B*ln(e*((b*x+a)/(d*x+c))
^n)*a^3*b^5*c*d^7*n+16*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*c^3*d^5*n+24*A*x^
2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6*d^8+24*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)
*b^8*c^2*d^6-36*A*x^2*a^2*b^6*d^8*n+36*A*x^2*b^8*c^2*d^6*n+12*B*x*a^3*b^5*
d^8*n^2+12*B*x*b^8*c^3*d^5*n^2-8*A*x*a^3*b^5*d^8*n+8*A*x*b^8*c^3*d^5*n+12*
B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^6*c^2*d^6+2*B*ln(e*((b*x+a)/(d*x+c))^n
)*a^4*b^4*d^8*n-2*B*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c^4*d^4*n+24*A*ln(e*((b*
x+a)/(d*x+c))^n)*a^2*b^6*c^2*d^6+16*B*a^3*b^5*c*d^7*n^2-30*B*a^2*b^6*c^2*d
^6*n^2+16*B*a*b^7*c^3*d^5*n^2-16*A*a^3*b^5*c*d^7*n+16*A*a*b^7*c^3*d^5*n-48
*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6*c*d^7*n+48*B*x*ln(e*((b*x+a)/(d*...
```

### 3.157.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(475) = 950$.

Time = 0.43 (sec) , antiderivative size = 1416, normalized size of antiderivative = 2.93

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output
```
-1/4*(2*A*b^4*c^4 - 16*A*a*b^3*c^3*d + 16*A*a^3*b*c*d^3 - 2*A*a^4*d^4 - 24
*(A*b^4*c*d^3 - A*a*b^3*d^4)*x^3 - 12*(3*A*b^4*c^2*d^2 - 3*A*a^2*b^2*d^4 +
 (B*b^4*c^2*d^2 - 2*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n)*x^2 - 12*(B*b^4*d^4*
n*x^4 + B*a^2*b^2*c^2*d^2*n + 2*(B*b^4*c*d^3 + B*a*b^3*d^4)*n*x^3 + (B*b^4
*c^2*d^2 + 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*n*x^2 + 2*(B*a*b^3*c^2*d^2 + B
*a^2*b^2*c*d^3)*n*x)*log((b*x + a)/(d*x + c))^2 + (B*b^4*c^4 - 16*B*a*b^3*
c^3*d + 30*B*a^2*b^2*c^2*d^2 - 16*B*a^3*b*c*d^3 + B*a^4*d^4)*n - 4*(2*A*b^
4*c^3*d + 12*A*a*b^3*c^2*d^2 - 12*A*a^2*b^2*c*d^3 - 2*A*a^3*b*d^4 + 3*(B*b
^4*c^3*d - B*a*b^3*c^2*d^2 - B*a^2*b^2*c*d^3 + B*a^3*b*d^4)*n)*x + 2*(B*b^
4*c^4 - 8*B*a*b^3*c^3*d + 8*B*a^3*b*c*d^3 - B*a^4*d^4 - 12*(B*b^4*c*d^3 -
B*a*b^3*d^4)*x^3 - 18*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*x^2 - 4*(B*b^4*c^3*d
 + 6*B*a*b^3*c^2*d^2 - 6*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*x - 12*(B*b^4*d^4*
x^4 + B*a^2*b^2*c^2*d^2 + 2*(B*b^4*c*d^3 + B*a*b^3*d^4)*x^3 + (B*b^4*c^2*d
^2 + 4*B*a*b^3*c*d^3 + B*a^2*b^2*d^4)*x^2 + 2*(B*a*b^3*c^2*d^2 + B*a^2*b^2
*c*d^3)*x)*log((b*x + a)/(d*x + c)))*log(e) - 2*(12*A*b^4*d^4*x^4 + 12*A*a
^2*b^2*c^2*d^2 + 12*(2*A*b^4*c*d^3 + 2*A*a*b^3*d^4 + (B*b^4*c*d^3 - B*a*b^
3*d^4)*n)*x^3 + 6*(2*A*b^4*c^2*d^2 + 8*A*a*b^3*c*d^3 + 2*A*a^2*b^2*d^4 + 3
*(B*b^4*c^2*d^2 - B*a^2*b^2*d^4)*n)*x^2 - (B*b^4*c^4 - 8*B*a*b^3*c^3*d + 8
*B*a^3*b*c*d^3 - B*a^4*d^4)*n + 4*(6*A*a*b^3*c^2*d^2 + 6*A*a^2*b^2*c*d^3 +
 (B*b^4*c^3*d + 6*B*a*b^3*c^2*d^2 - 6*B*a^2*b^2*c*d^3 - B*a^3*b*d^4)*n)...
```

### 3.157.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**3/(d*i*x+c*i)**3,x
)
```

output
```
Timed out
```

### 3.157.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2383 vs. $2(475) = 950$.

Time = 0.33 (sec) , antiderivative size = 2383, normalized size of antiderivative = 4.93

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output
```
1/2*B*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d^3
 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b*d
^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c*d
^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b^4
*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3 +
(b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 + a^
6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2 +
2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^6
- 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g^3
*i^3) + 12*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3
*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d^2
*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*
c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) - 1/4*(b^4*c^4 - 16*a*b^3*c^3*d + 30*a^2*b^2*c^2*d^2 - 16*a^3*b*
c*d^3 + a^4*d^4 - 12*(b^4*c^2*d^2 - 2*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 12*
(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*b^3*d^4)*x^3 + (b^4*c^2*
d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*c^2*d^2 + a^2*b^2*c*d^3)
*x)*log(b*x + a)^2 - 24*(b^4*d^4*x^4 + a^2*b^2*c^2*d^2 + 2*(b^4*c*d^3 + a*
b^3*d^4)*x^3 + (b^4*c^2*d^2 + 4*a*b^3*c*d^3 + a^2*b^2*d^4)*x^2 + 2*(a*b^3*
c^2*d^2 + a^2*b^2*c*d^3)*x)*log(b*x + a)*log(d*x + c) + 12*(b^4*d^4*x^4...
```

### 3.157.8 Giac [F]

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A}{(bgx+ag)^3(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/((b*g*x + a*g)^3*(d*i*x +
 c*i)^3), x)
```

### 3.157.9 Mupad [B] (verification not implemented)

Time = 4.60 (sec) , antiderivative size = 1341, normalized size of antiderivative = 2.78

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx$$

$$= \frac{2\,x\,(\cdots}{x^4\,\left(2\,a^3\,b^2\,d^5\,g^3\,i^3-6\,a^2\,b^3\,c\,d^4\,g^3\,i^3+6\,a\,b^4\,c^2\,d^3\,g^3\,i^3-2\,b^5\,c^3\,d^2\,g^3\,i^3\right)-x\,\left(-4\,a^5\,c\,d^4\,g^3\,i^3+8\,a^4\,b\,c^2\,d^3\,g\cdots\right.}$$

$$+\frac{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\,\left(x\,\left(\frac{3\,B\,b\,d\,{\left(a\,d+b\,c\right)}^2}{{\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}^2}-\frac{B\,b\,d}{a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2}+\frac{6\,B\,a\,b^2\,c\,d^2}{{\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}^2}\right)-\frac{B\,\left(a\,d+b\,c\right)}{2\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}+\cdots\right.}{x\,\left(2\,d\,a^2\,c\,g^3\,i^3+2\,b\,a\,c^2\,g^3\,i^3\right)+x^3\,\left(2\,c\,b^2\,d\,g^3\,i^3+2\,a\,b\,d^2\,g^3\,i^3\right)+x^2\,\left(a^2\,d^2\,g^3\,i^3+4\,a\,b\,c\cdots\right.}$$

$$-\frac{3\,B\,b^2\,d^2\,{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)}^2}{g^3\,i^3\,n\,{\left(a\,d-b\,c\right)}^5}$$

$$+\frac{A\,b^2\,d^2\,\mathrm{atan}\left(\frac{\left(a^5\,d^5\,g^3\,i^3-3\,a^4\,b\,c\,d^4\,g^3\,i^3+2\,a^3\,b^2\,c^2\,d^3\,g^3\,i^3+2\,a^2\,b^3\,c^3\,d^2\,g^3\,i^3-3\,a\,b^4\,c^4\,d\,g^3\,i^3+b^5\,c^5\,g^3\,i^3\right)\,1{}\mathrm{i}}{g^3\,i^3\,{\left(a\,d-b\,c\right)}^5}+\frac{b\,d\,x\,\left(a^4\,d^4\,g^3\,i^3-4\,a^3\cdots\right.}{\cdots}\right.}{g^3\,i^3\,{\left(a\,d-b\,c\right)}^5}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^3*(c*i + d*i*x)^
3),x)
```

output
```
((2*x*(2*A*a^2*b*d^3 + 2*A*b^3*c^2*d + 14*A*a*b^2*c*d^2 - 3*B*a^2*b*d^3*n
+ 3*B*b^3*c^2*d*n))/(a*d - b*c) - (2*A*a^3*d^3 + 2*A*b^3*c^3 - B*a^3*d^3*n
 + B*b^3*c^3*n - 14*A*a*b^2*c^2*d - 14*A*a^2*b*c*d^2 - 15*B*a*b^2*c^2*d*n
+ 15*B*a^2*b*c*d^2*n)/(2*(a*d - b*c)) + (6*x^2*(3*A*a*b^2*d^3 + 3*A*b^3*c*
d^2 - B*a*b^2*d^3*n + B*b^3*c*d^2*n))/(a*d - b*c) + (12*A*b^3*d^3*x^3)/(a*
d - b*c))/(x^4*(2*a^3*b^2*d^5*g^3*i^3 - 2*b^5*c^3*d^2*g^3*i^3 + 6*a*b^4*c^
2*d^3*g^3*i^3 - 6*a^2*b^3*c*d^4*g^3*i^3) - x*(4*a*b^4*c^5*g^3*i^3 - 4*a^5*
c*d^4*g^3*i^3 - 8*a^2*b^3*c^4*d*g^3*i^3 + 8*a^4*b*c^2*d^3*g^3*i^3) + x^3*(
4*a^4*b*d^5*g^3*i^3 - 4*b^5*c^4*d*g^3*i^3 + 8*a*b^4*c^3*d^2*g^3*i^3 - 8*a^
3*b^2*c*d^4*g^3*i^3) + x^2*(2*a^5*d^5*g^3*i^3 - 2*b^5*c^5*g^3*i^3 - 2*a*b^
4*c^4*d*g^3*i^3 + 2*a^4*b*c*d^4*g^3*i^3 + 16*a^2*b^3*c^3*d^2*g^3*i^3 - 16*
a^3*b^2*c^2*d^3*g^3*i^3) - 2*a^2*b^3*c^5*g^3*i^3 + 2*a^5*c^2*d^3*g^3*i^3 +
 6*a^3*b^2*c^4*d*g^3*i^3 - 6*a^4*b*c^3*d^2*g^3*i^3) + (log(e*((a + b*x)/(c
 + d*x))^n)*(x*((3*B*b*d*(a*d + b*c)^2)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2
- (B*b*d)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d) + (6*B*a*b^2*c*d^2)/(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d)^2) - (B*(a*d + b*c))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d
)) + (6*B*b^3*d^3*x^3)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2 + (9*B*b^2*d^2*x^
2*(a*d + b*c))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2 + (3*B*a*b*c*d*(a*d + b*c
))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2))/(x*(2*a*b*c^2*g^3*i^3 + 2*a^2*c*d*g
^3*i^3) + x^3*(2*a*b*d^2*g^3*i^3 + 2*b^2*c*d*g^3*i^3) + x^2*(a^2*d^2*g^...
```

3.157. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^3(ci+dix)^3}\,dx$

# 3.158 $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx$



## 3.158.1 Optimal result

Integrand size = 43, antiderivative size = 587

$$\begin{aligned}
\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx &= \frac{Bd^5n(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2} - \frac{5bBd^4n(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} \\
&- \frac{10b^3Bd^2n(c+dx)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4Bdn(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2} \\
&- \frac{b^5Bn(c+dx)^3}{9(bc-ad)^6g^4i^3(a+bx)^3} \\
&- \frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} \\
&+ \frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(c+dx)} \\
&- \frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} \\
&+ \frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} \\
&- \frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3(bc-ad)^6g^4i^3(a+bx)^3} \\
&- \frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3} \\
&+ \frac{5b^2Bd^3n\log^2\left(\frac{a+bx}{c+dx}\right)}{(bc-ad)^6g^4i^3}
\end{aligned}$$

output
```
1/4*B*d^5*n*(b*x+a)^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-5*b*B*d^4*n*(b*x+a)/(
-a*d+b*c)^6/g^4/i^3/(d*x+c)-10*b^3*B*d^2*n*(d*x+c)/(-a*d+b*c)^6/g^4/i^3/(b
*x+a)+5/4*b^4*B*d*n*(d*x+c)^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/9*b^5*B*n*(
d*x+c)^3/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-1/2*d^5*(b*x+a)^2*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2+5*b*d^4*(b*x+a)*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(d*x+c)-10*b^3*d^2*(d*x+c)*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*d*(d*x+c)^
2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/3*b^5*(
d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-10
*b^2*d^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((b*x+a)/(d*x+c))/(-a*d+b*c)^6/
g^4/i^3+5*b^2*B*d^3*n*ln((b*x+a)/(d*x+c))^2/(-a*d+b*c)^6/g^4/i^3
```

## 3.158.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.91 (sec) , antiderivative size = 671, normalized size of antiderivative = 1.14

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx =$$

$$-\frac{\frac{4b^2B(bc-ad)^3n}{(a+bx)^3}-\frac{33b^2Bd(bc-ad)^2n}{(a+bx)^2}+\frac{216b^3Bcd^2n}{a+bx}-\frac{216ab^2Bd^3n}{a+bx}+\frac{66b^2Bd^2(bc-ad)n}{a+bx}-\frac{9Bd^3(bc-ad)^2n}{(c+dx)^2}-\frac{144b^2Bcd^3n}{c+dx}+\ldots}{\ldots}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d
*i*x)^3),x]
```

output
```
-1/36*((4*b^2*B*(b*c - a*d)^3*n)/(a + b*x)^3 - (33*b^2*B*d*(b*c - a*d)^2*n
)/(a + b*x)^2 + (216*b^3*B*c*d^2*n)/(a + b*x) - (216*a*b^2*B*d^3*n)/(a + b
*x) + (66*b^2*B*d^2*(b*c - a*d)*n)/(a + b*x) - (9*B*d^3*(b*c - a*d)^2*n)/(
c + d*x)^2 - (144*b^2*B*c*d^3*n)/(c + d*x) + (144*a*b*B*d^4*n)/(c + d*x) -
 (18*b*B*d^3*(b*c - a*d)*n)/(c + d*x) + 120*b^2*B*d^3*n*Log[a + b*x] + (12
*b^2*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^3 - (
54*b^2*d*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2
 + (216*b^2*d^2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b
*x) + (18*d^3*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d
*x)^2 + (144*b*d^3*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c
+ d*x) + 360*b^2*d^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) -
 120*b^2*B*d^3*n*Log[c + d*x] - 360*b^2*d^3*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])*Log[c + d*x] - 180*b^2*B*d^3*n*(Log[a + b*x]*(Log[a + b*x] - 2*Lo
g[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]
) + 180*b^2*B*d^3*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*
Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^6*g^
4*i^3)
```

### 3.158.3 Rubi [A] (verified)

Time = 0.60 (sec) , antiderivative size = 407, normalized size of antiderivative = 0.69, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.116, Rules used = {2961, 2772, 27, 2010, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{(ag+bgx)^4(ci+dix)^3}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i^3(bc-ad)^6}$$

$$\Big\downarrow \text{ 2772}$$

$$-Bn\int -\frac{(c+dx)^4\left(2b^5-\frac{15d(a+bx)b^4}{c+dx}+\frac{60d^2(a+bx)^2b^3}{(c+dx)^2}+\frac{60d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)b^2}{(c+dx)^3}-\frac{30d^4(a+bx)^4b}{(c+dx)^4}+\frac{3d^5(a+bx)^5}{(c+dx)^5}\right)}{6(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^5(c+dx)^3\left(B\log\left(e\left(\right.\right.\right.}{3(a+bx}$$

$$\Big\downarrow \text{27}$$

$$\frac{1}{6}Bn\int \frac{(c+dx)^4\left(2b^5-\frac{15d(a+bx)b^4}{c+dx}+\frac{60d^2(a+bx)^2b^3}{(c+dx)^2}+\frac{60d^3(a+bx)^3\log\left(\frac{a+bx}{c+dx}\right)b^2}{(c+dx)^3}-\frac{30d^4(a+bx)^4b}{(c+dx)^4}+\frac{3d^5(a+bx)^5}{(c+dx)^5}\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{b^5(c+dx)^3\left(B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{3(a+bx)^3}$$

$$\Big\downarrow \text{2010}$$

$$\frac{1}{6}Bn\int \left(\frac{\left(2b^5-\frac{15d(a+bx)b^4}{c+dx}+\frac{60d^2(a+bx)^2b^3}{(c+dx)^2}-\frac{30d^4(a+bx)^4b}{(c+dx)^4}+\frac{3d^5(a+bx)^5}{(c+dx)^5}\right)(c+dx)^4}{(a+bx)^4}+\frac{60b^2d^3\log\left(\frac{a+bx}{c+dx}\right)(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx}-\frac{b^5(c+dx)^3\left(B\,\mathrm{l}\right.}{3(}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^5(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}+\frac{5b^4d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{10b^3d^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-10b^2d^3\log\left(\frac{a-}{c-}\right.$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])/((a*g + b*g*x)^4*(c*i + d*i*x)^
3),x]
```

output
```
(-1/2*(d^5*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x)^2
 + (5*b*d^4*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(c + d*x) -
(10*b^3*d^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) +
(5*b^4*d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^
2) - (b^5*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)
^3) - 10*b^2*d^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[(a + b*x)/(c +
 d*x)] + (B*n*((3*d^5*(a + b*x)^2)/(2*(c + d*x)^2) - (30*b*d^4*(a + b*x))/
(c + d*x) - (60*b^3*d^2*(c + d*x))/(a + b*x) + (15*b^4*d*(c + d*x)^2)/(2*(
a + b*x)^2) - (2*b^5*(c + d*x)^3)/(3*(a + b*x)^3) + 30*b^2*d^3*Log[(a + b*
x)/(c + d*x)]^2))/6)/((b*c - a*d)^6*g^4*i^3)
```

3.158. $\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx$

### 3.158.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2010
```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

rule 2772
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_)^(r_
.))^(q_.), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x^r)^q, x]}, Simp[(a +
 b*Log[c*x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]]
/; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[q, 0] && IntegerQ[m] &&  !(EqQ[q
, 1] && EqQ[m, -1])
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.158.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2148 vs. $2(575) = 1150$.

Time = 206.92 (sec) , antiderivative size = 2149, normalized size of antiderivative = 3.66

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 2149 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,method=_
RETURNVERBOSE)
```

output
```
-1/36*(-180*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^6*c*d^8*n+360*B*ln(e*((b*x+a
)/(d*x+c))^n)*a^2*b^8*c^3*d^6*n-90*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c^4*d
^5*n+1080*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c*d^8*n+1620*B*x^2*ln(e*((
b*x+a)/(d*x+c))^n)*a*b^9*c^2*d^7*n-720*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b
^7*c*d^8*n+1080*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c^2*d^7*n+360*B*x*ln
(e*((b*x+a)/(d*x+c))^n)*a*b^9*c^3*d^6*n-9*B*a^5*b^5*d^9*n^2+4*B*b^10*c^5*d
^4*n^2+18*A*a^5*b^5*d^9*n+12*A*b^10*c^5*d^4*n+180*B*x^5*ln(e*((b*x+a)/(d*x
+c))^n)^2*b^10*d^9+360*A*x^5*ln(e*((b*x+a)/(d*x+c))^n)*b^10*d^9+600*B*x^4*
ln(e*((b*x+a)/(d*x+c))^n)*b^10*c*d^8*n+1080*B*x^3*ln(e*((b*x+a)/(d*x+c))^n
)^2*a*b^9*c*d^8-540*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*d^9*n+660*B*x^
3*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^2*d^7*n-240*B*x^3*a*b^9*c*d^8*n^2+2160*
A*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c*d^8+1260*A*x^2*a*b^9*c^2*d^7*n+360
*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^7*c*d^8+540*B*x*ln(e*((b*x+a)/(d*x+
c))^n)^2*a^2*b^8*c^2*d^7-780*B*x^2*a^2*b^8*c*d^8*n^2+420*B*x^2*a*b^9*c^2*d
^7*n^2+2160*A*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^8*c*d^8+1080*A*x^2*ln(e*
((b*x+a)/(d*x+c))^n)*a*b^9*c^2*d^7-720*A*x^2*a^2*b^8*c*d^8*n-90*B*x*ln(e*(
(b*x+a)/(d*x+c))^n)*a^4*b^6*d^9*n-30*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^
4*d^5*n+360*A*x^3*a*b^9*c*d^8*n+1080*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2
*b^8*c*d^8+540*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^9*c^2*d^7-540*B*x^2*l
n(e*((b*x+a)/(d*x+c))^n)*a^3*b^7*d^9*n+120*B*x^2*ln(e*((b*x+a)/(d*x+c))...
```

### 3.158.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2181 vs. $2(575) = 1150$.

Time = 0.46 (sec) , antiderivative size = 2181, normalized size of antiderivative = 3.72

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output
```
-1/36*(12*A*b^5*c^5 - 90*A*a*b^4*c^4*d + 360*A*a^2*b^3*c^3*d^2 - 120*A*a^3
*b^2*c^2*d^3 - 180*A*a^4*b*c*d^4 + 18*A*a^5*d^5 + 120*(3*A*b^5*c*d^4 - 3*A
*a*b^4*d^5 + (B*b^5*c*d^4 - B*a*b^4*d^5)*n)*x^4 + 60*(9*A*b^5*c^2*d^3 + 6*
A*a*b^4*c*d^4 - 15*A*a^2*b^3*d^5 + 2*(3*B*b^5*c^2*d^3 - 2*B*a*b^4*c*d^4 -
B*a^2*b^3*d^5)*n)*x^3 + 20*(6*A*b^5*c^3*d^2 + 63*A*a*b^4*c^2*d^3 - 36*A*a^
2*b^3*c*d^4 - 33*A*a^3*b^2*d^5 + (11*B*b^5*c^3*d^2 + 21*B*a*b^4*c^2*d^3 -
39*B*a^2*b^3*c*d^4 + 7*B*a^3*b^2*d^5)*n)*x^2 + 180*(B*b^5*d^5*n*x^5 + B*a^
3*b^2*c^2*d^3*n + (2*B*b^5*c*d^4 + 3*B*a*b^4*d^5)*n*x^4 + (B*b^5*c^2*d^3 +
 6*B*a*b^4*c*d^4 + 3*B*a^2*b^3*d^5)*n*x^3 + (3*B*a*b^4*c^2*d^3 + 6*B*a^2*b
^3*c*d^4 + B*a^3*b^2*d^5)*n*x^2 + (3*B*a^2*b^3*c^2*d^3 + 2*B*a^3*b^2*c*d^4
)*n*x)*log((b*x + a)/(d*x + c))^2 + (4*B*b^5*c^5 - 45*B*a*b^4*c^4*d + 360*
B*a^2*b^3*c^3*d^2 - 490*B*a^3*b^2*c^2*d^3 + 180*B*a^4*b*c*d^4 - 9*B*a^5*d^
5)*n - 5*(6*A*b^5*c^4*d - 72*A*a*b^4*c^3*d^2 - 144*A*a^2*b^3*c^2*d^3 + 192
*A*a^3*b^2*c*d^4 + 18*A*a^4*b*d^5 + (5*B*b^5*c^4*d - 108*B*a*b^4*c^3*d^2 +
 78*B*a^2*b^3*c^2*d^3 + 52*B*a^3*b^2*c*d^4 - 27*B*a^4*b*d^5)*n)*x + 6*(2*B
*b^5*c^5 - 15*B*a*b^4*c^4*d + 60*B*a^2*b^3*c^3*d^2 - 20*B*a^3*b^2*c^2*d^3
- 30*B*a^4*b*c*d^4 + 3*B*a^5*d^5 + 60*(B*b^5*c*d^4 - B*a*b^4*d^5)*x^4 + 30
*(3*B*b^5*c^2*d^3 + 2*B*a*b^4*c*d^4 - 5*B*a^2*b^3*d^5)*x^3 + 10*(2*B*b^5*c
^3*d^2 + 21*B*a*b^4*c^2*d^3 - 12*B*a^2*b^3*c*d^4 - 11*B*a^3*b^2*d^5)*x^2 -
 5*(B*b^5*c^4*d - 12*B*a*b^4*c^3*d^2 - 24*B*a^2*b^3*c^2*d^3 + 32*B*a^3*...
```

## 3.158.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))/(b*g*x+a*g)**4/(d*i*x+c*i)**3,x
)
```

output
```
Timed out
```

### 3.158.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3819 vs. 2(575) = 1150.

Time = 0.53 (sec) , antiderivative size = 3819, normalized size of antiderivative = 6.51

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output
```
-1/6*B*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^2
+ 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(2*
b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a*b
^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*c^
4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*b^
3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3 +
10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d^7
)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5*c
^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*b*
d^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25*a
^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 - a
^8*d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5*d
^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*d^
6)*g^4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*a^
6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log(b
*x + a)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*d^
3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*lo
g(d*x + c)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3
*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n) - 1/36*(4*b^5*c^5 - 45*a*b^4*c^4*d + 360*a^...
```

### 3.158.8 Giac [F]

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \int \frac{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}{(bgx+ag)^4(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)/((b*g*x + a*g)^4*(d*i*x +
 c*i)^3), x)
```

### 3.158.9 Mupad [B] (verification not implemented)

Time = 7.38 (sec) , antiderivative size = 2400, normalized size of antiderivative = 4.09

$$\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))/((a*g + b*g*x)^4*(c*i + d*i*x)^
3),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((x*((5*B*(2*a*b*d^2 + b^2*c*d)*(a*d + b*c)
)/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - (5*B*b*d)/(6*(a^2*d^2 + b^2*c^2
- 2*a*b*c*d)) + (5*B*a*b^2*c*d^2)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + x^2
*((5*B*b*d*(2*a*b*d^2 + b^2*c*d))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) +
(5*B*b^2*d^2*(a*d + b*c))/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) - (B*(3*a*d +
 2*b*c))/(6*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (5*B*a*c*(2*a*b*d^2 + b^2*c
*d))/(3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2) + (5*B*b^3*d^3*x^3)/(a^2*d^2 +
b^2*c^2 - 2*a*b*c*d)^2)/(x*(2*a^3*c*d*g^4*i^3 + 3*a^2*b*c^2*g^4*i^3) + x^2
*(a^3*d^2*g^4*i^3 + 3*a*b^2*c^2*g^4*i^3 + 6*a^2*b*c*d*g^4*i^3) + x^3*(b^3*
c^2*g^4*i^3 + 3*a^2*b*d^2*g^4*i^3 + 6*a*b^2*c*d*g^4*i^3) + x^4*(2*b^3*c*d*
g^4*i^3 + 3*a*b^2*d^2*g^4*i^3) + a^3*c^2*g^4*i^3 + b^3*d^2*g^4*i^3*x^5) +
(10*B*b^2*d^3*(x^2*((g^4*i^3*n*(a*d + b*c)^2*(a*d - b*c))/d + 2*a*b*c*g^4*
i^3*n*(a*d - b*c)) + b^2*d*g^4*i^3*n*x^4*(a*d - b*c) + (a^2*c^2*g^4*i^3*n*
(a*d - b*c))/d + 2*b*g^4*i^3*n*x^3*(a*d + b*c)*(a*d - b*c) + (2*a*c*g^4*i^
3*n*x*(a*d + b*c)*(a*d - b*c))/d))/(g^4*i^3*n*(a*d - b*c)^6*(x*(2*a^3*c*d*
g^4*i^3 + 3*a^2*b*c^2*g^4*i^3) + x^2*(a^3*d^2*g^4*i^3 + 3*a*b^2*c^2*g^4*i^
3 + 6*a^2*b*c*d*g^4*i^3) + x^3*(b^3*c^2*g^4*i^3 + 3*a^2*b*d^2*g^4*i^3 + 6*
a*b^2*c*d*g^4*i^3) + x^4*(2*b^3*c*d*g^4*i^3 + 3*a*b^2*d^2*g^4*i^3) + a^3*c
^2*g^4*i^3 + b^3*d^2*g^4*i^3*x^5))) + ((12*A*b^4*c^4 - 18*A*a^4*d^4 + 9*B*
a^4*d^4*n + 4*B*b^4*c^4*n + 282*A*a^2*b^2*c^2*d^2 - 78*A*a*b^3*c^3*d + ...
```

# 3.159 $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$



## 3.159.1 Optimal result

Integrand size = 43, antiderivative size = 584

$$\begin{aligned}
&\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx\\
&=\frac{3B^2(bc-ad)^4g^3in^2x}{10bd^3}-\frac{3B^2(bc-ad)^3g^3in^2(c+dx)^2}{20d^4}\\
&\quad+\frac{bB^2(bc-ad)^2g^3in^2(c+dx)^3}{30d^4}-\frac{B(bc-ad)^2g^3in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^2d}\\
&\quad-\frac{B(bc-ad)g^3in(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2}\\
&\quad+\frac{(bc-ad)g^3i(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2}\\
&\quad+\frac{g^3i(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b}\\
&\quad+\frac{B(bc-ad)^3g^3in(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^2d^2}\\
&\quad-\frac{B(bc-ad)^4g^3in(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^2d^3}\\
&\quad-\frac{B(bc-ad)^5g^3in\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{60b^2d^4}\\
&\quad-\frac{B^2(bc-ad)^5g^3in^2\log(c+dx)}{10b^2d^4}-\frac{B^2(bc-ad)^5g^3in^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^2d^4}
\end{aligned}$$

output
```
3/10*B^2*(-a*d+b*c)^4*g^3*i*n^2*x/b/d^3-3/20*B^2*(-a*d+b*c)^3*g^3*i*n^2*(d
*x+c)^2/d^4+1/30*b*B^2*(-a*d+b*c)^2*g^3*i*n^2*(d*x+c)^3/d^4-1/30*B*(-a*d+b
*c)^2*g^3*i*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/10*B*(-a*d
+b*c)*g^3*i*n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/20*(-a*d+b*c
)*g^3*i*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2+1/5*g^3*i*(b*x+a)^
4*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/60*B*(-a*d+b*c)^3*g^3*i*n*
(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d^2-1/60*B*(-a*d+b*c
)^4*g^3*i*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d^3-1/60
*B*(-a*d+b*c)^5*g^3*i*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*
d+b*c)/b/(d*x+c))/b^2/d^4-1/10*B^2*(-a*d+b*c)^5*g^3*i*n^2*ln(d*x+c)/b^2/d^
4-1/10*B^2*(-a*d+b*c)^5*g^3*i*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^4
```

### 3.159.2 Mathematica [A] (verified)

Time = 0.47 (sec) , antiderivative size = 949, normalized size of antiderivative = 1.62

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{g^3i\left(5(bc-ad)(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+4d(a+bx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{5B(bc-ad)^2n\left(6A\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2,x]
```

output
```
(g^3*i*(5*(b*c - a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
 + 4*d*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (5*B*(b*c -
a*d)^2*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((
a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*Log[e*
((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c - a*d
)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a*d)^2*
n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*Log[(d*(
a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(
c + d*x))/(b*c - a*d)])))/(3*d^4) + (B*(b*c - a*d)*n*(24*A*b*d*(b*c - a*d)
^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - 12*
d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*d
^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*d^4*
(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*n*
Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log
[c + d*x] + 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2
*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*
d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d)^3*Log[c
 + d*x]) + 12*B*(b*c - a*d)^3*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) +...
```

### 3.159.3 Rubi [A] (verified)

Time = 1.43 (sec) , antiderivative size = 646, normalized size of antiderivative = 1.11, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.256$, Rules used = {2961, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2961}$$

$$g^3i(bc-ad)^5\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$g^3 i(bc-ad)^5\left(-\frac{2Bn\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{5b(c+dx)^4\left(b-\right.}\right.$$

$\downarrow$ 2773

$$g^3 i(bc-ad)^5\left(-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int \frac{(a+bx)^3}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}\right)}{5b}+\frac{\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}\right.$$

$\downarrow$ 49

$$g^3 i(bc-ad)^5\left(-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int \left(\frac{b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{1}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+b}{c+d}}{4b}\right.}{5b}\right.$$

$\downarrow$ 2009

$$g^3 i(bc-ad)^5\left(\frac{\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{b^3}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3b^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2781

3.159. $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^3 i(bc-ad)^5 \left( \frac{\frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{2b}}{5b} - \frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn}{\cdots}\right.}{\cdots} \right.$$

$\downarrow$ 2784

$$g^3 i(bc-ad)^5 \left( \frac{\frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\int \frac{(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} d\frac{a+bx}{c+dx}}{3d}\right)}{2b}}{5b} - \frac{2Bn\left(\cdots\right.}{\cdots} \right.$$

$\downarrow$ 2784

$$g^3 i(bc-ad)^5 \left( \frac{\frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{\int \frac{(a+bx)\left(6A+5Bn+6\cdots\right.}{(c+dx)\left(b-\cdots\right.}}{\cdots}}{3d}\right)}{2b}}{5b} \cdots \right.$$

$\downarrow$ 2784

3.159. $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$g^3 i(bc-ad)^5\left(\frac{\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(e\left(\frac{a+b}{c+d}\cdots\right.\right.\right.}{d(c+dx)\left(b-\frac{d}{\cdots}\right.}\cdots}{3d\cdots}\cdots\right.}{2b\cdots}\cdots}{5b\cdots}\cdots\right.$$

$\downarrow$ 2754

$$g^3 i(bc-ad)^5\left(\frac{\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{\frac{(a+bx)^2\left(3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3A+Bn\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(6B\log\left(e\left(\frac{a+b}{c+d}\cdots\right.\right.\right.}{d(c+dx)\left(b-\frac{d}{\cdots}\right.}\cdots}{\cdots}\right.}{\cdots}\cdots}{5b\cdots}\cdots\right.$$

$\downarrow$ 2838

3.159. $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^3 i(bc - ad)^5 \left( -\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{b^3}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{3b^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2} + \frac{3b}{d^4\left(b-\frac{d(a+bx)}{c+dx}\right)} + \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^4}\right)}{4b}\right)}{5b} + \right.$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
,x]
```

output
```
(b*c - a*d)^5*g^3*i*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/(5*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*n*(((a + b*x)^4
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b*(c + d*x)^4*(b - (d*(a + b*x
))/(c + d*x))^4) - (B*n*(b^3/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) - (3*
b^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3*b)/(d^4*(b - (d*(a + b*x
))/(c + d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]/d^4))/(4*b)))/(5*b) + ((
(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b*(c + d*x)^4*(b
- (d*(a + b*x))/(c + d*x))^4) - (B*n*(((a + b*x)^3*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a
+ b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^
2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(6*A + 5*B*n + 6*B*Log[e*
((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (
-(((6*A + 11*B*n + 6*B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x
))/(b*(c + d*x))])/d) - (6*B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)
/d)/(2*d))/(3*d)))/(2*b))/(5*b))
```

## 3.159. $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

### 3.159.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

## 3.159. $\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.159.4 Maple [F]

$$\int (bgx+ag)^3 (dix+ci) \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

### 3.159.5 Fricas [F]

$$\int (ag+bgx)^3(ci+dix) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (bgx+ag)^3(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="fricas")
```

output
```
integral(A^2*b^3*d*g^3*i*x^4 + A^2*a^3*c*g^3*i + (A^2*b^3*c + 3*A^2*a*b^2*
d)*g^3*i*x^3 + 3*(A^2*a*b^2*c + A^2*a^2*b*d)*g^3*i*x^2 + (3*A^2*a^2*b*c +
A^2*a^3*d)*g^3*i*x + (B^2*b^3*d*g^3*i*x^4 + B^2*a^3*c*g^3*i + (B^2*b^3*c +
 3*B^2*a*b^2*d)*g^3*i*x^3 + 3*(B^2*a*b^2*c + B^2*a^2*b*d)*g^3*i*x^2 + (3*B
^2*a^2*b*c + B^2*a^3*d)*g^3*i*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B
*b^3*d*g^3*i*x^4 + A*B*a^3*c*g^3*i + (A*B*b^3*c + 3*A*B*a*b^2*d)*g^3*i*x^3
 + 3*(A*B*a*b^2*c + A*B*a^2*b*d)*g^3*i*x^2 + (3*A*B*a^2*b*c + A*B*a^3*d)*g
^3*i*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

## 3.159.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x
)
```

output
```
Timed out
```

## 3.159.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3764 vs. $2(559) = 1118$.

Time = 0.75 (sec) , antiderivative size = 3764, normalized size of antiderivative = 6.45

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="maxima")
```

output
```
2/5*A*B*b^3*d*g^3*i*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*b
^3*d*g^3*i*x^5 + 1/2*A*B*b^3*c*g^3*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + 3/2*A*B*a*b^2*d*g^3*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
 1/4*A^2*b^3*c*g^3*i*x^4 + 3/4*A^2*a*b^2*d*g^3*i*x^4 + 2*A*B*a*b^2*c*g^3*i
*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a^2*b*d*g^3*i*x^3*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*b^2*c*g^3*i*x^3 + A^2*a^2*b*d*g
^3*i*x^3 + 3*A*B*a^2*b*c*g^3*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ A*B*a^3*d*g^3*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A^2*a^2
*b*c*g^3*i*x^2 + 1/2*A^2*a^3*d*g^3*i*x^2 + 1/30*A*B*b^3*d*g^3*i*n*(12*a^5*
log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^
4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12
*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/12*A*B*b^3*c*g^3*i*n*(6*a^4*log(b*x
 + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b
^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/4*A*B*
a*b^2*d*g^3*i*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3
*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3
*d^3)*x)/(b^3*d^3)) + A*B*a*b^2*c*g^3*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*
log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^
2*d^2)) + A*B*a^2*b*d*g^3*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) -...
```

### 3.159.8 Giac [F]

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (bgx+ag)^3(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n)
+ A)^2, x)
```

### 3.159.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
,x)
```

output
```
int((a*g + b*g*x)^3*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
, x)
```

# 3.160 $\int (ag+bgx)^2(ci+dix) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$



## 3.160.1 Optimal result

Integrand size = 43, antiderivative size = 487

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= -\frac{B^2(bc-ad)^3g^2in^2x}{3bd^2}+\frac{B^2(bc-ad)^2g^2in^2(c+dx)^2}{12d^3} \\
&\quad -\frac{B(bc-ad)^2g^2in(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2d} \\
&\quad -\frac{B(bc-ad)g^2in(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^2} \\
&\quad +\frac{(bc-ad)g^2i(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^2} \\
&\quad +\frac{g^2i(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b} \\
&\quad +\frac{B(bc-ad)^3g^2in(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{12b^2d^2} \\
&\quad +\frac{B(bc-ad)^4g^2in\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{12b^2d^3} \\
&\quad +\frac{B^2(bc-ad)^4g^2in^2\log(c+dx)}{6b^2d^3}+\frac{B^2(bc-ad)^4g^2in^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^2d^3}
\end{aligned}$$

output
```
-1/3*B^2*(-a*d+b*c)^3*g^2*i*n^2*x/b/d^2+1/12*B^2*(-a*d+b*c)^2*g^2*i*n^2*(d
*x+c)^2/d^3-1/12*B*(-a*d+b*c)^2*g^2*i*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))/b^2/d-1/6*B*(-a*d+b*c)*g^2*i*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/b^2+1/12*(-a*d+b*c)*g^2*i*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
^2/b^2+1/4*g^2*i*(b*x+a)^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/1
2*B*(-a*d+b*c)^3*g^2*i*n*(b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n))/b
^2/d^2+1/12*B*(-a*d+b*c)^4*g^2*i*n*(2*A+3*B*n+2*B*ln(e*((b*x+a)/(d*x+c))^n
))*ln((-a*d+b*c)/b/(d*x+c))/b^2/d^3+1/6*B^2*(-a*d+b*c)^4*g^2*i*n^2*ln(d*x+
c)/b^2/d^3+1/6*B^2*(-a*d+b*c)^4*g^2*i*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b
^2/d^3
```

## 3.160.2 Mathematica [A] (verified)

Time = 0.34 (sec) , antiderivative size = 716, normalized size of antiderivative = 1.47

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$

$$=\frac{g^2i\Big(4(bc-ad)(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+3d(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{4B(bc-ad)^2n\big(2A\cdots}{\cdots}}{\cdots}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2,x]
```

output
```
(g^2*i*(4*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
 + 3*d*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (4*B*(b*c -
a*d)^2*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a +
b*x)/(c + d*x))^n] - d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
) - 2*B*(b*c - a*d)^2*n*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*Log[e*((a +
b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(b*d*x + (-(b*c) + a*d)
*Log[c + d*x]) + B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] -
 Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/d
^3 - (B*(b*c - a*d)*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a +
b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b
*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(
b*c - a*d)^2*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n
*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*Po
lyLog[2, (b*(c + d*x))/(b*c - a*d)]))/d^3))/(12*b^2)
```

## 3.160.3 Rubi [A] (verified)

Time = 1.19 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.11, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.233$, Rules used = {2961, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2961}$$

$$g^2i(bc-ad)^4\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$g^2i(bc-ad)^4\left(-\frac{Bn\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}+\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{4b(c+dx)^3\left(b-\frac{d(}{c}\right.}\right.$$

$$\Big\downarrow \text{2773}$$

$$g^2i(bc-ad)^4\left(-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\frac{(a+bx)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right)}{2b}+\frac{\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}\right.$$

$$\Big\downarrow \text{49}$$

$$g^2i(bc-ad)^4\left(-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\left(\frac{b^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{1}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}}{3b}\right)}{2b}+\frac{\int\frac{(a+bx)^2}{(c+\ldots}\ldots}{\ldots}\right.$$

$$\Big\downarrow \text{2009}$$

$$g^2i(bc-ad)^4\left(\frac{\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{b^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\ldots\right)}{3b}\ldots\right)}{2b}\ldots\right.$$

$$\Big\downarrow \text{2781}$$

3.160. $\int(ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^2 i(bc-ad)^4 \left( \frac{\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}}{4b} - \frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn}{\ldots}\right.}{\ldots}\right.$$

$\downarrow$ 2784

$$g^2 i(bc-ad)^4 \left( \frac{\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\int\frac{(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{3b}}{4b} - \frac{Bn\left(\ldots\right.}{\ldots}\right.$$

$\downarrow$ 2784

$$g^2 i(bc-ad)^4 \left( \frac{\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{2A+3Bn+2B\log\left(e\left(\ldots\right.\right.}{b-\frac{d(a+bx)}{c+dx}}\ldots}{d}}{2d}\ldots\right.}{3b}}{4b}\ldots\right.$$

$\downarrow$ 2754

3.160. $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^2 i(bc-ad)^4\left(\frac{\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\frac{(a+bx)\left(2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+2A+Bn\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2Bn\int\frac{(c+dx)\log\left(1-\frac{c}{\cdots}\right.}{\frac{a+bx}{d}}}{\cdots}}{\cdots}\right.}{3b}}{4b}\right.$$

$\downarrow$ 2838

$$g^2 i(bc-ad)^4\left(-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{b^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^3}\right)}{3b}\right)}{2b}+\frac{(a+bx)^3\left(B\log\left(e\left(\cdots\right.\right.\right.}{3b(c+dx)^3\left(b-\cdots\right.}\right.$$

input

```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
,x]
```

3.160. $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

output
```
(b*c - a*d)^4*g^2*i*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/(4*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*(((a + b*x)^3*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b*(c + d*x)^3*(b - (d*(a + b*x))
/(c + d*x))^3) - (B*n*(b^2/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*b)
/(d^3*(b - (d*(a + b*x))/(c + d*x))) - Log[b - (d*(a + b*x))/(c + d*x)]/d^
3))/(3*b)))/(2*b) + (((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a + b*x)^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x
))/(c + d*x))^2) - (((a + b*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((2*A + 3*B*n + 2*
B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d)
 - (2*B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d)))/(3*b))/(4
*b))
```

### 3.160.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.160. $\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

### 3.160.4 Maple [F]

$$\int (bgx+ag)^2 (dix+ci) \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

### 3.160.5 Fricas [F]

$$\int (ag+bgx)^2(ci+dix) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^2(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="fricas")
```

output
```
integral(A^2*b^2*d*g^2*i*x^3 + A^2*a^2*c*g^2*i + (A^2*b^2*c + 2*A^2*a*b*d)
*g^2*i*x^2 + (2*A^2*a*b*c + A^2*a^2*d)*g^2*i*x + (B^2*b^2*d*g^2*i*x^3 + B^
2*a^2*c*g^2*i + (B^2*b^2*c + 2*B^2*a*b*d)*g^2*i*x^2 + (2*B^2*a*b*c + B^2*a
^2*d)*g^2*i*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^2*d*g^2*i*x^3 +
 A*B*a^2*c*g^2*i + (A*B*b^2*c + 2*A*B*a*b*d)*g^2*i*x^2 + (2*A*B*a*b*c + A*
B*a^2*d)*g^2*i*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

### 3.160.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix) \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x
)
```

output
```
Timed out
```

### 3.160.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2691 vs. 2(466) = 932.

Time = 0.73 (sec) , antiderivative size = 2691, normalized size of antiderivative = 5.53

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="maxima")
```

output
```
1/2*A*B*b^2*d*g^2*i*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*b
^2*d*g^2*i*x^4 + 2/3*A*B*b^2*c*g^2*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + 4/3*A*B*a*b*d*g^2*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1
/3*A^2*b^2*c*g^2*i*x^3 + 2/3*A^2*a*b*d*g^2*i*x^3 + 2*A*B*a*b*c*g^2*i*x^2*l
og(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*B*a^2*d*g^2*i*x^2*log(e*(b*x/(d*
x + c) + a/(d*x + c))^n) + A^2*a*b*c*g^2*i*x^2 + 1/2*A^2*a^2*d*g^2*i*x^2 -
 1/12*A*B*b^2*d*g^2*i*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 +
 (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c
^3 - a^3*d^3)*x)/(b^3*d^3)) + 1/3*A*B*b^2*c*g^2*i*n*(2*a^3*log(b*x + a)/b^
3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d
^2)*x)/(b^2*d^2)) + 2/3*A*B*a*b*d*g^2*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*
log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^
2*d^2)) - 2*A*B*a*b*c*g^2*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2
 + (b*c - a*d)*x/(b*d)) - A*B*a^2*d*g^2*i*n*(a^2*log(b*x + a)/b^2 - c^2*lo
g(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a^2*c*g^2*i*n*(a*log(b*x + a
)/b - c*log(d*x + c)/d) + 2*A*B*a^2*c*g^2*i*x*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) + A^2*a^2*c*g^2*i*x - 1/12*(2*a^3*c*d^3*g^2*i*n^2 + (g^2*i*n^2
+ 2*g^2*i*n*log(e))*b^3*c^4 - 2*(g^2*i*n^2 + 4*g^2*i*n*log(e))*a*b^2*c^3*d
 - (g^2*i*n^2 - 12*g^2*i*n*log(e))*a^2*b*c^2*d^2)*B^2*log(d*x + c)/(b*d^3)
 - 1/6*(b^4*c^4*g^2*i*n^2 - 4*a*b^3*c^3*d*g^2*i*n^2 + 6*a^2*b^2*c^2*d^2...
```

### 3.160.8 Giac [F]

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (bgx+ag)^2(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n)
+ A)^2, x)
```

### 3.160.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
,x)
```

output
```
int((a*g + b*g*x)^2*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
, x)
```

# 3.161 $\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$



## 3.161.1 Optimal result

Integrand size = 41, antiderivative size = 372

$$\begin{aligned}
&\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \frac{B^2(bc-ad)^2gin^2x}{3bd}-\frac{B(bc-ad)^2gin(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2d} \\
&\quad -\frac{B(bc-ad)gin(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2} \\
&\quad +\frac{(bc-ad)gi(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2} \\
&\quad +\frac{gi(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3b} \\
&\quad -\frac{B(bc-ad)^3gin\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{3b^2d^2} \\
&\quad -\frac{B^2(bc-ad)^3gin^2\log(c+dx)}{3b^2d^2}-\frac{B^2(bc-ad)^3gin^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{3b^2d^2}
\end{aligned}$$

output
```
1/3*B^2*(-a*d+b*c)^2*g*i*n^2*x/b/d-1/3*B*(-a*d+b*c)^2*g*i*n*(b*x+a)*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))/b^2/d-1/3*B*(-a*d+b*c)*g*i*n*(b*x+a)^2*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))/b^2+1/6*(-a*d+b*c)*g*i*(b*x+a)^2*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))^2/b^2+1/3*g*i*(b*x+a)^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))^2/b-1/3*B*(-a*d+b*c)^3*g*i*n*(A+B*n+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-
a*d+b*c)/b/(d*x+c))/b^2/d^2-1/3*B^2*(-a*d+b*c)^3*g*i*n^2*ln(d*x+c)/b^2/d^2
-1/3*B^2*(-a*d+b*c)^3*g*i*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/d^2
```

### 3.161.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 937 vs. 2(372) = 744.

Time = 0.44 (sec) , antiderivative size = 937, normalized size of antiderivative = 2.52

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{gi\Big(-6Ab^2Bcd(bc-ad)nx+6aAbBd^2(-bc+ad)nx+4AbBd(bc-ad)(bc+ad)nx-6bB^2cd(bc-ad)n}{}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])^2,x]
```

output
```
(g*i*(-6*A*b^2*B*c*d*(b*c - a*d)*n*x + 6*a*A*b*B*d^2*(-(b*c) + a*d)*n*x +
4*A*b*B*d*(b*c - a*d)*(b*c + a*d)*n*x - 6*b*B^2*c*d*(b*c - a*d)*n*(a + b*x
)*Log[e*((a + b*x)/(c + d*x))^n] + 6*a*B^2*d^2*(-(b*c) + a*d)*n*(a + b*x)*
Log[e*((a + b*x)/(c + d*x))^n] + 4*B^2*d*(b*c - a*d)*(b*c + a*d)*n*(a + b*
x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*b^2*B*d^2*(b*c - a*d)*n*x^2*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]) + 6*a^2*b*B*c*d^2*n*Log[a + b*x]*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) - 2*a^3*B*d^3*n*Log[a + b*x]*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]) + 6*a*b^2*c*d^2*x*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2 + 3*b^2*d^2*(b*c + a*d)*x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2 + 2*b^3*d^3*x^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 6*b*B^2*c*(b
*c - a*d)^2*n^2*Log[c + d*x] + 6*a*B^2*d*(b*c - a*d)^2*n^2*Log[c + d*x] -
4*B^2*(b*c - a*d)^2*(b*c + a*d)*n^2*Log[c + d*x] + 2*b^3*B*c^3*n*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 6*a*b^2*B*c^2*d*n*(A + B*Log[
e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 2*B^2*(b*c - a*d)*n^2*(a^2*d^2*
Log[a + b*x] - b*(d*(-(b*c) + a*d)*x + b*c^2*Log[c + d*x])) - 3*a^2*b*B^2*
c*d^2*n^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)])
- 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + a^3*B^2*d^3*n^2*(Log[a + b
*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)]) - b^3*B^2*c^3*n^2*((2*Log[(d*(a + b*x))/(-(b*c)
+ a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c ...
```

### 3.161.3 Rubi [A] (verified)

Time = 0.88 (sec) , antiderivative size = 426, normalized size of antiderivative = 1.15, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.220, Rules used = {2961, 2783, 2773, 49, 2009, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2961}$$

$$gi(bc-ad)^3\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$gi(bc-ad)^3\left(-\frac{2Bn\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{3b(c+dx)^2\left(b-\frac{d(a}{c\right.}}\right.$$

$$\Big\downarrow \text{2773}$$

$$gi(bc-ad)^3\left(-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int \frac{a+bx}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\right.$$

$$\Big\downarrow \text{49}$$

$$gi(bc-ad)^3\left(-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{b}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}+\frac{1}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{3b}\right.$$

$\downarrow$ 2009

$$gi(bc-ad)^3\left(\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^2}\right)}{2b}\right.}{3b}\right.$$

$\downarrow$ 2781

$$gi(bc-ad)^3\left(\frac{\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{3b}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\right.}{}\right.}{3b}\right.$$

$\downarrow$ 2784

---

3.161. $\int(ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$gi(bc-ad)^3\left(\frac{\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}}{3b}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\right.}{2b(c+d\right.}}{}\right.$$

$\downarrow$ 2754

$$gi(bc-ad)^3\left(\frac{\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\right.\right.}{d}}{d}\right.}{b}}{3b}\right.$$

$\downarrow$ 2838

$$gi(bc-ad)^3\left(-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^2}\right)}{2b}\right)}{3b}+\frac{\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-}{}\right.$$

input

```
Int[(a*g + b*g*x)*(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x
]
```

3.161. $\int(ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

output
```
(b*c - a*d)^3*g*i*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(3*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a + b*x)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b*(c + d*x)^2*(b - (d*(a + b*x))
/(c + d*x))^2) - (B*n*(b/(d^2*(b - (d*(a + b*x))/(c + d*x))) + Log[b - (d*
(a + b*x))/(c + d*x)]/d^2))/(2*b)))/(3*b) + (((a + b*x)^2*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2)/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2
) - (B*n*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*
(b - (d*(a + b*x))/(c + d*x))) - (-(((A + B*n + B*Log[e*((a + b*x)/(c + d*
x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*n*PolyLog[2, (d*(a +
 b*x))/(b*(c + d*x))])/d)/d))/b)/(3*b))
```

### 3.161.3.1 Defintions of rubi rules used

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

## 3.161. $\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.161.4 Maple [F]

$$\int (bgx+ag)\,(dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}dx$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

### 3.161.5 Fricas [F]

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$=\int (bgx+ag)(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, al
gorithm="fricas")
```

output
```
integral(A^2*b*d*g*i*x^2 + A^2*a*c*g*i + (A^2*b*c + A^2*a*d)*g*i*x + (B^2*
b*d*g*i*x^2 + B^2*a*c*g*i + (B^2*b*c + B^2*a*d)*g*i*x)*log(e*((b*x + a)/(d
*x + c))^n)^2 + 2*(A*B*b*d*g*i*x^2 + A*B*a*c*g*i + (A*B*b*c + A*B*a*d)*g*i
*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

### 3.161.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

### 3.161.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1542 vs. 2(355) = 710.

Time = 0.71 (sec) , antiderivative size = 1542, normalized size of antiderivative = 4.15

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, al
gorithm="maxima")
```

output
```
2/3*A*B*b*d*g*i*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*b*d*g
*i*x^3 + A*B*b*c*g*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A*B*a*d*
g*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*b*c*g*i*x^2 + 1/2
*A^2*a*d*g*i*x^2 + 1/3*A*B*b*d*g*i*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d
*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2
)) - A*B*b*c*g*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a
*d)*x/(b*d)) - A*B*a*d*g*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2
+ (b*c - a*d)*x/(b*d)) + 2*A*B*a*c*g*i*n*(a*log(b*x + a)/b - c*log(d*x + c
)/d) + 2*A*B*a*c*g*i*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*c*g*
i*x - 1/3*(a^2*c*d^2*g*i*n^2 - b^2*c^3*g*i*n*log(e) - (g*i*n^2 - 3*g*i*n*l
og(e))*a*b*c^2*d)*B^2*log(d*x + c)/(b*d^2) + 1/3*(b^3*c^3*g*i*n^2 - 3*a*b^
2*c^2*d*g*i*n^2 + 3*a^2*b*c*d^2*g*i*n^2 - a^3*d^3*g*i*n^2)*(log(b*x + a)*l
og((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2
/(b^2*d^2) + 1/6*(2*B^2*b^3*d^3*g*i*x^3*log(e)^2 - ((2*g*i*n*log(e) - 3*g*
i*log(e)^2)*b^3*c*d^2 - (2*g*i*n*log(e) + 3*g*i*log(e)^2)*a*b^2*d^3)*B^2*x
^2 - (3*a^2*b*c*d^2*g*i*n^2 - a^3*d^3*g*i*n^2)*B^2*log(b*x + a)^2 - 2*(b^3
*c^3*g*i*n^2 - 3*a*b^2*c^2*d*g*i*n^2)*B^2*log(b*x + a)*log(d*x + c) + (b^3
*c^3*g*i*n^2 - 3*a*b^2*c^2*d*g*i*n^2)*B^2*log(d*x + c)^2 + 2*((g*i*n^2 - g
*i*n*log(e))*b^3*c^2*d - (2*g*i*n^2 - 3*g*i*log(e)^2)*a*b^2*c*d^2 + (g*i*n
^2 + g*i*n*log(e))*a^2*b*d^3)*B^2*x - 2*(a*b^2*c^2*d*g*i*n^2 + a^3*d^3*...
```

### 3.161.8 Giac [F]

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (bgx+ag)(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)*(d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) +
A)^2, x)
```

### 3.161.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)\ (c\,i+d\,i\,x)\ \left(A+B\ \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x
)
```

output
```
int((a*g + b*g*x)*(c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,
x)
```

# 3.162 $\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$



## 3.162.1 Optimal result

Integrand size = 33, antiderivative size = 220

$$\begin{aligned}
&\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= -\frac{B(bc-ad)in(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2}+\frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d} \\
&\quad+\frac{B^2(bc-ad)^2in^2\log(c+dx)}{b^2d}+\frac{B(bc-ad)^2in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d} \\
&\quad-\frac{B^2(bc-ad)^2in^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2d}
\end{aligned}$$

output
```
-B*(-a*d+b*c)*i*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/2*i*(d*x+c
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+B^2*(-a*d+b*c)^2*i*n^2*ln(d*x+c)/b
^2/d+B*(-a*d+b*c)^2*i*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(
b*x+a))/b^2/d-B^2*(-a*d+b*c)^2*i*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/d
```

### 3.162.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 216, normalized size of antiderivative = 0.98

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \frac{i\left((c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{B(bc-ad)n\left(B(bc-ad)n\log^2(a+bx)-2\left(Abdx+Bd(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B(-bc+ad)n\right.\right.}{}\right.}{2d}$$

input
```
Integrate[(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(i*((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*(b*c - a*d)*
n*(B*(b*c - a*d)*n*Log[a + b*x]^2 - 2*(A*b*d*x + B*d*(a + b*x)*Log[e*((a +
 b*x)/(c + d*x))^n] + B*(-(b*c) + a*d)*n*Log[c + d*x]) - 2*(b*c - a*d)*Log
[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(b*(c + d*x))/(b
*c - a*d)]) + 2*B*(-(b*c) + a*d)*n*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)
]))/b^2))/(2*d)
```

### 3.162.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 239, normalized size of antiderivative = 1.09, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {2951, 2756, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2951}$$

$$i(bc-ad)^2\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2756}$$

$$i(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{d}\right)$$

$\downarrow$ 2789

$$i(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{d}\right)$$

$\downarrow$ 2751

$$i(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}\right.}{d}\right.$$

$\downarrow$ 16

$$i(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+}{c+}\right.}{bd}\right.}{b}\right.}{d}\right.$$

$\downarrow$ 2779

3.162. $\int(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$i(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\cdots}{b(c\cdots}\right.}{\cdots}\right.}{d}\right.$$

$\Big\downarrow$ 2838

$$i(bc-ad)^2\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\frac{Bn\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+\cdots}{c+\cdots}\right.\right.\right.}{b(c+dx)\left(b-\frac{d(a\cdots}{c\cdots}\right.}\right.}{\cdots}\right.}{d}\right.$$

input
```
Int[(c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^2*i*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*d*(b - (d*(a
+ b*x))/(c + d*x))^2) - (B*n*((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b - (d*(
a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x)
)/(d*(a + b*x))])/b)/b))/d)
```

3.162. $\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

### 3.162.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

## 3.162. $\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

### 3.162.4 Maple [F]

$$\int (dix+ci)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2dx$$

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

### 3.162.5 Fricas [F]

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx=\int (dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="fri
cas")
```

output
```
integral(A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log(e*((b*x + a)/(d*x
 + c))^n)^2 + 2*(A*B*d*i*x + A*B*c*i)*log(e*((b*x + a)/(d*x + c))^n), x)
```

### 3.162.6 Sympy [F(-1)]

Timed out.

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

### 3.162.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 825 vs. $2(217) = 434$.

Time = 0.69 (sec) , antiderivative size = 825, normalized size of antiderivative = 3.75

$$\int (ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = ABdix^2\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right) + \frac{1}{2}A^2dix^2 - ABdin\left(\frac{a^2\log(bx+a)}{b^2} - \frac{c^2\log(dx+c)}{d^2} + \frac{(bc-ad)x}{bd}\right) + 2ABcin\left(\frac{a\log(bx+a)}{b} - \frac{c\log(dx+c)}{d}\right) + 2ABcix\log\left(e\left(\frac{bx}{dx+c} + \frac{a}{dx+c}\right)^n\right) + A^2cix - \frac{(acdin^2 - (in^2 - in\log(e))bc^2)B^2\log(dx+c)}{bd} - \frac{(b^2c^2in^2 - 2abcdin^2 + a^2d^2in^2)\left(\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)\right)B^2}{b^2d} + \frac{2B^2b^2c^2in^2\log(bx+a)\log(dx+c) - B^2b^2c^2in^2\log(dx+c)^2 + B^2b^2d^2ix^2\log(e)^2 - (2abcdin^2 - a^2d^2}{}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="max
ima")
```

output
```
A*B*d*i*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*d*i*x^2 - A*B
*d*i*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d))
 + 2*A*B*c*i*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c*i*x*log(e*(
b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*i*x - (a*c*d*i*n^2 - (i*n^2 - i*n*
log(e))*b*c^2)*B^2*log(d*x + c)/(b*d) - (b^2*c^2*i*n^2 - 2*a*b*c*d*i*n^2 +
 a^2*d^2*i*n^2)*(log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-
(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^2*d) + 1/2*(2*B^2*b^2*c^2*i*n^2*log(b*x
 + a)*log(d*x + c) - B^2*b^2*c^2*i*n^2*log(d*x + c)^2 + B^2*b^2*d^2*i*x^2*
log(e)^2 - (2*a*b*c*d*i*n^2 - a^2*d^2*i*n^2)*B^2*log(b*x + a)^2 + 2*(a*b*d
^2*i*n*log(e) - (i*n*log(e) - i*log(e)^2)*b^2*c*d)*B^2*x - 2*((i*n^2 - 2*i
*n*log(e))*a*b*c*d - (i*n^2 - i*n*log(e))*a^2*d^2)*B^2*log(b*x + a) + (B^2
*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x)*log((b*x + a)^n)^2 + (B^2*b^2*d^2*i*x^
2 + 2*B^2*b^2*c*d*i*x)*log((d*x + c)^n)^2 + 2*(B^2*b^2*d^2*i*x^2*log(e) -
B^2*b^2*c^2*i*n*log(d*x + c) + (a*b*d^2*i*n - (i*n - 2*i*log(e))*b^2*c*d)*
B^2*x + (2*a*b*c*d*i*n - a^2*d^2*i*n)*B^2*log(b*x + a))*log((b*x + a)^n) -
 2*(B^2*b^2*d^2*i*x^2*log(e) - B^2*b^2*c^2*i*n*log(d*x + c) + (a*b*d^2*i*n
 - (i*n - 2*i*log(e))*b^2*c*d)*B^2*x + (2*a*b*c*d*i*n - a^2*d^2*i*n)*B^2*l
og(b*x + a) + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i*x)*log((b*x + a)^n))*lo
g((d*x + c)^n))/(b^2*d)
```

### 3.162.8 Giac [F]

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\int (dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="gia
c")
```

output
```
integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

### 3.162.9 Mupad [F(-1)]

Timed out.

$$\int (ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (c\,i+d\,i\,x)\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

# 3.163 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$



## 3.163.1 Optimal result

Integrand size = 43, antiderivative size = 306

$$\begin{aligned}
&\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx \\
&= \frac{di(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g} \\
&\quad + \frac{2B(bc-ad)in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^2g} \\
&\quad - \frac{(bc-ad)i\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} \\
&\quad + \frac{2B^2(bc-ad)in^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^2g} \\
&\quad + \frac{2B(bc-ad)in\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g} \\
&\quad + \frac{2B^2(bc-ad)in^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g}
\end{aligned}$$

output
```
d*i*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2/g+2*B*(-a*d+b*c)*i*n*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^2/g-(-a*d+b*c)*i*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^2/g+2*B^2*(-a
*d+b*c)*i*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^2/g+2*B*(-a*d+b*c)*i*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^2/g+2*B^2*(-a
*d+b*c)*i*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^2/g
```

## 3.163.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1372 vs. $2(306) = 612$.

Time = 0.59 (sec) , antiderivative size = 1372, normalized size of antiderivative = 4.48

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \text{Too large to display}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
b*g*x),x]
```

output
```
(i*(3*b*d*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c +
 d*x)])^2 + 3*(b*c - a*d)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^
n] - B*n*Log[(a + b*x)/(c + d*x)])^2 - 3*B*n*(A + B*Log[e*((a + b*x)/(c +
d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(a*d*Log[a/b + x]^2 - 2*a*d*Log[a
/b + x]*(1 + Log[a + b*x]) + 2*(-(b*c) + a*d + Log[c/d + x]*(b*c + a*d*Log
[a + b*x] - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) + (-(b*d*x) + a*d*Log[a
 + b*x])*Log[(a + b*x)/(c + d*x)]) - 2*a*d*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)]) + 3*b*B*c*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b
*x)/(c + d*x)])*(Log[a/b + x]^2 - 2*Log[a + b*x]*(Log[a/b + x] - Log[c/d +
 x] - Log[(a + b*x)/(c + d*x)]) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c
) + a*d)] + PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - B^2*n^2*(a*d*Log[a/b
 + x]^3 - 3*d*(2*b*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x)*Log[a/b + x]^2
) - 3*b*(2*d*x - 2*(c + d*x)*Log[c/d + x] + (c + d*x)*Log[c/d + x]^2) - 3*
d*(b*x - a*Log[a + b*x])*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c
+ d*x)])^2 + 6*(a*d + 2*b*d*x - b*d*x*Log[c/d + x] - b*c*Log[c + d*x] + Lo
g[a/b + x]*(-(d*(a + b*x)) + d*(a + b*x)*Log[c/d + x] + (b*c - a*d)*Log[(b
*(c + d*x))/(b*c - a*d)]) + (b*c - a*d)*PolyLog[2, (d*(a + b*x))/(-(b*c) +
 a*d)]) - 3*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(-2*b
*c + 2*a*d - 2*d*(a + b*x)*Log[a/b + x] + a*d*Log[a/b + x]^2 + 2*Log[c/d +
 x]*(b*(c + d*x) - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) - 2*a*d*PolyL...
```

### 3.163.3 Rubi [A] (verified)

Time = 1.05 (sec) , antiderivative size = 288, normalized size of antiderivative = 0.94, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {2961, 2789, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ag+bgx}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{i(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \text{ 2789}$$

$$\frac{i(bc-ad)\left(\frac{d\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \text{ 2755}$$

$$\frac{i(bc-ad)\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2Bn\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \text{ 2754}$$

$$\frac{i(bc-ad)\left(\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2Bn\left(\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\cdots\right.}{\cdots}\right.}{b}\right.}{b}\right.}{g}$$

$\Big\downarrow$ 2779

$$\frac{i(bc-ad)\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2Bn\left(\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}\right)}{b}\right)}{b}+\frac{2Bn\cdots}{\cdots}\right.}{g}$$

$\Big\downarrow$ 2821

$$\frac{i(bc-ad)\left(\frac{\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int \frac{(c+dx)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\cdots\right.}{b}}{b}\right.}{g}$$

3.163. $\displaystyle\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$\downarrow$ 2838

$$i(bc-ad)\left(\frac{\frac{2Bn\left(\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int\frac{(c+dx)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{b}\right.$$
$$\overline{\qquad\qquad g}$$

$\downarrow$ 7143

$$i(bc-ad)\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2Bn\left(-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d}-\frac{Bn\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d}\right)}{b}\right)}{b}+\frac{2Bn\left(\operatorname{PolyLog}\left(\right.\right.}{}\right.$$
$$\overline{\qquad\qquad g}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)
,x]
```

output
```
((b*c - a*d)*i*((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(
b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (2*B*n*(-(((A + B*Log[e*((a +
 b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*n*PolyL
og[2, (d*(a + b*x))/(b*(c + d*x))])/d))/b))/b + (-(((A + B*Log[e*((a + b*x
)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (2*B*n*((A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))]
 + B*n*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b)/b))/g
```

3.163. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

### 3.163.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.163. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.163.4 Maple [F]

$$\int \frac{(dix + ci) \left(A + B \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{bgx + ag} dx$$

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

### 3.163.5 Fricas [F]

$$\int \frac{(ci + dix) \left(A + B \log \left(e \left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{ag + bgx} dx = \int \frac{(dix + ci)\left(B \log \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right) + A\right)^{2}}{bgx + ag} dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, al
gorithm="fricas")
```

output
```
integral((A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log(e*((b*x + a)/(d*
x + c))^n)^2 + 2*(A*B*d*i*x + A*B*c*i)*log(e*((b*x + a)/(d*x + c))^n))/(b*
g*x + a*g), x)
```

### 3.163.6 Sympy [F]

$$\int \frac{(ci + dix) \left(A + B \log \left(e \left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{ag + bgx} dx$$
$$= \frac{i\left(\int \frac{A^2 c}{a+bx} dx + \int \frac{A^2 dx}{a+bx} dx + \int \frac{B^2 c \log \left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)^{2}}{a+bx} dx + \int \frac{2ABc \log \left(e\left(\frac{a}{c+dx} + \frac{bx}{c+dx}\right)^{n}\right)}{a+bx} dx + \int \frac{B^2 dx \log \left(e\left(\frac{a}{c+dx} + \right.\right.}{a+bx}\right.}{g}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x)
```

output
```
i*(Integral(A**2*c/(a + b*x), x) + Integral(A**2*d*x/(a + b*x), x) + Integ
ral(B**2*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a + b*x), x) + Inte
gral(2*A*B*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x) + Integ
ral(B**2*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a + b*x), x) + In
tegral(2*A*B*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a + b*x), x))/g
```

### 3.163.7 Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, al
gorithm="maxima")
```

output
```
A^2*d*i*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + A^2*c*i*log(b*g*x + a*g)/(b*g
) + (B^2*b*d*i*x + (b*c*i - a*d*i)*B^2*log(b*x + a))*log((d*x + c)^n)^2/(b
^2*g) - integrate(-(B^2*b^2*c^2*i*log(e)^2 + 2*A*B*b^2*c^2*i*log(e) + (B^2
*b^2*d^2*i*log(e)^2 + 2*A*B*b^2*d^2*i*log(e))*x^2 + (B^2*b^2*d^2*i*x^2 + 2
*B^2*b^2*c*d*i*x + B^2*b^2*c^2*i)*log((b*x + a)^n)^2 + 2*(B^2*b^2*c*d*i*lo
g(e)^2 + 2*A*B*b^2*c*d*i*log(e))*x + 2*(B^2*b^2*c^2*i*log(e) + A*B*b^2*c^2
*i + (B^2*b^2*d^2*i*log(e) + A*B*b^2*d^2*i)*x^2 + 2*(B^2*b^2*c*d*i*log(e)
+ A*B*b^2*c*d*i)*x)*log((b*x + a)^n) - 2*(B^2*b^2*c^2*i*log(e) + A*B*b^2*c
^2*i + ((i*n + i*log(e))*B^2*b^2*d^2 + A*B*b^2*d^2*i)*x^2 + (2*A*B*b^2*c*d
*i + (a*b*d^2*i*n + 2*b^2*c*d*i*log(e))*B^2)*x + ((b^2*c*d*i*n - a*b*d^2*i
*n)*B^2*x + (a*b*c*d*i*n - a^2*d^2*i*n)*B^2)*log(b*x + a) + (B^2*b^2*d^2*i
*x^2 + 2*B^2*b^2*c*d*i*x + B^2*b^2*c^2*i)*log((b*x + a)^n))*log((d*x + c)^
n))/(b^3*d*g*x^2 + a*b^2*c*g + (b^3*c*g + a*b^2*d*g)*x), x)
```

### 3.163.8 Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x, al
gorithm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x +
a*g), x)
```

### 3.163.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
,x)
```

output
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
, x)
```

# 3.164 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$



## 3.164.1 Optimal result

Integrand size = 43, antiderivative size = 261

$$\begin{aligned}
&\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx \\
&= -\frac{2B^2in^2(c+dx)}{bg^2(a+bx)} - \frac{2Bin(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{bg^2(a+bx)} \\
&\quad - \frac{i(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{bg^2(a+bx)} - \frac{di\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} \\
&\quad + \frac{2Bdin\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2} + \frac{2B^2din^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^2g^2}
\end{aligned}$$

output
```
-2*B^2*i*n^2*(d*x+c)/b/g^2/(b*x+a)-2*B*i*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))/b/g^2/(b*x+a)-i*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b/g^2/(
b*x+a)-d*i*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^2
/g^2+2*B*d*i*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+
a))/b^2/g^2+2*B^2*d*i*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^2/g^2
```

### 3.164.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1564 vs. $2(261) = 522$.

Time = 1.09 (sec) , antiderivative size = 1564, normalized size of antiderivative = 5.99

$$\int \frac{(ci + dix) \left(A + B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^2} \, dx = \text{Too large to display}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
b*g*x)^2,x]
```

output
```
(i*((-3*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b
*x)/(c + d*x)])^2)/(a + b*x) + 3*d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 + (6*b*B*c*n*(-A - B*Log[e*(
(a + b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d*x)])*(-(d*(a + b*x)*Log
[c/d + x]) + d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)] + (b*c - a*d)*(
1 + Log[(a + b*x)/(c + d*x)])))/((b*c - a*d)*(a + b*x)) + (3*b*B^2*c*n^2*(
-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(a + b*x)/
(c + d*x)] - 2*d*(a + b*x)*Log[a + b*x]*Log[(a + b*x)/(c + d*x)] - (b*c -
a*d)*Log[(a + b*x)/(c + d*x)]^2 + 2*d*(a + b*x)*Log[c + d*x] - 2*d*(a + b*
x)*Log[(a + b*x)/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] + d*(a + b*x)*(
Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog
[2, (d*(a + b*x))/(-(b*c) + a*d)]) + d*(a + b*x)*(Log[(b*c - a*d)/(b*c + b
*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x
)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(a + b*x)) +
 3*B*d*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*
x)])*(Log[a/b + x]^2 - 2*Log[a/b + x]*Log[a + b*x] - 2*Log[c/d + x]*Log[(d
*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a + b*x]*((a*d)/(b*c - a*d) + Log[c/d
+ x] + Log[(a + b*x)/(c + d*x)]) + 2*a*((a + b*x)^(-1) + Log[(a + b*x)/(c
+ d*x)]/(a + b*x) + (d*Log[c + d*x])/(-(b*c) + a*d)) - 2*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)]) + (B^2*d*n^2*((b*c - a*d)*(a + b*x)*Log[a/b + x]^...
```

### 3.164.3 Rubi [A] (verified)

Time = 0.91 (sec) , antiderivative size = 234, normalized size of antiderivative = 0.90, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2961, 2780, 2742, 2741, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.164. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2} \, dx$

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{2780}$$

$$\frac{i\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^2}$$

$$\Big\downarrow \text{2742}$$

$$\frac{i\left(\frac{2Bn\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^2}$$

$$\Big\downarrow \text{2741}$$

$$\frac{i\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{b}\right)}{g^2}$$

$$\Big\downarrow \text{2779}$$

$$\frac{i\left(\frac{d\left(\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}\right)}{b}+\frac{2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.}{a+bx}\right.}{}\right.}{g^2}$$

$$\Big\downarrow \text{2821}$$

---

3.164. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}dx$

$$i\left(\frac{d\left(\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int\frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}\right)}{b}+\right.$$
$$\overline{\qquad\qquad g^2\qquad\qquad}$$

$$\downarrow 7143$$

$$i\left(\frac{d\left(\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+Bn\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}\right)}{b}+\frac{2Bn\left(-\frac{(c+dx)(}{}\right.}{}\right.$$
$$\overline{\qquad\qquad g^2\qquad\qquad}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)
^2,x]
```

output
```
(i*((-(((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)) + 2
*B*n*(-((B*n*(c + d*x))/(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]))/(a + b*x)))/b + (d*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (2*B*n*((A + B*Log[e*((a +
b*x)/(c + d*x))^n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + B*n*PolyLog[
3, (b*(c + d*x))/(d*(a + b*x))]))/b))/b))/g^2
```

### 3.164.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

3.164. $\int\frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.164.4 Maple [F]

$$\int \frac{(dix + ci)\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{(bgx + ag)^{2}}dx$$

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

### 3.164.5 Fricas [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,
algorithm="fricas")
```

output
```
integral((A^2*d*i*x + A^2*c*i + (B^2*d*i*x + B^2*c*i)*log(e*((b*x + a)/(d*
x + c))^n)^2 + 2*(A*B*d*i*x + A*B*c*i)*log(e*((b*x + a)/(d*x + c))^n))/(b^
2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

### 3.164.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2,x
)
```

output
```
Timed out
```

### 3.164.7 Maxima [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,
algorithm="maxima")
```

output
```
-2*A*B*c*i*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d)*g^
2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) + A^2*d*i*(a/(b^3*g^2*x + a*b^2
*g^2) + log(b*x + a)/(b^2*g^2)) - 2*A*B*c*i*log(e*(b*x/(d*x + c) + a/(d*x
+ c))^n)/(b^2*g^2*x + a*b*g^2) - A^2*c*i/(b^2*g^2*x + a*b*g^2) - ((b*c*i -
 a*d*i)*B^2 - (B^2*b*d*i*x + B^2*a*d*i)*log(b*x + a))*log((d*x + c)^n)^2/(
b^3*g^2*x + a*b^2*g^2) - integrate(-(B^2*b^2*c^2*i*log(e)^2 + (B^2*b^2*d^2
*i*log(e)^2 + 2*A*B*b^2*d^2*i*log(e))*x^2 + (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2
*c*d*i*x + B^2*b^2*c^2*i)*log((b*x + a)^n)^2 + 2*(B^2*b^2*c*d*i*log(e)^2 +
 A*B*b^2*c*d*i*log(e))*x + 2*(B^2*b^2*c^2*i*log(e) + (B^2*b^2*d^2*i*log(e)
 + A*B*b^2*d^2*i)*x^2 + (2*B^2*b^2*c*d*i*log(e) + A*B*b^2*c*d*i)*x)*log((b
*x + a)^n) + 2*((a*b*c*d*i*n - a^2*d^2*i*n - b^2*c^2*i*log(e))*B^2 - (B^2*
b^2*d^2*i*log(e) + A*B*b^2*d^2*i)*x^2 - (A*B*b^2*c*d*i + (a*b*d^2*i*n - (i
*n - 2*i*log(e))*b^2*c*d)*B^2)*x - (B^2*b^2*d^2*i*n*x^2 + 2*B^2*a*b*d^2*i*
n*x + B^2*a^2*d^2*i*n)*log(b*x + a) - (B^2*b^2*d^2*i*x^2 + 2*B^2*b^2*c*d*i
*x + B^2*b^2*c^2*i)*log((b*x + a)^n))*log((d*x + c)^n))/(b^4*d*g^2*x^3 + a
^2*b^2*c*g^2 + (b^4*c*g^2 + 2*a*b^3*d*g^2)*x^2 + (2*a*b^3*c*g^2 + a^2*b^2*
d*g^2)*x), x)
```

### 3.164.8 Giac [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x,
algorithm="giac")
```

output
```
integrate((d*i*x + c*i)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x +
a*g)^2, x)
```

### 3.164.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(c\,i+d\,i\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{{(a\,g+b\,g\,x)}^2}\,dx$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
^2,x)
```

output
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
^2, x)
```

3.164. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

# 3.165 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$



## 3.165.1 Optimal result

Integrand size = 43, antiderivative size = 151

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = -\frac{B^2in^2(c+dx)^2}{4(bc-ad)g^3(a+bx)^2} - \frac{Bin(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)g^3(a+bx)^2} - \frac{i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)g^3(a+bx)^2}$$

output
```
-1/4*B^2*i*n^2*(d*x+c)^2/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*B*i*n*(d*x+c)^2*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^3/(b*x+a)^2-1/2*i*(d*x+c)^2*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/g^3/(b*x+a)^2
```

## 3.165.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.50 (sec) , antiderivative size = 801, normalized size of antiderivative = 5.30

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{i\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - 4d(-bc+ad)(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + 4Bdn(a+\cdots\right.}{\cdots}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
b*g*x)^3,x]
```

output
```
-1/4*(i*(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 4*d*(-
(b*c) + a*d)*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 4*B*d*n*
(a + b*x)*(2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d*(a +
 b*x)*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + 2*B*n*(b*c - a*d + d
*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d*x]) - B*d*n*(a + b*x)*(Log
[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2,
 (d*(a + b*x))/(-(b*c) + a*d)]) + B*d*n*(a + b*x)*((2*Log[(d*(a + b*x))/(-
(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b
*c - a*d)])) + B*n*(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
 + 4*d*(-(b*c) + a*d)*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4
*d^2*(a + b*x)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d
^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 4*B*d
*n*(a + b*x)*(b*c - a*d + d*(a + b*x)*Log[a + b*x] - d*(a + b*x)*Log[c + d
*x]) + B*n*((b*c - a*d)^2 + 2*d*(-(b*c) + a*d)*(a + b*x) - 2*d^2*(a + b*x)
^2*Log[a + b*x] + 2*d^2*(a + b*x)^2*Log[c + d*x]) + 2*B*d^2*n*(a + b*x)^2*
(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLo
g[2, (d*(a + b*x))/(-(b*c) + a*d)]) - 2*B*d^2*n*(a + b*x)^2*((2*Log[(d*(a
+ b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c
+ d*x))/(b*c - a*d)]))))/(b^2*(b*c - a*d)*g^3*(a + b*x)^2)
```

### 3.165.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 122, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3}dx$$

$$\Big\downarrow \; 2961$$

$$\frac{i\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3(bc-ad)}$$

$$\Big\downarrow \text{2742}$$

$$\frac{i\left(Bn\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}\right)}{g^3(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{i\left(Bn\left(-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}\right)-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}\right)}{g^3(bc-ad)}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)
^3,x]
```

output
```
(i*(-1/2*((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^
2 + B*n*(-1/4*(B*n*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2))))/((b*c - a*d)*g^3)
```

### 3.165.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))    Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

---

3.165. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

### 3.165.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(145) = 290$.

Time = 4.83 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.51

| method | result |
|---|---|
| parallelrisch | $-\frac{-8ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}c\,d^{2}in-4B^{2}x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{4}c\,d^{2}i\,n^{2}+4ABxa\,b^{3}d^{3}i\,n^{2}-4ABx\,b^{4}c\,d^{2}i\,n^{2}-4AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}{}$ |

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,method=_
RETURNVERBOSE)
```

output
```
-1/4*(-8*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^2*i*n-4*B^2*x*ln(e*((b*x+
a)/(d*x+c))^n)*b^4*c*d^2*i*n^2+4*A*B*x*a*b^3*d^3*i*n^2-4*A*B*x*b^4*c*d^2*i
*n^2-4*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d*i*n-4*A*B*x^2*ln(e*((b*x+a)
/(d*x+c))^n)*b^4*d^3*i*n-4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*c*d^2*i*n
+2*A*B*a^2*b^2*d^3*i*n^2+4*A^2*x*a*b^3*d^3*i*n-4*A^2*x*b^4*c*d^2*i*n-2*B^2
*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*d^3*i*n-2*B^2*x^2*ln(e*((b*x+a)/(d*x+
c))^n)*b^4*d^3*i*n^2-2*A*B*b^4*c^2*d*i*n^2+2*B^2*x*a*b^3*d^3*i*n^3-2*B^2*x
*b^4*c*d^2*i*n^3-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*c^2*d*i*n-2*B^2*ln(
e*((b*x+a)/(d*x+c))^n)*b^4*c^2*d*i*n^2+B^2*a^2*b^2*d^3*i*n^3-B^2*b^4*c^2*d
*i*n^3+2*A^2*a^2*b^2*d^3*i*n-2*A^2*b^4*c^2*d*i*n)/g^3/(b*x+a)^2/b^4/d/n/(a
*d-b*c)
```

### 3.165.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. $2(145) = 290$.

Time = 0.35 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.97

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(ag+bgx)^{3}}\,dx =$$
$$-\frac{(B^{2}b^{2}c^{2}-B^{2}a^{2}d^{2})in^{2}+2\,(ABb^{2}c^{2}-ABa^{2}d^{2})in+2\,(2\,(B^{2}b^{2}cd-B^{2}abd^{2})ix+(B^{2}b^{2}c^{2}-B^{2}a^{2}d^{2})i)\,\mathrm{l}}{}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,
algorithm="fricas")
```

output
```
-1/4*((B^2*b^2*c^2 - B^2*a^2*d^2)*i*n^2 + 2*(A*B*b^2*c^2 - A*B*a^2*d^2)*i*
n + 2*(2*(B^2*b^2*c*d - B^2*a*b*d^2)*i*x + (B^2*b^2*c^2 - B^2*a^2*d^2)*i)*
log(e)^2 + 2*(B^2*b^2*d^2*i*n^2*x^2 + 2*B^2*b^2*c*d*i*n^2*x + B^2*b^2*c^2*
i*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A^2*b^2*c^2 - A^2*a^2*d^2)*i + 2*((
B^2*b^2*c*d - B^2*a*b*d^2)*i*n^2 + 2*(A*B*b^2*c*d - A*B*a*b*d^2)*i*n + 2*(
A^2*b^2*c*d - A^2*a*b*d^2)*i)*x + 2*((B^2*b^2*c^2 - B^2*a^2*d^2)*i*n + 2*(
A*B*b^2*c^2 - A*B*a^2*d^2)*i + 2*((B^2*b^2*c*d - B^2*a*b*d^2)*i*n + 2*(A*B
*b^2*c*d - A*B*a*b*d^2)*i)*x + 2*(B^2*b^2*d^2*i*n*x^2 + 2*B^2*b^2*c*d*i*n*
x + B^2*b^2*c^2*i*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*b^2*c^2*i*n
^2 + 2*A*B*b^2*c^2*i*n + (B^2*b^2*d^2*i*n^2 + 2*A*B*b^2*d^2*i*n)*x^2 + 2*(
B^2*b^2*c*d*i*n^2 + 2*A*B*b^2*c*d*i*n)*x)*log((b*x + a)/(d*x + c)))/((b^5*
c - a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^
2*d)*g^3)
```

### 3.165.6 Sympy [F]

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{i\left(\int \frac{A^2c}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{A^2dx}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{B^2c\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2ABc\log\left(e\left(\frac{a}{c+dx}+\frac{}{c\cdot}\right.\right.}{a^3+3a^2bx+3ab^2x^2+}\right.}{g^3}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3,x
)
```

output
```
i*(Integral(A**2*c/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + I
ntegral(A**2*d*x/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Int
egral(B**2*c*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**3 + 3*a**2*b*x
 + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*c*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + I
ntegral(B**2*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a**3 + 3*a**2
*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*d*x*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
x))/g**3
```

### 3.165.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2017 vs. 2(145) = 290.

Time = 0.31 (sec) , antiderivative size = 2017, normalized size of antiderivative = 13.36

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,
algorithm="maxima")
```

output
```
-1/2*A*B*d*i*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^4*
d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3)
+ 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*
g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*
d^2)*g^3)) + 1/2*A*B*c*i*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3
*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*
log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c
)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*(2*b*x + a)*B^2*d*i*log
(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b
^2*g^3) + 1/4*(2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)*g^3*x^2 + 2
*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*d^2*log(b*x
+ a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x + c)/((b^3*
c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
) - (b^2*c^2 - 8*a*b*c*d + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*
d^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(d*x + c)
^2 - 6*(b^2*c*d - a*b*d^2)*x - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log
(b*x + a) + 2*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2*d^2*x^2 +
2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c))*n^2/(a^2*b^3*c^2*g^3 -
2*a^3*b^2*c*d*g^3 + a^4*b*d^2*g^3 + (b^5*c^2*g^3 - 2*a*b^4*c*d*g^3 + a^2*b
^3*d^2*g^3)*x^2 + 2*(a*b^4*c^2*g^3 - 2*a^2*b^3*c*d*g^3 + a^3*b^2*d^2*g^...
```

### 3.165.8 Giac [A] (verification not implemented)

Time = 1.70 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.29

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx =$$

$$-\frac{1}{4}\left(\frac{2\,(dx+c)^2B^2in^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)^2g^3}+\frac{2\,(B^2in^2+2\,B^2in\log(e)+2\,ABin)(dx+c)^2\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^2g^3}+\frac{(B^2in^2\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x,
algorithm="giac")
```

output
```
-1/4*(2*(d*x + c)^2*B^2*i*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)^2*g^3)
 + 2*(B^2*i*n^2 + 2*B^2*i*n*log(e) + 2*A*B*i*n)*(d*x + c)^2*log((b*x + a)/
(d*x + c))/((b*x + a)^2*g^3) + (B^2*i*n^2 + 2*B^2*i*n*log(e) + 2*B^2*i*log
(e)^2 + 2*A*B*i*n + 4*A*B*i*log(e) + 2*A^2*i)*(d*x + c)^2/((b*x + a)^2*g^3
))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.165.9 Mupad [B] (verification not implemented)

Time = 3.31 (sec) , antiderivative size = 561, normalized size of antiderivative = 3.72

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{\frac{B^2\,c\,i}{2\,b}+\frac{B^2\,d\,i\,x}{b}+\frac{B^2\,a\,d\,i}{2\,b^2}}{a^2\,g^3+2\,a\,b\,g^3\,x+b^2\,g^3\,x^2}-\frac{B^2\,d^2\,i}{2\,b^2\,g^3\,(a\,d-b\,c)}\right)$$
$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B\,a\,d\,i+A\,B\,b\,c\,i-B^2\,a\,d\,i\,n+B^2\,b\,c\,i\,n+2\,A\,B\,b\,d\,i\,x}{a^2\,b^2\,g^3+2\,a\,b^3\,g^3\,x+b^4\,g^3\,x^2}\right.$$
$$\left.+\frac{B^2\,d^2\,i\left(\frac{a\,b^2\,g^3\,n\,(a\,d-b\,c)}{2\,d}+\frac{b^3\,g^3\,n\,x\,(a\,d-b\,c)}{d}+\frac{b^2\,g^3\,n\,(a\,d-b\,c)\,(2\,a\,d-b\,c)}{2\,d^2}\right)}{b^2\,g^3\,(a\,d-b\,c)\,(a^2\,b^2\,g^3+2\,a\,b^3\,g^3\,x+b^4\,g^3\,x^2)}\right)$$
$$-\frac{x\,(2\,b\,d\,i\,A^2+2\,b\,d\,i\,A\,B\,n+b\,d\,i\,B^2\,n^2)+A^2\,a\,d\,i+A^2\,b\,c\,i+\frac{B^2\,a\,d\,i\,n^2}{2}+\frac{B^2\,b\,c\,i\,n^2}{2}+A\,B\,a\,d\,i\,n+A\,B\,b}{2\,a^2\,b^2\,g^3+4\,a\,b^3\,g^3\,x+2\,b^4\,g^3\,x^2}$$
$$-\frac{B\,d^2\,i\,n\,\mathrm{atan}\left(\frac{B\,d^2\,i\,n\,(2\,A+B\,n)\left(\frac{c\,b^3\,g^3+a\,d\,b^2\,g^3}{b^2\,g^3}+2\,b\,d\,x\right)1\mathrm{i}}{(a\,d-b\,c)\,(i\,B^2\,d^2\,n^2+2\,A\,i\,B\,d^2\,n)}\right)(2\,A+B\,n)\,1\mathrm{i}}{b^2\,g^3\,(a\,d-b\,c)}$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
^3,x)
```

output
```
- log(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*c*i)/(2*b) + (B^2*d*i*x)/b + (B^
2*a*d*i)/(2*b^2))/(a^2*g^3 + b^2*g^3*x^2 + 2*a*b*g^3*x) - (B^2*d^2*i)/(2*b
^2*g^3*(a*d - b*c))) - log(e*((a + b*x)/(c + d*x))^n)*((A*B*a*d*i + A*B*b*
c*i - B^2*a*d*i*n + B^2*b*c*i*n + 2*A*B*b*d*i*x)/(a^2*b^2*g^3 + b^4*g^3*x^
2 + 2*a*b^3*g^3*x) + (B^2*d^2*i*((a*b^2*g^3*n*(a*d - b*c))/(2*d) + (b^3*g^
3*n*x*(a*d - b*c))/d + (b^2*g^3*n*(a*d - b*c)*(2*a*d - b*c))/(2*d^2)))/(b^
2*g^3*(a*d - b*c)*(a^2*b^2*g^3 + b^4*g^3*x^2 + 2*a*b^3*g^3*x))) - (x*(2*A^
2*b*d*i + B^2*b*d*i*n^2 + 2*A*B*b*d*i*n) + A^2*a*d*i + A^2*b*c*i + (B^2*a*
d*i*n^2)/2 + (B^2*b*c*i*n^2)/2 + A*B*a*d*i*n + A*B*b*c*i*n)/(2*a^2*b^2*g^3
 + 2*b^4*g^3*x^2 + 4*a*b^3*g^3*x) - (B*d^2*i*n*atan((B*d^2*i*n*(2*A + B*n)
*((b^3*c*g^3 + a*b^2*d*g^3)/(b^2*g^3) + 2*b*d*x)*1i)/((a*d - b*c)*(B^2*d^2
*i*n^2 + 2*A*B*d^2*i*n)))*(2*A + B*n)*1i)/(b^2*g^3*(a*d - b*c))
```

3.165. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

# 3.166 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$



## 3.166.1 Optimal result

Integrand size = 43, antiderivative size = 307

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \frac{B^2din^2(c+dx)^2}{4(bc-ad)^2g^4(a+bx)^2} - \frac{2bB^2in^2(c+dx)^3}{27(bc-ad)^2g^4(a+bx)^3} + \frac{Bdin(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2g^4(a+bx)^2} - \frac{2bBin(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^2g^4(a+bx)^3} + \frac{di(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2g^4(a+bx)^2} - \frac{bi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^2g^4(a+bx)^3}$$

output
```
1/4*B^2*d*i*n^2*(d*x+c)^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-2/27*b*B^2*i*n^2*(d*x
+c)^3/(-a*d+b*c)^2/g^4/(b*x+a)^3+1/2*B*d*i*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/(-a*d+b*c)^2/g^4/(b*x+a)^2-2/9*b*B*i*n*(d*x+c)^3*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^4/(b*x+a)^3+1/2*d*i*(d*x+c)^2*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^4/(b*x+a)^2-1/3*b*i*(d*x+c)^3*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^4/(b*x+a)^3
```

### 3.166.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.63 (sec) , antiderivative size = 1082, normalized size of antiderivative = 3.52

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{i\left(36(bc-ad)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+54d(bc-ad)^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+2Bn\left(12A\right.\right.}{}$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
b*g*x)^4,x]
```

output
```
-1/108*(i*(36*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 54*
d*(b*c - a*d)^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 2*B*n
*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d*(b*c - a*d)^2*(a + b*x
) - 15*B*d*(b*c - a*d)^2*n*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2 +
66*B*d^2*(b*c - a*d)*n*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] + 6
6*B*d^3*n*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*n*(a + b*x)^3*Log[a + b*x]^2
 + 12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n] - 18*B*d*(b*c - a*d)^
2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^2*(b*c - a*d)*(a + b*x
)^2*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log
[e*((a + b*x)/(c + d*x))^n] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3
*n*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*n*(a + b*x)^3*Log[(d*(a + b*x))/(-(
b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[e*((a + b*x)/(c + d*x
))^n]*Log[c + d*x] - 18*B*d^3*n*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*n*(a
 + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*n*(a + b*
x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*n*(a + b*x)^3*Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)]) + 27*B*d*n*(a + b*x)*(2*(b*c - a*d)^2*
(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d*(-(b*c) + a*d)*(a + b*x)*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*d^2*(a + b*x)^2*Log[a + b*x]*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]) + 4*d^2*(a + b*x)^2*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])*Log[c + d*x] - 4*B*d*n*(a + b*x)*(b*c - a*d + d*(a + ...
```

## 3.166.3 Rubi [A] (verified)

Time = 0.45 (sec) , antiderivative size = 238, normalized size of antiderivative = 0.78, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{i\int \left(\frac{b(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}-\frac{d(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}}{g^4(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i\left(-\frac{b(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}-\frac{2bBn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3}+\frac{d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}+\frac{Bdn(c+dx)^2\left(B\right.}{2(\ldots}\right.}{g^4(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)
^4,x]
```

output
```
(i*((B^2*d*n^2*(c + d*x)^2)/(4*(a + b*x)^2) - (2*b*B^2*n^2*(c + d*x)^3)/(2
7*(a + b*x)^3) + (B*d*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(2*(a + b*x)^2) - (2*b*B*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(9*(a + b*x)^3) + (d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2)/(2*(a + b*x)^2) - (b*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^2)/(3*(a + b*x)^3)))/((b*c - a*d)^2*g^4)
```

### 3.166.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.166.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1098 vs. $2(295) = 590$.

Time = 10.49 (sec) , antiderivative size = 1099, normalized size of antiderivative = 3.58

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1099 |

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,method=_
RETURNVERBOSE)
```

output
```
-1/108*(-108*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^4*i*n-108*B^2*x*ln(
e*((b*x+a)/(d*x+c))^n)^2*a*b^5*c*d^3*i*n-108*B^2*x*ln(e*((b*x+a)/(d*x+c))^
n)*a*b^5*c*d^3*i*n^2+108*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^2*d^2*i*n-1
08*A*B*x*a*b^5*c*d^3*i*n^2-108*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c^2*d^2
*i*n-216*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*c*d^3*i*n-54*B^2*x*a*b^5*c*
d^3*i*n^3+90*A*B*x*a^2*b^4*d^4*i*n^2+18*A*B*x*b^6*c^2*d^2*i*n^2-54*B^2*ln(
e*((b*x+a)/(d*x+c))^n)^2*a*b^5*c^2*d^2*i*n-54*B^2*ln(e*((b*x+a)/(d*x+c))^n
)*a*b^5*c^2*d^2*i*n^2-108*A^2*x*a*b^5*c*d^3*i*n+72*A*B*ln(e*((b*x+a)/(d*x+
c))^n)*b^6*c^3*d*i*n+19*B^2*a^3*b^3*d^4*i*n^3+8*B^2*b^6*c^3*d*i*n^3+18*A^2
*a^3*b^3*d^4*i*n+36*A^2*b^6*c^3*d*i*n-36*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)
*b^6*d^4*i*n-54*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*d^4*i*n-54*B^2*x
^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^4*i*n^2-36*B^2*x^2*ln(e*((b*x+a)/(d*x
+c))^n)*b^6*c*d^3*i*n^2+36*A*B*x^2*a*b^5*d^4*i*n^2-36*A*B*x^2*b^6*c*d^3*i*
n^2+54*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*c^2*d^2*i*n+18*B^2*x*ln(e*((b
*x+a)/(d*x+c))^n)*b^6*c^2*d^2*i*n^2-18*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2
*b^6*d^4*i*n-30*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^4*i*n^2+30*B^2*x^2
*a*b^5*d^4*i*n^3-30*B^2*x^2*b^6*c*d^3*i*n^3+57*B^2*x*a^2*b^4*d^4*i*n^3-3*B
^2*x*b^6*c^2*d^2*i*n^3+36*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*c^3*d*i*n+24
*B^2*ln(e*((b*x+a)/(d*x+c))^n)*b^6*c^3*d*i*n^2+54*A^2*x*a^2*b^4*d^4*i*n+54
*A^2*x*b^6*c^2*d^2*i*n-54*A*B*a*b^5*c^2*d^2*i*n^2-27*B^2*a*b^5*c^2*d^2*...
```

### 3.166.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1167 vs. $2(295) = 590$.

Time = 0.40 (sec) , antiderivative size = 1167, normalized size of antiderivative = 3.80

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,
algorithm="fricas")
```

output
```
-1/108*((8*B^2*b^3*c^3 - 27*B^2*a*b^2*c^2*d + 19*B^2*a^3*d^3)*i*n^2 + 6*(4
*A*B*b^3*c^3 - 9*A*B*a*b^2*c^2*d + 5*A*B*a^3*d^3)*i*n - 6*(5*(B^2*b^3*c*d^
2 - B^2*a*b^2*d^3)*i*n^2 + 6*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*i*n)*x^2 + 18
*(3*(B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*i*x + (2*B^2*b^3*c
^3 - 3*B^2*a*b^2*c^2*d + B^2*a^3*d^3)*i)*log(e)^2 - 18*(B^2*b^3*d^3*i*n^2*
x^3 + 3*B^2*a*b^2*d^3*i*n^2*x^2 - 3*(B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^2)*i*
n^2*x - (2*B^2*b^3*c^3 - 3*B^2*a*b^2*c^2*d)*i*n^2)*log((b*x + a)/(d*x + c)
)^2 + 18*(2*A^2*b^3*c^3 - 3*A^2*a*b^2*c^2*d + A^2*a^3*d^3)*i - 3*((B^2*b^3
*c^2*d + 18*B^2*a*b^2*c*d^2 - 19*B^2*a^2*b*d^3)*i*n^2 - 6*(A*B*b^3*c^2*d -
 6*A*B*a*b^2*c*d^2 + 5*A*B*a^2*b*d^3)*i*n - 18*(A^2*b^3*c^2*d - 2*A^2*a*b^
2*c*d^2 + A^2*a^2*b*d^3)*i)*x - 6*(6*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*i*n*x
^2 - (4*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 5*B^2*a^3*d^3)*i*n - 6*(2*A*B*b^
3*c^3 - 3*A*B*a*b^2*c^2*d + A*B*a^3*d^3)*i - 3*((B^2*b^3*c^2*d - 6*B^2*a*b
^2*c*d^2 + 5*B^2*a^2*b*d^3)*i*n + 6*(A*B*b^3*c^2*d - 2*A*B*a*b^2*c*d^2 + A
*B*a^2*b*d^3)*i)*x + 6*(B^2*b^3*d^3*i*n*x^3 + 3*B^2*a*b^2*d^3*i*n*x^2 - 3*
(B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^2)*i*n*x - (2*B^2*b^3*c^3 - 3*B^2*a*b^2*c
^2*d)*i*n)*log((b*x + a)/(d*x + c)))*log(e) + 6*((4*B^2*b^3*c^3 - 9*B^2*a*
b^2*c^2*d)*i*n^2 - (5*B^2*b^3*d^3*i*n^2 + 6*A*B*b^3*d^3*i*n)*x^3 + 6*(2*A*
B*b^3*c^3 - 3*A*B*a*b^2*c^2*d)*i*n - 3*(6*A*B*a*b^2*d^3*i*n + (2*B^2*b^3*c
*d^2 + 3*B^2*a*b^2*d^3)*i*n^2)*x^2 + 3*((B^2*b^3*c^2*d - 6*B^2*a*b^2*c*...
```

## 3.166.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4,x
)
```

output
```
Timed out
```

### 3.166.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3312 vs. $2(295) = 590$.

Time = 0.41 (sec) , antiderivative size = 3312, normalized size of antiderivative = 10.79

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,
algorithm="maxima")
```

output
```
-1/9*A*B*c*i*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b
^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*
(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3
*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*
g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c*d^2 - a^
3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*c^2*d + 3*a^2*b^2*c
*d^2 - a^3*b*d^3)*g^4)) - 1/18*A*B*d*i*n*((5*a*b^2*c^2 - 22*a^2*b*c*d + 5*
a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c^2 - 16*a*b^2*c*d + 5*
a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)*g^4*x^3 + 3*(a*b^6*c^
2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^5*c^2 - 2*a^3*b^4*c*d
+ a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d + a^5*b^2*d^2)*g^4) -
6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b^3*c
*d^2 - a^3*b^2*d^3)*g^4) + 6*(3*b*c*d^2 - a*d^3)*log(d*x + c)/((b^5*c^3 -
3*a*b^4*c^2*d + 3*a^2*b^3*c*d^2 - a^3*b^2*d^3)*g^4)) - 1/6*(3*b*x + a)*B^2
*d*i*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(b^5*g^4*x^3 + 3*a*b^4*g^4*x
^2 + 3*a^2*b^3*g^4*x + a^3*b^2*g^4) - 1/54*(6*n*((6*b^2*d^2*x^2 + 2*b^2*c^
2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^
5*c*d + a^2*b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)
*g^4*x^2 + 3*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*
c^2 - 2*a^4*b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - ...
```

### 3.166.8 Giac [A] (verification not implemented)

Time = 2.31 (sec) , antiderivative size = 501, normalized size of antiderivative = 1.63

$$\int \frac{(ci + dix)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^4}\,dx =$$

$$-\frac{1}{108}\left(\frac{18\left(2B^2bin^2 - \frac{3(bx+a)B^2din^2}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^3bcg^4}{(dx+c)^3} - \frac{(bx+a)^3adg^4}{(dx+c)^3}} + \frac{6\left(4B^2bin^2 - \frac{9(bx+a)B^2din^2}{dx+c} + 12B^2bin\log(e) - \frac{18(\cdots}{}\right.}{\frac{(bx+a)^3bcg^4}{(dx+c)^3}\cdots}\right.$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,
algorithm="giac")
```

output
```
-1/108*(18*(2*B^2*b*i*n^2 - 3*(b*x + a)*B^2*d*i*n^2/(d*x + c))*log((b*x +
a)/(d*x + c))^2/((b*x + a)^3*b*c*g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^4/(d*
x + c)^3) + 6*(4*B^2*b*i*n^2 - 9*(b*x + a)*B^2*d*i*n^2/(d*x + c) + 12*B^2*
b*i*n*log(e) - 18*(b*x + a)*B^2*d*i*n*log(e)/(d*x + c) + 12*A*B*b*i*n - 18
*(b*x + a)*A*B*d*i*n/(d*x + c))*log((b*x + a)/(d*x + c))/((b*x + a)^3*b*c*
g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^4/(d*x + c)^3) + (8*B^2*b*i*n^2 - 27*(
b*x + a)*B^2*d*i*n^2/(d*x + c) + 24*B^2*b*i*n*log(e) - 54*(b*x + a)*B^2*d*
i*n*log(e)/(d*x + c) + 36*B^2*b*i*log(e)^2 - 54*(b*x + a)*B^2*d*i*log(e)^2
/(d*x + c) + 24*A*B*b*i*n - 54*(b*x + a)*A*B*d*i*n/(d*x + c) + 72*A*B*b*i*
log(e) - 108*(b*x + a)*A*B*d*i*log(e)/(d*x + c) + 36*A^2*b*i - 54*(b*x + a
)*A^2*d*i/(d*x + c))/((b*x + a)^3*b*c*g^4/(d*x + c)^3 - (b*x + a)^3*a*d*g^
4/(d*x + c)^3))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.166.9 Mupad [B] (verification not implemented)

Time = 4.26 (sec) , antiderivative size = 993, normalized size of antiderivative = 3.23

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{\frac{18\,i\,A^2\,a^2\,d^2+18\,i\,A^2\,a\,b\,c\,d-36\,i\,A^2\,b^2\,c^2+30\,i\,A\,B\,a^2\,d^2\,n+30\,i\,A\,B\,a\,b\,c\,d\,n-24\,i\,A\,B\,b^2\,c^2\,n+19\,i\,B^2\,a^2\,d^2\,n^2+19\,i\,B^2\,a\,b\,c\,d\,n^2-8\,i\,B^2\,b^2\,c^2}{6\,(a\,d-b\,c)}}{18\,a^3\,b^2\,g^4+54\,a^2\,b^3\,g^4\,x}$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{\frac{B^2\,c\,i}{3\,b}+\frac{B^2\,d\,i\,x}{2\,b}+\frac{B^2\,a\,d\,i}{6\,b^2}}{a^3\,g^4+3\,a^2\,b\,g^4\,x+3\,a\,b^2\,g^4\,x^2+b^3\,g^4\,x^3} - \frac{B^2\,d^3\,i}{6\,b^2\,g^4\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B\,a\,d\,i+2\,A\,B\,b\,c\,i-B^2\,a\,d\,i\,n+B^2\,b\,c\,i\,n+3\,A\,B\,b\,d\,i\,x}{3\,a^3\,b^2\,g^4+9\,a^2\,b^3\,g^4\,x+9\,a\,b^4\,g^4\,x^2+3\,b^5\,g^4\,x^3}\right.$$

$$+\frac{B^2\,d^3\,i\,\left(x\left(b\left(\frac{a\,b^2\,g^4\,n\,(a\,d-b\,c)}{d}+\frac{b^2\,g^4\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{2\,d^2}\right)+\frac{2\,a\,b^3\,g^4\,n\,(a\,d-b\,c)}{d}+\frac{b^3\,g^4\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{d^2}\right)+a\left(\frac{a\,b^2}{}\right.\right.}{3\,b^2\,g^4\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(3\,a^3\,b^2\,g^4+9\,a^2\right.}$$

$$-\frac{B\,d^3\,i\,n\,\mathrm{atan}\left(\frac{B\,d^3\,i\,n\,(6\,A+5\,B\,n)\,\left(2\,b\,d\,x-\frac{b^4\,c^2\,g^4-a^2\,b^2\,d^2\,g^4}{b^2\,g^4\,(a\,d-b\,c)}\right)\,1\mathrm{i}}{(a\,d-b\,c)\,\left(5\,i\,B^2\,d^3\,n^2+6\,A\,i\,B\,d^3\,n\right)}\right)\,\left(6\,A+5\,B\,n\right)\,1\mathrm{i}}{9\,b^2\,g^4\,\left(a\,d-b\,c\right)^2}$$

---

3.166. $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
^4,x)
```

output
```
- ((18*A^2*a^2*d^2*i - 36*A^2*b^2*c^2*i + 19*B^2*a^2*d^2*i*n^2 - 8*B^2*b^2
*c^2*i*n^2 + 30*A*B*a^2*d^2*i*n - 24*A*B*b^2*c^2*i*n + 18*A^2*a*b*c*d*i +
19*B^2*a*b*c*d*i*n^2 + 30*A*B*a*b*c*d*i*n)/(6*(a*d - b*c)) + (x*(18*A^2*a*
b*d^2*i - 18*A^2*b^2*c*d*i + 19*B^2*a*b*d^2*i*n^2 + B^2*b^2*c*d*i*n^2 + 30
*A*B*a*b*d^2*i*n - 6*A*B*b^2*c*d*i*n))/(2*(a*d - b*c)) + (x^2*(5*B^2*b^2*d
^2*i*n^2 + 6*A*B*b^2*d^2*i*n))/(a*d - b*c))/(18*a^3*b^2*g^4 + 18*b^5*g^4*x
^3 + 54*a^2*b^3*g^4*x + 54*a*b^4*g^4*x^2) - log(e*((a + b*x)/(c + d*x))^n)
^2*(((B^2*c*i)/(3*b) + (B^2*d*i*x)/(2*b) + (B^2*a*d*i)/(6*b^2))/(a^3*g^4 +
 b^3*g^4*x^3 + 3*a*b^2*g^4*x^2 + 3*a^2*b*g^4*x) - (B^2*d^3*i)/(6*b^2*g^4*(
a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - log(e*((a + b*x)/(c + d*x))^n)*((A*B*a*
d*i + 2*A*B*b*c*i - B^2*a*d*i*n + B^2*b*c*i*n + 3*A*B*b*d*i*x)/(3*a^3*b^2*
g^4 + 3*b^5*g^4*x^3 + 9*a^2*b^3*g^4*x + 9*a*b^4*g^4*x^2) + (B^2*d^3*i*(x*(
b*((a*b^2*g^4*n*(a*d - b*c))/d + (b^2*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*
d^2)) + (2*a*b^3*g^4*n*(a*d - b*c))/d + (b^3*g^4*n*(a*d - b*c)*(3*a*d - b*
c))/d^2) + a*((a*b^2*g^4*n*(a*d - b*c))/d + (b^2*g^4*n*(a*d - b*c)*(3*a*d
- b*c))/(2*d^2)) + (3*b^4*g^4*n*x^2*(a*d - b*c))/d + (b^2*g^4*n*(a*d - b*c
)*(3*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/d^3))/(3*b^2*g^4*(a^2*d^2 + b^2*c^2 -
 2*a*b*c*d)*(3*a^3*b^2*g^4 + 3*b^5*g^4*x^3 + 9*a^2*b^3*g^4*x + 9*a*b^4*g^4
*x^2))) - (B*d^3*i*n*atan((B*d^3*i*n*(6*A + 5*B*n)*(2*b*d*x - (b^4*c^2*g^4
 - a^2*b^2*d^2*g^4)/(b^2*g^4*(a*d - b*c)))*1i)/((a*d - b*c)*(5*B^2*d^3*...
```

---

3.166. $$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$$

# 3.167 $\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$



## 3.167.1 Optimal result

Integrand size = 43, antiderivative size = 475

$$\begin{aligned}\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx =& -\frac{B^2d^2in^2(c+dx)^2}{4(bc-ad)^3g^5(a+bx)^2}\\ &+\frac{4bB^2din^2(c+dx)^3}{27(bc-ad)^3g^5(a+bx)^3}\\ &-\frac{b^2B^2in^2(c+dx)^4}{32(bc-ad)^3g^5(a+bx)^4}\\ &-\frac{Bd^2in(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^5(a+bx)^2}\\ &+\frac{4bBdin(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^5(a+bx)^3}\\ &-\frac{b^2Bin(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^3g^5(a+bx)^4}\\ &-\frac{d^2i(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^5(a+bx)^2}\\ &+\frac{2bdi(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^5(a+bx)^3}\\ &-\frac{b^2i(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^3g^5(a+bx)^4}\end{aligned}$$

output
```
-1/4*B^2*d^2*i*n^2*(d*x+c)^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+4/27*b*B^2*d*i*n^2
*(d*x+c)^3/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/32*b^2*B^2*i*n^2*(d*x+c)^4/(-a*d+b
*c)^3/g^5/(b*x+a)^4-1/2*B*d^2*i*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)/(-a*d+b*c)^3/g^5/(b*x+a)^2+4/9*b*B*d*i*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/8*b^2*B*i*n*(d*x+c)^4*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^5/(b*x+a)^4-1/2*d^2*i*(d*x+c)^2*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^5/(b*x+a)^2+2/3*b*d*i*(d*x+c)^3*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^5/(b*x+a)^3-1/4*b^2*i*(d*
x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^5/(b*x+a)^4
```

### 3.167.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.71 (sec) , antiderivative size = 1319, normalized size of antiderivative = 2.78

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{i\Big(216(bc-ad)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-288d(-bc+ad)^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+16Bd$$

input
```
Integrate[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g +
b*g*x)^5,x]
```

output
```
-1/864*(i*(216*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 28
8*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 +
16*B*d*n*(a + b*x)*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d*(b*c
 - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*n*(a + b*x) + 36*A*d^2*(b*c - a
*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*n*(a + b*x)^2 + 36*A*d^3*(a + b*x)^
3*Log[a + b*x] + 66*B*d^3*n*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*n*(a + b*x
)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n] - 1
8*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^2*(b
*c - a*d)*(a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^3*(a + b*x)^
3*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n] - 36*A*d^3*(a + b*x)^3*Log[c
 + d*x] - 66*B*d^3*n*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*n*(a + b*x)^3*Log
[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[e*(
(a + b*x)/(c + d*x))^n]*Log[c + d*x] - 18*B*d^3*n*(a + b*x)^3*Log[c + d*x]
^2 + 36*B*d^3*n*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] +
36*B*d^3*n*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3
*n*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 3*B*n*(36*A*(b*c -
 a*d)^4 + 9*B*(b*c - a*d)^4*n + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 28*B*d
*(-(b*c) + a*d)^3*n*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 + 78*B*
d^2*(b*c - a*d)^2*n*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*x)^3 + 3
00*B*d^3*(-(b*c) + a*d)*n*(a + b*x)^3 - 144*A*d^4*(a + b*x)^4*Log[a + b...
```

### 3.167.3 Rubi [A] (verified)

Time = 0.54 (sec) , antiderivative size = 364, normalized size of antiderivative = 0.77, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^5}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{i\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^3}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{i\int\left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^5}{(a+bx)^5}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^4}{(a+bx)^4}+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^3}{(a+bx)^3}\right)d\frac{a+bx}{c+dx}}{g^5(bc-ad)^3}$$

$\big\downarrow$ 2009

$$i\left(-\frac{b^2(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4(a+bx)^4}-\frac{b^2Bn(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8(a+bx)^4}-\frac{d^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}-\frac{Bd^2n(c+dx)^2}{}\right.$$

input
```
Int[((c*i + d*i*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*x)
^5,x]
```

output
```
(i*(-1/4*(B^2*d^2*n^2*(c + d*x)^2)/(a + b*x)^2 + (4*b*B^2*d*n^2*(c + d*x)^
3)/(27*(a + b*x)^3) - (b^2*B^2*n^2*(c + d*x)^4)/(32*(a + b*x)^4) - (B*d^2*
n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + (4
*b*B*d*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)^
3) - (b^2*B*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(8*(a +
b*x)^4) - (d^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a
 + b*x)^2) + (2*b*d*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(3*(a + b*x)^3) - (b^2*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2)/(4*(a + b*x)^4)))/((b*c - a*d)^3*g^5)
```

### 3.167.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

3.167. $\int\frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.167.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2196 vs. $2(457) = 914$.

Time = 28.62 (sec) , antiderivative size = 2197, normalized size of antiderivative = 4.63

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 2197 |

input
```
int((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,method=_
RETURNVERBOSE)
```

output
```
1/864*(480*A*B*x^3*a^7*b*c*d^4*i*n^2+144*A*B*x^3*a^6*b^2*c^2*d^3*i*n^2-172
8*A*B*x^3*a^5*b^3*c^3*d^2*i*n^2+1536*A*B*x^3*a^4*b^4*c^4*d*i*n^2+576*B^2*x
^2*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^2*d^3*i*n^2-72*B^2*x^2*ln(e*((b*x+a)/
(d*x+c))^n)*a^6*b^2*c^3*d^2*i*n^2+864*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^
8*c*d^4*i*n+576*A*B*x^2*a^7*b*c^2*d^3*i*n^2-2664*A*B*x^2*a^6*b^2*c^3*d^2*i
*n^2+2304*A*B*x^2*a^5*b^3*c^4*d*i*n^2-864*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^
2*a^7*b*c^3*d^2*i*n+288*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^2*c^4*d*i*
n-288*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^3*d^2*i*n^2+48*B^2*x*ln(e*((
b*x+a)/(d*x+c))^n)*a^6*b^2*c^4*d*i*n^2+1728*A*B*x*ln(e*((b*x+a)/(d*x+c))^n
)*a^8*c^2*d^3*i*n-2016*A*B*x*a^7*b*c^3*d^2*i*n^2+1584*A*B*x*a^6*b^2*c^4*d*
i*n^2-1152*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^4*d*i*n+72*B^2*x^4*ln(e*(
(b*x+a)/(d*x+c))^n)^2*a^6*b^2*c*d^4*i*n+156*B^2*x^4*ln(e*((b*x+a)/(d*x+c))
^n)*a^6*b^2*c*d^4*i*n^2+156*A*B*x^4*a^6*b^2*c*d^4*i*n^2-432*A*B*x^4*a^4*b^
4*c^3*d^2*i*n^2+384*A*B*x^4*a^3*b^5*c^4*d*i*n^2+288*B^2*x^3*ln(e*((b*x+a)/
(d*x+c))^n)^2*a^7*b*c*d^4*i*n+480*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*
c*d^4*i*n^2+144*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^2*d^3*i*n^2-17
28*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c^3*d^2*i*n+576*A*B*x*ln(e*((b*x+
a)/(d*x+c))^n)*a^6*b^2*c^4*d*i*n+144*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6
*b^2*c*d^4*i*n+576*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*c*d^4*i*n+288*A
^2*x^3*a^7*b*c*d^4*i*n-1728*A^2*x^3*a^5*b^3*c^3*d^2*i*n+2304*A^2*x^3*a^...
```

### 3.167.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1868 vs. $2(457) = 914$.

Time = 0.42 (sec) , antiderivative size = 1868, normalized size of antiderivative = 3.93

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,
algorithm="fricas")
```

output
```
-1/864*((27*B^2*b^4*c^4 - 128*B^2*a*b^3*c^3*d + 216*B^2*a^2*b^2*c^2*d^2 -
115*B^2*a^4*d^4)*i*n^2 + 12*(13*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*i*n^2 + 12
*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*i*n)*x^3 + 12*(9*A*B*b^4*c^4 - 32*A*B*a*b
^3*c^3*d + 36*A*B*a^2*b^2*c^2*d^2 - 13*A*B*a^4*d^4)*i*n - 6*((B^2*b^4*c^2*
d^2 - 80*B^2*a*b^3*c*d^3 + 79*B^2*a^2*b^2*d^4)*i*n^2 + 12*(A*B*b^4*c^2*d^2
 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)*i*n)*x^2 + 72*(4*(B^2*b^4*c^3*d
- 3*B^2*a*b^3*c^2*d^2 + 3*B^2*a^2*b^2*c*d^3 - B^2*a^3*b*d^4)*i*x + (3*B^2*
b^4*c^4 - 8*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^2 - B^2*a^4*d^4)*i)*log(
e)^2 + 72*(B^2*b^4*d^4*i*n^2*x^4 + 4*B^2*a*b^3*d^4*i*n^2*x^3 + 6*B^2*a^2*b
^2*d^4*i*n^2*x^2 + 4*(B^2*b^4*c^3*d - 3*B^2*a*b^3*c^2*d^2 + 3*B^2*a^2*b^2*
c*d^3)*i*n^2*x + (3*B^2*b^4*c^4 - 8*B^2*a*b^3*c^3*d + 6*B^2*a^2*b^2*c^2*d^
2)*i*n^2)*log((b*x + a)/(d*x + c))^2 + 72*(3*A^2*b^4*c^4 - 8*A^2*a*b^3*c^3
*d + 6*A^2*a^2*b^2*c^2*d^2 - A^2*a^4*d^4)*i - 4*((5*B^2*b^4*c^3*d - 12*B^2
*a*b^3*c^2*d^2 - 108*B^2*a^2*b^2*c*d^3 + 115*B^2*a^3*b*d^4)*i*n^2 - 12*(A*
B*b^4*c^3*d - 6*A*B*a*b^3*c^2*d^2 + 18*A*B*a^2*b^2*c*d^3 - 13*A*B*a^3*b*d^
4)*i*n - 72*(A^2*b^4*c^3*d - 3*A^2*a*b^3*c^2*d^2 + 3*A^2*a^2*b^2*c*d^3 - A
^2*a^3*b*d^4)*i)*x + 12*(12*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*i*n*x^3 - 6*(B
^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 7*B^2*a^2*b^2*d^4)*i*n*x^2 + (9*B^2*b
^4*c^4 - 32*B^2*a*b^3*c^3*d + 36*B^2*a^2*b^2*c^2*d^2 - 13*B^2*a^4*d^4)*i*n
 + 12*(3*A*B*b^4*c^4 - 8*A*B*a*b^3*c^3*d + 6*A*B*a^2*b^2*c^2*d^2 - A*B*...
```

### 3.167.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**5,x
)
```

output
```
Timed out
```

### 3.167.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4838 vs. 2(457) = 914.

Time = 0.54 (sec) , antiderivative size = 4838, normalized size of antiderivative = 10.19

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,
algorithm="maxima")
```

output
```
1/24*A*B*c*i*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*b*c*
d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*a*b^
2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2 - a
^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2 - a
^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d^2 -
 a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c*d^2
 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d^2 -
 a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b
^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/((b^5
*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^
5)) - 1/72*A*B*d*i*n*((7*a*b^3*c^3 - 33*a^2*b^2*c^2*d + 75*a^3*b*c*d^2 - 1
3*a^4*d^3 + 12*(4*b^4*c*d^2 - a*b^3*d^3)*x^3 - 6*(4*b^4*c^2*d - 29*a*b^3*c
*d^2 + 7*a^2*b^2*d^3)*x^2 + 4*(4*b^4*c^3 - 21*a*b^3*c^2*d + 57*a^2*b^2*c*d
^2 - 13*a^3*b*d^3)*x)/((b^9*c^3 - 3*a*b^8*c^2*d + 3*a^2*b^7*c*d^2 - a^3*b^
6*d^3)*g^5*x^4 + 4*(a*b^8*c^3 - 3*a^2*b^7*c^2*d + 3*a^3*b^6*c*d^2 - a^4*b^
5*d^3)*g^5*x^3 + 6*(a^2*b^7*c^3 - 3*a^3*b^6*c^2*d + 3*a^4*b^5*c*d^2 - a^5*
b^4*d^3)*g^5*x^2 + 4*(a^3*b^6*c^3 - 3*a^4*b^5*c^2*d + 3*a^5*b^4*c*d^2 - a^
6*b^3*d^3)*g^5*x + (a^4*b^5*c^3 - 3*a^5*b^4*c^2*d + 3*a^6*b^3*c*d^2 - a^7*
b^2*d^3)*g^5) + 12*(4*b*c*d^3 - a*d^4)*log(b*x + a)/((b^6*c^4 - 4*a*b^5*c^
3*d + 6*a^2*b^4*c^2*d^2 - 4*a^3*b^3*c*d^3 + a^4*b^2*d^4)*g^5) - 12*(4*b...
```

### 3.167.8 Giac [A] (verification not implemented)

Time = 3.07 (sec) , antiderivative size = 871, normalized size of antiderivative = 1.83

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx =$$

$$-\frac{1}{864}\left(\frac{72\left(3B^2b^2in^2-\frac{8(bx+a)B^2bdin^2}{dx+c}+\frac{6(bx+a)^2B^2d^2in^2}{(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^4b^2c^2g^5}{(dx+c)^4}-\frac{2(bx+a)^4abcdg^5}{(dx+c)^4}+\frac{(bx+a)^4a^2d^2g^5}{(dx+c)^4}}+\frac{12\left(9B^2b^2in^2-\frac{32(bx+a)B^2bdin^2}{dx+c}+\right.}{}\right.$$

input

```
integrate((d*i*x+c*i)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,
algorithm="giac")
```

output

```
-1/864*(72*(3*B^2*b^2*i*n^2 - 8*(b*x + a)*B^2*b*d*i*n^2/(d*x + c) + 6*(b*x
 + a)^2*B^2*d^2*i*n^2/(d*x + c)^2)*log((b*x + a)/(d*x + c))^2/((b*x + a)^4
*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*x + a)^4*a*b*c*d*g^5/(d*x + c)^4 + (b*x +
a)^4*a^2*d^2*g^5/(d*x + c)^4) + 12*(9*B^2*b^2*i*n^2 - 32*(b*x + a)*B^2*b*d
*i*n^2/(d*x + c) + 36*(b*x + a)^2*B^2*d^2*i*n^2/(d*x + c)^2 + 36*B^2*b^2*i
*n*log(e) - 96*(b*x + a)*B^2*b*d*i*n*log(e)/(d*x + c) + 72*(b*x + a)^2*B^2
*d^2*i*n*log(e)/(d*x + c)^2 + 36*A*B*b^2*i*n - 96*(b*x + a)*A*B*b*d*i*n/(d
*x + c) + 72*(b*x + a)^2*A*B*d^2*i*n/(d*x + c)^2)*log((b*x + a)/(d*x + c))
/((b*x + a)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*x + a)^4*a*b*c*d*g^5/(d*x + c
)^4 + (b*x + a)^4*a^2*d^2*g^5/(d*x + c)^4) + (27*B^2*b^2*i*n^2 - 128*(b*x
+ a)*B^2*b*d*i*n^2/(d*x + c) + 216*(b*x + a)^2*B^2*d^2*i*n^2/(d*x + c)^2 +
 108*B^2*b^2*i*n*log(e) - 384*(b*x + a)*B^2*b*d*i*n*log(e)/(d*x + c) + 432
*(b*x + a)^2*B^2*d^2*i*n*log(e)/(d*x + c)^2 + 216*B^2*b^2*i*log(e)^2 - 576
*(b*x + a)*B^2*b*d*i*log(e)^2/(d*x + c) + 432*(b*x + a)^2*B^2*d^2*i*log(e)
^2/(d*x + c)^2 + 108*A*B*b^2*i*n - 384*(b*x + a)*A*B*b*d*i*n/(d*x + c) + 4
32*(b*x + a)^2*A*B*d^2*i*n/(d*x + c)^2 + 432*A*B*b^2*i*log(e) - 1152*(b*x
+ a)*A*B*b*d*i*log(e)/(d*x + c) + 864*(b*x + a)^2*A*B*d^2*i*log(e)/(d*x +
c)^2 + 216*A^2*b^2*i - 576*(b*x + a)*A^2*b*d*i/(d*x + c) + 432*(b*x + a)^2
*A^2*d^2*i/(d*x + c)^2)/((b*x + a)^4*b^2*c^2*g^5/(d*x + c)^4 - 2*(b*x + a)
^4*a*b*c*d*g^5/(d*x + c)^4 + (b*x + a)^4*a^2*d^2*g^5/(d*x + c)^4))*(b*c...
```

### 3.167.9 Mupad [B] (verification not implemented)

Time = 6.36 (sec) , antiderivative size = 1794, normalized size of antiderivative = 3.78

$$\int \frac{(ci+dix)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*x)
^5,x)
```

output
```
((72*A^2*a^3*d^3*i + 216*A^2*b^3*c^3*i + 115*B^2*a^3*d^3*i*n^2 + 27*B^2*b^
3*c^3*i*n^2 + 156*A*B*a^3*d^3*i*n + 108*A*B*b^3*c^3*i*n - 360*A^2*a*b^2*c^
2*d*i + 72*A^2*a^2*b*c*d^2*i - 101*B^2*a*b^2*c^2*d*i*n^2 + 115*B^2*a^2*b*c
*d^2*i*n^2 - 276*A*B*a*b^2*c^2*d*i*n + 156*A*B*a^2*b*c*d^2*i*n)/(12*(a*d -
 b*c)) + (x^2*(79*B^2*a*b^2*d^3*i*n^2 - B^2*b^3*c*d^2*i*n^2 + 84*A*B*a*b^2
*d^3*i*n - 12*A*B*b^3*c*d^2*i*n))/(2*(a*d - b*c)) + (x*(72*A^2*a^2*b*d^3*i
 + 72*A^2*b^3*c^2*d*i + 115*B^2*a^2*b*d^3*i*n^2 - 5*B^2*b^3*c^2*d*i*n^2 -
144*A^2*a*b^2*c*d^2*i + 7*B^2*a*b^2*c*d^2*i*n^2 + 156*A*B*a^2*b*d^3*i*n +
12*A*B*b^3*c^2*d*i*n - 60*A*B*a*b^2*c*d^2*i*n))/(3*(a*d - b*c)) + (d*x^3*(
13*B^2*b^3*d^2*i*n^2 + 12*A*B*b^3*d^2*i*n))/(a*d - b*c))/(x*(288*a^3*b^4*c
*g^5 - 288*a^4*b^3*d*g^5) - x^3*(288*a^2*b^5*d*g^5 - 288*a*b^6*c*g^5) + x^
4*(72*b^7*c*g^5 - 72*a*b^6*d*g^5) + x^2*(432*a^2*b^5*c*g^5 - 432*a^3*b^4*d
*g^5) + 72*a^4*b^3*c*g^5 - 72*a^5*b^2*d*g^5) - log(e*((a + b*x)/(c + d*x))
^n)^2*(((B^2*c*i)/(4*b) + (B^2*d*i*x)/(3*b) + (B^2*a*d*i)/(12*b^2))/(a^4*g
^5 + b^4*g^5*x^4 + 4*a*b^3*g^5*x^3 + 6*a^2*b^2*g^5*x^2 + 4*a^3*b*g^5*x) -
(B^2*d^4*i)/(12*b^2*g^5*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
))) - log(e*((a + b*x)/(c + d*x))^n)*((A*B*a*d*i + 3*A*B*b*c*i - B^2*a*d*i
*n + B^2*b*c*i*n + 4*A*B*b*d*i*x)/(6*a^4*b^2*g^5 + 6*b^6*g^5*x^4 + 24*a^3*
b^3*g^5*x + 24*a*b^5*g^5*x^3 + 36*a^2*b^4*g^5*x^2) + (B^2*d^4*i*(x^2*(b*(b
*((3*a*b^2*g^5*n*(a*d - b*c))/(2*d) + (b^2*g^5*n*(a*d - b*c)*(4*a*d - b...
```

# 3.168 $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

### 3.168.1 Optimal result

Integrand size = 45, antiderivative size = 766

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$
$$= \frac{3B^2(bc-ad)^5g^3i^2n^2x}{20b^2d^3}+\frac{B^2(bc-ad)^2g^3i^2n^2(a+bx)^4}{60b^3}-\frac{3B^2(bc-ad)^4g^3i^2n^2(c+dx)^2}{40bd^4}$$
$$+\frac{B^2(bc-ad)^3g^3i^2n^2(c+dx)^3}{60d^4}-\frac{B(bc-ad)^3g^3i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{90b^3d}$$
$$-\frac{B(bc-ad)^2g^3i^2n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^3}$$
$$-\frac{B(bc-ad)g^3i^2n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^2}$$
$$+\frac{(bc-ad)^2g^3i^2(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^3}$$
$$+\frac{(bc-ad)g^3i^2(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{15b^2}$$
$$+\frac{g^3i^2(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b}$$
$$+\frac{B(bc-ad)^4g^3i^2n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{180b^3d^2}$$
$$-\frac{B(bc-ad)^5g^3i^2n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{180b^3d^3}$$
$$-\frac{B(bc-ad)^6g^3i^2n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{180b^3d^4}$$
$$-\frac{B^2(bc-ad)^6g^3i^2n^2\log(c+dx)}{20b^3d^4}-\frac{B^2(bc-ad)^6g^3i^2n^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{30b^3d^4}$$

output
```
3/20*B^2*(-a*d+b*c)^5*g^3*i^2*n^2*x/b^2/d^3+1/60*B^2*(-a*d+b*c)^2*g^3*i^2*
n^2*(b*x+a)^4/b^3-3/40*B^2*(-a*d+b*c)^4*g^3*i^2*n^2*(d*x+c)^2/b/d^4+1/60*B
^2*(-a*d+b*c)^3*g^3*i^2*n^2*(d*x+c)^3/d^4-1/90*B*(-a*d+b*c)^3*g^3*i^2*n*(b
*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d-1/20*B*(-a*d+b*c)^2*g^3*i^2*
n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3-1/15*B*(-a*d+b*c)*g^3*i^2*
n*(b*x+a)^4*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2+1/60*(-a*d+b*c)^2*
g^3*i^2*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3+1/15*(-a*d+b*c)*g^
3*i^2*(b*x+a)^4*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2+1/6*g^3*i^2*
(b*x+a)^4*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/180*B*(-a*d+b*c)
^4*g^3*i^2*n*(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d^2-1/1
80*B*(-a*d+b*c)^5*g^3*i^2*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^
n))/b^3/d^3-1/180*B*(-a*d+b*c)^6*g^3*i^2*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(
d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^3/d^4-1/20*B^2*(-a*d+b*c)^6*g^3*i^2
*n^2*ln(d*x+c)/b^3/d^4-1/30*B^2*(-a*d+b*c)^6*g^3*i^2*n^2*polylog(2,d*(b*x+
a)/b/(d*x+c))/b^3/d^4
```

### 3.168.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1634 vs. $2(766) = 1532$.

Time = 0.84 (sec) , antiderivative size = 1634, normalized size of antiderivative = 2.13

$$\int (ag + bgx)^3 (ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2,x]
```

output
```
(g^3*i^2*(15*(b*c - a*d)^2*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2 + 24*d*(b*c - a*d)*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2 + 10*d^2*(a + b*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (5*B*(
b*c - a*d)^3*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)*Lo
g[e*((a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(b*c
 - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c - a
*d)^2*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2*Lo
g[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2
, (b*(c + d*x))/(b*c - a*d)])))/d^4 + (2*B*(b*c - a*d)^2*n*(24*A*b*d*(b*c
- a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n]
 - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
 + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) -
6*d^4*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d
)^4*n*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n
])*Log[c + d*x] + 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)
^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*
x + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2 + 2*d^3*(a + b*x)^3 - 6*(b*c - a*d...
```

### 3.168.3 Rubi [A] (verified)

Time = 2.19 (sec) , antiderivative size = 1003, normalized size of antiderivative = 1.31, number of steps used = 18, number of rules used = 17, $\frac{\text{number of rules}}{\text{integrand size}} = 0.378$, Rules used = {2961, 2783, 2782, 27, 87, 49, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^3(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2961}$$

$$g^3i^2(bc-ad)^6\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$g^3 i^2 (bc - ad)^6 \left( -\frac{Bn \int \frac{(a+bx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{3b} + \frac{\int \frac{(a+bx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{3b} + \frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{6b(c+dx)^4 \left(b-\frac{d(}{c}\right.} \right.$$

$\downarrow$ 2782

$$g^3 i^2 (bc - ad)^6 \left( -\frac{Bn\left(-Bn \int \frac{(a+bx)^3 \left(5b-\frac{d(a+bx)}{c+dx}\right)}{20b^2 (c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx} + \frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{20b^2 (c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)}{3b} + \right.$$

$\downarrow$ 27

$$g^3 i^2 (bc - ad)^6 \left( -\frac{Bn\left(-\frac{Bn \int \frac{(a+bx)^3 \left(5b-\frac{d(a+bx)}{c+dx}\right)}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{20b^2} + \frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{20b^2 (c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right)}{3b} + \frac{\int \frac{(a+}{}}{} \right.$$

$\downarrow$ 87

$$g^3 i^2 (bc - ad)^6 \left( -\frac{Bn\left(-\frac{Bn\left(\int \frac{(a+bx)^3}{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx} + \frac{(a+bx)^4}{(c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{20b^2} + \frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{20b^2 (c+dx)^4 \left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{(a+bx)^4 \left(B\log\left(e\left(\right.\right.\right.}{5b(c+dx)^4 \left(b-\right.}\right.}{3b} \right.$$

$\downarrow$ 49

3.168. $\int (ag+bgx)^3 (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$g^3i^2(bc-ad)^6\left(-\frac{Bn\left(-\frac{Bn\left(\int\left(\frac{b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{1}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}+\frac{(a+bx)^4}{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{20b^2}+\frac{(a\text{-}}{}\right.}{3b}\right.$$

$\downarrow$ 2009

$$g^3i^2(bc-ad)^6\left(\frac{\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{3b}-\frac{Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{Bn}{}\right.}{}\right.$$

$\downarrow$ 2783

$$g^3i^2(bc-ad)^6\left(\frac{-\frac{2Bn\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}}{3b}-\frac{B}{}\right.$$

$\downarrow$ 2773

3.168. $\int(ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^3i^2(bc- ad)^6\left(\frac{-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\frac{(a+bx)^3}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}\right)}{5b}+\frac{\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^4(}{5b(c+}}{3b}\right.$$

$\downarrow$ 49

$$g^3i^2(bc- ad)^6\left(\frac{-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int\left(\frac{b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{1}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}}{4b}\right)}{5b}+}{3b}\right.$$

$\downarrow$ 2009

$$g^3i^2(bc- ad)^6\left(\frac{\frac{\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}-\frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{b^3}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3b^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3}{d^4\left(b-\frac{d(}{}\right)}\right.}{4b}\right.}{5b}}{3b}\right.$$

$\downarrow$ 2781

3.168. $\int(ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^3 i^2 (bc - ad)^6 \left( \frac{\frac{\frac{(a+bx)^4 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx}}{2b}}{5b} - \frac{2Bn\left(\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{b^3}{3d^4\left(b-\frac{d(a+}{c+d}\right.}\right.}{}\right.}{}}{3b}\right.$$

$$\Big\downarrow \text{2784}$$

$$(bc - ad)^6 g^3 i^2 \left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \right.}{}\right.$$

$$\Big\downarrow \text{2784}$$

3.168. $\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$(bc-ad)^6g^3i^2\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\right.}{}\right.$$

$\downarrow$ 2784

$$(bc-ad)^6g^3i^2\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\right.}{}\right.$$

$\downarrow$ 2754

---

3.168. $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-$$

$$ad)^6g^3i^2\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\right.}{}\right.$$

$\downarrow$ 2838

$$(bc-$$

$$ad)^6g^3i^2\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^4}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{20b^2(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)(a+bx)^4}{5b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\right.}{}\right.$$

3.168. $\int(ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2,x]
```

output
```
(b*c - a*d)^6*g^3*i^2*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(6*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^6) - (B*n*(((a + b*x)^4
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b*(c + d*x)^4*(b - (d*(a + b*x
))/(c + d*x))^5) + ((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2
0*b^2*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*((a + b*x)^4/((c
 + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) + b^3/(3*d^4*(b - (d*(a + b*x))
/(c + d*x))^3) - (3*b^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3*b)/(
d^4*(b - (d*(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]/d^4)
)/(20*b^2)))/(3*b) + (((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2)/(5*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*n*(((a + b*x)^
4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*b*(c + d*x)^4*(b - (d*(a + b*
x))/(c + d*x))^4) - (B*n*(b^3/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) - (3
*b^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) + (3*b)/(d^4*(b - (d*(a + b*
x))/(c + d*x))) + Log[b - (d*(a + b*x))/(c + d*x)]/d^4))/(4*b)))/(5*b) + (
((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b*(c + d*x)^4*(b
 - (d*(a + b*x))/(c + d*x))^4) - (B*n*(((a + b*x)^3*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(3*d*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (((a
 + b*x)^2*(3*A + B*n + 3*B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)
^2*(b - (d*(a + b*x))/(c + d*x))^2) - (((a + b*x)*(6*A + 5*B*n + 6*B*Log[e
*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)))...
```

### 3.168.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

## 3.168. $\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.168.4 Maple [F]

$$\int (bgx+ag)^3 (dix+ci)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

### 3.168.5 Fricas [F]

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (bgx+ag)^3(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2\,dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="fricas")
```

output
```
integral(A^2*b^3*d^2*g^3*i^2*x^5 + A^2*a^3*c^2*g^3*i^2 + (2*A^2*b^3*c*d +
3*A^2*a*b^2*d^2)*g^3*i^2*x^4 + (A^2*b^3*c^2 + 6*A^2*a*b^2*c*d + 3*A^2*a^2*
b*d^2)*g^3*i^2*x^3 + (3*A^2*a*b^2*c^2 + 6*A^2*a^2*b*c*d + A^2*a^3*d^2)*g^3
*i^2*x^2 + (3*A^2*a^2*b*c^2 + 2*A^2*a^3*c*d)*g^3*i^2*x + (B^2*b^3*d^2*g^3*
i^2*x^5 + B^2*a^3*c^2*g^3*i^2 + (2*B^2*b^3*c*d + 3*B^2*a*b^2*d^2)*g^3*i^2*
x^4 + (B^2*b^3*c^2 + 6*B^2*a*b^2*c*d + 3*B^2*a^2*b*d^2)*g^3*i^2*x^3 + (3*B
^2*a*b^2*c^2 + 6*B^2*a^2*b*c*d + B^2*a^3*d^2)*g^3*i^2*x^2 + (3*B^2*a^2*b*c
^2 + 2*B^2*a^3*c*d)*g^3*i^2*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b
^3*d^2*g^3*i^2*x^5 + A*B*a^3*c^2*g^3*i^2 + (2*A*B*b^3*c*d + 3*A*B*a*b^2*d^
2)*g^3*i^2*x^4 + (A*B*b^3*c^2 + 6*A*B*a*b^2*c*d + 3*A*B*a^2*b*d^2)*g^3*i^2
*x^3 + (3*A*B*a*b^2*c^2 + 6*A*B*a^2*b*c*d + A*B*a^3*d^2)*g^3*i^2*x^2 + (3*
A*B*a^2*b*c^2 + 2*A*B*a^3*c*d)*g^3*i^2*x)*log(e*((b*x + a)/(d*x + c))^n),
x)
```

### 3.168.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**
2,x)
```

output
```
Timed out
```

### 3.168.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5952 vs. $2(735) = 1470$.

Time = 0.82 (sec) , antiderivative size = 5952, normalized size of antiderivative = 7.77

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="maxima")
```

output
```
1/3*A*B*b^3*d^2*g^3*i^2*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/6*A
^2*b^3*d^2*g^3*i^2*x^6 + 4/5*A*B*b^3*c*d*g^3*i^2*x^5*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n) + 6/5*A*B*a*b^2*d^2*g^3*i^2*x^5*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) + 2/5*A^2*b^3*c*d*g^3*i^2*x^5 + 3/5*A^2*a*b^2*d^2*g^3*i^2*x
^5 + 1/2*A*B*b^3*c^2*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
3*A*B*a*b^2*c*d*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A
*B*a^2*b*d^2*g^3*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*
b^3*c^2*g^3*i^2*x^4 + 3/2*A^2*a*b^2*c*d*g^3*i^2*x^4 + 3/4*A^2*a^2*b*d^2*g^
3*i^2*x^4 + 2*A*B*a*b^2*c^2*g^3*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + 4*A*B*a^2*b*c*d*g^3*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 2/3*A*B*a^3*d^2*g^3*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2
*a*b^2*c^2*g^3*i^2*x^3 + 2*A^2*a^2*b*c*d*g^3*i^2*x^3 + 1/3*A^2*a^3*d^2*g^3
*i^2*x^3 + 3*A*B*a^2*b*c^2*g^3*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))
^n) + 2*A*B*a^3*c*d*g^3*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3
/2*A^2*a^2*b*c^2*g^3*i^2*x^2 + A^2*a^3*c*d*g^3*i^2*x^2 - 1/180*A*B*b^3*d^2
*g^3*i^2*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*log(d*x + c)/d^6 + (12*(b^5*c
*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d
^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5
*d^5)*x)/(b^5*d^5)) + 1/15*A*B*b^3*c*d*g^3*i^2*n*(12*a^5*log(b*x + a)/b^5
- 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c^2...
```

### 3.168.8 Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="giac")
```

output
```
Timed out
```

### 3.168.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2,x)
```

output
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2, x)
```

# 3.169 $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

## 3.169.1 Optimal result

Integrand size = 45, antiderivative size = 819

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= -\frac{B^2(bc-ad)^4g^2i^2n^2x}{10b^2d^2} - \frac{B^2(bc-ad)^3g^2i^2n^2(c+dx)^2}{20bd^3}$$
$$+ \frac{B^2(bc-ad)^2g^2i^2n^2(c+dx)^3}{30d^3} - \frac{B(bc-ad)^3g^2i^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^3d}$$
$$- \frac{B(bc-ad)^2g^2i^2n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15b^3}$$
$$- \frac{B(bc-ad)^3g^2i^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{5bd^3}$$
$$+ \frac{4B(bc-ad)^2g^2i^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3}$$
$$- \frac{bB(bc-ad)g^2i^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^3}$$
$$+ \frac{(bc-ad)^2g^2i^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{30b^3}$$
$$+ \frac{(bc-ad)g^2i^2(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2}$$
$$+ \frac{g^2i^2(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b}$$
$$+ \frac{B(bc-ad)^4g^2i^2n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^3d^2}$$
$$+ \frac{B(bc-ad)^5g^2i^2n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{30b^3d^3}$$
$$+ \frac{B^2(bc-ad)^5g^2i^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{30b^3d^3} + \frac{B^2(bc-ad)^5g^2i^2n^2\log(c+dx)}{10b^3d^3}$$
$$+ \frac{B^2(bc-ad)^5g^2i^2n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{15b^3d^3}$$

output

```
-1/10*B^2*(-a*d+b*c)^4*g^2*i^2*n^2*x/b^2/d^2-1/20*B^2*(-a*d+b*c)^3*g^2*i^2
*n^2*(d*x+c)^2/b/d^3+1/30*B^2*(-a*d+b*c)^2*g^2*i^2*n^2*(d*x+c)^3/d^3-1/30*
B*(-a*d+b*c)^3*g^2*i^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/d-1
/15*B*(-a*d+b*c)^2*g^2*i^2*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3
-1/5*B*(-a*d+b*c)^3*g^2*i^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/
d^3+4/15*B*(-a*d+b*c)^2*g^2*i^2*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)/d^3-1/10*b*B*(-a*d+b*c)*g^2*i^2*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))/d^3+1/30*(-a*d+b*c)^2*g^2*i^2*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)^2/b^3+1/10*(-a*d+b*c)*g^2*i^2*(b*x+a)^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))^2/b^2+1/5*g^2*i^2*(b*x+a)^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))^2/b+1/30*B*(-a*d+b*c)^4*g^2*i^2*n*(b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(
d*x+c))^n))/b^3/d^2+1/30*B*(-a*d+b*c)^5*g^2*i^2*n*(2*A+3*B*n+2*B*ln(e*((b*
x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^3/d^3+1/30*B^2*(-a*d+b*c)^5*g
^2*i^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d^3+1/10*B^2*(-a*d+b*c)^5*g^2*i^2*n^2*l
n(d*x+c)/b^3/d^3+1/15*B^2*(-a*d+b*c)^5*g^2*i^2*n^2*polylog(2,d*(b*x+a)/b/(
d*x+c))/b^3/d^3
```

### 3.169.2 Mathematica [A] (verified)

Time = 0.59 (sec) , antiderivative size = 1254, normalized size of antiderivative = 1.53

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{g^2i^2\Big(20d^3(bc-ad)^2(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+30d^4(bc-ad)(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{}$$

input

```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2,x]
```

output
```
(g^2*i^2*(20*d^3*(b*c - a*d)^2*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^2 + 30*d^4*(b*c - a*d)*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^2 + 12*d^5*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 +
20*B*(b*c - a*d)^3*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c - a*d)*(a + b*x)*
Log[e*((a + b*x)/(c + d*x))^n] - d^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c + d*x] - 2*(b*c - a*d)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(b*d*x + (-
(b*c) + a*d)*Log[c + d*x]) + B*(b*c - a*d)^2*n*((2*Log[(d*(a + b*x))/(-(b*
c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c
- a*d)])) - 10*B*(b*c - a*d)^2*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a
*d)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a +
 b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*
c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c -
a*d)*n*(2*b*d*(b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*
x]) + 3*B*(b*c - a*d)^2*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c
 - a*d)^3*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) + B*(b*c - a*d)*n*(24*A*b
*d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c +
d*x))^n] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c ...
```

### 3.169.3 Rubi [A] (verified)

Time = 2.07 (sec) , antiderivative size = 891, normalized size of antiderivative = 1.09, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.333$, Rules used = {2961, 2783, 2782, 27, 1195, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2961}$$

$$g^2i^2(bc-ad)^5\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$g^2i^2(bc-ad)^5\left(-\frac{2Bn\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{2\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^3\left(B\log\left(e\left(\right.\right.\right.}{5b(c+dx)^3\left(b-\right.}\right.$$

$\downarrow$ 2782

$$g^2i^2(bc-ad)^5\left(-\frac{2Bn\left(-Bn\int\frac{(c+dx)\left(b^2-\frac{4d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{12d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{2b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\,l}{2}\right.}{5b}\right.$$

$\downarrow$ 27

$$g^2i^2(bc-ad)^5\left(-\frac{2Bn\left(-\frac{Bn\int\frac{(c+dx)\left(b^2-\frac{4d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{12d^3}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{2b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\log\left(\right.}{2d^3\left(\right.}\right.}{5b}\right.$$

$\downarrow$ 1195

$$g^2i^2(bc-ad)^5\left(-\frac{2Bn\left(-\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{5d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{12d^3}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right.}{5b}\right.$$

$\downarrow$ 2009

---

3.169. $\int(ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^2i^2(bc-ad)^5\left(\frac{2\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}-\frac{2Bn\left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{2b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\log\left(e\left(\frac{a+b}{c+d}\right.\right.}{2d^3\left(b-\frac{d(a}{c}\right.}\right.}{}\right.$$

$\downarrow$ 2783

$$g^2i^2(bc-ad)^5\left(\frac{2\left(-\frac{Bn\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}+\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b}\right.$$

$\downarrow$ 2773

$$g^2i^2(bc-ad)^5\left(\frac{2\left(-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int \frac{(a+bx)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}\right)}{2b}+\frac{\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}+\frac{(a+bx)}{4b}\right.}{5b}\right.$$

$\downarrow$ 49

3.169. $\int(ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^2i^2(bc-ad)^5\left(\frac{2\left(-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\left(\frac{b^2}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{1}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)d\frac{a+bx}{c+dx}}{3b}\right)}{2b}+\frac{\int\frac{(a+bx)^2\left(A+B\log\ldots\right.}{(c+dx)^2\left(b\ldots\right.}}{\ldots}\right)}{5b}\ldots\right.$$

$\downarrow$ 2009

$$g^2i^2(bc-ad)^5\left(\frac{2\left(\frac{\int\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{4b}-\frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{b^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\log\left(b-\ldots\right.}{\ldots}\right.}{3b}\ldots\right.}{2b}\ldots\right.}{5b}\ldots\right.$$

$\downarrow$ 2781

---

3.169. $\int(ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^2 i^2 (bc - ad)^5 \left( \frac{2\left( \frac{\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}}{4b} - \frac{Bn\left(\frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{Bn\left(\frac{b}{2d^3\left(b-\frac{d}{\ldots}\right.}\ldots\right.}{\ldots}\right.}{2b\ldots}\right.}{5b\ldots}\right.$$

$\downarrow$ 2784

$$(bc - ad)^5 g^2 i^2 \left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{A+B\log\ldots}{2d^3\left(b-\ldots\right.}\ldots\right.}{\ldots}\right.$$

$\downarrow$ 2784

3.169. $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc - ad)^5 g^2 i^2 \left( \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{5b(c+dx)^3\left(b - \frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{A+B\log}{2d^3\left(b-\right.}\right.}{}\right.$$

$\downarrow$ 2754

3.169. $\int (ag+bgx)^2(ci+dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

$$(bc-ad)^5 g^2 i^2 \left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{A+B\log}{2d^3\left(b-\right.}\right.}{}\right.$$

$\downarrow$ 2838

---

3.169. $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-ad)^5 g^2 i^2 \left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{A+B\log}{2d^3\left(b-\right.}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2,x]
```

3.169. $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

output
```
(b*c - a*d)^5*g^2*i^2*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(5*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*n*((b^2*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d^3*(b - (d*(a + b*x))/(c + d*x))^4)
 - (2*b*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d^3*(b - (d*(a + b*x))/
(c + d*x))^3) + (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(2*d^3*(b - (d*(a +
 b*x))/(c + d*x))^2) - (B*n*(b/(b - (d*(a + b*x))/(c + d*x))^3 - 5/(2*(b -
 (d*(a + b*x))/(c + d*x))^2) + 1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(
a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(12*d^3))
)/(5*b) + (2*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b*
(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*(((a + b*x)^3*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]))/(3*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d
*x))^3) - (B*n*(b^2/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*b)/(d^3*(
b - (d*(a + b*x))/(c + d*x))) - Log[b - (d*(a + b*x))/(c + d*x)]/d^3))/(3*
b)))/(2*b) + (((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*b*
(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a + b*x)^2*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2*(b - (d*(a + b*x))/(c +
 d*x))^2) - (((a + b*x)*(2*A + B*n + 2*B*Log[e*((a + b*x)/(c + d*x))^n]))/
(d*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((2*A + 3*B*n + 2*B*Log[e
*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (2*B
*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d)/d)/(2*d)))/(3*b))/(4*b))...
```

### 3.169.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.169. $\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

## 3.169. $\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.169.4 Maple [F]

$$\int (bgx+ag)^{2} (dix+ci)^{2} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}dx$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

### 3.169.5 Fricas [F]

$$\begin{aligned}
&\int (ag+bgx)^2(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= \int (bgx+ag)^{2}(dix+ci)^{2}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}dx
\end{aligned}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="fricas")
```

output
```
integral(A^2*b^2*d^2*g^2*i^2*x^4 + A^2*a^2*c^2*g^2*i^2 + 2*(A^2*b^2*c*d +
A^2*a*b*d^2)*g^2*i^2*x^3 + (A^2*b^2*c^2 + 4*A^2*a*b*c*d + A^2*a^2*d^2)*g^2
*i^2*x^2 + 2*(A^2*a*b*c^2 + A^2*a^2*c*d)*g^2*i^2*x + (B^2*b^2*d^2*g^2*i^2*
x^4 + B^2*a^2*c^2*g^2*i^2 + 2*(B^2*b^2*c*d + B^2*a*b*d^2)*g^2*i^2*x^3 + (B
^2*b^2*c^2 + 4*B^2*a*b*c*d + B^2*a^2*d^2)*g^2*i^2*x^2 + 2*(B^2*a*b*c^2 + B
^2*a^2*c*d)*g^2*i^2*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^2*d^2*g
^2*i^2*x^4 + A*B*a^2*c^2*g^2*i^2 + 2*(A*B*b^2*c*d + A*B*a*b*d^2)*g^2*i^2*x
^3 + (A*B*b^2*c^2 + 4*A*B*a*b*c*d + A*B*a^2*d^2)*g^2*i^2*x^2 + 2*(A*B*a*b*
c^2 + A*B*a^2*c*d)*g^2*i^2*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

### 3.169.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**
2,x)
```

output
```
Timed out
```

### 3.169.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4247 vs. $2(786) = 1572$.

Time = 0.78 (sec) , antiderivative size = 4247, normalized size of antiderivative = 5.19

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="maxima")
```

output
```
2/5*A*B*b^2*d^2*g^2*i^2*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A
^2*b^2*d^2*g^2*i^2*x^5 + A*B*b^2*c*d*g^2*i^2*x^4*log(e*(b*x/(d*x + c) + a/
(d*x + c))^n) + A*B*a*b*d^2*g^2*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + 1/2*A^2*b^2*c*d*g^2*i^2*x^4 + 1/2*A^2*a*b*d^2*g^2*i^2*x^4 + 2/3*A*B
*b^2*c^2*g^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 8/3*A*B*a*b*
c*d*g^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2/3*A*B*a^2*d^2*g
^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*b^2*c^2*g^2*i^
2*x^3 + 4/3*A^2*a*b*c*d*g^2*i^2*x^3 + 1/3*A^2*a^2*d^2*g^2*i^2*x^3 + 2*A*B*
a*b*c^2*g^2*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a^2*c*d
*g^2*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*b*c^2*g^2*i^2*
x^2 + A^2*a^2*c*d*g^2*i^2*x^2 + 1/30*A*B*b^2*d^2*g^2*i^2*n*(12*a^5*log(b*x
 + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(
b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12*(b^4*c
^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/6*A*B*b^2*c*d*g^2*i^2*n*(6*a^4*log(b*x + a
)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c
^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/6*A*B*a*b*
d^2*g^2*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c
*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d
^3)*x)/(b^3*d^3)) + 1/3*A*B*b^2*c^2*g^2*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*
c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)...
```

### 3.169.8 Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="giac")
```

output
```
Timed out
```

### 3.169.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2,x)
```

output
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2, x)
```

# 3.170 $\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

## 3.170.1 Optimal result

Integrand size = 43, antiderivative size = 635

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{B^2(bc-ad)^3gi^2n^2x}{12b^2d}+\frac{B^2(bc-ad)^2gi^2n^2(c+dx)^2}{12bd^2}$$
$$-\frac{B(bc-ad)^3gi^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^3d}$$
$$-\frac{B(bc-ad)^2gi^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6b^3}$$
$$+\frac{B(bc-ad)^2gi^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4bd^2}$$
$$-\frac{B(bc-ad)gi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6d^2}$$
$$+\frac{(bc-ad)^2gi^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{12b^3}$$
$$+\frac{(bc-ad)gi^2(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b^2}$$
$$+\frac{gi^2(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4b}$$
$$-\frac{B(bc-ad)^4gi^2n\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{6b^3d^2}$$
$$-\frac{B^2(bc-ad)^4gi^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^3d^2}-\frac{B^2(bc-ad)^4gi^2n^2\log(c+dx)}{4b^3d^2}$$
$$-\frac{B^2(bc-ad)^4gi^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{6b^3d^2}$$

output
```
1/12*B^2*(-a*d+b*c)^3*g*i^2*n^2*x/b^2/d+1/12*B^2*(-a*d+b*c)^2*g*i^2*n^2*(d
*x+c)^2/b/d^2-1/6*B*(-a*d+b*c)^3*g*i^2*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/b^3/d-1/6*B*(-a*d+b*c)^2*g*i^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))/b^3+1/4*B*(-a*d+b*c)^2*g*i^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))/b/d^2-1/6*B*(-a*d+b*c)*g*i^2*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))/d^2+1/12*(-a*d+b*c)^2*g*i^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)^2/b^3+1/6*(-a*d+b*c)*g*i^2*(b*x+a)^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))^2/b^2+1/4*g*i^2*(b*x+a)^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2
/b-1/6*B*(-a*d+b*c)^4*g*i^2*n*(A+B*n+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d
+b*c)/b/(d*x+c))/b^3/d^2-1/12*B^2*(-a*d+b*c)^4*g*i^2*n^2*ln((b*x+a)/(d*x+c
))/b^3/d^2-1/4*B^2*(-a*d+b*c)^4*g*i^2*n^2*ln(d*x+c)/b^3/d^2-1/6*B^2*(-a*d+
b*c)^4*g*i^2*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/d^2
```

### 3.170.2 Mathematica [A] (verified)

Time = 0.36 (sec) , antiderivative size = 713, normalized size of antiderivative = 1.12

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{gi^2\left(-4(bc-ad)(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+3b(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{4B(bc-ad)^2n\left(\right.}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2,x]
```

output
```
(g*i^2*(-4*(b*c - a*d)*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2 + 3*b*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (4*B*(b*c -
 a*d)^2*n*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Lo
g[a + b*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] +
 b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*
Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*
Log[c + d*x] - B*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
 + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3
 - (B*(b*c - a*d)*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x
+ (b*c - a*d)*Log[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(
c + d*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)
*Log[e*((a + b*x)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]) + 6*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a
+ b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d
*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(12*d^2)
```

### 3.170.3 Rubi [A] (verified)

Time = 1.49 (sec) , antiderivative size = 702, normalized size of antiderivative = 1.11, number of steps used = 15, number of rules used = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.326$, Rules used = {2961, 2783, 2782, 27, 86, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2961}$$

$$gi^2(bc-ad)^4\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$gi^2(bc-ad)^4\left(-\frac{Bn\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{4b(c+dx)^2\left(b-\frac{d(a+}{c+}\right.}\right.$$

$\downarrow$ 2782

$$gi^2(bc-ad)^4\left(-\frac{Bn\left(-Bn\int -\frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{6d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int \frac{(a+bx)\left(A+B\log\right.}{(c+dx)\left(b\right.}}{}\right.$$

$\downarrow$ 27

$$gi^2(bc-ad)^4\left(-\frac{Bn\left(\frac{Bn\int \frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{2b}\right.$$

$\downarrow$ 86

$$gi^2(bc-ad)^4\left(-\frac{Bn\left(\frac{Bn\int \left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{3d^2\left(b-\frac{d(a+bx}{c+dx}\right.}\right.}{2b}\right.$$

$\downarrow$ 2009

---

3.170. $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$gi^2(bc-ad)^4\left(\frac{\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}-\frac{Bn\left(-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log}{}\right.}{}\right.}{2b}\right.$$

$\downarrow$ 2783

$$gi^2(bc-ad)^4\left(\frac{-\frac{2Bn\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}}{2b}-\frac{Bn}{}\right.$$

$\downarrow$ 2773

$$gi^2(bc-ad)^4\left(\frac{-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int \frac{a+bx}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^2\left(B\right.}{3b(c+d}}{2b}\right.$$

$\downarrow$ 49

3.170. $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$gi^2(bc-ad)^4\left(\frac{-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{b}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}+\frac{1}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{2b}\right)}{3b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}+}{2b}\right.$$

$$\Big\downarrow \text{2009}$$

$$gi^2(bc-ad)^4\left(\frac{\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3b}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{d^2}\right)}{2b}\right)}{3b}+\frac{(a+bx)}{3b}}{2b}\right.$$

$$\Big\downarrow \text{2781}$$

$$gi^2(bc-ad)^4\left(\frac{\frac{\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{3b}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{b}{d^2\left(b-\frac{d(a+bx)}{c+dx}\right)}\right.}{}\right.}{3b}}{2b}\right.$$

$$\Big\downarrow \text{2784}$$

3.170. $\int(ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$gi^2(bc-ad)^4\left(\frac{\frac{\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\int\frac{A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{d}\right)}{b}}{3b}-\frac{2Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{2b(c+dx)^2\left(b-\frac{d(a-}{c+}\right.}\right.}{2b}\right.$$

$\downarrow$ 2754

$$gi^2(bc-ad)^4\left(\frac{\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{d}}{d}\right.}{b}}{3b}\right.$$

$\downarrow$ 2838

$$gi^2(bc-ad)^4\left(-\frac{Bn\left(-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{1}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right.}{6d^2}\right.}{2b}\right.$$

---

3.170. $\int(ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
,x]
```

output
```
(b*c - a*d)^4*g*i^2*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/(4*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*((b*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]))/(3*d^2*(b - (d*(a + b*x))/(c + d*x))^3) - (A
+ B*Log[e*((a + b*x)/(c + d*x))^n])/(2*d^2*(b - (d*(a + b*x))/(c + d*x))^2
) + (B*n*(-(b - (d*(a + b*x))/(c + d*x))^(-2) + 1/(b*(b - (d*(a + b*x))/(c
 + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)
]/b^2))/(6*d^2)))/(2*b) + (((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2)/(3*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a +
b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*b*(c + d*x)^2*(b - (d*(a
 + b*x))/(c + d*x))^2) - (B*n*(b/(d^2*(b - (d*(a + b*x))/(c + d*x))) + Log
[b - (d*(a + b*x))/(c + d*x)]/d^2))/(2*b)))/(3*b) + (((a + b*x)^2*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])^2)/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c +
 d*x))^2) - (B*n*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c
 + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (-(((A + B*n + B*Log[e*((a + b*x)
/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d) - (B*n*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))])/d)/d))/b)/(3*b))/(2*b))
```

### 3.170.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

---

3.170. $\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

## 3.170. $\int (ag+bgx)(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.170.4 Maple [F]

$$\int (bgx+ag)(dix+ci)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2dx$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

### 3.170.5 Fricas [F]

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$
$$=\int (bgx+ag)(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="fricas")
```

output
```
integral(A^2*b*d^2*g*i^2*x^3 + A^2*a*c^2*g*i^2 + (2*A^2*b*c*d + A^2*a*d^2)
*g*i^2*x^2 + (A^2*b*c^2 + 2*A^2*a*c*d)*g*i^2*x + (B^2*b*d^2*g*i^2*x^3 + B^
2*a*c^2*g*i^2 + (2*B^2*b*c*d + B^2*a*d^2)*g*i^2*x^2 + (B^2*b*c^2 + 2*B^2*a
*c*d)*g*i^2*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b*d^2*g*i^2*x^3 +
 A*B*a*c^2*g*i^2 + (2*A*B*b*c*d + A*B*a*d^2)*g*i^2*x^2 + (A*B*b*c^2 + 2*A*
B*a*c*d)*g*i^2*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

### 3.170.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x
)
```

output
```
Timed out
```

### 3.170.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2662 vs. $2(608) = 1216$.

Time = 0.74 (sec) , antiderivative size = 2662, normalized size of antiderivative = 4.19

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="maxima")
```

output
```
1/2*A*B*b*d^2*g*i^2*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*b
*d^2*g*i^2*x^4 + 4/3*A*B*b*c*d*g*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c
))^n) + 2/3*A*B*a*d^2*g*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2
/3*A^2*b*c*d*g*i^2*x^3 + 1/3*A^2*a*d^2*g*i^2*x^3 + A*B*b*c^2*g*i^2*x^2*log
(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a*c*d*g*i^2*x^2*log(e*(b*x/(d*
x + c) + a/(d*x + c))^n) + 1/2*A^2*b*c^2*g*i^2*x^2 + A^2*a*c*d*g*i^2*x^2 -
 1/12*A*B*b*d^2*g*i^2*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 +
 (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c
^3 - a^3*d^3)*x)/(b^3*d^3)) + 2/3*A*B*b*c*d*g*i^2*n*(2*a^3*log(b*x + a)/b^
3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d
^2)*x)/(b^2*d^2)) + 1/3*A*B*a*d^2*g*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*
log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^
2*d^2)) - A*B*b*c^2*g*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 +
 (b*c - a*d)*x/(b*d)) - 2*A*B*a*c*d*g*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*lo
g(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*a*c^2*g*i^2*n*(a*log(b*x + a
)/b - c*log(d*x + c)/d) + 2*A*B*a*c^2*g*i^2*x*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) + A^2*a*c^2*g*i^2*x - 1/12*(7*a^2*b*c^2*d^2*g*i^2*n^2 - 2*a^3*c
*d^3*g*i^2*n^2 + (g*i^2*n^2 - 2*g*i^2*n*log(e))*b^3*c^4 - 2*(3*g*i^2*n^2 -
 4*g*i^2*n*log(e))*a*b^2*c^3*d)*B^2*log(d*x + c)/(b^2*d^2) + 1/6*(b^4*c^4*
g*i^2*n^2 - 4*a*b^3*c^3*d*g*i^2*n^2 + 6*a^2*b^2*c^2*d^2*g*i^2*n^2 - 4*a...
```

### 3.170.8 Giac [F]

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx$$
$$=\int (bgx+ag)(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="giac")
```

output
```
integrate((b*g*x + a*g)*(d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n)
+ A)^2, x)
```

### 3.170.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)\,(c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
,x)
```

output
```
int((a*g + b*g*x)*(c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
, x)
```

# 3.171 $\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$



## 3.171.1 Optimal result

Integrand size = 35, antiderivative size = 361

$$\begin{aligned}
\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx
&= \frac{B^2(bc-ad)^2i^2n^2x}{3b^2} - \frac{2B(bc-ad)^2i^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^3} \\
&\quad - \frac{B(bc-ad)i^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3bd} \\
&\quad + \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d} \\
&\quad + \frac{B^2(bc-ad)^3i^2n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^3d} + \frac{B^2(bc-ad)^3i^2n^2\log(c+dx)}{b^3d} \\
&\quad + \frac{2B(bc-ad)^3i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d} \\
&\quad - \frac{2B^2(bc-ad)^3i^2n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^3d}
\end{aligned}$$

output
```
1/3*B^2*(-a*d+b*c)^2*i^2*n^2*x/b^2-2/3*B*(-a*d+b*c)^2*i^2*n*(b*x+a)*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))/b^3-1/3*B*(-a*d+b*c)*i^2*n*(d*x+c)^2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/b/d+1/3*i^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
^2/d+1/3*B^2*(-a*d+b*c)^3*i^2*n^2*ln((b*x+a)/(d*x+c))/b^3/d+B^2*(-a*d+b*c)
^3*i^2*n^2*ln(d*x+c)/b^3/d+2/3*B*(-a*d+b*c)^3*i^2*n*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/d-2/3*B^2*(-a*d+b*c)^3*i^2*n^2*pol
ylog(2,b*(d*x+c)/d/(b*x+a))/b^3/d
```

### 3.171.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 303, normalized size of antiderivative = 0.84

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{i^2\left((c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{B(bc-ad)n\left(2Abd(bc-ad)x-B(bc-ad)n(bdx+(bc-ad)\log(a+bx))+2Bd(bc-ad)(a+bx)\right.}{}\right.}{}$$

input
```
Integrate[(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(i^2*((c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d
)*n*(2*A*b*d*(b*c - a*d)*x - B*(b*c - a*d)*n*(b*d*x + (b*c - a*d)*Log[a +
b*x]) + 2*B*d*(b*c - a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + b^2*(
c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*(b*c - a*d)^2*Log[a
+ b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c
+ d*x] - B*(b*c - a*d)^2*n*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x
))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])))/b^3))/(3*
d)
```

### 3.171.3 Rubi [A] (verified)

Time = 0.94 (sec) , antiderivative size = 376, normalized size of antiderivative = 1.04, number of steps used = 12, number of rules used = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {2951, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \ 2951$$

$$i^2(bc-ad)^3\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2756$$

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)$$

$\downarrow$ 2789

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{3d}\right)$$

$\downarrow$ 2756

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{3d}\right)$$

$\downarrow$ 54

---

3.171. $\int(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\right.}{3d}\right.$$

$\downarrow$ 2009

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log(}{}\right.}{}\right.}{b}\right.}{3d}\right.$$

$\downarrow$ 2789

3.171. $\int(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+}{c+}\right.\right.}{2d\left(b-\frac{d(a}{c}\right.}\right.}{}\right.}{3d}\right.$$

$\downarrow$ 2751

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}}{b}\right.}{3d}\right.$$

$\downarrow$ 16

---

3.171. $\int(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+b}{c+d}\right.}{bd}\right.}{b}}{b}\right.}{3d}\right.$$

$\downarrow$ 2779

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(\right.}{b(c-}\right.}{}\right.}{}\right.$$

$\downarrow$ 2838

---

3.171. $\int(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$i^2(bc-ad)^3\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{Bn\,\cdots}{\cdots}\cdots\right)}{\cdots}\right.$$

input
```
Int[(c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(b*c - a*d)^3*i^2*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(3*d*(b - (d*(
a + b*x))/(c + d*x))^3) - (2*B*n*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n
])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(1/(b*(b - (d*(a + b*x))/(
c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x
)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b - (d*(a + b*x)
)/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1
 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x))/(d*(a
+ b*x))])/b)/b)/b))/(3*d))
```

### 3.171.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

## 3.171. $\int (ci+dix)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

### 3.171.4 Maple [F]

$$\int (dix+ci)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

### 3.171.5 Fricas [F]

$$\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\ = \int (dix+ci)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x^
2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A
*B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log(e*((b*x + a)/(d*x + c)
)^n), x)
```

### 3.171.6 Sympy [F(-1)]

Timed out.

$$\int (ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

### 3.171.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1473 vs. $2(346) = 692$.

Time = 0.72 (sec) , antiderivative size = 1473, normalized size of antiderivative = 4.08

$$\int (ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
2/3*A*B*d^2*i^2*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*A^2*d^2*i
^2*x^3 + 2*A*B*c*d*i^2*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c*
d*i^2*x^2 + 1/3*A*B*d^2*i^2*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 2*
A*B*c*d*i^2*n*(a^2*log(b*x + a)/b^2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x
/(b*d)) + 2*A*B*c^2*i^2*n*(a*log(b*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^
2*i^2*x*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*c^2*i^2*x - 1/3*(5*a*
b*c^2*d*i^2*n^2 - 2*a^2*c*d^2*i^2*n^2 - (3*i^2*n^2 - 2*i^2*n*log(e))*b^2*c
^3)*B^2*log(d*x + c)/(b^2*d) - 2/3*(b^3*c^3*i^2*n^2 - 3*a*b^2*c^2*d*i^2*n^
2 + 3*a^2*b*c*d^2*i^2*n^2 - a^3*d^3*i^2*n^2)*(log(b*x + a)*log((b*d*x + a*
d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))*B^2/(b^3*d) + 1/3
*(B^2*b^3*d^3*i^2*x^3*log(e)^2 + 2*B^2*b^3*c^3*i^2*n^2*log(b*x + a)*log(d*
x + c) - B^2*b^3*c^3*i^2*n^2*log(d*x + c)^2 + (a*b^2*d^3*i^2*n*log(e) - (i
^2*n*log(e) - 3*i^2*log(e)^2)*b^3*c*d^2)*B^2*x^2 - (3*a*b^2*c^2*d*i^2*n^2
- 3*a^2*b*c*d^2*i^2*n^2 + a^3*d^3*i^2*n^2)*B^2*log(b*x + a)^2 + ((i^2*n^2
- 4*i^2*n*log(e) + 3*i^2*log(e)^2)*b^3*c^2*d - 2*(i^2*n^2 - 3*i^2*n*log(e)
)*a*b^2*c*d^2 + (i^2*n^2 - 2*i^2*n*log(e))*a^2*b*d^3)*B^2*x - (2*(2*i^2*n^
2 - 3*i^2*n*log(e))*a*b^2*c^2*d - (7*i^2*n^2 - 6*i^2*n*log(e))*a^2*b*c*d^2
 + (3*i^2*n^2 - 2*i^2*n*log(e))*a^3*d^3)*B^2*log(b*x + a) + (B^2*b^3*d^3*i
^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x)*log((b*x + a)...
```

### 3.171.8 Giac [F]

$$\int (ci + dix)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \int (dix + ci)^2 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

### 3.171.9 Mupad [F(-1)]

Timed out.

$$\int (ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (c\,i+d\,i\,x)^2\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

# 3.172 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$



## 3.172.1 Optimal result

Integrand size = 45, antiderivative size = 572

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx \\
&= -\frac{Bd(bc-ad)i^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g} \\
&\quad+\frac{d(bc-ad)i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g}+\frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg} \\
&\quad+\frac{2B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^3g}+\frac{B^2(bc-ad)^2i^2n^2\log(c+dx)}{b^3g} \\
&\quad+\frac{B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad-\frac{(bc-ad)^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad+\frac{2B^2(bc-ad)^2i^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g}-\frac{B^2(bc-ad)^2i^2n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad+\frac{2B(bc-ad)^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g} \\
&\quad+\frac{2B^2(bc-ad)^2i^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g}
\end{aligned}$$

output
```
-B*d*(-a*d+b*c)*i^2*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g+d*(-a*
d+b*c)*i^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3/g+1/2*i^2*(d*x+c)
^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b/g+2*B*(-a*d+b*c)^2*i^2*n*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^3/g+B^2*(-a*d+b*c)^2*i^2*
n^2*ln(d*x+c)/b^3/g+B*(-a*d+b*c)^2*i^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*l
n(1-b*(d*x+c)/d/(b*x+a))/b^3/g-(-a*d+b*c)^2*i^2*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g+2*B^2*(-a*d+b*c)^2*i^2*n^2*polylog
(2,d*(b*x+a)/b/(d*x+c))/b^3/g-B^2*(-a*d+b*c)^2*i^2*n^2*polylog(2,b*(d*x+c)
/d/(b*x+a))/b^3/g+2*B*(-a*d+b*c)^2*i^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*p
olylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g+2*B^2*(-a*d+b*c)^2*i^2*n^2*polylog(3,b
*(d*x+c)/d/(b*x+a))/b^3/g
```

### 3.172.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2852 vs. $2(572) = 1144$.

Time = 2.09 (sec) , antiderivative size = 2852, normalized size of antiderivative = 4.99

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x),x]
```

output
```
(i^2*(12*b*d*(2*b*c - a*d)*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*L
og[(a + b*x)/(c + d*x)])^2 + 6*b^2*d^2*x^2*(A + B*Log[e*((a + b*x)/(c + d*
x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 + 12*(b*c - a*d)^2*Log[a + b*x]*(
A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 - 2
4*b*B*c*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d
*x)])*(a*d*Log[a/b + x]^2 - 2*a*d*Log[a/b + x]*(1 + Log[a + b*x]) + 2*(-(b
*c) + a*d + Log[c/d + x]*(b*c + a*d*Log[a + b*x] - a*d*Log[(d*(a + b*x))/(
-(b*c) + a*d)]) + (-(b*d*x) + a*d*Log[a + b*x])*Log[(a + b*x)/(c + d*x)])
- 2*a*d*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 12*b^2*B*c^2*n*(A + B*Log
[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(Log[a/b + x]^
2 - 2*Log[a + b*x]*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)]
) - 2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + PolyLog[2, (b*(c +
 d*x))/(b*c - a*d)])) + 6*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*
Log[(a + b*x)/(c + d*x)])*(-4*a*d^2*(a + b*x)*(-1 + Log[a/b + x]) + 2*a^2*
d^2*Log[a/b + x]^2 + 4*a*b*d*(c + d*x)*(-1 + Log[c/d + x]) + d^2*(b*x*(2*a
 - b*x) + 2*b^2*x^2*Log[a/b + x] - 2*a^2*Log[a + b*x]) - 2*d^2*(b*x*(-2*a
+ b*x) + 2*a^2*Log[a + b*x])*(Log[a/b + x] - Log[c/d + x] - Log[(a + b*x)/
(c + d*x)]) + b^2*(d*x*(-2*c + d*x) - 2*d^2*x^2*Log[c/d + x] + 2*c^2*Log[c
 + d*x]) - 4*a^2*d^2*(Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Pol
yLog[2, (b*(c + d*x))/(b*c - a*d)])) - 8*b*B^2*c*n^2*(a*d*Log[a/b + x]^...
```

### 3.172.3 Rubi [A] (verified)

Time = 1.98 (sec) , antiderivative size = 529, normalized size of antiderivative = 0.92, number of steps used = 13, number of rules used = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {2961, 2789, 2756, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ag+bgx}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i^2(bc-ad)^2\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \text{2789}$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$\downarrow$ 2756

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$\downarrow$ 2789

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{d}\right)}{b}+\frac{d\int \frac{\left(A+B\log\left(\right.\right.}{\left(b-\frac{d}{}\right.}}{}\right)}{g}$$

$\downarrow$ 2751

3.172. $\displaystyle\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\cdots}{b}\cdots\right)}{d}\cdots\right)}{b}\cdots\right)}{g}\cdots$$

$$\Big\downarrow \ 16$$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\cdots\right)}{b}\cdots\right)}{d}\cdots\right)}{b}\cdots\right)}{g}\cdots$$

$$\Big\downarrow \ 2755$$

---

3.172. $\displaystyle\int\frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$\frac{i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\cdots\right)}{b}\cdots\right)}{d}\cdots\right)}{b}\cdots\right)}{g\cdots}$$

$\big\downarrow$ 2754

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\cdots\right)}{b}\cdots\right)}{d}\cdots\right)}{b}\cdots\right)\cdots$$

$\big\downarrow$ 2779

---

3.172. $\displaystyle \int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\right.}{b(c+dx)}\right.}{}\right.}{d}\right)}{b}\right.$$

$\downarrow$ 2821

$$i^2(bc-ad)^2\left(\frac{\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int\frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{b}}{b}\right.$$

$\downarrow$ 2838

3.172. $\int\frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$i^2(bc-ad)^2\left(\frac{\frac{\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int\frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\cdots}{b}}{b}\right.$$

$\downarrow$ 7143

$$i^2(bc-ad)^2\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\frac{Bn\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\cdots\right.\right.}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}\cdots\right.}{\cdots}\right.}{d}\right.}{b}\right.$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x),x]
```

---

3.172. $\int\frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

output
```
((b*c - a*d)^2*i^2*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*d*(b -
 (d*(a + b*x))/(c + d*x))^2) - (B*n*((d*(((a + b*x)*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b
 - (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c +
d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c
 + d*x))/(d*(a + b*x))])/b)/b))/d))/b + ((d*(((a + b*x)*(A + B*Log[e*((a +
 b*x)/(c + d*x))^n])^2)/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (2*B
*n*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c
+ d*x))])/d) - (B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d))/b))/b + (
-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a +
b*x))])/b) + (2*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b*
(c + d*x))/(d*(a + b*x))] + B*n*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/
b)/b)/b))/g
```

### 3.172.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

---

## 3.172. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

### 3.172.4 Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{bgx+ag}dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

### 3.172.5 Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x,
algorithm="fricas")
```

output
```
integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x
^2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(
A*B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log(e*((b*x + a)/(d*x + c
))^n))/(b*g*x + a*g), x)
```

### 3.172.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x
)
```

output
```
Timed out
```

### 3.172.7 Maxima [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x,
algorithm="maxima")
```

output
```
2*A^2*c*d*i^2*(x/(b*g) - a*log(b*x + a)/(b^2*g)) + 1/2*A^2*d^2*i^2*(2*a^2*
log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A^2*c^2*i^2*log(b*g*x +
a*g)/(b*g) + 1/2*(B^2*b^2*d^2*i^2*x^2 + 2*(2*b^2*c*d*i^2 - a*b*d^2*i^2)*B^
2*x + 2*(b^2*c^2*i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B^2*log(b*x + a))*log(
(d*x + c)^n)^2/(b^3*g) - integrate(-(B^2*b^3*c^3*i^2*log(e)^2 + 2*A*B*b^3*
c^3*i^2*log(e) + (B^2*b^3*d^3*i^2*log(e)^2 + 2*A*B*b^3*d^3*i^2*log(e))*x^3
 + 3*(B^2*b^3*c*d^2*i^2*log(e)^2 + 2*A*B*b^3*c*d^2*i^2*log(e))*x^2 + (B^2*
b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + B^2*b^
3*c^3*i^2)*log((b*x + a)^n)^2 + 3*(B^2*b^3*c^2*d*i^2*log(e)^2 + 2*A*B*b^3*
c^2*d*i^2*log(e))*x + 2*(B^2*b^3*c^3*i^2*log(e) + A*B*b^3*c^3*i^2 + (B^2*b
^3*d^3*i^2*log(e) + A*B*b^3*d^3*i^2)*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(e) + A
*B*b^3*c*d^2*i^2)*x^2 + 3*(B^2*b^3*c^2*d*i^2*log(e) + A*B*b^3*c^2*d*i^2)*x
)*log((b*x + a)^n) - (2*B^2*b^3*c^3*i^2*log(e) + 2*A*B*b^3*c^3*i^2 + (2*A*
B*b^3*d^3*i^2 + (i^2*n + 2*i^2*log(e))*B^2*b^3*d^3)*x^3 + (6*A*B*b^3*c*d^2
*i^2 - (a*b^2*d^3*i^2*n - 2*(2*i^2*n + 3*i^2*log(e))*b^3*c*d^2)*B^2)*x^2 +
 2*(3*A*B*b^3*c^2*d*i^2 + (2*a*b^2*c*d^2*i^2*n - a^2*b*d^3*i^2*n + 3*b^3*c
^2*d*i^2*log(e))*B^2)*x + 2*((b^3*c^2*d*i^2*n - 2*a*b^2*c*d^2*i^2*n + a^2*
b*d^3*i^2*n)*B^2*x + (a*b^2*c^2*d*i^2*n - 2*a^2*b*c*d^2*i^2*n + a^3*d^3*i^
2*n)*B^2)*log(b*x + a) + 2*(B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*c*d^2*i^2*x^2
+ 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^2)*log((b*x + a)^n))*log((d*x +...
```

### 3.172.8 Giac [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x,
algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x
+ a*g), x)
```

### 3.172.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{{\left(c\,i+d\,i\,x\right)}^2\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x),x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x), x)
```

# 3.173 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$



## 3.173.1 Optimal result

Integrand size = 45, antiderivative size = 472

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx \\
&= -\frac{2B^2(bc-ad)i^2n^2(c+dx)}{b^2g^2(a+bx)} - \frac{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^2(a+bx)} \\
&\quad + \frac{d^2i^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^2} - \frac{(bc-ad)i^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g^2(a+bx)} \\
&\quad + \frac{2Bd(bc-ad)i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^3g^2} \\
&\quad - \frac{2d(bc-ad)i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{2B^2d(bc-ad)i^2n^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^3g^2} \\
&\quad + \frac{4Bd(bc-ad)i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2} \\
&\quad + \frac{4B^2d(bc-ad)i^2n^2\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^2}
\end{aligned}$$

output
```
-2*B^2*(-a*d+b*c)*i^2*n^2*(d*x+c)/b^2/g^2/(b*x+a)-2*B*(-a*d+b*c)*i^2*n*(d*
x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^2/(b*x+a)+d^2*i^2*(b*x+a)*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))^2/b^3/g^2-(-a*d+b*c)*i^2*(d*x+c)*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))^2/b^2/g^2/(b*x+a)+2*B*d*(-a*d+b*c)*i^2*n*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^3/g^2-2*d*(-a*d+b*c)*i^2*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^2+2*B^2*d*(-a
*d+b*c)*i^2*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^3/g^2+4*B*d*(-a*d+b*c)*i^
2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^2
+4*B^2*d*(-a*d+b*c)*i^2*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^3/g^2
```

## 3.173.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2885 vs. $2(472) = 944$.

Time = 2.78 (sec) , antiderivative size = 2885, normalized size of antiderivative = 6.11

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^2,x]
```

output
```
(i^2*(3*b*d^2*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/
(c + d*x)])^2 - (3*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B
*n*Log[(a + b*x)/(c + d*x)])^2)/(a + b*x) + 6*d*(b*c - a*d)*Log[a + b*x]*(
A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 + (
6*b^2*B*c^2*n*(-A - B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(
c + d*x)])*(-(d*(a + b*x)*Log[c/d + x]) + d*(a + b*x)*Log[(d*(a + b*x))/(-
(b*c) + a*d)] + (b*c - a*d)*(1 + Log[(a + b*x)/(c + d*x)])))/((b*c - a*d)*
(a + b*x)) + (3*b^2*B^2*c^2*n^2*(-2*b*c + 2*a*d - 2*d*(a + b*x)*Log[a + b*
x] - 2*(b*c - a*d)*Log[(a + b*x)/(c + d*x)] - 2*d*(a + b*x)*Log[a + b*x]*L
og[(a + b*x)/(c + d*x)] - (b*c - a*d)*Log[(a + b*x)/(c + d*x)]^2 + 2*d*(a
+ b*x)*Log[c + d*x] - 2*d*(a + b*x)*Log[(a + b*x)/(c + d*x)]*Log[(b*c - a*
d)/(b*c + b*d*x)] + d*(a + b*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c
+ d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + d*(a
 + b*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a
*d)])))/((b*c - a*d)*(a + b*x)) + 6*b*B*c*d*n*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(Log[a/b + x]^2 - 2*Log[a/b + x]
*Log[a + b*x] - 2*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a
 + b*x]*((a*d)/(b*c - a*d) + Log[c/d + x] + Log[(a + b*x)/(c + d*x)]) + 2*
a*((a + b*x)^(-1) + Log[(a + b*x)/(c + d*x)]/(a + b*x) + (d*Log[c + d*x...
```

### 3.173.3 Rubi [A] (verified)

Time = 0.72 (sec) , antiderivative size = 396, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i^2(bc-ad)\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{i^2(bc-ad)\int\left(\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2(a+bx)^2}+\frac{2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right)d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i^2(bc-ad)\left(\frac{d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^3(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{4Bdn\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^3}+\frac{2Bdn\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(\right.\right.}{b^3}\right.}{}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^2,x]
```

output
```
((b*c - a*d)*i^2*((-2*B^2*n^2*(c + d*x))/(b^2*(a + b*x)) - (2*B*n*(c + d*x
)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^2*(a + b*x)) - ((c + d*x)*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(b^2*(a + b*x)) + (d^2*(a + b*x)*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(b^3*(c + d*x)*(b - (d*(a + b*x))/
(c + d*x))) + (2*B*d*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(
a + b*x))/(b*(c + d*x))])/b^3 - (2*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b^3 + (2*B^2*d*n^2*PolyLog[2, (d
*(a + b*x))/(b*(c + d*x))])/b^3 + (4*B*d*n*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b^3 + (4*B^2*d*n^2*PolyLo
g[3, (b*(c + d*x))/(d*(a + b*x))])/b^3))/g^2
```

### 3.173.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

3.173. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.173.4 Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^2}dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

## 3.173.5 Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x
, algorithm="fricas")
```

output
```
integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x
^2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(
A*B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log(e*((b*x + a)/(d*x + c
))^n))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

### 3.173.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
2,x)
```

output
```
Timed out
```

### 3.173.7 Maxima [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x
, algorithm="maxima")
```

output
```
-2*A*B*c^2*i^2*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d
)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - A^2*(a^2/(b^4*g^2*x + a*b
^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2))*d^2*i^2 + 2*A^2*c*d*i^
2*(a/(b^3*g^2*x + a*b^2*g^2) + log(b*x + a)/(b^2*g^2)) - 2*A*B*c^2*i^2*log
(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^2*g^2*x + a*b*g^2) - A^2*c^2*i^2/(b
^2*g^2*x + a*b*g^2) + (B^2*b^2*d^2*i^2*x^2 + B^2*a*b*d^2*i^2*x - (b^2*c^2*
i^2 - 2*a*b*c*d*i^2 + a^2*d^2*i^2)*B^2 + 2*((b^2*c*d*i^2 - a*b*d^2*i^2)*B^
2*x + (a*b*c*d*i^2 - a^2*d^2*i^2)*B^2)*log(b*x + a))*log((d*x + c)^n)^2/(b
^4*g^2*x + a*b^3*g^2) - integrate(-(B^2*b^3*c^3*i^2*log(e)^2 + (B^2*b^3*d^
3*i^2*log(e)^2 + 2*A*B*b^3*d^3*i^2*log(e))*x^3 + 3*(B^2*b^3*c*d^2*i^2*log(
e)^2 + 2*A*B*b^3*c*d^2*i^2*log(e))*x^2 + (B^2*b^3*d^3*i^2*x^3 + 3*B^2*b^3*
c*d^2*i^2*x^2 + 3*B^2*b^3*c^2*d*i^2*x + B^2*b^3*c^3*i^2)*log((b*x + a)^n)^
2 + (3*B^2*b^3*c^2*d*i^2*log(e)^2 + 4*A*B*b^3*c^2*d*i^2*log(e))*x + 2*(B^2
*b^3*c^3*i^2*log(e) + (B^2*b^3*d^3*i^2*log(e) + A*B*b^3*d^3*i^2)*x^3 + 3*(
B^2*b^3*c*d^2*i^2*log(e) + A*B*b^3*c*d^2*i^2)*x^2 + (3*B^2*b^3*c^2*d*i^2*l
og(e) + 2*A*B*b^3*c^2*d*i^2)*x)*log((b*x + a)^n) - 2*((A*B*b^3*d^3*i^2 + (
i^2*n + i^2*log(e))*B^2*b^3*d^3)*x^3 - (a*b^2*c^2*d*i^2*n - 2*a^2*b*c*d^2*
i^2*n + a^3*d^3*i^2*n - b^3*c^3*i^2*log(e))*B^2 + (3*A*B*b^3*c*d^2*i^2 + (
2*a*b^2*d^3*i^2*n + 3*b^3*c*d^2*i^2*log(e))*B^2)*x^2 + (2*A*B*b^3*c^2*d*i^
2 + (2*a*b^2*c*d^2*i^2*n - (i^2*n - 3*i^2*log(e))*b^3*c^2*d)*B^2)*x + 2...
```

### 3.173.8 Giac [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x
, algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x
+ a*g)^2, x)
```

### 3.173.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{\left(c\,i+d\,i\,x\right)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{\left(a\,g+b\,g\,x\right)^2}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^2,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^2, x)
```

# 3.174 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$



## 3.174.1 Optimal result

Integrand size = 45, antiderivative size = 417

$$\begin{aligned}
&\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx \\
&= -\frac{2B^2di^2n^2(c+dx)}{b^2g^3(a+bx)}-\frac{B^2i^2n^2(c+dx)^2}{4bg^3(a+bx)^2}-\frac{2Bdi^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^2g^3(a+bx)} \\
&\quad-\frac{Bi^2n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2bg^3(a+bx)^2}-\frac{di^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^2g^3(a+bx)} \\
&\quad-\frac{i^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2bg^3(a+bx)^2}-\frac{d^2i^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} \\
&\quad+\frac{2Bd^2i^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3} \\
&\quad+\frac{2B^2d^2i^2n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^3g^3}
\end{aligned}$$

output
```
-2*B^2*d*i^2*n^2*(d*x+c)/b^2/g^3/(b*x+a)-1/4*B^2*i^2*n^2*(d*x+c)^2/b/g^3/(
b*x+a)^2-2*B*d*i^2*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^3/(b*x+
a)-1/2*B*i^2*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^3/(b*x+a)^2-d
*i^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2/g^3/(b*x+a)-1/2*i^2*(d*
x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b/g^3/(b*x+a)^2-d^2*i^2*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^3/g^3+2*B*d^2*i^2*n*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^3/g^3+2*B^
2*d^2*i^2*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^3/g^3
```

### 3.174.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3662 vs. $2(417) = 834$.

Time = 4.48 (sec) , antiderivative size = 3662, normalized size of antiderivative = 8.78

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^3,x]
```

output
```
(i^2*((-6*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a
 + b*x)/(c + d*x)])^2)/(a + b*x)^2 + (24*d*(-(b*c) + a*d)*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2)/(a + b*x) + 12*d^
2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(
c + d*x)])^2 + (6*b^2*B*c^2*n*(b^2*c^2 - 4*a*b*c*d + a^2*d^2 - 2*b^2*c*d*x
 - 2*a*b*d^2*x - 2*b^2*d^2*x^2 + 2*d^2*(a + b*x)^2*Log[c/d + x] - 2*d^2*(a
 + b*x)^2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 2*b^2*c^2*Log[(a + b*x)/(c +
 d*x)] - 4*a*b*c*d*Log[(a + b*x)/(c + d*x)] + 2*a^2*d^2*Log[(a + b*x)/(c +
 d*x)])*(-A - B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d*
x)]))/((b*c - a*d)^2*(a + b*x)^2) + (12*b*B*c*d*n*(-A - B*Log[e*((a + b*x)
/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d*x)])*(3*a*b^2*c^2 - 4*a^2*b*c*d
+ a^3*d^2 + 4*b^3*c^2*x - 6*a*b^2*c*d*x + 2*a^2*b*d^2*x - 2*d*(-2*b*c + a*
d)*(a + b*x)^2*Log[a + b*x] + 2*(b*c - a*d)^2*(a + 2*b*x)*Log[(a + b*x)/(c
 + d*x)] - 4*a^2*b*c*d*Log[c + d*x] + 2*a^3*d^2*Log[c + d*x] - 8*a*b^2*c*d
*x*Log[c + d*x] + 4*a^2*b*d^2*x*Log[c + d*x] - 4*b^3*c*d*x^2*Log[c + d*x]
+ 2*a*b^2*d^2*x^2*Log[c + d*x]))/((b*c - a*d)^2*(a + b*x)^2) + (3*b^2*B^2*
c^2*n^2*(-(b*c - a*d)^2 + 6*d*(b*c - a*d)*(a + b*x) + 6*d^2*(a + b*x)^2*Lo
g[a + b*x] - 2*d^2*(a + b*x)^2*Log[a + b*x]^2 - 2*(b*c - a*d)^2*Log[(a + b
*x)/(c + d*x)] + 4*d*(b*c - a*d)*(a + b*x)*Log[(a + b*x)/(c + d*x)] + 4*d^
2*(a + b*x)^2*Log[a + b*x]*Log[(a + b*x)/(c + d*x)] - 2*(b*c - a*d)^2*L...
```

### 3.174.3 Rubi [A] (verified)

Time = 1.39 (sec) , antiderivative size = 353, normalized size of antiderivative = 0.85, number of steps used = 11, number of rules used = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2961, 2780, 2742, 2741, 2780, 2742, 2741, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3} \, dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i^2 \int \frac{(c+dx)^3 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^3}$$

$$\Big\downarrow \text{2780}$$

$$\frac{i^2 \left(\frac{\int \frac{(c+dx)^3 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d \int \frac{(c+dx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^3}$$

$$\Big\downarrow \text{2742}$$

$$\frac{i^2 \left(\frac{Bn \int \frac{(c+dx)^3 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{(c+dx)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}}{b}+\frac{d \int \frac{(c+dx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^3}$$

$$\Big\downarrow \text{2741}$$

$$\frac{i^2 \left(\frac{d \int \frac{(c+dx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{Bn \left(-\frac{(c+dx)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}\right)-\frac{(c+dx)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}}{b}\right)}{g^3}$$

$$\Big\downarrow \text{2780}$$

$$\frac{i^2\left(\frac{d\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{Bn\left(-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx}{4(a+bx)}\right.}{b}}{g^3}$$

$\downarrow$ 2742

$$\frac{i^2\left(\frac{d\left(\frac{2Bn\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{Bn\left(-\frac{(c+dx}{}\right.}{}}{g^3}$$

$\downarrow$ 2741

$$\frac{i^2\left(\frac{d\left(\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{b}\right)}{b}+\frac{Bn\left(-\right.}{}}{g^3}$$

$\downarrow$ 2779

## 3.174. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

$$i^2\left(\frac{d\left(\frac{d\left(\frac{2Bn\int \frac{\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}}{b}d\frac{a+bx}{c+dx}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}\right)}{b}+\frac{2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{a+bx}\right.}{}\right)}{b}\right.}{g}$$

$\big\downarrow$ 2821

$$i^2\left(\frac{d\left(\frac{d\left(\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int \frac{(c+dx)\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}\right)}{b}\right.}{b}\right.$$

$\big\downarrow$ 7143

---

3.174. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

$$i^2\left(\frac{d\left(\frac{d\left(\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+Bn\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}\right)}{b}+\frac{2Bn\left(-\frac{(c+dx)}{\cdots}\right.}{\cdots}}{b}\right.$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^3,x]
```

output
```
(i^2*((-1/2*((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*
x)^2 + B*n*(-1/4*(B*n*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2)))/b + (d*((-(((c + d*x)*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)) + 2*B*n*(-((B*n*(c + d*x))/
(a + b*x)) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x))
)/b + (d*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x)
)/(d*(a + b*x))])/b) + (2*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Poly
Log[2, (b*(c + d*x))/(d*(a + b*x))] + B*n*PolyLog[3, (b*(c + d*x))/(d*(a +
 b*x))]))/b))/b))/b))/g^3
```

### 3.174.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.174.4 Maple [F]

$$\int \frac{(dix+ci)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^3}dx$$

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

output
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

### 3.174.5 Fricas [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x
, algorithm="fricas")
```

output
```
integral((A^2*d^2*i^2*x^2 + 2*A^2*c*d*i^2*x + A^2*c^2*i^2 + (B^2*d^2*i^2*x
^2 + 2*B^2*c*d*i^2*x + B^2*c^2*i^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(
A*B*d^2*i^2*x^2 + 2*A*B*c*d*i^2*x + A*B*c^2*i^2)*log(e*((b*x + a)/(d*x + c
))^n))/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

### 3.174.6 Sympy [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= \frac{i^2\left(\int \frac{A^2c^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{A^2d^2x^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{B^2c^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^3+3a^2bx+3ab^2x^2+b^3x^3}\,dx + \int \frac{2ABc^2\log\left(e\left(\frac{a}{c+dx}\right.\right.}{a^3+3a^2bx+3ab^2\ldots}\right.}{\ldots}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
3,x)
```

output
```
i**2*(Integral(A**2*c**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3),
x) + Integral(A**2*d**2*x**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**
3), x) + Integral(B**2*c**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a*
*3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*A*B*c**2*log
(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 +
b**3*x**3), x) + Integral(2*A**2*c*d*x/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2
+ b**3*x**3), x) + Integral(B**2*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral
(2*A*B*d**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3 + 3*a**2*b*
x + 3*a*b**2*x**2 + b**3*x**3), x) + Integral(2*B**2*c*d*x*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)**2/(a**3 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3
), x) + Integral(4*A*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a**3
 + 3*a**2*b*x + 3*a*b**2*x**2 + b**3*x**3), x))/g**3
```

### 3.174.7 Maxima [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x
, algorithm="maxima")
```

output
```
-A*B*c*d*i^2*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c - a*b^4*
d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)*g^3)
+ 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*
g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*
d^2)*g^3)) + 1/2*A*B*c^2*i^2*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*b^3*d)
*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3) + 2*
d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*log(d*x
 + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) + 1/2*A^2*d^2*i^2*((4*a*b
*x + 3*a^2)/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(
b^3*g^3)) - 2*(2*b*x + a)*A*B*c*d*i^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)/(b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - (2*b*x + a)*A^2*c*d*i^2/(
b^4*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - A*B*c^2*i^2*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2*
c^2*i^2/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*(4*(b^2*c*d*i^2 -
a*b*d^2*i^2)*B^2*x + (b^2*c^2*i^2 + 2*a*b*c*d*i^2 - 3*a^2*d^2*i^2)*B^2 - 2
*(B^2*b^2*d^2*i^2*x^2 + 2*B^2*a*b*d^2*i^2*x + B^2*a^2*d^2*i^2)*log(b*x + a
))*log((d*x + c)^n)^2/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) - integr
ate(-(3*B^2*b^3*c^2*d*i^2*x*log(e)^2 + B^2*b^3*c^3*i^2*log(e)^2 + (B^2*b^3
*d^3*i^2*log(e)^2 + 2*A*B*b^3*d^3*i^2*log(e))*x^3 + (3*B^2*b^3*c*d^2*i^2*l
og(e)^2 + 2*A*B*b^3*c*d^2*i^2*log(e))*x^2 + (B^2*b^3*d^3*i^2*x^3 + 3*B^...
```

### 3.174.8 Giac [F]

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x
, algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x
+ a*g)^3, x)
```

### 3.174.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^3}\,dx$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^3,x)
```

output
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^3, x)
```

# 3.175 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$



## 3.175.1 Optimal result

Integrand size = 45, antiderivative size = 157

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = -\frac{2B^2i^2n^2(c+dx)^3}{27(bc-ad)g^4(a+bx)^3} - \frac{2Bi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)g^4(a+bx)^3} - \frac{i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)g^4(a+bx)^3}$$

output
```
-2/27*B^2*i^2*n^2*(d*x+c)^3/(-a*d+b*c)/g^4/(b*x+a)^3-2/9*B*i^2*n*(d*x+c)^3
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/g^4/(b*x+a)^3-1/3*i^2*(d*x+c)^
3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/g^4/(b*x+a)^3
```

## 3.175.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.18 (sec) , antiderivative size = 1418, normalized size of antiderivative = 9.03

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^4,x]
```

output
```
-1/54*(i^2*(18*(b*c - a*d)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 54
*d*(b*c - a*d)^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 54*d
^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + B
*n*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d*(b*c - a*d)^2*(a + b
*x) - 15*B*d*(b*c - a*d)^2*n*(a + b*x) + 36*A*d^2*(b*c - a*d)*(a + b*x)^2
+ 66*B*d^2*(b*c - a*d)*n*(a + b*x)^2 + 36*A*d^3*(a + b*x)^3*Log[a + b*x] +
 66*B*d^3*n*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*n*(a + b*x)^3*Log[a + b*x]
^2 + 12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n] - 18*B*d*(b*c - a*d
)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^2*(b*c - a*d)*(a + b
*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^3*(a + b*x)^3*Log[a + b*x]*L
og[e*((a + b*x)/(c + d*x))^n] - 36*A*d^3*(a + b*x)^3*Log[c + d*x] - 66*B*d
^3*n*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*n*(a + b*x)^3*Log[(d*(a + b*x))/(
-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Log[e*((a + b*x)/(c + d
*x))^n]*Log[c + d*x] - 18*B*d^3*n*(a + b*x)^3*Log[c + d*x]^2 + 36*B*d^3*n*
(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d)] + 36*B*d^3*n*(a +
b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*B*d^3*n*(a + b*x)^3*P
olyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 54*B*d^2*n*(a + b*x)^2*(2*(b*c - a
*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*d*(a + b*x)*Log[a + b*x]*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*d*(a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])*Log[c + d*x] + 2*B*n*(b*c - a*d + d*(a + b*x)*Log[a +...
```

### 3.175.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 127, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2961, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4}dx$$

$$\Big\downarrow \ 2961$$

$$\frac{i^2\int \frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4(bc-ad)}$$

$$\Big\downarrow \text{2742}$$

$$\frac{i^2\left(\frac{2}{3}Bn\int\frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}\right)}{g^4(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{i^2\left(\frac{2}{3}Bn\left(-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}-\frac{Bn(c+dx)^3}{9(a+bx)^3}\right)-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}\right)}{g^4(bc-ad)}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^4,x]
```

output
```
(i^2*(-1/3*((c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x
)^3 + (2*B*n*(-1/9*(B*n*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Log
[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3)))/3))/((b*c - a*d)*g^4)
```

### 3.175.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

3.175. $\int\frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}dx$

### 3.175.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 824 vs. $2(151) = 302$.

Time = 9.98 (sec) , antiderivative size = 825, normalized size of antiderivative = 5.25

| method | result |
|---|---|
| parallelrisch | $-\frac{-54AB\,x^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^5c\,d^3i^2n-54ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^5c^2d^2i^2n-9B^2x^3\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^2b^5d^4i^2n-6B^2x^3\ln\left(e\left(\frac{bx+}{dx+}\right.\right.}{}$ |

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x,method
=_RETURNVERBOSE)
```

output
```
-1/27*(-54*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^3*i^2*n-54*A*B*x*ln(e
*((b*x+a)/(d*x+c))^n)*b^5*c^2*d^2*i^2*n-9*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n
)^2*b^5*d^4*i^2*n-6*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^4*i^2*n^2+6*B^
2*x^2*a*b^4*d^4*i^2*n^3-6*B^2*x^2*b^5*c*d^3*i^2*n^3+6*B^2*x*a^2*b^3*d^4*i^
2*n^3-6*B^2*x*b^5*c^2*d^2*i^2*n^3+27*A^2*x^2*a*b^4*d^4*i^2*n-27*A^2*x^2*b^
5*c*d^3*i^2*n-9*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^5*c^3*d*i^2*n-18*A*B*x^3
*ln(e*((b*x+a)/(d*x+c))^n)*b^5*d^4*i^2*n-27*B^2*x^2*ln(e*((b*x+a)/(d*x+c))
^n)^2*b^5*c*d^3*i^2*n-18*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c*d^3*i^2*n
^2+18*A*B*x^2*a*b^4*d^4*i^2*n^2-18*A*B*x^2*b^5*c*d^3*i^2*n^2-27*B^2*x*ln(e
*((b*x+a)/(d*x+c))^n)^2*b^5*c^2*d^2*i^2*n-18*B^2*x*ln(e*((b*x+a)/(d*x+c))^
n)*b^5*c^2*d^2*i^2*n^2+18*A*B*x*a^2*b^3*d^4*i^2*n^2-18*A*B*x*b^5*c^2*d^2*i
^2*n^2-18*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^5*c^3*d*i^2*n+6*A*B*a^3*b^2*d^4*
i^2*n^2-6*A*B*b^5*c^3*d*i^2*n^2-9*A^2*b^5*c^3*d*i^2*n-6*B^2*ln(e*((b*x+a)/
(d*x+c))^n)*b^5*c^3*d*i^2*n^2+27*A^2*x*a^2*b^3*d^4*i^2*n-27*A^2*x*b^5*c^2*
d^2*i^2*n+2*B^2*a^3*b^2*d^4*i^2*n^3-2*B^2*b^5*c^3*d*i^2*n^3+9*A^2*a^3*b^2*
d^4*i^2*n)/g^4/(b*x+a)^3/b^5/d/n/(a*d-b*c)
```

### 3.175.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 975 vs. $2(151) = 302$.

Time = 0.35 (sec) , antiderivative size = 975, normalized size of antiderivative = 6.21

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{2\left(B^2b^3c^3-B^2a^3d^3\right)i^2n^2+6\left(ABb^3c^3-ABa^3d^3\right)i^2n+9\left(A^2b^3c^3-A^2a^3d^3\right)i^2+3\left(2\left(B^2b^3cd^2-B^2ab^2\right.\right.}{}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x
, algorithm="fricas")
```

output
```
-1/27*(2*(B^2*b^3*c^3 - B^2*a^3*d^3)*i^2*n^2 + 6*(A*B*b^3*c^3 - A*B*a^3*d^
3)*i^2*n + 9*(A^2*b^3*c^3 - A^2*a^3*d^3)*i^2 + 3*(2*(B^2*b^3*c*d^2 - B^2*a
*b^2*d^3)*i^2*n^2 + 6*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*i^2*n + 9*(A^2*b^3*c
*d^2 - A^2*a*b^2*d^3)*i^2)*x^2 + 9*(3*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*i^2*
x^2 + 3*(B^2*b^3*c^2*d - B^2*a^2*b*d^3)*i^2*x + (B^2*b^3*c^3 - B^2*a^3*d^3
)*i^2)*log(e)^2 + 9*(B^2*b^3*d^3*i^2*n^2*x^3 + 3*B^2*b^3*c*d^2*i^2*n^2*x^2
 + 3*B^2*b^3*c^2*d*i^2*n^2*x + B^2*b^3*c^3*i^2*n^2)*log((b*x + a)/(d*x + c
))^2 + 3*(2*(B^2*b^3*c^2*d - B^2*a^2*b*d^3)*i^2*n^2 + 6*(A*B*b^3*c^2*d - A
*B*a^2*b*d^3)*i^2*n + 9*(A^2*b^3*c^2*d - A^2*a^2*b*d^3)*i^2)*x + 6*((B^2*b
^3*c^3 - B^2*a^3*d^3)*i^2*n + 3*(A*B*b^3*c^3 - A*B*a^3*d^3)*i^2 + 3*((B^2*
b^3*c*d^2 - B^2*a*b^2*d^3)*i^2*n + 3*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*i^2)*
x^2 + 3*((B^2*b^3*c^2*d - B^2*a^2*b*d^3)*i^2*n + 3*(A*B*b^3*c^2*d - A*B*a^
2*b*d^3)*i^2)*x + 3*(B^2*b^3*d^3*i^2*n*x^3 + 3*B^2*b^3*c*d^2*i^2*n*x^2 + 3
*B^2*b^3*c^2*d*i^2*n*x + B^2*b^3*c^3*i^2*n)*log((b*x + a)/(d*x + c)))*log(
e) + 6*(B^2*b^3*c^3*i^2*n^2 + 3*A*B*b^3*c^3*i^2*n + (B^2*b^3*d^3*i^2*n^2 +
 3*A*B*b^3*d^3*i^2*n)*x^3 + 3*(B^2*b^3*c*d^2*i^2*n^2 + 3*A*B*b^3*c*d^2*i^2
*n)*x^2 + 3*(B^2*b^3*c^2*d*i^2*n^2 + 3*A*B*b^3*c^2*d*i^2*n)*x)*log((b*x +
a)/(d*x + c)))/((b^7*c - a*b^6*d)*g^4*x^3 + 3*(a*b^6*c - a^2*b^5*d)*g^4*x^
2 + 3*(a^2*b^5*c - a^3*b^4*d)*g^4*x + (a^3*b^4*c - a^4*b^3*d)*g^4)
```

### 3.175.6 Sympy [F]

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$$
$$= \frac{i^2\left(\int \frac{A^2c^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{A^2d^2x^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \int \frac{B^2c^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{a^4+4a^3bx+6a^2b^2x^2+4ab^3x^3+b^4x^4}\,dx + \right.}{}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
4,x)
```

output
```
i**2*(Integral(A**2*c**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*
x**3 + b**4*x**4), x) + Integral(A**2*d**2*x**2/(a**4 + 4*a**3*b*x + 6*a**
2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B**2*c**2*log(e*(a
/(c + d*x) + b*x/(c + d*x))**n)**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 +
 4*a*b**3*x**3 + b**4*x**4), x) + Integral(2*A*B*c**2*log(e*(a/(c + d*x) +
 b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3
+ b**4*x**4), x) + Integral(2*A**2*c*d*x/(a**4 + 4*a**3*b*x + 6*a**2*b**2*
x**2 + 4*a*b**3*x**3 + b**4*x**4), x) + Integral(B**2*d**2*x**2*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4
*a*b**3*x**3 + b**4*x**4), x) + Integral(2*A*B*d**2*x**2*log(e*(a/(c + d*x
) + b*x/(c + d*x))**n)/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x*
*3 + b**4*x**4), x) + Integral(2*B**2*c*d*x*log(e*(a/(c + d*x) + b*x/(c +
d*x))**n)**2/(a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*
x**4), x) + Integral(4*A*B*c*d*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(
a**4 + 4*a**3*b*x + 6*a**2*b**2*x**2 + 4*a*b**3*x**3 + b**4*x**4), x))/g**
4
```

### 3.175.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5588 vs. 2(151) = 302.

Time = 0.52 (sec) , antiderivative size = 5588, normalized size of antiderivative = 35.59

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x
, algorithm="maxima")
```

output
```
-1/9*A*B*d^2*i^2*n*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4*c
^2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2*a
^3*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c^2
 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d +
 a^4*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) + 6
*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*c^
2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2 +
 a^2*d^3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b
^3*d^3)*g^4)) - 1/9*A*B*c^2*i^2*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d
+ 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*b^
4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3*(
a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*b^
2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d + 3
*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^3*
c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/9*A*B*c*d*i^2*n*((5*a*b^2*c
^2 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b^3*c
^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*d^2)
*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^2*b^
5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*c*d
+ a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 - 3*...
```

### 3.175.8 Giac [A] (verification not implemented)

Time = 3.77 (sec) , antiderivative size = 216, normalized size of antiderivative = 1.38

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$
$$-\frac{1}{27}\left(\frac{9\,(dx+c)^3B^2i^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)^3g^4}+\frac{6\,(B^2i^2n^2+3\,B^2i^2n\log(e)+3\,ABi^2n)(dx+c)^3\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^3g^4}+\frac{(2\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x
, algorithm="giac")
```

output
```
-1/27*(9*(d*x + c)^3*B^2*i^2*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)^3*g
^4) + 6*(B^2*i^2*n^2 + 3*B^2*i^2*n*log(e) + 3*A*B*i^2*n)*(d*x + c)^3*log((
b*x + a)/(d*x + c))/((b*x + a)^3*g^4) + (2*B^2*i^2*n^2 + 6*B^2*i^2*n*log(e
) + 9*B^2*i^2*log(e)^2 + 6*A*B*i^2*n + 18*A*B*i^2*log(e) + 9*A^2*i^2)*(d*x
 + c)^3/((b*x + a)^3*g^4))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.175.9 Mupad [B] (verification not implemented)

Time = 3.73 (sec) , antiderivative size = 1195, normalized size of antiderivative = 7.61

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx =$$

$$-\frac{x\left(9\,c\,A^2\,b^2\,d\,i^2+9\,a\,A^2\,b\,d^2\,i^2+6\,c\,A\,B\,b^2\,d\,i^2\,n+6\,a\,A\,B\,b\,d^2\,i^2\,n+2\,c\,B^2\,b^2\,d\,i^2\,n^2+2\,a\,B^2\,b\,d^2\,i^2\,n^2\right)}{}$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{a\left(-a\,n\,B^2\,d^2\,i^2+b\,c\,n\,B^2\,d\,i^2+2\,A\,a\,B\,d^2\,i^2+2\,A\,b\,c\,B\,d\,i^2\right)+x\left(b\left(-a\,n\,B^2\,d^2\right.\right.}{}\right.$$

$$+\frac{2\,B^2\,d^3\,i^2\left(x\left(b\left(\frac{a\,b^3\,g^4\,n\,(a\,d-b\,c)}{d}+\frac{b^3\,g^4\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{2\,d^2}\right)+\frac{2\,a\,b^4\,g^4\,n\,(a\,d-b\,c)}{d}+\frac{b^4\,g^4\,n\,(a\,d-b\,c)\,(3\,a\,d-b\,c)}{d^2}\right)+a\left(\right.\right.}{3\,b^3\,g^4\,(a\,d-b\,c)\,(3\,a^3\,b^3\,g^4+9\,a^2\,b^4\,g^4}$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{a\left(\frac{B^2\,c\,d\,i^2}{3\,b^2}+\frac{B^2\,a\,d^2\,i^2}{3\,b^3}\right)+x\left(b\left(\frac{B^2\,c\,d\,i^2}{3\,b^2}+\frac{B^2\,a\,d^2\,i^2}{3\,b^3}\right)+\frac{2\,B^2\,c\,d\,i^2}{3\,b}+\frac{2\,B^2\,a\,d^2\,i^2}{3\,b^2}\right)+\frac{B^2\,c^2}{3\,b}}{a^3\,g^4+3\,a^2\,b\,g^4\,x+3\,a\,b^2\,g^4\,x^2+b^3\,g^4\,x^3}\right.$$

$$\left.-\frac{B^2\,d^3\,i^2}{3\,b^3\,g^4\,(a\,d-b\,c)}\right)-\frac{B\,d^3\,i^2\,n\,\mathrm{atan}\left(\frac{\left(\frac{9\,c\,b^4\,g^4+9\,a\,d\,b^3\,g^4}{9\,b^3\,g^4}+2\,b\,d\,x\right)1\mathrm{i}}{a\,d-b\,c}\right)(3\,A+B\,n)\,4\mathrm{i}}{9\,b^3\,g^4\,(a\,d-b\,c)}$$

input

```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^4,x)
```

output
```
- (x*(9*A^2*a*b*d^2*i^2 + 9*A^2*b^2*c*d*i^2 + 2*B^2*a*b*d^2*i^2*n^2 + 2*B^
2*b^2*c*d*i^2*n^2 + 6*A*B*a*b*d^2*i^2*n + 6*A*B*b^2*c*d*i^2*n) + x^2*(9*A^
2*b^2*d^2*i^2 + 2*B^2*b^2*d^2*i^2*n^2 + 6*A*B*b^2*d^2*i^2*n) + 3*A^2*a^2*d
^2*i^2 + 3*A^2*b^2*c^2*i^2 + (2*B^2*a^2*d^2*i^2*n^2)/3 + (2*B^2*b^2*c^2*i^
2*n^2)/3 + 3*A^2*a*b*c*d*i^2 + 2*A*B*a^2*d^2*i^2*n + 2*A*B*b^2*c^2*i^2*n +
 (2*B^2*a*b*c*d*i^2*n^2)/3 + 2*A*B*a*b*c*d*i^2*n)/(9*a^3*b^3*g^4 + 9*b^6*g
^4*x^3 + 27*a^2*b^4*g^4*x + 27*a*b^5*g^4*x^2) - log(e*((a + b*x)/(c + d*x)
)^n)*((a*(2*A*B*a*d^2*i^2 - B^2*a*d^2*i^2*n + B^2*b*c*d*i^2*n + 2*A*B*b*c*
d*i^2) + x*(b*(2*A*B*a*d^2*i^2 - B^2*a*d^2*i^2*n + B^2*b*c*d*i^2*n + 2*A*B
*b*c*d*i^2) + 4*A*B*a*b*d^2*i^2 + 4*A*B*b^2*c*d*i^2 - 2*B^2*a*b*d^2*i^2*n
+ 2*B^2*b^2*c*d*i^2*n) + 2*A*B*b^2*c^2*i^2 - 2*B^2*a^2*d^2*i^2*n + 6*A*B*b
^2*d^2*i^2*x^2 + 2*B^2*a*b*c*d*i^2*n)/(3*a^3*b^3*g^4 + 3*b^6*g^4*x^3 + 9*a
^2*b^4*g^4*x + 9*a*b^5*g^4*x^2) + (2*B^2*d^3*i^2*(x*(b*((a*b^3*g^4*n*(a*d
- b*c))/d + (b^3*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2)) + (2*a*b^4*g^4*
n*(a*d - b*c))/d + (b^4*g^4*n*(a*d - b*c)*(3*a*d - b*c))/d^2) + a*((a*b^3*
g^4*n*(a*d - b*c))/d + (b^3*g^4*n*(a*d - b*c)*(3*a*d - b*c))/(2*d^2)) + (3
*b^5*g^4*n*x^2*(a*d - b*c))/d + (b^3*g^4*n*(a*d - b*c)*(3*a^2*d^2 + b^2*c^
2 - 3*a*b*c*d))/d^3))/(3*b^3*g^4*(a*d - b*c)*(3*a^3*b^3*g^4 + 3*b^6*g^4*x^
3 + 9*a^2*b^4*g^4*x + 9*a*b^5*g^4*x^2))) - log(e*((a + b*x)/(c + d*x))^n)^
2*((a*((B^2*c*d*i^2)/(3*b^2) + (B^2*a*d^2*i^2)/(3*b^3)) + x*(b*((B^2*c*...
```

3.175. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

# 3.176 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$



## 3.176.1 Optimal result

Integrand size = 45, antiderivative size = 319

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \frac{2B^2di^2n^2(c+dx)^3}{27(bc-ad)^2g^5(a+bx)^3} - \frac{bB^2i^2n^2(c+dx)^4}{32(bc-ad)^2g^5(a+bx)^4} + \frac{2Bdi^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^2g^5(a+bx)^3} - \frac{bBi^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{8(bc-ad)^2g^5(a+bx)^4} + \frac{di^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^2g^5(a+bx)^3} - \frac{bi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4(bc-ad)^2g^5(a+bx)^4}$$

output
```
2/27*B^2*d*i^2*n^2*(d*x+c)^3/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/32*b*B^2*i^2*n^2
*(d*x+c)^4/(-a*d+b*c)^2/g^5/(b*x+a)^4+2/9*B*d*i^2*n*(d*x+c)^3*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/8*b*B*i^2*n*(d*x+c)^4*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^5/(b*x+a)^4+1/3*d*i^2*(d*x+c)^
3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^5/(b*x+a)^3-1/4*b*i^2*(
d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^5/(b*x+a)^4
```

### 3.176.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.63 (sec) , antiderivative size = 1787, normalized size of antiderivative = 5.60

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^5,x]
```

output
```
-1/864*(i^2*(216*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 -
576*d*(-(b*c) + a*d)^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
+ 432*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2 + 32*B*d*n*(a + b*x)*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d)^3*n - 18*A*d
*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*n*(a + b*x) + 36*A*d^2*(b*
c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*n*(a + b*x)^2 + 36*A*d^3*(a +
b*x)^3*Log[a + b*x] + 66*B*d^3*n*(a + b*x)^3*Log[a + b*x] - 18*B*d^3*n*(a
+ b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n
] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d
^2*(b*c - a*d)*(a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 36*B*d^3*(a +
b*x)^3*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n] - 36*A*d^3*(a + b*x)^3*
Log[c + d*x] - 66*B*d^3*n*(a + b*x)^3*Log[c + d*x] + 36*B*d^3*n*(a + b*x)^
3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*(a + b*x)^3*Lo
g[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] - 18*B*d^3*n*(a + b*x)^3*Log[c +
 d*x]^2 + 36*B*d^3*n*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*x))/(b*c - a*d
)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 36*
B*d^3*n*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 3*B*n*(36*A*(
b*c - a*d)^4 + 9*B*(b*c - a*d)^4*n + 48*A*d*(-(b*c) + a*d)^3*(a + b*x) + 2
8*B*d*(-(b*c) + a*d)^3*n*(a + b*x) + 72*A*d^2*(b*c - a*d)^2*(a + b*x)^2 +
78*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2 + 144*A*d^3*(-(b*c) + a*d)*(a + b*...
```

### 3.176.3 Rubi [A] (verified)

Time = 0.49 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.75, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

---

3.176. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^5}\,dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{i^2\int \frac{(c+dx)^5\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^5}d\frac{a+bx}{c+dx}}{g^5(bc-ad)^2}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{i^2\int \left(\frac{b(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^5}-\frac{d(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}}{g^5(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{i^2\left(-\frac{b(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4(a+bx)^4}-\frac{bBn(c+dx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{8(a+bx)^4}+\frac{d(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}+\frac{2Bdn(c+dx)^3\left(\right.}{9}\right.}{g^5(bc-ad)^2}$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^5,x]
```

output
```
(i^2*((2*B^2*d*n^2*(c + d*x)^3)/(27*(a + b*x)^3) - (b*B^2*n^2*(c + d*x)^4)
/(32*(a + b*x)^4) + (2*B*d*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n]))/(9*(a + b*x)^3) - (b*B*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(8*(a + b*x)^4) + (d*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^2)/(3*(a + b*x)^3) - (b*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^2)/(4*(a + b*x)^4)))/((b*c - a*d)^2*g^5)
```

### 3.176.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

---

3.176. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.176.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1883 vs. $2(307) = 614$.

Time = 27.47 (sec) , antiderivative size = 1884, normalized size of antiderivative = 5.91

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1884 |

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x,method
=_RETURNVERBOSE)
```

output
```
1/864*(-1152*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^4*d*i^2*n+144*A*B*x^4
*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c*d^4*i^2*n-864*A*B*x^2*ln(e*((b*x+a)/(d*
x+c))^n)*a^6*b*c^3*d^2*i^2*n-192*A*B*x^4*a^3*b^4*c^4*d*i^2*n^2+144*B^2*x^3
*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^2*d^3*i^2*n^2+576*A*B*x^3*ln(e*((b*x+a)
/(d*x+c))^n)*a^7*c*d^4*i^2*n+144*A*B*x^3*a^6*b*c^2*d^3*i^2*n^2-768*A*B*x^3
*a^4*b^3*c^4*d*i^2*n^2-432*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b*c^3*d
^2*i^2*n-72*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^3*d^2*i^2*n^2+1728*A
*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^2*d^3*i^2*n-72*A*B*x^2*a^6*b*c^3*d^
2*i^2*n^2-1152*A*B*x^2*a^5*b^2*c^4*d*i^2*n^2-576*B^2*x*ln(e*((b*x+a)/(d*x+
c))^n)^2*a^6*b*c^4*d*i^2*n-240*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c^4*d
*i^2*n^2+1728*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*c^3*d^2*i^2*n-1008*A*B*x
*a^6*b*c^4*d*i^2*n^2+72*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b*c*d^4*i^
2*n+84*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b*c*d^4*i^2*n^2+84*A*B*x^4*a^
6*b*c*d^4*i^2*n^2+108*A*B*x^4*a^2*b^5*c^5*i^2*n^2+288*B^2*x^3*ln(e*((b*x+a
)/(d*x+c))^n)^2*a^7*c*d^4*i^2*n+192*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^7*
c*d^4*i^2*n^2+84*B^2*x^3*a^6*b*c^2*d^3*i^2*n^3-256*B^2*x^3*a^4*b^3*c^4*d*i
^2*n^3+72*A^2*x^4*a^6*b*c*d^4*i^2*n-288*A^2*x^4*a^3*b^4*c^4*d*i^2*n+192*A*
B*x^3*a^7*c*d^4*i^2*n^2+432*A*B*x^3*a^3*b^4*c^5*i^2*n^2+864*B^2*x^2*ln(e*(
(b*x+a)/(d*x+c))^n)^2*a^7*c^2*d^3*i^2*n+576*B^2*x^2*ln(e*((b*x+a)/(d*x+c))
^n)*a^7*c^2*d^3*i^2*n^2+30*B^2*x^2*a^6*b*c^3*d^2*i^2*n^3-384*B^2*x^2*a^...
```

### 3.176.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1729 vs. $2(307) = 614$.

Time = 0.38 (sec) , antiderivative size = 1729, normalized size of antiderivative = 5.42

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x
, algorithm="fricas")
```

output
```
-1/864*((27*B^2*b^4*c^4 - 64*B^2*a*b^3*c^3*d + 37*B^2*a^4*d^4)*i^2*n^2 + 1
2*(9*A*B*b^4*c^4 - 16*A*B*a*b^3*c^3*d + 7*A*B*a^4*d^4)*i^2*n - 12*(7*(B^2*
b^4*c*d^3 - B^2*a*b^3*d^4)*i^2*n^2 + 12*(A*B*b^4*c*d^3 - A*B*a*b^3*d^4)*i^
2*n)*x^3 + 72*(3*A^2*b^4*c^4 - 4*A^2*a*b^3*c^3*d + A^2*a^4*d^4)*i^2 - 6*((
5*B^2*b^4*c^2*d^2 + 32*B^2*a*b^3*c*d^3 - 37*B^2*a^2*b^2*d^4)*i^2*n^2 - 12*
(A*B*b^4*c^2*d^2 - 8*A*B*a*b^3*c*d^3 + 7*A*B*a^2*b^2*d^4)*i^2*n - 72*(A^2*
b^4*c^2*d^2 - 2*A^2*a*b^3*c*d^3 + A^2*a^2*b^2*d^4)*i^2)*x^2 + 72*(6*(B^2*b
^4*c^2*d^2 - 2*B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*i^2*x^2 + 4*(2*B^2*b^4*c
^3*d - 3*B^2*a*b^3*c^2*d^2 + B^2*a^3*b*d^4)*i^2*x + (3*B^2*b^4*c^4 - 4*B^2
*a*b^3*c^3*d + B^2*a^4*d^4)*i^2)*log(e)^2 - 72*(B^2*b^4*d^4*i^2*n^2*x^4 +
4*B^2*a*b^3*d^4*i^2*n^2*x^3 - 6*(B^2*b^4*c^2*d^2 - 2*B^2*a*b^3*c*d^3)*i^2*
n^2*x^2 - 4*(2*B^2*b^4*c^3*d - 3*B^2*a*b^3*c^2*d^2)*i^2*n^2*x - (3*B^2*b^4
*c^4 - 4*B^2*a*b^3*c^3*d)*i^2*n^2)*log((b*x + a)/(d*x + c))^2 + 4*((11*B^2
*b^4*c^3*d - 48*B^2*a*b^3*c^2*d^2 + 37*B^2*a^3*b*d^4)*i^2*n^2 + 12*(5*A*B*
b^4*c^3*d - 12*A*B*a*b^3*c^2*d^2 + 7*A*B*a^3*b*d^4)*i^2*n + 72*(2*A^2*b^4*
c^3*d - 3*A^2*a*b^3*c^2*d^2 + A^2*a^3*b*d^4)*i^2)*x - 12*(12*(B^2*b^4*c*d^
3 - B^2*a*b^3*d^4)*i^2*n*x^3 - (9*B^2*b^4*c^4 - 16*B^2*a*b^3*c^3*d + 7*B^2
*a^4*d^4)*i^2*n - 12*(3*A*B*b^4*c^4 - 4*A*B*a*b^3*c^3*d + A*B*a^4*d^4)*i^2
 - 6*((B^2*b^4*c^2*d^2 - 8*B^2*a*b^3*c*d^3 + 7*B^2*a^2*b^2*d^4)*i^2*n + 12
*(A*B*b^4*c^2*d^2 - 2*A*B*a*b^3*c*d^3 + A*B*a^2*b^2*d^4)*i^2)*x^2 - 4*(...
```

### 3.176.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
5,x)
```

output
```
Timed out
```

### 3.176.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 8087 vs. 2(307) = 614.

Time = 0.73 (sec) , antiderivative size = 8087, normalized size of antiderivative = 25.35

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x
, algorithm="maxima")
```

output
```
1/24*A*B*c^2*i^2*n*((12*b^3*d^3*x^3 - 3*b^3*c^3 + 13*a*b^2*c^2*d - 23*a^2*
b*c*d^2 + 25*a^3*d^3 - 6*(b^3*c*d^2 - 7*a*b^2*d^3)*x^2 + 4*(b^3*c^2*d - 5*
a*b^2*c*d^2 + 13*a^2*b*d^3)*x)/((b^8*c^3 - 3*a*b^7*c^2*d + 3*a^2*b^6*c*d^2
 - a^3*b^5*d^3)*g^5*x^4 + 4*(a*b^7*c^3 - 3*a^2*b^6*c^2*d + 3*a^3*b^5*c*d^2
 - a^4*b^4*d^3)*g^5*x^3 + 6*(a^2*b^6*c^3 - 3*a^3*b^5*c^2*d + 3*a^4*b^4*c*d
^2 - a^5*b^3*d^3)*g^5*x^2 + 4*(a^3*b^5*c^3 - 3*a^4*b^4*c^2*d + 3*a^5*b^3*c
*d^2 - a^6*b^2*d^3)*g^5*x + (a^4*b^4*c^3 - 3*a^5*b^3*c^2*d + 3*a^6*b^2*c*d
^2 - a^7*b*d^3)*g^5) + 12*d^4*log(b*x + a)/((b^5*c^4 - 4*a*b^4*c^3*d + 6*a
^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4)*g^5) - 12*d^4*log(d*x + c)/(
(b^5*c^4 - 4*a*b^4*c^3*d + 6*a^2*b^3*c^2*d^2 - 4*a^3*b^2*c*d^3 + a^4*b*d^4
)*g^5)) - 1/72*A*B*d^2*i^2*n*((13*a^2*b^3*c^3 - 75*a^3*b^2*c^2*d + 33*a^4*
b*c*d^2 - 7*a^5*d^3 - 12*(6*b^5*c^2*d - 4*a*b^4*c*d^2 + a^2*b^3*d^3)*x^3 +
 6*(6*b^5*c^3 - 46*a*b^4*c^2*d + 29*a^2*b^3*c*d^2 - 7*a^3*b^2*d^3)*x^2 + 4
*(10*a*b^4*c^3 - 63*a^2*b^3*c^2*d + 33*a^3*b^2*c*d^2 - 7*a^4*b*d^3)*x)/((b
^10*c^3 - 3*a*b^9*c^2*d + 3*a^2*b^8*c*d^2 - a^3*b^7*d^3)*g^5*x^4 + 4*(a*b^
9*c^3 - 3*a^2*b^8*c^2*d + 3*a^3*b^7*c*d^2 - a^4*b^6*d^3)*g^5*x^3 + 6*(a^2*
b^8*c^3 - 3*a^3*b^7*c^2*d + 3*a^4*b^6*c*d^2 - a^5*b^5*d^3)*g^5*x^2 + 4*(a^
3*b^7*c^3 - 3*a^4*b^6*c^2*d + 3*a^5*b^5*c*d^2 - a^6*b^4*d^3)*g^5*x + (a^4*
b^6*c^3 - 3*a^5*b^5*c^2*d + 3*a^6*b^4*c*d^2 - a^7*b^3*d^3)*g^5) - 12*(6*b^
2*c^2*d^2 - 4*a*b*c*d^3 + a^2*d^4)*log(b*x + a)/((b^7*c^4 - 4*a*b^6*c^3...
```

### 3.176.8 Giac [A] (verification not implemented)

Time = 4.55 (sec) , antiderivative size = 541, normalized size of antiderivative = 1.70

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx =$$
$$-\frac{1}{864}\left(\frac{72\left(3\,B^2bi^2n^2-\frac{4\,(bx+a)B^2di^2n^2}{dx+c}\right)\log\left(\frac{bx+a}{dx+c}\right)^2}{\frac{(bx+a)^4bcg^5}{(dx+c)^4}-\frac{(bx+a)^4adg^5}{(dx+c)^4}}+\frac{12\left(9\,B^2bi^2n^2-\frac{16\,(bx+a)B^2di^2n^2}{dx+c}+36\,B^2bi^2n\log\left(e\right)\right.}{\frac{(bx+a}{(dx}}\right.$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^5,x
, algorithm="giac")
```

output
```
-1/864*(72*(3*B^2*b*i^2*n^2 - 4*(b*x + a)*B^2*d*i^2*n^2/(d*x + c))*log((b*
x + a)/(d*x + c))^2/((b*x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5
/(d*x + c)^4) + 12*(9*B^2*b*i^2*n^2 - 16*(b*x + a)*B^2*d*i^2*n^2/(d*x + c)
 + 36*B^2*b*i^2*n*log(e) - 48*(b*x + a)*B^2*d*i^2*n*log(e)/(d*x + c) + 36*
A*B*b*i^2*n - 48*(b*x + a)*A*B*d*i^2*n/(d*x + c))*log((b*x + a)/(d*x + c))
/((b*x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5/(d*x + c)^4) + (27
*B^2*b*i^2*n^2 - 64*(b*x + a)*B^2*d*i^2*n^2/(d*x + c) + 108*B^2*b*i^2*n*lo
g(e) - 192*(b*x + a)*B^2*d*i^2*n*log(e)/(d*x + c) + 216*B^2*b*i^2*log(e)^2
 - 288*(b*x + a)*B^2*d*i^2*log(e)^2/(d*x + c) + 108*A*B*b*i^2*n - 192*(b*x
 + a)*A*B*d*i^2*n/(d*x + c) + 432*A*B*b*i^2*log(e) - 576*(b*x + a)*A*B*d*i
^2*log(e)/(d*x + c) + 216*A^2*b*i^2 - 288*(b*x + a)*A^2*d*i^2/(d*x + c))/(
(b*x + a)^4*b*c*g^5/(d*x + c)^4 - (b*x + a)^4*a*d*g^5/(d*x + c)^4))*(b*c/(
b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.176.9 Mupad [B] (verification not implemented)

Time = 5.31 (sec) , antiderivative size = 1934, normalized size of antiderivative = 6.06

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^5,x)
```

output
```
- log(e*((a + b*x)/(c + d*x))^n)*((a*(A*B*a*d^2*i^2 - (B^2*a*d^2*i^2*n)/2
+ (B^2*b*c*d*i^2*n)/2 + 2*A*B*b*c*d*i^2) + x*(b*(A*B*a*d^2*i^2 - (B^2*a*d^
2*i^2*n)/2 + (B^2*b*c*d*i^2*n)/2 + 2*A*B*b*c*d*i^2) + 3*A*B*a*b*d^2*i^2 +
6*A*B*b^2*c*d*i^2 - (3*B^2*a*b*d^2*i^2*n)/2 + (3*B^2*b^2*c*d*i^2*n)/2) + 3
*A*B*b^2*c^2*i^2 - B^2*a^2*d^2*i^2*n + (B^2*b^2*c^2*i^2*n)/2 + 6*A*B*b^2*d
^2*i^2*x^2 + (B^2*a*b*c*d*i^2*n)/2)/(6*a^4*b^3*g^5 + 6*b^7*g^5*x^4 + 24*a^
3*b^4*g^5*x + 24*a*b^6*g^5*x^3 + 36*a^2*b^5*g^5*x^2) + (B^2*d^4*i^2*(x^2*(
b*(b*((3*a*b^3*g^5*n*(a*d - b*c))/(2*d) + (b^3*g^5*n*(a*d - b*c)*(4*a*d -
b*c))/(2*d^2)) + (3*a*b^4*g^5*n*(a*d - b*c))/d + (b^4*g^5*n*(a*d - b*c)*(4
*a*d - b*c))/d^2) + (9*a*b^5*g^5*n*(a*d - b*c))/(2*d) + (3*b^5*g^5*n*(a*d
- b*c)*(4*a*d - b*c))/(2*d^2)) + a*(a*((3*a*b^3*g^5*n*(a*d - b*c))/(2*d) +
 (b^3*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(2*d^2)) + (b^3*g^5*n*(a*d - b*c)*(
6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3)) + x*(a*(b*((3*a*b^3*g^5*n*(a*d
- b*c))/(2*d) + (b^3*g^5*n*(a*d - b*c)*(4*a*d - b*c))/(2*d^2)) + (3*a*b^4*
g^5*n*(a*d - b*c))/d + (b^4*g^5*n*(a*d - b*c)*(4*a*d - b*c))/d^2) + b*(a*(
(3*a*b^3*g^5*n*(a*d - b*c))/(2*d) + (b^3*g^5*n*(a*d - b*c)*(4*a*d - b*c))/
(2*d^2)) + (b^3*g^5*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^
3)) + (3*b^4*g^5*n*(a*d - b*c)*(6*a^2*d^2 + b^2*c^2 - 4*a*b*c*d))/(2*d^3))
 + (3*b^3*g^5*n*(a*d - b*c)*(4*a^3*d^3 - b^3*c^3 + 4*a*b^2*c^2*d - 6*a^2*b
*c*d^2))/(2*d^4) + (6*b^6*g^5*n*x^3*(a*d - b*c))/d))/(6*b^3*g^5*(a^2*d^...
```

3.176. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^5}\,dx$

# 3.177 $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx$



## 3.177.1 Optimal result

Integrand size = 45, antiderivative size = 493

$$\begin{aligned}\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx &= -\frac{2B^2d^2i^2n^2(c+dx)^3}{27(bc-ad)^3g^6(a+bx)^3} \\ &+ \frac{bB^2di^2n^2(c+dx)^4}{16(bc-ad)^3g^6(a+bx)^4} \\ &- \frac{2b^2B^2i^2n^2(c+dx)^5}{125(bc-ad)^3g^6(a+bx)^5} \\ &- \frac{2Bd^2i^2n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^3g^6(a+bx)^3} \\ &+ \frac{bBdi^2n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4(bc-ad)^3g^6(a+bx)^4} \\ &- \frac{2b^2Bi^2n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{25(bc-ad)^3g^6(a+bx)^5} \\ &- \frac{d^2i^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^3g^6(a+bx)^3} \\ &+ \frac{bdi^2(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^6(a+bx)^4} \\ &- \frac{b^2i^2(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5(bc-ad)^3g^6(a+bx)^5}\end{aligned}$$

output
```
-2/27*B^2*d^2*i^2*n^2*(d*x+c)^3/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/16*b*B^2*d*i^
2*n^2*(d*x+c)^4/(-a*d+b*c)^3/g^6/(b*x+a)^4-2/125*b^2*B^2*i^2*n^2*(d*x+c)^5
/(-a*d+b*c)^3/g^6/(b*x+a)^5-2/9*B*d^2*i^2*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(
d*x+c))^n))/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/4*b*B*d*i^2*n*(d*x+c)^4*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^6/(b*x+a)^4-2/25*b^2*B*i^2*n*(d*x+c)
^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^6/(b*x+a)^5-1/3*d^2*i^2*
(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^6/(b*x+a)^3+1/2
*b*d*i^2*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^6/(b*x
+a)^4-1/5*b^2*i^2*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3
/g^6/(b*x+a)^5
```

### 3.177.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 1.80 (sec) , antiderivative size = 2112, normalized size of antiderivative = 4.28

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^6,x]
```

output
```
-1/54000*(i^2*(10800*(b*c - a*d)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2 + 27000*d*(b*c - a*d)^4*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2 - 18000*d^2*(-(b*c) + a*d)^3*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^2 + 1000*B*d^2*n*(a + b*x)^2*(12*A*(b*c - a*d)^3 + 4*B*(b*c - a*d
)^3*n - 18*A*d*(b*c - a*d)^2*(a + b*x) - 15*B*d*(b*c - a*d)^2*n*(a + b*x)
+ 36*A*d^2*(b*c - a*d)*(a + b*x)^2 + 66*B*d^2*(b*c - a*d)*n*(a + b*x)^2 +
36*A*d^3*(a + b*x)^3*Log[a + b*x] + 66*B*d^3*n*(a + b*x)^3*Log[a + b*x] -
18*B*d^3*n*(a + b*x)^3*Log[a + b*x]^2 + 12*B*(b*c - a*d)^3*Log[e*((a + b*x
)/(c + d*x))^n] - 18*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x)/(c + d*x
))^n] + 36*B*d^2*(b*c - a*d)*(a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] +
36*B*d^3*(a + b*x)^3*Log[a + b*x]*Log[e*((a + b*x)/(c + d*x))^n] - 36*A*d^
3*(a + b*x)^3*Log[c + d*x] - 66*B*d^3*n*(a + b*x)^3*Log[c + d*x] + 36*B*d^
3*n*(a + b*x)^3*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 36*B*d^3*
(a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n]*Log[c + d*x] - 18*B*d^3*n*(a +
b*x)^3*Log[c + d*x]^2 + 36*B*d^3*n*(a + b*x)^3*Log[a + b*x]*Log[(b*(c + d*
x))/(b*c - a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (d*(a + b*x))/(-(b*c)
 + a*d)] + 36*B*d^3*n*(a + b*x)^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) +
 375*B*d*n*(a + b*x)*(36*A*(b*c - a*d)^4 + 9*B*(b*c - a*d)^4*n + 48*A*d*(-
(b*c) + a*d)^3*(a + b*x) + 28*B*d*(-(b*c) + a*d)^3*n*(a + b*x) + 72*A*d^2*
(b*c - a*d)^2*(a + b*x)^2 + 78*B*d^2*(b*c - a*d)^2*n*(a + b*x)^2 + 144*...
```

### 3.177.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 366, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^6}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i^2\int \frac{(c+dx)^6\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^6}d\frac{a+bx}{c+dx}}{g^6(bc-ad)^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{i^2 \int \left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^6}{(a+bx)^6}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^5}{(a+bx)^5}+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^4}{(a+bx)^4}\right)d\frac{a+bx}{c+dx}}{g^6(bc-ad)^3}$$

$$\Big\downarrow \text{2009}$$

$$i^2\left(-\frac{b^2(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5(a+bx)^5}-\frac{2b^2Bn(c+dx)^5\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{25(a+bx)^5}-\frac{d^2(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}-\frac{2Bd^2n(c+d}{}\right.$$

input
```
Int[((c*i + d*i*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^6,x]
```

output
```
(i^2*((-2*B^2*d^2*n^2*(c + d*x)^3)/(27*(a + b*x)^3) + (b*B^2*d*n^2*(c + d*
x)^4)/(16*(a + b*x)^4) - (2*b^2*B^2*n^2*(c + d*x)^5)/(125*(a + b*x)^5) - (
2*B*d^2*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)
^3) + (b*B*d*n*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*(a +
 b*x)^4) - (2*b^2*B*n*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(25*(a + b*x)^5) - (d^2*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(3*(a + b*x)^3) + (b*d*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2)/(2*(a + b*x)^4) - (b^2*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2)/(5*(a + b*x)^5)))/((b*c - a*d)^3*g^6)
```

### 3.177.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

3.177. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.177.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2527 vs. $2(475) = 950$.

Time = 56.95 (sec) , antiderivative size = 2528, normalized size of antiderivative = 5.13

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 2528 |

input
```
int((d*i*x+c*i)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x,method
=_RETURNVERBOSE)
```

output
```
-1/54000*(9000*A*B*x^2*a*b^8*c^2*d^4*i^2*n^2-54000*B^2*x*ln(e*((b*x+a)/(d*
x+c))^n)^2*a^2*b^7*c^2*d^4*i^2*n+72000*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a
*b^8*c^3*d^3*i^2*n-36000*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c^2*d^4*i
^2*n^2+30000*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^3*d^3*i^2*n^2-54000*A
*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^4*d^2*i^2*n-36000*A*B*x*a^2*b^7*c^2*d
^4*i^2*n^2+30000*A*B*x*a*b^8*c^3*d^3*i^2*n^2-36000*A*B*ln(e*((b*x+a)/(d*x+
c))^n)*a^2*b^7*c^3*d^3*i^2*n+54000*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^4
*d^2*i^2*n-18000*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*d^6*i^2*n-18000*B
^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c*d^5*i^2*n^2-36000*A*B*x^3*ln(e*((
b*x+a)/(d*x+c))^n)*a^2*b^7*d^6*i^2*n-18000*A*B*x^3*a*b^8*c*d^5*i^2*n^2-540
00*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^7*c*d^5*i^2*n+54000*B^2*x^2*l
n(e*((b*x+a)/(d*x+c))^n)^2*a*b^8*c^2*d^4*i^2*n-36000*B^2*x^2*ln(e*((b*x+a)
/(d*x+c))^n)*a^2*b^7*c*d^5*i^2*n^2+9000*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*
a*b^8*c^2*d^4*i^2*n^2-36000*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^3*d^3*
i^2*n-36000*A*B*x^2*a^2*b^7*c*d^5*i^2*n^2-4000*B^2*a^2*b^7*c^3*d^3*i^2*n^3
+3375*B^2*a*b^8*c^4*d^2*i^2*n^3+2820*A*B*a^5*b^4*d^6*i^2*n^2-4320*A*B*b^9*
c^5*d*i^2*n^2-18000*A^2*a^2*b^7*c^3*d^3*i^2*n+27000*A^2*a*b^8*c^4*d^2*i^2*
n-108000*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c*d^5*i^2*n+108000*A*B*
x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c^2*d^4*i^2*n-108000*A*B*x*ln(e*((b*x+
a)/(d*x+c))^n)*a^2*b^7*c^2*d^4*i^2*n+144000*A*B*x*ln(e*((b*x+a)/(d*x+c)...
```

### 3.177.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2633 vs. $2(475) = 950$.

Time = 0.47 (sec) , antiderivative size = 2633, normalized size of antiderivative = 5.34

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x
, algorithm="fricas")
```

output
```
-1/54000*((864*B^2*b^5*c^5 - 3375*B^2*a*b^4*c^4*d + 4000*B^2*a^2*b^3*c^3*d
^2 - 1489*B^2*a^5*d^5)*i^2*n^2 + 60*(47*(B^2*b^5*c*d^4 - B^2*a*b^4*d^5)*i^
2*n^2 + 60*(A*B*b^5*c*d^4 - A*B*a*b^4*d^5)*i^2*n)*x^4 + 60*(72*A*B*b^5*c^5
 - 225*A*B*a*b^4*c^4*d + 200*A*B*a^2*b^3*c^3*d^2 - 47*A*B*a^5*d^5)*i^2*n +
 30*((13*B^2*b^5*c^2*d^3 + 350*B^2*a*b^4*c*d^4 - 363*B^2*a^2*b^3*d^5)*i^2*
n^2 - 60*(A*B*b^5*c^2*d^3 - 10*A*B*a*b^4*c*d^4 + 9*A*B*a^2*b^3*d^5)*i^2*n)
*x^3 + 1800*(6*A^2*b^5*c^5 - 15*A^2*a*b^4*c^4*d + 10*A^2*a^2*b^3*c^3*d^2 -
 A^2*a^5*d^5)*i^2 - 10*((86*B^2*b^5*c^3*d^2 - 375*B^2*a*b^4*c^2*d^3 - 1200
*B^2*a^2*b^3*c*d^4 + 1489*B^2*a^3*b^2*d^5)*i^2*n^2 - 60*(2*A*B*b^5*c^3*d^2
 - 15*A*B*a*b^4*c^2*d^3 + 60*A*B*a^2*b^3*c*d^4 - 47*A*B*a^3*b^2*d^5)*i^2*n
 - 1800*(A^2*b^5*c^3*d^2 - 3*A^2*a*b^4*c^2*d^3 + 3*A^2*a^2*b^3*c*d^4 - A^2
*a^3*b^2*d^5)*i^2)*x^2 + 1800*(10*(B^2*b^5*c^3*d^2 - 3*B^2*a*b^4*c^2*d^3 +
 3*B^2*a^2*b^3*c*d^4 - B^2*a^3*b^2*d^5)*i^2*x^2 + 5*(3*B^2*b^5*c^4*d - 8*B
^2*a*b^4*c^3*d^2 + 6*B^2*a^2*b^3*c^2*d^3 - B^2*a^4*b*d^5)*i^2*x + (6*B^2*b
^5*c^5 - 15*B^2*a*b^4*c^4*d + 10*B^2*a^2*b^3*c^3*d^2 - B^2*a^5*d^5)*i^2)*l
og(e)^2 + 1800*(B^2*b^5*d^5*i^2*n^2*x^5 + 5*B^2*a*b^4*d^5*i^2*n^2*x^4 + 10
*B^2*a^2*b^3*d^5*i^2*n^2*x^3 + 10*(B^2*b^5*c^3*d^2 - 3*B^2*a*b^4*c^2*d^3 +
 3*B^2*a^2*b^3*c*d^4)*i^2*n^2*x^2 + 5*(3*B^2*b^5*c^4*d - 8*B^2*a*b^4*c^3*d
^2 + 6*B^2*a^2*b^3*c^2*d^3)*i^2*n^2*x + (6*B^2*b^5*c^5 - 15*B^2*a*b^4*c^4*
d + 10*B^2*a^2*b^3*c^3*d^2)*i^2*n^2)*log((b*x + a)/(d*x + c))^2 + 5*((1...
```

### 3.177.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
6,x)
```

output
```
Timed out
```

### 3.177.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 10936 vs. 2(475) = 950.

Time = 1.02 (sec) , antiderivative size = 10936, normalized size of antiderivative = 22.18

$$\int \frac{(ci+dix)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x
, algorithm="maxima")
```

output
```
-1/150*A*B*c^2*i^2*n*((60*b^4*d^4*x^4 + 12*b^4*c^4 - 63*a*b^3*c^3*d + 137*
a^2*b^2*c^2*d^2 - 163*a^3*b*c*d^3 + 137*a^4*d^4 - 30*(b^4*c*d^3 - 9*a*b^3*
d^4)*x^3 + 10*(2*b^4*c^2*d^2 - 13*a*b^3*c*d^3 + 47*a^2*b^2*d^4)*x^2 - 5*(3
*b^4*c^3*d - 17*a*b^3*c^2*d^2 + 43*a^2*b^2*c*d^3 - 77*a^3*b*d^4)*x)/((b^10
*c^4 - 4*a*b^9*c^3*d + 6*a^2*b^8*c^2*d^2 - 4*a^3*b^7*c*d^3 + a^4*b^6*d^4)*
g^6*x^5 + 5*(a*b^9*c^4 - 4*a^2*b^8*c^3*d + 6*a^3*b^7*c^2*d^2 - 4*a^4*b^6*c
*d^3 + a^5*b^5*d^4)*g^6*x^4 + 10*(a^2*b^8*c^4 - 4*a^3*b^7*c^3*d + 6*a^4*b^
6*c^2*d^2 - 4*a^5*b^5*c*d^3 + a^6*b^4*d^4)*g^6*x^3 + 10*(a^3*b^7*c^4 - 4*a
^4*b^6*c^3*d + 6*a^5*b^5*c^2*d^2 - 4*a^6*b^4*c*d^3 + a^7*b^3*d^4)*g^6*x^2
+ 5*(a^4*b^6*c^4 - 4*a^5*b^5*c^3*d + 6*a^6*b^4*c^2*d^2 - 4*a^7*b^3*c*d^3 +
 a^8*b^2*d^4)*g^6*x + (a^5*b^5*c^4 - 4*a^6*b^4*c^3*d + 6*a^7*b^3*c^2*d^2 -
 4*a^8*b^2*c*d^3 + a^9*b*d^4)*g^6) + 60*d^5*log(b*x + a)/((b^6*c^5 - 5*a*b
^5*c^4*d + 10*a^2*b^4*c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5
*b*d^5)*g^6) - 60*d^5*log(d*x + c)/((b^6*c^5 - 5*a*b^5*c^4*d + 10*a^2*b^4*
c^3*d^2 - 10*a^3*b^3*c^2*d^3 + 5*a^4*b^2*c*d^4 - a^5*b*d^5)*g^6)) - 1/900*
A*B*d^2*i^2*n*((47*a^2*b^4*c^4 - 278*a^3*b^3*c^3*d + 822*a^4*b^2*c^2*d^2 -
 278*a^5*b*c*d^3 + 47*a^6*d^4 + 60*(10*b^6*c^2*d^2 - 5*a*b^5*c*d^3 + a^2*b
^4*d^4)*x^4 - 30*(10*b^6*c^3*d - 95*a*b^5*c^2*d^2 + 46*a^2*b^4*c*d^3 - 9*a
^3*b^3*d^4)*x^3 + 10*(20*b^6*c^4 - 140*a*b^5*c^3*d + 537*a^2*b^4*c^2*d^2 -
 248*a^3*b^3*c*d^3 + 47*a^4*b^2*d^4)*x^2 + 5*(35*a*b^5*c^4 - 218*a^2*b^...
```

### 3.177.8 Giac [A] (verification not implemented)

Time = 6.30 (sec) , antiderivative size = 931, normalized size of antiderivative = 1.89

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^6,x
, algorithm="giac")
```

output
```
-1/54000*(1800*(6*B^2*b^2*i^2*n^2 - 15*(b*x + a)*B^2*b*d*i^2*n^2/(d*x + c)
 + 10*(b*x + a)^2*B^2*d^2*i^2*n^2/(d*x + c)^2)*log((b*x + a)/(d*x + c))^2/
((b*x + a)^5*b^2*c^2*g^6/(d*x + c)^5 - 2*(b*x + a)^5*a*b*c*d*g^6/(d*x + c)
^5 + (b*x + a)^5*a^2*d^2*g^6/(d*x + c)^5) + 60*(72*B^2*b^2*i^2*n^2 - 225*(
b*x + a)*B^2*b*d*i^2*n^2/(d*x + c) + 200*(b*x + a)^2*B^2*d^2*i^2*n^2/(d*x
+ c)^2 + 360*B^2*b^2*i^2*n*log(e) - 900*(b*x + a)*B^2*b*d*i^2*n*log(e)/(d*
x + c) + 600*(b*x + a)^2*B^2*d^2*i^2*n*log(e)/(d*x + c)^2 + 360*A*B*b^2*i^
2*n - 900*(b*x + a)*A*B*b*d*i^2*n/(d*x + c) + 600*(b*x + a)^2*A*B*d^2*i^2*
n/(d*x + c)^2)*log((b*x + a)/(d*x + c))/((b*x + a)^5*b^2*c^2*g^6/(d*x + c)
^5 - 2*(b*x + a)^5*a*b*c*d*g^6/(d*x + c)^5 + (b*x + a)^5*a^2*d^2*g^6/(d*x
+ c)^5) + (864*B^2*b^2*i^2*n^2 - 3375*(b*x + a)*B^2*b*d*i^2*n^2/(d*x + c)
+ 4000*(b*x + a)^2*B^2*d^2*i^2*n^2/(d*x + c)^2 + 4320*B^2*b^2*i^2*n*log(e)
 - 13500*(b*x + a)*B^2*b*d*i^2*n*log(e)/(d*x + c) + 12000*(b*x + a)^2*B^2*
d^2*i^2*n*log(e)/(d*x + c)^2 + 10800*B^2*b^2*i^2*log(e)^2 - 27000*(b*x + a
)*B^2*b*d*i^2*log(e)^2/(d*x + c) + 18000*(b*x + a)^2*B^2*d^2*i^2*log(e)^2/
(d*x + c)^2 + 4320*A*B*b^2*i^2*n - 13500*(b*x + a)*A*B*b*d*i^2*n/(d*x + c)
 + 12000*(b*x + a)^2*A*B*d^2*i^2*n/(d*x + c)^2 + 21600*A*B*b^2*i^2*log(e)
- 54000*(b*x + a)*A*B*b*d*i^2*log(e)/(d*x + c) + 36000*(b*x + a)^2*A*B*d^2
*i^2*log(e)/(d*x + c)^2 + 10800*A^2*b^2*i^2 - 27000*(b*x + a)*A^2*b*d*i^2/
(d*x + c) + 18000*(b*x + a)^2*A^2*d^2*i^2/(d*x + c)^2)/((b*x + a)^5*b^2...
```

### 3.177.9 Mupad [B] (verification not implemented)

Time = 7.85 (sec) , antiderivative size = 3296, normalized size of antiderivative = 6.69

$$\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx = \text{Too large to display}$$

input
```
int(((c*i + d*i*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^6,x)
```

output
```
((1800*A^2*a^4*d^4*i^2 + 10800*A^2*b^4*c^4*i^2 + 1489*B^2*a^4*d^4*i^2*n^2
+ 864*B^2*b^4*c^4*i^2*n^2 - 16200*A^2*a*b^3*c^3*d*i^2 + 1800*A^2*a^3*b*c*d
^3*i^2 + 2820*A*B*a^4*d^4*i^2*n + 4320*A*B*b^4*c^4*i^2*n + 1800*A^2*a^2*b^
2*c^2*d^2*i^2 + 1489*B^2*a^2*b^2*c^2*d^2*i^2*n^2 - 2511*B^2*a*b^3*c^3*d*i^
2*n^2 + 1489*B^2*a^3*b*c*d^3*i^2*n^2 + 2820*A*B*a^2*b^2*c^2*d^2*i^2*n - 91
80*A*B*a*b^3*c^3*d*i^2*n + 2820*A*B*a^3*b*c*d^3*i^2*n)/(60*(a*d - b*c)) +
(x*(1800*A^2*a^3*b*d^4*i^2 + 5400*A^2*b^4*c^3*d*i^2 - 9000*A^2*a*b^3*c^2*d
^2*i^2 + 1800*A^2*a^2*b^2*c*d^3*i^2 + 1489*B^2*a^3*b*d^4*i^2*n^2 + 189*B^2
*b^4*c^3*d*i^2*n^2 + 1620*A*B*b^4*c^3*d*i^2*n - 911*B^2*a*b^3*c^2*d^2*i^2*
n^2 + 1489*B^2*a^2*b^2*c*d^3*i^2*n^2 + 2820*A*B*a^3*b*d^4*i^2*n - 4380*A*B
*a*b^3*c^2*d^2*i^2*n + 2820*A*B*a^2*b^2*c*d^3*i^2*n))/(12*(a*d - b*c)) + (
x^2*(1800*A^2*a^2*b^2*d^4*i^2 + 1800*A^2*b^4*c^2*d^2*i^2 - 3600*A^2*a*b^3*
c*d^3*i^2 + 1489*B^2*a^2*b^2*d^4*i^2*n^2 - 86*B^2*b^4*c^2*d^2*i^2*n^2 + 28
20*A*B*a^2*b^2*d^4*i^2*n + 120*A*B*b^4*c^2*d^2*i^2*n + 289*B^2*a*b^3*c*d^3
*i^2*n^2 - 780*A*B*a*b^3*c*d^3*i^2*n))/(6*(a*d - b*c)) + (x^3*(363*B^2*a*b
^3*d^4*i^2*n^2 + 13*B^2*b^4*c*d^3*i^2*n^2 - 60*A*B*b^4*c*d^3*i^2*n + 540*A
*B*a*b^3*d^4*i^2*n))/(2*(a*d - b*c)) + (d*x^4*(47*B^2*b^4*d^3*i^2*n^2 + 60
*A*B*b^4*d^3*i^2*n))/(a*d - b*c))/(x*(4500*a^4*b^5*c*g^6 - 4500*a^5*b^4*d*
g^6) - x^4*(4500*a^2*b^7*d*g^6 - 4500*a*b^8*c*g^6) + x^5*(900*b^9*c*g^6 -
900*a*b^8*d*g^6) + x^2*(9000*a^3*b^6*c*g^6 - 9000*a^4*b^5*d*g^6) + x^3*...
```

3.177. $\int \frac{(ci+dix)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^6}\,dx$

# 3.178 $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

### 3.178.1 Optimal result

Integrand size = 45, antiderivative size = 1172

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \frac{5B^2(bc-ad)^6g^3i^3n^2x}{84b^3d^3}+\frac{B^2(bc-ad)^3g^3i^3n^2(a+bx)^4}{140b^4}-\frac{29B^2(bc-ad)^5g^3i^3n^2(c+dx)^2}{840b^2d^4}$$
$$+\frac{47B^2(bc-ad)^4g^3i^3n^2(c+dx)^3}{1260bd^4}-\frac{13B^2(bc-ad)^3g^3i^3n^2(c+dx)^4}{420d^4}$$
$$+\frac{bB^2(bc-ad)^2g^3i^3n^2(c+dx)^5}{105d^4}-\frac{B(bc-ad)^4g^3i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{210b^4d}$$
$$-\frac{3B(bc-ad)^3g^3i^3n(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{140b^4}$$
$$-\frac{B(bc-ad)^2g^3i^3n(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35b^3}$$
$$+\frac{2B(bc-ad)^4g^3i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21bd^4}$$
$$-\frac{3B(bc-ad)^3g^3i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{14d^4}$$
$$+\frac{6bB(bc-ad)^2g^3i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{35d^4}$$
$$-\frac{b^2B(bc-ad)g^3i^3n(c+dx)^6\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{21d^4}$$
$$+\frac{(bc-ad)^3g^3i^3(a+bx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{140b^4}$$
$$+\frac{(bc-ad)^2g^3i^3(a+bx)^4(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{35b^3}$$
$$+\frac{(bc-ad)g^3i^3(a+bx)^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{14b^2}$$
$$+\frac{g^3i^3(a+bx)^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{7b}$$
$$+\frac{B(bc-ad)^5g^3i^3n(a+bx)^2\left(3A+Bn+3B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{420b^4d^2}$$
$$-\frac{B(bc-ad)^6g^3i^3n(a+bx)\left(6A+5Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{420b^4d^3}$$
$$-\frac{B(bc-ad)^7g^3i^3n\left(6A+11Bn+6B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{420b^4d^4}$$
$$-\frac{B^2(bc-ad)^7g^3i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{210b^4d^4}-\frac{11B^2(bc-ad)^7g^3i^3n^2\log(c+dx)}{420b^4d^4}$$
$$-\frac{B^2(bc-ad)^7g^3i^3n^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{70b^4d^4}$$

output
```
5/84*B^2*(-a*d+b*c)^6*g^3*i^3*n^2*x/b^3/d^3+1/140*B^2*(-a*d+b*c)^3*g^3*i^3
*n^2*(b*x+a)^4/b^4-29/840*B^2*(-a*d+b*c)^5*g^3*i^3*n^2*(d*x+c)^2/b^2/d^4+4
7/1260*B^2*(-a*d+b*c)^4*g^3*i^3*n^2*(d*x+c)^3/b/d^4-13/420*B^2*(-a*d+b*c)^
3*g^3*i^3*n^2*(d*x+c)^4/d^4+1/105*b*B^2*(-a*d+b*c)^2*g^3*i^3*n^2*(d*x+c)^5
/d^4-1/210*B*(-a*d+b*c)^4*g^3*i^3*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))/b^4/d-3/140*B*(-a*d+b*c)^3*g^3*i^3*n*(b*x+a)^4*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))/b^4-1/35*B*(-a*d+b*c)^2*g^3*i^3*n*(b*x+a)^4*(d*x+c)*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))/b^3+2/21*B*(-a*d+b*c)^4*g^3*i^3*n*(d*x+c)^3*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))/b/d^4-3/14*B*(-a*d+b*c)^3*g^3*i^3*n*(d*x+c)^4*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))/d^4+6/35*b*B*(-a*d+b*c)^2*g^3*i^3*n*(d*x+c)^5*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/d^4-1/21*b^2*B*(-a*d+b*c)*g^3*i^3*n*(d*x+c)^6
*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4+1/140*(-a*d+b*c)^3*g^3*i^3*(b*x+a)^4*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4+1/35*(-a*d+b*c)^2*g^3*i^3*(b*x+a)^4*
(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3+1/14*(-a*d+b*c)*g^3*i^3*(b*x
+a)^4*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2+1/7*g^3*i^3*(b*x+a)^
4*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b+1/420*B*(-a*d+b*c)^5*g^3*i
^3*n*(b*x+a)^2*(3*A+B*n+3*B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d^2-1/420*B*(-a
*d+b*c)^6*g^3*i^3*n*(b*x+a)*(6*A+5*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/
d^3-1/420*B*(-a*d+b*c)^7*g^3*i^3*n*(6*A+11*B*n+6*B*ln(e*((b*x+a)/(d*x+c))^
n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/d^4-1/210*B^2*(-a*d+b*c)^7*g^3*i^3*n^2...
```

### 3.178.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2448 vs. $2(1172) = 2344$.

Time = 1.85 (sec) , antiderivative size = 2448, normalized size of antiderivative = 2.09

$$\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Result too large to show}$$

input
```
Integrate[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2,x]
```

output
```
(g^3*i^3*(35*(b*c - a*d)^3*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2 + 84*d*(b*c - a*d)^2*(a + b*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2 + 70*d^2*(b*c - a*d)*(a + b*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2 + 20*d^3*(a + b*x)^7*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (35*
B*(b*c - a*d)^4*n*(6*A*b*d*(b*c - a*d)^2*x + 6*B*d*(b*c - a*d)^2*(a + b*x)
*Log[e*((a + b*x)/(c + d*x))^n] + 3*d^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]) + 2*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x
)/(c + d*x))^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 6*(b*c - a*d)^3*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*(2*b*d*(
b*c - a*d)*x - d^2*(a + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + 3*B*(b*c
- a*d)^2*n*(b*d*x + (-(b*c) + a*d)*Log[c + d*x]) + 3*B*(b*c - a*d)^3*n*((2
*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLo
g[2, (b*(c + d*x))/(b*c - a*d)])))/(3*d^4) + (7*B*(b*c - a*d)^3*n*(24*A*b*
d*(b*c - a*d)^3*x + 24*B*d*(b*c - a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d
*x))^n] - 12*d^2*(b*c - a*d)^2*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]) + 8*d^3*(b*c - a*d)*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]) - 6*d^4*(a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 24*B*(b*
c - a*d)^4*n*Log[c + d*x] - 24*(b*c - a*d)^4*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])*Log[c + d*x] + 4*B*(b*c - a*d)^2*n*(2*b*d*(b*c - a*d)*x - d^2*(a
 + b*x)^2 - 2*(b*c - a*d)^2*Log[c + d*x]) + B*(b*c - a*d)*n*(6*b*d*(b*c...
```

### 3.178.3 Rubi [A] (verified)

Time = 3.52 (sec) , antiderivative size = 1455, normalized size of antiderivative = 1.24, number of steps used = 23, number of rules used = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.489$, Rules used = {2961, 2783, 2782, 27, 2123, 2009, 2783, 2782, 27, 87, 49, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (agx+bgx)^3(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2961}$$

$$g^3i^3(bc-ad)^7\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^8}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$g^3i^3(bc-ad)^7\left(-\frac{2Bn\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}}{7b}+\frac{3\int\frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}}{7b}+\frac{(a+bx)^4\left(B\log\left(e\left(\right.\right.\right.}{7b(c+dx)^4\left(b-\right.}\right.$$

$\downarrow$ 2782

$$g^3i^3(bc-ad)^7\left(-\frac{2Bn\left(-Bn\int-\frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{60d^4(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}+\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3b^2\left(B\log\left(e\left(\right.\right.\right.}{5d^4\left(b-\right.}\right.}{7b}\right.$$

$\downarrow$ 27

$$g^3i^3(bc-ad)^7\left(-\frac{2Bn\left(\frac{Bn\int\frac{(c+dx)\left(b^3-\frac{6d(a+bx)b^2}{c+dx}+\frac{15d^2(a+bx)^2b}{(c+dx)^2}-\frac{20d^3(a+bx)^3}{(c+dx)^3}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{60d^4}+\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}\right.}{7b}\right.$$

$\downarrow$ 2123

$$g^3i^3(bc-ad)^7\left(-\frac{2Bn\left(\frac{Bn\int\left(-\frac{10db^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^6}+\frac{26db}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{19d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3b}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2b^2}+\frac{c+dx}{(a+bx)b^3}+\frac{d}{\left(b-\frac{d(a+bx)}{c+dx}\right)b^3}\right)}{60d^4}\right.}{7b}\right.$$

$\downarrow$ 2009

3.178. $\int(ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$g^3i^3(bc-ad)^7\left(\frac{3\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}}{7b}-\frac{2Bn\left(\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3b\left(B\log\left(e\right.\right.}{4d^4\left(b-\right.}\right.}{}\right.$$

$\downarrow$ 2783

$$g^3i^3(bc-ad)^7\left(\frac{3\left(-\frac{Bn\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{3b}+\frac{\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}}{3b}+\frac{(a+bx)^4\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{6b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}\right)}{7b}\right.$$

$\downarrow$ 2782

$$(bc-ad)^7g^3i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$\downarrow$ 27

3.178. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$$\Big\downarrow \; 87$$

$$(bc-ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$$\Big\downarrow \; 49$$

3.178. $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

$$(bc-ad)^7g^3i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-ad)^7g^3i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$\downarrow$ 2783

3.178. $\int(ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-ad)^7g^3i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$\downarrow$ 2773

$$(bc-ad)^7g^3i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$\downarrow$ 49

3.178. $\int(ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc - ad)^7 g^3 i^3 \left( \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4 \left(b - \frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$\downarrow$ 2009

3.178. $\int (ag+bgx)^3(ci+dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

$$(bc-ad)^7 g^3 i^3 \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$\downarrow$ 2781

3.178. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc - ad)^7 g^3 i^3 \left( \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b - \frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$\downarrow$ 2784

---

3.178. $\int (ag + bgx)^3 (ci + dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

$$(bc-ad)^7 g^3 i^3 \left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$$\Big\downarrow \; 2784$$

---

3.178. $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

$$(bc-$$

$$ad)^7g^3i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7}-\frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$\downarrow$ 2784

3.178. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-ad)^7 g^3 i^3 \left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$$\downarrow 2754$$

3.178. $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

$$(bc-ad)^7 g^3 i^3 \left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

$\downarrow$ 2838

3.178. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-ad)^7 g^3 i^3 \left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^4}{7b(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^7} - \frac{2Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^3}{6d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5} + \frac{3\left(A+B\right.}{4d^4}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2,x]
```

3.178. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

output
```
(b*c - a*d)^7*g^3*i^3*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(7*b*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^7) - (2*B*n*((b^3*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(6*d^4*(b - (d*(a + b*x))/(c + d*x))^6)
 - (3*b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*d^4*(b - (d*(a + b*x)
)/(c + d*x))^5) + (3*b*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d^4*(b -
 (d*(a + b*x))/(c + d*x))^4) - (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*d
^4*(b - (d*(a + b*x))/(c + d*x))^3) + (B*n*((-2*b^2)/(b - (d*(a + b*x))/(c
 + d*x))^5 + (13*b)/(2*(b - (d*(a + b*x))/(c + d*x))^4) - 19/(3*(b - (d*(a
 + b*x))/(c + d*x))^3) + 1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b^2*
(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[b - (d
*(a + b*x))/(c + d*x)]/b^3))/(60*d^4)))/(7*b) + (3*(((a + b*x)^4*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n])^2)/(6*b*(c + d*x)^4*(b - (d*(a + b*x))/(c +
d*x))^6) - (B*n*(((a + b*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(5*b
*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^5) + ((a + b*x)^4*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]))/(20*b^2*(c + d*x)^4*(b - (d*(a + b*x))/(c + d*x
))^4) - (B*n*((a + b*x)^4/((c + d*x)^4*(b - (d*(a + b*x))/(c + d*x))^4) +
b^3/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) - (3*b^2)/(2*d^4*(b - (d*(a +
b*x))/(c + d*x))^2) + (3*b)/(d^4*(b - (d*(a + b*x))/(c + d*x))) + Log[b -
(d*(a + b*x))/(c + d*x)]/d^4))/(20*b^2)))/(3*b) + (((a + b*x)^4*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2)/(5*b*(c + d*x)^4*(b - (d*(a + b*x))/(c ...
```

#### 3.178.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 87
```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_] :> Simp[(-(b*e - a*f))*(c + d*x)^(n + 1)*((e + f*x)^(p + 1)/(f*(p
+ 1)*(c*f - d*e))), x] - Simp[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f*(p
+ 1)))/(f*(p + 1)*(c*f - d*e))   Int[(c + d*x)^n*(e + f*x)^(p + 1), x], x]
/; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Intege
rQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.178. $\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

rule 2123
```
Int[(Px_)*((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol]
:> Int[ExpandIntegrand[Px*(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c
, d, m, n}, x] && PolyQ[Px, x] && (IntegersQ[m, n] || IGtQ[m, -2])
```

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

---

## 3.178. $\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.178.4 Maple [F]

$$\int (bgx+ag)^3 (dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

### 3.178.5 Fricas [F]

$$\int (ag+bgx)^3(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^3(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="fricas")
```

output
```
integral(A^2*b^3*d^3*g^3*i^3*x^6 + A^2*a^3*c^3*g^3*i^3 + 3*(A^2*b^3*c*d^2
+ A^2*a*b^2*d^3)*g^3*i^3*x^5 + 3*(A^2*b^3*c^2*d + 3*A^2*a*b^2*c*d^2 + A^2*
a^2*b*d^3)*g^3*i^3*x^4 + (A^2*b^3*c^3 + 9*A^2*a*b^2*c^2*d + 9*A^2*a^2*b*c*
d^2 + A^2*a^3*d^3)*g^3*i^3*x^3 + 3*(A^2*a*b^2*c^3 + 3*A^2*a^2*b*c^2*d + A^
2*a^3*c*d^2)*g^3*i^3*x^2 + 3*(A^2*a^2*b*c^3 + A^2*a^3*c^2*d)*g^3*i^3*x + (
B^2*b^3*d^3*g^3*i^3*x^6 + B^2*a^3*c^3*g^3*i^3 + 3*(B^2*b^3*c*d^2 + B^2*a*b
^2*d^3)*g^3*i^3*x^5 + 3*(B^2*b^3*c^2*d + 3*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3
)*g^3*i^3*x^4 + (B^2*b^3*c^3 + 9*B^2*a*b^2*c^2*d + 9*B^2*a^2*b*c*d^2 + B^2
*a^3*d^3)*g^3*i^3*x^3 + 3*(B^2*a*b^2*c^3 + 3*B^2*a^2*b*c^2*d + B^2*a^3*c*d
^2)*g^3*i^3*x^2 + 3*(B^2*a^2*b*c^3 + B^2*a^3*c^2*d)*g^3*i^3*x)*log(e*((b*x
 + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*d^3*g^3*i^3*x^6 + A*B*a^3*c^3*g^3*i^3 +
 3*(A*B*b^3*c*d^2 + A*B*a*b^2*d^3)*g^3*i^3*x^5 + 3*(A*B*b^3*c^2*d + 3*A*B*
a*b^2*c*d^2 + A*B*a^2*b*d^3)*g^3*i^3*x^4 + (A*B*b^3*c^3 + 9*A*B*a*b^2*c^2*
d + 9*A*B*a^2*b*c*d^2 + A*B*a^3*d^3)*g^3*i^3*x^3 + 3*(A*B*a*b^2*c^3 + 3*A*
B*a^2*b*c^2*d + A*B*a^3*c*d^2)*g^3*i^3*x^2 + 3*(A*B*a^2*b*c^3 + A*B*a^3*c^
2*d)*g^3*i^3*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

### 3.178.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**
2,x)
```

output
```
Timed out
```

### 3.178.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 7845 vs. 2(1125) = 2250.

Time = 0.83 (sec) , antiderivative size = 7845, normalized size of antiderivative = 6.69

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="maxima")
```

output
```
2/7*A*B*b^3*d^3*g^3*i^3*x^7*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/7*A
^2*b^3*d^3*g^3*i^3*x^7 + A*B*b^3*c*d^2*g^3*i^3*x^6*log(e*(b*x/(d*x + c) +
a/(d*x + c))^n) + A*B*a*b^2*d^3*g^3*i^3*x^6*log(e*(b*x/(d*x + c) + a/(d*x
+ c))^n) + 1/2*A^2*b^3*c*d^2*g^3*i^3*x^6 + 1/2*A^2*a*b^2*d^3*g^3*i^3*x^6 +
 6/5*A*B*b^3*c^2*d*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 18
/5*A*B*a*b^2*c*d^2*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 6/
5*A*B*a^2*b*d^3*g^3*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/5*A
^2*b^3*c^2*d*g^3*i^3*x^5 + 9/5*A^2*a*b^2*c*d^2*g^3*i^3*x^5 + 3/5*A^2*a^2*b
*d^3*g^3*i^3*x^5 + 1/2*A*B*b^3*c^3*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n) + 9/2*A*B*a*b^2*c^2*d*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*
x + c))^n) + 9/2*A*B*a^2*b*c*d^2*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x
 + c))^n) + 1/2*A*B*a^3*d^3*g^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + 1/4*A^2*b^3*c^3*g^3*i^3*x^4 + 9/4*A^2*a*b^2*c^2*d*g^3*i^3*x^4 + 9/4
*A^2*a^2*b*c*d^2*g^3*i^3*x^4 + 1/4*A^2*a^3*d^3*g^3*i^3*x^4 + 2*A*B*a*b^2*c
^3*g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 6*A*B*a^2*b*c^2*d*
g^3*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a^3*c*d^2*g^3*i
^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*a*b^2*c^3*g^3*i^3*x^3
+ 3*A^2*a^2*b*c^2*d*g^3*i^3*x^3 + A^2*a^3*c*d^2*g^3*i^3*x^3 + 3*A*B*a^2*b*
c^3*g^3*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3*A*B*a^3*c^2*d*g
^3*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/2*A^2*a^2*b*c^3*g...
```

### 3.178.8 Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)^3*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="giac")
```

output
```
Timed out
```

### 3.178.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^3(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^3\,(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2,x)
```

output
```
int((a*g + b*g*x)^3*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2, x)
```

# 3.179 $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

### 3.179.1 Optimal result

Integrand size = 45, antiderivative size = 976

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= -\frac{7B^2(bc-ad)^5g^2i^3n^2x}{180b^3d^2}-\frac{7B^2(bc-ad)^4g^2i^3n^2(c+dx)^2}{360b^2d^3}-\frac{B^2(bc-ad)^3g^2i^3n^2(c+dx)^3}{60bd^3}$$
$$+\frac{B^2(bc-ad)^2g^2i^3n^2(c+dx)^4}{60d^3}-\frac{B(bc-ad)^4g^2i^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^4d}$$
$$-\frac{B(bc-ad)^3g^2i^3n(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30b^4}$$
$$-\frac{B(bc-ad)^4g^2i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^2d^3}$$
$$+\frac{B(bc-ad)^3g^2i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{45bd^3}$$
$$+\frac{7B(bc-ad)^2g^2i^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60d^3}$$
$$-\frac{bB(bc-ad)g^2i^3n(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{15d^3}$$
$$+\frac{(bc-ad)^3g^2i^3(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{60b^4}$$
$$+\frac{(bc-ad)^2g^2i^3(a+bx)^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^3}$$
$$+\frac{(bc-ad)g^2i^3(a+bx)^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^2}$$
$$+\frac{g^2i^3(a+bx)^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{6b}$$
$$+\frac{B(bc-ad)^5g^2i^3n(a+bx)\left(2A+Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{60b^4d^2}$$
$$+\frac{B(bc-ad)^6g^2i^3n\left(2A+3Bn+2B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{60b^4d^3}$$
$$+\frac{B^2(bc-ad)^6g^2i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{36b^4d^3}+\frac{11B^2(bc-ad)^6g^2i^3n^2\log(c+dx)}{180b^4d^3}$$
$$+\frac{B^2(bc-ad)^6g^2i^3n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{30b^4d^3}$$

output
```
-7/180*B^2*(-a*d+b*c)^5*g^2*i^3*n^2*x/b^3/d^2-7/360*B^2*(-a*d+b*c)^4*g^2*i
^3*n^2*(d*x+c)^2/b^2/d^3-1/60*B^2*(-a*d+b*c)^3*g^2*i^3*n^2*(d*x+c)^3/b/d^3
+1/60*B^2*(-a*d+b*c)^2*g^2*i^3*n^2*(d*x+c)^4/d^3-1/60*B*(-a*d+b*c)^4*g^2*i
^3*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d-1/30*B*(-a*d+b*c)^3*g
^2*i^3*n*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4-1/10*B*(-a*d+b*c)^4
*g^2*i^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d^3+1/45*B*(-a*d+
b*c)^3*g^2*i^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^3+7/60*B*(-
a*d+b*c)^2*g^2*i^3*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3-1/15*b*
B*(-a*d+b*c)*g^2*i^3*n*(d*x+c)^5*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3+1/60*
(-a*d+b*c)^3*g^2*i^3*(b*x+a)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4+1/20*
(-a*d+b*c)^2*g^2*i^3*(b*x+a)^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b
^3+1/10*(-a*d+b*c)*g^2*i^3*(b*x+a)^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))
^n))^2/b^2+1/6*g^2*i^3*(b*x+a)^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
^2/b+1/60*B*(-a*d+b*c)^5*g^2*i^3*n*(b*x+a)*(2*A+B*n+2*B*ln(e*((b*x+a)/(d*x
+c))^n))/b^4/d^2+1/60*B*(-a*d+b*c)^6*g^2*i^3*n*(2*A+3*B*n+2*B*ln(e*((b*x+a
)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/d^3+1/36*B^2*(-a*d+b*c)^6*g^2*
i^3*n^2*ln((b*x+a)/(d*x+c))/b^4/d^3+11/180*B^2*(-a*d+b*c)^6*g^2*i^3*n^2*ln
(d*x+c)/b^4/d^3+1/30*B^2*(-a*d+b*c)^6*g^2*i^3*n^2*polylog(2,d*(b*x+a)/b/(d
*x+c))/b^4/d^3
```

### 3.179.2 Mathematica [A] (verified)

Time = 0.84 (sec) , antiderivative size = 1627, normalized size of antiderivative = 1.67

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
Integrate[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2,x]
```

output
```
(g^2*i^3*(15*(b*c - a*d)^2*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^
n])^2 - 24*b*(b*c - a*d)*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2 + 10*b^2*(c + d*x)^6*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (5*B*(
b*c - a*d)^3*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*
c - a*d)*Log[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d
*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[
e*((a + b*x)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]) + 6*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]) - 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x
]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)])))/b^4 + (2*B*(b*c - a*d)^2*n*(24*A*b*d*(b*c - a*d)
^3*x - 12*B*(b*c - a*d)^3*n*(b*d*x + (b*c - a*d)*Log[a + b*x]) - 4*B*(b*c
- a*d)^2*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a
+ b*x]) - B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c +
d*x)^2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) + 24*B*d*(b*c -
 a*d)^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 12*b^2*(b*c - a*d)^2*(c
 + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*b^3*(b*c - a*d)*(c +
d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*b^4*(c + d*x)^4*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]) + 24*(b*c - a*d)^4*Log[a + b*x]*(A + B*L...
```

### 3.179.3 Rubi [A] (verified)

Time = 3.02 (sec) , antiderivative size = 1266, normalized size of antiderivative = 1.30, number of steps used = 21, number of rules used = 20, $\frac{\text{number of rules}}{\text{integrand size}} = 0.444$, Rules used = {2961, 2783, 2782, 27, 1195, 2009, 2783, 2782, 27, 1195, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^2(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2961}$$

$$g^2i^3(bc-ad)^6\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^7}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$g^2 i^3 (bc-ad)^6 \left(-\frac{Bn \int \frac{(a+bx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{3b} + \frac{\int \frac{(a+bx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{2b} + \frac{(a+bx)^3 \left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{6b(c+dx)^3\left(b-\frac{d(}{c}\right.}\right.$$

$\downarrow$ 2782

$$g^2 i^3 (bc-ad)^6 \left(-\frac{Bn\left(-Bn\int \frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{30d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx} + \frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\log}{3d}\right.}{3b}\right.$$

$\downarrow$ 27

$$g^2 i^3 (bc-ad)^6 \left(-\frac{Bn\left(-\frac{Bn\int \frac{(c+dx)\left(b^2-\frac{5d(a+bx)b}{c+dx}+\frac{10d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{30d^3} + \frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\log\left(e\right.}{3d^3\left(b-\right.}\right.}{3b}\right.$$

$\downarrow$ 1195

$$g^2 i^3 (bc-ad)^6 \left(-\frac{Bn\left(-\frac{Bn\int \left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{9d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{6bd}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}+\frac{c+dx}{b^3(a+bx)}\right) d\frac{a+bx}{c+dx}}{30d^3} + \frac{b^2\left(B\log\right.}{5d}\right.}{3b}\right.$$

$\downarrow$ 2009

3.179. $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$g^2 i^3 (bc - ad)^6 \left( \frac{\int \frac{(a+bx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^6} d\frac{a+bx}{c+dx}}{2b} - \frac{Bn \left( \frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n \right.}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right.} \right.}{} \right.$$

$\downarrow$ 2783

$$g^2 i^3 (bc - ad)^6 \left( \frac{-\frac{2Bn\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{2\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5} d\frac{a+bx}{c+dx}}{5b} + \frac{(a+bx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{5b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}}{2b} - \right.$$

$\downarrow$ 2782

$$g^2 i^3 (bc - ad)^6 \left( \frac{-\frac{2Bn\left(-Bn\int \frac{(c+dx)\left(b^2-\frac{4d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{12d^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4} d\frac{a+bx}{c+dx} + \frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{2b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right)}{5b}}{2b} \right.$$

$\downarrow$ 27

---

3.179. $\int (ag+bgx)^2 (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$g^2i^3(bc-ad)^6\left(-\frac{\frac{2Bn\left(-\frac{Bn\int\frac{(c+dx)\left(b^2-\frac{4d(a+bx)b}{c+dx}+\frac{6d^2(a+bx)^2}{(c+dx)^2}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{12d^3}+\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{2b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\right)}{5b}}{2b}\right.$$

$\Big\downarrow$ 1195

$$(bc-ad)^6g^2i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{A+B\log\left(e\ldots\right.}{3d^3\left(b-\frac{d}{\ldots}\right.}\right.}{\ldots}\right.$$

$\Big\downarrow$ 2009

$$(bc-ad)^6g^2i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{A+B\log\left(e\ldots\right.}{3d^3\left(b-\frac{d}{\ldots}\right.}\right.}{\ldots}\right.$$

$\Big\downarrow$ 2783

---

3.179. $\int(ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-ad)^6g^2i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{A+B\log\left(e\right.}{3d^3\left(b-\frac{d}{}\right.}\right.}{}\right.$$

$\downarrow$ 2773

$$(bc-ad)^6g^2i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{A+B\log\left(e\right.}{3d^3\left(b-\frac{d}{}\right.}\right.}{}\right.$$

$\downarrow$ 49

3.179. $\int(ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-$$

$$ad)^6g^2i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{A+B\log\left(e\right.}{3d^3\left(b-\frac{d}{}\right.}\right.}{}\right.$$

$\downarrow$ 2009

$$(bc-$$

$$ad)^6g^2i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{A+B\log\left(e\right.}{3d^3\left(b-\frac{d}{}\right.}\right.}{}\right.$$

---

3.179. $\int(ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$\Big\downarrow \text{2781}$$

$$(bc-ad)^6g^2i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{A+B\log\left(e\right.}{3d^3\left(b-\frac{d}{}\right.}\right.}{}\right.$$

$$\Big\downarrow \text{2784}$$

3.179. $\int(ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-$$

$$ad)^6g^2i^3\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6}-\frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{A+B\log\left(e\right.}{3d^3\left(b-\frac{d}{}\right.}\right.}{}\right.$$

$\downarrow$ 2784

---

3.179. $\int(ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-ad)^6 g^2 i^3 \left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(e\right.}{3d^3\left(b-\frac{d}{}\right.}\right.}{}\right.$$

$\downarrow$ 2754

---

3.179. $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-ad)^6 g^2 i^3 \left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^6} - \frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(e\right.}{3d^3\left(b-\frac{d}{}\right.}\right.}{}\right.$$

$\downarrow$ 2838

---

3.179. $\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

$$(bc - ad)^6 g^2 i^3 \left( \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 (a+bx)^3}{6b(c+dx)^3\left(b - \frac{d(a+bx)}{c+dx}\right)^6} - \frac{Bn\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b^2}{5d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^5} - \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)b}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{A+B\log\left(e\right.}{3d^3\left(b-\frac{d}{}\right.}\right.}{}\right.$$

input
```
Int[(a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2,x]
```

3.179. $\int (ag + bgx)^2 (ci + dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

output
```
(b*c - a*d)^6*g^2*i^3*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
^2)/(6*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^6) - (B*n*((b^2*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(5*d^3*(b - (d*(a + b*x))/(c + d*x))^5) -
 (b*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d^3*(b - (d*(a + b*x))/(c +
 d*x))^4) + (A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*d^3*(b - (d*(a + b*x
))/(c + d*x))^3) - (B*n*((3*b)/(2*(b - (d*(a + b*x))/(c + d*x))^4) - 3/(b
- (d*(a + b*x))/(c + d*x))^3 + 1/(2*b*(b - (d*(a + b*x))/(c + d*x))^2) + 1
/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^3 - Log[
b - (d*(a + b*x))/(c + d*x)]/b^3))/(30*d^3)))/(3*b) + (((a + b*x)^3*(A + B
*Log[e*((a + b*x)/(c + d*x))^n])^2)/(5*b*(c + d*x)^3*(b - (d*(a + b*x))/(c
 + d*x))^5) - (2*B*n*((b^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(4*d^3*
(b - (d*(a + b*x))/(c + d*x))^4) - (2*b*(A + B*Log[e*((a + b*x)/(c + d*x))
^n]))/(3*d^3*(b - (d*(a + b*x))/(c + d*x))^3) + (A + B*Log[e*((a + b*x)/(c
 + d*x))^n])/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(b/(b - (d*(a
+ b*x))/(c + d*x))^3 - 5/(2*(b - (d*(a + b*x))/(c + d*x))^2) + 1/(b*(b - (
d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a +
b*x))/(c + d*x)]/b^2))/(12*d^3)))/(5*b) + (2*(((a + b*x)^3*(A + B*Log[e*((
a + b*x)/(c + d*x))^n])^2)/(4*b*(c + d*x)^3*(b - (d*(a + b*x))/(c + d*x))^
4) - (B*n*(((a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*b*(c +
d*x)^3*(b - (d*(a + b*x))/(c + d*x))^3) - (B*n*(b^2/(2*d^3*(b - (d*(a +...
```

### 3.179.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 1195
```
Int[((d_.) + (e_.)*(x_))^(m_.)*((f_.) + (g_.)*(x_))^(n_.)*((a_.) + (b_.)*(x
_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :> Int[ExpandIntegrand[(d + e*x)^m*(f +
 g*x)^n*(a + b*x + c*x^2)^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n}, x
] && IGtQ[p, 0]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.179. $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

## 3.179. $\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.179.4 Maple [F]

$$\int (bgx+ag)^2 (dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

### 3.179.5 Fricas [F]

$$\int (ag+bgx)^2(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (bgx+ag)^2(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="fricas")
```

output
```
integral(A^2*b^2*d^3*g^2*i^3*x^5 + A^2*a^2*c^3*g^2*i^3 + (3*A^2*b^2*c*d^2
+ 2*A^2*a*b*d^3)*g^2*i^3*x^4 + (3*A^2*b^2*c^2*d + 6*A^2*a*b*c*d^2 + A^2*a^
2*d^3)*g^2*i^3*x^3 + (A^2*b^2*c^3 + 6*A^2*a*b*c^2*d + 3*A^2*a^2*c*d^2)*g^2
*i^3*x^2 + (2*A^2*a*b*c^3 + 3*A^2*a^2*c^2*d)*g^2*i^3*x + (B^2*b^2*d^3*g^2*
i^3*x^5 + B^2*a^2*c^3*g^2*i^3 + (3*B^2*b^2*c*d^2 + 2*B^2*a*b*d^3)*g^2*i^3*
x^4 + (3*B^2*b^2*c^2*d + 6*B^2*a*b*c*d^2 + B^2*a^2*d^3)*g^2*i^3*x^3 + (B^2
*b^2*c^3 + 6*B^2*a*b*c^2*d + 3*B^2*a^2*c*d^2)*g^2*i^3*x^2 + (2*B^2*a*b*c^3
 + 3*B^2*a^2*c^2*d)*g^2*i^3*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b
^2*d^3*g^2*i^3*x^5 + A*B*a^2*c^3*g^2*i^3 + (3*A*B*b^2*c*d^2 + 2*A*B*a*b*d^
3)*g^2*i^3*x^4 + (3*A*B*b^2*c^2*d + 6*A*B*a*b*c*d^2 + A*B*a^2*d^3)*g^2*i^3
*x^3 + (A*B*b^2*c^3 + 6*A*B*a*b*c^2*d + 3*A*B*a^2*c*d^2)*g^2*i^3*x^2 + (2*
A*B*a*b*c^3 + 3*A*B*a^2*c^2*d)*g^2*i^3*x)*log(e*((b*x + a)/(d*x + c))^n),
x)
```

### 3.179.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**
2,x)
```

output
```
Timed out
```

### 3.179.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5931 vs. $2(937) = 1874$.

Time = 0.78 (sec) , antiderivative size = 5931, normalized size of antiderivative = 6.08

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="maxima")
```

output
```
1/3*A*B*b^2*d^3*g^2*i^3*x^6*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/6*A
^2*b^2*d^3*g^2*i^3*x^6 + 6/5*A*B*b^2*c*d^2*g^2*i^3*x^5*log(e*(b*x/(d*x + c
) + a/(d*x + c))^n) + 4/5*A*B*a*b*d^3*g^2*i^3*x^5*log(e*(b*x/(d*x + c) + a
/(d*x + c))^n) + 3/5*A^2*b^2*c*d^2*g^2*i^3*x^5 + 2/5*A^2*a*b*d^3*g^2*i^3*x
^5 + 3/2*A*B*b^2*c^2*d*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 3*A*B*a*b*c*d^2*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2
*A*B*a^2*d^3*g^2*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3/4*A^2*
b^2*c^2*d*g^2*i^3*x^4 + 3/2*A^2*a*b*c*d^2*g^2*i^3*x^4 + 1/4*A^2*a^2*d^3*g^
2*i^3*x^4 + 2/3*A*B*b^2*c^3*g^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c)
)^n) + 4*A*B*a*b*c^2*d*g^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
+ 2*A*B*a^2*c*d^2*g^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3
*A^2*b^2*c^3*g^2*i^3*x^3 + 2*A^2*a*b*c^2*d*g^2*i^3*x^3 + A^2*a^2*c*d^2*g^2
*i^3*x^3 + 2*A*B*a*b*c^3*g^2*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
) + 3*A*B*a^2*c^2*d*g^2*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A
^2*a*b*c^3*g^2*i^3*x^2 + 3/2*A^2*a^2*c^2*d*g^2*i^3*x^2 - 1/180*A*B*b^2*d^3
*g^2*i^3*n*(60*a^6*log(b*x + a)/b^6 - 60*c^6*log(d*x + c)/d^6 + (12*(b^5*c
*d^4 - a*b^4*d^5)*x^5 - 15*(b^5*c^2*d^3 - a^2*b^3*d^5)*x^4 + 20*(b^5*c^3*d
^2 - a^3*b^2*d^5)*x^3 - 30*(b^5*c^4*d - a^4*b*d^5)*x^2 + 60*(b^5*c^5 - a^5
*d^5)*x)/(b^5*d^5)) + 1/10*A*B*b^2*c*d^2*g^2*i^3*n*(12*a^5*log(b*x + a)/b^
5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^4 - 4*(b^4*c...
```

### 3.179.8 Giac [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)^2*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x
, algorithm="giac")
```

output
```
Timed out
```

### 3.179.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^2(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)^2\,(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2,x)
```

output
```
int((a*g + b*g*x)^2*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))
^2, x)
```

# 3.180 $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

## 3.180.1 Optimal result

Integrand size = 43, antiderivative size = 786

$$
\begin{aligned}
&\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx \\
&= \frac{B^2(bc-ad)^4gi^3n^2x}{60b^3d}+\frac{B^2(bc-ad)^3gi^3n^2(c+dx)^2}{30b^2d^2}+\frac{B^2(bc-ad)^2gi^3n^2(c+dx)^3}{30bd^2} \\
&\quad-\frac{B(bc-ad)^4gi^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^4d} \\
&\quad-\frac{B(bc-ad)^3gi^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10b^4} \\
&\quad+\frac{3B(bc-ad)^3gi^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{20b^2d^2} \\
&\quad+\frac{B(bc-ad)^2gi^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{30bd^2} \\
&\quad-\frac{B(bc-ad)gi^3n(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{10d^2} \\
&\quad+\frac{(bc-ad)^3gi^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^4} \\
&\quad+\frac{(bc-ad)^2gi^3(a+bx)^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{10b^3} \\
&\quad+\frac{3(bc-ad)gi^3(a+bx)^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{20b^2} \\
&\quad+\frac{gi^3(a+bx)^2(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b} \\
&\quad-\frac{B(bc-ad)^5gi^3n\left(A+Bn+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{10b^4d^2} \\
&\quad-\frac{B^2(bc-ad)^5gi^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d^2}-\frac{11B^2(bc-ad)^5gi^3n^2\log(c+dx)}{60b^4d^2} \\
&\quad-\frac{B^2(bc-ad)^5gi^3n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{10b^4d^2}
\end{aligned}
$$

output
```
1/60*B^2*(-a*d+b*c)^4*g*i^3*n^2*x/b^3/d+1/30*B^2*(-a*d+b*c)^3*g*i^3*n^2*(d
*x+c)^2/b^2/d^2+1/30*B^2*(-a*d+b*c)^2*g*i^3*n^2*(d*x+c)^3/b/d^2-1/10*B*(-a
*d+b*c)^4*g*i^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/d-1/10*B*(-a
*d+b*c)^3*g*i^3*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4+3/20*B*(-a
*d+b*c)^3*g*i^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/d^2+1/30*B
*(-a*d+b*c)^2*g*i^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d^2-1/10
*B*(-a*d+b*c)*g*i^3*n*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^2+1/20*(
-a*d+b*c)^3*g*i^3*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4+1/10*(-a
*d+b*c)^2*g*i^3*(b*x+a)^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3+3/
20*(-a*d+b*c)*g*i^3*(b*x+a)^2*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/
b^2+1/5*g*i^3*(b*x+a)^2*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b-1/10
*B*(-a*d+b*c)^5*g*i^3*n*(A+B*n+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/
b/(d*x+c))/b^4/d^2-1/12*B^2*(-a*d+b*c)^5*g*i^3*n^2*ln((b*x+a)/(d*x+c))/b^4
/d^2-11/60*B^2*(-a*d+b*c)^5*g*i^3*n^2*ln(d*x+c)/b^4/d^2-1/10*B^2*(-a*d+b*c
)^5*g*i^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/d^2
```

### 3.180.2 Mathematica [A] (verified)

Time = 0.44 (sec) , antiderivative size = 945, normalized size of antiderivative = 1.20

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{gi^3\left(-5(bc-ad)(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+4b(c+dx)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{5B(bc-ad)^2n(}{}\right.}{}$$

input
```
Integrate[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])^2,x]
```

output
```
(g*i^3*(-5*(b*c - a*d)*(c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2 + 4*b*(c + d*x)^5*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (5*B*(b*c -
 a*d)^2*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a
*d)*Log[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2
 + 2*(b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a
 + b*x)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^
n]) + 6*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
- 6*B*(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x]*(Lo
g[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)])))/(3*b^4) - (B*(b*c - a*d)*n*(24*A*b*d*(b*c - a*d)^3*x
- 12*B*(b*c - a*d)^3*n*(b*d*x + (b*c - a*d)*Log[a + b*x]) - 4*B*(b*c - a*d
)^2*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(b*c - a*d)^2*Log[a + b*x
]) - B*(b*c - a*d)*n*(6*b*d*(b*c - a*d)^2*x + 3*b^2*(b*c - a*d)*(c + d*x)^
2 + 2*b^3*(c + d*x)^3 + 6*(b*c - a*d)^3*Log[a + b*x]) + 24*B*d*(b*c - a*d)
^3*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] + 12*b^2*(b*c - a*d)^2*(c + d*
x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 8*b^3*(b*c - a*d)*(c + d*x)^
3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 6*b^4*(c + d*x)^4*(A + B*Log[e*
((a + b*x)/(c + d*x))^n]) + 24*(b*c - a*d)^4*Log[a + b*x]*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]) - 24*B*(b*c - a*d)^4*n*Log[c + d*x] - 12*B*(b*c ...
```

## 3.180.3 Rubi [A] (verified)

Time = 2.15 (sec) , antiderivative size = 1002, normalized size of antiderivative = 1.27, number of steps used = 20, number of rules used = 19, $\frac{\text{number of rules}}{\text{integrand size}} = 0.442$, Rules used = {2961, 2783, 2782, 27, 86, 2009, 2783, 2782, 27, 86, 2009, 2783, 2773, 49, 2009, 2781, 2784, 2754, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \text{2961}$$

$$gi^3(bc-ad)^5\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^6}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2783}$$

$$gi^3(bc-ad)^5\left(-\frac{2Bn\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{3\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}+\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{5b(c+dx)^2\left(b-\frac{d}{}\right.}\right.$$

$\downarrow$ 2782

$$gi^3(bc-ad)^5\left(-\frac{2Bn\left(-Bn\int -\frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{12d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b}+\frac{3\int \frac{(a+bx)\left(A\right.}{(c+d}}{}\right.$$

$\downarrow$ 27

$$gi^3(bc-ad)^5\left(-\frac{2Bn\left(\frac{Bn\int \frac{(c+dx)\left(b-\frac{4d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b}+\frac{3\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a}{c}\right.\right.\right.}{(c+dx)\left(b-\frac{d(a+b}{c+d}\right.}}{5b}\right.$$

$\downarrow$ 86

$$gi^3(bc-ad)^5\left(-\frac{2Bn\left(\frac{Bn\int \left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{c+dx}{b^3(a+bx)}\right)d\frac{a+bx}{c+dx}}{12d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\right.}{5b}\right.$$

$\downarrow$ 2009

---

3.180. $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$gi^3(bc-ad)^5\left(\frac{3\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}}{5b}-\frac{2Bn\left(-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2783

$$gi^3(bc-ad)^5\left(\frac{3\left(-\frac{Bn\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b}+\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}\right)}{5b}-\right.$$

$\downarrow$ 2782

$$gi^3(bc-ad)^5\left(\frac{3\left(-\frac{Bn\left(-Bn\int -\frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{6d^2(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}}{2b}\right.}{5b}\right.$$

$\downarrow$ 27

3.180. $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$gi^3(bc-ad)^5\left(\frac{3\left(-\frac{Bn\left(\frac{Bn\int\frac{(c+dx)\left(b-\frac{3d(a+bx)}{c+dx}\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}+\frac{\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+}{c+}}{2b}\right.}{5b}\right.$$

$\downarrow$ 86

$$gi^3(bc-ad)^5\left(\frac{3\left(-\frac{Bn\left(\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2d}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{6d^2}-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)}{2b}\right.}{5b}\right.$$

$\downarrow$ 2009

3.180. $\int(ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$gi^3(bc-$$

$$ad)^5 \left( \frac{3\left( \frac{\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2b} - \frac{Bn\left(-\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}\right.}{6}\right.}{2b}\right.}{5b}\right.$$

$\downarrow$ 2783

$$(bc-$$

$$ad)^5gi^3\left(\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2773

3.180. $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-ad)^5gi^3\left(\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 49

$$(bc-ad)^5gi^3\left(\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}\right.}{}\right.}{5b}\right.$$

---

3.180. $\int(ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$\downarrow$ 2009

$$(bc-ad)^5gi^3\left(\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2781

---

3.180. $\int(ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc - ad)^5 g i^3 \left( \frac{(a+bx)^2 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2\left(b - \frac{d(a+bx)}{c+dx}\right)^5} - \frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3} + \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4} + \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2784

---

3.180. $\int (ag+bgx)(ci+dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

$$(bc-ad)^5gi^3\left(\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2754

---

3.180. $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-ad)^5gi^3\left(\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}\right.}{}\right.}{5b}\right.$$

$\downarrow$ 2838

3.180. $\int(ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$(bc-ad)^5gi^3\left(\frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{5b(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^5}-\frac{2Bn\left(-\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4d^2\left(b-\frac{d(a+bx)}{c+dx}\right)^4}+\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}\right.}{}\right.}{5b}\right.$$

input
```
Int[(a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
,x]
```

3.180. $\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

output
```
(b*c - a*d)^5*g*i^3*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2
)/(5*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^5) - (2*B*n*((b*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(4*d^2*(b - (d*(a + b*x))/(c + d*x))^4) - (
A + B*Log[e*((a + b*x)/(c + d*x))^n])/(3*d^2*(b - (d*(a + b*x))/(c + d*x))
^3) + (B*n*(-(b - (d*(a + b*x))/(c + d*x))^(-3) + 1/(2*b*(b - (d*(a + b*x)
)/(c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c
 + d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(12*d^2)))/(5*b) + (
3*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(4*b*(c + d*x)^2
*(b - (d*(a + b*x))/(c + d*x))^4) - (B*n*((b*(A + B*Log[e*((a + b*x)/(c +
d*x))^n]))/(3*d^2*(b - (d*(a + b*x))/(c + d*x))^3) - (A + B*Log[e*((a + b*
x)/(c + d*x))^n])/(2*d^2*(b - (d*(a + b*x))/(c + d*x))^2) + (B*n*(-(b - (d
*(a + b*x))/(c + d*x))^(-2) + 1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(a
 + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(6*d^2)))/
(2*b) + (((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*b*(c +
d*x)^2*(b - (d*(a + b*x))/(c + d*x))^3) - (2*B*n*(((a + b*x)^2*(A + B*Log[
e*((a + b*x)/(c + d*x))^n]))/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x)
)^2) - (B*n*(b/(d^2*(b - (d*(a + b*x))/(c + d*x))) + Log[b - (d*(a + b*x))
/(c + d*x)]/d^2))/(2*b)))/(3*b) + (((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2)/(2*b*(c + d*x)^2*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(
((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(d*(c + d*x)*(b - (d...
```

### 3.180.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 49
```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x]
&& IGtQ[m, 0] && IGtQ[m + n + 2, 0]
```

rule 86
```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x], x] /;
 FreeQ[{a, b, c, d, e, f, n}, x] && ((ILtQ[n, 0] && ILtQ[p, 0]) || EqQ[p, 1
] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p + 5*(n + 2), 0] || GeQ[n + p
+ 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b, c, d, e, f])))
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

3.180. $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2773
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_)^(r_.))^(q_), x_Symbol] :> Simp[(f*x)^(m + 1)*(d + e*x^r)^(q + 1)*((a +
 b*Log[c*x^n])/(d*f*(m + 1))), x] - Simp[b*(n/(d*(m + 1)))   Int[(f*x)^m*(d
 + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, f, m, n, q, r}, x] && Eq
Q[m + r*(q + 1) + 1, 0] && NeQ[m, -1]
```

rule 2781
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + Simp[b*n*(p/(d*(q + 1)))   Int[(f*x)
^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b, c,
d, e, f, m, n, q}, x] && EqQ[m + q + 2, 0] && IGtQ[p, 0] && LtQ[q, -1]
```

rule 2782
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*(x_)^(m_.)*((d_) + (e_.)*(x_))^(q
_), x_Symbol] :> With[{u = IntHide[x^m*(d + e*x)^q, x]}, Simp[(a + b*Log[c*
x^n])   u, x] - Simp[b*n   Int[SimplifyIntegrand[u/x, x], x], x]] /; FreeQ[
{a, b, c, d, e, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[m, 0]
```

rule 2783
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_))^(q_), x_Symbol] :> Simp[(-(f*x)^(m + 1))*(d + e*x)^(q + 1)*((a
+ b*Log[c*x^n])^p/(d*f*(q + 1))), x] + (Simp[(m + q + 2)/(d*(q + 1))   Int[
(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^p, x], x] + Simp[b*n*(p/(d*(q
+ 1)))   Int[(f*x)^m*(d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p - 1), x], x])
/; FreeQ[{a, b, c, d, e, f, n}, x] && ILtQ[m + q + 2, 0] && IGtQ[p, 0] && L
tQ[q, -1] && GtQ[m, 0]
```

rule 2784
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((f_.)*(x_))^(m_.)*((d_) + (e_.)*
(x_))^(q_.), x_Symbol] :> Simp[(f*x)^m*(d + e*x)^(q + 1)*((a + b*Log[c*x^n]
)/(e*(q + 1))), x] - Simp[f/(e*(q + 1))   Int[(f*x)^(m - 1)*(d + e*x)^(q +
1)*(a*m + b*n + b*m*Log[c*x^n]), x], x] /; FreeQ[{a, b, c, d, e, f, m, n},
x] && ILtQ[q, -1] && GtQ[m, 0]
```

## 3.180. $\int (ag+bgx)(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.180.4 Maple [F]

$$\int (bgx+ag)\,(dix+ci)^{3} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}dx$$

input
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

### 3.180.5 Fricas [F]

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = \int (bgx+ag)(dix+ci)^{3}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}\,dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="fricas")
```

output
```
integral(A^2*b*d^3*g*i^3*x^4 + A^2*a*c^3*g*i^3 + (3*A^2*b*c*d^2 + A^2*a*d^
3)*g*i^3*x^3 + 3*(A^2*b*c^2*d + A^2*a*c*d^2)*g*i^3*x^2 + (A^2*b*c^3 + 3*A^
2*a*c^2*d)*g*i^3*x + (B^2*b*d^3*g*i^3*x^4 + B^2*a*c^3*g*i^3 + (3*B^2*b*c*d
^2 + B^2*a*d^3)*g*i^3*x^3 + 3*(B^2*b*c^2*d + B^2*a*c*d^2)*g*i^3*x^2 + (B^2
*b*c^3 + 3*B^2*a*c^2*d)*g*i^3*x)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B
*b*d^3*g*i^3*x^4 + A*B*a*c^3*g*i^3 + (3*A*B*b*c*d^2 + A*B*a*d^3)*g*i^3*x^3
 + 3*(A*B*b*c^2*d + A*B*a*c*d^2)*g*i^3*x^2 + (A*B*b*c^3 + 3*A*B*a*c^2*d)*g
*i^3*x)*log(e*((b*x + a)/(d*x + c))^n), x)
```

### 3.180.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x
)
```

output
```
Timed out
```

### 3.180.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3724 vs. $2(753) = 1506$.

Time = 0.76 (sec) , antiderivative size = 3724, normalized size of antiderivative = 4.74

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx=\text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="maxima")
```

output
```
2/5*A*B*b*d^3*g*i^3*x^5*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/5*A^2*b
*d^3*g*i^3*x^5 + 3/2*A*B*b*c*d^2*g*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x +
 c))^n) + 1/2*A*B*a*d^3*g*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) +
 3/4*A^2*b*c*d^2*g*i^3*x^4 + 1/4*A^2*a*d^3*g*i^3*x^4 + 2*A*B*b*c^2*d*g*i^3
*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 2*A*B*a*c*d^2*g*i^3*x^3*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*b*c^2*d*g*i^3*x^3 + A^2*a*c*d^2*g
*i^3*x^3 + A*B*b*c^3*g*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 3*
A*B*a*c^2*d*g*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/2*A^2*b*c
^3*g*i^3*x^2 + 3/2*A^2*a*c^2*d*g*i^3*x^2 + 1/30*A*B*b*d^3*g*i^3*n*(12*a^5*
log(b*x + a)/b^5 - 12*c^5*log(d*x + c)/d^5 - (3*(b^4*c*d^3 - a*b^3*d^4)*x^
4 - 4*(b^4*c^2*d^2 - a^2*b^2*d^4)*x^3 + 6*(b^4*c^3*d - a^3*b*d^4)*x^2 - 12
*(b^4*c^4 - a^4*d^4)*x)/(b^4*d^4)) - 1/4*A*B*b*c*d^2*g*i^3*n*(6*a^4*log(b*
x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(
b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) - 1/12*A*
B*a*d^3*g*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*c^4*log(d*x + c)/d^4 + (2*(b^3
*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2*b*d^3)*x^2 + 6*(b^3*c^3 - a^3
*d^3)*x)/(b^3*d^3)) + A*B*b*c^2*d*g*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*
log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^
2*d^2)) + A*B*a*c*d^2*g*i^3*n*(2*a^3*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)
/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) -...
```

### 3.180.8 Giac [F]

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2dx = \int (bgx+ag)(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2dx$$

input
```
integrate((b*g*x+a*g)*(d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x,
algorithm="giac")
```

output
```
integrate((b*g*x + a*g)*(d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n)
+ A)^2, x)
```

### 3.180.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int (a\,g+b\,g\,x)\,\left(c\,i+d\,i\,x\right)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2\,dx$$

input
```
int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
,x)
```

output
```
int((a*g + b*g*x)*(c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2
, x)
```

# 3.181 $\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$



## 3.181.1 Optimal result

Integrand size = 35, antiderivative size = 454

$$\begin{aligned}\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx &= \frac{5B^2(bc-ad)^3i^3n^2x}{12b^3}+\frac{B^2(bc-ad)^2i^3n^2(c+dx)^2}{12b^2d} \\ &- \frac{B(bc-ad)^3i^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^4} \\ &- \frac{B(bc-ad)^2i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{4b^2d} \\ &- \frac{B(bc-ad)i^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{6bd} \\ &+ \frac{i^3(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{4d} \\ &+ \frac{5B^2(bc-ad)^4i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{12b^4d}+\frac{11B^2(bc-ad)^4i^3n^2\log(c+dx)}{12b^4d} \\ &+ \frac{B(bc-ad)^4i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d} \\ &- \frac{B^2(bc-ad)^4i^3n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{2b^4d}\end{aligned}$$

output
```
5/12*B^2*(-a*d+b*c)^3*i^3*n^2*x/b^3+1/12*B^2*(-a*d+b*c)^2*i^3*n^2*(d*x+c)^
2/b^2/d-1/2*B*(-a*d+b*c)^3*i^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b
^4-1/4*B*(-a*d+b*c)^2*i^3*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/
d-1/6*B*(-a*d+b*c)*i^3*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/d+1/4
*i^3*(d*x+c)^4*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d+5/12*B^2*(-a*d+b*c)^4*i
^3*n^2*ln((b*x+a)/(d*x+c))/b^4/d+11/12*B^2*(-a*d+b*c)^4*i^3*n^2*ln(d*x+c)/
b^4/d+1/2*B*(-a*d+b*c)^4*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x
+c)/d/(b*x+a))/b^4/d-1/2*B^2*(-a*d+b*c)^4*i^3*n^2*polylog(2,b*(d*x+c)/d/(b
*x+a))/b^4/d
```

### 3.181.2 Mathematica [A] (verified)

Time = 0.12 (sec) , antiderivative size = 409, normalized size of antiderivative = 0.90

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$

$$= \frac{i^3\left(\left(c+dx\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{B(bc-ad)n\left(6Abd(bc-ad)^2x-3B(bc-ad)^2n(bdx+(bc-ad)\log(a+bx))-B(bc-ad)n(2bd}{}\right.}{}$$

input
```
Integrate[(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

output
```
(i^3*((c + d*x)^4*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - (B*(b*c - a*d
)*n*(6*A*b*d*(b*c - a*d)^2*x - 3*B*(b*c - a*d)^2*n*(b*d*x + (b*c - a*d)*Lo
g[a + b*x]) - B*(b*c - a*d)*n*(2*b*d*(b*c - a*d)*x + b^2*(c + d*x)^2 + 2*(
b*c - a*d)^2*Log[a + b*x]) + 6*B*d*(b*c - a*d)^2*(a + b*x)*Log[e*((a + b*x
)/(c + d*x))^n] + 3*b^2*(b*c - a*d)*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n]) + 2*b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) +
6*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 6*B*
(b*c - a*d)^3*n*Log[c + d*x] - 3*B*(b*c - a*d)^3*n*(Log[a + b*x]*(Log[a +
b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b
*c) + a*d)])))/(3*b^4)))/(4*d)
```

### 3.181.3 Rubi [A] (verified)

Time = 1.34 (sec) , antiderivative size = 535, normalized size of antiderivative = 1.18, number of steps used = 16, number of rules used = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {2951, 2756, 2789, 2756, 54, 2009, 2789, 2756, 54, 2009, 2789, 2751, 16, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ci+dix)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2 dx$$

$$\Big\downarrow \text{2951}$$

$$i^3(bc-ad)^4\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^5}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2756}$$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{2d}\right)$$

$$\Big\downarrow \text{2789}$$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{2d}\right)$$

$$\Big\downarrow \text{2756}$$

$$i^3(bc - ad)^4 \left( \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right.}}{b}}{2d}\right.$$

$$\downarrow 54$$

$$i^3(bc - ad)^4 \left( \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\int\left(\frac{d}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{c+}{b^3(a}\right.}{3d}\right.}{b}}{2d}\right.$$

$$\downarrow 2009$$

3.181. $\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^3}-\frac{\log\left(b\right.}{}\right.}{}\right.}{}\right)}{2d}\right)$$

$$\Big\downarrow \text{ 2789}$$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+b}{c+d}\right.\right.}{3d\left(b-\frac{d(a+}{c+}\right.}\right.}{}\right)}{2d}\right)$$

$$\Big\downarrow \text{ 2756}$$

3.181. $\int(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}}{b}}{b}\right.}{}\right.$$

$\downarrow$ 54

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\int\cdots}{b}\right.}{}\right.$$

$\downarrow$ 2009

3.181. $\int(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$i^3(bc - ad)^4 \left( \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}{4d\left(b - \frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left( \frac{\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b} + \frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d}{\ldots}\right.}{b\ldots}\right.}{\ldots}\right.}{b}}{b}\right.}{\ldots}\right.$$

$\downarrow$ 2789

$$i^3(bc - ad)^4 \left( \frac{\left(B \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right) + A\right)^2}{4d\left(b - \frac{d(a+bx)}{c+dx}\right)^4} - \frac{Bn\left( \frac{\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b} + \frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b} + \frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{2d\left(b-\frac{d(a+bx}{c+dx}\right.}\right.}{b}\right.}{\ldots}\right.$$

$\downarrow$ 2751

3.181. $\int (ci + dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}}{b}}{b}}{b}\right)$$

$\downarrow$ 16

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\right)}{b}}{b}}{b}\right)$$

$\downarrow$ 2779

---

3.181. $\int(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{\frac{\frac{Bn\int\frac{(c+dx)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b}}{b}}{b}+\frac{d\left(\frac{(a+bx)(B\cdots}{b(c+\cdots}\cdots\right.}{\cdots}\cdots\right.}{\cdots}\right.$$

$\downarrow$ 2838

$$i^3(bc-ad)^4\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{4d\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{\frac{Bn\,\mathrm{PolyL}\cdots}{\cdots}}{\cdots}\cdots\right.}{\cdots}\right.$$

input
```
Int[(c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2,x]
```

## 3.181. $\int (ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$

output
```
(b*c - a*d)^4*i^3*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(4*d*(b - (d*(
a + b*x))/(c + d*x))^4) - (B*n*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n])
/(3*d*(b - (d*(a + b*x))/(c + d*x))^3) - (B*n*(1/(2*b*(b - (d*(a + b*x))/(
c + d*x))^2) + 1/(b^2*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c +
d*x)]/b^3 - Log[b - (d*(a + b*x))/(c + d*x)]/b^3))/(3*d)))/b + ((d*((A + B
*Log[e*((a + b*x)/(c + d*x))^n])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (
B*n*(1/(b*(b - (d*(a + b*x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 -
Log[b - (d*(a + b*x))/(c + d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*
Log[e*((a + b*x)/(c + d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)
)) + (B*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*P
olyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b)/b)/b))/(2*d))
```

### 3.181.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

## 3.181. $\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

### 3.181.4 Maple [F]

$$\int (dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2 dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

### 3.181.5 Fricas [F]

$$\int (ci + dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (dix + ci)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) + A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="f
ricas")
```

output
```
integral(A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*c
^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2*
c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*d
^2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^
n), x)
```

### 3.181.6 Sympy [F(-1)]

Timed out.

$$\int (ci + dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2,x)
```

output
```
Timed out
```

### 3.181.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2129 vs. $2(433) = 866$.

Time = 0.73 (sec) , antiderivative size = 2129, normalized size of antiderivative = 4.69

$$\int (ci + dix)^3 \left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx = \text{Too large to display}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="m
axima")
```

output
```
1/2*A*B*d^3*i^3*x^4*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/4*A^2*d^3*i
^3*x^4 + 2*A*B*c*d^2*i^3*x^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + A^2*
c*d^2*i^3*x^3 + 3*A*B*c^2*d*i^3*x^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
 + 3/2*A^2*c^2*d*i^3*x^2 - 1/12*A*B*d^3*i^3*n*(6*a^4*log(b*x + a)/b^4 - 6*
c^4*log(d*x + c)/d^4 + (2*(b^3*c*d^2 - a*b^2*d^3)*x^3 - 3*(b^3*c^2*d - a^2
*b*d^3)*x^2 + 6*(b^3*c^3 - a^3*d^3)*x)/(b^3*d^3)) + A*B*c*d^2*i^3*n*(2*a^3
*log(b*x + a)/b^3 - 2*c^3*log(d*x + c)/d^3 - ((b^2*c*d - a*b*d^2)*x^2 - 2*
(b^2*c^2 - a^2*d^2)*x)/(b^2*d^2)) - 3*A*B*c^2*d*i^3*n*(a^2*log(b*x + a)/b^
2 - c^2*log(d*x + c)/d^2 + (b*c - a*d)*x/(b*d)) + 2*A*B*c^3*i^3*n*(a*log(b
*x + a)/b - c*log(d*x + c)/d) + 2*A*B*c^3*i^3*x*log(e*(b*x/(d*x + c) + a/(
d*x + c))^n) + A^2*c^3*i^3*x - 1/12*(26*a*b^2*c^3*d*i^3*n^2 - 21*a^2*b*c^2
*d^2*i^3*n^2 + 6*a^3*c*d^3*i^3*n^2 - (11*i^3*n^2 - 6*i^3*n*log(e))*b^3*c^4
)*B^2*log(d*x + c)/(b^3*d) - 1/2*(b^4*c^4*i^3*n^2 - 4*a*b^3*c^3*d*i^3*n^2
+ 6*a^2*b^2*c^2*d^2*i^3*n^2 - 4*a^3*b*c*d^3*i^3*n^2 + a^4*d^4*i^3*n^2)*(lo
g(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))*B^2/(b^4*d) + 1/12*(3*B^2*b^4*d^4*i^3*x^4*log(e)^2 + 6*B^2*b^4*c^
4*i^3*n^2*log(b*x + a)*log(d*x + c) - 3*B^2*b^4*c^4*i^3*n^2*log(d*x + c)^2
 + 2*(a*b^3*d^4*i^3*n*log(e) - (i^3*n*log(e) - 6*i^3*log(e)^2)*b^4*c*d^3)*
B^2*x^3 + ((i^3*n^2 - 9*i^3*n*log(e) + 18*i^3*log(e)^2)*b^4*c^2*d^2 - 2*(i
^3*n^2 - 6*i^3*n*log(e))*a*b^3*c*d^3 + (i^3*n^2 - 3*i^3*n*log(e))*a^2*b...
```

### 3.181.8 Giac [F]

$$\int (ci+dix)^3 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (dix+ci)^3 \left(B\log \left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2,x, algorithm="g
iac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2, x)
```

### 3.181.9 Mupad [F(-1)]

Timed out.

$$\int (ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int (c\,i+d\,i\,x)^3 \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2 dx$$

input
```
int((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2,x)
```

output
```
int((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2, x)
```

# 3.182 $\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

## 3.182.1 Optimal result

Integrand size = 45, antiderivative size = 762

$$\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx \\
&= \frac{B^2d(bc-ad)^2i^3n^2x}{3b^3g} - \frac{5Bd(bc-ad)^2i^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^4g} \\
&\quad - \frac{B(bc-ad)i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3b^2g} \\
&\quad + \frac{d(bc-ad)^2i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g} \\
&\quad + \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g} + \frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg} \\
&\quad + \frac{2B(bc-ad)^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g} \\
&\quad + \frac{B^2(bc-ad)^3i^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{3b^4g} + \frac{2B^2(bc-ad)^3i^3n^2\log(c+dx)}{b^4g} \\
&\quad + \frac{5B(bc-ad)^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g} \\
&\quad - \frac{(bc-ad)^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&\quad + \frac{2B^2(bc-ad)^3i^3n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g} - \frac{5B^2(bc-ad)^3i^3n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3b^4g} \\
&\quad + \frac{2B(bc-ad)^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g} \\
&\quad + \frac{2B^2(bc-ad)^3i^3n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g}
\end{aligned}$$

output
```
1/3*B^2*d*(-a*d+b*c)^2*i^3*n^2*x/b^3/g-5/3*B*d*(-a*d+b*c)^2*i^3*n*(b*x+a)*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/g-1/3*B*(-a*d+b*c)*i^3*n*(d*x+c)^2*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g+d*(-a*d+b*c)^2*i^3*(b*x+a)*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))^2/b^4/g+1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*((b*x+a
)/(d*x+c))^n))^2/b^2/g+1/3*i^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2
/b/g+2*B*(-a*d+b*c)^3*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/
b/(d*x+c))/b^4/g+1/3*B^2*(-a*d+b*c)^3*i^3*n^2*ln((b*x+a)/(d*x+c))/b^4/g+2*
B^2*(-a*d+b*c)^3*i^3*n^2*ln(d*x+c)/b^4/g+5/3*B*(-a*d+b*c)^3*i^3*n*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g-(-a*d+b*c)^3*i^3*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g+2*B^2*(-a
*d+b*c)^3*i^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g-5/3*B^2*(-a*d+b*c)^
3*i^3*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g+2*B*(-a*d+b*c)^3*i^3*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g+2*B^2*(-a
*d+b*c)^3*i^3*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^4/g
```

### 3.182.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4969 vs. $2(762) = 1524$.

Time = 7.27 (sec) , antiderivative size = 4969, normalized size of antiderivative = 6.52

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x),x]
```

output
```
(i^3*(36*b*d*(3*b^2*c^2 - 3*a*b*c*d + a^2*d^2)*x*(A + B*Log[e*((a + b*x)/(
c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 + 18*b^2*d^2*(3*b*c - a*d)*
x^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^
2 + 12*b^3*d^3*x^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*
x)/(c + d*x)])^2 + 36*(b*c - a*d)^3*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(
c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 - 108*b^2*B*c^2*n*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(a*d*Log[a/b
+ x]^2 - 2*a*d*Log[a/b + x]*(1 + Log[a + b*x]) + 2*(-(b*c) + a*d + Log[c/d
 + x]*(b*c + a*d*Log[a + b*x] - a*d*Log[(d*(a + b*x))/(-(b*c) + a*d)]) + (
-(b*d*x) + a*d*Log[a + b*x])*Log[(a + b*x)/(c + d*x)]) - 2*a*d*PolyLog[2,
(b*(c + d*x))/(b*c - a*d)]) - 12*B*n*(-A - B*Log[e*((a + b*x)/(c + d*x))^n
] + B*n*Log[(a + b*x)/(c + d*x)])*(6*a^2*b*c*d^2 - 6*a^3*d^3 + 2*b^3*c^2*d
*x + 3*a*b^2*c*d^2*x - 5*a^2*b*d^3*x - b^3*c*d^2*x^2 + a*b^2*d^3*x^2 - 3*a
^3*d^3*Log[a/b + x]^2 - 6*a^2*b*c*d^2*Log[c/d + x] + 5*a^3*d^3*Log[a + b*x
] - 6*a^3*d^3*Log[c/d + x]*Log[a + b*x] + 6*a^3*d^3*Log[a/b + x]*(1 + Log[
a + b*x]) + 6*a^3*d^3*Log[c/d + x]*Log[(d*(a + b*x))/(-(b*c) + a*d)] + 6*a
^2*b*d^3*x*Log[(a + b*x)/(c + d*x)] - 3*a*b^2*d^3*x^2*Log[(a + b*x)/(c + d
*x)] + 2*b^3*d^3*x^3*Log[(a + b*x)/(c + d*x)] - 6*a^3*d^3*Log[a + b*x]*Log
[(a + b*x)/(c + d*x)] - 2*b^3*c^3*Log[c + d*x] - 3*a*b^2*c^2*d*Log[c + d*x
] + 6*a^3*d^3*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 36*b^3*B*c^3*n*(...
```

### 3.182.3 Rubi [A] (verified)

Time = 3.08 (sec) , antiderivative size = 901, normalized size of antiderivative = 1.18, number of steps used = 17, number of rules used = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.356$, Rules used = {2961, 2789, 2756, 2789, 2756, 54, 2009, 2789, 2751, 16, 2755, 2754, 2779, 2821, 2838, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ag+bgx}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{i^3(bc-ad)^3\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{g}$$

$$\Big\downarrow \text{ 2789}$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \text{ 2756}$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{3d}\right)}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}\right)}{g}$$

$$\Big\downarrow \text{ 2789}$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\int \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{b}+\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}\right)}{3d}\right)}{b}+\frac{d\int \frac{\left(A+B\log \cdots\right.}{\left(b-\cdots\right.}}{\cdots}\cdots\right)}{g}$$

$$\Big\downarrow \text{ 2756}$$

3.182. $\displaystyle\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\frac{c+dx}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\cdots\right.}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}\cdots}{b}\cdots}{3d}\cdots\right)}{b}\cdots\right)}{g\cdots}$$

$\Big\downarrow$ 54

---

3.182. $$\int\frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\int\left(\frac{d}{b^2\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{d}{b\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{c+dx}{b^2(a+bx)}\right)d\frac{a+bx}{c+dx}}{2d}\right)}{b}+\frac{\int\frac{(\cdots}{\cdots}}{\cdots}\cdots\right)}{3d}\cdots\right)}{b}\cdots\right)}{\cdots}$$

$\downarrow$ 2009

3.182. $\displaystyle\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$i^3(bc-ad)^3 \left( \frac{d \left( \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left( \frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2} d\frac{a+bx}{c+dx}}{b} + \frac{d\left( \frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(\cdots}{c\cdots}\right.}{b^2} \cdots\right.}{2d\cdots} \right.}{b} \cdots \right.}{3d} \cdots \right.}{b} \cdots \right.$$

$\downarrow$ 2789

3.182. $\displaystyle \int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{\frac{d\int\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}}{b}+\frac{d\left(\frac{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right.}{2d\left(b-\frac{d(a+bx)}{c+dx}\right.}\right.}{\ }\right.}{3d}\right.}{b}\right.}{\ }$$

$\downarrow$ 2751

3.182. $\int\frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bn\int\frac{1}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}\cdots}{b}\cdots}{b}\cdots\right)}{3d}\cdots\right)}{b}\cdots\right)}{\cdots}$$

$\downarrow$ 16

---

3.182. $\displaystyle\int\frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$\frac{i^3(bc-ad)^3\left(\frac{d\left(\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{Bn\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{bd}\cdots}{b}\cdots\right)}{3d}\cdots\right)}{b}\cdots\right)}{\cdots}$$

$\downarrow$ 2755

---

3.182. $\displaystyle\int\frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$(bc-ad)^3 i^3 \left( \frac{d\left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left( \frac{d\left( \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left( \frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)} \right)}{2d} \right)}{b} + \frac{d\left(\frac{(a+bx)(\cdots}{b(c\cdots}\right.}{\cdots} \right.}{3d} \right.}{b} \right.$$

$\downarrow$ 2754

---

3.182. $\displaystyle \int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$\frac{(bc-ad)^3 i^3 \left(\frac{d\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{2Bn\left(\frac{d\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2}-\frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2}+\frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b}+\frac{d\left(\frac{(a+bx)(\cdots}{b(c\cdots}\right.}{\cdots}\right.}{3d}\right.}{b}\right.}{\cdots}$$

$\downarrow$ 2779

---

3.182. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$(bc-ad)^3 i^3 \left( \frac{d\left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left( \frac{d\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b} + \frac{d\left(\frac{(a+bx)(\cdots}{b(c\cdots}\right.}{\cdots} \cdots \right)}{\cdots} \cdots \right)}{\cdots} \cdots \right)$$

$\downarrow$ 2821

---

3.182. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$(bc-ad)^3 i^3 \left( \frac{d\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left(\frac{d\left(\frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d}\right)}{b} + \frac{d\left(\frac{(a+bx)(\cdots}{b(c\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots}{\cdots}\right.\cdots$$

$\downarrow$ 2838

---

3.182. $\displaystyle\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$(bc-ad)^3 i^3 \left( \frac{d\left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn\left( \frac{d\left( \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d} \right)}{b} + \frac{d\left(\frac{(a+bx)(\cdots}{b(c\cdots}\right.}{\cdots} \right.}{\cdots}\right.}{b} \cdots \right.$$

↓ 7143

3.182. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

$$(bc-ad)^3 i^3 \left( \frac{d \left( \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d\left(b-\frac{d(a+bx)}{c+dx}\right)^3} - \frac{2Bn \left( \frac{d \left( \frac{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{2d\left(b-\frac{d(a+bx)}{c+dx}\right)^2} - \frac{Bn\left(\frac{\log\left(\frac{a+bx}{c+dx}\right)}{b^2} - \frac{\log\left(b-\frac{d(a+bx)}{c+dx}\right)}{b^2} + \frac{1}{b\left(b-\frac{d(a+bx)}{c+dx}\right)}\right)}{2d} \right)}{b} + \cdots \right)}{\cdots} \right)}{b} \cdots \right)$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x),x]
```

## 3.182. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

output
```
((b*c - a*d)^3*i^3*((d*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(3*d*(b -
 (d*(a + b*x))/(c + d*x))^3) - (2*B*n*((d*((A + B*Log[e*((a + b*x)/(c + d*
x))^n])/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*(1/(b*(b - (d*(a + b*
x))/(c + d*x))) + Log[(a + b*x)/(c + d*x)]/b^2 - Log[b - (d*(a + b*x))/(c
+ d*x)]/b^2))/(2*d)))/b + ((d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (B*n*Log[b - (d*(a +
 b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*
Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*n*PolyLog[2, (b*(c + d*x))/(
d*(a + b*x))])/b)/b)/b))/(3*d)))/b + ((d*((A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2/(2*d*(b - (d*(a + b*x))/(c + d*x))^2) - (B*n*((d*(((a + b*x)*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n]))/(b*(c + d*x)*(b - (d*(a + b*x))/(c + d
*x))) + (B*n*Log[b - (d*(a + b*x))/(c + d*x)])/(b*d)))/b + (-(((A + B*Log[
e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b) + (B*
n*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b)/b))/d))/b + ((d*(((a + b*x)*
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(b*(c + d*x)*(b - (d*(a + b*x))/
(c + d*x))) - (2*B*n*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d
*(a + b*x))/(b*(c + d*x))])/d) - (B*n*PolyLog[2, (d*(a + b*x))/(b*(c + d*x
))])/d))/b))/b + (-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(
c + d*x))/(d*(a + b*x))])/b) + (2*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^
n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))] + B*n*PolyLog[3, (b*(c + d*...
```

#### 3.182.3.1 Defintions of rubi rules used

rule 16
```
Int[(c_.)/((a_.) + (b_.)*(x_)), x_Symbol] :> Simp[c*(Log[RemoveContent[a +
b*x, x]]/b), x] /; FreeQ[{a, b, c}, x]
```

rule 54
```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int[E
xpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d}, x] &&
 ILtQ[m, 0] && IntegerQ[n] &&  !(IGtQ[n, 0] && LtQ[m + n + 2, 0])
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2751
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_) + (e_.)*(x_)^(r_.))^(q_), x
_Symbol] :> Simp[x*(d + e*x^r)^(q + 1)*((a + b*Log[c*x^n])/d), x] - Simp[b*
(n/d)   Int[(d + e*x^r)^(q + 1), x], x] /; FreeQ[{a, b, c, d, e, n, q, r},
x] && EqQ[r*(q + 1) + 1, 0]
```

---

3.182. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2755
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_))^2, x_Sy
mbol] :> Simp[x*((a + b*Log[c*x^n])^p/(d*(d + e*x))), x] - Simp[b*n*(p/d)
 Int[(a + b*Log[c*x^n])^(p - 1)/(d + e*x), x], x] /; FreeQ[{a, b, c, d, e,
n, p}, x] && GtQ[p, 0]
```

rule 2756
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.),
x_Symbol] :> Simp[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/(e*(q + 1))), x]
- Simp[b*n*(p/(e*(q + 1)))   Int[((d + e*x)^(q + 1)*(a + b*Log[c*x^n])^(p -
 1))/x, x], x] /; FreeQ[{a, b, c, d, e, n, p, q}, x] && GtQ[p, 0] && NeQ[q,
 -1] && (EqQ[p, 1] || (IntegersQ[2*p, 2*q] &&  !IGtQ[q, 0]) || (EqQ[p, 2] &
& NeQ[q, 1]))
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2789
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_))/
(x_), x_Symbol] :> Simp[1/d   Int[(d + e*x)^(q + 1)*((a + b*Log[c*x^n])^p/x
), x], x] - Simp[e/d   Int[(d + e*x)^q*(a + b*Log[c*x^n])^p, x], x] /; Free
Q[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && LtQ[q, -1] && IntegerQ[2*q]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

---

## 3.182. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.182.4 Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{bgx+ag}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x)
```

### 3.182.5 Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x,
algorithm="fricas")
```

output
```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*
d^2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))
^n))/(b*g*x + a*g), x)
```

### 3.182.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g),x
)
```

output
```
Timed out
```

### 3.182.7 Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x,
algorithm="maxima")
```

output
```
3*A^2*c^2*d*i^3*(x/(b*g) - a*log(b*x + a)/(b^2*g)) - 1/6*A^2*d^3*i^3*(6*a^
3*log(b*x + a)/(b^4*g) - (2*b^2*x^3 - 3*a*b*x^2 + 6*a^2*x)/(b^3*g)) + 3/2*
A^2*c*d^2*i^3*(2*a^2*log(b*x + a)/(b^3*g) + (b*x^2 - 2*a*x)/(b^2*g)) + A^2
*c^3*i^3*log(b*g*x + a*g)/(b*g) + 1/6*(2*B^2*b^3*d^3*i^3*x^3 + 3*(3*b^3*c*
d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 6*(3*b^3*c^2*d*i^3 - 3*a*b^2*c*d^2*i^3
+ a^2*b*d^3*i^3)*B^2*x + 6*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3*a^2*b*c*d^
2*i^3 - a^3*d^3*i^3)*B^2*log(b*x + a))*log((d*x + c)^n)^2/(b^4*g) - integr
ate(-1/3*(3*B^2*b^4*c^4*i^3*log(e)^2 + 6*A*B*b^4*c^4*i^3*log(e) + 3*(B^2*b
^4*d^4*i^3*log(e)^2 + 2*A*B*b^4*d^4*i^3*log(e))*x^4 + 12*(B^2*b^4*c*d^3*i^
3*log(e)^2 + 2*A*B*b^4*c*d^3*i^3*log(e))*x^3 + 18*(B^2*b^4*c^2*d^2*i^3*log
(e)^2 + 2*A*B*b^4*c^2*d^2*i^3*log(e))*x^2 + 3*(B^2*b^4*d^4*i^3*x^4 + 4*B^2
*b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B
^2*b^4*c^4*i^3)*log((b*x + a)^n)^2 + 12*(B^2*b^4*c^3*d*i^3*log(e)^2 + 2*A*
B*b^4*c^3*d*i^3*log(e))*x + 6*(B^2*b^4*c^4*i^3*log(e) + A*B*b^4*c^4*i^3 +
(B^2*b^4*d^4*i^3*log(e) + A*B*b^4*d^4*i^3)*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(
e) + A*B*b^4*c*d^3*i^3)*x^3 + 6*(B^2*b^4*c^2*d^2*i^3*log(e) + A*B*b^4*c^2*
d^2*i^3)*x^2 + 4*(B^2*b^4*c^3*d*i^3*log(e) + A*B*b^4*c^3*d*i^3)*x)*log((b*
x + a)^n) - (6*B^2*b^4*c^4*i^3*log(e) + 6*A*B*b^4*c^4*i^3 + 2*(3*A*B*b^4*d
^4*i^3 + (i^3*n + 3*i^3*log(e))*B^2*b^4*d^4)*x^4 + (24*A*B*b^4*c*d^3*i^3 -
 (a*b^3*d^4*i^3*n - 3*(3*i^3*n + 8*i^3*log(e))*b^4*c*d^3)*B^2)*x^3 + 3*...
```

### 3.182.8 Giac [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{bgx+ag}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g),x,
algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x
+ a*g), x)
```

### 3.182.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ag+bgx}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{a\,g+b\,g\,x}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x),x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x), x)
```

# 3.183 $\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

### 3.183.1 Optimal result

Integrand size = 45, antiderivative size = 739

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$$
$$= -\frac{2B^2(bc-ad)^2i^3n^2(c+dx)}{b^3g^2(a+bx)} - \frac{Bd^2(bc-ad)i^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^4g^2}$$
$$- \frac{2B(bc-ad)^2i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^2(a+bx)}$$
$$+ \frac{2d^2(bc-ad)i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g^2}$$
$$- \frac{(bc-ad)^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^2(a+bx)} + \frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^2}$$
$$+ \frac{4Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g^2}$$
$$+ \frac{B^2d(bc-ad)^2i^3n^2\log(c+dx)}{b^4g^2}$$
$$+ \frac{Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$- \frac{3d(bc-ad)^2i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{4B^2d(bc-ad)^2i^3n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^2} - \frac{B^2d(bc-ad)^2i^3n^2\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{6Bd(bc-ad)^2i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$
$$+ \frac{6B^2d(bc-ad)^2i^3n^2\,\mathrm{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^2}$$

output
```
-2*B^2*(-a*d+b*c)^2*i^3*n^2*(d*x+c)/b^3/g^2/(b*x+a)-B*d^2*(-a*d+b*c)*i^3*n
*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^4/g^2-2*B*(-a*d+b*c)^2*i^3*n*(d
*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g^2/(b*x+a)+2*d^2*(-a*d+b*c)*i^3
*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^4/g^2-(-a*d+b*c)^2*i^3*(d*x+c
)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^3/g^2/(b*x+a)+1/2*d*i^3*(d*x+c)^2*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))^2/b^2/g^2+4*B*d*(-a*d+b*c)^2*i^3*n*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/g^2+B^2*d*(-a*d+b*c)^2
*i^3*n^2*ln(d*x+c)/b^4/g^2+B*d*(-a*d+b*c)^2*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^2-3*d*(-a*d+b*c)^2*i^3*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^2+4*B^2*d*(-a*d+b*c
)^2*i^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g^2-B^2*d*(-a*d+b*c)^2*i^3*
n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^2+6*B*d*(-a*d+b*c)^2*i^3*n*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^2+6*B^2*d*(
-a*d+b*c)^2*i^3*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/b^4/g^2
```

### 3.183.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 4818 vs. $2(739) = 1478$.

Time = 6.33 (sec) , antiderivative size = 4818, normalized size of antiderivative = 6.52

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^2,x]
```

output
```
(i^3*(4*b*d^2*(3*b*c - 2*a*d)*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*
n*Log[(a + b*x)/(c + d*x)])^2 + 2*b^2*d^3*x^2*(A + B*Log[e*((a + b*x)/(c +
 d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2 - (4*(b*c - a*d)^3*(A + B*Log[
e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2)/(a + b*x) +
12*d*(b*c - a*d)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*
n*Log[(a + b*x)/(c + d*x)])^2 + (8*b^3*B*c^3*n*(-A - B*Log[e*((a + b*x)/(c
 + d*x))^n] + B*n*Log[(a + b*x)/(c + d*x)])*(-(d*(a + b*x)*Log[c/d + x]) +
 d*(a + b*x)*Log[(d*(a + b*x))/(-(b*c) + a*d)] + (b*c - a*d)*(1 + Log[(a +
 b*x)/(c + d*x)])))/((b*c - a*d)*(a + b*x)) + (4*b^3*B^2*c^3*n^2*(-2*b*c +
 2*a*d - 2*d*(a + b*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(a + b*x)/(c + d*x
)] - 2*d*(a + b*x)*Log[a + b*x]*Log[(a + b*x)/(c + d*x)] - (b*c - a*d)*Log
[(a + b*x)/(c + d*x)]^2 + 2*d*(a + b*x)*Log[c + d*x] - 2*d*(a + b*x)*Log[(
a + b*x)/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] + d*(a + b*x)*(Log[a +
b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(
a + b*x))/(-(b*c) + a*d)]) + d*(a + b*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(
2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*
PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(a + b*x)) + 12*b^2*
B*c^2*d*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d
*x)])*(Log[a/b + x]^2 - 2*Log[a/b + x]*Log[a + b*x] - 2*Log[c/d + x]*Log[(
d*(a + b*x))/(-(b*c) + a*d)] + 2*Log[a + b*x]*((a*d)/(b*c - a*d) + Log[...
```

### 3.183.3 Rubi [A] (verified)

Time = 0.99 (sec) , antiderivative size = 624, normalized size of antiderivative = 0.84, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i^3(bc-ad)^2\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{g^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{i^3(bc-ad)^2\int\left(\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3(a+bx)^2}+\frac{3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{d^2\left(A+B\log\right.}{b^2\left(b-\right.}\right.}{g^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{i^3(bc-ad)^2\left(\frac{2d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{Bd^2n(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{6Bdn\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{b^4}\right.}{}$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^2,x]
```

output
```
((b*c - a*d)^2*i^3*((-2*B^2*n^2*(c + d*x))/(b^3*(a + b*x)) - (2*B*n*(c + d
*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^3*(a + b*x)) - (B*d^2*n*(a
+ b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(b^4*(c + d*x)*(b - (d*(a +
 b*x))/(c + d*x))) - ((c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/
(b^3*(a + b*x)) + (d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*b^2*(b -
 (d*(a + b*x))/(c + d*x))^2) + (2*d^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2)/(b^4*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - (B^2*d*n^2
*Log[b - (d*(a + b*x))/(c + d*x)])/b^4 + (4*B*d*n*(A + B*Log[e*((a + b*x)/
(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/b^4 + (B*d*n*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b^4 -
 (3*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a
 + b*x))])/b^4 + (4*B^2*d*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/b^4
 - (B^2*d*n^2*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b^4 + (6*B*d*n*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))]
)/b^4 + (6*B^2*d*n^2*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))])/b^4))/g^2
```

3.183. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx$

### 3.183.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.183.4 Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^2}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x)
```

## 3.183.5 Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x
, algorithm="fricas")
```

output
```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*
d^2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))
^n))/(b^2*g^2*x^2 + 2*a*b*g^2*x + a^2*g^2), x)
```

### 3.183.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
2,x)
```

output
```
Timed out
```

### 3.183.7 Maxima [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x
, algorithm="maxima")
```

output
```
-2*A*B*c^3*i^3*n*(1/(b^2*g^2*x + a*b*g^2) + d*log(b*x + a)/((b^2*c - a*b*d
)*g^2) - d*log(d*x + c)/((b^2*c - a*b*d)*g^2)) - 3*A^2*(a^2/(b^4*g^2*x + a
*b^3*g^2) - x/(b^2*g^2) + 2*a*log(b*x + a)/(b^3*g^2))*c*d^2*i^3 + 1/2*(2*a
^3/(b^5*g^2*x + a*b^4*g^2) + 6*a^2*log(b*x + a)/(b^4*g^2) + (b*x^2 - 4*a*x
)/(b^3*g^2))*A^2*d^3*i^3 + 3*A^2*c^2*d*i^3*(a/(b^3*g^2*x + a*b^2*g^2) + lo
g(b*x + a)/(b^2*g^2)) - 2*A*B*c^3*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)/(b^2*g^2*x + a*b*g^2) - A^2*c^3*i^3/(b^2*g^2*x + a*b*g^2) + 1/2*(B^2*b^
3*d^3*i^3*x^3 + 3*(2*b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 2*(3*a*b^2*c
*d^2*i^3 - 2*a^2*b*d^3*i^3)*B^2*x - 2*(b^3*c^3*i^3 - 3*a*b^2*c^2*d*i^3 + 3
*a^2*b*c*d^2*i^3 - a^3*d^3*i^3)*B^2 + 6*((b^3*c^2*d*i^3 - 2*a*b^2*c*d^2*i^
3 + a^2*b*d^3*i^3)*B^2*x + (a*b^2*c^2*d*i^3 - 2*a^2*b*c*d^2*i^3 + a^3*d^3*
i^3)*B^2)*log(b*x + a))*log((d*x + c)^n)^2/(b^5*g^2*x + a*b^4*g^2) - integ
rate(-(B^2*b^4*c^4*i^3*log(e)^2 + (B^2*b^4*d^4*i^3*log(e)^2 + 2*A*B*b^4*d^
4*i^3*log(e))*x^4 + 4*(B^2*b^4*c*d^3*i^3*log(e)^2 + 2*A*B*b^4*c*d^3*i^3*lo
g(e))*x^3 + 6*(B^2*b^4*c^2*d^2*i^3*log(e)^2 + 2*A*B*b^4*c^2*d^2*i^3*log(e)
)*x^2 + (B^2*b^4*d^4*i^3*x^4 + 4*B^2*b^4*c*d^3*i^3*x^3 + 6*B^2*b^4*c^2*d^2
*i^3*x^2 + 4*B^2*b^4*c^3*d*i^3*x + B^2*b^4*c^4*i^3)*log((b*x + a)^n)^2 + 2
*(2*B^2*b^4*c^3*d*i^3*log(e)^2 + 3*A*B*b^4*c^3*d*i^3*log(e))*x + 2*(B^2*b^
4*c^4*i^3*log(e) + (B^2*b^4*d^4*i^3*log(e) + A*B*b^4*d^4*i^3)*x^4 + 4*(B^2
*b^4*c*d^3*i^3*log(e) + A*B*b^4*c*d^3*i^3)*x^3 + 6*(B^2*b^4*c^2*d^2*i^3...
```

## 3.183.8 Giac [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(dix+ci)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2,x
, algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x
+ a*g)^2, x)
```

### 3.183.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2}\,dx = \int \frac{(c\,i+d\,i\,x)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^2}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^2,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^2, x)
```

# 3.184 $\int \frac{(ci+dix)^3 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3} \, dx$

### 3.184.1 Optimal result

Integrand size = 45, antiderivative size = 644

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$$
$$= -\frac{4B^2d(bc-ad)i^3n^2(c+dx)}{b^3g^3(a+bx)} - \frac{B^2(bc-ad)i^3n^2(c+dx)^2}{4b^2g^3(a+bx)^2}$$
$$- \frac{4Bd(bc-ad)i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^3(a+bx)}$$
$$- \frac{B(bc-ad)i^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^3(a+bx)^2}$$
$$+ \frac{d^3i^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^4g^3}$$
$$- \frac{2d(bc-ad)i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^3(a+bx)}$$
$$- \frac{(bc-ad)i^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^3(a+bx)^2}$$
$$+ \frac{2Bd^2(bc-ad)i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{b^4g^3}$$
$$- \frac{3d^2(bc-ad)i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$
$$+ \frac{2B^2d^2(bc-ad)i^3n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{b^4g^3}$$
$$+ \frac{6Bd^2(bc-ad)i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$
$$+ \frac{6B^2d^2(bc-ad)i^3n^2\,\mathrm{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^3}$$

output
```
-4*B^2*d*(-a*d+b*c)*i^3*n^2*(d*x+c)/b^3/g^3/(b*x+a)-1/4*B^2*(-a*d+b*c)*i^3
*n^2*(d*x+c)^2/b^2/g^3/(b*x+a)^2-4*B*d*(-a*d+b*c)*i^3*n*(d*x+c)*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))/b^3/g^3/(b*x+a)-1/2*B*(-a*d+b*c)*i^3*n*(d*x+c)^2*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^3/(b*x+a)^2+d^3*i^3*(b*x+a)*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))^2/b^4/g^3-2*d*(-a*d+b*c)*i^3*(d*x+c)*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))^2/b^3/g^3/(b*x+a)-1/2*(-a*d+b*c)*i^3*(d*x+c)^2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))^2/b^2/g^3/(b*x+a)^2+2*B*d^2*(-a*d+b*c)*i^3*n*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/b^4/g^3-3*d^2*(-a*d+b*c)*
i^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^3+2*
B^2*d^2*(-a*d+b*c)*i^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/b^4/g^3+6*B*d^2*
(-a*d+b*c)*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,b*(d*x+c)/d/(b*
x+a))/b^4/g^3+6*B^2*d^2*(-a*d+b*c)*i^3*n^2*polylog(3,b*(d*x+c)/d/(b*x+a))/
b^4/g^3
```

### 3.184.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6938 vs. $2(644) = 1288$.

Time = 6.86 (sec) , antiderivative size = 6938, normalized size of antiderivative = 10.77

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^3,x]
```

output
```
Result too large to show
```

### 3.184.3 Rubi [A] (verified)

Time = 0.82 (sec) , antiderivative size = 526, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3}\,dx$$

$$\Big\downarrow \; 2961$$

$$\frac{i^3(bc-ad)\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{g^3}$$

$$\Big\downarrow \; 2795$$

$$\frac{i^3(bc-ad)\int \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d^3}{b^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d^2}{b^3(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d}{b^3(a+bx)^2}+\frac{(c+dx)^3\left(\right.}{}\right.}{g^3}$$

$$\Big\downarrow \; 2009$$

$$i^3(bc-ad)\left(\frac{d^3(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b^4(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}+\frac{6Bd^2n\,\mathrm{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{b^4}+\frac{2Bd^2n\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\right.}{b^4}\right.$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^3,x]
```

output
```
((b*c - a*d)*i^3*((-4*B^2*d*n^2*(c + d*x))/(b^3*(a + b*x)) - (B^2*n^2*(c +
 d*x)^2)/(4*b^2*(a + b*x)^2) - (4*B*d*n*(c + d*x)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(b^3*(a + b*x)) - (B*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(2*b^2*(a + b*x)^2) - (2*d*(c + d*x)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])^2)/(b^3*(a + b*x)) - ((c + d*x)^2*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])^2)/(2*b^2*(a + b*x)^2) + (d^3*(a + b*x)*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2)/(b^4*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) +
(2*B*d^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b
*(c + d*x))])/b^4 - (3*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1
- (b*(c + d*x))/(d*(a + b*x))])/b^4 + (2*B^2*d^2*n^2*PolyLog[2, (d*(a + b*
x))/(b*(c + d*x))])/b^4 + (6*B*d^2*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*PolyLog[2, (b*(c + d*x))/(d*(a + b*x))])/b^4 + (6*B^2*d^2*n^2*PolyLog[3,
 (b*(c + d*x))/(d*(a + b*x))])/b^4))/g^3
```

## 3.184. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx$

### 3.184.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.184.4 Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^3}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x)
```

## 3.184.5 Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x
, algorithm="fricas")
```

output
```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*
d^2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))
^n))/(b^3*g^3*x^3 + 3*a*b^2*g^3*x^2 + 3*a^2*b*g^3*x + a^3*g^3), x)
```

## 3.184.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
3,x)
```

output
```
Timed out
```

## 3.184.7 Maxima [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x
, algorithm="maxima")
```

output
```
-3/2*A*B*c^2*d*i^3*n*((3*a*b*c - a^2*d + 2*(2*b^2*c - a*b*d)*x)/((b^5*c -
a*b^4*d)*g^3*x^2 + 2*(a*b^4*c - a^2*b^3*d)*g^3*x + (a^2*b^3*c - a^3*b^2*d)
*g^3) + 2*(2*b*c*d - a*d^2)*log(b*x + a)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2
*d^2)*g^3) - 2*(2*b*c*d - a*d^2)*log(d*x + c)/((b^4*c^2 - 2*a*b^3*c*d + a^
2*b^2*d^2)*g^3)) + 1/2*A*B*c^3*i^3*n*((2*b*d*x - b*c + 3*a*d)/((b^4*c - a*
b^3*d)*g^3*x^2 + 2*(a*b^3*c - a^2*b^2*d)*g^3*x + (a^2*b^2*c - a^3*b*d)*g^3
) + 2*d^2*log(b*x + a)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3) - 2*d^2*l
og(d*x + c)/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^3)) - 1/2*A^2*d^3*i^3*(
(6*a^2*b*x + 5*a^3)/(b^6*g^3*x^2 + 2*a*b^5*g^3*x + a^2*b^4*g^3) - 2*x/(b^3
*g^3) + 6*a*log(b*x + a)/(b^4*g^3)) + 3/2*A^2*c*d^2*i^3*((4*a*b*x + 3*a^2)
/(b^5*g^3*x^2 + 2*a*b^4*g^3*x + a^2*b^3*g^3) + 2*log(b*x + a)/(b^3*g^3)) -
 3*(2*b*x + a)*A*B*c^2*d*i^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(b^4*g
^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - 3/2*(2*b*x + a)*A^2*c^2*d*i^3/(b^4
*g^3*x^2 + 2*a*b^3*g^3*x + a^2*b^2*g^3) - A*B*c^3*i^3*log(e*(b*x/(d*x + c)
 + a/(d*x + c))^n)/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) - 1/2*A^2*c^3
*i^3/(b^3*g^3*x^2 + 2*a*b^2*g^3*x + a^2*b*g^3) + 1/2*(2*B^2*b^3*d^3*i^3*x^
3 + 4*B^2*a*b^2*d^3*i^3*x^2 - 2*(3*b^3*c^2*d*i^3 - 6*a*b^2*c*d^2*i^3 + 2*a
^2*b*d^3*i^3)*B^2*x - (b^3*c^3*i^3 + 3*a*b^2*c^2*d*i^3 - 9*a^2*b*c*d^2*i^3
 + 5*a^3*d^3*i^3)*B^2 + 6*((b^3*c*d^2*i^3 - a*b^2*d^3*i^3)*B^2*x^2 + 2*(a*
b^2*c*d^2*i^3 - a^2*b*d^3*i^3)*B^2*x + (a^2*b*c*d^2*i^3 - a^3*d^3*i^3)*...
```

### 3.184.8 Giac [F]

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3,x
, algorithm="giac")
```

output
```
integrate((d*i*x + c*i)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(b*g*x
+ a*g)^3, x)
```

### 3.184.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3}\,dx = \int \frac{(c\,i+d\,i\,x)^3\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^3}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^3,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^3, x)
```

# 3.185 $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$



## 3.185.1 Optimal result

Integrand size = 45, antiderivative size = 561

$$
\begin{aligned}
&\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx \\
&= -\frac{2B^2d^2i^3n^2(c+dx)}{b^3g^4(a+bx)}-\frac{B^2di^3n^2(c+dx)^2}{4b^2g^4(a+bx)^2}-\frac{2B^2i^3n^2(c+dx)^3}{27bg^4(a+bx)^3} \\
&\quad-\frac{2Bd^2i^3n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{b^3g^4(a+bx)}-\frac{Bdi^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2b^2g^4(a+bx)^2} \\
&\quad-\frac{2Bi^3n(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9bg^4(a+bx)^3}-\frac{d^2i^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b^3g^4(a+bx)} \\
&\quad-\frac{di^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2b^2g^4(a+bx)^2}-\frac{i^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3bg^4(a+bx)^3} \\
&\quad-\frac{d^3i^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} \\
&\quad+\frac{2Bd^3i^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4} \\
&\quad+\frac{2B^2d^3i^3n^2\operatorname{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)}{b^4g^4}
\end{aligned}
$$

output
```
-2*B^2*d^2*i^3*n^2*(d*x+c)/b^3/g^4/(b*x+a)-1/4*B^2*d*i^3*n^2*(d*x+c)^2/b^2
/g^4/(b*x+a)^2-2/27*B^2*i^3*n^2*(d*x+c)^3/b/g^4/(b*x+a)^3-2*B*d^2*i^3*n*(d
*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^3/g^4/(b*x+a)-1/2*B*d*i^3*n*(d*x+c
)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b^2/g^4/(b*x+a)^2-2/9*B*i^3*n*(d*x+c)^
3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/b/g^4/(b*x+a)^3-d^2*i^3*(d*x+c)*(A+B*ln(
e*((b*x+a)/(d*x+c))^n))^2/b^3/g^4/(b*x+a)-1/2*d*i^3*(d*x+c)^2*(A+B*ln(e*((
b*x+a)/(d*x+c))^n))^2/b^2/g^4/(b*x+a)^2-1/3*i^3*(d*x+c)^3*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))^2/b/g^4/(b*x+a)^3-d^3*i^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^
2*ln(1-b*(d*x+c)/d/(b*x+a))/b^4/g^4+2*B*d^3*i^3*n*(A+B*ln(e*((b*x+a)/(d*x+
c))^n))*polylog(2,b*(d*x+c)/d/(b*x+a))/b^4/g^4+2*B^2*d^3*i^3*n^2*polylog(3
,b*(d*x+c)/d/(b*x+a))/b^4/g^4
```

### 3.185.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 8570 vs. $2(561) = 1122$.

Time = 7.50 (sec) , antiderivative size = 8570, normalized size of antiderivative = 15.28

$$\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Result too large to show}$$

input
```
Integrate[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g
+ b*g*x)^4,x]
```

output
```
Result too large to show
```

### 3.185.3 Rubi [A] (verified)

Time = 1.93 (sec) , antiderivative size = 473, normalized size of antiderivative = 0.84, number of steps used = 14, number of rules used = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.289$, Rules used = {2961, 2780, 2742, 2741, 2780, 2742, 2741, 2780, 2742, 2741, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ci+dix)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4}\,dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{i^3\int\frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{g^4}$$

$\Big\downarrow$ 2780

$$\frac{i^3\left(\frac{\int\frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{b}+\frac{d\int\frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^4}$$

$\Big\downarrow$ 2742

$$\frac{i^3\left(\frac{\frac{2}{3}Bn\int\frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^4}d\frac{a+bx}{c+dx}-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}}{b}+\frac{d\int\frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{g^4}$$

$\Big\downarrow$ 2741

$$\frac{i^3\left(\frac{d\int\frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{\frac{2}{3}Bn\left(-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}-\frac{Bn(c+dx)^3}{9(a+bx)^3}\right)-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}}{b}\right)}{g^4}$$

$\Big\downarrow$ 2780

$$\frac{i^3\left(\frac{d\left(\frac{\int\frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{b}+\frac{d\int\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\frac{2}{3}Bn\left(-\frac{(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3(a+bx)^3}-\frac{Bn(c+}{9(a+}\right.}{b}\right.}{g^4}$$

$\Big\downarrow$ 2742

3.185. $\displaystyle\int\frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

$$\frac{i^3\left(\frac{d\left(\frac{Bn\int \frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^3}d\frac{a+bx}{c+dx}-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}}{b}+\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{\frac{2}{3}Bn\left(-\frac{(c+\cdots}{\cdots}\cdots\right.}{\cdots}\cdots\right)}{g^4}$$

$\downarrow$ 2741

$$\frac{i^3\left(\frac{d\left(\frac{d\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{Bn\left(-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)^2}{4(a+bx)^2}\right)-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}}{b}\right)}{b}+\frac{\frac{2}{3}\cdots}{\cdots}\cdots\right)}{g^4}$$

$\downarrow$ 2780

$$\frac{i^3\left(\frac{d\left(\frac{d\left(\frac{\int \frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{b}+\frac{d\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{Bn\left(-\frac{(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}-\frac{Bn(c+dx)\cdots}{4(a+bx)^2\cdots}\cdots\right.}{b}\cdots\right)}{b}\cdots\right)}{g^4}$$

$\downarrow$ 2742

---

3.185. $\displaystyle\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

$$i^3\left(\frac{d\left(\frac{d\left(\frac{2Bn\int\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)^2}d\frac{a+bx}{c+dx}-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{b}+\frac{d\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}\right)}{b}+\frac{Bn\left(-\frac{(c+dx)}{\cdots}\right.}{\cdots}\right)}{b}\cdots\right.$$

$\downarrow$ 2741

$$i^3\left(\frac{d\left(\frac{d\left(\frac{d\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{b}+\frac{2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{Bn(c+dx)}{a+bx}\right)-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}}{b}\right)}{b}+\frac{Bn\left(-\cdots\right.}{\cdots}\right)}{b}\cdots\right.$$

$\downarrow$ 2779

3.185. $$\int\frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$$

$$i^3\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{2Bn\int\frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}\right)}{b}+\frac{2Bn\left(-\frac{(c+dx)\left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{a+bx}\right.}{\ldots}}{b}}{\ldots}}{\ldots}\right.$$

$$\Big\downarrow \text{2821}$$

3.185. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

$$i^3\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int\frac{(c+dx)\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}\right.}{b}}{b}}{}\right.$$

$\downarrow$ 7143

---

3.185. $\displaystyle\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

$$i^3\left(\frac{d\left(\frac{d\left(\frac{d\left(\frac{2Bn\left(\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+Bn\,\text{PolyLog}\left(3,\frac{b(c+dx)}{d(a+bx)}\right)\right)}{b}-\frac{\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{b}\right)}{b}+\frac{2Bn\left(-\frac{(c+\cdots}{\cdots}\right.}{\cdots}}{b}\cdots}{\cdots}\cdots}{\cdots}\right.$$

input
```
Int[((c*i + d*i*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a*g + b*g*
x)^4,x]
```

output
```
(i^3*((-1/3*((c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*
x)^3 + (2*B*n*(-1/9*(B*n*(c + d*x)^3)/(a + b*x)^3 - ((c + d*x)^3*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]))/(3*(a + b*x)^3)))/3)/b + (d*((-1/2*((c + d*
x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^2 + B*n*(-1/4*(B*
n*(c + d*x)^2)/(a + b*x)^2 - ((c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x
))^n]))/(2*(a + b*x)^2)))/b + (d*((-(((c + d*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2)/(a + b*x)) + 2*B*n*(-((B*n*(c + d*x))/(a + b*x)) - ((c + d
*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)))/b + (d*(-(((A + B*
Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/b)
 + (2*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (b*(c + d*x))
/(d*(a + b*x))] + B*n*PolyLog[3, (b*(c + d*x))/(d*(a + b*x))]))/b))/b))/b)
)/b))/g^4
```

3.185. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

### 3.185.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2780
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*(x_)^(m_.))/((d_) + (e_.)*
(x_)^(r_.)), x_Symbol] :> Simp[1/d   Int[x^m*(a + b*Log[c*x^n])^p, x], x] -
 Simp[e/d   Int[(x^(m + r)*(a + b*Log[c*x^n])^p)/(d + e*x^r), x], x] /; Fre
eQ[{a, b, c, d, e, m, n, r}, x] && IGtQ[p, 0] && IGtQ[r, 0] && ILtQ[m, -1]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

---

3.185. $\int \frac{(ci+dix)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx$

### 3.185.4 Maple [F]

$$\int \frac{(dix+ci)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(bgx+ag)^4}dx$$

input
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x)
```

output
```
int((d*i*x+c*i)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x)
```

### 3.185.5 Fricas [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^4}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x
, algorithm="fricas")
```

output
```
integral((A^2*d^3*i^3*x^3 + 3*A^2*c*d^2*i^3*x^2 + 3*A^2*c^2*d*i^3*x + A^2*
c^3*i^3 + (B^2*d^3*i^3*x^3 + 3*B^2*c*d^2*i^3*x^2 + 3*B^2*c^2*d*i^3*x + B^2
*c^3*i^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*d^3*i^3*x^3 + 3*A*B*c*
d^2*i^3*x^2 + 3*A*B*c^2*d*i^3*x + A*B*c^3*i^3)*log(e*((b*x + a)/(d*x + c))
^n))/(b^4*g^4*x^4 + 4*a*b^3*g^4*x^3 + 6*a^2*b^2*g^4*x^2 + 4*a^3*b*g^4*x +
a^4*g^4), x)
```

### 3.185.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx = \text{Timed out}$$

input
```
integrate((d*i*x+c*i)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**
4,x)
```

output
```
Timed out
```

### 3.185.7 Maxima [F]

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx=\int \frac{(dix+ci)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^4}\,dx$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x
, algorithm="maxima")
```

output
```
-1/3*A*B*c*d^2*i^3*n*((11*a^2*b^2*c^2 - 7*a^3*b*c*d + 2*a^4*d^2 + 6*(3*b^4
*c^2 - 3*a*b^3*c*d + a^2*b^2*d^2)*x^2 + 3*(9*a*b^3*c^2 - 7*a^2*b^2*c*d + 2
*a^3*b*d^2)*x)/((b^8*c^2 - 2*a*b^7*c*d + a^2*b^6*d^2)*g^4*x^3 + 3*(a*b^7*c
^2 - 2*a^2*b^6*c*d + a^3*b^5*d^2)*g^4*x^2 + 3*(a^2*b^6*c^2 - 2*a^3*b^5*c*d
 + a^4*b^4*d^2)*g^4*x + (a^3*b^5*c^2 - 2*a^4*b^4*c*d + a^5*b^3*d^2)*g^4) +
 6*(3*b^2*c^2*d - 3*a*b*c*d^2 + a^2*d^3)*log(b*x + a)/((b^6*c^3 - 3*a*b^5*
c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4) - 6*(3*b^2*c^2*d - 3*a*b*c*d^2
 + a^2*d^3)*log(d*x + c)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3
*b^3*d^3)*g^4)) - 1/9*A*B*c^3*i^3*n*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*
d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^2 - 2*a*b^5*c*d + a^2*
b^4*d^2)*g^4*x^3 + 3*(a*b^5*c^2 - 2*a^2*b^4*c*d + a^3*b^3*d^2)*g^4*x^2 + 3
*(a^2*b^4*c^2 - 2*a^3*b^3*c*d + a^4*b^2*d^2)*g^4*x + (a^3*b^3*c^2 - 2*a^4*
b^2*c*d + a^5*b*d^2)*g^4) + 6*d^3*log(b*x + a)/((b^4*c^3 - 3*a*b^3*c^2*d +
 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4) - 6*d^3*log(d*x + c)/((b^4*c^3 - 3*a*b^
3*c^2*d + 3*a^2*b^2*c*d^2 - a^3*b*d^3)*g^4)) - 1/6*A*B*c^2*d*i^3*n*((5*a*b
^2*c^2 - 22*a^2*b*c*d + 5*a^3*d^2 - 6*(3*b^3*c*d - a*b^2*d^2)*x^2 + 3*(3*b
^3*c^2 - 16*a*b^2*c*d + 5*a^2*b*d^2)*x)/((b^7*c^2 - 2*a*b^6*c*d + a^2*b^5*
d^2)*g^4*x^3 + 3*(a*b^6*c^2 - 2*a^2*b^5*c*d + a^3*b^4*d^2)*g^4*x^2 + 3*(a^
2*b^5*c^2 - 2*a^3*b^4*c*d + a^4*b^3*d^2)*g^4*x + (a^3*b^4*c^2 - 2*a^4*b^3*
c*d + a^5*b^2*d^2)*g^4) - 6*(3*b*c*d^2 - a*d^3)*log(b*x + a)/((b^5*c^3 ...
```

### 3.185.8 Giac [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx=\text{Timed out}$$

input
```
integrate((d*i*x+c*i)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4,x
, algorithm="giac")
```

output
```
Timed out
```

### 3.185.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ci+dix)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4}\,dx=\int \frac{{\left(c\,i+d\,i\,x\right)}^3\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}{{\left(a\,g+b\,g\,x\right)}^4}\,dx$$

input
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^4,x)
```

output
```
int(((c*i + d*i*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^4, x)
```

# 3.186 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$

## 3.186.1 Optimal result

Integrand size = 45, antiderivative size = 768

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{bB^2(bc-ad)^2g^3n^2x}{3d^3i} + \frac{7B(bc-ad)^2g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^3i}$$
$$- \frac{b^2B(bc-ad)g^3n(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{3d^4i}$$
$$+ \frac{3(bc-ad)^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i}$$
$$- \frac{3b^2(bc-ad)g^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4i}$$
$$+ \frac{b^3g^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3d^4i}$$
$$+ \frac{6B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i}$$
$$+ \frac{(bc-ad)^3g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i}$$
$$+ \frac{B^2(bc-ad)^3g^3n^2\log\left(\frac{a+bx}{c+dx}\right)}{3d^4i} - \frac{2B^2(bc-ad)^3g^3n^2\log(c+dx)}{d^4i}$$
$$- \frac{7B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i}$$
$$+ \frac{6B^2(bc-ad)^3g^3n^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$
$$+ \frac{2B(bc-ad)^3g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$
$$+ \frac{7B^2(bc-ad)^3g^3n^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{3d^4i} - \frac{2B^2(bc-ad)^3g^3n^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i}$$

output
```
1/3*b*B^2*(-a*d+b*c)^2*g^3*n^2*x/d^3/i+7/3*B*(-a*d+b*c)^2*g^3*n*(b*x+a)*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))/d^3/i-1/3*b^2*B*(-a*d+b*c)*g^3*n*(d*x+c)^2*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^4/i+3*(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))^2/d^3/i-3/2*b^2*(-a*d+b*c)*g^3*(d*x+c)^2*(A+B*ln(e*(
(b*x+a)/(d*x+c))^n))^2/d^4/i+1/3*b^3*g^3*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))^2/d^4/i+6*B*(-a*d+b*c)^3*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln
((-a*d+b*c)/b/(d*x+c))/d^4/i+(-a*d+b*c)^3*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))^2*ln((-a*d+b*c)/b/(d*x+c))/d^4/i+1/3*B^2*(-a*d+b*c)^3*g^3*n^2*ln((b*x+
a)/(d*x+c))/d^4/i-2*B^2*(-a*d+b*c)^3*g^3*n^2*ln(d*x+c)/d^4/i-7/3*B*(-a*d+b
*c)^3*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/d^4/
i+6*B^2*(-a*d+b*c)^3*g^3*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i+2*B*(-a*
d+b*c)^3*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+
c))/d^4/i+7/3*B^2*(-a*d+b*c)^3*g^3*n^2*polylog(2,b*(d*x+c)/d/(b*x+a))/d^4/
i-2*B^2*(-a*d+b*c)^3*g^3*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^4/i
```

### 3.186.2 Mathematica [A] (verified)

Time = 1.03 (sec) , antiderivative size = 1072, normalized size of antiderivative = 1.40

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$

$$= \frac{g^3\Big(6bd(bc-ad)^2x\big(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\big)^2+3d^2(-bc+ad)(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+2d^3(a+}{}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i
+ d*i*x),x]
```

output
```
(g^3*(6*b*d*(b*c - a*d)^2*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 3*d
^2*(-(b*c) + a*d)*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 2
*d^3*(a + b*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 6*A^2*(b*c - a
*d)^3*Log[c + d*x] + 12*A*B*(b*c - a*d)^3*Log[e*((a + b*x)/(c + d*x))^n]*L
og[(b*c - a*d)/(b*c + b*d*x)] + 6*B^2*(b*c - a*d)^3*Log[e*((a + b*x)/(c +
d*x))^n]^2*Log[(b*c - a*d)/(b*c + b*d*x)] - 6*A*B*(b*c - a*d)^3*n*(Log[(b*
c - a*d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c -
a*d)/(b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) + 6*B*(b*c
 - a*d)^2*n*(2*a*d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2
*b*c*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - a*B*d*n*(Log[a
+ b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d
*(a + b*x))/(-(b*c) + a*d)]) + b*B*c*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d
)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])
) + 3*B*(b*c - a*d)^2*n*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c +
 d*x))^n] - 2*B*(b*c - a*d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((
a + b*x)/(c + d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x
))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x
))/(b*c - a*d)])) + 2*B*(b*c - a*d)*n*(2*A*b*d*(b*c - a*d)*x + 2*B*d*(b*c
- a*d)*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] - d^2*(a + b*x)^2*(A + B*L
og[e*((a + b*x)/(c + d*x))^n]) - 2*B*(b*c - a*d)^2*n*Log[c + d*x] - 2*(...
```

### 3.186.3 Rubi [A] (verified)

Time = 1.10 (sec) , antiderivative size = 679, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ci+dix}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{g^3(bc-ad)^3\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \text{2795}$$

$$\frac{g^3(bc-ad)^3\int\left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4}-\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}+\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}}{i}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g^3(bc-ad)^3\left(\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{b^2Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{3d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{2Bn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right.}{}\right.}{}$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*
x),x]
```

output
```
((b*c - a*d)^3*g^3*((b*B^2*n^2)/(3*d^4*(b - (d*(a + b*x))/(c + d*x))) - (b
^2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3*d^4*(b - (d*(a + b*x))/(
c + d*x))^2) + (7*B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(3
*d^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (b^3*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])^2)/(3*d^4*(b - (d*(a + b*x))/(c + d*x))^3) - (3*b^2*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))
^2) + (3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^3*(c + d*x
)*(b - (d*(a + b*x))/(c + d*x))) + (B^2*n^2*Log[(a + b*x)/(c + d*x)])/(3*d
^4) + (2*B^2*n^2*Log[b - (d*(a + b*x))/(c + d*x)])/d^4 + (6*B*n*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 + (
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x
))])/d^4 - (7*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (b*(c + d
*x))/(d*(a + b*x))])/(3*d^4) + (6*B^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c +
 d*x))])/d^4 + (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d
*(a + b*x))/(b*(c + d*x))])/d^4 + (7*B^2*n^2*PolyLog[2, (b*(c + d*x))/(d*(
a + b*x))])/(3*d^4) - (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/
d^4))/i
```

## 3.186. $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$

### 3.186.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.186.4 Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{dix+ci}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

## 3.186.5 Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x,
algorithm="fricas")
```

output
```
integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*
a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*
b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))
^n))/(d*i*x + c*i), x)
```

### 3.186.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i),x
)
```

output
```
Timed out
```

### 3.186.7 Maxima [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x,
algorithm="maxima")
```

output
```
3*A^2*a^2*b*g^3*(x/(d*i) - c*log(d*x + c)/(d^2*i)) - 1/6*A^2*b^3*g^3*(6*c^
3*log(d*x + c)/(d^4*i) - (2*d^2*x^3 - 3*c*d*x^2 + 6*c^2*x)/(d^3*i)) + 3/2*
A^2*a*b^2*g^3*(2*c^2*log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A^2
*a^3*g^3*log(d*i*x + c*i)/(d*i) + 1/6*(2*B^2*b^3*d^3*g^3*x^3 - 3*(b^3*c*d^
2*g^3 - 3*a*b^2*d^3*g^3)*B^2*x^2 + 6*(b^3*c^2*d*g^3 - 3*a*b^2*c*d^2*g^3 +
3*a^2*b*d^3*g^3)*B^2*x - 6*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^
2*g^3 - a^3*d^3*g^3)*B^2*log(d*x + c))*log((d*x + c)^n)^2/(d^4*i) - integr
ate(-1/3*(3*B^2*a^3*d^3*g^3*log(e)^2 + 6*A*B*a^3*d^3*g^3*log(e) + 3*(B^2*b
^3*d^3*g^3*log(e)^2 + 2*A*B*b^3*d^3*g^3*log(e))*x^3 + 9*(B^2*a*b^2*d^3*g^3
*log(e)^2 + 2*A*B*a*b^2*d^3*g^3*log(e))*x^2 + 3*(B^2*b^3*d^3*g^3*x^3 + 3*B
^2*a*b^2*d^3*g^3*x^2 + 3*B^2*a^2*b*d^3*g^3*x + B^2*a^3*d^3*g^3)*log((b*x +
 a)^n)^2 + 9*(B^2*a^2*b*d^3*g^3*log(e)^2 + 2*A*B*a^2*b*d^3*g^3*log(e))*x +
 6*(B^2*a^3*d^3*g^3*log(e) + A*B*a^3*d^3*g^3 + (B^2*b^3*d^3*g^3*log(e) + A
*B*b^3*d^3*g^3)*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e) + A*B*a*b^2*d^3*g^3)*x^2
 + 3*(B^2*a^2*b*d^3*g^3*log(e) + A*B*a^2*b*d^3*g^3)*x)*log((b*x + a)^n) -
(6*B^2*a^3*d^3*g^3*log(e) + 6*A*B*a^3*d^3*g^3 + 2*(3*A*B*b^3*d^3*g^3 + (g^
3*n + 3*g^3*log(e))*B^2*b^3*d^3)*x^3 - 6*(b^3*c^3*g^3*n - 3*a*b^2*c^2*d*g^
3*n + 3*a^2*b*c*d^2*g^3*n - a^3*d^3*g^3*n)*B^2*log(d*x + c) + 3*(6*A*B*a*b
^2*d^3*g^3 - (b^3*c*d^2*g^3*n - 3*(g^3*n + 2*g^3*log(e))*a*b^2*d^3)*B^2)*x
^2 + 6*(3*A*B*a^2*b*d^3*g^3 + (b^3*c^2*d*g^3*n - 3*a*b^2*c*d^2*g^3*n + ...
```

### 3.186.8 Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x,
algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x
+ c*i), x)
```

### 3.186.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(a\,g+b\,g\,x)^3 \left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x),x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x), x)
```

# 3.187 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$



## 3.187.1 Optimal result

Integrand size = 45, antiderivative size = 573

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx \\
&= -\frac{B(bc-ad)g^2n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^2i} \\
&\quad -\frac{2(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i} \\
&\quad +\frac{b^2g^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^3i} \\
&\quad -\frac{4B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i} \\
&\quad -\frac{(bc-ad)^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i}+\frac{B^2(bc-ad)^2g^2n^2\log(c+dx)}{d^3i} \\
&\quad +\frac{B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i} \\
&\quad -\frac{4B^2(bc-ad)^2g^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} \\
&\quad -\frac{2B(bc-ad)^2g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i} \\
&\quad -\frac{B^2(bc-ad)^2g^2n^2\operatorname{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^3i}+\frac{2B^2(bc-ad)^2g^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i}
\end{aligned}$$

output
```
-B*(-a*d+b*c)*g^2*n*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d^2/i-2*(-a*d+
b*c)*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^2/i+1/2*b^2*g^2*(d*x+
c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^3/i-4*B*(-a*d+b*c)^2*g^2*n*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^3/i-(-a*d+b*c)^2*g^2*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d^3/i+B^2*(-a*d
+b*c)^2*g^2*n^2*ln(d*x+c)/d^3/i+B*(-a*d+b*c)^2*g^2*n*(A+B*ln(e*((b*x+a)/(d
*x+c))^n))*ln(1-b*(d*x+c)/d/(b*x+a))/d^3/i-4*B^2*(-a*d+b*c)^2*g^2*n^2*poly
log(2,d*(b*x+a)/b/(d*x+c))/d^3/i-2*B*(-a*d+b*c)^2*g^2*n*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i-B^2*(-a*d+b*c)^2*g^2*n^
2*polylog(2,b*(d*x+c)/d/(b*x+a))/d^3/i+2*B^2*(-a*d+b*c)^2*g^2*n^2*polylog(
3,d*(b*x+a)/b/(d*x+c))/d^3/i
```

### 3.187.2 Mathematica [A] (verified)

Time = 0.81 (sec) , antiderivative size = 763, normalized size of antiderivative = 1.33

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{g^2\Big(-2bd(bc-ad)x\Big(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\Big)^2+d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+2A^2(bc-ad)^2\log(}{}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i
+ d*i*x),x]
```

output
```
(g^2*(-2*b*d*(b*c - a*d)*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + d^2*
(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 2*A^2*(b*c - a*d)^2
*Log[c + d*x] - 4*A*B*(b*c - a*d)^2*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*
c - a*d)/(b*c + b*d*x)] - 2*B^2*(b*c - a*d)^2*Log[e*((a + b*x)/(c + d*x))^
n]^2*Log[(b*c - a*d)/(b*c + b*d*x)] + 2*A*B*(b*c - a*d)^2*n*(Log[(b*c - a*
d)/(b*c + b*d*x)]*(2*Log[(d*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(
b*c + b*d*x)]) - 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)]) - 2*B*(b*c - a*d
)*n*(2*a*d*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*c*(A
+ B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - a*B*d*n*(Log[a + b*x]*(
Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*
x))/(-(b*c) + a*d)]) + b*B*c*n*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log
[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])) - B*(b
*c - a*d)*n*(2*A*b*d*x + 2*B*d*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n] -
2*B*(b*c - a*d)*n*Log[c + d*x] - 2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n])*Log[c + d*x] + B*(b*c - a*d)*n*((2*Log[(d*(a + b*x))/(-(b*c) +
 a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*
d)])) + 4*B^2*(b*c - a*d)^2*n*(-(Log[e*((a + b*x)/(c + d*x))^n]*PolyLog[2,
 (d*(a + b*x))/(b*(c + d*x))]) + n*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]
)))/(2*d^3*i)
```

## 3.187.3 Rubi [A] (verified)

Time = 0.81 (sec) , antiderivative size = 507, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2 \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ci+dix} \, dx$$

$$\Big\downarrow \ 2961$$

$$\frac{g^2(bc-ad)^2 \int \frac{(a+bx)^2 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \ 2795$$

$$\frac{g^2(bc-ad)^2 \int \left(\frac{\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2 \left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{2b\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{b^2\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2 \left(b-\frac{d(a+bx)}{c+dx}\right)^3}\right)d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow 2009$$

$$\frac{g^2(bc-ad)^2\left(\frac{b^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{2Bn\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3}-\frac{4Bn\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{d^3}}{}$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*
x),x]
```

output
```
((b*c - a*d)^2*g^2*(-((B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
))/(d^2*(c + d*x)*(b - (d*(a + b*x))/(c + d*x)))) + (b^2*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n])^2)/(2*d^3*(b - (d*(a + b*x))/(c + d*x))^2) - (2*(a +
 b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^2*(c + d*x)*(b - (d*(a
+ b*x))/(c + d*x))) - (B^2*n^2*Log[b - (d*(a + b*x))/(c + d*x)])/d^3 - (4*
B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d
*x))])/d^3 - ((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x
))/(b*(c + d*x))])/d^3 + (B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1
 - (b*(c + d*x))/(d*(a + b*x))])/d^3 - (4*B^2*n^2*PolyLog[2, (d*(a + b*x))
/(b*(c + d*x))])/d^3 - (2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyL
og[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 - (B^2*n^2*PolyLog[2, (b*(c + d*x)
)/(d*(a + b*x))])/d^3 + (2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))]
)/d^3))/i
```

### 3.187.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.187.4 Maple [F]

$$\int \frac{(bgx+ag)^2\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{dix+ci}dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

## 3.187.5 Fricas [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x,
algorithm="fricas")
```

output
```
integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x
^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(
A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c
))^n))/(d*i*x + c*i), x)
```

### 3.187.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i),x
)
```

output
```
Timed out
```

### 3.187.7 Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x,
algorithm="maxima")
```

output
```
2*A^2*a*b*g^2*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + 1/2*A^2*b^2*g^2*(2*c^2*
log(d*x + c)/(d^3*i) + (d*x^2 - 2*c*x)/(d^2*i)) + A^2*a^2*g^2*log(d*i*x +
c*i)/(d*i) + 1/2*(B^2*b^2*d^2*g^2*x^2 - 2*(b^2*c*d*g^2 - 2*a*b*d^2*g^2)*B^
2*x + 2*(b^2*c^2*g^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B^2*log(d*x + c))*log(
(d*x + c)^n)^2/(d^3*i) - integrate(-(B^2*a^2*d^2*g^2*log(e)^2 + 2*A*B*a^2*
d^2*g^2*log(e) + (B^2*b^2*d^2*g^2*log(e)^2 + 2*A*B*b^2*d^2*g^2*log(e))*x^2
 + (B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log((b*x
+ a)^n)^2 + 2*(B^2*a*b*d^2*g^2*log(e)^2 + 2*A*B*a*b*d^2*g^2*log(e))*x + 2*
(B^2*a^2*d^2*g^2*log(e) + A*B*a^2*d^2*g^2 + (B^2*b^2*d^2*g^2*log(e) + A*B*
b^2*d^2*g^2)*x^2 + 2*(B^2*a*b*d^2*g^2*log(e) + A*B*a*b*d^2*g^2)*x)*log((b*
x + a)^n) - (2*B^2*a^2*d^2*g^2*log(e) + 2*A*B*a^2*d^2*g^2 + 2*(b^2*c^2*g^2
*n - 2*a*b*c*d*g^2*n + a^2*d^2*g^2*n)*B^2*log(d*x + c) + (2*A*B*b^2*d^2*g^
2 + (g^2*n + 2*g^2*log(e))*B^2*b^2*d^2)*x^2 + 2*(2*A*B*a*b*d^2*g^2 - (b^2*
c*d*g^2*n - 2*(g^2*n + g^2*log(e))*a*b*d^2)*B^2)*x + 2*(B^2*b^2*d^2*g^2*x^
2 + 2*B^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log((b*x + a)^n))*log((d*x + c)
^n))/(d^3*i*x + c*d^2*i), x)
```

### 3.187.8 Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x,
algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x
+ c*i), x)
```

### 3.187.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)^2\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x),x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x), x)
```

# 3.188 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$



## 3.188.1 Optimal result

Integrand size = 43, antiderivative size = 303

$$\begin{aligned}
\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx
&= \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{di} \\
&+ \frac{2B(bc-ad)gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i} \\
&+ \frac{(bc-ad)g\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i} \\
&+ \frac{2B^2(bc-ad)gn^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i} \\
&+ \frac{2B(bc-ad)gn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i} \\
&- \frac{2B^2(bc-ad)gn^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i}
\end{aligned}$$

output
```
g*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d/i+2*B*(-a*d+b*c)*g*n*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/(d*x+c))/d^2/i+(-a*d+b*c)*g*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d^2/i+2*B^2*(-a*d+b*
c)*g*n^2*polylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i+2*B*(-a*d+b*c)*g*n*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i-2*B^2*(-a*d+b*
c)*g*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^2/i
```

### 3.188.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1385 vs. $2(303) = 606$.

Time = 0.56 (sec) , antiderivative size = 1385, normalized size of antiderivative = 4.57

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = \text{Too large to display}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i +
d*i*x),x]
```

output
```
(g*(3*b*d*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c +
 d*x)])^2 - 3*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[
(a + b*x)/(c + d*x)])^2*Log[c + d*x] - 3*a*B*d*n*(A + B*Log[e*((a + b*x)/(
c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(Log[c/d + x]^2 + 2*(Log[a/b
+ x] - Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*Log[c + d*x] - 2*(Log[a/b
+ x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a
*d)])) - 3*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(
c + d*x)])*(-2*d*(a + b*x)*(-1 + Log[a/b + x]) + 2*b*(c + d*x)*(-1 + Log[c
/d + x]) - b*c*Log[c/d + x]^2 + 2*b*(Log[a/b + x] - Log[c/d + x] - Log[(a
+ b*x)/(c + d*x)])*(d*x - c*Log[c + d*x]) + 2*b*c*(Log[a/b + x]*Log[(b*(c
+ d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) + a*B^2*
d*n^2*(Log[c/d + x]^3 + 3*Log[c/d + x]^2*(-Log[a/b + x] + Log[(d*(a + b*x)
)/(-(b*c) + a*d)]) + 3*(-Log[a/b + x] + Log[c/d + x] + Log[(a + b*x)/(c +
d*x)])^2*Log[c + d*x] + 3*Log[a/b + x]^2*Log[(b*(c + d*x))/(b*c - a*d)] +
6*Log[a/b + x]*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)] + 3*(Log[a/b + x]
- Log[c/d + x] - Log[(a + b*x)/(c + d*x)])*(Log[c/d + x]^2 - 2*(Log[a/b +
x]*Log[(b*(c + d*x))/(b*c - a*d)] + PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d
)])) + 6*Log[c/d + x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 6*PolyLog[3,
 (d*(a + b*x))/(-(b*c) + a*d)] - 6*PolyLog[3, (b*(c + d*x))/(b*c - a*d)])
+ B^2*n^2*(3*d*(2*b*x - 2*(a + b*x)*Log[a/b + x] + (a + b*x)*Log[a/b + ...
```

### 3.188.3 Rubi [A] (verified)

Time = 0.52 (sec) , antiderivative size = 280, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ci+dix}\,dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{g(bc-ad)\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \text{2795}$$

$$\frac{g(bc-ad)\int \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)^2}\right)d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \text{2009}$$

$$\frac{g(bc-ad)\left(\frac{2Bn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2}+\frac{2Bn\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2}+\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log \cdots\right.}{d^2 \cdots}\right.}{i}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x)
,x]
```

output
```
((b*c - a*d)*g*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d*(c
 + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (2*B*n*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 + ((A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^2 + (2*B^
2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^2 + (2*B*n*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^2 - (2
*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^2))/i
```

### 3.188.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.188.4 Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{dix+ci}dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

## 3.188.5 Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{ci+dix}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, al
gorithm="fricas")
```

output
```
integral((A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(d*
x + c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n))/(d*
i*x + c*i), x)
```

### 3.188.6 Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$=\frac{g\left(\int \frac{A^2a}{c+dx}\,dx+\int \frac{A^2bx}{c+dx}\,dx+\int \frac{B^2a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c+dx}\,dx+\int \frac{2ABa\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c+dx}\,dx+\int \frac{B^2bx\log\left(e\left(\frac{a}{c+dx}+\right.\right.}{c+dx}\right.}{i}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i),x)
```

output
```
g*(Integral(A**2*a/(c + d*x), x) + Integral(A**2*b*x/(c + d*x), x) + Integ
ral(B**2*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c + d*x), x) + Inte
gral(2*A*B*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x) + Integ
ral(B**2*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c + d*x), x) + In
tegral(2*A*B*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c + d*x), x))/i
```

### 3.188.7 Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, al
gorithm="maxima")
```

output
```
A^2*b*g*(x/(d*i) - c*log(d*x + c)/(d^2*i)) + A^2*a*g*log(d*i*x + c*i)/(d*i
) + (B^2*b*d*g*x - (b*c*g - a*d*g)*B^2*log(d*x + c))*log((d*x + c)^n)^2/(d
^2*i) - integrate(-(B^2*a*d*g*log(e)^2 + 2*A*B*a*d*g*log(e) + (B^2*b*d*g*x
 + B^2*a*d*g)*log((b*x + a)^n)^2 + (B^2*b*d*g*log(e)^2 + 2*A*B*b*d*g*log(e
))*x + 2*(B^2*a*d*g*log(e) + A*B*a*d*g + (B^2*b*d*g*log(e) + A*B*b*d*g)*x)
*log((b*x + a)^n) - 2*(B^2*a*d*g*log(e) + A*B*a*d*g - (b*c*g*n - a*d*g*n)*
B^2*log(d*x + c) + ((g*n + g*log(e))*B^2*b*d + A*B*b*d*g)*x + (B^2*b*d*g*x
 + B^2*a*d*g)*log((b*x + a)^n))*log((d*x + c)^n))/(d^2*i*x + c*d*i), x)
```

### 3.188.8 Giac [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, al
gorithm="giac")
```

output
```
integrate((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x +
c*i), x)
```

### 3.188.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x)
,x)
```

output
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x)
, x)
```

# 3.189 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$



## 3.189.1 Optimal result

Integrand size = 35, antiderivative size = 137

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx = -\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{di} - \frac{2Bn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{di} + \frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{di}$$

output
```
-(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d/i-2*B*n*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d/i+2*B^2*n^2*p
olylog(3,d*(b*x+a)/b/(d*x+c))/d/i
```

## 3.189.2 Mathematica [A] (verified)

Time = 0.52 (sec) , antiderivative size = 268, normalized size of antiderivative = 1.96

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx$$
$$= \frac{A^2\log(c+dx) + 2ABn\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log\left(\frac{bc-ad}{bc+bdx}\right) - 2AB\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\log\left(\frac{bc-ad}{bc+bdx}\right) - B^2\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x),x]
```

output
```
(A^2*Log[c + d*x] + 2*A*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[(b*c - a
*d)/(b*c + b*d*x)] - 2*A*B*Log[e*((a + b*x)/(c + d*x))^n]*Log[(b*c - a*d)/
(b*c + b*d*x)] - B^2*Log[e*((a + b*x)/(c + d*x))^n]^2*Log[(b*c - a*d)/(b*c
 + b*d*x)] + A*B*n*Log[(b*c - a*d)/(b*c + b*d*x)]^2 - 2*B^2*n*Log[e*((a +
b*x)/(c + d*x))^n]*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))] - 2*A*B*n*PolyL
og[2, (b*(c + d*x))/(b*c - a*d)] + 2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(
c + d*x))])/(d*i)
```

## 3.189.3 Rubi [A] (verified)

Time = 0.42 (sec) , antiderivative size = 128, normalized size of antiderivative = 0.93, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {2951, 2754, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{ci+dix} \, dx$$

$$\Big\downarrow \text{2951}$$

$$\frac{\int \frac{\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{b-\frac{d(a+bx)}{c+dx}}d\frac{a+bx}{c+dx}}{i}$$

$$\Big\downarrow \text{2754}$$

$$\frac{\frac{2Bn\int \frac{(c+dx)\left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right) \log \left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{d}-\frac{\log \left(1-\frac{d(a+bx)}{b(c+dx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}}{i}$$

$$\Big\downarrow \text{2821}$$

$$\frac{\frac{2Bn\left(Bn\int \frac{(c+dx) \text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)}{d}-\frac{\log \left(1-\frac{d(a+bx)}{b(c+dx)}\right) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}}{i}$$

$$\Big\downarrow \text{7143}$$

$$\frac{\frac{2Bn\left(Bn\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)-\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)}{d}-\frac{\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d}}{i}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x),x]
```

output
```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c +
 d*x))])/d) + (2*B*n*(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2,
(d*(a + b*x))/(b*(c + d*x))]) + B*n*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))
]))/d)/i
```

### 3.189.3.1 Defintions of rubi rules used

rule 2754
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((d_) + (e_.)*(x_)), x_Symb
ol] :> Simp[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^p/e), x] - Simp[b*n*(p/e)
  Int[Log[1 + e*(x/d)]*((a + b*Log[c*x^n])^(p - 1)/x), x], x] /; FreeQ[{a,
b, c, d, e, n}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.189.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{dix+ci}dx$$

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

output
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x)
```

### 3.189.5 Fricas [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{ci+dix}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{dix+ci}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, algorithm="fri
cas")
```

output
```
integral((B^2*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*A*B*log(e*((b*x + a)/(d
*x + c))^n) + A^2)/(d*i*x + c*i), x)
```

### 3.189.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{ci+dix}\,dx = \frac{\int \frac{A^2}{c+dx}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)^{2}}{c+dx}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^{n}\right)}{c+dx}\,dx}{i}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i),x)
```

output
```
(Integral(A**2/(c + d*x), x) + Integral(B**2*log(e*(a/(c + d*x) + b*x/(c +
 d*x))**n)**2/(c + d*x), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c +
 d*x))**n)/(c + d*x), x))/i
```

### 3.189.7 Maxima [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, algorithm="max
ima")
```

output
```
B^2*log(d*x + c)*log((d*x + c)^n)^2/(d*i) + A^2*log(d*i*x + c*i)/(d*i) - i
ntegrate(-(B^2*log((b*x + a)^n)^2 + B^2*log(e)^2 + 2*A*B*log(e) + 2*(B^2*l
og(e) + A*B)*log((b*x + a)^n) - 2*(B^2*n*log(d*x + c) + B^2*log((b*x + a)^
n) + B^2*log(e) + A*B)*log((d*x + c)^n))/(d*i*x + c*i), x)
```

### 3.189.8 Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{dix+ci}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i),x, algorithm="gia
c")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x + c*i), x)
```

### 3.189.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{ci+dix}\,dx=\int \frac{\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{c\,i+d\,i\,x}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*i + d*i*x),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*i + d*i*x), x)
```

# 3.190 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$



## 3.190.1 Optimal result

Integrand size = 45, antiderivative size = 50

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx=\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)gin}$$

output
```
1/3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)/g/i/n
```

## 3.190.2 Mathematica [A] (verified)

Time = 0.17 (sec) , antiderivative size = 90, normalized size of antiderivative = 1.80

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx =\frac{3A^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+3AB\log^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+B^2\log^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{3bcgin-3adgin}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d
*i*x)),x]
```

output
```
(3*A^2*Log[e*((a + b*x)/(c + d*x))^n] + 3*A*B*Log[e*((a + b*x)/(c + d*x))^
n]^2 + B^2*Log[e*((a + b*x)/(c + d*x))^n]^3)/(3*b*c*g*i*n - 3*a*d*g*i*n)
```

### 3.190.3 Rubi [A] (warning: unable to verify)

Time = 0.34 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.80, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)(ci+dix)}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}}{gi(bc-ad)}$$

$$\Big\downarrow \text{2739}$$

$$\frac{\int \frac{(a+bx)^2}{(c+dx)^2}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bgin(bc-ad)}$$

$$\Big\downarrow \text{15}$$

$$\frac{(a+bx)^3}{3Bgin(c+dx)^3(bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d*i*x))
,x]
```

output
```
(a + b*x)^3/(3*B*(b*c - a*d)*g*i*n*(c + d*x)^3)
```

#### 3.190.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.190.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(48) = 96$.

Time = 1.44 (sec) , antiderivative size = 118, normalized size of antiderivative = 2.36

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{B^{2} a^{2} c^{2} \ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)^{3}+3 A B \,a^{2} c^{2} \ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)^{2}+3 A^{2} \ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right) a^{2} c^{2}}{3 i g \,c^{2} a^{2} n \left(a d -c b \right)}$ | 118 |
| default | $\frac{A^{2} \left(\frac{\ln \left(d x +c \right)}{a d -c b}-\frac{\ln \left(b x +a \right)}{a d -c b}\right)}{g i}-\frac{B^{2} \ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)^{3}}{3 g i n \left(a d -c b \right)}-\frac{A B \ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)^{2}}{g i n \left(a d -c b \right)}$ | 135 |
| parts | $\frac{A^{2} \left(\frac{\ln \left(d x +c \right)}{a d -c b}-\frac{\ln \left(b x +a \right)}{a d -c b}\right)}{g i}-\frac{B^{2} \ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)^{3}}{3 g i n \left(a d -c b \right)}-\frac{A B \ln \left(e \left(\frac{b x +a}{d x +c}\right)^{n}\right)^{2}}{g i n \left(a d -c b \right)}$ | 135 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i),x,method=_RE
TURNVERBOSE)
```

output
```
-1/3*(B^2*a^2*c^2*ln(e*((b*x+a)/(d*x+c))^n)^3+3*A*B*a^2*c^2*ln(e*((b*x+a)/
(d*x+c))^n)^2+3*A^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*c^2)/i/g/c^2/a^2/n/(a*d-
b*c)
```

## 3.190.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 149 vs. $2(48) = 96$.

Time = 0.32 (sec) , antiderivative size = 149, normalized size of antiderivative = 2.98

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$$
$$=\frac{B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^3+3\,B^2\log(e)^2\log\left(\frac{bx+a}{dx+c}\right)+3\,ABn\log\left(\frac{bx+a}{dx+c}\right)^2+3\,A^2\log\left(\frac{bx+a}{dx+c}\right)+3\left(B^2n\log\left(\frac{bx+a}{dx+c}\right)^2+\right.}{3\,(bc-ad)gi}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i),x, al
gorithm="fricas")
```

output
```
1/3*(B^2*n^2*log((b*x + a)/(d*x + c))^3 + 3*B^2*log(e)^2*log((b*x + a)/(d*
x + c)) + 3*A*B*n*log((b*x + a)/(d*x + c))^2 + 3*A^2*log((b*x + a)/(d*x +
c)) + 3*(B^2*n*log((b*x + a)/(d*x + c))^2 + 2*A*B*log((b*x + a)/(d*x + c))
)*log(e))/((b*c - a*d)*g*i)
```

### 3.190.6 Sympy [F]

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{\int \frac{A^2}{ac+adx+bcx+bdx^2}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{ac+adx+bcx+bdx^2}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{ac+adx+bcx+bdx^2}\,dx}{gi}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)/(d*i*x+c*i),x)
```

output
```
(Integral(A**2/(a*c + a*d*x + b*c*x + b*d*x**2), x) + Integral(B**2*log(e*
(a/(c + d*x) + b*x/(c + d*x))**n)**2/(a*c + a*d*x + b*c*x + b*d*x**2), x)
+ Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(a*c + a*d*x + b*
c*x + b*d*x**2), x))/(g*i)
```

### 3.190.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(48) = 96$.

Time = 0.22 (sec) , antiderivative size = 407, normalized size of antiderivative = 8.14

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx$$
$$= B^2\left(\frac{\log(bx+a)}{(bc-ad)gi}-\frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2$$
$$+2\,AB\left(\frac{\log(bx+a)}{(bc-ad)gi}-\frac{\log(dx+c)}{(bc-ad)gi}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)$$
$$+\frac{1}{3}\left(\frac{(\log(bx+a)^3-3\,\log(bx+a)^2\log(dx+c)+3\,\log(bx+a)\log(dx+c)^2-\log(dx+c)^3)n^2}{bcgi-adgi}-\frac{3\,(\right.$$
$$-\frac{(\log(bx+a)^2-2\,\log(bx+a)\log(dx+c)+\log(dx+c)^2)ABn}{bcgi-adgi}$$
$$+A^2\left(\frac{\log(bx+a)}{(bc-ad)gi}-\frac{\log(dx+c)}{(bc-ad)gi}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i),x, al
gorithm="maxima")
```

output
```
B^2*(log(b*x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n)^2 + 2*A*B*(log(b*x + a)/((b*c - a*d)*g*
i) - log(d*x + c)/((b*c - a*d)*g*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
) + 1/3*((log(b*x + a)^3 - 3*log(b*x + a)^2*log(d*x + c) + 3*log(b*x + a)*
log(d*x + c)^2 - log(d*x + c)^3)*n^2/(b*c*g*i - a*d*g*i) - 3*(log(b*x + a)
^2 - 2*log(b*x + a)*log(d*x + c) + log(d*x + c)^2)*n*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(b*c*g*i - a*d*g*i))*B^2 - (log(b*x + a)^2 - 2*log(b*x +
 a)*log(d*x + c) + log(d*x + c)^2)*A*B*n/(b*c*g*i - a*d*g*i) + A^2*(log(b*
x + a)/((b*c - a*d)*g*i) - log(d*x + c)/((b*c - a*d)*g*i))
```

### 3.190.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 167 vs. 2(48) = 96.

Time = 0.55 (sec) , antiderivative size = 167, normalized size of antiderivative = 3.34

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = \frac{\left(B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^3+3\,B^2n\log(e)\log\left(\frac{bx+a}{dx+c}\right)^2+3\,B^2\log(e)^2\log\left(\frac{bx+a}{dx+c}\right)+3\,ABn\log\left(\frac{bx+a}{dx+c}\right)^2+6\,AB\log(e\right.}{3\,gi}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i),x, al
gorithm="giac")
```

output
```
1/3*(B^2*n^2*log((b*x + a)/(d*x + c))^3 + 3*B^2*n*log(e)*log((b*x + a)/(d*
x + c))^2 + 3*B^2*log(e)^2*log((b*x + a)/(d*x + c)) + 3*A*B*n*log((b*x + a
)/(d*x + c))^2 + 6*A*B*log(e)*log((b*x + a)/(d*x + c)) + 3*A^2*log((b*x +
a)/(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)/(g*i)
```

### 3.190.9 Mupad [B] (verification not implemented)

Time = 2.60 (sec) , antiderivative size = 122, normalized size of antiderivative = 2.44

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)}\,dx = -\frac{\frac{B^2\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^3}{3}+A\,B\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2}{g\,i\,n\,(a\,d-b\,c)} + \frac{A^2\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{g\,i\,(a\,d-b\,c)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)*(c*i + d*i*x))
,x)
```

output
```
(A^2*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(g*i*(a*d - b*c))
- ((B^2*log(e*((a + b*x)/(c + d*x))^n)^3)/3 + A*B*log(e*((a + b*x)/(c + d*
x))^n)^2)/(g*i*n*(a*d - b*c))
```

# 3.191 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$



## 3.191.1 Optimal result

Integrand size = 45, antiderivative size = 199

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = &-\frac{2bB^2n^2(c+dx)}{(bc-ad)^2g^2i(a+bx)}\\ &-\frac{2bBn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^2g^2i(a+bx)}\\ &-\frac{b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2g^2i(a+bx)}\\ &-\frac{d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^2g^2in}\end{aligned}$$

output
```
-2*b*B^2*n^2*(d*x+c)/(-a*d+b*c)^2/g^2/i/(b*x+a)-2*b*B*n*(d*x+c)*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/g^2/i/(b*x+a)-b*(d*x+c)*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g^2/i/(b*x+a)-1/3*d*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))^3/B/(-a*d+b*c)^2/g^2/i/n
```

### 3.191.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 793 vs. 2(199) = 398.

Time = 0.45 (sec) , antiderivative size = 793, normalized size of antiderivative = 3.98

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$
$$= -\frac{B^2dn^2\log^3\left(\frac{a+bx}{c+dx}\right)}{3(bc-ad)^2g^2i} + \frac{2Bn\log\left(\frac{a+bx}{c+dx}\right)\left(A+Bn+B\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)\right)}{(-bc+ad)g^2i(a+bx)}$$
$$+ \frac{\log^2\left(\frac{a+bx}{c+dx}\right)\left(-aABdn-bB^2cn^2-AbBdnx-bB^2dn^2x-aB^2dn\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)-bB\right.}{(-bc+ad)^2g^2i(a+bx)}$$
$$+ \frac{-A^2-2ABn-2B^2n^2-2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)-2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)}{(bc-ad)g^2i(a+bx)}$$
$$- \frac{d\log(a+bx)\left(A^2+2ABn+2B^2n^2+2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)+2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-\right.\right.}{(bc-ad)^2g^2i}$$
$$+ \frac{d\left(A^2+2ABn+2B^2n^2+2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)+2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)\right.}{(bc-ad)^2g^2i}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i +
 d*i*x)),x]
```

output
```
-1/3*(B^2*d*n^2*Log[(a + b*x)/(c + d*x)]^3)/((b*c - a*d)^2*g^2*i) + (2*B*n
*Log[(a + b*x)/(c + d*x)]*(A + B*n + B*(Log[e*((a + b*x)/(c + d*x))^n] - n
*Log[(a + b*x)/(c + d*x)])))/((-(b*c) + a*d)*g^2*i*(a + b*x)) + (Log[(a +
b*x)/(c + d*x)]^2*(-(a*A*B*d*n) - b*B^2*c*n^2 - A*b*B*d*n*x - b*B^2*d*n^2*
x - a*B^2*d*n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]
) - b*B^2*d*n*x*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x
)])))/((-(b*c) + a*d)^2*g^2*i*(a + b*x)) + (-A^2 - 2*A*B*n - 2*B^2*n^2 - 2
*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - 2*B^2
*n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - B^2*(Lo
g[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2)/((b*c - a*d)
*g^2*i*(a + b*x)) - (d*Log[a + b*x]*(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Lo
g[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + 2*B^2*n*(Log[
e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a
+ b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2))/((b*c - a*d)^2*g^2*
i) + (d*(A^2 + 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b*x)/(c + d*x))^n]
 - n*Log[(a + b*x)/(c + d*x)]) + 2*B^2*n*(Log[e*((a + b*x)/(c + d*x))^n] -
 n*Log[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log
[(a + b*x)/(c + d*x)])^2)*Log[c + d*x])/((b*c - a*d)^2*g^2*i)
```

---

3.191. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$

### 3.191.3 Rubi [A] (verified)

Time = 0.46 (sec) , antiderivative size = 152, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2(ci+dix)}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i(bc-ad)^2}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left(\frac{b(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}-\frac{d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}\right)d\frac{a+bx}{c+dx}}{g^2i(bc-ad)^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{-\frac{d\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bn}-\frac{b(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}-\frac{2bBn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}-\frac{2bB^2n^2(c+dx)}{a+bx}}{g^2i(bc-ad)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i + d*i*x
)),x]
```

output
```
((-2*b*B^2*n^2*(c + d*x))/(a + b*x) - (2*b*B*n*(c + d*x)*(A + B*Log[e*((a
+ b*x)/(c + d*x))^n]))/(a + b*x) - (b*(c + d*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2)/(a + b*x) - (d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(
3*B*n))/((b*c - a*d)^2*g^2*i)
```

### 3.191.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.191.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(197) = 394$.

Time = 4.86 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.56

| method | result |
|---|---|
| parallelrisch | $-\frac{-6B^2ab^3d^3n^3+6B^2b^4cd^2n^3-3A^2ab^3d^3n+3A^2b^4cd^2n+B^2x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3b^4d^3+B^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3ab^3d^3+3A^2x\ln\left(\right.}{}$ |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x,method=_
RETURNVERBOSE)
```

output
```
-1/3*(-6*B^2*a*b^3*d^3*n^3+6*B^2*b^4*c*d^2*n^3-3*A^2*a*b^3*d^3*n+3*A^2*b^4
*c*d^2*n+B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*b^4*d^3+B^2*ln(e*((b*x+a)/(d*x+
c))^n)^3*a*b^3*d^3+3*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^3+3*A^2*ln(e*((
b*x+a)/(d*x+c))^n)*a*b^3*d^3+6*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^4*d^3*n+6
*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^4*c*d^2*n-6*A*B*a*b^3*d^3*n^2+6*A*B*b^4*c
*d^2*n^2+3*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*d^3*n+6*B^2*x*ln(e*((b*x+
a)/(d*x+c))^n)*b^4*d^3*n^2+3*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*d^3+3*B
^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^4*c*d^2*n+6*B^2*ln(e*((b*x+a)/(d*x+c))^n)
*b^4*c*d^2*n^2+3*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^3*d^3)/i/g^2/(b*x+a)/
n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b^3/d^2
```

### 3.191.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(197) = 394$.

Time = 0.34 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.15

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = -\frac{3A^2bc-3A^2ad+(B^2bdn^2x+B^2adn^2)\log\left(\frac{bx+a}{dx+c}\right)^3+6(B^2bc-B^2ad)n^2+3\left(B^2bc-B^2ad+(B^2bdx\right.}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x,
algorithm="fricas")
```

output
```
-1/3*(3*A^2*b*c - 3*A^2*a*d + (B^2*b*d*n^2*x + B^2*a*d*n^2)*log((b*x + a)/
(d*x + c))^3 + 6*(B^2*b*c - B^2*a*d)*n^2 + 3*(B^2*b*c - B^2*a*d + (B^2*b*d
*x + B^2*a*d)*log((b*x + a)/(d*x + c)))*log(e)^2 + 3*(B^2*b*c*n^2 + A*B*a*
d*n + (B^2*b*d*n^2 + A*B*b*d*n)*x)*log((b*x + a)/(d*x + c))^2 + 6*(A*B*b*c
 - A*B*a*d)*n + 3*(2*A*B*b*c - 2*A*B*a*d + (B^2*b*d*n*x + B^2*a*d*n)*log((
b*x + a)/(d*x + c))^2 + 2*(B^2*b*c - B^2*a*d)*n + 2*(B^2*b*c*n + A*B*a*d +
 (B^2*b*d*n + A*B*b*d)*x)*log((b*x + a)/(d*x + c)))*log(e) + 3*(2*B^2*b*c*
n^2 + 2*A*B*b*c*n + A^2*a*d + (2*B^2*b*d*n^2 + 2*A*B*b*d*n + A^2*b*d)*x)*l
og((b*x + a)/(d*x + c)))/((b^3*c^2 - 2*a*b^2*c*d + a^2*b*d^2)*g^2*i*x + (a
*b^2*c^2 - 2*a^2*b*c*d + a^3*d^2)*g^2*i)
```

### 3.191.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2/(d*i*x+c*i),x
)
```

output
```
Timed out
```

### 3.191.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1018 vs. $2(197) = 394$.

Time = 0.27 (sec) , antiderivative size = 1018, normalized size of antiderivative = 5.12

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx =$$
$$-B^2\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log\Big(e$$
$$-2\,AB\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)\log$$
$$-\frac{1}{3}\Bigg(\frac{((bdx+ad)\log(bx+a)^3-(bdx+ad)\log(dx+c)^3-3\,(bdx+ad)\log(bx+a)^2-3\,(bdx+ad-(}{a}$$
$$+\frac{((bdx+ad)\log(bx+a)^2+(bdx+ad)\log(dx+c)^2-2\,bc+2\,ad-2\,(bdx+ad)\log(bx+a)+2\,(bdx}{ab^2c^2g^2i-2\,a^2bcdg^2i+a^3d^2g^2i+(b^3c^2g^2i-2\,ab^2cdg^2i+a^2b}$$
$$-A^2\left(\frac{1}{(b^2c-abd)g^2ix+(abc-a^2d)g^2i}+\frac{d\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}-\frac{d\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)g^2i}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x,
algorithm="maxima")
```

output
```
-B^2*(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)
/((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*
b*c*d + a^2*d^2)*g^2*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2 - 2*A*B*
(1/((b^2*c - a*b*d)*g^2*i*x + (a*b*c - a^2*d)*g^2*i) + d*log(b*x + a)/((b^
2*c^2 - 2*a*b*c*d + a^2*d^2)*g^2*i) - d*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d
 + a^2*d^2)*g^2*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/3*(((b*d*x
+ a*d)*log(b*x + a)^3 - (b*d*x + a*d)*log(d*x + c)^3 - 3*(b*d*x + a*d)*log
(b*x + a)^2 - 3*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x + c)^2
+ 6*b*c - 6*a*d + 6*(b*d*x + a*d)*log(b*x + a) - 3*(2*b*d*x + (b*d*x + a*d
)*log(b*x + a)^2 + 2*a*d - 2*(b*d*x + a*d)*log(b*x + a))*log(d*x + c))*n^2
/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^2*g^2*i + (b^3*c^2*g^2*i - 2
*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x) - 3*((b*d*x + a*d)*log(b*x + a)^2 +
 (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2*a*d - 2*(b*d*x + a*d)*log(b*x +
a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b*x + a))*log(d*x + c))*n*log(e*(b
*x/(d*x + c) + a/(d*x + c))^n)/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*
d^2*g^2*i + (b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x))*B^2
+ ((b*d*x + a*d)*log(b*x + a)^2 + (b*d*x + a*d)*log(d*x + c)^2 - 2*b*c + 2
*a*d - 2*(b*d*x + a*d)*log(b*x + a) + 2*(b*d*x + a*d - (b*d*x + a*d)*log(b
*x + a))*log(d*x + c))*A*B*n/(a*b^2*c^2*g^2*i - 2*a^2*b*c*d*g^2*i + a^3*d^
2*g^2*i + (b^3*c^2*g^2*i - 2*a*b^2*c*d*g^2*i + a^2*b*d^2*g^2*i)*x) - A^...
```

### 3.191.8 Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2(dix+ci)}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i),x,
algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/((b*g*x + a*g)^2*(d*i*x
 + c*i)), x)
```

### 3.191.9 Mupad [B] (verification not implemented)

Time = 2.55 (sec) , antiderivative size = 361, normalized size of antiderivative = 1.81

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)}\,dx$$
$$= \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{(a\,d-b\,c)\,\left(a\,g^2\,i+b\,g^2\,i\,x\right)}-\frac{B\,d\,(A+B\,n)}{g^2\,i\,n\,(a\,d-b\,c)^2}\right)$$
$$+\frac{A^2+2\,A\,B\,n+2\,B^2\,n^2}{(a\,d-b\,c)\,\left(a\,g^2\,i+b\,g^2\,i\,x\right)}+\frac{2\,B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(A+B\,n)}{(a\,d-b\,c)\,\left(a\,g^2\,i+b\,g^2\,i\,x\right)}-\frac{B^2\,d\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^3}{3\,g^2\,i\,n\,(a\,d-b\,c)^2}$$
$$+\frac{d\,\mathrm{atan}\left(\frac{d\left(2\,b\,d\,x+\frac{a^2\,d^2\,g^2\,i-b^2\,c^2\,g^2\,i}{g^2\,i\,(a\,d-b\,c)}\right)\,\left(A^2+2\,A\,B\,n+2\,B^2\,n^2\right)\,1\mathrm{i}}{(a\,d-b\,c)\,\left(d\,A^2+2\,d\,A\,B\,n+2\,d\,B^2\,n^2\right)}\right)\,\left(A^2+2\,A\,B\,n+2\,B^2\,n^2\right)\,2\mathrm{i}}{g^2\,i\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^2*(c*i + d*i*x
)),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/((a*d - b*c)*(a*g^2*i + b*g^2*i*x))
- (B*d*(A + B*n))/(g^2*i*n*(a*d - b*c)^2)) + (A^2 + 2*B^2*n^2 + 2*A*B*n)/(
(a*d - b*c)*(a*g^2*i + b*g^2*i*x)) + (2*B*log(e*((a + b*x)/(c + d*x))^n)*(
A + B*n))/((a*d - b*c)*(a*g^2*i + b*g^2*i*x)) + (d*atan((d*(2*b*d*x + (a^2
*d^2*g^2*i - b^2*c^2*g^2*i)/(g^2*i*(a*d - b*c)))*(A^2 + 2*B^2*n^2 + 2*A*B*
n)*1i)/((a*d - b*c)*(A^2*d + 2*B^2*d*n^2 + 2*A*B*d*n)))*(A^2 + 2*B^2*n^2 +
 2*A*B*n)*2i)/(g^2*i*(a*d - b*c)^2) - (B^2*d*log(e*((a + b*x)/(c + d*x))^n
)^3)/(3*g^2*i*n*(a*d - b*c)^2)
```

# 3.192 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$



## 3.192.1 Optimal result

Integrand size = 45, antiderivative size = 369

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx &= \frac{4bB^2dn^2(c+dx)}{(bc-ad)^3g^3i(a+bx)}-\frac{b^2B^2n^2(c+dx)^2}{4(bc-ad)^3g^3i(a+bx)^2}\\ &+\frac{4bBdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^3i(a+bx)}\\ &-\frac{b^2Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3g^3i(a+bx)^2}\\ &+\frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^3i(a+bx)}\\ &-\frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3g^3i(a+bx)^2}\\ &+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3g^3in}\end{aligned}$$

output
```
4*b*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/4*b^2*B^2*n^2*(d*x+c)^2
/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+4*b*B*d*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/2*b^2*B*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+2*b*d*(d*x+c)*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))^2/(-a*d+b*c)^3/g^3/i/(b*x+a)-1/2*b^2*(d*x+c)^2*(A+B*ln(e*((b
*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^3/i/(b*x+a)^2+1/3*d^2*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))^3/B/(-a*d+b*c)^3/g^3/i/n
```

### 3.192.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 975 vs. 2(369) = 738.

Time = 0.77 (sec) , antiderivative size = 975, normalized size of antiderivative = 2.64

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$$
$$= \frac{4B^2d^2n^2(a+bx)^2\log^3\left(\frac{a+bx}{c+dx}\right)+6Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(2a^2Ad^2-b^2Bc^2n+4abBcdn+4aAbd^2x+2b^2Bcdnx-\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i +
 d*i*x)),x]
```

output
```
(4*B^2*d^2*n^2*(a + b*x)^2*Log[(a + b*x)/(c + d*x)]^3 + 6*B*n*Log[(a + b*x
)/(c + d*x)]^2*(2*a^2*A*d^2 - b^2*B*c^2*n + 4*a*b*B*c*d*n + 4*a*A*b*d^2*x
+ 2*b^2*B*c*d*n*x + 4*a*b*B*d^2*n*x + 2*A*b^2*d^2*x^2 + 3*b^2*B*d^2*n*x^2
+ 2*B*d^2*(a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n] - 2*B*d^2*n*(a + b*x)
^2*Log[(a + b*x)/(c + d*x)]) - 6*B*(b*c - a*d)*n*Log[(a + b*x)/(c + d*x)]*
(2*A*b*c - 6*a*A*d + b*B*c*n - 7*a*B*d*n - 4*A*b*d*x - 6*b*B*d*n*x + 2*B*(
-3*a*d + b*(c - 2*d*x))*Log[e*((a + b*x)/(c + d*x))^n] + 2*B*n*(-(b*c) + 3
*a*d + 2*b*d*x)*Log[(a + b*x)/(c + d*x)]) - 3*(b*c - a*d)^2*(2*A^2 + 2*A*B
*n + B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + B*n)*
Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[
e*((a + b*x)/(c + d*x))^n]*(2*A + B*n - 2*B*n*Log[(a + b*x)/(c + d*x)])) +
 6*d*(b*c - a*d)*(a + b*x)*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*B^2*Log[e*((a
+ b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + 3*B*n)*Log[(a + b*x)/(c + d*x)] + 2*
B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(2
*A + 3*B*n - 2*B*n*Log[(a + b*x)/(c + d*x)])) + 6*d^2*(a + b*x)^2*Log[a +
b*x]*(2*A^2 + 6*A*B*n + 7*B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2
 - 2*B*n*(2*A + 3*B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/
(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(2*A + 3*B*n - 2*B*n*Log
[(a + b*x)/(c + d*x)])) - 6*d^2*(a + b*x)^2*(2*A^2 + 6*A*B*n + 7*B^2*n^2 +
 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + 3*B*n)*Log[(a + ...
```

### 3.192.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 274, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3(ci+dix)}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i(bc-ad)^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^3}{(a+bx)^3}-\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^2}{(a+bx)^2}+\frac{d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)}{a+bx}\right)d\frac{a+bx}{c+dx}}{g^3i(bc-ad)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^2(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}-\frac{b^2Bn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{d^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{3Bn}+\frac{2bd(c+dx)\left(B\log\left(e\left(\frac{a+b\ldots}{c+d\ldots}\right.\right.\right.}{a+bx}}{g^3i(bc-ad)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i + d*i*x
)),x]
```

output
```
((4*b*B^2*d*n^2*(c + d*x))/(a + b*x) - (b^2*B^2*n^2*(c + d*x)^2)/(4*(a + b
*x)^2) + (4*b*B*d*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a +
 b*x) - (b^2*B*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a
 + b*x)^2) + (2*b*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a
 + b*x) - (b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a
 + b*x)^2) + (d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*B*n))/((b*c
 - a*d)^3*g^3*i)
```

#### 3.192.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.192.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1258 vs. $2(361) = 722$.

Time = 11.70 (sec) , antiderivative size = 1259, normalized size of antiderivative = 3.41

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1259 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x,method=_
RETURNVERBOSE)
```

output
```
-1/12*(24*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^3*d*n+12*B^2*x*ln(e*((
b*x+a)/(d*x+c))^n)^2*a^4*b^2*c^3*d*n+36*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*
a^4*b^2*c^2*d^2*n+48*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b*c^2*d^2*n+48*B^
2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b*c^2*d^2*n^2+36*B^2*x*ln(e*((b*x+a)/(d*
x+c))^n)*a^4*b^2*c^3*d*n^2+24*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b*c^2*
d^2+48*A*B*x*a^5*b*c^2*d^2*n^2-60*A*B*x*a^4*b^2*c^3*d*n^2+48*A*B*ln(e*((b*
x+a)/(d*x+c))^n)*a^5*b*c^3*d*n+18*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*
b^2*c^2*d^2*n+42*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^2*d^2*n^2+12*
A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^2*c^2*d^2+42*A*B*x^2*a^4*b^2*c^2
*d^2*n^2-48*A*B*x^2*a^3*b^3*c^3*d*n^2+24*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2
*a^5*b*c^2*d^2*n+4*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^4*b^2*c^2*d^2+45*
B^2*x^2*a^4*b^2*c^2*d^2*n^3-48*B^2*x^2*a^3*b^3*c^3*d*n^3+6*A*B*x^2*a^2*b^4
*c^4*n^2+8*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^5*b*c^2*d^2+48*B^2*x*a^5*b*
c^2*d^2*n^3-54*B^2*x*a^4*b^2*c^3*d*n^3+12*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n
)*a^4*b^2*c^2*d^2+18*A^2*x^2*a^4*b^2*c^2*d^2*n-24*A^2*x^2*a^3*b^3*c^3*d*n+
12*A*B*x*a^3*b^3*c^4*n^2+24*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b*c^3*d*n+
48*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b*c^3*d*n^2+24*A^2*x*ln(e*((b*x+a)/(d
*x+c))^n)*a^5*b*c^2*d^2+24*A^2*x*a^5*b*c^2*d^2*n-36*A^2*x*a^4*b^2*c^3*d*n-
12*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^2*c^4*n+3*B^2*x^2*a^2*b^4*c^4*n^3+6
*B^2*x*a^3*b^3*c^4*n^3+6*A^2*x^2*a^2*b^4*c^4*n-6*B^2*ln(e*((b*x+a)/(d*x...
```

### 3.192.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1068 vs. $2(361) = 722$.

Time = 0.35 (sec) , antiderivative size = 1068, normalized size of antiderivative = 2.89

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx =$$

$$-\frac{6\,A^2b^2c^2 - 24\,A^2abcd + 18\,A^2a^2d^2 - 4\left(B^2b^2d^2n^2x^2 + 2\,B^2abd^2n^2x + B^2a^2d^2n^2\right)\log\left(\frac{bx+a}{dx+c}\right)^3 + 3\,(B^2b^2c}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x,
algorithm="fricas")
```

output
```
-1/12*(6*A^2*b^2*c^2 - 24*A^2*a*b*c*d + 18*A^2*a^2*d^2 - 4*(B^2*b^2*d^2*n^
2*x^2 + 2*B^2*a*b*d^2*n^2*x + B^2*a^2*d^2*n^2)*log((b*x + a)/(d*x + c))^3
+ 3*(B^2*b^2*c^2 - 16*B^2*a*b*c*d + 15*B^2*a^2*d^2)*n^2 + 6*(B^2*b^2*c^2 -
 4*B^2*a*b*c*d + 3*B^2*a^2*d^2 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*x - 2*(B^2*
b^2*d^2*x^2 + 2*B^2*a*b*d^2*x + B^2*a^2*d^2)*log((b*x + a)/(d*x + c)))*log
(e)^2 - 6*(2*A*B*a^2*d^2*n - (B^2*b^2*c^2 - 4*B^2*a*b*c*d)*n^2 + (3*B^2*b^
2*d^2*n^2 + 2*A*B*b^2*d^2*n)*x^2 + 2*(2*A*B*a*b*d^2*n + (B^2*b^2*c*d + 2*B
^2*a*b*d^2)*n^2)*x)*log((b*x + a)/(d*x + c))^2 + 6*(A*B*b^2*c^2 - 8*A*B*a*
b*c*d + 7*A*B*a^2*d^2)*n - 6*(2*A^2*b^2*c*d - 2*A^2*a*b*d^2 + 7*(B^2*b^2*c
*d - B^2*a*b*d^2)*n^2 + 6*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x + 6*(2*A*B*b^2*
c^2 - 8*A*B*a*b*c*d + 6*A*B*a^2*d^2 - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*a*b*d^2
*n*x + B^2*a^2*d^2*n)*log((b*x + a)/(d*x + c))^2 + (B^2*b^2*c^2 - 8*B^2*a*
b*c*d + 7*B^2*a^2*d^2)*n - 2*(2*A*B*b^2*c*d - 2*A*B*a*b*d^2 + 3*(B^2*b^2*c
*d - B^2*a*b*d^2)*n)*x - 2*(2*A*B*a^2*d^2 + (3*B^2*b^2*d^2*n + 2*A*B*b^2*d
^2)*x^2 - (B^2*b^2*c^2 - 4*B^2*a*b*c*d)*n + 2*(2*A*B*a*b*d^2 + (B^2*b^2*c*
d + 2*B^2*a*b*d^2)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) - 6*(2*A^2*a^2*d
^2 - (B^2*b^2*c^2 - 8*B^2*a*b*c*d)*n^2 + (7*B^2*b^2*d^2*n^2 + 6*A*B*b^2*d^
2*n + 2*A^2*b^2*d^2)*x^2 - 2*(A*B*b^2*c^2 - 4*A*B*a*b*c*d)*n + 2*(2*A^2*a*
b*d^2 + (3*B^2*b^2*c*d + 4*B^2*a*b*d^2)*n^2 + 2*(A*B*b^2*c*d + 2*A*B*a*b*d
^2)*n)*x)*log((b*x + a)/(d*x + c)))/((b^5*c^3 - 3*a*b^4*c^2*d + 3*a^2*b...
```

### 3.192.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3/(d*i*x+c*i),x
)
```

output
```
Timed out
```

### 3.192.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2126 vs. 2(361) = 722.

Time = 0.34 (sec) , antiderivative size = 2126, normalized size of antiderivative = 5.76

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x,
algorithm="maxima")
```

output
```
1/2*B^2*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^2*b^2*d^2)*g^
3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x + (a^2*b^2*c^2
 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/((b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(e*(b*x/(d*x + c) + a/(
d*x + c))^n)^2 + A*B*((2*b*d*x - b*c + 3*a*d)/((b^4*c^2 - 2*a*b^3*c*d + a^
2*b^2*d^2)*g^3*i*x^2 + 2*(a*b^3*c^2 - 2*a^2*b^2*c*d + a^3*b*d^2)*g^3*i*x +
 (a^2*b^2*c^2 - 2*a^3*b*c*d + a^4*d^2)*g^3*i) + 2*d^2*log(b*x + a)/((b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i) - 2*d^2*log(d*x + c)/
((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^3*i))*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) - 1/12*((3*b^2*c^2 - 48*a*b*c*d + 45*a^2*d^2 - 4
*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a)^3 + 4*(b^2*d^2*x^2 + 2
*a*b*d^2*x + a^2*d^2)*log(d*x + c)^3 + 18*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2
*d^2)*log(b*x + a)^2 + 6*(3*b^2*d^2*x^2 + 6*a*b*d^2*x + 3*a^2*d^2 - 2*(b^2
*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a))*log(d*x + c)^2 - 42*(b^2*c
*d - a*b*d^2)*x - 42*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b*x + a) +
6*(7*b^2*d^2*x^2 + 14*a*b*d^2*x + 7*a^2*d^2 + 2*(b^2*d^2*x^2 + 2*a*b*d^2*x
 + a^2*d^2)*log(b*x + a)^2 - 6*(b^2*d^2*x^2 + 2*a*b*d^2*x + a^2*d^2)*log(b
*x + a))*log(d*x + c))*n^2/(a^2*b^3*c^3*g^3*i - 3*a^3*b^2*c^2*d*g^3*i + 3*
a^4*b*c*d^2*g^3*i - a^5*d^3*g^3*i + (b^5*c^3*g^3*i - 3*a*b^4*c^2*d*g^3*...
```

### 3.192.8 Giac [A] (verification not implemented)

Time = 277.82 (sec) , antiderivative size = 196, normalized size of antiderivative = 0.53

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx =$$
$$-\frac{1}{4}\left(\frac{2\,(dx+c)^2B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)^2g^3i} + \frac{2\,(B^2n^2+2\,B^2n\log(e)+2\,ABn)(dx+c)^2\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)^2g^3i} + \frac{(B^2n^2+2\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i),x,
algorithm="giac")
```

output
```
-1/4*(2*(d*x + c)^2*B^2*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)^2*g^3*i)
 + 2*(B^2*n^2 + 2*B^2*n*log(e) + 2*A*B*n)*(d*x + c)^2*log((b*x + a)/(d*x +
 c))/((b*x + a)^2*g^3*i) + (B^2*n^2 + 2*B^2*n*log(e) + 2*B^2*log(e)^2 + 2*
A*B*n + 4*A*B*log(e) + 2*A^2)*(d*x + c)^2/((b*x + a)^2*g^3*i))*(b*c/(b*c -
 a*d)^2 - a*d/(b*c - a*d)^2)^2
```

### 3.192.9 Mupad [B] (verification not implemented)

Time = 5.25 (sec) , antiderivative size = 1011, normalized size of antiderivative = 2.74

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$$

$$=\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B^2\,n}{x^2\,\left(b^3\,c\,g^3\,i-a\,b^2\,d\,g^3\,i\right)+x\,\left(2\,a\,b^2\,c\,g^3\,i-2\,a^2\,b\,d\,g^3\,i\right)-a^3\,d\,g^3\,i+a^2\,b\,c\,g^3\,i}\right.$$
$$-\frac{d^2\,\left(3\,n\,B^2+2\,A\,B\right)\,\left(\frac{a\,g^3\,i\,n\,{\left(a\,d-b\,c\right)}^2}{2\,d}+\frac{g^3\,i\,n\,{\left(a\,d-b\,c\right)}^2\,\left(2\,a\,d-b\,c\right)}{2\,d^2}+\frac{b\,g^3\,i\,n\,x\,{\left(a\,d-b\,c\right)}^2}{d}\right)}{g^3\,i\,n\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(x^2\,\left(b^3\,c\,g^3\,i-a\,b^2\,d\,g^3\,i\right)+x\,\left(2\,a\,b^2\,c\,g^3\,i-2\,a^2\,b\,d\,g^3\,i\right)-a^3\right.}$$
$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{d^2\,\left(3\,n\,B^2+2\,A\,B\right)}{2\,g^3\,i\,n\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right.$$
$$\left.-\frac{B^2\,d^2\,\left(\frac{g^3\,i\,n\,\left(a\,d-b\,c\right)\,\left(2\,a\,d-b\,c\right)}{2\,d^2}+\frac{a\,g^3\,i\,n\,\left(a\,d-b\,c\right)}{2\,d}+\frac{b\,g^3\,i\,n\,x\,\left(a\,d-b\,c\right)}{d}\right)}{g^3\,i\,n\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(i\,a^2\,g^3+2\,i\,a\,b\,g^3\,x+i\,b^2\,g^3\,x^2\right)}\right)$$
$$-\frac{\frac{6\,A^2\,a\,d-2\,A^2\,b\,c+15\,B^2\,a\,d\,n^2-B^2\,b\,c\,n^2+14\,A\,B\,a\,d\,n-2\,A\,B\,b\,c\,n}{2\,\left(a\,d-b\,c\right)}+\frac{x\,\left(2\,b\,d\,A^2+6\,b\,d\,A\,B\,n+7\,b\,d\,B^2\,n^2\right)}{a\,d-b\,c}}{x^2\,\left(2\,b^3\,c\,g^3\,i-2\,a\,b^2\,d\,g^3\,i\right)+x\,\left(4\,a\,b^2\,c\,g^3\,i-4\,a^2\,b\,d\,g^3\,i\right)-2\,a^3\,d\,g^3\,i+2\,a^2\,b\,c\,g^3\,i}$$
$$-\frac{B^2\,d^2\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^3}{3\,g^3\,i\,n\,\left(a\,d-b\,c\right)\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}$$
$$+\frac{d^2\,\mathrm{atan}\left(\frac{d^2\,\left(\frac{i\,a^3\,d^3\,g^3-i\,a^2\,b\,c\,d^2\,g^3-i\,a\,b^2\,c^2\,d\,g^3+i\,b^3\,c^3\,g^3}{i\,a^2\,d^2\,g^3-2\,i\,a\,b\,c\,d\,g^3+i\,b^2\,c^2\,g^3}+2\,b\,d\,x\right)\,\left(A^2+3\,A\,B\,n+\frac{7\,B^2\,n^2}{2}\right)\,\left(i\,a^2\,d^2\,g^3-2\,i\,a\,b\,c\,d\,g^3+i\,b^2\,c^2\,g^3\right)\,2{}\mathrm{i}}{g^3\,i\,{\left(a\,d-b\,c\right)}^3\,\left(2\,A^2\,d^2+6\,A\,B\,d^2\,n+7\,B^2\,d^2\,n^2\right)}\right)\,\left(A\right.}{g^3\,i\,{\left(a\,d-b\,c\right)}^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^3*(c*i + d*i*x
)),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((B^2*n)/(x^2*(b^3*c*g^3*i - a*b^2*d*g^3*i)
 + x*(2*a*b^2*c*g^3*i - 2*a^2*b*d*g^3*i) - a^3*d*g^3*i + a^2*b*c*g^3*i) -
(d^2*(3*B^2*n + 2*A*B)*((a*g^3*i*n*(a*d - b*c)^2)/(2*d) + (g^3*i*n*(a*d -
b*c)^2*(2*a*d - b*c))/(2*d^2) + (b*g^3*i*n*x*(a*d - b*c)^2)/d))/(g^3*i*n*(
a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(x^2*(b^3*c*g^3*i - a*b^2*d*g^3
*i) + x*(2*a*b^2*c*g^3*i - 2*a^2*b*d*g^3*i) - a^3*d*g^3*i + a^2*b*c*g^3*i)
)) - log(e*((a + b*x)/(c + d*x))^n)^2*((d^2*(3*B^2*n + 2*A*B))/(2*g^3*i*n*
(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) - (B^2*d^2*((g^3*i*n*(a*d - b
*c)*(2*a*d - b*c))/(2*d^2) + (a*g^3*i*n*(a*d - b*c))/(2*d) + (b*g^3*i*n*x*
(a*d - b*c))/d))/(g^3*i*n*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(a^2
*g^3*i + b^2*g^3*i*x^2 + 2*a*b*g^3*i*x))) - ((6*A^2*a*d - 2*A^2*b*c + 15*B
^2*a*d*n^2 - B^2*b*c*n^2 + 14*A*B*a*d*n - 2*A*B*b*c*n)/(2*(a*d - b*c)) + (
x*(2*A^2*b*d + 7*B^2*b*d*n^2 + 6*A*B*b*d*n))/(a*d - b*c))/(x^2*(2*b^3*c*g^
3*i - 2*a*b^2*d*g^3*i) + x*(4*a*b^2*c*g^3*i - 4*a^2*b*d*g^3*i) - 2*a^3*d*g
^3*i + 2*a^2*b*c*g^3*i) + (d^2*atan((d^2*((a^3*d^3*g^3*i + b^3*c^3*g^3*i -
 a*b^2*c^2*d*g^3*i - a^2*b*c*d^2*g^3*i)/(a^2*d^2*g^3*i + b^2*c^2*g^3*i - 2
*a*b*c*d*g^3*i) + 2*b*d*x)*(A^2 + (7*B^2*n^2)/2 + 3*A*B*n)*(a^2*d^2*g^3*i
+ b^2*c^2*g^3*i - 2*a*b*c*d*g^3*i)*2i)/(g^3*i*(a*d - b*c)^3*(2*A^2*d^2 + 7
*B^2*d^2*n^2 + 6*A*B*d^2*n)))*(A^2 + (7*B^2*n^2)/2 + 3*A*B*n)*2i)/(g^3*i*(
a*d - b*c)^3) - (B^2*d^2*log(e*((a + b*x)/(c + d*x))^n)^3)/(3*g^3*i*n*(...
```

3.192. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)}\,dx$

# 3.193 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$



## 3.193.1 Optimal result

Integrand size = 45, antiderivative size = 543

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx &= -\frac{6bB^2d^2n^2(c+dx)}{(bc-ad)^4g^4i(a+bx)}+\frac{3b^2B^2dn^2(c+dx)^2}{4(bc-ad)^4g^4i(a+bx)^2}\\ &-\frac{2b^3B^2n^2(c+dx)^3}{27(bc-ad)^4g^4i(a+bx)^3}\\ &-\frac{6bBd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^4i(a+bx)}\\ &+\frac{3b^2Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^4i(a+bx)^2}\\ &-\frac{2b^3Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^4g^4i(a+bx)^3}\\ &-\frac{3bd^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^4i(a+bx)}\\ &+\frac{3b^2d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^4i(a+bx)^2}\\ &-\frac{b^3(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^4g^4i(a+bx)^3}\\ &-\frac{d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^4g^4in}\end{aligned}$$

output
```
-6*b*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/4*b^2*B^2*d*n^2*(d*x
+c)^2/(-a*d+b*c)^4/g^4/i/(b*x+a)^2-2/27*b^3*B^2*n^2*(d*x+c)^3/(-a*d+b*c)^4
/g^4/i/(b*x+a)^3-6*b*B*d^2*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d
+b*c)^4/g^4/i/(b*x+a)+3/2*b^2*B*d*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^
n))/(-a*d+b*c)^4/g^4/i/(b*x+a)^2-2/9*b^3*B*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/
(d*x+c))^n))/(-a*d+b*c)^4/g^4/i/(b*x+a)^3-3*b*d^2*(d*x+c)*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^4/i/(b*x+a)+3/2*b^2*d*(d*x+c)^2*(A+B*ln(e
*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^4/i/(b*x+a)^2-1/3*b^3*(d*x+c)^3*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^4/i/(b*x+a)^3-1/3*d^3*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^4/g^4/i/n
```

### 3.193.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1295 vs. $2(543) = 1086$.

Time = 1.00 (sec) , antiderivative size = 1295, normalized size of antiderivative = 2.38

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx =$$
$$-\frac{36B^2d^3n^2(a+bx)^3\log^3\left(\frac{a+bx}{c+dx}\right)+18Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(6a^3Ad^3+2b^3Bc^3n-9ab^2Bc^2dn+18a^2bBcd^2n+\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i +
 d*i*x)),x]
```

output
```
-1/108*(36*B^2*d^3*n^2*(a + b*x)^3*Log[(a + b*x)/(c + d*x)]^3 + 18*B*n*Log
[(a + b*x)/(c + d*x)]^2*(6*a^3*A*d^3 + 2*b^3*B*c^3*n - 9*a*b^2*B*c^2*d*n +
 18*a^2*b*B*c*d^2*n + 18*a^2*A*b*d^3*x - 3*b^3*B*c^2*d*n*x + 18*a*b^2*B*c*
d^2*n*x + 18*a^2*b*B*d^3*n*x + 18*a*A*b^2*d^3*x^2 + 6*b^3*B*c*d^2*n*x^2 +
27*a*b^2*B*d^3*n*x^2 + 6*A*b^3*d^3*x^3 + 11*b^3*B*d^3*n*x^3 + 6*B*d^3*(a +
 b*x)^3*Log[e*((a + b*x)/(c + d*x))^n] - 6*B*d^3*n*(a + b*x)^3*Log[(a + b*
x)/(c + d*x)]) - 3*d*(b*c - a*d)^2*(a + b*x)*(18*A^2 + 30*A*B*n + 19*B^2*n
^2 + 18*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(6*A + 5*B*n)*Log[(a
+ b*x)/(c + d*x)] + 18*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a
+ b*x)/(c + d*x))^n]*(6*A + 5*B*n - 6*B*n*Log[(a + b*x)/(c + d*x)])) + 6*d
^2*(b*c - a*d)*(a + b*x)^2*(18*A^2 + 66*A*B*n + 85*B^2*n^2 + 18*B^2*Log[e*
((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(6*A + 11*B*n)*Log[(a + b*x)/(c + d*x)]
 + 18*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a + b*x)/(c + d*x))
^n]*(6*A + 11*B*n - 6*B*n*Log[(a + b*x)/(c + d*x)])) + 6*d^3*(a + b*x)^3*L
og[a + b*x]*(18*A^2 + 66*A*B*n + 85*B^2*n^2 + 18*B^2*Log[e*((a + b*x)/(c +
 d*x))^n]^2 - 6*B*n*(6*A + 11*B*n)*Log[(a + b*x)/(c + d*x)] + 18*B^2*n^2*L
og[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a + b*x)/(c + d*x))^n]*(6*A + 11*B
*n - 6*B*n*Log[(a + b*x)/(c + d*x)])) + 4*(b*c - a*d)^3*(9*A^2 + 6*A*B*n +
 2*B^2*n^2 + 9*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(3*A + B*n)*Lo
g[(a + b*x)/(c + d*x)] + 9*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log...
```

### 3.193.3 Rubi [A] (verified)

Time = 0.65 (sec) , antiderivative size = 400, normalized size of antiderivative = 0.74, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4(ci+dix)}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i(bc-ad)^4}$$

$$\Big\downarrow \text{ 2795}$$

$$\int \left( \frac{\frac{b^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^4}{(a+bx)^4} - \frac{3b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^3}{(a+bx)^3} + \frac{3bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2(c+dx)^2}{(a+bx)^2} - \frac{d^3\left(A+B\log\right.}{}}{g^4i(bc-ad)^4}\right.$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^3(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3} - \frac{2b^3Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3} + \frac{3b^2d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2} + \frac{3b^2Bdn(c+dx}{}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i + d*i*x
)),x]
```

output
```
((-6*b*B^2*d^2*n^2*(c + d*x))/(a + b*x) + (3*b^2*B^2*d*n^2*(c + d*x)^2)/(4
*(a + b*x)^2) - (2*b^3*B^2*n^2*(c + d*x)^3)/(27*(a + b*x)^3) - (6*b*B*d^2*
n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (3*b^2*B*d
*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) - (
2*b^3*B*n*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)
^3) - (3*b*d^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*
x) + (3*b^2*d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a
+ b*x)^2) - (b^3*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*
(a + b*x)^3) - (d^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*B*n))/((b
*c - a*d)^4*g^4*i)
```

### 3.193.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

3.193. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.193.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2215 vs. 2(529) = 1058.

Time = 25.75 (sec) , antiderivative size = 2216, normalized size of antiderivative = 4.08

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 2216 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x,method=_
RETURNVERBOSE)
```

output
```
-1/108*(-8*B^2*x^3*a^2*b^6*c^5*n^3-24*B^2*x^2*a^3*b^5*c^5*n^3-36*A^2*x^3*a
^2*b^6*c^5*n-24*B^2*x*a^4*b^4*c^5*n^3-108*A^2*x^2*a^3*b^5*c^5*n+36*B^2*ln(
e*((b*x+a)/(d*x+c))^n)^2*a^5*b^3*c^5*n+24*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^
5*b^3*c^5*n^2-108*A^2*x*a^4*b^4*c^5*n+108*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*
a^8*c^2*d^3+396*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^2*d^3*n+972*A*
B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^2*d^3*n+216*A*B*x^2*ln(e*((b*x+a
)/(d*x+c))^n)*a^5*b^3*c^3*d^2*n+648*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^7*b*
c^2*d^3*n+648*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^6*b^2*c^3*d^2*n-108*A*B*x*
ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^4*d*n+198*B^2*x^3*ln(e*((b*x+a)/(d*x+c
))^n)^2*a^5*b^3*c^2*d^3*n+510*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a^5*b^3*c^
2*d^3*n^2+108*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b^3*c^2*d^3+510*A*B*
x^3*a^5*b^3*c^2*d^3*n^2-648*A*B*x^3*a^4*b^4*c^3*d^2*n^2+162*A*B*x^3*a^3*b^
5*c^4*d*n^2+486*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^2*c^2*d^3*n+108*
B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^5*b^3*c^3*d^2*n+1134*B^2*x^2*ln(e*((
b*x+a)/(d*x+c))^n)*a^6*b^2*c^2*d^3*n^2+396*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^
n)*a^5*b^3*c^3*d^2*n^2+324*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^2*c^2
*d^3+1134*A*B*x^2*a^6*b^2*c^2*d^3*n^2-1548*A*B*x^2*a^5*b^3*c^3*d^2*n^2+486
*A*B*x^2*a^4*b^4*c^4*d*n^2+324*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^7*b*c^2
*d^3*n+324*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^6*b^2*c^3*d^2*n-54*B^2*x*ln
(e*((b*x+a)/(d*x+c))^n)^2*a^5*b^3*c^4*d*n+648*B^2*x*ln(e*((b*x+a)/(d*x+...
```

### 3.193.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1910 vs. $2(529) = 1058$.

Time = 0.42 (sec) , antiderivative size = 1910, normalized size of antiderivative = 3.52

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x,
algorithm="fricas")
```

output
```
-1/108*(36*A^2*b^3*c^3 - 162*A^2*a*b^2*c^2*d + 324*A^2*a^2*b*c*d^2 - 198*A
^2*a^3*d^3 + 36*(B^2*b^3*d^3*n^2*x^3 + 3*B^2*a*b^2*d^3*n^2*x^2 + 3*B^2*a^2
*b*d^3*n^2*x + B^2*a^3*d^3*n^2)*log((b*x + a)/(d*x + c))^3 + (8*B^2*b^3*c^
3 - 81*B^2*a*b^2*c^2*d + 648*B^2*a^2*b*c*d^2 - 575*B^2*a^3*d^3)*n^2 + 6*(1
8*A^2*b^3*c*d^2 - 18*A^2*a*b^2*d^3 + 85*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n^
2 + 66*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*n)*x^2 + 18*(2*B^2*b^3*c^3 - 9*B^2*
a*b^2*c^2*d + 18*B^2*a^2*b*c*d^2 - 11*B^2*a^3*d^3 + 6*(B^2*b^3*c*d^2 - B^2
*a*b^2*d^3)*x^2 - 3*(B^2*b^3*c^2*d - 6*B^2*a*b^2*c*d^2 + 5*B^2*a^2*b*d^3)*
x + 6*(B^2*b^3*d^3*x^3 + 3*B^2*a*b^2*d^3*x^2 + 3*B^2*a^2*b*d^3*x + B^2*a^3
*d^3)*log((b*x + a)/(d*x + c)))*log(e)^2 + 18*(6*A*B*a^3*d^3*n + (11*B^2*b
^3*d^3*n^2 + 6*A*B*b^3*d^3*n)*x^3 + (2*B^2*b^3*c^3 - 9*B^2*a*b^2*c^2*d + 1
8*B^2*a^2*b*c*d^2)*n^2 + 3*(6*A*B*a*b^2*d^3*n + (2*B^2*b^3*c*d^2 + 9*B^2*a
*b^2*d^3)*n^2)*x^2 + 3*(6*A*B*a^2*b*d^3*n - (B^2*b^3*c^2*d - 6*B^2*a*b^2*c
*d^2 - 6*B^2*a^2*b*d^3)*n^2)*x)*log((b*x + a)/(d*x + c))^2 + 6*(4*A*B*b^3*
c^3 - 27*A*B*a*b^2*c^2*d + 108*A*B*a^2*b*c*d^2 - 85*A*B*a^3*d^3)*n - 3*(18
*A^2*b^3*c^2*d - 108*A^2*a*b^2*c*d^2 + 90*A^2*a^2*b*d^3 + (19*B^2*b^3*c^2*
d - 378*B^2*a*b^2*c*d^2 + 359*B^2*a^2*b*d^3)*n^2 + 6*(5*A*B*b^3*c^2*d - 54
*A*B*a*b^2*c*d^2 + 49*A*B*a^2*b*d^3)*n)*x + 6*(12*A*B*b^3*c^3 - 54*A*B*a*b
^2*c^2*d + 108*A*B*a^2*b*c*d^2 - 66*A*B*a^3*d^3 + 6*(6*A*B*b^3*c*d^2 - 6*A
*B*a*b^2*d^3 + 11*(B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*n)*x^2 + 18*(B^2*b^3*...
```

### 3.193.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4/(d*i*x+c*i),x
)
```

output
```
Timed out
```

### 3.193.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3445 vs. 2(529) = 1058.

Time = 0.46 (sec) , antiderivative size = 3445, normalized size of antiderivative = 6.34

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x,
algorithm="maxima")
```

output
```
-1/6*B^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b*c*d + 11*a^2*d^2 - 3*(b^2*c*d
 - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d + 3*a^2*b^4*c*d^2 - a^3*b^3*d^3
)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d + 3*a^3*b^3*c*d^2 - a^4*b^2*d
^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c^2*d + 3*a^4*b^2*c*d^2 - a^5*b
*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2*d + 3*a^5*b*c*d^2 - a^6*d^3)*
g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2
- 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*log(d*x + c)/((b^4*c^4 - 4*a*b^3
*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i))*log(e*(b*x/(
d*x + c) + a/(d*x + c))^n)^2 - 1/3*A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 - 7*a*b
*c*d + 11*a^2*d^2 - 3*(b^2*c*d - 5*a*b*d^2)*x)/((b^6*c^3 - 3*a*b^5*c^2*d +
 3*a^2*b^4*c*d^2 - a^3*b^3*d^3)*g^4*i*x^3 + 3*(a*b^5*c^3 - 3*a^2*b^4*c^2*d
 + 3*a^3*b^3*c*d^2 - a^4*b^2*d^3)*g^4*i*x^2 + 3*(a^2*b^4*c^3 - 3*a^3*b^3*c
^2*d + 3*a^4*b^2*c*d^2 - a^5*b*d^3)*g^4*i*x + (a^3*b^3*c^3 - 3*a^4*b^2*c^2
*d + 3*a^5*b*c*d^2 - a^6*d^3)*g^4*i) + 6*d^3*log(b*x + a)/((b^4*c^4 - 4*a*
b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^4*i) - 6*d^3*lo
g(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 +
 a^4*d^4)*g^4*i))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/108*((8*b^3*c
^3 - 81*a*b^2*c^2*d + 648*a^2*b*c*d^2 - 575*a^3*d^3 + 36*(b^3*d^3*x^3 + 3*
a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(b*x + a)^3 - 36*(b^3*d^3*x^3
+ 3*a*b^2*d^3*x^2 + 3*a^2*b*d^3*x + a^3*d^3)*log(d*x + c)^3 + 510*(b^3*...
```

### 3.193.8 Giac [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i),x,
algorithm="giac")
```

output
```
Timed out
```

### 3.193.9 Mupad [B] (verification not implemented)

Time = 7.20 (sec) , antiderivative size = 1921, normalized size of antiderivative = 3.54

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^4*(c*i + d*i*x
)),x)
```

output
```
((198*A^2*a^2*d^2 + 36*A^2*b^2*c^2 + 575*B^2*a^2*d^2*n^2 + 8*B^2*b^2*c^2*n
^2 - 126*A^2*a*b*c*d + 510*A*B*a^2*d^2*n + 24*A*B*b^2*c^2*n - 73*B^2*a*b*c
*d*n^2 - 138*A*B*a*b*c*d*n)/(6*(a*d - b*c)) + (x^2*(18*A^2*b^2*d^2 + 85*B^
2*b^2*d^2*n^2 + 66*A*B*b^2*d^2*n))/(a*d - b*c) + (x*(90*A^2*a*b*d^2 - 18*A
^2*b^2*c*d + 359*B^2*a*b*d^2*n^2 - 19*B^2*b^2*c*d*n^2 + 294*A*B*a*b*d^2*n
- 30*A*B*b^2*c*d*n))/(2*(a*d - b*c)))/(x*(54*a^4*b*d^2*g^4*i + 54*a^2*b^3*
c^2*g^4*i - 108*a^3*b^2*c*d*g^4*i) + x^2*(54*a*b^4*c^2*g^4*i + 54*a^3*b^2*
d^2*g^4*i - 108*a^2*b^3*c*d*g^4*i) + x^3*(18*b^5*c^2*g^4*i + 18*a^2*b^3*d^
2*g^4*i - 36*a*b^4*c*d*g^4*i) + 18*a^5*d^2*g^4*i + 18*a^3*b^2*c^2*g^4*i -
36*a^4*b*c*d*g^4*i) - log(e*((a + b*x)/(c + d*x))^n)^2*((d^3*(11*B^2*n + 6
*A*B))/(6*g^4*i*n*(a*d - b*c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b
*c*d^2)) - (B^2*d^3*(x*(b*((g^4*i*n*(a*d - b*c)*(3*a*d - b*c))/(6*d^2) + (
a*g^4*i*n*(a*d - b*c))/(3*d)) + (2*a*b*g^4*i*n*(a*d - b*c))/(3*d) + (b*g^4
*i*n*(a*d - b*c)*(3*a*d - b*c))/(3*d^2)) + a*((g^4*i*n*(a*d - b*c)*(3*a*d
- b*c))/(6*d^2) + (a*g^4*i*n*(a*d - b*c))/(3*d)) + (g^4*i*n*(a*d - b*c)*(3
*a^2*d^2 + b^2*c^2 - 3*a*b*c*d))/(3*d^3) + (b^2*g^4*i*n*x^2*(a*d - b*c))/d
))/(g^4*i*n*(a*d - b*c)*(a^3*d^3 - b^3*c^3 + 3*a*b^2*c^2*d - 3*a^2*b*c*d^2
)*(a^3*g^4*i + b^3*g^4*i*x^3 + 3*a^2*b*g^4*i*x + 3*a*b^2*g^4*i*x^2))) - lo
g(e*((a + b*x)/(c + d*x))^n)*((6*B^2*a*d*n - 3*B^2*b*c*n + 3*B^2*b*d*n*x)/
(x*(9*a^4*b*d^2*g^4*i + 9*a^2*b^3*c^2*g^4*i - 18*a^3*b^2*c*d*g^4*i) + x...
```

3.193. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)}\,dx$

# 3.194 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$

### 3.194.1 Optimal result

Integrand size = 45, antiderivative size = 770

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$$
$$= \frac{2AB(bc-ad)^2g^3n(a+bx)}{d^3i^2(c+dx)} - \frac{2B^2(bc-ad)^2g^3n^2(a+bx)}{d^3i^2(c+dx)}$$
$$+ \frac{2B^2(bc-ad)^2g^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^2(c+dx)}$$
$$- \frac{bB(bc-ad)g^3n(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{d^3i^2}$$
$$- \frac{3b(bc-ad)g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^2}$$
$$- \frac{(bc-ad)^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^2(c+dx)}$$
$$+ \frac{b^3g^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^4i^2}$$
$$- \frac{6bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^2}$$
$$- \frac{3b(bc-ad)^2g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^2} + \frac{bB^2(bc-ad)^2g^3n^2\log(c+dx)}{d^4i^2}$$
$$+ \frac{bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2}$$
$$- \frac{6bB^2(bc-ad)^2g^3n^2\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$
$$- \frac{6bB(bc-ad)^2g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$
$$- \frac{bB^2(bc-ad)^2g^3n^2\,\text{PolyLog}\left(2,\frac{b(c+dx)}{d(a+bx)}\right)}{d^4i^2} + \frac{6bB^2(bc-ad)^2g^3n^2\,\text{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^2}$$

output
```
2*A*B*(-a*d+b*c)^2*g^3*n*(b*x+a)/d^3/i^2/(d*x+c)-2*B^2*(-a*d+b*c)^2*g^3*n^
2*(b*x+a)/d^3/i^2/(d*x+c)+2*B^2*(-a*d+b*c)^2*g^3*n*(b*x+a)*ln(e*((b*x+a)/(
d*x+c))^n)/d^3/i^2/(d*x+c)-b*B*(-a*d+b*c)*g^3*n*(b*x+a)*(A+B*ln(e*((b*x+a)
/(d*x+c))^n))/d^3/i^2-3*b*(-a*d+b*c)*g^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))^2/d^3/i^2-(-a*d+b*c)^2*g^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^
2/d^3/i^2/(d*x+c)+1/2*b^3*g^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/
d^4/i^2-6*b*B*(-a*d+b*c)^2*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+
b*c)/b/(d*x+c))/d^4/i^2-3*b*(-a*d+b*c)^2*g^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))^2*ln((-a*d+b*c)/b/(d*x+c))/d^4/i^2+b*B^2*(-a*d+b*c)^2*g^3*n^2*ln(d*x+c)
/d^4/i^2+b*B*(-a*d+b*c)^2*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln(1-b*(d*
x+c)/d/(b*x+a))/d^4/i^2-6*b*B^2*(-a*d+b*c)^2*g^3*n^2*polylog(2,d*(b*x+a)/b
/(d*x+c))/d^4/i^2-6*b*B*(-a*d+b*c)^2*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^2-b*B^2*(-a*d+b*c)^2*g^3*n^2*polylog
(2,b*(d*x+c)/d/(b*x+a))/d^4/i^2+6*b*B^2*(-a*d+b*c)^2*g^3*n^2*polylog(3,d*(
b*x+a)/b/(d*x+c))/d^4/i^2
```

### 3.194.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 5850 vs. $2(770) = 1540$.

Time = 6.98 (sec) , antiderivative size = 5850, normalized size of antiderivative = 7.60

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i
+ d*i*x)^2,x]
```

output
```
Result too large to show
```

### 3.194.3 Rubi [A] (verified)

Time = 0.87 (sec) , antiderivative size = 640, normalized size of antiderivative = 0.83, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^2}\,dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{g^3(bc-ad)^2 \int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{g^3(bc-ad)^2 \int \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^3}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^3}-\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^2}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}+\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b}{d^3\left(b-\frac{d(a+bx)}{c+dx}\right)}-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right.}{d^3}\right.}{i^2}$$

$$\Big\downarrow \text{2009}$$

$$g^3(bc-ad)^2\left(\frac{b^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2d^4\left(b-\frac{d(a+bx)}{c+dx}\right)^2}-\frac{6bBn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^4}-\frac{3b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{d^4}\right.$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*
x)^2,x]
```

output
```
((b*c - a*d)^2*g^3*((2*A*B*n*(a + b*x))/(d^3*(c + d*x)) - (2*B^2*n^2*(a +
b*x))/(d^3*(c + d*x)) + (2*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])
/(d^3*(c + d*x)) - (b*B*n*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])
)/(d^3*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) - ((a + b*x)*(A + B*Log[e*
((a + b*x)/(c + d*x))^n])^2)/(d^3*(c + d*x)) + (b^3*(A + B*Log[e*((a + b*x
)/(c + d*x))^n])^2)/(2*d^4*(b - (d*(a + b*x))/(c + d*x))^2) - (3*b*(a + b*
x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^3*(c + d*x)*(b - (d*(a + b
*x))/(c + d*x))) - (b*B^2*n^2*Log[b - (d*(a + b*x))/(c + d*x)])/d^4 - (6*b
*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c +
d*x))])/d^4 - (3*b*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a
+ b*x))/(b*(c + d*x))])/d^4 + (b*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)*Log[1 - (b*(c + d*x))/(d*(a + b*x))])/d^4 - (6*b*B^2*n^2*PolyLog[2, (d*(
a + b*x))/(b*(c + d*x))])/d^4 - (6*b*B*n*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 - (b*B^2*n^2*PolyLog[2,
 (b*(c + d*x))/(d*(a + b*x))])/d^4 + (6*b*B^2*n^2*PolyLog[3, (d*(a + b*x))
/(b*(c + d*x))])/d^4))/i^2
```

---

3.194. $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$

### 3.194.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.194.4 Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(dix+ci)^2}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

## 3.194.5 Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x
, algorithm="fricas")
```

output
```
integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*
a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*
b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))
^n))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

### 3.194.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**
2,x)
```

output
```
Timed out
```

### 3.194.7 Maxima [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^3 \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x
, algorithm="maxima")
```

output
```
2*A*B*a^3*g^3*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)
*i^2) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) + 1/2*(2*c^3/(d^5*i^2*x + c*
d^4*i^2) + 6*c^2*log(d*x + c)/(d^4*i^2) + (d*x^2 - 4*c*x)/(d^3*i^2))*A^2*b
^3*g^3 - 3*A^2*a*b^2*(c^2/(d^4*i^2*x + c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(
d*x + c)/(d^3*i^2))*g^3 + 3*A^2*a^2*b*g^3*(c/(d^3*i^2*x + c*d^2*i^2) + log
(d*x + c)/(d^2*i^2)) - 2*A*B*a^3*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
)/(d^2*i^2*x + c*d*i^2) - A^2*a^3*g^3/(d^2*i^2*x + c*d*i^2) + 1/2*(B^2*b^3
*d^3*g^3*x^3 - 3*(b^3*c*d^2*g^3 - 2*a*b^2*d^3*g^3)*B^2*x^2 - 2*(2*b^3*c^2*
d*g^3 - 3*a*b^2*c*d^2*g^3)*B^2*x + 2*(b^3*c^3*g^3 - 3*a*b^2*c^2*d*g^3 + 3*
a^2*b*c*d^2*g^3 - a^3*d^3*g^3)*B^2 + 6*((b^3*c^2*d*g^3 - 2*a*b^2*c*d^2*g^3
 + a^2*b*d^3*g^3)*B^2*x + (b^3*c^3*g^3 - 2*a*b^2*c^2*d*g^3 + a^2*b*c*d^2*g
^3)*B^2)*log(d*x + c))*log((d*x + c)^n)^2/(d^5*i^2*x + c*d^4*i^2) - integr
ate(-(B^2*a^3*d^3*g^3*log(e)^2 + (B^2*b^3*d^3*g^3*log(e)^2 + 2*A*B*b^3*d^3
*g^3*log(e))*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e)^2 + 2*A*B*a*b^2*d^3*g^3*log
(e))*x^2 + (B^2*b^3*d^3*g^3*x^3 + 3*B^2*a*b^2*d^3*g^3*x^2 + 3*B^2*a^2*b*d^
3*g^3*x + B^2*a^3*d^3*g^3)*log((b*x + a)^n)^2 + 3*(B^2*a^2*b*d^3*g^3*log(e
)^2 + 2*A*B*a^2*b*d^3*g^3*log(e))*x + 2*(B^2*a^3*d^3*g^3*log(e) + (B^2*b^3
*d^3*g^3*log(e) + A*B*b^3*d^3*g^3)*x^3 + 3*(B^2*a*b^2*d^3*g^3*log(e) + A*B
*a*b^2*d^3*g^3)*x^2 + 3*(B^2*a^2*b*d^3*g^3*log(e) + A*B*a^2*b*d^3*g^3)*x)*
log((b*x + a)^n) - ((2*A*B*b^3*d^3*g^3 + (g^3*n + 2*g^3*log(e))*B^2*b^3...
```

### 3.194.8 Giac [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x
, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x
+ c*i)^2, x)
```

### 3.194.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{{\left(a\,g+b\,g\,x\right)}^3\,{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}^2}{{\left(c\,i+d\,i\,x\right)}^2}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^2,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^2, x)
```

# 3.195 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$



## 3.195.1 Optimal result

Integrand size = 45, antiderivative size = 500

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx \\
&= -\frac{2AB(bc-ad)g^2n(a+bx)}{d^2i^2(c+dx)}+\frac{2B^2(bc-ad)g^2n^2(a+bx)}{d^2i^2(c+dx)} \\
&\quad -\frac{2B^2(bc-ad)g^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^2(c+dx)}+\frac{bg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^2} \\
&\quad +\frac{(bc-ad)g^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^2(c+dx)} \\
&\quad +\frac{2bB(bc-ad)g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} \\
&\quad +\frac{2b(bc-ad)g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^2} \\
&\quad +\frac{2bB^2(bc-ad)g^2n^2\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&\quad +\frac{4bB(bc-ad)g^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2} \\
&\quad -\frac{4bB^2(bc-ad)g^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^2}
\end{aligned}$$

output
```
-2*A*B*(-a*d+b*c)*g^2*n*(b*x+a)/d^2/i^2/(d*x+c)+2*B^2*(-a*d+b*c)*g^2*n^2*(
b*x+a)/d^2/i^2/(d*x+c)-2*B^2*(-a*d+b*c)*g^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c
))^n)/d^2/i^2/(d*x+c)+b*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^2/
i^2+(-a*d+b*c)*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^2/i^2/(d*x+
c)+2*b*B*(-a*d+b*c)*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a*d+b*c)/b/
(d*x+c))/d^3/i^2+2*b*(-a*d+b*c)*g^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((
-a*d+b*c)/b/(d*x+c))/d^3/i^2+2*b*B^2*(-a*d+b*c)*g^2*n^2*polylog(2,d*(b*x+a
)/b/(d*x+c))/d^3/i^2+4*b*B*(-a*d+b*c)*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)*polylog(2,d*(b*x+a)/b/(d*x+c))/d^3/i^2-4*b*B^2*(-a*d+b*c)*g^2*n^2*polylo
g(3,d*(b*x+a)/b/(d*x+c))/d^3/i^2
```

### 3.195.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2859 vs. $2(500) = 1000$.

Time = 3.97 (sec) , antiderivative size = 2859, normalized size of antiderivative = 5.72

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i
+ d*i*x)^2,x]
```

output
```
(g^2*(3*b^2*d*x*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/
(c + d*x)])^2 - (3*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B
*n*Log[(a + b*x)/(c + d*x)])^2)/(c + d*x) - 6*b*(b*c - a*d)*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2*Log[c + d*x] + (
6*a^2*B*d^2*n*(-A - B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(
c + d*x)])*(b*c - a*d + b*(c + d*x)*Log[a/b + x] + (-(b*c) + a*d)*Log[(a +
 b*x)/(c + d*x)] - b*c*Log[(b*(c + d*x))/(b*c - a*d)] - b*d*x*Log[(b*(c +
d*x))/(b*c - a*d)]))/((-(b*c) + a*d)*(c + d*x)) + 6*a*b*B*d*n*(A + B*Log[e
*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])*(-Log[c/d + x]^2
 + 2*Log[c/d + x]*Log[c + d*x] + 2*(-(c/(c + d*x)) + (b*c*Log[a + b*x])/(-
(b*c) + a*d) + (b*c*Log[c + d*x])/(b*c - a*d) - Log[a/b + x]*Log[c + d*x]
+ Log[(a + b*x)/(c + d*x)]*(c/(c + d*x) + Log[c + d*x]) + Log[a/b + x]*Log
[(b*(c + d*x))/(b*c - a*d)]) + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)])
 + 6*b^2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c
+ d*x)])*(d*(a/b + x)*(-1 + Log[a/b + x]) - (c^2*Log[a/b + x])/(c + d*x) -
 (c + d*x)*(-1 + Log[c/d + x]) + c*Log[c/d + x]^2 + (c^2*(1 + Log[c/d + x]
))/(c + d*x) + (b*c^2*(Log[a + b*x] - Log[c + d*x]))/(b*c - a*d) + (-Log[a
/b + x] + Log[c/d + x] + Log[(a + b*x)/(c + d*x)])*(d*x - c^2/(c + d*x) -
2*c*Log[c + d*x]) - 2*c*(Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] + Pol
yLog[2, (d*(a + b*x))/(-(b*c) + a*d)])) - (3*a^2*B^2*d^2*n^2*(2*b*c - 2...
```

### 3.195.3 Rubi [A] (verified)

Time = 0.67 (sec) , antiderivative size = 411, normalized size of antiderivative = 0.82, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^2}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{g^2(bc-ad)\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{2795}$$

$$\frac{g^2(bc-ad)\int \left(\frac{2b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2}+\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)^2}\right)d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^2(bc-ad)\left(\frac{4bBn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3}+\frac{2bBn\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3}+\frac{2b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)}{}\right.}{}$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*
x)^2,x]
```

output
```
((b*c - a*d)*g^2*((-2*A*B*n*(a + b*x))/(d^2*(c + d*x)) + (2*B^2*n^2*(a + b
*x))/(d^2*(c + d*x)) - (2*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/
(d^2*(c + d*x)) + ((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^
2*(c + d*x)) + (b*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^2
*(c + d*x)*(b - (d*(a + b*x))/(c + d*x))) + (2*b*B*n*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 + (2*b*(A + B*
Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^
3 + (2*b*B^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^3 + (4*b*B*n*(
A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x
))])/d^3 - (4*b*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^3))/i^2
```

### 3.195.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

---

3.195. $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.195.4 Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(dix+ci)^2}dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

## 3.195.5 Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x
, algorithm="fricas")
```

output
```
integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x
^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(
A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c
))^n))/(d^2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

### 3.195.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**
2,x)
```

output
```
Timed out
```

### 3.195.7 Maxima [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x
, algorithm="maxima")
```

output
```
2*A*B*a^2*g^2*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)
*i^2) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) - A^2*b^2*(c^2/(d^4*i^2*x +
c*d^3*i^2) - x/(d^2*i^2) + 2*c*log(d*x + c)/(d^3*i^2))*g^2 + 2*A^2*a*b*g^2
*(c/(d^3*i^2*x + c*d^2*i^2) + log(d*x + c)/(d^2*i^2)) - 2*A*B*a^2*g^2*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^2*i^2*x + c*d*i^2) - A^2*a^2*g^2/(d^
2*i^2*x + c*d*i^2) + (B^2*b^2*d^2*g^2*x^2 + B^2*b^2*c*d*g^2*x - (b^2*c^2*g
^2 - 2*a*b*c*d*g^2 + a^2*d^2*g^2)*B^2 - 2*((b^2*c*d*g^2 - a*b*d^2*g^2)*B^2
*x + (b^2*c^2*g^2 - a*b*c*d*g^2)*B^2)*log(d*x + c))*log((d*x + c)^n)^2/(d^
4*i^2*x + c*d^3*i^2) - integrate(-(B^2*a^2*d^2*g^2*log(e)^2 + (B^2*b^2*d^2
*g^2*log(e)^2 + 2*A*B*b^2*d^2*g^2*log(e))*x^2 + (B^2*b^2*d^2*g^2*x^2 + 2*B
^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log((b*x + a)^n)^2 + 2*(B^2*a*b*d^2*g^
2*log(e)^2 + 2*A*B*a*b*d^2*g^2*log(e))*x + 2*(B^2*a^2*d^2*g^2*log(e) + (B^
2*b^2*d^2*g^2*log(e) + A*B*b^2*d^2*g^2)*x^2 + 2*(B^2*a*b*d^2*g^2*log(e) +
A*B*a*b*d^2*g^2)*x)*log((b*x + a)^n) + 2*((b^2*c^2*g^2*n - 2*a*b*c*d*g^2*n
 + (g^2*n - g^2*log(e))*a^2*d^2)*B^2 - (A*B*b^2*d^2*g^2 + (g^2*n + g^2*log
(e))*B^2*b^2*d^2)*x^2 - (2*A*B*a*b*d^2*g^2 + (b^2*c*d*g^2*n + 2*a*b*d^2*g^
2*log(e))*B^2)*x + 2*((b^2*c*d*g^2*n - a*b*d^2*g^2*n)*B^2*x + (b^2*c^2*g^2
*n - a*b*c*d*g^2*n)*B^2)*log(d*x + c) - (B^2*b^2*d^2*g^2*x^2 + 2*B^2*a*b*d
^2*g^2*x + B^2*a^2*d^2*g^2)*log((b*x + a)^n))*log((d*x + c)^n))/(d^4*i^2*x
^2 + 2*c*d^3*i^2*x + c^2*d^2*i^2), x)
```

### 3.195.8 Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x
, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x
+ c*i)^2, x)
```

### 3.195.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(a\,g+b\,g\,x)^2\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(c\,i+d\,i\,x)^2}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^2,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^2, x)
```

# 3.196 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$



## 3.196.1 Optimal result

Integrand size = 43, antiderivative size = 282

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \frac{2ABgn(a+bx)}{di^2(c+dx)}-\frac{2B^2gn^2(a+bx)}{di^2(c+dx)}+\frac{2B^2gn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{di^2(c+dx)} - \frac{g(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{di^2(c+dx)}-\frac{bg\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^2i^2} - \frac{2bBgn\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}+\frac{2bB^2gn^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^2i^2}$$

output
```
2*A*B*g*n*(b*x+a)/d/i^2/(d*x+c)-2*B^2*g*n^2*(b*x+a)/d/i^2/(d*x+c)+2*B^2*g*
n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/d/i^2/(d*x+c)-g*(b*x+a)*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))^2/d/i^2/(d*x+c)-b*g*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln
((-a*d+b*c)/b/(d*x+c))/d^2/i^2-2*b*B*g*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*p
olylog(2,d*(b*x+a)/b/(d*x+c))/d^2/i^2+2*b*B^2*g*n^2*polylog(3,d*(b*x+a)/b/
(d*x+c))/d^2/i^2
```

### 3.196.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1570 vs. $2(282) = 564$.

Time = 1.01 (sec) , antiderivative size = 1570, normalized size of antiderivative = 5.57

$$\int \frac{(ag + bgx)\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci + dix)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i +
d*i*x)^2,x]
```

output
```
(g*((3*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*
x)/(c + d*x)])^2)/(c + d*x) + 3*b*(A + B*Log[e*((a + b*x)/(c + d*x))^n] -
B*n*Log[(a + b*x)/(c + d*x)])^2*Log[c + d*x] + (6*a*B*d*n*(-A - B*Log[e*((
a + b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d*x)])*(b*c - a*d + b*(c +
 d*x)*Log[a/b + x] + (-(b*c) + a*d)*Log[(a + b*x)/(c + d*x)] - b*c*Log[(b*
(c + d*x))/(b*c - a*d)] - b*d*x*Log[(b*(c + d*x))/(b*c - a*d)]))/((-(b*c)
+ a*d)*(c + d*x)) + 3*b*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Lo
g[(a + b*x)/(c + d*x)])*(-Log[c/d + x]^2 + 2*Log[c/d + x]*Log[c + d*x] + 2
*(-(c/(c + d*x)) + (b*c*Log[a + b*x])/(-(b*c) + a*d) + (b*c*Log[c + d*x])/
(b*c - a*d) - Log[a/b + x]*Log[c + d*x] + Log[(a + b*x)/(c + d*x)]*(c/(c +
 d*x) + Log[c + d*x]) + Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)]) + 2*P
olyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) - (3*a*B^2*d*n^2*(2*b*c - 2*a*d +
 2*b*(c + d*x)*Log[a + b*x] - 2*(b*c - a*d)*Log[(a + b*x)/(c + d*x)] - 2*b
*(c + d*x)*Log[a + b*x]*Log[(a + b*x)/(c + d*x)] + (b*c - a*d)*Log[(a + b*
x)/(c + d*x)]^2 - 2*b*(c + d*x)*Log[c + d*x] - 2*b*(c + d*x)*Log[(a + b*x)
/(c + d*x)]*Log[(b*c - a*d)/(b*c + b*d*x)] + b*(c + d*x)*(Log[a + b*x]*(Lo
g[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x)
)/(-(b*c) + a*d)]) + b*(c + d*x)*(Log[(b*c - a*d)/(b*c + b*d*x)]*(2*Log[(d
*(a + b*x))/(-(b*c) + a*d)] + Log[(b*c - a*d)/(b*c + b*d*x)]) - 2*PolyLog[
2, (b*(c + d*x))/(b*c - a*d)])))/((b*c - a*d)*(c + d*x)) + (b*B^2*n^2*(...
```

### 3.196.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 260, normalized size of antiderivative = 0.92, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.070$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

---

3.196. $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$

$$\int \frac{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^2}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{g\int \left(-\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d}-\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d\left(\frac{d(a+bx)}{c+dx}-b\right)}\right)d\frac{a+bx}{c+dx}}{i^2}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g\left(-\frac{2bBn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^2}-\frac{b\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^2}-\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d(c+dx)}\right.}{i^2}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x)
^2,x]
```

output
```
(g*((2*A*B*n*(a + b*x))/(d*(c + d*x)) - (2*B^2*n^2*(a + b*x))/(d*(c + d*x)
) + (2*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d*(c + d*x)) - ((a
 + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d*(c + d*x)) - (b*(A +
B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/
d^2 - (2*b*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b
*x))/(b*(c + d*x))])/d^2 + (2*b*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d
*x))])/d^2))/i^2
```

3.196. $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}dx$

### 3.196.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.196.4 Maple [F]

$$\int \frac{(bgx+ag)\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{2}}{(dix+ci)^{2}}dx$$

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

output
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x)
```

## 3.196.5 Fricas [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(ci+dix)^{2}}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{2}}{(dix+ci)^{2}}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output
```
integral((A^2*b*g*x + A^2*a*g + (B^2*b*g*x + B^2*a*g)*log(e*((b*x + a)/(d*
x + c))^n)^2 + 2*(A*B*b*g*x + A*B*a*g)*log(e*((b*x + a)/(d*x + c))^n))/(d^
2*i^2*x^2 + 2*c*d*i^2*x + c^2*i^2), x)
```

### 3.196.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**2,x
)
```

output
```
Timed out
```

### 3.196.7 Maxima [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output
```
2*A*B*a*g*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2
) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) + A^2*b*g*(c/(d^3*i^2*x + c*d^2*
i^2) + log(d*x + c)/(d^2*i^2)) - 2*A*B*a*g*log(e*(b*x/(d*x + c) + a/(d*x +
 c))^n)/(d^2*i^2*x + c*d*i^2) - A^2*a*g/(d^2*i^2*x + c*d*i^2) + ((b*c*g -
a*d*g)*B^2 + (B^2*b*d*g*x + B^2*b*c*g)*log(d*x + c))*log((d*x + c)^n)^2/(d
^3*i^2*x + c*d^2*i^2) - integrate(-(B^2*a*d*g*log(e)^2 + (B^2*b*d*g*x + B^
2*a*d*g)*log((b*x + a)^n)^2 + (B^2*b*d*g*log(e)^2 + 2*A*B*b*d*g*log(e))*x
+ 2*(B^2*a*d*g*log(e) + (B^2*b*d*g*log(e) + A*B*b*d*g)*x)*log((b*x + a)^n)
 - 2*((b*c*g*n - (g*n - g*log(e))*a*d)*B^2 + (B^2*b*d*g*log(e) + A*B*b*d*g
)*x + (B^2*b*d*g*n*x + B^2*b*c*g*n)*log(d*x + c) + (B^2*b*d*g*x + B^2*a*d*
g)*log((b*x + a)^n))*log((d*x + c)^n))/(d^3*i^2*x^2 + 2*c*d^2*i^2*x + c^2*
d*i^2), x)
```

### 3.196.8 Giac [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(bgx+ag)\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^2}\,dx$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output
```
integrate((b*g*x + a*g)*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x +
c*i)^2, x)
```

### 3.196.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx=\int \frac{(a\,g+b\,g\,x)\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{\left(c\,i+d\,i\,x\right)^2}\,dx$$

input
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x)
^2,x)
```

output
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x)
^2, x)
```

# 3.197 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$



## 3.197.1 Optimal result

Integrand size = 35, antiderivative size = 163

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = -\frac{2ABn(a+bx)}{(bc-ad)i^2(c+dx)} + \frac{2B^2n^2(a+bx)}{(bc-ad)i^2(c+dx)} - \frac{2B^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)i^2(c+dx)} + \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(c+dx)}$$

output
```
-2*A*B*n*(b*x+a)/(-a*d+b*c)/i^2/(d*x+c)+2*B^2*n^2*(b*x+a)/(-a*d+b*c)/i^2/(
d*x+c)-2*B^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)/i^2/(d*x+c)+(b
*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^2/(d*x+c)
```

## 3.197.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.23 (sec) , antiderivative size = 331, normalized size of antiderivative = 2.03

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx$$

$$= \frac{-\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + \frac{Bn\left(2(bc-ad)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+2b(c+dx)\log(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)-2b(c+dx)\left(A+B\right.\right.}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x)^2,x]
```

output
```
(-(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)*(A + B*Lo
g[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Log[a + b*x]*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))
^n])*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d
*x)*Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b
*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) +
b*B*n*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[
c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d))/(d*i^2*
(c + d*x))
```

### 3.197.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 119, normalized size of antiderivative = 0.73, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2951, 2733, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^2} \, dx$$
$$\Big\downarrow \text{2951}$$
$$\frac{\int \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d\frac{a+bx}{c+dx}}{i^2 (bc-ad)}$$
$$\Big\downarrow \text{2733}$$
$$\frac{\frac{(a+bx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2 B n \int \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}}{i^2 (bc-ad)}$$
$$\Big\downarrow \text{2009}$$
$$\frac{\frac{(a+bx) \left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2 B n \left(\frac{A (a+bx)}{c+dx}+\frac{B (a+bx) \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{B n (a+bx)}{c+dx}\right)}{i^2 (bc-ad)}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x)^2,x]
```

output
```
(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*x) - 2*B*n*((
A*(a + b*x))/(c + d*x) - (B*n*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[e*((
a + b*x)/(c + d*x))^n])/(c + d*x)))/((b*c - a*d)*i^2)
```

### 3.197.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

## 3.197.4 Maple [A] (verified)

Time = 2.16 (sec) , antiderivative size = 294, normalized size of antiderivative = 1.80

| method | result |
|---|---|
| parallelrisch | $-\frac{2B^2ab\,d^3n^3-2B^2b^2c\,d^2n^3+A^2ab\,d^3n-A^2b^2c\,d^2n+2ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)b^2d^3n+2AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)ab\,d^3n-2ABab\,d^3n^2}{i^2(dx-}$ |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x,method=_RETURNVERBOS
E)
```

output
```
-(2*B^2*a*b*d^3*n^3-2*B^2*b^2*c*d^2*n^3+A^2*a*b*d^3*n-A^2*b^2*c*d^2*n+2*A*
B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^3*n+2*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*
b*d^3*n-2*A*B*a*b*d^3*n^2+2*A*B*b^2*c*d^2*n^2+B^2*x*ln(e*((b*x+a)/(d*x+c))
^n)^2*b^2*d^3*n-2*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^3*n^2+B^2*ln(e*((b
*x+a)/(d*x+c))^n)^2*a*b*d^3*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^3*n^2)
/i^2/(d*x+c)/b/d^3/n/(a*d-b*c)
```

### 3.197.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 263, normalized size of antiderivative = 1.61

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = -\frac{A^2bc-A^2ad+2\left(B^2bc-B^2ad\right)n^2+\left(B^2bc-B^2ad\right)\log\left(e\right)^2-\left(B^2bdn^2x+B^2adn^2\right)\log\left(\frac{bx+a}{dx+c}\right)^2-2\,(A}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, algorithm="f
ricas")
```

output
```
-(A^2*b*c - A^2*a*d + 2*(B^2*b*c - B^2*a*d)*n^2 + (B^2*b*c - B^2*a*d)*log(
e)^2 - (B^2*b*d*n^2*x + B^2*a*d*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(A*B*b
*c - A*B*a*d)*n + 2*(A*B*b*c - A*B*a*d - (B^2*b*c - B^2*a*d)*n - (B^2*b*d*
n*x + B^2*a*d*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*a*d*n^2 - A*B*a
*d*n + (B^2*b*d*n^2 - A*B*b*d*n)*x)*log((b*x + a)/(d*x + c)))/((b*c*d^2 -
a*d^3)*i^2*x + (b*c^2*d - a*c*d^2)*i^2)
```

### 3.197.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \frac{\int \frac{A^2}{c^2+2cdx+d^2x^2}\,dx+\int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^2+2cdx+d^2x^2}\,dx+\int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^2+2cdx+d^2x^2}\,dx}{i^2}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**2,x)
```

output
```
(Integral(A**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integral(B**2*log(e*(a/(
c + d*x) + b*x/(c + d*x))**n)**2/(c**2 + 2*c*d*x + d**2*x**2), x) + Integr
al(2*A*B*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**2 + 2*c*d*x + d**2*x*
*2), x))/i**2
```

### 3.197.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 428 vs. $2(163) = 326$.

Time = 0.21 (sec) , antiderivative size = 428, normalized size of antiderivative = 2.63

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = 2\,ABn\left(\frac{1}{d^2i^2x+cdi^2} + \frac{b\log(bx+a)}{(bcd-ad^2)i^2} - \frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right) + \left(2\,n\left(\frac{1}{d^2i^2x+cdi^2} + \frac{b\log(bx+a)}{(bcd-ad^2)i^2} - \frac{b\log(dx+c)}{(bcd-ad^2)i^2}\right)\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right) - \frac{((bdx+bc)\log}{} \right. - \frac{B^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2}{d^2i^2x+cdi^2} - \frac{2\,AB\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^2i^2x+cdi^2} - \frac{A^2}{d^2i^2x+cdi^2}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, algorithm="m
axima")
```

output
```
2*A*B*n*(1/(d^2*i^2*x + c*d*i^2) + b*log(b*x + a)/((b*c*d - a*d^2)*i^2) -
b*log(d*x + c)/((b*c*d - a*d^2)*i^2)) + (2*n*(1/(d^2*i^2*x + c*d*i^2) + b*
log(b*x + a)/((b*c*d - a*d^2)*i^2) - b*log(d*x + c)/((b*c*d - a*d^2)*i^2))
*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - ((b*d*x + b*c)*log(b*x + a)^2 +
(b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a
) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n^2/(b*c^2*
d*i^2 - a*c*d^2*i^2 + (b*c*d^2*i^2 - a*d^3*i^2)*x))*B^2 - B^2*log(e*(b*x/(
d*x + c) + a/(d*x + c))^n)^2/(d^2*i^2*x + c*d*i^2) - 2*A*B*log(e*(b*x/(d*x
 + c) + a/(d*x + c))^n)/(d^2*i^2*x + c*d*i^2) - A^2/(d^2*i^2*x + c*d*i^2)
```

### 3.197.8 Giac [A] (verification not implemented)

Time = 0.99 (sec) , antiderivative size = 175, normalized size of antiderivative = 1.07

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \left(\frac{(bx+a)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(dx+c)i^2} - \frac{2\left(B^2n^2 - B^2n\log(e) - ABn\right)(bx+a)\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)i^2} + \frac{\left(2\,B^2n^2 - 2\,B^2n\log(\right.}{}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^2,x, algorithm="g
iac")
```

output
```
((b*x + a)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((d*x + c)*i^2) - 2*(B^2*n^2
 - B^2*n*log(e) - A*B*n)*(b*x + a)*log((b*x + a)/(d*x + c))/((d*x + c)*i^2
) + (2*B^2*n^2 - 2*B^2*n*log(e) + B^2*log(e)^2 - 2*A*B*n + 2*A*B*log(e) +
A^2)*(b*x + a)/((d*x + c)*i^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.197.9 Mupad [B] (verification not implemented)

Time = 1.99 (sec) , antiderivative size = 237, normalized size of antiderivative = 1.45

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^2}\,dx = \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{2\,B^2\,n}{x\,d^2\,i^2+c\,d\,i^2}-\frac{2\,A\,B}{x\,d^2\,i^2+c\,d\,i^2}\right) - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{d\,\left(c\,i^2+d\,i^2\,x\right)} + \frac{B^2\,b}{d\,i^2\,\left(a\,d-b\,c\right)}\right) - \frac{A^2-2\,A\,B\,n+2\,B^2\,n^2}{x\,d^2\,i^2+c\,d\,i^2} + \frac{B\,b\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{a\,d^2\,i^2+b\,c\,d\,i^2}{d\,i^2}\right)\,1\mathrm{i}}{a\,d-b\,c}\right)\,\left(A-B\,n\right)\,4\mathrm{i}}{d\,i^2\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*i + d*i*x)^2,x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((2*B^2*n)/(d^2*i^2*x + c*d*i^2) - (2*A*B)/
(d^2*i^2*x + c*d*i^2)) - log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(d*(c*i^2 +
 d*i^2*x)) + (B^2*b)/(d*i^2*(a*d - b*c))) - (A^2 + 2*B^2*n^2 - 2*A*B*n)/(d
^2*i^2*x + c*d*i^2) + (B*b*n*atan(((2*b*d*x + (a*d^2*i^2 + b*c*d*i^2)/(d*i
^2))*1i)/(a*d - b*c))*(A - B*n)*4i)/(d*i^2*(a*d - b*c))
```

# 3.198 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$



## 3.198.1 Optimal result

Integrand size = 45, antiderivative size = 231

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \frac{2ABdn(a+bx)}{(bc-ad)^2gi^2(c+dx)} - \frac{2B^2dn^2(a+bx)}{(bc-ad)^2gi^2(c+dx)} + \frac{2B^2dn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2gi^2(c+dx)} - \frac{d(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2gi^2(c+dx)} + \frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^2gi^2n}$$

output
```
2*A*B*d*n*(b*x+a)/(-a*d+b*c)^2/g/i^2/(d*x+c)-2*B^2*d*n^2*(b*x+a)/(-a*d+b*c
)^2/g/i^2/(d*x+c)+2*B^2*d*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2
/g/i^2/(d*x+c)-d*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/g/
i^2/(d*x+c)+1/3*b*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^2/g/i^2/n
```

### 3.198.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 789 vs. 2(231) = 462.

Time = 0.40 (sec) , antiderivative size = 789, normalized size of antiderivative = 3.42

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{bB^2n^2\log^3\left(\frac{a+bx}{c+dx}\right)}{3(bc-ad)^2gi^2} - \frac{2Bn\log\left(\frac{a+bx}{c+dx}\right)\left(-A+Bn-B\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)\right)}{(bc-ad)gi^2(c+dx)}$$
$$+ \frac{\log^2\left(\frac{a+bx}{c+dx}\right)\left(AbBcn-aB^2dn^2+AbBdnx-bB^2dn^2x+bB^2cn\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)+bB^2d\right.}{(bc-ad)^2gi^2(c+dx)}$$
$$+ \frac{A^2-2ABn+2B^2n^2+2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)-2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)+}{(bc-ad)gi^2(c+dx)}$$
$$+ \frac{b\log(a+bx)\left(A^2-2ABn+2B^2n^2+2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)-2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\right.\right.}{(bc-ad)^2gi^2}$$
$$- \frac{b\left(A^2-2ABn+2B^2n^2+2AB\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)-2B^2n\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)-n\log\left(\frac{a+bx}{c+dx}\right)\right)\right.}{(bc-ad)^2gi^2}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d
*i*x)^2),x]
```

output
```
(b*B^2*n^2*Log[(a + b*x)/(c + d*x)]^3)/(3*(b*c - a*d)^2*g*i^2) - (2*B*n*Lo
g[(a + b*x)/(c + d*x)]*(-A + B*n - B*(Log[e*((a + b*x)/(c + d*x))^n] - n*L
og[(a + b*x)/(c + d*x)])))/((b*c - a*d)*g*i^2*(c + d*x)) + (Log[(a + b*x)/
(c + d*x)]^2*(A*b*B*c*n - a*B^2*d*n^2 + A*b*B*d*n*x - b*B^2*d*n^2*x + b*B^
2*c*n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + b*B^
2*d*n*x*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])))/((
b*c - a*d)^2*g*i^2*(c + d*x)) + (A^2 - 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Log[e*
((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - 2*B^2*n*(Log[e*((
a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*
x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2)/((b*c - a*d)*g*i^2*(c +
d*x)) + (b*Log[a + b*x]*(A^2 - 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b*
x)/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) - 2*B^2*n*(Log[e*((a + b*x)
/(c + d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c +
d*x))^n] - n*Log[(a + b*x)/(c + d*x)])^2))/((b*c - a*d)^2*g*i^2) - (b*(A^2
 - 2*A*B*n + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a
+ b*x)/(c + d*x)]) - 2*B^2*n*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a +
b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(
c + d*x)])^2)*Log[c + d*x])/((b*c - a*d)^2*g*i^2)
```

### 3.198.3 Rubi [A] (warning: unable to verify)

Time = 0.53 (sec) , antiderivative size = 151, normalized size of antiderivative = 0.65, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {2961, 2788, 2733, 2009, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)(ci+dix)^2}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{2788}$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d\frac{a+bx}{c+dx}}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{2733}$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}\right)}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-B\right.\right.}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{2739}$$

$$\frac{\frac{b\int \frac{(a+bx)^2}{(c+dx)^2}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}-d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{Bn(a+bx}{c+dx}\right.\right.}{gi^2(bc-ad)^2}$$

$$\Big\downarrow \text{15}$$

$$\frac{\frac{b(a+bx)^3}{3Bn(c+dx)^3}-d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{Bn(a+bx)}{c+dx}\right)\right)}{gi^2(bc-ad)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d*i*x)^
2),x]
```

output
```
((b*(a + b*x)^3)/(3*B*n*(c + d*x)^3) - d*(((a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])^2)/(c + d*x) - 2*B*n*((A*(a + b*x))/(c + d*x) - (B*n*(a
+ b*x))/(c + d*x) + (B*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(c + d*x)
)))/((b*c - a*d)^2*g*i^2)
```

### 3.198.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

3.198. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.198.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 508 vs. $2(229) = 458$.

Time = 5.06 (sec) , antiderivative size = 509, normalized size of antiderivative = 2.20

| method | result |
|---|---|
| parallelrisch | $\frac{-6B^2ab^2d^4n^3+6B^2b^3cd^3n^3-3A^2ab^2d^4n+3A^2b^3cd^3n+B^2x\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3b^3d^4+B^2\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^3b^3cd^3+3A^2x\ln\left(e\left(\right.\right.}{}$ |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x,method=_
RETURNVERBOSE)
```

output
```
1/3*(-6*B^2*a*b^2*d^4*n^3+6*B^2*b^3*c*d^3*n^3-3*A^2*a*b^2*d^4*n+3*A^2*b^3*
c*d^3*n+B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*b^3*d^4+B^2*ln(e*((b*x+a)/(d*x+c
))^n)^3*b^3*c*d^3+3*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^4+3*A^2*ln(e*((b
*x+a)/(d*x+c))^n)*b^3*c*d^3-6*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^4*n-6*
A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^4*n+6*A*B*a*b^2*d^4*n^2-6*A*B*b^3*c*
d^3*n^2-3*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^4*n+6*B^2*x*ln(e*((b*x+a
)/(d*x+c))^n)*b^3*d^4*n^2+3*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^4-3*B^
2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^2*d^4*n+6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*
a*b^2*d^4*n^2+3*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*c*d^3)/i^2/g/(d*x+c)/(
a^2*d^2-2*a*b*c*d+b^2*c^2)/b^2/d^3/n
```

### 3.198.5 Fricas [A] (verification not implemented)

Time = 0.36 (sec) , antiderivative size = 432, normalized size of antiderivative = 1.87

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= \frac{3A^2bc-3A^2ad+(B^2bdn^2x+B^2bcn^2)\log\left(\frac{bx+a}{dx+c}\right)^3+6(B^2bc-B^2ad)n^2+3\left(B^2bc-B^2ad+(B^2bdx+\right.}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x,
algorithm="fricas")
```

output
```
1/3*(3*A^2*b*c - 3*A^2*a*d + (B^2*b*d*n^2*x + B^2*b*c*n^2)*log((b*x + a)/(
d*x + c))^3 + 6*(B^2*b*c - B^2*a*d)*n^2 + 3*(B^2*b*c - B^2*a*d + (B^2*b*d*
x + B^2*b*c)*log((b*x + a)/(d*x + c)))*log(e)^2 - 3*(B^2*a*d*n^2 - A*B*b*c
*n + (B^2*b*d*n^2 - A*B*b*d*n)*x)*log((b*x + a)/(d*x + c))^2 - 6*(A*B*b*c
- A*B*a*d)*n + 3*(2*A*B*b*c - 2*A*B*a*d + (B^2*b*d*n*x + B^2*b*c*n)*log((b
*x + a)/(d*x + c))^2 - 2*(B^2*b*c - B^2*a*d)*n - 2*(B^2*a*d*n - A*B*b*c +
(B^2*b*d*n - A*B*b*d)*x)*log((b*x + a)/(d*x + c)))*log(e) + 3*(2*B^2*a*d*n
^2 - 2*A*B*a*d*n + A^2*b*c + (2*B^2*b*d*n^2 - 2*A*B*b*d*n + A^2*b*d)*x)*lo
g((b*x + a)/(d*x + c)))/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*g*i^2*x + (b^
2*c^3 - 2*a*b*c^2*d + a^2*c*d^2)*g*i^2)
```

### 3.198.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)/(d*i*x+c*i)**2,x
)
```

output
```
Timed out
```

### 3.198.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1014 vs. 2(229) = 458.

Time = 0.26 (sec) , antiderivative size = 1014, normalized size of antiderivative = 4.39

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$= B^2\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)\log\left(e\left(\right.\right.$$
$$+2\,AB\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)\log$$
$$+\frac{1}{3}\left(\frac{\left((bdx+bc)\log(bx+a)^3-(bdx+bc)\log(dx+c)^3+3\,(bdx+bc)\log(bx+a)^2+3\,(bdx+bc+(bd\right.}{b^2c}\right.$$
$$-\frac{\left((bdx+bc)\log(bx+a)^2+(bdx+bc)\log(dx+c)^2+2\,bc-2\,ad+2\,(bdx+bc)\log(bx+a)-2\,(bdx+\right.}{b^2c^3gi^2-2\,abc^2dgi^2+a^2cd^2gi^2+(b^2c^2dgi^2-2\,abcd^2gi^2+a^2d}$$
$$+A^2\left(\frac{1}{(bcd-ad^2)gi^2x+(bc^2-acd)gi^2}+\frac{b\log(bx+a)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}-\frac{b\log(dx+c)}{(b^2c^2-2\,abcd+a^2d^2)gi^2}\right)$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x,
algorithm="maxima")
```

output
```
B^2*(1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/
((b^2*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b
*c*d + a^2*d^2)*g*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2 + 2*A*B*(
1/((b*c*d - a*d^2)*g*i^2*x + (b*c^2 - a*c*d)*g*i^2) + b*log(b*x + a)/((b^2
*c^2 - 2*a*b*c*d + a^2*d^2)*g*i^2) - b*log(d*x + c)/((b^2*c^2 - 2*a*b*c*d
+ a^2*d^2)*g*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) + 1/3*(((b*d*x +
 b*c)*log(b*x + a)^3 - (b*d*x + b*c)*log(d*x + c)^3 + 3*(b*d*x + b*c)*log(
b*x + a)^2 + 3*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c)^2 +
 6*b*c - 6*a*d + 6*(b*d*x + b*c)*log(b*x + a) - 3*(2*b*d*x + (b*d*x + b*c)
*log(b*x + a)^2 + 2*b*c + 2*(b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n^2/
(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2*g*i^2 + (b^2*c^2*d*g*i^2 -
2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x) - 3*((b*d*x + b*c)*log(b*x + a)^2 +
(b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*a*d + 2*(b*d*x + b*c)*log(b*x + a
) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*x + a))*log(d*x + c))*n*log(e*(b*
x/(d*x + c) + a/(d*x + c))^n)/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d
^2*g*i^2 + (b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x))*B^2 -
 ((b*d*x + b*c)*log(b*x + a)^2 + (b*d*x + b*c)*log(d*x + c)^2 + 2*b*c - 2*
a*d + 2*(b*d*x + b*c)*log(b*x + a) - 2*(b*d*x + b*c + (b*d*x + b*c)*log(b*
x + a))*log(d*x + c))*A*B*n/(b^2*c^3*g*i^2 - 2*a*b*c^2*d*g*i^2 + a^2*c*d^2
*g*i^2 + (b^2*c^2*d*g*i^2 - 2*a*b*c*d^2*g*i^2 + a^2*d^3*g*i^2)*x) + A^2...
```

### 3.198.8 Giac [A] (verification not implemented)

Time = 0.90 (sec) , antiderivative size = 368, normalized size of antiderivative = 1.59

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx$$
$$=\frac{1}{3}\left(\frac{B^2bn^2\log\left(\frac{bx+a}{dx+c}\right)^3}{bcgi^2-adgi^2}-3\left(\frac{(bx+a)B^2dn^2}{(bcgi^2-adgi^2)(dx+c)}-\frac{B^2bn\log(e)+ABbn}{bcgi^2-adgi^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2+\frac{3\left(B^2b\log\right.}{}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^2,x,
algorithm="giac")
```

output
```
1/3*(B^2*b*n^2*log((b*x + a)/(d*x + c))^3/(b*c*g*i^2 - a*d*g*i^2) - 3*((b*
x + a)*B^2*d*n^2/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) - (B^2*b*n*log(e) + A
*B*b*n)/(b*c*g*i^2 - a*d*g*i^2))*log((b*x + a)/(d*x + c))^2 + 3*(B^2*b*log
(e)^2 + 2*A*B*b*log(e) + A^2*b)*log((b*x + a)/(d*x + c))/(b*c*g*i^2 - a*d*
g*i^2) + 6*(B^2*d*n^2 - B^2*d*n*log(e) - A*B*d*n)*(b*x + a)*log((b*x + a)/
(d*x + c))/((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)) - 3*(2*B^2*d*n^2 - 2*B^2*d*
n*log(e) + B^2*d*log(e)^2 - 2*A*B*d*n + 2*A*B*d*log(e) + A^2*d)*(b*x + a)/
((b*c*g*i^2 - a*d*g*i^2)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^
2)
```

### 3.198.9 Mupad [B] (verification not implemented)

Time = 1.96 (sec) , antiderivative size = 365, normalized size of antiderivative = 1.58

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^2}\,dx = \frac{B^2\,b\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^3}{3\,g\,i^2\,n\,(a\,d-b\,c)^2} - \frac{A^2-2\,A\,B\,n+2\,B^2\,n^2}{(a\,d-b\,c)\,\left(c\,g\,i^2+d\,g\,i^2\,x\right)} - \frac{2\,B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(A-B\,n)}{(a\,d-b\,c)\,\left(c\,g\,i^2+d\,g\,i^2\,x\right)} - \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{(a\,d-b\,c)\,\left(c\,g\,i^2+d\,g\,i^2\,x\right)} - \frac{B\,b\,(A-B\,n)}{g\,i^2\,n\,(a\,d-b\,c)^2}\right) - \frac{b\,\mathrm{atan}\left(\frac{b\left(2\,b\,d\,x+\frac{a^2\,d^2\,g\,i^2-b^2\,c^2\,g\,i^2}{g\,i^2\,(a\,d-b\,c)}\right)\,\left(A^2-2\,A\,B\,n+2\,B^2\,n^2\right)\,1\mathrm{i}}{(a\,d-b\,c)\,\left(b\,A^2-2\,b\,A\,B\,n+2\,b\,B^2\,n^2\right)}\right)\,\left(A^2-2\,A\,B\,n+2\,B^2\,n^2\right)\,2\mathrm{i}}{g\,i^2\,(a\,d-b\,c)^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)*(c*i + d*i*x)^
2),x)
```

output
```
(B^2*b*log(e*((a + b*x)/(c + d*x))^n)^3)/(3*g*i^2*n*(a*d - b*c)^2) - (A^2
+ 2*B^2*n^2 - 2*A*B*n)/((a*d - b*c)*(c*g*i^2 + d*g*i^2*x)) - (2*B*log(e*((
a + b*x)/(c + d*x))^n)*(A - B*n))/((a*d - b*c)*(c*g*i^2 + d*g*i^2*x)) - (b
*atan((b*(2*b*d*x + (a^2*d^2*g*i^2 - b^2*c^2*g*i^2)/(g*i^2*(a*d - b*c)))*(
A^2 + 2*B^2*n^2 - 2*A*B*n)*1i)/((a*d - b*c)*(A^2*b + 2*B^2*b*n^2 - 2*A*B*b
*n)))*(A^2 + 2*B^2*n^2 - 2*A*B*n)*2i)/(g*i^2*(a*d - b*c)^2) - log(e*((a +
b*x)/(c + d*x))^n)^2*(B^2/((a*d - b*c)*(c*g*i^2 + d*g*i^2*x)) - (B*b*(A -
B*n))/(g*i^2*n*(a*d - b*c)^2))
```

# 3.199 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx$



## 3.199.1 Optimal result

Integrand size = 45, antiderivative size = 392

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx &= -\frac{2ABd^2n(a+bx)}{(bc-ad)^3g^2i^2(c+dx)}+\frac{2B^2d^2n^2(a+bx)}{(bc-ad)^3g^2i^2(c+dx)} \\ &\quad -\frac{2b^2B^2n^2(c+dx)}{(bc-ad)^3g^2i^2(a+bx)}-\frac{2B^2d^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^3g^2i^2(c+dx)} \\ &\quad -\frac{2b^2Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^3g^2i^2(a+bx)} \\ &\quad +\frac{d^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^2i^2(c+dx)} \\ &\quad -\frac{b^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3g^2i^2(a+bx)} \\ &\quad -\frac{2bd\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3g^2i^2n}\end{aligned}$$

output
```
-2*A*B*d^2*n*(b*x+a)/(-a*d+b*c)^3/g^2/i^2/(d*x+c)+2*B^2*d^2*n^2*(b*x+a)/(-
a*d+b*c)^3/g^2/i^2/(d*x+c)-2*b^2*B^2*n^2*(d*x+c)/(-a*d+b*c)^3/g^2/i^2/(b*x
+a)-2*B^2*d^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^3/g^2/i^2/(d*
x+c)-2*b^2*B*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g^2/i^
2/(b*x+a)+d^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^2/i
^2/(d*x+c)-b^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^3/g^2/
i^2/(b*x+a)-2/3*b*d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^3/g^2/i
^2/n
```

### 3.199.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 870 vs. 2(392) = 784.

Time = 0.62 (sec) , antiderivative size = 870, normalized size of antiderivative = 2.22

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx =$$

$$-\frac{2bB^2dn^2(a+bx)(c+dx)\log^3\left(\frac{a+bx}{c+dx}\right)+3Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(2aAbcd+b^2Bc^2n-a^2Bd^2n+2Ab^2cdx+2a\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i +
 d*i*x)^2),x]
```

output
```
-1/3*(2*b*B^2*d*n^2*(a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]^3 + 3*B*n
*Log[(a + b*x)/(c + d*x)]^2*(2*a*A*b*c*d + b^2*B*c^2*n - a^2*B*d^2*n + 2*A
*b^2*c*d*x + 2*a*A*b*d^2*x + 2*b^2*B*c*d*n*x - 2*a*b*B*d^2*n*x + 2*A*b^2*d
^2*x^2 + 2*b*B*d*(a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n] - 2*b*
B*d*n*(a + b*x)*(c + d*x)*Log[(a + b*x)/(c + d*x)]) + 6*B*(b*c - a*d)*n*Lo
g[(a + b*x)/(c + d*x)]*(A*b*c + a*A*d + b*B*c*n - a*B*d*n + 2*A*b*d*x + B*
(a*d + b*(c + 2*d*x))*Log[e*((a + b*x)/(c + d*x))^n] - B*n*(b*c + a*d + 2*
b*d*x)*Log[(a + b*x)/(c + d*x)]) + 6*b*d*(a + b*x)*(c + d*x)*Log[a + b*x]*
(A^2 + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)
/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*x)/(c +
d*x)])^2) + 3*b*(b*c - a*d)*(c + d*x)*(A^2 + 2*A*B*n + 2*B^2*n^2 + B^2*Log
[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(A + B*n)*Log[(a + b*x)/(c + d*x)] +
 B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(
A + B*n - B*n*Log[(a + b*x)/(c + d*x)])) + 3*d*(b*c - a*d)*(a + b*x)*(A^2
- 2*A*B*n + 2*B^2*n^2 + B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-A +
 B*n)*Log[(a + b*x)/(c + d*x)] + B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*
Log[e*((a + b*x)/(c + d*x))^n]*(-A + B*n + B*n*Log[(a + b*x)/(c + d*x)]))
- 6*b*d*(a + b*x)*(c + d*x)*(A^2 + 2*B^2*n^2 + 2*A*B*(Log[e*((a + b*x)/(c
+ d*x))^n] - n*Log[(a + b*x)/(c + d*x)]) + B^2*(Log[e*((a + b*x)/(c + d*x)
)^n] - n*Log[(a + b*x)/(c + d*x)])^2)*Log[c + d*x])/((b*c - a*d)^3*g^2*...
```

### 3.199.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 281, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2(ci+dix)^2}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i^2(bc-ad)^3}$$

$$\Big\downarrow \text{2795}$$

$$\frac{\int \left(d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}-\frac{2bd(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}\right)d\frac{a+bx}{c+dx}}{g^2i^2(bc-ad)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{-\frac{b^2(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}-\frac{2b^2Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx}+\frac{d^2(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-\frac{2ABd^2n(a+bx)}{c+dx}-}{g^2i^2(bc-ad)^3}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i + d*i*x
)^2),x]
```

output
```
((-2*A*B*d^2*n*(a + b*x))/(c + d*x) + (2*B^2*d^2*n^2*(a + b*x))/(c + d*x)
- (2*b^2*B^2*n^2*(c + d*x))/(a + b*x) - (2*B^2*d^2*n*(a + b*x)*Log[e*((a +
 b*x)/(c + d*x))^n])/(c + d*x) - (2*b^2*B*n*(c + d*x)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(a + b*x) + (d^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
+ d*x))^n])^2)/(c + d*x) - (b^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x
))^n])^2)/(a + b*x) - (2*b*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*
B*n))/((b*c - a*d)^3*g^2*i^2)
```

---

3.199. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}dx$

### 3.199.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.199.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1306 vs. $2(390) = 780$.

Time = 9.72 (sec) , antiderivative size = 1307, normalized size of antiderivative = 3.33

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1307 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x,method
=_RETURNVERBOSE)
```

output
```
1/3*(-12*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^3*d^2*n+12*A*B*x*ln(e*((b
*x+a)/(d*x+c))^n)*a^3*b^2*c^4*d*n+12*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3
*b^2*c^3*d^2*n^2+6*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^2*c^3*d^2-6*A
*B*x^2*a^4*b*c^2*d^3*n^2+12*A*B*x^2*a^3*b^2*c^3*d^2*n^2-6*A*B*x^2*a^2*b^3*
c^4*d*n^2-6*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b*c^3*d^2*n+6*B^2*x*ln(e
*((b*x+a)/(d*x+c))^n)^2*a^3*b^2*c^4*d*n+12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)
*a^4*b*c^3*d^2*n^2+12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^4*d*n^2+6*
A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b*c^3*d^2+6*A*B*x*ln(e*((b*x+a)/(d*x
+c))^n)^2*a^3*b^2*c^4*d+6*A*B*x*a^4*b*c^3*d^2*n^2+6*A*B*x*a^3*b^2*c^4*d*n^
2+6*B^2*x^2*a^4*b*c^2*d^3*n^3-6*B^2*x^2*a^2*b^3*c^4*d*n^3+2*B^2*x*ln(e*((b
*x+a)/(d*x+c))^n)^3*a^4*b*c^3*d^2+2*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^3*
b^2*c^4*d-6*B^2*x*a^4*b*c^3*d^2*n^3+6*B^2*x*a^3*b^2*c^4*d*n^3+6*A^2*x^2*ln
(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^3*d^2+3*A^2*x^2*a^4*b*c^2*d^3*n-3*A^2*x^
2*a^2*b^3*c^4*d*n-6*A*B*x*a^5*c^2*d^3*n^2-6*A*B*x*a^2*b^3*c^5*n^2+6*A^2*x*
ln(e*((b*x+a)/(d*x+c))^n)*a^4*b*c^3*d^2+6*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*
a^3*b^2*c^4*d-3*A^2*x*a^4*b*c^3*d^2*n+3*A^2*x*a^3*b^2*c^4*d*n+6*A*B*ln(e*(
(b*x+a)/(d*x+c))^n)^2*a^4*b*c^4*d-6*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^5*c^3*
d^2*n+6*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^5*n+3*B^2*ln(e*((b*x+a)/(d
*x+c))^n)^2*a^3*b^2*c^5*n+6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^5*c^3*d^2*n^2+
6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^2*c^5*n^2+3*A^2*x*a^5*c^2*d^3*n-3...
```

### 3.199.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(390) = 780$.

Time = 0.38 (sec) , antiderivative size = 983, normalized size of antiderivative = 2.51

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx =$$
$$-\frac{3\,A^2b^2c^2-3\,A^2a^2d^2+2\,(B^2b^2d^2n^2x^2+B^2abcdn^2+(B^2b^2cd+B^2abd^2)n^2x)\log\left(\frac{bx+a}{dx+c}\right)^3+6\,(B^2b^2c^2-}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x
, algorithm="fricas")
```

output
```
-1/3*(3*A^2*b^2*c^2 - 3*A^2*a^2*d^2 + 2*(B^2*b^2*d^2*n^2*x^2 + B^2*a*b*c*d
*n^2 + (B^2*b^2*c*d + B^2*a*b*d^2)*n^2*x)*log((b*x + a)/(d*x + c))^3 + 6*(
B^2*b^2*c^2 - B^2*a^2*d^2)*n^2 + 3*(B^2*b^2*c^2 - B^2*a^2*d^2 + 2*(B^2*b^2
*c*d - B^2*a*b*d^2)*x + 2*(B^2*b^2*d^2*x^2 + B^2*a*b*c*d + (B^2*b^2*c*d +
B^2*a*b*d^2)*x)*log((b*x + a)/(d*x + c)))*log(e)^2 + 3*(2*A*B*b^2*d^2*n*x^
2 + 2*A*B*a*b*c*d*n + (B^2*b^2*c^2 - B^2*a^2*d^2)*n^2 + 2*((B^2*b^2*c*d -
B^2*a*b*d^2)*n^2 + (A*B*b^2*c*d + A*B*a*b*d^2)*n)*x)*log((b*x + a)/(d*x +
c))^2 + 6*(A*B*b^2*c^2 - 2*A*B*a*b*c*d + A*B*a^2*d^2)*n + 6*(A^2*b^2*c*d -
 A^2*a*b*d^2 + 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2)*x + 6*(A*B*b^2*c^2 - A*B
*a^2*d^2 + (B^2*b^2*d^2*n*x^2 + B^2*a*b*c*d*n + (B^2*b^2*c*d + B^2*a*b*d^2
)*n*x)*log((b*x + a)/(d*x + c))^2 + (B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2
*d^2)*n + 2*(A*B*b^2*c*d - A*B*a*b*d^2)*x + (2*A*B*b^2*d^2*x^2 + 2*A*B*a*b
*c*d + (B^2*b^2*c^2 - B^2*a^2*d^2)*n + 2*(A*B*b^2*c*d + A*B*a*b*d^2 + (B^2
*b^2*c*d - B^2*a*b*d^2)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) + 6*(A^2*a*
b*c*d + (B^2*b^2*c^2 + B^2*a^2*d^2)*n^2 + (2*B^2*b^2*d^2*n^2 + A^2*b^2*d^2
)*x^2 + (A*B*b^2*c^2 - A*B*a^2*d^2)*n + (A^2*b^2*c*d + A^2*a*b*d^2 + 2*(B^
2*b^2*c*d + B^2*a*b*d^2)*n^2 + 2*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x)*log((b*
x + a)/(d*x + c)))/((b^4*c^3*d - 3*a*b^3*c^2*d^2 + 3*a^2*b^2*c*d^3 - a^3*b
*d^4)*g^2*i^2*x^2 + (b^4*c^4 - 2*a*b^3*c^3*d + 2*a^3*b*c*d^3 - a^4*d^4)*g^
2*i^2*x + (a*b^3*c^4 - 3*a^2*b^2*c^3*d + 3*a^3*b*c^2*d^2 - a^4*c*d^3)*g...
```

## 3.199.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2/(d*i*x+c*i)**
2,x)
```

output
```
Timed out
```

### 3.199.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2006 vs. $2(390) = 780$.

Time = 0.31 (sec) , antiderivative size = 2006, normalized size of antiderivative = 5.12

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x
, algorithm="maxima")
```

output
```
-B^2*((2*b*d*x + b*c + a*d)/((b^3*c^2*d - 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i
^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^
2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^2) + 2*b*d*log(b*x + a)/((b^3*c^3
 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2) - 2*b*d*log(d*x + c)/
((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g^2*i^2))*log(e*(b*x/
(d*x + c) + a/(d*x + c))^n)^2 - 2*A*B*((2*b*d*x + b*c + a*d)/((b^3*c^2*d -
 2*a*b^2*c*d^2 + a^2*b*d^3)*g^2*i^2*x^2 + (b^3*c^3 - a*b^2*c^2*d - a^2*b*c
*d^2 + a^3*d^3)*g^2*i^2*x + (a*b^2*c^3 - 2*a^2*b*c^2*d + a^3*c*d^2)*g^2*i^
2) + 2*b*d*log(b*x + a)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^
3)*g^2*i^2) - 2*b*d*log(d*x + c)/((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2
 - a^3*d^3)*g^2*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 2/3*((3*b^2
*c^2 - 3*a^2*d^2 + (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x
 + a)^3 + 3*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)*l
og(d*x + c)^2 - (b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b*d^2)*x)*log(d*x +
c)^3 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + a*b*c*d + (b^2*c*d + a*b
*d^2)*x)*log(b*x + a) - 3*(2*b^2*d^2*x^2 + 2*a*b*c*d + (b^2*d^2*x^2 + a*b*
c*d + (b^2*c*d + a*b*d^2)*x)*log(b*x + a)^2 + 2*(b^2*c*d + a*b*d^2)*x)*log
(d*x + c))*n^2/(a*b^3*c^4*g^2*i^2 - 3*a^2*b^2*c^3*d*g^2*i^2 + 3*a^3*b*c^2*
d^2*g^2*i^2 - a^4*c*d^3*g^2*i^2 + (b^4*c^3*d*g^2*i^2 - 3*a*b^3*c^2*d^2*g^2
*i^2 + 3*a^2*b^2*c*d^3*g^2*i^2 - a^3*b*d^4*g^2*i^2)*x^2 + (b^4*c^4*g^2*...
```

### 3.199.8 Giac [A] (verification not implemented)

Time = 285.30 (sec) , antiderivative size = 185, normalized size of antiderivative = 0.47

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx = -\left(\frac{(dx+c)B^2n^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(bx+a)g^2i^2} + \frac{2\left(B^2n^2+B^2n\log(e)+ABn\right)(dx+c)\log\left(\frac{bx+a}{dx+c}\right)}{(bx+a)g^2i^2} + \frac{\left(2B^2n^2+2B^2n\log\right.}{}\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^2,x
, algorithm="giac")
```

output
```
-((d*x + c)*B^2*n^2*log((b*x + a)/(d*x + c))^2/((b*x + a)*g^2*i^2) + 2*(B^
2*n^2 + B^2*n*log(e) + A*B*n)*(d*x + c)*log((b*x + a)/(d*x + c))/((b*x + a
)*g^2*i^2) + (2*B^2*n^2 + 2*B^2*n*log(e) + B^2*log(e)^2 + 2*A*B*n + 2*A*B*
log(e) + A^2)*(d*x + c)/((b*x + a)*g^2*i^2))*(b*c/(b*c - a*d)^2 - a*d/(b*c
 - a*d)^2)^2
```

### 3.199.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^2}\,dx=\int \frac{\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)^2}{{\left(a\,g+b\,g\,x\right)}^2\,{\left(c\,i+d\,i\,x\right)}^2}\,dx$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^2*(c*i + d*i*x
)^2),x)
```

output
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^2*(c*i + d*i*x
)^2), x)
```

# 3.200 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$



## 3.200.1 Optimal result

Integrand size = 45, antiderivative size = 560

$$\begin{aligned}
\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx &= \frac{2ABd^3n(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} - \frac{2B^2d^3n^2(a+bx)}{(bc-ad)^4g^3i^2(c+dx)} \\
&+ \frac{6b^2B^2dn^2(c+dx)}{(bc-ad)^4g^3i^2(a+bx)} - \frac{b^3B^2n^2(c+dx)^2}{4(bc-ad)^4g^3i^2(a+bx)^2} \\
&+ \frac{2B^2d^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^4g^3i^2(c+dx)} \\
&+ \frac{6b^2Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^3i^2(a+bx)} \\
&- \frac{b^3Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^3i^2(a+bx)^2} \\
&- \frac{d^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^3i^2(c+dx)} \\
&+ \frac{3b^2d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^3i^2(a+bx)} \\
&- \frac{b^3(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^3i^2(a+bx)^2} \\
&+ \frac{bd^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^4g^3i^2n}
\end{aligned}$$

output
```
2*A*B*d^3*n*(b*x+a)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)-2*B^2*d^3*n^2*(b*x+a)/(-a
*d+b*c)^4/g^3/i^2/(d*x+c)+6*b^2*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^4/g^3/i^2/(b*
x+a)-1/4*b^3*B^2*n^2*(d*x+c)^2/(-a*d+b*c)^4/g^3/i^2/(b*x+a)^2+2*B^2*d^3*n*
(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^4/g^3/i^2/(d*x+c)+6*b^2*B*d*n
*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(b*x+a)-1/2*
b^3*B*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^3/i^2/(b*
x+a)^2-d^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^3/i^2/
(d*x+c)+3*b^2*d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4/g^3
/i^2/(b*x+a)-1/2*b^3*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c
)^4/g^3/i^2/(b*x+a)^2+b*d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)
^4/g^3/i^2/n
```

### 3.200.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1340 vs. $2(560) = 1120$.

Time = 0.93 (sec) , antiderivative size = 1340, normalized size of antiderivative = 2.39

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$$
$$= \frac{4bB^2d^2n^2(a+bx)^2(c+dx)\log^3\left(\frac{a+bx}{c+dx}\right)+2Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(6a^2Abcd^2-b^3Bc^3n+6ab^2Bc^2dn-2a^3Bd^3n\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i +
 d*i*x)^2),x]
```

output
```
(4*b*B^2*d^2*n^2*(a + b*x)^2*(c + d*x)*Log[(a + b*x)/(c + d*x)]^3 + 2*B*n*
Log[(a + b*x)/(c + d*x)]^2*(6*a^2*A*b*c*d^2 - b^3*B*c^3*n + 6*a*b^2*B*c^2*
d*n - 2*a^3*B*d^3*n + 12*a*A*b^2*c*d^2*x + 6*a^2*A*b*d^3*x + 3*b^3*B*c^2*d
*n*x + 12*a*b^2*B*c*d^2*n*x - 6*a^2*b*B*d^3*n*x + 6*A*b^3*c*d^2*x^2 + 12*a
*A*b^2*d^3*x^2 + 9*b^3*B*c*d^2*n*x^2 + 6*A*b^3*d^3*x^3 + 3*b^3*B*d^3*n*x^3
 + 6*b*B*d^2*(a + b*x)^2*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n] - 6*b*B*
d^2*n*(a + b*x)^2*(c + d*x)*Log[(a + b*x)/(c + d*x)]) + 2*b*d*(b*c - a*d)*
(a + b*x)*(c + d*x)*(4*A^2 + 10*A*B*n + 11*B^2*n^2 + 4*B^2*Log[e*((a + b*x
)/(c + d*x))^n]^2 - 2*B*n*(4*A + 5*B*n)*Log[(a + b*x)/(c + d*x)] + 4*B^2*n
^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(4*A +
5*B*n - 4*B*n*Log[(a + b*x)/(c + d*x)])) - b*(b*c - a*d)^2*(c + d*x)*(2*A^
2 + 2*A*B*n + B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*
A + B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 +
 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(2*A + B*n - 2*B*n*Log[(a + b*x)/(c +
d*x)])) + 6*b*d^2*(a + b*x)^2*(c + d*x)*Log[a + b*x]*(2*A^2 + 2*A*B*n + 5*
B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + B*n)*Log[(
a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a
 + b*x)/(c + d*x))^n]*(2*A + B*n - 2*B*n*Log[(a + b*x)/(c + d*x)])) + 2*B*
(b*c - a*d)*n*Log[(a + b*x)/(c + d*x)]*(2*b*d*(a + b*x)*(c + d*x)*(4*A + 5
*B*n + 4*B*Log[e*((a + b*x)/(c + d*x))^n] - 4*B*n*Log[(a + b*x)/(c + d*...
```

### 3.200.3 Rubi [A] (verified)

Time = 0.62 (sec) , antiderivative size = 401, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3 (ci+dix)^2} \, dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{\int \frac{(c+dx)^3 \left(b-\frac{d(a+bx)}{c+dx}\right)^3 \left(A+B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3} d\frac{a+bx}{c+dx}}{g^3 i^2 (bc-ad)^4}$$

$$\Big\downarrow \text{ 2795}$$

$$\int \frac{\left(-\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d^3+\frac{3b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d^2}{a+bx}-\frac{3b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d}{(a+bx)^2}+\frac{b^3(c+dx)^3}{}\right.}{g^3i^2(bc-ad)^4}$$

$$\Big\downarrow \text{ 2009}$$

$$-\frac{b^3(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2}-\frac{b^3Bn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2}+\frac{3b^2d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx}+\frac{6b^2Bdn(c+dx)\left(\right.}{}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i + d*i*x
)^2),x]
```

output
```
((2*A*B*d^3*n*(a + b*x))/(c + d*x) - (2*B^2*d^3*n^2*(a + b*x))/(c + d*x) +
 (6*b^2*B^2*d*n^2*(c + d*x))/(a + b*x) - (b^3*B^2*n^2*(c + d*x)^2)/(4*(a +
 b*x)^2) + (2*B^2*d^3*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(c + d*x
) + (6*b^2*B*d*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*
x) - (b^3*B*n*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a +
b*x)^2) - (d^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*
x) + (3*b^2*d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x
) - (b^3*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x
)^2) + (b*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(B*n))/((b*c - a*d
)^4*g^3*i^2)
```

### 3.200.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

3.200. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.200.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2000 vs. $2(554) = 1108$.

Time = 28.10 (sec) , antiderivative size = 2001, normalized size of antiderivative = 3.57

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 2001 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x,method
=_RETURNVERBOSE)
```

output
```
1/4*(24*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^6*c*d^5*n-15*B^2*a^2*b^5*c*d
^5*n^3+24*B^2*a*b^6*c^2*d^4*n^3+8*A*B*a^3*b^4*d^6*n^2-2*A*B*b^7*c^3*d^3*n^
2-6*A^2*a^2*b^5*c*d^5*n+12*A^2*a*b^6*c^2*d^4*n+48*A*B*x*ln(e*((b*x+a)/(d*x
+c))^n)*a*b^6*c*d^5*n-30*A*B*a^2*b^5*c*d^5*n^2+24*A*B*a*b^6*c^2*d^4*n^2+12
*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^7*d^6*n+18*B^2*x^2*ln(e*((b*x+a)/(d*x
+c))^n)^2*b^7*c*d^5*n+48*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*d^6*n^2+4
2*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c*d^5*n^2+24*A*B*x^2*ln(e*((b*x+a)
/(d*x+c))^n)^2*a*b^6*d^6+12*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^7*c*d^5-
12*A*B*x^2*a*b^6*d^6*n^2+12*A*B*x^2*b^7*c*d^5*n^2+8*B^2*x*ln(e*((b*x+a)/(d
*x+c))^n)^3*a*b^6*c*d^5-12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*d^6*n
+6*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^7*c^2*d^4*n+24*B^2*x*ln(e*((b*x+a)/
(d*x+c))^n)*a^2*b^5*d^6*n^2+18*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^7*c^2*d^4
*n^2+4*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*b^7*d^6+12*A^2*x^3*ln(e*((b*x+a
)/(d*x+c))^n)*b^7*d^6-8*B^2*a^3*b^4*d^6*n^3-B^2*b^7*c^3*d^3*n^3-4*A^2*a^3*
b^4*d^6*n-2*A^2*b^7*c^3*d^3*n+18*B^2*x*a*b^6*c*d^5*n^3+12*A*B*x*ln(e*((b*x
+a)/(d*x+c))^n)^2*a^2*b^5*d^6-6*A*B*x*a^2*b^5*d^6*n^2+18*A*B*x*b^7*c^2*d^4
*n^2+12*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^6*c^2*d^4*n+24*B^2*ln(e*((b*x+
a)/(d*x+c))^n)*a*b^6*c^2*d^4*n^2+24*A^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^6*
c*d^5+12*A^2*x*a*b^6*c*d^5*n+12*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^5*c*
d^5-8*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^4*d^6*n-4*A*B*ln(e*((b*x+a)/(...
```

### 3.200.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2052 vs. $2(554) = 1108$.

Time = 0.37 (sec) , antiderivative size = 2052, normalized size of antiderivative = 3.66

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x
, algorithm="fricas")
```

output
```
-1/4*(2*A^2*b^3*c^3 - 12*A^2*a*b^2*c^2*d + 6*A^2*a^2*b*c*d^2 + 4*A^2*a^3*d
^3 - 4*(B^2*b^3*d^3*n^2*x^3 + B^2*a^2*b*c*d^2*n^2 + (B^2*b^3*c*d^2 + 2*B^2
*a*b^2*d^3)*n^2*x^2 + (2*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*n^2*x)*log((b*x
+ a)/(d*x + c))^3 + (B^2*b^3*c^3 - 24*B^2*a*b^2*c^2*d + 15*B^2*a^2*b*c*d^2
 + 8*B^2*a^3*d^3)*n^2 - 6*(2*A^2*b^3*c*d^2 - 2*A^2*a*b^2*d^3 + 5*(B^2*b^3*
c*d^2 - B^2*a*b^2*d^3)*n^2 + 2*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*n)*x^2 + 2*
(B^2*b^3*c^3 - 6*B^2*a*b^2*c^2*d + 3*B^2*a^2*b*c*d^2 + 2*B^2*a^3*d^3 - 6*(
B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*x^2 - 3*(B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2
- 3*B^2*a^2*b*d^3)*x - 6*(B^2*b^3*d^3*x^3 + B^2*a^2*b*c*d^2 + (B^2*b^3*c*d
^2 + 2*B^2*a*b^2*d^3)*x^2 + (2*B^2*a*b^2*c*d^2 + B^2*a^2*b*d^3)*x)*log((b*
x + a)/(d*x + c)))*log(e)^2 - 2*(6*A*B*a^2*b*c*d^2*n + 3*(B^2*b^3*d^3*n^2
+ 2*A*B*b^3*d^3*n)*x^3 - (B^2*b^3*c^3 - 6*B^2*a*b^2*c^2*d + 2*B^2*a^3*d^3)
*n^2 + 3*(3*B^2*b^3*c*d^2*n^2 + 2*(A*B*b^3*c*d^2 + 2*A*B*a*b^2*d^3)*n)*x^2
 + 3*((B^2*b^3*c^2*d + 4*B^2*a*b^2*c*d^2 - 2*B^2*a^2*b*d^3)*n^2 + 2*(2*A*B
*a*b^2*c*d^2 + A*B*a^2*b*d^3)*n)*x)*log((b*x + a)/(d*x + c))^2 + 2*(A*B*b^
3*c^3 - 12*A*B*a*b^2*c^2*d + 15*A*B*a^2*b*c*d^2 - 4*A*B*a^3*d^3)*n - 3*(2*
A^2*b^3*c^2*d + 4*A^2*a*b^2*c*d^2 - 6*A^2*a^2*b*d^3 + (7*B^2*b^3*c^2*d + 6
*B^2*a*b^2*c*d^2 - 13*B^2*a^2*b*d^3)*n^2 + 2*(3*A*B*b^3*c^2*d - 2*A*B*a*b^
2*c*d^2 - A*B*a^2*b*d^3)*n)*x + 2*(2*A*B*b^3*c^3 - 12*A*B*a*b^2*c^2*d + 6*
A*B*a^2*b*c*d^2 + 4*A*B*a^3*d^3 - 6*(2*A*B*b^3*c*d^2 - 2*A*B*a*b^2*d^3 ...
```

### 3.200.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3/(d*i*x+c*i)**
2,x)
```

output
```
Timed out
```

### 3.200.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4198 vs. 2(554) = 1108.

Time = 0.51 (sec) , antiderivative size = 4198, normalized size of antiderivative = 7.50

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x
, algorithm="maxima")
```

output
```
1/2*B^2*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3
*a*b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4
)*g^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d
^3 - 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2
*c^2*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3
*d + 3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^
4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2)
- 6*b*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a
^3*b*c*d^3 + a^4*d^4)*g^3*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2 +
 A*B*((6*b^2*d^2*x^2 - b^2*c^2 + 5*a*b*c*d + 2*a^2*d^2 + 3*(b^2*c*d + 3*a*
b*d^2)*x)/((b^5*c^3*d - 3*a*b^4*c^2*d^2 + 3*a^2*b^3*c*d^3 - a^3*b^2*d^4)*g
^3*i^2*x^3 + (b^5*c^4 - a*b^4*c^3*d - 3*a^2*b^3*c^2*d^2 + 5*a^3*b^2*c*d^3
- 2*a^4*b*d^4)*g^3*i^2*x^2 + (2*a*b^4*c^4 - 5*a^2*b^3*c^3*d + 3*a^3*b^2*c^
2*d^2 + a^4*b*c*d^3 - a^5*d^4)*g^3*i^2*x + (a^2*b^3*c^4 - 3*a^3*b^2*c^3*d
+ 3*a^4*b*c^2*d^2 - a^5*c*d^3)*g^3*i^2) + 6*b*d^2*log(b*x + a)/((b^4*c^4 -
 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^3*i^2) - 6
*b*d^2*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*
b*c*d^3 + a^4*d^4)*g^3*i^2))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/4*
((b^3*c^3 - 24*a*b^2*c^2*d + 15*a^2*b*c*d^2 + 8*a^3*d^3 - 4*(b^3*d^3*x^3 +
 a^2*b*c*d^2 + (b^3*c*d^2 + 2*a*b^2*d^3)*x^2 + (2*a*b^2*c*d^2 + a^2*b*d...
```

### 3.200.8 Giac [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^2,x
, algorithm="giac")
```

output
```
Timed out
```

### 3.200.9 Mupad [B] (verification not implemented)

Time = 7.25 (sec) , antiderivative size = 1784, normalized size of antiderivative = 3.19

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx=\text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^3*(c*i + d*i*x
)^2),x)
```

output
```
(B^2*b*d^2*log(e*((a + b*x)/(c + d*x))^n)^3)/(g^3*i^2*n*(a*d - b*c)^4) - l
og(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*(2*a*d + b*c))/(2*(a^2*d^2 + b^2*c^
2 - 2*a*b*c*d)) + (3*B^2*b*d*x)/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)))/(x*(a
^2*d*g^3*i^2 + 2*a*b*c*g^3*i^2) + x^2*(b^2*c*g^3*i^2 + 2*a*b*d*g^3*i^2) +
a^2*c*g^3*i^2 + b^2*d*g^3*i^2*x^3) - (3*B*b*d^2*(2*A + B*n))/(2*g^3*i^2*n*
(a*d - b*c)^4) + (3*B^2*b*d^2*(b*g^3*i^2*n*x^2*(a*d - b*c) + (a*c*g^3*i^2*
n*(a*d - b*c))/d + (g^3*i^2*n*x*(a*d + b*c)*(a*d - b*c))/d))/(g^3*i^2*n*(a
*d - b*c)^4*(x*(a^2*d*g^3*i^2 + 2*a*b*c*g^3*i^2) + x^2*(b^2*c*g^3*i^2 + 2*
a*b*d*g^3*i^2) + a^2*c*g^3*i^2 + b^2*d*g^3*i^2*x^3))) - ((4*A^2*a^2*d^2 -
2*A^2*b^2*c^2 + 8*B^2*a^2*d^2*n^2 - B^2*b^2*c^2*n^2 + 10*A^2*a*b*c*d - 8*A
*B*a^2*d^2*n - 2*A*B*b^2*c^2*n + 23*B^2*a*b*c*d*n^2 + 22*A*B*a*b*c*d*n)/(2
*(a*d - b*c)) + (3*x^2*(2*A^2*b^2*d^2 + 5*B^2*b^2*d^2*n^2 + 2*A*B*b^2*d^2*
n))/(a*d - b*c) + (3*x*(6*A^2*a*b*d^2 + 2*A^2*b^2*c*d + 13*B^2*a*b*d^2*n^2
 + 7*B^2*b^2*c*d*n^2 + 2*A*B*a*b*d^2*n + 6*A*B*b^2*c*d*n))/(2*(a*d - b*c))
)/(x*(2*a^4*d^3*g^3*i^2 + 4*a*b^3*c^3*g^3*i^2 - 6*a^2*b^2*c^2*d*g^3*i^2) +
 x^2*(2*b^4*c^3*g^3*i^2 + 4*a^3*b*d^3*g^3*i^2 - 6*a^2*b^2*c*d^2*g^3*i^2) +
 x^3*(2*a^2*b^2*d^3*g^3*i^2 + 2*b^4*c^2*d*g^3*i^2 - 4*a*b^3*c*d^2*g^3*i^2)
 + 2*a^2*b^2*c^3*g^3*i^2 + 2*a^4*c*d^2*g^3*i^2 - 4*a^3*b*c^2*d*g^3*i^2) -
(b*d^2*atan((b*d^2*(2*A^2 + 5*B^2*n^2 + 2*A*B*n)*(2*a^4*d^4*g^3*i^2 - 2*b^
4*c^4*g^3*i^2 + 4*a*b^3*c^3*d*g^3*i^2 - 4*a^3*b*c*d^3*g^3*i^2)*3i)/(2*g...
```

3.200. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^2}\,dx$

# 3.201 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$

### 3.201.1 Optimal result

Integrand size = 45, antiderivative size = 729

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = -\frac{2ABd^4n(a+bx)}{(bc-ad)^5g^4i^2(c+dx)}+\frac{2B^2d^4n^2(a+bx)}{(bc-ad)^5g^4i^2(c+dx)}-\frac{12b^2B^2d^2n^2(c+dx)}{(bc-ad)^5g^4i^2(a+bx)}+\frac{b^3B^2dn^2(c+dx)^2}{(bc-ad)^5g^4i^2(a+bx)^2}-\frac{2b^4B^2n^2(c+dx)^3}{27(bc-ad)^5g^4i^2(a+bx)^3}-\frac{2B^2d^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^5g^4i^2(c+dx)}-\frac{12b^2Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)}+\frac{2b^3Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^4i^2(a+bx)^2}-\frac{2b^4Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^5g^4i^2(a+bx)^3}+\frac{d^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(c+dx)}-\frac{6b^2d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)}+\frac{2b^3d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^4i^2(a+bx)^2}-\frac{b^4(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^5g^4i^2(a+bx)^3}-\frac{4bd^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^5g^4i^2n}$$

output
```
-2*A*B*d^4*n*(b*x+a)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)+2*B^2*d^4*n^2*(b*x+a)/(-
a*d+b*c)^5/g^4/i^2/(d*x+c)-12*b^2*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^5/g^4/i^2
/(b*x+a)+b^3*B^2*d*n^2*(d*x+c)^2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-2/27*b^4*B
^2*n^2*(d*x+c)^3/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3-2*B^2*d^4*n*(b*x+a)*ln(e*(
(b*x+a)/(d*x+c))^n)/(-a*d+b*c)^5/g^4/i^2/(d*x+c)-12*b^2*B*d^2*n*(d*x+c)*(A
+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)+2*b^3*B*d*n*(d*
x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^2-2/9*
b^4*B*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^4/i^2/(b*
x+a)^3+d^4*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^5/g^4/i^2/
(d*x+c)-6*b^2*d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^5/g
^4/i^2/(b*x+a)+2*b^3*d*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b
*c)^5/g^4/i^2/(b*x+a)^2-1/3*b^4*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^
2/(-a*d+b*c)^5/g^4/i^2/(b*x+a)^3-4/3*b*d^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
^3/B/(-a*d+b*c)^5/g^4/i^2/n
```

### 3.201.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1695 vs. 2(729) = 1458.

Time = 1.41 (sec) , antiderivative size = 1695, normalized size of antiderivative = 2.33

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i +
 d*i*x)^2),x]
```

output
```
-1/27*(36*b*B^2*d^3*n^2*(a + b*x)^3*(c + d*x)*Log[(a + b*x)/(c + d*x)]^3 +
 9*B*n*Log[(a + b*x)/(c + d*x)]^2*(12*a^3*A*b*c*d^3 + b^4*B*c^4*n - 6*a*b^
3*B*c^3*d*n + 18*a^2*b^2*B*c^2*d^2*n - 3*a^4*B*d^4*n + 36*a^2*A*b^2*c*d^3*
x + 12*a^3*A*b*d^4*x - 2*b^4*B*c^3*d*n*x + 18*a*b^3*B*c^2*d^2*n*x + 36*a^2
*b^2*B*c*d^3*n*x - 12*a^3*b*B*d^4*n*x + 36*a*A*b^3*c*d^3*x^2 + 36*a^2*A*b^
2*d^4*x^2 + 6*b^4*B*c^2*d^2*n*x^2 + 54*a*b^3*B*c*d^3*n*x^2 + 12*A*b^4*c*d^
3*x^3 + 36*a*A*b^3*d^4*x^3 + 22*b^4*B*c*d^3*n*x^3 + 18*a*b^3*B*d^4*n*x^3 +
 12*A*b^4*d^4*x^4 + 10*b^4*B*d^4*n*x^4 + 12*b*B*d^3*(a + b*x)^3*(c + d*x)*
Log[e*((a + b*x)/(c + d*x))^n] - 12*b*B*d^3*n*(a + b*x)^3*(c + d*x)*Log[(a
 + b*x)/(c + d*x)]) + 3*b*d^2*(b*c - a*d)*(a + b*x)^2*(c + d*x)*(27*A^2 +
78*A*B*n + 92*B^2*n^2 + 27*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(9
*A + 13*B*n)*Log[(a + b*x)/(c + d*x)] + 27*B^2*n^2*Log[(a + b*x)/(c + d*x)
]^2 + 6*B*Log[e*((a + b*x)/(c + d*x))^n]*(9*A + 13*B*n - 9*B*n*Log[(a + b*
x)/(c + d*x)])) + 6*b*d^3*(a + b*x)^3*(c + d*x)*Log[a + b*x]*(18*A^2 + 30*
A*B*n + 55*B^2*n^2 + 18*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(6*A
+ 5*B*n)*Log[(a + b*x)/(c + d*x)] + 18*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2
+ 6*B*Log[e*((a + b*x)/(c + d*x))^n]*(6*A + 5*B*n - 6*B*n*Log[(a + b*x)/(c
 + d*x)])) + b*(b*c - a*d)^3*(c + d*x)*(9*A^2 + 6*A*B*n + 2*B^2*n^2 + 9*B^
2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(3*A + B*n)*Log[(a + b*x)/(c +
d*x)] + 9*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a + b*x)/(c ...
```

### 3.201.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 522, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4(ci+dix)^2}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i^2(bc-ad)^5}$$

$$\Big\downarrow \text{ 2795}$$

$$\int \frac{\left(\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d^4-\frac{4b(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d^3}{a+bx}+\frac{6b^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d^2}{(a+bx)^2}-\frac{4b^3(c+dx)^3}{\cdots}\right.}{g^4i^2(bc-ad)^5}$$

$\downarrow$ 2009

$$-\frac{b^4(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3}-\frac{2b^4Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3}+\frac{2b^3d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(a+bx)^2}+\frac{2b^3Bdn(c+dx}{\cdots}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i + d*i*x
)^2),x]
```

output
```
((-2*A*B*d^4*n*(a + b*x))/(c + d*x) + (2*B^2*d^4*n^2*(a + b*x))/(c + d*x)
- (12*b^2*B^2*d^2*n^2*(c + d*x))/(a + b*x) + (b^3*B^2*d*n^2*(c + d*x)^2)/(
a + b*x)^2 - (2*b^4*B^2*n^2*(c + d*x)^3)/(27*(a + b*x)^3) - (2*B^2*d^4*n*(
a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(c + d*x) - (12*b^2*B*d^2*n*(c +
d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (2*b^3*B*d*n*(c +
 d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x)^2 - (2*b^4*B*n*(
c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)^3) + (d^4*
(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*x) - (6*b^2*d^2
*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) + (2*b^3*d*
(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x)^2 - (b^4*(
c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*(a + b*x)^3) - (4*
b*d^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*B*n))/((b*c - a*d)^5*g^
4*i^2)
```

### 3.201.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

---

3.201. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.201.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3040 vs. 2(721) = 1442.

Time = 44.82 (sec) , antiderivative size = 3041, normalized size of antiderivative = 4.17

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 3041 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x,method
=_RETURNVERBOSE)
```

output
```
1/27*(-342*A*B*x*a^2*b^7*c*d^6*n^2+36*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)^3*
b^9*d^7+108*A^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^9*d^7-54*B^2*a^4*b^5*d^7*n
^3+2*B^2*b^9*c^4*d^3*n^3-27*A^2*a^4*b^5*d^7*n+9*A^2*b^9*c^4*d^3*n+180*A*B*
x^3*b^9*c*d^6*n^2+108*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^8*c*d^6+54*B
^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^9*c^2*d^5*n+648*B^2*x^2*ln(e*((b*x+a)
/(d*x+c))^n)*a^2*b^7*d^7*n^2+198*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^2
*d^5*n^2+324*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^7*c^2*d^5*n^2-54*B^2*ln(e
*((b*x+a)/(d*x+c))^n)*a*b^8*c^3*d^4*n^2+324*A^2*x*ln(e*((b*x+a)/(d*x+c))^n
)*a^2*b^7*c*d^6+54*A^2*x*a^2*b^7*c*d^6*n+162*A^2*x*a*b^8*c^2*d^5*n+108*A*B
*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^6*c*d^6-54*A*B*ln(e*((b*x+a)/(d*x+c))^n
)*a^4*b^5*d^7*n+18*A*B*ln(e*((b*x+a)/(d*x+c))^n)*b^9*c^4*d^3*n-330*A*B*a^3
*b^6*c*d^6*n^2+324*A*B*a^2*b^7*c^2*d^5*n^2-54*A*B*a*b^8*c^3*d^4*n^2+180*A*
B*x^4*ln(e*((b*x+a)/(d*x+c))^n)*b^9*d^7*n+162*B^2*x^3*ln(e*((b*x+a)/(d*x+c
))^n)^2*a*b^8*d^7*n+198*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^9*c*d^6*n+81
0*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*d^7*n^2+510*B^2*x^3*ln(e*((b*x+a
)/(d*x+c))^n)*b^9*c*d^6*n^2+324*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^8*
d^7+108*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^9*c*d^6-180*A*B*x^3*a*b^8*d^
7*n^2+972*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^8*c*d^6*n+648*A*B*x*ln(e*(
(b*x+a)/(d*x+c))^n)*a^2*b^7*c*d^6*n+324*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*
b^8*c^2*d^5*n+480*B^2*x^2*a*b^8*c*d^6*n^3+324*A*B*x^2*ln(e*((b*x+a)/(d*...
```

### 3.201.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3183 vs. $2(721) = 1442$.

Time = 0.43 (sec) , antiderivative size = 3183, normalized size of antiderivative = 4.37

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x
, algorithm="fricas")
```

output
```
-1/27*(9*A^2*b^4*c^4 - 54*A^2*a*b^3*c^3*d + 162*A^2*a^2*b^2*c^2*d^2 - 90*A
^2*a^3*b*c*d^3 - 27*A^2*a^4*d^4 + 6*(18*A^2*b^4*c*d^3 - 18*A^2*a*b^3*d^4 +
 55*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)*n^2 + 30*(A*B*b^4*c*d^3 - A*B*a*b^3*d^
4)*n)*x^3 + 36*(B^2*b^4*d^4*n^2*x^4 + B^2*a^3*b*c*d^3*n^2 + (B^2*b^4*c*d^3
 + 3*B^2*a*b^3*d^4)*n^2*x^3 + 3*(B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*n^2*x^
2 + (3*B^2*a^2*b^2*c*d^3 + B^2*a^3*b*d^4)*n^2*x)*log((b*x + a)/(d*x + c))^
3 + (2*B^2*b^4*c^4 - 27*B^2*a*b^3*c^3*d + 324*B^2*a^2*b^2*c^2*d^2 - 245*B^
2*a^3*b*c*d^3 - 54*B^2*a^4*d^4)*n^2 + 3*(18*A^2*b^4*c^2*d^2 + 72*A^2*a*b^3
*c*d^3 - 90*A^2*a^2*b^2*d^4 + 5*(17*B^2*b^4*c^2*d^2 + 32*B^2*a*b^3*c*d^3 -
 49*B^2*a^2*b^2*d^4)*n^2 + 6*(11*A*B*b^4*c^2*d^2 + 8*A*B*a*b^3*c*d^3 - 19*
A*B*a^2*b^2*d^4)*n)*x^2 + 9*(B^2*b^4*c^4 - 6*B^2*a*b^3*c^3*d + 18*B^2*a^2*
b^2*c^2*d^2 - 10*B^2*a^3*b*c*d^3 - 3*B^2*a^4*d^4 + 12*(B^2*b^4*c*d^3 - B^2
*a*b^3*d^4)*x^3 + 6*(B^2*b^4*c^2*d^2 + 4*B^2*a*b^3*c*d^3 - 5*B^2*a^2*b^2*d
^4)*x^2 - 2*(B^2*b^4*c^3*d - 9*B^2*a*b^3*c^2*d^2 - 3*B^2*a^2*b^2*c*d^3 + 1
1*B^2*a^3*b*d^4)*x + 12*(B^2*b^4*d^4*x^4 + B^2*a^3*b*c*d^3 + (B^2*b^4*c*d^
3 + 3*B^2*a*b^3*d^4)*x^3 + 3*(B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*x^2 + (3*
B^2*a^2*b^2*c*d^3 + B^2*a^3*b*d^4)*x)*log((b*x + a)/(d*x + c)))*log(e)^2 +
 9*(12*A*B*a^3*b*c*d^3*n + 2*(5*B^2*b^4*d^4*n^2 + 6*A*B*b^4*d^4*n)*x^4 + 2
*((11*B^2*b^4*c*d^3 + 9*B^2*a*b^3*d^4)*n^2 + 6*(A*B*b^4*c*d^3 + 3*A*B*a*b^
3*d^4)*n)*x^3 + (B^2*b^4*c^4 - 6*B^2*a*b^3*c^3*d + 18*B^2*a^2*b^2*c^2*d...
```

### 3.201.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4/(d*i*x+c*i)**
2,x)
```

output
```
Timed out
```

### 3.201.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 6171 vs. 2(721) = 1442.

Time = 0.68 (sec) , antiderivative size = 6171, normalized size of antiderivative = 8.47

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^4(ci + dix)^2}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x
, algorithm="maxima")
```

output
```
-1/3*B^2*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a
^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 - 2*(b^3*c^2*d - 8*a*b^2*c*d^2 -
11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3
*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5
*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*
x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d
^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i^2*x^2 + (3*a^2*b^5*c^5 - 11*a^3*b^
4*c^4*d + 14*a^4*b^3*c^3*d^2 - 6*a^5*b^2*c^2*d^3 - a^6*b*c*d^4 + a^7*d^5)*
g^4*i^2*x + (a^3*b^4*c^5 - 4*a^4*b^3*c^4*d + 6*a^5*b^2*c^3*d^2 - 4*a^6*b*c
^2*d^3 + a^7*c*d^4)*g^4*i^2) + 12*b*d^3*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c
^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*
g^4*i^2) - 12*b*d^3*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^
3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^4*i^2))*log(e*(b*x
/(d*x + c) + a/(d*x + c))^n)^2 - 2/3*A*B*((12*b^3*d^3*x^3 + b^3*c^3 - 5*a*
b^2*c^2*d + 13*a^2*b*c*d^2 + 3*a^3*d^3 + 6*(b^3*c*d^2 + 5*a*b^2*d^3)*x^2 -
 2*(b^3*c^2*d - 8*a*b^2*c*d^2 - 11*a^2*b*d^3)*x)/((b^7*c^4*d - 4*a*b^6*c^3
*d^2 + 6*a^2*b^5*c^2*d^3 - 4*a^3*b^4*c*d^4 + a^4*b^3*d^5)*g^4*i^2*x^4 + (b
^7*c^5 - a*b^6*c^4*d - 6*a^2*b^5*c^3*d^2 + 14*a^3*b^4*c^2*d^3 - 11*a^4*b^3
*c*d^4 + 3*a^5*b^2*d^5)*g^4*i^2*x^3 + 3*(a*b^6*c^5 - 3*a^2*b^5*c^4*d + 2*a
^3*b^4*c^3*d^2 + 2*a^4*b^3*c^2*d^3 - 3*a^5*b^2*c*d^4 + a^6*b*d^5)*g^4*i...
```

### 3.201.8 Giac [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^2,x
, algorithm="giac")
```

output
```
Timed out
```

### 3.201.9 Mupad [B] (verification not implemented)

Time = 8.35 (sec) , antiderivative size = 3157, normalized size of antiderivative = 4.33

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx=\text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^4*(c*i + d*i*x
)^2),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((x*(8*A*B*b^2*c*d - 8*A*B*a*b*d^2 + 12*B^2
*b*d*n*(a*d + b*c) + (16*B^2*a*b*d^2*n)/3 - (16*B^2*b^2*c*d*n)/3) - 6*A*B*
a^2*d^2 + 2*A*B*b^2*c^2 + 6*B^2*a^2*d^2*n + (2*B^2*b^2*c^2*n)/3 + 12*B^2*b
^2*d^2*n*x^2 + 4*A*B*a*b*c*d + (16*B^2*a*b*c*d*n)/3)/(x*(3*a^6*d^4*g^4*i^2
 - 9*a^2*b^4*c^4*g^4*i^2 + 24*a^3*b^3*c^3*d*g^4*i^2 - 18*a^4*b^2*c^2*d^2*g
^4*i^2) - x^2*(9*a*b^5*c^4*g^4*i^2 - 9*a^5*b*d^4*g^4*i^2 - 18*a^2*b^4*c^3*
d*g^4*i^2 + 18*a^4*b^2*c*d^3*g^4*i^2) - x^3*(3*b^6*c^4*g^4*i^2 - 9*a^4*b^2
*d^4*g^4*i^2 + 24*a^3*b^3*c*d^3*g^4*i^2 - 18*a^2*b^4*c^2*d^2*g^4*i^2) + x^
4*(3*a^3*b^3*d^4*g^4*i^2 - 3*b^6*c^3*d*g^4*i^2 + 9*a*b^5*c^2*d^2*g^4*i^2 -
 9*a^2*b^4*c*d^3*g^4*i^2) - 3*a^3*b^3*c^4*g^4*i^2 + 3*a^6*c*d^3*g^4*i^2 +
9*a^4*b^2*c^3*d*g^4*i^2 - 9*a^5*b*c^2*d^2*g^4*i^2) - (4*d^3*(6*A*B*b + 5*B
^2*b*n)*(x*((a*d + b*c)*((3*a*g^4*i^2*n*(a*d - b*c)^4)/(2*d) + (3*g^4*i^2*
n*(a*d - b*c)^4*(2*a*d - b*c))/(2*d^2)) + (3*a*b*c*g^4*i^2*n*(a*d - b*c)^4
)/d) + x^2*(b*d*((3*a*g^4*i^2*n*(a*d - b*c)^4)/(2*d) + (3*g^4*i^2*n*(a*d -
 b*c)^4*(2*a*d - b*c))/(2*d^2)) + (3*b*g^4*i^2*n*(a*d + b*c)*(a*d - b*c)^4
)/d) + a*c*((3*a*g^4*i^2*n*(a*d - b*c)^4)/(2*d) + (3*g^4*i^2*n*(a*d - b*c)
^4*(2*a*d - b*c))/(2*d^2)) + 3*b^2*g^4*i^2*n*x^3*(a*d - b*c)^4))/(3*g^4*i^
2*n*(a*d - b*c)^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(x*(3*a^6*d^4*g^4*i^2 -
9*a^2*b^4*c^4*g^4*i^2 + 24*a^3*b^3*c^3*d*g^4*i^2 - 18*a^4*b^2*c^2*d^2*g^4*
i^2) - x^2*(9*a*b^5*c^4*g^4*i^2 - 9*a^5*b*d^4*g^4*i^2 - 18*a^2*b^4*c^3*...
```

3.201. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^2}\,dx$

# 3.202 $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

### 3.202.1 Optimal result

Integrand size = 45, antiderivative size = 676

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{B^2(bc-ad)g^3n^2(a+bx)^2}{4d^2i^3(c+dx)^2} - \frac{4AbB(bc-ad)g^3n(a+bx)}{d^3i^3(c+dx)}$$
$$+ \frac{4bB^2(bc-ad)g^3n^2(a+bx)}{d^3i^3(c+dx)} - \frac{4bB^2(bc-ad)g^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^3i^3(c+dx)}$$
$$- \frac{B(bc-ad)g^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2d^2i^3(c+dx)^2}$$
$$+ \frac{b^2g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^3} + \frac{(bc-ad)g^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2d^2i^3(c+dx)^2}$$
$$+ \frac{2b(bc-ad)g^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^3i^3(c+dx)}$$
$$+ \frac{2b^2B(bc-ad)g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3}$$
$$+ \frac{3b^2(bc-ad)g^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^4i^3}$$
$$+ \frac{2b^2B^2(bc-ad)g^3n^2\,\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}$$
$$+ \frac{6b^2B(bc-ad)g^3n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\mathrm{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}$$
$$- \frac{6b^2B^2(bc-ad)g^3n^2\,\mathrm{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^4i^3}$$

output
```
1/4*B^2*(-a*d+b*c)*g^3*n^2*(b*x+a)^2/d^2/i^3/(d*x+c)^2-4*A*b*B*(-a*d+b*c)*
g^3*n*(b*x+a)/d^3/i^3/(d*x+c)+4*b*B^2*(-a*d+b*c)*g^3*n^2*(b*x+a)/d^3/i^3/(
d*x+c)-4*b*B^2*(-a*d+b*c)*g^3*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/d^3/i^3/
(d*x+c)-1/2*B*(-a*d+b*c)*g^3*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/d
^2/i^3/(d*x+c)^2+b^2*g^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^3/i^3
+1/2*(-a*d+b*c)*g^3*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^2/i^3/(d
*x+c)^2+2*b*(-a*d+b*c)*g^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/d^3/i
^3/(d*x+c)+2*b^2*B*(-a*d+b*c)*g^3*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*ln((-a
*d+b*c)/b/(d*x+c))/d^4/i^3+3*b^2*(-a*d+b*c)*g^3*(A+B*ln(e*((b*x+a)/(d*x+c)
)^n))^2*ln((-a*d+b*c)/b/(d*x+c))/d^4/i^3+2*b^2*B^2*(-a*d+b*c)*g^3*n^2*poly
log(2,d*(b*x+a)/b/(d*x+c))/d^4/i^3+6*b^2*B*(-a*d+b*c)*g^3*n*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))*polylog(2,d*(b*x+a)/b/(d*x+c))/d^4/i^3-6*b^2*B^2*(-a*d+b
*c)*g^3*n^2*polylog(3,d*(b*x+a)/b/(d*x+c))/d^4/i^3
```

### 3.202.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 6878 vs. $2(676) = 1352$.

Time = 7.02 (sec) , antiderivative size = 6878, normalized size of antiderivative = 10.17

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i
+ d*i*x)^3,x]
```

output
```
Result too large to show
```

### 3.202.3 Rubi [A] (verified)

Time = 0.75 (sec) , antiderivative size = 545, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^3 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^3}\,dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{g^3(bc-ad)\int \frac{(a+bx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^2}d\frac{a+bx}{c+dx}}{i^3}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{g^3(bc-ad)\int \left(\frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^3}{d^3\left(\frac{d(a+bx)}{c+dx}-b\right)^2}+\frac{3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^2}{d^3\left(\frac{d(a+bx)}{c+dx}-b\right)}+\frac{2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b}{d^3}+\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{d^2(c+dx)}\right.}{i^3}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{g^3(bc-ad)\left(\frac{6b^2Bn\,\text{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^4}+\frac{3b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^4}+\frac{2b^2Bn\log\left(1-\frac{d(a+}{b(c+}\right.}{}\right.}{}$$

input
```
Int[((a*g + b*g*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*
x)^3,x]
```

output
```
((b*c - a*d)*g^3*((B^2*n^2*(a + b*x)^2)/(4*d^2*(c + d*x)^2) - (4*A*b*B*n*(
a + b*x))/(d^3*(c + d*x)) + (4*b*B^2*n^2*(a + b*x))/(d^3*(c + d*x)) - (4*b
*B^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d^3*(c + d*x)) - (B*n*(a
 + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d^2*(c + d*x)^2) + ((
a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*d^2*(c + d*x)^2) +
 (2*b*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^3*(c + d*x))
+ (b^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^3*(c + d*x)*
(b - (d*(a + b*x))/(c + d*x))) + (2*b^2*B*n*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 + (3*b^2*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^4 + (2*
b^2*B^2*n^2*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))])/d^4 + (6*b^2*B*n*(A +
 B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x))/(b*(c + d*x))]
)/d^4 - (6*b^2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*x))])/d^4))/i^3
```

3.202. $\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

### 3.202.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.202.4 Maple [F]

$$\int \frac{(bgx+ag)^3 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(dix+ci)^3}dx$$

input
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

output
```
int((b*g*x+a*g)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

## 3.202.5 Fricas [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x
, algorithm="fricas")
```

output
```
integral((A^2*b^3*g^3*x^3 + 3*A^2*a*b^2*g^3*x^2 + 3*A^2*a^2*b*g^3*x + A^2*
a^3*g^3 + (B^2*b^3*g^3*x^3 + 3*B^2*a*b^2*g^3*x^2 + 3*B^2*a^2*b*g^3*x + B^2
*a^3*g^3)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(A*B*b^3*g^3*x^3 + 3*A*B*a*
b^2*g^3*x^2 + 3*A*B*a^2*b*g^3*x + A*B*a^3*g^3)*log(e*((b*x + a)/(d*x + c))
^n))/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

### 3.202.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**3*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**
3,x)
```

output
```
Timed out
```

### 3.202.7 Maxima [F]

$$\int \frac{(ag+bgx)^3 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x
, algorithm="maxima")
```

output
```
3/2*A*B*a^2*b*g^3*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 -
 a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*
i^3) + 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*
d^4)*i^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 +
 a^2*d^4)*i^3)) + 1/2*A*B*a^3*g^3*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a
*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3)
 + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*lo
g(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*A^2*b^3*g^3*((
6*c^2*d*x + 5*c^3)/(d^6*i^3*x^2 + 2*c*d^5*i^3*x + c^2*d^4*i^3) - 2*x/(d^3*
i^3) + 6*c*log(d*x + c)/(d^4*i^3)) + 3/2*A^2*a*b^2*g^3*((4*c*d*x + 3*c^2)/
(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x + c)/(d^3*i^3)) -
3*(2*d*x + c)*A*B*a^2*b*g^3*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)/(d^4*i^
3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - 3/2*(2*d*x + c)*A^2*a^2*b*g^3/(d^4*
i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - A*B*a^3*g^3*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A^2*a^3*
g^3/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 1/2*(2*B^2*b^3*d^3*g^3*x^3
 + 4*B^2*b^3*c*d^2*g^3*x^2 - 2*(2*b^3*c^2*d*g^3 - 6*a*b^2*c*d^2*g^3 + 3*a^
2*b*d^3*g^3)*B^2*x - (5*b^3*c^3*g^3 - 9*a*b^2*c^2*d*g^3 + 3*a^2*b*c*d^2*g^
3 + a^3*d^3*g^3)*B^2 - 6*((b^3*c*d^2*g^3 - a*b^2*d^3*g^3)*B^2*x^2 + 2*(b^3
*c^2*d*g^3 - a*b^2*c*d^2*g^3)*B^2*x + (b^3*c^3*g^3 - a*b^2*c^2*d*g^3)*B...
```

### 3.202.8 Giac [F]

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^3\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^3*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x
, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^3*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x
+ c*i)^3, x)
```

### 3.202.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{\left(a\,g+b\,g\,x\right)^3\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{\left(c\,i+d\,i\,x\right)^3}\,dx$$

input
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^3,x)
```

output
```
int(((a*g + b*g*x)^3*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^3, x)
```

# 3.203 $\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$



## 3.203.1 Optimal result

Integrand size = 45, antiderivative size = 441

$$\begin{aligned}
&\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx \\
&= -\frac{B^2g^2n^2(a+bx)^2}{4di^3(c+dx)^2}+\frac{2AbBg^2n(a+bx)}{d^2i^3(c+dx)}-\frac{2bB^2g^2n^2(a+bx)}{d^2i^3(c+dx)} \\
&\quad+\frac{2bB^2g^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{d^2i^3(c+dx)}+\frac{Bg^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2di^3(c+dx)^2} \\
&\quad-\frac{g^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2di^3(c+dx)^2}-\frac{bg^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2i^3(c+dx)} \\
&\quad-\frac{b^2g^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(\frac{bc-ad}{b(c+dx)}\right)}{d^3i^3} \\
&\quad-\frac{2b^2Bg^2n\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3} \\
&\quad+\frac{2b^2B^2g^2n^2\operatorname{PolyLog}\left(3,\frac{d(a+bx)}{b(c+dx)}\right)}{d^3i^3}
\end{aligned}$$

output
```
-1/4*B^2*g^2*n^2*(b*x+a)^2/d/i^3/(d*x+c)^2+2*A*b*B*g^2*n*(b*x+a)/d^2/i^3/(
d*x+c)-2*b*B^2*g^2*n^2*(b*x+a)/d^2/i^3/(d*x+c)+2*b*B^2*g^2*n*(b*x+a)*ln(e*
((b*x+a)/(d*x+c))^n)/d^2/i^3/(d*x+c)+1/2*B*g^2*n*(b*x+a)^2*(A+B*ln(e*((b*x
+a)/(d*x+c))^n))/d/i^3/(d*x+c)^2-1/2*g^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x
+c))^n))^2/d/i^3/(d*x+c)^2-b*g^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2
/d^2/i^3/(d*x+c)-b^2*g^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2*ln((-a*d+b*c)/b
/(d*x+c))/d^3/i^3-2*b^2*B*g^2*n*(A+B*ln(e*((b*x+a)/(d*x+c))^n))*polylog(2,
d*(b*x+a)/b/(d*x+c))/d^3/i^3+2*b^2*B^2*g^2*n^2*polylog(3,d*(b*x+a)/b/(d*x+
c))/d^3/i^3
```

### 3.203.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 3622 vs. 2(441) = 882.

Time = 4.78 (sec) , antiderivative size = 3622, normalized size of antiderivative = 8.21

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i
+ d*i*x)^3,x]
```

output
```
(g^2*((-6*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a
 + b*x)/(c + d*x)])^2)/(c + d*x)^2 + (24*b*(b*c - a*d)*(A + B*Log[e*((a +
b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2)/(c + d*x) + 12*b^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c + d*x)])^2*Log
[c + d*x] + (12*a*b*B*d*n*(-A - B*Log[e*((a + b*x)/(c + d*x))^n] + B*n*Log
[(a + b*x)/(c + d*x)])*(-(b^2*c^3) + 4*a*b*c^2*d - 3*a^2*c*d^2 - 2*b^2*c^2
*d*x + 6*a*b*c*d^2*x - 4*a^2*d^3*x - 2*b*(b*c - 2*a*d)*(c + d*x)^2*Log[a +
 b*x] + 2*(b*c - a*d)^2*(c + 2*d*x)*Log[(a + b*x)/(c + d*x)] + 2*b^2*c^3*L
og[c + d*x] - 4*a*b*c^2*d*Log[c + d*x] + 4*b^2*c^2*d*x*Log[c + d*x] - 8*a*
b*c*d^2*x*Log[c + d*x] + 2*b^2*c*d^2*x^2*Log[c + d*x] - 4*a*b*d^3*x^2*Log[
c + d*x]))/((b*c - a*d)^2*(c + d*x)^2) + (6*a^2*B*d^2*n*(-A - B*Log[e*((a
+ b*x)/(c + d*x))^n] + B*n*Log[(a + b*x)/(c + d*x)])*(-(b^2*c^2) + 4*a*b*c
*d - a^2*d^2 + 2*b^2*c*d*x + 2*a*b*d^2*x + 2*b^2*d^2*x^2 - 2*b^2*(c + d*x)
^2*Log[a/b + x] + 2*(b*c - a*d)^2*Log[(a + b*x)/(c + d*x)] + 2*b^2*c^2*Log
[(b*(c + d*x))/(b*c - a*d)] + 4*b^2*c*d*x*Log[(b*(c + d*x))/(b*c - a*d)] +
 2*b^2*d^2*x^2*Log[(b*(c + d*x))/(b*c - a*d)]))/((b*c - a*d)^2*(c + d*x)^2
) + 6*b^2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c
 + d*x)])*(-2*Log[c/d + x]^2 - (8*c*(1 + Log[c/d + x]))/(c + d*x) + (c^2*(
1 + 2*Log[c/d + x]))/(c + d*x)^2 + 8*c*(Log[a/b + x]/(c + d*x) + (b*(Log[a
 + b*x] - Log[c + d*x]))/(-(b*c) + a*d)) + 2*(-Log[a/b + x] + Log[c/d +...
```

### 3.203.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 389, normalized size of antiderivative = 0.88, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^2 \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^3}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{g^2\int \frac{(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{i^3}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{g^2\int\left(-\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d(c+dx)}-\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2\left(\frac{d(a+bx)}{c+dx}-b\right)}-\frac{b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{d^2}\right)d\frac{a+bx}{c+dx}}{i^3}$$

$$\Big\downarrow \text{2009}$$

$$g^2\left(-\frac{2b^2Bn\operatorname{PolyLog}\left(2,\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{d^3}-\frac{b^2\log\left(1-\frac{d(a+bx)}{b(c+dx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{d^3}-\frac{b(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{d^2(c+dx)}\right.$$

input
```
Int[((a*g + b*g*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*
x)^3,x]
```

output
```
(g^2*(-1/4*(B^2*n^2*(a + b*x)^2)/(d*(c + d*x)^2) + (2*A*b*B*n*(a + b*x))/(
d^2*(c + d*x)) - (2*b*B^2*n^2*(a + b*x))/(d^2*(c + d*x)) + (2*b*B^2*n*(a +
 b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(d^2*(c + d*x)) + (B*n*(a + b*x)^2*(
A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*d*(c + d*x)^2) - ((a + b*x)^2*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*d*(c + d*x)^2) - (b*(a + b*x)*(
A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(d^2*(c + d*x)) - (b^2*(A + B*Log
[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - (d*(a + b*x))/(b*(c + d*x))])/d^3 -
 (2*b^2*B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, (d*(a + b*x)
)/(b*(c + d*x))])/d^3 + (2*b^2*B^2*n^2*PolyLog[3, (d*(a + b*x))/(b*(c + d*
x))])/d^3))/i^3
```

### 3.203.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.203.4 Maple [F]

$$\int \frac{(bgx+ag)^2 \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}{(dix+ci)^3}dx$$

input
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

output
```
int((b*g*x+a*g)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x)
```

### 3.203.5 Fricas [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x
, algorithm="fricas")
```

output
```
integral((A^2*b^2*g^2*x^2 + 2*A^2*a*b*g^2*x + A^2*a^2*g^2 + (B^2*b^2*g^2*x
^2 + 2*B^2*a*b*g^2*x + B^2*a^2*g^2)*log(e*((b*x + a)/(d*x + c))^n)^2 + 2*(
A*B*b^2*g^2*x^2 + 2*A*B*a*b*g^2*x + A*B*a^2*g^2)*log(e*((b*x + a)/(d*x + c
))^n))/(d^3*i^3*x^3 + 3*c*d^2*i^3*x^2 + 3*c^2*d*i^3*x + c^3*i^3), x)
```

### 3.203.6 Sympy [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \frac{g^2\left(\int \frac{A^2a^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{A^2b^2x^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{B^2a^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx+\int \frac{2ABa^2\log\left(e\left(\frac{a}{c+dx}\right.\right.}{c^3+3c^2dx+3cd^2\ldots}\right.}{\ldots}$$

input
```
integrate((b*g*x+a*g)**2*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**
3,x)
```

output
```
g**2*(Integral(A**2*a**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x) + Integral(A**2*b**2*x**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**
3), x) + Integral(B**2*a**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c*
*3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*a**2*log
(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 +
d**3*x**3), x) + Integral(2*A**2*a*b*x/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2
+ d**3*x**3), x) + Integral(B**2*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d
*x))**n)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral
(2*A*B*b**2*x**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*
x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*B**2*a*b*x*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3
), x) + Integral(4*A*B*a*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)/(c**3
 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

### 3.203.7 Maxima [F]

$$\int \frac{(ag+bgx)^2 \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x
, algorithm="maxima")
```

output
```
A*B*a*b*g^2*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d^5
)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3) +
 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i
^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d
^4)*i^3)) + 1/2*A*B*a^2*g^2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*
i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b
^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x
+ c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) + 1/2*A^2*b^2*g^2*((4*c*d*
x + 3*c^2)/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) + 2*log(d*x + c)/(d
^3*i^3)) - 2*(2*d*x + c)*A*B*a*b*g^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^n
)/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - (2*d*x + c)*A^2*a*b*g^2/(d
^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^2*i^3) - A*B*a^2*g^2*log(e*(b*x/(d*x +
c) + a/(d*x + c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A^2*a
^2*g^2/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) + 1/2*(4*(b^2*c*d*g^2 - a
*b*d^2*g^2)*B^2*x + (3*b^2*c^2*g^2 - 2*a*b*c*d*g^2 - a^2*d^2*g^2)*B^2 + 2*
(B^2*b^2*d^2*g^2*x^2 + 2*B^2*b^2*c*d*g^2*x + B^2*b^2*c^2*g^2)*log(d*x + c)
)*log((d*x + c)^n)^2/(d^5*i^3*x^2 + 2*c*d^4*i^3*x + c^2*d^3*i^3) - integra
te(-(2*B^2*a*b*d^2*g^2*x*log(e)^2 + B^2*a^2*d^2*g^2*log(e)^2 + (B^2*b^2*d^
2*g^2*log(e)^2 + 2*A*B*b^2*d^2*g^2*log(e))*x^2 + (B^2*b^2*d^2*g^2*x^2 + 2*
B^2*a*b*d^2*g^2*x + B^2*a^2*d^2*g^2)*log((b*x + a)^n)^2 + 2*(2*B^2*a*b*...
```

### 3.203.8 Giac [F]

$$\int \frac{(ag+bgx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx=\int \frac{(bgx+ag)^2\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(dix+ci)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^2*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x
, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^2*(B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/(d*i*x
+ c*i)^3, x)
```

### 3.203.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^2 \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \int \frac{\left(a\,g+b\,g\,x\right)^2\,\left(A+B\,\ln \left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{\left(c\,i+d\,i\,x\right)^3}\,dx$$

input
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^3,x)
```

output
```
int(((a*g + b*g*x)^2*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^3, x)
```

# 3.204 $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$



## 3.204.1 Optimal result

Integrand size = 43, antiderivative size = 151

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \frac{B^2gn^2(a+bx)^2}{4(bc-ad)i^3(c+dx)^2} - \frac{Bgn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)i^3(c+dx)^2} + \frac{g(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)i^3(c+dx)^2}$$

output
```
1/4*B^2*g*n^2*(b*x+a)^2/(-a*d+b*c)/i^3/(d*x+c)^2-1/2*B*g*n*(b*x+a)^2*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^3/(d*x+c)^2+1/2*g*(b*x+a)^2*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^3/(d*x+c)^2
```

## 3.204.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.56 (sec) , antiderivative size = 803, normalized size of antiderivative = 5.32

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$

$$= \frac{g\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 - 4b(bc-ad)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 + 4bBn(c+dx)\left(\right.\right.}{}$$

input
```
Integrate[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i +
d*i*x)^3,x]
```

output
```
(g*(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 - 4*b*(b*c -
a*d)*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + 4*b*B*n*(c + d*x
)*(2*(b*c - a*d)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 2*b*(c + d*x)*Lo
g[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 2*b*(c + d*x)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 2*B*n*(b*c - a*d + b*(c + d*
x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - b*B*n*(c + d*x)*(Log[a + b*x
]*(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a +
 b*x))/(-(b*c) + a*d)]) + b*B*n*(c + d*x)*((2*Log[(d*(a + b*x))/(-(b*c) +
a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d
)])) - B*n*(2*(b*c - a*d)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(
b*c - a*d)*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d
*x)^2*Log[a + b*x]*(A + B*Log[e*((a + b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x
)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[c + d*x] - 4*b*B*n*(c + d*x
)*(b*c - a*d + b*(c + d*x)*Log[a + b*x] - b*(c + d*x)*Log[c + d*x]) - B*n*
((b*c - a*d)^2 + 2*b*(b*c - a*d)*(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x
] - 2*b^2*(c + d*x)^2*Log[c + d*x]) - 2*b^2*B*n*(c + d*x)^2*(Log[a + b*x]*
(Log[a + b*x] - 2*Log[(b*(c + d*x))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b
*x))/(-(b*c) + a*d)]) + 2*b^2*B*n*(c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c
) + a*d)] - Log[c + d*x])*Log[c + d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c -
 a*d)]))))/(4*d^2*(b*c - a*d)*i^3*(c + d*x)^2)
```

### 3.204.3 Rubi [A] (verified)

Time = 0.33 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.81, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.070, Rules used = {2961, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^3}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{g\int \frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{c+dx}d\frac{a+bx}{c+dx}}{i^3(bc-ad)}$$

$$\Big\downarrow \text{2742}$$

$$\frac{g\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(c+dx)^2}-Bn\int\frac{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{c+dx}d\frac{a+bx}{c+dx}\right)}{i^3(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{g\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(c+dx)^2}-Bn\left(\frac{(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}-\frac{Bn(a+bx)^2}{4(c+dx)^2}\right)\right)}{i^3(bc-ad)}$$

input
```
Int[((a*g + b*g*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c*i + d*i*x)
^3,x]
```

output
```
(g*(((a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(c + d*x)^2)
 - B*n*(-1/4*(B*n*(a + b*x)^2)/(c + d*x)^2 + ((a + b*x)^2*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]))/(2*(c + d*x)^2))))/((b*c - a*d)*i^3)
```

### 3.204.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

---

3.204. $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

### 3.204.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 529 vs. 2(145) = 290.

Time = 4.68 (sec) , antiderivative size = 530, normalized size of antiderivative = 3.51

| method | result |
|---|---|
| parallelrisch | $-\frac{8ABx\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a b^{2}d^{4}gn+4ABx b^{3}c d^{3}g n^{2}+4AB\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)a^{2}b d^{4}gn+4AB x^{2}\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)b^{3}d^{4}gn+4B^{2}x\ln}{}$ |

input
```
int((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,method=_
RETURNVERBOSE)
```

output
```
-1/4*(8*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^4*g*n+4*A*B*x*b^3*c*d^3*g*
n^2+4*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^4*g*n+4*A*B*x^2*ln(e*((b*x+a)/
(d*x+c))^n)*b^3*d^4*g*n+4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^2*d^4*g*n-
4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*d^4*g*n^2-4*A*B*x*a*b^2*d^4*g*n^2+
B^2*a^2*b*d^4*g*n^3-B^2*b^3*c^2*d^2*g*n^3+2*A^2*a^2*b*d^4*g*n-2*A^2*b^3*c^
2*d^2*g*n-2*A*B*a^2*b*d^4*g*n^2+2*A*B*b^3*c^2*d^2*g*n^2+2*B^2*x^2*ln(e*((b
*x+a)/(d*x+c))^n)^2*b^3*d^4*g*n-2*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^
4*g*n^2+2*B^2*x*a*b^2*d^4*g*n^3-2*B^2*x*b^3*c*d^3*g*n^3+2*B^2*ln(e*((b*x+a
)/(d*x+c))^n)^2*a^2*b*d^4*g*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^4*g*
n^2+4*A^2*x*a*b^2*d^4*g*n-4*A^2*x*b^3*c*d^3*g*n)/i^3/(d*x+c)^2/b/d^4/n/(a*
d-b*c)
```

### 3.204.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 600 vs. 2(145) = 290.

Time = 0.35 (sec) , antiderivative size = 600, normalized size of antiderivative = 3.97

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{2}}{(ci+dix)^{3}}\,dx =$$
$$-\frac{(B^{2}b^{2}c^{2}-B^{2}a^{2}d^{2})gn^{2}-2\left(ABb^{2}c^{2}-ABa^{2}d^{2}\right)gn+2\left(2\left(B^{2}b^{2}cd-B^{2}abd^{2}\right)gx+\left(B^{2}b^{2}c^{2}-B^{2}a^{2}d^{2}\right)g\right)}{}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output
```
-1/4*((B^2*b^2*c^2 - B^2*a^2*d^2)*g*n^2 - 2*(A*B*b^2*c^2 - A*B*a^2*d^2)*g*
n + 2*(2*(B^2*b^2*c*d - B^2*a*b*d^2)*g*x + (B^2*b^2*c^2 - B^2*a^2*d^2)*g)*
log(e)^2 - 2*(B^2*b^2*d^2*g*n^2*x^2 + 2*B^2*a*b*d^2*g*n^2*x + B^2*a^2*d^2*
g*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A^2*b^2*c^2 - A^2*a^2*d^2)*g + 2*((
B^2*b^2*c*d - B^2*a*b*d^2)*g*n^2 - 2*(A*B*b^2*c*d - A*B*a*b*d^2)*g*n + 2*(
A^2*b^2*c*d - A^2*a*b*d^2)*g)*x - 2*((B^2*b^2*c^2 - B^2*a^2*d^2)*g*n - 2*(
A*B*b^2*c^2 - A*B*a^2*d^2)*g + 2*((B^2*b^2*c*d - B^2*a*b*d^2)*g*n - 2*(A*B
*b^2*c*d - A*B*a*b*d^2)*g)*x + 2*(B^2*b^2*d^2*g*n*x^2 + 2*B^2*a*b*d^2*g*n*
x + B^2*a^2*d^2*g*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*(B^2*a^2*d^2*g*n
^2 - 2*A*B*a^2*d^2*g*n + (B^2*b^2*d^2*g*n^2 - 2*A*B*b^2*d^2*g*n)*x^2 + 2*(
B^2*a*b*d^2*g*n^2 - 2*A*B*a*b*d^2*g*n)*x)*log((b*x + a)/(d*x + c)))/((b*c*
d^4 - a*d^5)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*
d^3)*i^3)
```

### 3.204.6 Sympy [F]

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{g\left(\int \frac{A^2a}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{A^2bx}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2a\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2ABa\log\left(e\left(\frac{a}{c+dx}+\right.\right.}{c^3+3c^2dx+3cd^2x^2+}\right.}{i^3}$$

input
```
integrate((b*g*x+a*g)*(A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**3,x
)
```

output
```
g*(Integral(A**2*a/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + I
ntegral(A**2*b*x/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Int
egral(B**2*a*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x
 + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*a*log(e*(a/(c + d*x) +
b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + I
ntegral(B**2*b*x*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2
*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*b*x*log(e*(a/(c + d
*x) + b*x/(c + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3),
x))/i**3
```

### 3.204.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1995 vs. 2(145) = 290.

Time = 0.31 (sec) , antiderivative size = 1995, normalized size of antiderivative = 13.21

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output
```
1/2*A*B*b*g*n*((b*c^2 - 3*a*c*d + 2*(b*c*d - 2*a*d^2)*x)/((b*c*d^4 - a*d^5
)*i^3*x^2 + 2*(b*c^2*d^3 - a*c*d^4)*i^3*x + (b*c^3*d^2 - a*c^2*d^3)*i^3) +
 2*(b^2*c - 2*a*b*d)*log(b*x + a)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*i
^3) - 2*(b^2*c - 2*a*b*d)*log(d*x + c)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d
^4)*i^3)) + 1/2*A*B*a*g*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*
x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*l
og(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)
/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3)) - 1/2*(2*d*x + c)*B^2*b*g*log(
e*(b*x/(d*x + c) + a/(d*x + c))^n)^2/(d^4*i^3*x^2 + 2*c*d^3*i^3*x + c^2*d^
2*i^3) + 1/4*(2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*
(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x +
 a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c
^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)
 - (7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c
^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(d*x + c)^
2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(
b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*n^2/(b^2*c^4*d*i^3 - 2*a
*b*c^3*d^2*i^3 + a^2*c^2*d^3*i^3 + (b^2*c^2*d^3*i^3 - 2*a*b*c*d^4*i^3 + a^
2*d^5*i^3)*x^2 + 2*(b^2*c^3*d^2*i^3 - 2*a*b*c^2*d^3*i^3 + a^2*c*d^4*i^3...
```

### 3.204.8 Giac [A] (verification not implemented)

Time = 1.91 (sec) , antiderivative size = 195, normalized size of antiderivative = 1.29

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{1}{4}\left(\frac{2\,(bx+a)^2B^2gn^2\log\left(\frac{bx+a}{dx+c}\right)^2}{(dx+c)^2i^3} - \frac{2\,(B^2gn^2-2\,B^2gn\log(e)-2\,ABgn)(bx+a)^2\log\left(\frac{bx+a}{dx+c}\right)}{(dx+c)^2i^3} + \frac{(B^2gn^2}{}\right.$$

input
```
integrate((b*g*x+a*g)*(A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output
```
1/4*(2*(b*x + a)^2*B^2*g*n^2*log((b*x + a)/(d*x + c))^2/((d*x + c)^2*i^3)
- 2*(B^2*g*n^2 - 2*B^2*g*n*log(e) - 2*A*B*g*n)*(b*x + a)^2*log((b*x + a)/(
d*x + c))/((d*x + c)^2*i^3) + (B^2*g*n^2 - 2*B^2*g*n*log(e) + 2*B^2*g*log(
e)^2 - 2*A*B*g*n + 4*A*B*g*log(e) + 2*A^2*g)*(b*x + a)^2/((d*x + c)^2*i^3)
)*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

## 3.204.9 Mupad [B] (verification not implemented)

Time = 4.17 (sec) , antiderivative size = 565, normalized size of antiderivative = 3.74

$$\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{\frac{B^2\,a\,g}{2\,d}+\frac{B^2\,b\,g\,x}{d}+\frac{B^2\,b\,c\,g}{2\,d^2}}{c^2\,i^3+2\,c\,d\,i^3\,x+d^2\,i^3\,x^2}+\frac{B^2\,b^2\,g}{2\,d^2\,i^3\,\left(a\,d-b\,c\right)}\right)$$
$$-\frac{x\left(2\,b\,d\,g\,A^2-2\,b\,d\,g\,A\,B\,n+b\,d\,g\,B^2\,n^2\right)+A^2\,a\,d\,g+A^2\,b\,c\,g+\frac{B^2\,a\,d\,g\,n^2}{2}+\frac{B^2\,b\,c\,g\,n^2}{2}-A\,B\,a\,d\,g\,n-A}{2\,c^2\,d^2\,i^3+4\,c\,d^3\,i^3\,x+2\,d^4\,i^3\,x^2}$$
$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B\,a\,d\,g+A\,B\,b\,c\,g-B^2\,a\,d\,g\,n+B^2\,b\,c\,g\,n+2\,A\,B\,b\,d\,g\,x}{c^2\,d^2\,i^3+2\,c\,d^3\,i^3\,x+d^4\,i^3\,x^2}\right.$$
$$\left.-\frac{B^2\,b^2\,g\left(\frac{c\,d^2\,i^3\,n\,(a\,d-b\,c)}{2\,b}+\frac{d^3\,i^3\,n\,x\,(a\,d-b\,c)}{b}-\frac{d^2\,i^3\,n\,(a\,d-b\,c)\,(a\,d-2\,b\,c)}{2\,b^2}\right)}{d^2\,i^3\,\left(a\,d-b\,c\right)\,\left(c^2\,d^2\,i^3+2\,c\,d^3\,i^3\,x+d^4\,i^3\,x^2\right)}\right)$$
$$-\frac{B\,b^2\,g\,n\,\mathrm{atan}\left(\frac{B\,b^2\,g\,n\,(2\,A-B\,n)\left(\frac{a\,d^3\,i^3+b\,c\,d^2\,i^3}{d^2\,i^3}+2\,b\,d\,x\right)1\mathrm{i}}{(a\,d-b\,c)\,(B^2\,b^2\,g\,n^2-2\,A\,B\,b^2\,g\,n)}\right)\,\left(2\,A-B\,n\right)\,1\mathrm{i}}{d^2\,i^3\,\left(a\,d-b\,c\right)}$$

input
```
int(((a*g + b*g*x)*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*x)
^3,x)
```

output
```
- log(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*a*g)/(2*d) + (B^2*b*g*x)/d + (B^
2*b*c*g)/(2*d^2))/(c^2*i^3 + d^2*i^3*x^2 + 2*c*d*i^3*x) + (B^2*b^2*g)/(2*d
^2*i^3*(a*d - b*c))) - (x*(2*A^2*b*d*g + B^2*b*d*g*n^2 - 2*A*B*b*d*g*n) +
A^2*a*d*g + A^2*b*c*g + (B^2*a*d*g*n^2)/2 + (B^2*b*c*g*n^2)/2 - A*B*a*d*g*
n - A*B*b*c*g*n)/(2*c^2*d^2*i^3 + 2*d^4*i^3*x^2 + 4*c*d^3*i^3*x) - log(e*(
(a + b*x)/(c + d*x))^n)*((A*B*a*d*g + A*B*b*c*g - B^2*a*d*g*n + B^2*b*c*g*
n + 2*A*B*b*d*g*x)/(c^2*d^2*i^3 + d^4*i^3*x^2 + 2*c*d^3*i^3*x) - (B^2*b^2*
g*((c*d^2*i^3*n*(a*d - b*c))/(2*b) + (d^3*i^3*n*x*(a*d - b*c))/b - (d^2*i^
3*n*(a*d - b*c)*(a*d - 2*b*c))/(2*b^2)))/(d^2*i^3*(a*d - b*c)*(c^2*d^2*i^3
 + d^4*i^3*x^2 + 2*c*d^3*i^3*x))) - (B*b^2*g*n*atan((B*b^2*g*n*(2*A - B*n)
*((a*d^3*i^3 + b*c*d^2*i^3)/(d^2*i^3) + 2*b*d*x)*1i)/((a*d - b*c)*(B^2*b^2
*g*n^2 - 2*A*B*b^2*g*n)))*(2*A - B*n)*1i)/(d^2*i^3*(a*d - b*c))
```

3.204. $\int \frac{(ag+bgx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

# 3.205 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$



## 3.205.1 Optimal result

Integrand size = 35, antiderivative size = 317

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = -\frac{B^2dn^2(a+bx)^2}{4(bc-ad)^2i^3(c+dx)^2} - \frac{2AbBn(a+bx)}{(bc-ad)^2i^3(c+dx)} + \frac{2bB^2n^2(a+bx)}{(bc-ad)^2i^3(c+dx)} - \frac{2bB^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^2i^3(c+dx)} + \frac{Bdn(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^2i^3(c+dx)^2} - \frac{d(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^2i^3(c+dx)^2} + \frac{b(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^2i^3(c+dx)}$$

output
```
-1/4*B^2*d*n^2*(b*x+a)^2/(-a*d+b*c)^2/i^3/(d*x+c)^2-2*A*b*B*n*(b*x+a)/(-a*
d+b*c)^2/i^3/(d*x+c)+2*b*B^2*n^2*(b*x+a)/(-a*d+b*c)^2/i^3/(d*x+c)-2*b*B^2*
n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^2/i^3/(d*x+c)+1/2*B*d*n*(b*
x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^2/i^3/(d*x+c)^2-1/2*d*(b
*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/i^3/(d*x+c)^2+b*(b*
x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^2/i^3/(d*x+c)
```

### 3.205.2 Mathematica [C] (verified)

Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.

Time = 0.22 (sec) , antiderivative size = 464, normalized size of antiderivative = 1.46

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{-2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2+\frac{Bn\left(2(bc-ad)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b(bc-ad)(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)+4b^2(c+dx)^2\log}{}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x)^3,x]
```

output
```
(-2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2 + (B*n*(2*(b*c - a*d)^2*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]) + 4*b*(b*c - a*d)*(c + d*x)*(A + B*Log[e
*((a + b*x)/(c + d*x))^n]) + 4*b^2*(c + d*x)^2*Log[a + b*x]*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]) - 4*b^2*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c +
d*x))^n])*Log[c + d*x] - 4*b*B*n*(c + d*x)*(b*c - a*d + b*(c + d*x)*Log[a
+ b*x] - b*(c + d*x)*Log[c + d*x]) - B*n*((b*c - a*d)^2 + 2*b*(b*c - a*d)*
(c + d*x) + 2*b^2*(c + d*x)^2*Log[a + b*x] - 2*b^2*(c + d*x)^2*Log[c + d*x
]) - 2*b^2*B*n*(c + d*x)^2*(Log[a + b*x]*(Log[a + b*x] - 2*Log[(b*(c + d*x
))/(b*c - a*d)]) - 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*d)]) + 2*b^2*B*n
*(c + d*x)^2*((2*Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[c + d*x])*Log[c +
 d*x] + 2*PolyLog[2, (b*(c + d*x))/(b*c - a*d)])))/(b*c - a*d)^2)/(4*d*i^3
*(c + d*x)^2)
```

### 3.205.3 Rubi [A] (verified)

Time = 0.35 (sec) , antiderivative size = 240, normalized size of antiderivative = 0.76, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2951, 2767, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ci+dix)^3}\,dx$$
$$\Big\downarrow \text{2951}$$

$$\frac{\int \left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+bx}{c+dx}}{i^3(bc-ad)^2}$$

$$\Big\downarrow \text{2767}$$

$$\frac{\int \left(b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{d(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{c+dx}\right) d\frac{a+bx}{c+dx}}{i^3(bc-ad)^2}$$

$$\Big\downarrow \text{2009}$$

$$\frac{\frac{Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}+\frac{b(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-\frac{d(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(c+dx)^2}-\frac{2AbBn(a+bx)}{c+dx}-\frac{2bB^2}{}}{i^3(bc-ad)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(c*i + d*i*x)^3,x]
```

output
```
(-1/4*(B^2*d*n^2*(a + b*x)^2)/(c + d*x)^2 - (2*A*b*B*n*(a + b*x))/(c + d*x
) + (2*b*B^2*n^2*(a + b*x))/(c + d*x) - (2*b*B^2*n*(a + b*x)*Log[e*((a + b
*x)/(c + d*x))^n])/(c + d*x) + (B*d*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(2*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^2)/(2*(c + d*x)^2) + (b*(a + b*x)*(A + B*Log[e*((a + b*x)/(c
 + d*x))^n])^2)/(c + d*x))/((b*c - a*d)^2*i^3)
```

### 3.205.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

3.205. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

### 3.205.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 671 vs. $2(311) = 622$.

Time = 4.86 (sec) , antiderivative size = 672, normalized size of antiderivative = 2.12

| method | result |
|---|---|
| parallelrisch | $-\frac{-8B^2ab^2cd^4n^3-2ABa^2bd^5n^2-6ABb^3c^2d^3n^2-4A^2ab^2cd^4n+2A^2b^3c^2d^3n+2A^2a^2bd^5n+7B^2b^3c^2d^3n^3+B^2a^2bd^5n^3-4}{}$ |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x,method=_RETURNVERBOS
E)
```

output
```
-1/4*(-8*B^2*a*b^2*c*d^4*n^3-2*A*B*a^2*b*d^5*n^2-6*A*B*b^3*c^2*d^3*n^2-4*A
^2*a*b^2*c*d^4*n+2*A^2*b^3*c^2*d^3*n+2*A^2*a^2*b*d^5*n+7*B^2*b^3*c^2*d^3*n
^3+B^2*a^2*b*d^5*n^3-4*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^3*d^5*n-4*B^2*x
*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*c*d^4*n+4*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)
*a*b^2*d^5*n^2+8*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*b^3*c*d^4*n^2+4*A*B*x*a*b
^2*d^5*n^2-4*A*B*x*b^3*c*d^4*n^2-4*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^2*c
*d^4*n+8*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n^2+4*A*B*ln(e*((b*x+a)
/(d*x+c))^n)*a^2*b*d^5*n+8*A*B*a*b^2*c*d^4*n^2-8*A*B*x*ln(e*((b*x+a)/(d*x+
c))^n)*b^3*c*d^4*n-8*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a*b^2*c*d^4*n-2*B^2*x^2
*ln(e*((b*x+a)/(d*x+c))^n)^2*b^3*d^5*n+6*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)
*b^3*d^5*n^2-6*B^2*x*a*b^2*d^5*n^3+6*B^2*x*b^3*c*d^4*n^3+2*B^2*ln(e*((b*x+
a)/(d*x+c))^n)^2*a^2*b*d^5*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b*d^5*n^2
)/i^3/(d*x+c)^2/n/(a^2*d^2-2*a*b*c*d+b^2*c^2)/b/d^4
```

### 3.205.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 654 vs. $2(311) = 622$.

Time = 0.33 (sec) , antiderivative size = 654, normalized size of antiderivative = 2.06

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = -\frac{2A^2b^2c^2-4A^2abcd+2A^2a^2d^2+(7B^2b^2c^2-8B^2abcd+B^2a^2d^2)n^2+2(B^2b^2c^2-2B^2abcd+B^2a^2d}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x, algorithm="f
ricas")
```

output
```
-1/4*(2*A^2*b^2*c^2 - 4*A^2*a*b*c*d + 2*A^2*a^2*d^2 + (7*B^2*b^2*c^2 - 8*B
^2*a*b*c*d + B^2*a^2*d^2)*n^2 + 2*(B^2*b^2*c^2 - 2*B^2*a*b*c*d + B^2*a^2*d
^2)*log(e)^2 - 2*(B^2*b^2*d^2*n^2*x^2 + 2*B^2*b^2*c*d*n^2*x + (2*B^2*a*b*c
*d - B^2*a^2*d^2)*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(3*A*B*b^2*c^2 - 4*A
*B*a*b*c*d + A*B*a^2*d^2)*n + 2*(3*(B^2*b^2*c*d - B^2*a*b*d^2)*n^2 - 2*(A*
B*b^2*c*d - A*B*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - 4*A*B*a*b*c*d + 2*A*B*a
^2*d^2 - 2*(B^2*b^2*c*d - B^2*a*b*d^2)*n*x - (3*B^2*b^2*c^2 - 4*B^2*a*b*c*
d + B^2*a^2*d^2)*n - 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*b^2*c*d*n*x + (2*B^2*a*b
*c*d - B^2*a^2*d^2)*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*((4*B^2*a*b*c*
d - B^2*a^2*d^2)*n^2 + (3*B^2*b^2*d^2*n^2 - 2*A*B*b^2*d^2*n)*x^2 - 2*(2*A*
B*a*b*c*d - A*B*a^2*d^2)*n - 2*(2*A*B*b^2*c*d*n - (2*B^2*b^2*c*d + B^2*a*b
*d^2)*n^2)*x)*log((b*x + a)/(d*x + c)))/((b^2*c^2*d^3 - 2*a*b*c*d^4 + a^2*
d^5)*i^3*x^2 + 2*(b^2*c^3*d^2 - 2*a*b*c^2*d^3 + a^2*c*d^4)*i^3*x + (b^2*c^
4*d - 2*a*b*c^3*d^2 + a^2*c^2*d^3)*i^3)
```

### 3.205.6 Sympy [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx = \frac{\int \frac{A^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{B^2\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)^2}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx + \int \frac{2AB\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}{c^3+3c^2dx+3cd^2x^2+d^3x^3}\,dx}{i^3}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(d*i*x+c*i)**3,x)
```

output
```
(Integral(A**2/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x) + Integ
ral(B**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)**2/(c**3 + 3*c**2*d*x + 3
*c*d**2*x**2 + d**3*x**3), x) + Integral(2*A*B*log(e*(a/(c + d*x) + b*x/(c
 + d*x))**n)/(c**3 + 3*c**2*d*x + 3*c*d**2*x**2 + d**3*x**3), x))/i**3
```

### 3.205.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 861 vs. $2(311) = 622$.

Time = 0.24 (sec) , antiderivative size = 861, normalized size of antiderivative = 2.72

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= \frac{1}{2}ABn\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}+\frac{2\,b^2\log(bx+a)}{(b^2c^2d-2\,abcd^2+a^2d^3)i^3}-\frac{2}{(b^2c^2d\cdots}\right.$$
$$+\frac{1}{4}\left(2\,n\left(\frac{2\,bdx+3\,bc-ad}{(bcd^3-ad^4)i^3x^2+2\,(bc^2d^2-acd^3)i^3x+(bc^3d-ac^2d^2)i^3}+\frac{2\,b^2\log(bx+a)}{(b^2c^2d-2\,abcd^2+a^2d^3)i^3}-\frac{}{(b^2c\cdots}\right.\right.$$
$$-\frac{B^2\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)^2}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}-\frac{AB\log\left(e\left(\frac{bx}{dx+c}+\frac{a}{dx+c}\right)^n\right)}{d^3i^3x^2+2\,cd^2i^3x+c^2di^3}-\frac{A^2}{2\,(d^3i^3x^2+2\,cd^2i^3x+c^2di^3)}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x, algorithm="m
axima")
```

output
```
1/2*A*B*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3 - a*d^4)*i^3*x^2 + 2*(b*c^2*d
^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i^3) + 2*b^2*log(b*x + a)/((b^
2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2*log(d*x + c)/((b^2*c^2*d - 2
*a*b*c*d^2 + a^2*d^3)*i^3)) + 1/4*(2*n*((2*b*d*x + 3*b*c - a*d)/((b*c*d^3
- a*d^4)*i^3*x^2 + 2*(b*c^2*d^2 - a*c*d^3)*i^3*x + (b*c^3*d - a*c^2*d^2)*i
^3) + 2*b^2*log(b*x + a)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3) - 2*b^2
*log(d*x + c)/((b^2*c^2*d - 2*a*b*c*d^2 + a^2*d^3)*i^3))*log(e*(b*x/(d*x +
 c) + a/(d*x + c))^n) - (7*b^2*c^2 - 8*a*b*c*d + a^2*d^2 + 2*(b^2*d^2*x^2
+ 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b
^2*c^2)*log(d*x + c)^2 + 6*(b^2*c*d - a*b*d^2)*x + 6*(b^2*d^2*x^2 + 2*b^2*
c*d*x + b^2*c^2)*log(b*x + a) - 2*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2
 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c))*n^2
/(b^2*c^4*d*i^3 - 2*a*b*c^3*d^2*i^3 + a^2*c^2*d^3*i^3 + (b^2*c^2*d^3*i^3 -
 2*a*b*c*d^4*i^3 + a^2*d^5*i^3)*x^2 + 2*(b^2*c^3*d^2*i^3 - 2*a*b*c^2*d^3*i
^3 + a^2*c*d^4*i^3)*x))*B^2 - 1/2*B^2*log(e*(b*x/(d*x + c) + a/(d*x + c))^
n)^2/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - A*B*log(e*(b*x/(d*x + c)
+ a/(d*x + c))^n)/(d^3*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3) - 1/2*A^2/(d^3
*i^3*x^2 + 2*c*d^2*i^3*x + c^2*d*i^3)
```

### 3.205.8 Giac [A] (verification not implemented)

Time = 1.09 (sec) , antiderivative size = 407, normalized size of antiderivative = 1.28

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$=\frac{1}{4}\left(2\left(\frac{2\,(bx+a)B^2bn^2}{(bci^3-adi^3)(dx+c)}-\frac{(bx+a)^2B^2dn^2}{(bci^3-adi^3)(dx+c)^2}\right)\log\left(\frac{bx+a}{dx+c}\right)^2+2\left(\frac{(B^2dn^2-2\,B^2dn\log(e)-}{(bci^3-adi^3)(d}\right.\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(d*i*x+c*i)^3,x, algorithm="g
iac")
```

output
```
1/4*(2*(2*(b*x + a)*B^2*b*n^2/((b*c*i^3 - a*d*i^3)*(d*x + c)) - (b*x + a)^
2*B^2*d*n^2/((b*c*i^3 - a*d*i^3)*(d*x + c)^2))*log((b*x + a)/(d*x + c))^2
+ 2*((B^2*d*n^2 - 2*B^2*d*n*log(e) - 2*A*B*d*n)*(b*x + a)^2/((b*c*i^3 - a*
d*i^3)*(d*x + c)^2) - 4*(B^2*b*n^2 - B^2*b*n*log(e) - A*B*b*n)*(b*x + a)/(
(b*c*i^3 - a*d*i^3)*(d*x + c)))*log((b*x + a)/(d*x + c)) - (B^2*d*n^2 - 2*
B^2*d*n*log(e) + 2*B^2*d*log(e)^2 - 2*A*B*d*n + 4*A*B*d*log(e) + 2*A^2*d)*
(b*x + a)^2/((b*c*i^3 - a*d*i^3)*(d*x + c)^2) + 4*(2*B^2*b*n^2 - 2*B^2*b*n
*log(e) + B^2*b*log(e)^2 - 2*A*B*b*n + 2*A*B*b*log(e) + A^2*b)*(b*x + a)/(
(b*c*i^3 - a*d*i^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)
```

### 3.205.9 Mupad [B] (verification not implemented)

Time = 3.17 (sec) , antiderivative size = 505, normalized size of antiderivative = 1.59

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$$
$$= -\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{B^2}{2\,d\,\left(c^2\,i^3+2\,c\,d\,i^3\,x+d^2\,i^3\,x^2\right)} - \frac{B^2\,b^2}{2\,d\,i^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)}\right)$$
$$- \frac{\frac{2\,A^2\,a\,d-2\,A^2\,b\,c+B^2\,a\,d\,n^2-7\,B^2\,b\,c\,n^2-2\,A\,B\,a\,d\,n+6\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)} - \frac{b\,x\,\left(3\,B^2\,d\,n^2-2\,A\,B\,d\,n\right)}{a\,d-b\,c}}{2\,c^2\,d\,i^3+4\,c\,d^2\,i^3\,x+2\,d^3\,i^3\,x^2}$$
$$- \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{A\,B}{c^2\,d\,i^3+2\,c\,d^2\,i^3\,x+d^3\,i^3\,x^2} + \frac{B^2\,b^2\,\left(\frac{d^2\,i^3\,n\,x\,(a\,d-b\,c)}{b} - \frac{d\,i^3\,n\,(a\,d-b\,c)\,(a\,d-2\,b\,c)}{2\,b^2} + \frac{c\,d\,i^3\,n\,(a\,d-b\,c)}{2\,b}\right)}{d\,i^3\,\left(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2\right)\,\left(c^2\,d\,i^3+2\,c\,d^2\,i^3\,x+d^3\,i^3\,x^2\right)}\right)$$
$$- \frac{B\,b^2\,n\,\mathrm{atan}\left(\frac{\left(2\,b\,d\,x+\frac{2\,a^2\,d^3\,i^3-2\,b^2\,c^2\,d\,i^3}{2\,d\,i^3\,(a\,d-b\,c)}\right)\,1{}\mathrm{i}}{a\,d-b\,c}\right)\,\left(2\,A-3\,B\,n\right)\,1{}\mathrm{i}}{d\,i^3\,{\left(a\,d-b\,c\right)}^2}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/(c*i + d*i*x)^3,x)
```

output
```
- log(e*((a + b*x)/(c + d*x))^n)^2*(B^2/(2*d*(c^2*i^3 + d^2*i^3*x^2 + 2*c*
d*i^3*x)) - (B^2*b^2)/(2*d*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d))) - ((2*A^2
*a*d - 2*A^2*b*c + B^2*a*d*n^2 - 7*B^2*b*c*n^2 - 2*A*B*a*d*n + 6*A*B*b*c*n
)/(2*(a*d - b*c)) - (b*x*(3*B^2*d*n^2 - 2*A*B*d*n))/(a*d - b*c))/(2*c^2*d*
i^3 + 2*d^3*i^3*x^2 + 4*c*d^2*i^3*x) - log(e*((a + b*x)/(c + d*x))^n)*((A*
B)/(c^2*d*i^3 + d^3*i^3*x^2 + 2*c*d^2*i^3*x) + (B^2*b^2*((d^2*i^3*n*x*(a*d
 - b*c))/b - (d*i^3*n*(a*d - b*c)*(a*d - 2*b*c))/(2*b^2) + (c*d*i^3*n*(a*d
 - b*c))/(2*b)))/(d*i^3*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^2*d*i^3 + d^3*i
^3*x^2 + 2*c*d^2*i^3*x))) - (B*b^2*n*atan(((2*b*d*x + (2*a^2*d^3*i^3 - 2*b
^2*c^2*d*i^3)/(2*d*i^3*(a*d - b*c)))*1i)/(a*d - b*c))*(2*A - 3*B*n)*1i)/(d
*i^3*(a*d - b*c)^2)
```

3.205. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ci+dix)^3}\,dx$

# 3.206 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$



## 3.206.1 Optimal result

Integrand size = 45, antiderivative size = 402

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx &= \frac{B^2d^2n^2(a+bx)^2}{4(bc-ad)^3gi^3(c+dx)^2}+\frac{4AbBdn(a+bx)}{(bc-ad)^3gi^3(c+dx)}\\
&-\frac{4bB^2dn^2(a+bx)}{(bc-ad)^3gi^3(c+dx)}+\frac{4bB^2dn(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^3gi^3(c+dx)}\\
&-\frac{Bd^2n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^3gi^3(c+dx)^2}\\
&+\frac{d^2(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^3gi^3(c+dx)^2}\\
&-\frac{2bd(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^3gi^3(c+dx)}\\
&+\frac{b^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^3gi^3n}\end{aligned}$$

output

```
1/4*B^2*d^2*n^2*(b*x+a)^2/(-a*d+b*c)^3/g/i^3/(d*x+c)^2+4*A*b*B*d*n*(b*x+a)
/(-a*d+b*c)^3/g/i^3/(d*x+c)-4*b*B^2*d*n^2*(b*x+a)/(-a*d+b*c)^3/g/i^3/(d*x+
c)+4*b*B^2*d*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^3/g/i^3/(d*x+c
)-1/2*B*d^2*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^3/g/i^3
/(d*x+c)^2+1/2*d^2*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^
3/g/i^3/(d*x+c)^2-2*b*d*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*
c)^3/g/i^3/(d*x+c)+1/3*b^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^
3/g/i^3/n
```

### 3.206.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 971 vs. $2(402) = 804$.

Time = 0.52 (sec) , antiderivative size = 971, normalized size of antiderivative = 2.42

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{4b^2B^2n^2\log^3\left(\frac{a+bx}{c+dx}\right) - \frac{6Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(-2Ab^2c^2+4abBcdn-a^2Bd^2n-4Ab^2cdx+4b^2Bcdnx+2abBd^2nx-2Ab^2d^2x^2+3b^2Bd^2nx^2-2b^2\right.}{(c+dx)^2}}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d
*i*x)^3),x]
```

output
```
(4*b^2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^3 - (6*B*n*Log[(a + b*x)/(c + d*x)
]^2*(-2*A*b^2*c^2 + 4*a*b*B*c*d*n - a^2*B*d^2*n - 4*A*b^2*c*d*x + 4*b^2*B*
c*d*n*x + 2*a*b*B*d^2*n*x - 2*A*b^2*d^2*x^2 + 3*b^2*B*d^2*n*x^2 - 2*b^2*B*
(c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n] + 2*b^2*B*n*(c + d*x)^2*Log[(a
+ b*x)/(c + d*x)]))/(c + d*x)^2 - (6*B*(b*c - a*d)*n*Log[(a + b*x)/(c + d*
x)]*(-6*A*b*c + 2*a*A*d + 7*b*B*c*n - a*B*d*n - 4*A*b*d*x + 6*b*B*d*n*x +
2*B*(-3*b*c + a*d - 2*b*d*x)*Log[e*((a + b*x)/(c + d*x))^n] + 2*B*n*(3*b*c
 - a*d + 2*b*d*x)*Log[(a + b*x)/(c + d*x)]))/(c + d*x)^2 + (3*(b*c - a*d)^
2*(2*A^2 - 2*A*B*n + B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*
B*n*(-2*A + B*n)*Log[(a + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d
*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(-2*A + B*n + 2*B*n*Log[(a + b
*x)/(c + d*x)])))/(c + d*x)^2 + (6*b*(b*c - a*d)*(2*A^2 - 6*A*B*n + 7*B^2*
n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A + 3*B*n)*Log[(a
 + b*x)/(c + d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a
+ b*x)/(c + d*x))^n]*(-2*A + 3*B*n + 2*B*n*Log[(a + b*x)/(c + d*x)])))/(c
+ d*x) + 6*b^2*Log[a + b*x]*(2*A^2 - 6*A*B*n + 7*B^2*n^2 + 2*B^2*Log[e*((a
 + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A + 3*B*n)*Log[(a + b*x)/(c + d*x)] +
2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*x))^n]*
(-2*A + 3*B*n + 2*B*n*Log[(a + b*x)/(c + d*x)])) - 6*b^2*(2*A^2 - 6*A*B*n
+ 7*B^2*n^2 + 2*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A + 3*...
```

### 3.206.3 Rubi [A] (warning: unable to verify)

Time = 0.89 (sec) , antiderivative size = 383, normalized size of antiderivative = 0.95, number of steps used = 10, number of rules used = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {2961, 2788, 2767, 2009, 2788, 2733, 2009, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)(ci+dix)^3}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}}{gi^3(bc-ad)^3}$$

$$\Big\downarrow \text{2788}$$

$$\frac{b\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\int \left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d\frac{a+bx}{c+dx}}{gi^3(bc-ad)^3}$$

$$\Big\downarrow \text{2767}$$

$$\frac{b\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\int \left(b\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2-\frac{d(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{c+dx}\right.}{gi^3(bc-ad)^3}$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\int \frac{(c+dx)\left(b-\frac{d(a+bx)}{c+dx}\right)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(c+dx)^2}+\frac{b(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}\right.}{gi^3(bc-ad}$$

$$\Big\downarrow \text{2788}$$

$$\frac{b\left(b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d\frac{a+bx}{c+dx}\right)-d\left(\frac{Bdn(a+bx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{2(c+dx)^2}\right.}{}$$

$$\Big\downarrow \text{2733}$$

$$b\left(b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\int\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}\right.\right.$$

$$\Big\downarrow \text{ 2009}$$

$$b\left(b\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}-d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\right.\right.\right.$$

$$\Big\downarrow \text{ 2739}$$

$$b\left(\frac{b\int \frac{(a+bx)^2}{(c+dx)^2}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}-d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{Bn(a}{c+}\right.\right.\right.$$

$$\Big\downarrow \text{ 15}$$

$$b\left(\frac{b(a+bx)^3}{3Bn(c+dx)^3}-d\left(\frac{(a+bx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{c+dx}-2Bn\left(\frac{A(a+bx)}{c+dx}+\frac{B(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{c+dx}-\frac{Bn(a+bx)}{c+dx}\right)\right)\right)-d\left(\frac{Bdn}{}\right.$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)*(c*i + d*i*x)^
3),x]
```

output
```
(-(d*(-1/4*(B^2*d*n^2*(a + b*x)^2)/(c + d*x)^2 - (2*A*b*B*n*(a + b*x))/(c
+ d*x) + (2*b*B^2*n^2*(a + b*x))/(c + d*x) - (2*b*B^2*n*(a + b*x)*Log[e*((
a + b*x)/(c + d*x))^n])/(c + d*x) + (B*d*n*(a + b*x)^2*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]))/(2*(c + d*x)^2) - (d*(a + b*x)^2*(A + B*Log[e*((a + b
*x)/(c + d*x))^n])^2)/(2*(c + d*x)^2) + (b*(a + b*x)*(A + B*Log[e*((a + b*
x)/(c + d*x))^n])^2)/(c + d*x))) + b*((b*(a + b*x)^3)/(3*B*n*(c + d*x)^3)
- d*(((a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*x) - 2*B*
n*((A*(a + b*x))/(c + d*x) - (B*n*(a + b*x))/(c + d*x) + (B*(a + b*x)*Log[
e*((a + b*x)/(c + d*x))^n])/(c + d*x))))/((b*c - a*d)^3*g*i^3)
```

3.206. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$

### 3.206.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2733
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.), x_Symbol] :> Simp[x*(a + b
*Log[c*x^n])^p, x] - Simp[b*n*p   Int[(a + b*Log[c*x^n])^(p - 1), x], x] /;
 FreeQ[{a, b, c, n}, x] && GtQ[p, 0] && IntegerQ[2*p]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2767
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_)^(r_.))^(
q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[c*x^n])^p, (d + e*x
^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b, c, d, e, n, p, q, r}, x]
&& IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0] && IntegerQ[r]))
```

rule 2788
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_) + (e_.)*(x_))^(q_.))
/(x_), x_Symbol] :> Simp[d   Int[(d + e*x)^(q - 1)*((a + b*Log[c*x^n])^p/x)
, x], x] + Simp[e   Int[(d + e*x)^(q - 1)*(a + b*Log[c*x^n])^p, x], x] /; F
reeQ[{a, b, c, d, e, n}, x] && IGtQ[p, 0] && GtQ[q, 0] && IntegerQ[2*q]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

---

3.206. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$

### 3.206.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1235 vs. 2(394) = 788.

Time = 11.42 (sec) , antiderivative size = 1236, normalized size of antiderivative = 3.07

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 1236 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x,method=_
RETURNVERBOSE)
```

output
```
-1/12*(4*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^2*c^4*d^2+48*B^2*x^2*a^
3*b*c^3*d^3*n^3-45*B^2*x^2*a^2*b^2*c^4*d^2*n^3+6*A*B*x^2*a^4*c^2*d^4*n^2+8
*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^2*c^5*d+54*B^2*x*a^3*b*c^4*d^2*n^
3-48*B^2*x*a^2*b^2*c^5*d*n^3+12*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*
c^4*d^2+24*A^2*x^2*a^3*b*c^3*d^3*n-36*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^
2*b^2*c^4*d^2*n-24*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^4*d^2*n-48*A*B*
x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^5*d*n+4*B^2*ln(e*((b*x+a)/(d*x+c))^n
)^3*a^2*b^2*c^6+12*A^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^6-24*B^2*x*ln(e
*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^5*d*n+36*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)
*a^3*b*c^4*d^2*n^2+48*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^5*d*n^2+24
*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^5*d-60*A*B*x*a^3*b*c^4*d^2*n^
2+48*A*B*x*a^2*b^2*c^5*d*n^2-48*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b*c^5*d*
n-18*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^2*c^4*d^2*n+42*B^2*x^2*ln(e
*((b*x+a)/(d*x+c))^n)*a^2*b^2*c^4*d^2*n^2+12*A*B*x^2*ln(e*((b*x+a)/(d*x+c)
)^n)^2*a^2*b^2*c^4*d^2-48*A*B*x^2*a^3*b*c^3*d^3*n^2+42*A*B*x^2*a^2*b^2*c^4
*d^2*n^2-12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b*c^4*d^2*n-3*B^2*x^2*a^
4*c^2*d^4*n^3-6*B^2*x*a^4*c^3*d^3*n^3-6*A^2*x^2*a^4*c^2*d^4*n+6*B^2*ln(e*(
(b*x+a)/(d*x+c))^n)^2*a^4*c^4*d^2*n-6*B^2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*c^
4*d^2*n^2-12*A^2*x*a^4*c^3*d^3*n+12*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^
2*c^6-18*A^2*x^2*a^2*b^2*c^4*d^2*n+12*A*B*x*a^4*c^3*d^3*n^2-24*B^2*ln(e...
```

### 3.206.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 1076 vs. $2(394) = 788$.

Time = 0.32 (sec) , antiderivative size = 1076, normalized size of antiderivative = 2.68

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \frac{18\,A^2b^2c^2-24\,A^2abcd+6\,A^2a^2d^2+4\left(B^2b^2d^2n^2x^2+2\,B^2b^2cdn^2x+B^2b^2c^2n^2\right)\log\left(\frac{bx+a}{dx+c}\right)^3+3\left(15\,B^2b^2c^2\right.}{}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x,
algorithm="fricas")
```

output
```
1/12*(18*A^2*b^2*c^2 - 24*A^2*a*b*c*d + 6*A^2*a^2*d^2 + 4*(B^2*b^2*d^2*n^2
*x^2 + 2*B^2*b^2*c*d*n^2*x + B^2*b^2*c^2*n^2)*log((b*x + a)/(d*x + c))^3 +
 3*(15*B^2*b^2*c^2 - 16*B^2*a*b*c*d + B^2*a^2*d^2)*n^2 + 6*(3*B^2*b^2*c^2
- 4*B^2*a*b*c*d + B^2*a^2*d^2 + 2*(B^2*b^2*c*d - B^2*a*b*d^2)*x + 2*(B^2*b
^2*d^2*x^2 + 2*B^2*b^2*c*d*x + B^2*b^2*c^2)*log((b*x + a)/(d*x + c)))*log(
e)^2 + 6*(2*A*B*b^2*c^2*n - (4*B^2*a*b*c*d - B^2*a^2*d^2)*n^2 - (3*B^2*b^2
*d^2*n^2 - 2*A*B*b^2*d^2*n)*x^2 + 2*(2*A*B*b^2*c*d*n - (2*B^2*b^2*c*d + B^
2*a*b*d^2)*n^2)*x)*log((b*x + a)/(d*x + c))^2 - 6*(7*A*B*b^2*c^2 - 8*A*B*a
*b*c*d + A*B*a^2*d^2)*n + 6*(2*A^2*b^2*c*d - 2*A^2*a*b*d^2 + 7*(B^2*b^2*c*
d - B^2*a*b*d^2)*n^2 - 6*(A*B*b^2*c*d - A*B*a*b*d^2)*n)*x + 6*(6*A*B*b^2*c
^2 - 8*A*B*a*b*c*d + 2*A*B*a^2*d^2 + 2*(B^2*b^2*d^2*n*x^2 + 2*B^2*b^2*c*d*
n*x + B^2*b^2*c^2*n)*log((b*x + a)/(d*x + c))^2 - (7*B^2*b^2*c^2 - 8*B^2*a
*b*c*d + B^2*a^2*d^2)*n + 2*(2*A*B*b^2*c*d - 2*A*B*a*b*d^2 - 3*(B^2*b^2*c*
d - B^2*a*b*d^2)*n)*x + 2*(2*A*B*b^2*c^2 - (3*B^2*b^2*d^2*n - 2*A*B*b^2*d^
2)*x^2 - (4*B^2*a*b*c*d - B^2*a^2*d^2)*n + 2*(2*A*B*b^2*c*d - (2*B^2*b^2*c
*d + B^2*a*b*d^2)*n)*x)*log((b*x + a)/(d*x + c)))*log(e) + 6*(2*A^2*b^2*c^
2 + (8*B^2*a*b*c*d - B^2*a^2*d^2)*n^2 + (7*B^2*b^2*d^2*n^2 - 6*A*B*b^2*d^2
*n + 2*A^2*b^2*d^2)*x^2 - 2*(4*A*B*a*b*c*d - A*B*a^2*d^2)*n + 2*(2*A^2*b^2
*c*d + (4*B^2*b^2*c*d + 3*B^2*a*b*d^2)*n^2 - 2*(2*A*B*b^2*c*d + A*B*a*b*d^
2)*n)*x)*log((b*x + a)/(d*x + c)))/((b^3*c^3*d^2 - 3*a*b^2*c^2*d^3 + 3*...
```

### 3.206.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)/(d*i*x+c*i)**3,x
)
```

output
```
Timed out
```

### 3.206.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2126 vs. 2(394) = 788.

Time = 0.33 (sec) , antiderivative size = 2126, normalized size of antiderivative = 5.29

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x,
algorithm="maxima")
```

output
```
1/2*B^2*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3 + a^2*d^4)*g*
i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x + (b^2*c^4 - 2
*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c^3 - 3*a*b^2*
c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/((b^3*c^3 - 3
*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(e*(b*x/(d*x + c) + a/(
d*x + c))^n)^2 + A*B*((2*b*d*x + 3*b*c - a*d)/((b^2*c^2*d^2 - 2*a*b*c*d^3
+ a^2*d^4)*g*i^3*x^2 + 2*(b^2*c^3*d - 2*a*b*c^2*d^2 + a^2*c*d^3)*g*i^3*x +
 (b^2*c^4 - 2*a*b*c^3*d + a^2*c^2*d^2)*g*i^3) + 2*b^2*log(b*x + a)/((b^3*c
^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3) - 2*b^2*log(d*x + c)/
((b^3*c^3 - 3*a*b^2*c^2*d + 3*a^2*b*c*d^2 - a^3*d^3)*g*i^3))*log(e*(b*x/(d
*x + c) + a/(d*x + c))^n) + 1/12*((45*b^2*c^2 - 48*a*b*c*d + 3*a^2*d^2 + 4
*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a)^3 - 4*(b^2*d^2*x^2 + 2
*b^2*c*d*x + b^2*c^2)*log(d*x + c)^3 + 18*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2
*c^2)*log(b*x + a)^2 + 6*(3*b^2*d^2*x^2 + 6*b^2*c*d*x + 3*b^2*c^2 + 2*(b^2
*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a))*log(d*x + c)^2 + 42*(b^2*c
*d - a*b*d^2)*x + 42*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b*x + a) -
6*(7*b^2*d^2*x^2 + 14*b^2*c*d*x + 7*b^2*c^2 + 2*(b^2*d^2*x^2 + 2*b^2*c*d*x
 + b^2*c^2)*log(b*x + a)^2 + 6*(b^2*d^2*x^2 + 2*b^2*c*d*x + b^2*c^2)*log(b
*x + a))*log(d*x + c))*n^2/(b^3*c^5*g*i^3 - 3*a*b^2*c^4*d*g*i^3 + 3*a^2*b*
c^3*d^2*g*i^3 - a^3*c^2*d^3*g*i^3 + (b^3*c^3*d^2*g*i^3 - 3*a*b^2*c^2*d^...
```

## 3.206.8 Giac [A] (verification not implemented)

Time = 1.19 (sec) , antiderivative size = 747, normalized size of antiderivative = 1.86

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$
$$= \frac{1}{12}\left(\frac{4\,B^2b^2n^2\log\left(\frac{bx+a}{dx+c}\right)^3}{b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3}-6\left(\frac{4\,(bx+a)B^2bdn^2}{(b^2c^2gi^3-2\,abcdgi^3+a^2d^2gi^3)(dx+c)}-\frac{(bx+a)^2}{(b^2c^2gi^3-2\,abcdgi^3\right.\right.$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)/(d*i*x+c*i)^3,x,
algorithm="giac")
```

output
```
1/12*(4*B^2*b^2*n^2*log((b*x + a)/(d*x + c))^3/(b^2*c^2*g*i^3 - 2*a*b*c*d*
g*i^3 + a^2*d^2*g*i^3) - 6*(4*(b*x + a)*B^2*b*d*n^2/((b^2*c^2*g*i^3 - 2*a*
b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)) - (b*x + a)^2*B^2*d^2*n^2/((b^2*c^
2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)^2) - 2*(B^2*b^2*n*log
(e) + A*B*b^2*n)/(b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3))*log((b
*x + a)/(d*x + c))^2 - 6*((B^2*d^2*n^2 - 2*B^2*d^2*n*log(e) - 2*A*B*d^2*n)
*(b*x + a)^2/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)^
2) - 8*(B^2*b*d*n^2 - B^2*b*d*n*log(e) - A*B*b*d*n)*(b*x + a)/((b^2*c^2*g*
i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)))*log((b*x + a)/(d*x + c)
) + 12*(B^2*b^2*log(e)^2 + 2*A*B*b^2*log(e) + A^2*b^2)*log((b*x + a)/(d*x
+ c))/(b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3) + 3*(B^2*d^2*n^2 -
 2*B^2*d^2*n*log(e) + 2*B^2*d^2*log(e)^2 - 2*A*B*d^2*n + 4*A*B*d^2*log(e)
+ 2*A^2*d^2)*(b*x + a)^2/((b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3 + a^2*d^2*g*i^3
)*(d*x + c)^2) - 24*(2*B^2*b*d*n^2 - 2*B^2*b*d*n*log(e) + B^2*b*d*log(e)^2
 - 2*A*B*b*d*n + 2*A*B*b*d*log(e) + A^2*b*d)*(b*x + a)/((b^2*c^2*g*i^3 - 2
*a*b*c*d*g*i^3 + a^2*d^2*g*i^3)*(d*x + c)))*(b*c/(b*c - a*d)^2 - a*d/(b*c
- a*d)^2)
```

3.206. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$

### 3.206.9 Mupad [B] (verification not implemented)

Time = 5.35 (sec) , antiderivative size = 1007, normalized size of antiderivative = 2.50

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$$

$$= \ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^2\left(\frac{b^2\,(3\,B^2\,n-2\,A\,B)}{2\,g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)} + \frac{B^2\,b^2\left(\frac{c\,g\,i^3\,n\,(a\,d-b\,c)}{2\,b}-\frac{g\,i^3\,n\,(a\,d-b\,c)\,(a\,d-2\,b\,c)}{2\,b^2}+\frac{d\,g\,i^3\,n\,x\,(a\,d-b\,c)}{b}\right)}{g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)\,(g\,c^2\,i^3+2\,g\,c\,d\,i^3\,x+g\,d^2\,i^3\,x^2)}\right)$$

$$- \frac{\frac{2\,A^2\,a\,d-6\,A^2\,b\,c+B^2\,a\,d\,n^2-15\,B^2\,b\,c\,n^2-2\,A\,B\,a\,d\,n+14\,A\,B\,b\,c\,n}{2\,(a\,d-b\,c)}-\frac{x\,(2\,b\,d\,A^2-6\,b\,d\,A\,B\,n+7\,b\,d\,B^2\,n^2)}{a\,d-b\,c}}{x^2\,(2\,a\,d^3\,g\,i^3-2\,b\,c\,d^2\,g\,i^3)+x\,(4\,a\,c\,d^2\,g\,i^3-4\,b\,c^2\,d\,g\,i^3)-2\,b\,c^3\,g\,i^3+2\,a\,c^2\,d\,g\,i^3}$$

$$-\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\left(\frac{B^2\,n}{x^2\,(a\,d^3\,g\,i^3-b\,c\,d^2\,g\,i^3)+x\,(2\,a\,c\,d^2\,g\,i^3-2\,b\,c^2\,d\,g\,i^3)-b\,c^3\,g\,i^3+a\,c^2\,d\,g\,i^3}\right.$$

$$+\frac{b^2\,(3\,B^2\,n-2\,A\,B)\left(\frac{c\,g\,i^3\,n\,(a\,d-b\,c)^2}{2\,b}-\frac{g\,i^3\,n\,(a\,d-b\,c)^2\,(a\,d-2\,b\,c)}{2\,b^2}+\frac{d\,g\,i^3\,n\,x\,(a\,d-b\,c)^2}{b}\right)}{g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)\,(x^2\,(a\,d^3\,g\,i^3-b\,c\,d^2\,g\,i^3)+x\,(2\,a\,c\,d^2\,g\,i^3-2\,b\,c^2\,d\,g\,i^3)-b\,c}$$

$$-\frac{B^2\,b^2\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^3}{3\,g\,i^3\,n\,(a\,d-b\,c)\,(a^2\,d^2-2\,a\,b\,c\,d+b^2\,c^2)}$$

$$+\frac{b^2\,\mathrm{atan}\left(\frac{b^2\left(\frac{g\,a^3\,d^3\,i^3-g\,a^2\,b\,c\,d^2\,i^3-g\,a\,b^2\,c^2\,d\,i^3+g\,b^3\,c^3\,i^3}{g\,a^2\,d^2\,i^3-2\,g\,a\,b\,c\,d\,i^3+g\,b^2\,c^2\,i^3}+2\,b\,d\,x\right)\left(A^2-3\,A\,B\,n+\frac{7\,B^2\,n^2}{2}\right)(g\,a^2\,d^2\,i^3-2\,g\,a\,b\,c\,d\,i^3+g\,b^2\,c^2\,i^3)\,2\mathrm{i}}{g\,i^3\,(a\,d-b\,c)^3\,(2\,A^2\,b^2-6\,A\,B\,b^2\,n+7\,B^2\,b^2\,n^2)}\right)\,(A}{g\,i^3\,(a\,d-b\,c)^3}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)*(c*i + d*i*x)^
3),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)^2*((b^2*(3*B^2*n - 2*A*B))/(2*g*i^3*n*(a*d
- b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (B^2*b^2*((c*g*i^3*n*(a*d - b*c)
)/(2*b) - (g*i^3*n*(a*d - b*c)*(a*d - 2*b*c))/(2*b^2) + (d*g*i^3*n*x*(a*d
- b*c))/b))/(g*i^3*n*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(c^2*g*i^
3 + d^2*g*i^3*x^2 + 2*c*d*g*i^3*x))) - ((2*A^2*a*d - 6*A^2*b*c + B^2*a*d*n
^2 - 15*B^2*b*c*n^2 - 2*A*B*a*d*n + 14*A*B*b*c*n)/(2*(a*d - b*c)) - (x*(2*
A^2*b*d + 7*B^2*b*d*n^2 - 6*A*B*b*d*n))/(a*d - b*c))/(x^2*(2*a*d^3*g*i^3 -
 2*b*c*d^2*g*i^3) + x*(4*a*c*d^2*g*i^3 - 4*b*c^2*d*g*i^3) - 2*b*c^3*g*i^3
+ 2*a*c^2*d*g*i^3) - log(e*((a + b*x)/(c + d*x))^n)*((B^2*n)/(x^2*(a*d^3*g
*i^3 - b*c*d^2*g*i^3) + x*(2*a*c*d^2*g*i^3 - 2*b*c^2*d*g*i^3) - b*c^3*g*i^
3 + a*c^2*d*g*i^3) + (b^2*(3*B^2*n - 2*A*B)*((c*g*i^3*n*(a*d - b*c)^2)/(2*
b) - (g*i^3*n*(a*d - b*c)^2*(a*d - 2*b*c))/(2*b^2) + (d*g*i^3*n*x*(a*d - b
*c)^2)/b))/(g*i^3*n*(a*d - b*c)*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)*(x^2*(a*d^
3*g*i^3 - b*c*d^2*g*i^3) + x*(2*a*c*d^2*g*i^3 - 2*b*c^2*d*g*i^3) - b*c^3*g
*i^3 + a*c^2*d*g*i^3))) + (b^2*atan((b^2*((a^3*d^3*g*i^3 + b^3*c^3*g*i^3 -
 a*b^2*c^2*d*g*i^3 - a^2*b*c*d^2*g*i^3)/(a^2*d^2*g*i^3 + b^2*c^2*g*i^3 - 2
*a*b*c*d*g*i^3) + 2*b*d*x)*(A^2 + (7*B^2*n^2)/2 - 3*A*B*n)*(a^2*d^2*g*i^3
+ b^2*c^2*g*i^3 - 2*a*b*c*d*g*i^3)*2i)/(g*i^3*(a*d - b*c)^3*(2*A^2*b^2 + 7
*B^2*b^2*n^2 - 6*A*B*b^2*n)))*(A^2 + (7*B^2*n^2)/2 - 3*A*B*n)*2i)/(g*i^3*(
a*d - b*c)^3) - (B^2*b^2*log(e*((a + b*x)/(c + d*x))^n)^3)/(3*g*i^3*n*(...
```

3.206. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)(ci+dix)^3}\,dx$

# 3.207 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx$



## 3.207.1 Optimal result

Integrand size = 45, antiderivative size = 562

$$\begin{aligned}\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx &= -\frac{B^2d^3n^2(a+bx)^2}{4(bc-ad)^4g^2i^3(c+dx)^2}-\frac{6AbBd^2n(a+bx)}{(bc-ad)^4g^2i^3(c+dx)} \\ &\quad+\frac{6bB^2d^2n^2(a+bx)}{(bc-ad)^4g^2i^3(c+dx)}-\frac{2b^3B^2n^2(c+dx)}{(bc-ad)^4g^2i^3(a+bx)} \\ &\quad-\frac{6bB^2d^2n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^4g^2i^3(c+dx)} \\ &\quad+\frac{Bd^3n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^4g^2i^3(c+dx)^2} \\ &\quad-\frac{2b^3Bn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^4g^2i^3(a+bx)} \\ &\quad-\frac{d^3(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^4g^2i^3(c+dx)^2} \\ &\quad+\frac{3bd^2(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^2i^3(c+dx)} \\ &\quad-\frac{b^3(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^4g^2i^3(a+bx)} \\ &\quad-\frac{b^2d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^4g^2i^3n}\end{aligned}$$

output
```
-1/4*B^2*d^3*n^2*(b*x+a)^2/(-a*d+b*c)^4/g^2/i^3/(d*x+c)^2-6*A*b*B*d^2*n*(b
*x+a)/(-a*d+b*c)^4/g^2/i^3/(d*x+c)+6*b*B^2*d^2*n^2*(b*x+a)/(-a*d+b*c)^4/g^
2/i^3/(d*x+c)-2*b^3*B^2*n^2*(d*x+c)/(-a*d+b*c)^4/g^2/i^3/(b*x+a)-6*b*B^2*d
^2*n*(b*x+a)*ln(e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^4/g^2/i^3/(d*x+c)+1/2*B*
d^3*n*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^3/(d*x+
c)^2-2*b^3*B*n*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^4/g^2/i^
3/(b*x+a)-1/2*d^3*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^4
/g^2/i^3/(d*x+c)^2+3*b*d^2*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d
+b*c)^4/g^2/i^3/(d*x+c)-b^3*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*
d+b*c)^4/g^2/i^3/(b*x+a)-b^2*d*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b
*c)^4/g^2/i^3/n
```

### 3.207.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1334 vs. $2(562) = 1124$.

Time = 0.89 (sec) , antiderivative size = 1334, normalized size of antiderivative = 2.37

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx =$$
$$-\frac{4b^2B^2dn^2(a+bx)(c+dx)^2\log^3\left(\frac{a+bx}{c+dx}\right)+2Bn\log^2\left(\frac{a+bx}{c+dx}\right)\left(6aAb^2c^2d+2b^3Bc^3n-6a^2bBcd^2n+a^3Bd^3\right.}{}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i +
 d*i*x)^3),x]
```

output
```
-1/4*(4*b^2*B^2*d*n^2*(a + b*x)*(c + d*x)^2*Log[(a + b*x)/(c + d*x)]^3 + 2
*B*n*Log[(a + b*x)/(c + d*x)]^2*(6*a*A*b^2*c^2*d + 2*b^3*B*c^3*n - 6*a^2*b
*B*c*d^2*n + a^3*B*d^3*n + 6*A*b^3*c^2*d*x + 12*a*A*b^2*c*d^2*x + 6*b^3*B*
c^2*d*n*x - 12*a*b^2*B*c*d^2*n*x - 3*a^2*b*B*d^3*n*x + 12*A*b^3*c*d^2*x^2
+ 6*a*A*b^2*d^3*x^2 - 9*a*b^2*B*d^3*n*x^2 + 6*A*b^3*d^3*x^3 - 3*b^3*B*d^3*
n*x^3 + 6*b^2*B*d*(a + b*x)*(c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n] - 6
*b^2*B*d*n*(a + b*x)*(c + d*x)^2*Log[(a + b*x)/(c + d*x)]) + 4*b^2*(b*c -
a*d)*(c + d*x)^2*(A^2 + 2*A*B*n + 2*B^2*n^2 + B^2*Log[e*((a + b*x)/(c + d*
x))^n]^2 - 2*B*n*(A + B*n)*Log[(a + b*x)/(c + d*x)] + B^2*n^2*Log[(a + b*x
)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d*x))^n]*(A + B*n - B*n*Log[(a
+ b*x)/(c + d*x)])) + 2*B*(b*c - a*d)*n*Log[(a + b*x)/(c + d*x)]*(2*b*d*(a
 + b*x)*(c + d*x)*(4*A - 5*B*n + 4*B*Log[e*((a + b*x)/(c + d*x))^n] - 4*B*
n*Log[(a + b*x)/(c + d*x)]) + d*(b*c - a*d)*(a + b*x)*(2*A - B*n + 2*B*Log
[e*((a + b*x)/(c + d*x))^n] - 2*B*n*Log[(a + b*x)/(c + d*x)]) + 4*b^2*(c +
 d*x)^2*(A + B*n + B*Log[e*((a + b*x)/(c + d*x))^n] - B*n*Log[(a + b*x)/(c
 + d*x)])) + d*(b*c - a*d)^2*(a + b*x)*(2*A^2 - 2*A*B*n + B^2*n^2 + 2*B^2*
Log[e*((a + b*x)/(c + d*x))^n]^2 + 2*B*n*(-2*A + B*n)*Log[(a + b*x)/(c + d
*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 - 2*B*Log[e*((a + b*x)/(c + d*
x))^n]*(-2*A + B*n + 2*B*n*Log[(a + b*x)/(c + d*x)])) + 6*b^2*d*(a + b*x)*
(c + d*x)^2*Log[a + b*x]*(2*A^2 - 2*A*B*n + 5*B^2*n^2 + 2*B^2*Log[e*((a...
```

### 3.207.3 Rubi [A] (verified)

Time = 0.66 (sec) , antiderivative size = 403, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^2(ci+dix)^3} \, dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)^2\left(b-\frac{d(a+bx)}{c+dx}\right)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^2}d\frac{a+bx}{c+dx}}{g^2i^3(bc-ad)^4}$$

$$\Big\downarrow \text{2795}$$

$$\int \left( \frac{\frac{(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^3}{(a+bx)^2} - \frac{3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^2}{a+bx} + 3d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b - \frac{d^3(a+bx)\left(A\right.}{}}{g^2i^3(bc-ad)^4}\right.$$

$\downarrow$ 2009

$$-\frac{b^3(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx} - \frac{2b^3Bn(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{a+bx} - \frac{b^2d\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{Bn} - \frac{d^3(a+bx)^2\left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{2(c+dx)^2}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^2*(c*i + d*i*x
)^3),x]
```

output
```
(-1/4*(B^2*d^3*n^2*(a + b*x)^2)/(c + d*x)^2 - (6*A*b*B*d^2*n*(a + b*x))/(c
 + d*x) + (6*b*B^2*d^2*n^2*(a + b*x))/(c + d*x) - (2*b^3*B^2*n^2*(c + d*x)
)/(a + b*x) - (6*b*B^2*d^2*n*(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(c
+ d*x) + (B*d^3*n*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(
c + d*x)^2) - (2*b^3*B*n*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
/(a + b*x) - (d^3*(a + b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2
*(c + d*x)^2) + (3*b*d^2*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^
2)/(c + d*x) - (b^3*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a
 + b*x) - (b^2*d*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(B*n))/((b*c -
a*d)^4*g^2*i^3)
```

### 3.207.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

3.207. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.207.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 1999 vs. $2(556) = 1112$.

Time = 28.47 (sec) , antiderivative size = 2000, normalized size of antiderivative = 3.56

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 2000 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x,method
=_RETURNVERBOSE)
```

output
```
-1/4*(-36*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^5*d^7*n-48*A*B*x*ln(e*((b*
x+a)/(d*x+c))^n)*a*b^5*c*d^6*n+4*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*b^6*d
^7+12*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^7+24*A*B*a^2*b^4*c*d^6*n^2-3
0*A*B*a*b^5*c^2*d^5*n^2+B^2*a^3*b^3*d^7*n^3+8*B^2*b^6*c^3*d^4*n^3+2*A^2*a^
3*b^3*d^7*n+4*A^2*b^6*c^3*d^4*n-12*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d
^7*n-18*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*d^7*n+42*B^2*x^2*ln(e*((
b*x+a)/(d*x+c))^n)*a*b^5*d^7*n^2+48*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^6*
c*d^6*n^2+12*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^5*d^7+24*A*B*x^2*ln(e
*((b*x+a)/(d*x+c))^n)^2*b^6*c*d^6+12*A*B*x^2*a*b^5*d^7*n^2-12*A*B*x^2*b^6*
c*d^6*n^2+8*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^5*c*d^6-6*B^2*x*ln(e*((b
*x+a)/(d*x+c))^n)^2*a^2*b^4*d^7*n+12*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6
*c^2*d^5*n+18*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*d^7*n^2+24*B^2*x*ln(
e*((b*x+a)/(d*x+c))^n)*b^6*c^2*d^5*n^2-18*B^2*x*a*b^5*c*d^6*n^3+12*A*B*x*l
n(e*((b*x+a)/(d*x+c))^n)^2*b^6*c^2*d^5+18*A*B*x*a^2*b^4*d^7*n^2-6*A*B*x*b^
6*c^2*d^5*n^2-12*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^2*b^4*c*d^6*n+24*B^2*ln
(e*((b*x+a)/(d*x+c))^n)*a^2*b^4*c*d^6*n^2+24*A^2*x*ln(e*((b*x+a)/(d*x+c))^
n)*a*b^5*c*d^6-12*A^2*x*a*b^5*c*d^6*n+12*A*B*ln(e*((b*x+a)/(d*x+c))^n)^2*a
*b^5*c^2*d^5+4*A*B*ln(e*((b*x+a)/(d*x+c))^n)*a^3*b^3*d^7*n+8*A*B*ln(e*((b*
x+a)/(d*x+c))^n)*b^6*c^3*d^4*n-6*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^6*d
^7*n+30*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^6*d^7*n^2+12*A*B*x^3*ln(e*(...
```

### 3.207.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2057 vs. $2(556) = 1112$.

Time = 0.35 (sec) , antiderivative size = 2057, normalized size of antiderivative = 3.66

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x
, algorithm="fricas")
```

output
```
-1/4*(4*A^2*b^3*c^3 + 6*A^2*a*b^2*c^2*d - 12*A^2*a^2*b*c*d^2 + 2*A^2*a^3*d
^3 + 4*(B^2*b^3*d^3*n^2*x^3 + B^2*a*b^2*c^2*d*n^2 + (2*B^2*b^3*c*d^2 + B^2
*a*b^2*d^3)*n^2*x^2 + (B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2)*n^2*x)*log((b*x
+ a)/(d*x + c))^3 + (8*B^2*b^3*c^3 + 15*B^2*a*b^2*c^2*d - 24*B^2*a^2*b*c*d
^2 + B^2*a^3*d^3)*n^2 + 6*(2*A^2*b^3*c*d^2 - 2*A^2*a*b^2*d^3 + 5*(B^2*b^3*
c*d^2 - B^2*a*b^2*d^3)*n^2 - 2*(A*B*b^3*c*d^2 - A*B*a*b^2*d^3)*n)*x^2 + 2*
(2*B^2*b^3*c^3 + 3*B^2*a*b^2*c^2*d - 6*B^2*a^2*b*c*d^2 + B^2*a^3*d^3 + 6*(
B^2*b^3*c*d^2 - B^2*a*b^2*d^3)*x^2 + 3*(3*B^2*b^3*c^2*d - 2*B^2*a*b^2*c*d^
2 - B^2*a^2*b*d^3)*x + 6*(B^2*b^3*d^3*x^3 + B^2*a*b^2*c^2*d + (2*B^2*b^3*c
*d^2 + B^2*a*b^2*d^3)*x^2 + (B^2*b^3*c^2*d + 2*B^2*a*b^2*c*d^2)*x)*log((b*
x + a)/(d*x + c)))*log(e)^2 + 2*(6*A*B*a*b^2*c^2*d*n - 3*(B^2*b^3*d^3*n^2
- 2*A*B*b^3*d^3*n)*x^3 + (2*B^2*b^3*c^3 - 6*B^2*a^2*b*c*d^2 + B^2*a^3*d^3)
*n^2 - 3*(3*B^2*a*b^2*d^3*n^2 - 2*(2*A*B*b^3*c*d^2 + A*B*a*b^2*d^3)*n)*x^2
 + 3*((2*B^2*b^3*c^2*d - 4*B^2*a*b^2*c*d^2 - B^2*a^2*b*d^3)*n^2 + 2*(A*B*b
^3*c^2*d + 2*A*B*a*b^2*c*d^2)*n)*x)*log((b*x + a)/(d*x + c))^2 + 2*(4*A*B*
b^3*c^3 - 15*A*B*a*b^2*c^2*d + 12*A*B*a^2*b*c*d^2 - A*B*a^3*d^3)*n + 3*(6*
A^2*b^3*c^2*d - 4*A^2*a*b^2*c*d^2 - 2*A^2*a^2*b*d^3 + (13*B^2*b^3*c^2*d -
6*B^2*a*b^2*c*d^2 - 7*B^2*a^2*b*d^3)*n^2 - 2*(A*B*b^3*c^2*d + 2*A*B*a*b^2*
c*d^2 - 3*A*B*a^2*b*d^3)*n)*x + 2*(4*A*B*b^3*c^3 + 6*A*B*a*b^2*c^2*d - 12*
A*B*a^2*b*c*d^2 + 2*A*B*a^3*d^3 + 6*(2*A*B*b^3*c*d^2 - 2*A*B*a*b^2*d^3 ...
```

### 3.207.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**2/(d*i*x+c*i)**
3,x)
```

output
```
Timed out
```

### 3.207.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4199 vs. $2(556) = 1112$.

Time = 0.52 (sec) , antiderivative size = 4199, normalized size of antiderivative = 7.47

$$\int \frac{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag + bgx)^2(ci + dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x
, algorithm="maxima")
```

output
```
-1/2*B^2*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d
+ a*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^
5)*g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*
b*c*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^
2 + 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*
d + 3*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c
^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3)
 - 6*b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*
a^3*b*c*d^3 + a^4*d^4)*g^2*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n)^2
- A*B*((6*b^2*d^2*x^2 + 2*b^2*c^2 + 5*a*b*c*d - a^2*d^2 + 3*(3*b^2*c*d + a
*b*d^2)*x)/((b^4*c^3*d^2 - 3*a*b^3*c^2*d^3 + 3*a^2*b^2*c*d^4 - a^3*b*d^5)*
g^2*i^3*x^3 + (2*b^4*c^4*d - 5*a*b^3*c^3*d^2 + 3*a^2*b^2*c^2*d^3 + a^3*b*c
*d^4 - a^4*d^5)*g^2*i^3*x^2 + (b^4*c^5 - a*b^3*c^4*d - 3*a^2*b^2*c^3*d^2 +
 5*a^3*b*c^2*d^3 - 2*a^4*c*d^4)*g^2*i^3*x + (a*b^3*c^5 - 3*a^2*b^2*c^4*d +
 3*a^3*b*c^3*d^2 - a^4*c^2*d^3)*g^2*i^3) + 6*b^2*d*log(b*x + a)/((b^4*c^4
- 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3*b*c*d^3 + a^4*d^4)*g^2*i^3) -
6*b^2*d*log(d*x + c)/((b^4*c^4 - 4*a*b^3*c^3*d + 6*a^2*b^2*c^2*d^2 - 4*a^3
*b*c*d^3 + a^4*d^4)*g^2*i^3))*log(e*(b*x/(d*x + c) + a/(d*x + c))^n) - 1/4
*((8*b^3*c^3 + 15*a*b^2*c^2*d - 24*a^2*b*c*d^2 + a^3*d^3 + 4*(b^3*d^3*x^3
+ a*b^2*c^2*d + (2*b^3*c*d^2 + a*b^2*d^3)*x^2 + (b^3*c^2*d + 2*a*b^2*c*...
```

### 3.207.8 Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^2(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^2/(d*i*x+c*i)^3,x
, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/((b*g*x + a*g)^2*(d*i*x
 + c*i)^3), x)
```

### 3.207.9 Mupad [B] (verification not implemented)

Time = 7.09 (sec) , antiderivative size = 1785, normalized size of antiderivative = 3.18

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^2*(c*i + d*i*x
)^3),x)
```

output
```
((4*A^2*b^2*c^2 - 2*A^2*a^2*d^2 - B^2*a^2*d^2*n^2 + 8*B^2*b^2*c^2*n^2 + 10
*A^2*a*b*c*d + 2*A*B*a^2*d^2*n + 8*A*B*b^2*c^2*n + 23*B^2*a*b*c*d*n^2 - 22
*A*B*a*b*c*d*n)/(2*(a*d - b*c)) + (3*x^2*(2*A^2*b^2*d^2 + 5*B^2*b^2*d^2*n^
2 - 2*A*B*b^2*d^2*n))/(a*d - b*c) + (3*x*(2*A^2*a*b*d^2 + 6*A^2*b^2*c*d +
7*B^2*a*b*d^2*n^2 + 13*B^2*b^2*c*d*n^2 - 6*A*B*a*b*d^2*n - 2*A*B*b^2*c*d*n
))/(2*(a*d - b*c)))/(x*(2*b^3*c^4*g^2*i^3 + 4*a^3*c*d^3*g^2*i^3 - 6*a^2*b*
c^2*d^2*g^2*i^3) + x^2*(2*a^3*d^4*g^2*i^3 + 4*b^3*c^3*d*g^2*i^3 - 6*a*b^2*
c^2*d^2*g^2*i^3) + x^3*(2*b^3*c^2*d^2*g^2*i^3 + 2*a^2*b*d^4*g^2*i^3 - 4*a*
b^2*c*d^3*g^2*i^3) + 2*a^3*c^2*d^2*g^2*i^3 + 2*a*b^2*c^4*g^2*i^3 - 4*a^2*b
*c^3*d*g^2*i^3) - log(e*((a + b*x)/(c + d*x))^n)^2*(((B^2*(a*d + 2*b*c))/(
2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (3*B^2*b*d*x)/(2*(a^2*d^2 + b^2*c^2 -
 2*a*b*c*d)))/(x*(b*c^2*g^2*i^3 + 2*a*c*d*g^2*i^3) + x^2*(a*d^2*g^2*i^3 +
2*b*c*d*g^2*i^3) + a*c^2*g^2*i^3 + b*d^2*g^2*i^3*x^3) + (3*B*b^2*d*(2*A -
B*n))/(2*g^2*i^3*n*(a*d - b*c)^4) - (3*B^2*b^2*d*(d*g^2*i^3*n*x^2*(a*d - b
*c) + (a*c*g^2*i^3*n*(a*d - b*c))/b + (g^2*i^3*n*x*(a*d + b*c)*(a*d - b*c)
)/b))/(g^2*i^3*n*(a*d - b*c)^4*(x*(b*c^2*g^2*i^3 + 2*a*c*d*g^2*i^3) + x^2*
(a*d^2*g^2*i^3 + 2*b*c*d*g^2*i^3) + a*c^2*g^2*i^3 + b*d^2*g^2*i^3*x^3))) -
 log(e*((a + b*x)/(c + d*x))^n)*((x*((3*B^2*b*d*n)/2 + 3*A*B*b*d) - (B^2*a
*d*n)/2 + 2*B^2*b*c*n + A*B*a*d + 2*A*B*b*c)/(x*(b^3*c^4*g^2*i^3 + 2*a^3*c
*d^3*g^2*i^3 - 3*a^2*b*c^2*d^2*g^2*i^3) + x^2*(a^3*d^4*g^2*i^3 + 2*b^3*...
```

3.207. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^2(ci+dix)^3}\,dx$

# 3.208 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$

### 3.208.1 Optimal result

Integrand size = 45, antiderivative size = 732

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \frac{B^2d^4n^2(a+bx)^2}{4(bc-ad)^5g^3i^3(c+dx)^2}+\frac{8AbBd^3n(a+bx)}{(bc-ad)^5g^3i^3(c+dx)} - \frac{8bB^2d^3n^2(a+bx)}{(bc-ad)^5g^3i^3(c+dx)}+\frac{8b^3B^2dn^2(c+dx)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4B^2n^2(c+dx)^2}{4(bc-ad)^5g^3i^3(a+bx)^2} + \frac{8bB^2d^3n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^5g^3i^3(c+dx)} - \frac{Bd^4n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(c+dx)^2} + \frac{8b^3Bdn(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4Bn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{d^4(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^5g^3i^3(c+dx)^2} - \frac{4bd^3(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^3i^3(c+dx)} + \frac{4b^3d(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^5g^3i^3(a+bx)} - \frac{b^4(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^5g^3i^3(a+bx)^2} + \frac{2b^2d^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{B(bc-ad)^5g^3i^3n}$$

output
```
1/4*B^2*d^4*n^2*(b*x+a)^2/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+8*A*b*B*d^3*n*(b*
x+a)/(-a*d+b*c)^5/g^3/i^3/(d*x+c)-8*b*B^2*d^3*n^2*(b*x+a)/(-a*d+b*c)^5/g^3
/i^3/(d*x+c)+8*b^3*B^2*d*n^2*(d*x+c)/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/4*b^4*
B^2*n^2*(d*x+c)^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+8*b*B^2*d^3*n*(b*x+a)*ln(
e*((b*x+a)/(d*x+c))^n)/(-a*d+b*c)^5/g^3/i^3/(d*x+c)-1/2*B*d^4*n*(b*x+a)^2*
(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(d*x+c)^2+8*b^3*B*d*n
*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*
b^4*B*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^5/g^3/i^3/(b*
x+a)^2+1/2*d^4*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^5/g^
3/i^3/(d*x+c)^2-4*b*d^3*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*
c)^5/g^3/i^3/(d*x+c)+4*b^3*d*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a
*d+b*c)^5/g^3/i^3/(b*x+a)-1/2*b^4*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)^2/(-a*d+b*c)^5/g^3/i^3/(b*x+a)^2+2*b^2*d^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n
))^3/B/(-a*d+b*c)^5/g^3/i^3/n
```

### 3.208.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1653 vs. $2(732) = 1464$.

Time = 1.17 (sec) , antiderivative size = 1653, normalized size of antiderivative = 2.26

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i +
 d*i*x)^3),x]
```

output
```
(8*b^2*B^2*d^2*n^2*(a + b*x)^2*(c + d*x)^2*Log[(a + b*x)/(c + d*x)]^3 + 2*
B*n*Log[(a + b*x)/(c + d*x)]^2*(12*a^2*A*b^2*c^2*d^2 - b^4*B*c^4*n + 8*a*b
^3*B*c^3*d*n - 8*a^3*b*B*c*d^3*n + a^4*B*d^4*n + 24*a*A*b^3*c^2*d^2*x + 24
*a^2*A*b^2*c*d^3*x + 4*b^4*B*c^3*d*n*x + 24*a*b^3*B*c^2*d^2*n*x - 24*a^2*b
^2*B*c*d^3*n*x - 4*a^3*b*B*d^4*n*x + 12*A*b^4*c^2*d^2*x^2 + 48*a*A*b^3*c*d
^3*x^2 + 12*a^2*A*b^2*d^4*x^2 + 18*b^4*B*c^2*d^2*n*x^2 - 18*a^2*b^2*B*d^4*
n*x^2 + 24*A*b^4*c*d^3*x^3 + 24*a*A*b^3*d^4*x^3 + 12*b^4*B*c*d^3*n*x^3 - 1
2*a*b^3*B*d^4*n*x^3 + 12*A*b^4*d^4*x^4 + 12*b^2*B*d^2*(a + b*x)^2*(c + d*x
)^2*Log[e*((a + b*x)/(c + d*x))^n] - 12*b^2*B*d^2*n*(a + b*x)^2*(c + d*x)^
2*Log[(a + b*x)/(c + d*x)]) + 12*b^2*d^2*(a + b*x)^2*(c + d*x)^2*Log[a + b
*x]*(2*A^2 + 5*B^2*n^2 + 4*A*B*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a
+ b*x)/(c + d*x)]) + 2*B^2*(Log[e*((a + b*x)/(c + d*x))^n] - n*Log[(a + b*
x)/(c + d*x)])^2) + 2*b^2*d*(b*c - a*d)*(a + b*x)*(c + d*x)^2*(6*A^2 + 14*
A*B*n + 15*B^2*n^2 + 6*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(6*A +
 7*B*n)*Log[(a + b*x)/(c + d*x)] + 6*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 +
2*B*Log[e*((a + b*x)/(c + d*x))^n]*(6*A + 7*B*n - 6*B*n*Log[(a + b*x)/(c +
 d*x)])) - b^2*(b*c - a*d)^2*(c + d*x)^2*(2*A^2 + 2*A*B*n + B^2*n^2 + 2*B^
2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 2*B*n*(2*A + B*n)*Log[(a + b*x)/(c +
d*x)] + 2*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 2*B*Log[e*((a + b*x)/(c + d
*x))^n]*(2*A + B*n - 2*B*n*Log[(a + b*x)/(c + d*x)])) + 2*B*(b*c - a*d)...
```

### 3.208.3 Rubi [A] (verified)

Time = 0.70 (sec) , antiderivative size = 525, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.067$, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^3(ci+dix)^3}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{\int \frac{(c+dx)^3\left(b-\frac{d(a+bx)}{c+dx}\right)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^3}d\frac{a+bx}{c+dx}}{g^3i^3(bc-ad)^5}$$

$$\Big\downarrow \text{ 2795}$$

$$\int \frac{\left(\frac{(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^4}{(a+bx)^3} - \frac{4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^3}{(a+bx)^2} + \frac{6d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^2}{a+bx} - 4d^3\left(A+\right.\right.}{g^3 i^3 (bc-ad)^5}$$

$\downarrow$ 2009

$$-\frac{b^4(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2} - \frac{b^4Bn(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{2(a+bx)^2} + \frac{4b^3d(c+dx)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{a+bx} + \frac{8b^3Bdn(c+dx)\left(\right.}{}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^3*(c*i + d*i*x
)^3),x]
```

output
```
((B^2*d^4*n^2*(a + b*x)^2)/(4*(c + d*x)^2) + (8*A*b*B*d^3*n*(a + b*x))/(c
+ d*x) - (8*b*B^2*d^3*n^2*(a + b*x))/(c + d*x) + (8*b^3*B^2*d*n^2*(c + d*x
))/(a + b*x) - (b^4*B^2*n^2*(c + d*x)^2)/(4*(a + b*x)^2) + (8*b*B^2*d^3*n*
(a + b*x)*Log[e*((a + b*x)/(c + d*x))^n])/(c + d*x) - (B*d^4*n*(a + b*x)^2
*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(c + d*x)^2) + (8*b^3*B*d*n*(c
 + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) - (b^4*B*n*(c +
d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) + (d^4*(a +
 b*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(c + d*x)^2) - (4*b*d
^3*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*x) + (4*b^3*
d*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) - (b^4*(c
+ d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x)^2) + (2*b^
2*d^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(B*n))/((b*c - a*d)^5*g^3*
i^3)
```

### 3.208.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

3.208. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.208.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 3005 vs. $2(720) = 1440$.

Time = 54.53 (sec) , antiderivative size = 3006, normalized size of antiderivative = 4.11

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 3006 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x,method
=_RETURNVERBOSE)
```

output
```
-1/4*(-32*B^2*a^3*b^5*c*d^7*n^3+32*B^2*a*b^7*c^3*d^5*n^3-2*A*B*a^4*b^4*d^8
*n^2-2*A*B*b^8*c^4*d^4*n^2-16*A^2*a^3*b^5*c*d^7*n+16*A^2*a*b^7*c^3*d^5*n+1
6*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*b^8*c*d^7-60*B^2*x^3*a*b^7*d^8*n^3+6
0*B^2*x^3*b^8*c*d^7*n^3+8*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^6*d^8+
8*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*b^8*c^2*d^6-90*B^2*x^2*a^2*b^6*d^8*n
^3+90*B^2*x^2*b^8*c^2*d^6*n^3+48*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*d
^8+48*A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*b^8*c*d^7-24*A^2*x^3*a*b^7*d^8*n+2
4*A^2*x^3*b^8*c*d^7*n-28*B^2*x*a^3*b^5*d^8*n^3+28*B^2*x*b^8*c^3*d^5*n^3+24
*A^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b^6*d^8+24*A^2*x^2*ln(e*((b*x+a)/(d
*x+c))^n)*b^8*c^2*d^6-36*A^2*x^2*a^2*b^6*d^8*n+36*A^2*x^2*b^8*c^2*d^6*n+8*
B^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^6*c^2*d^6+2*B^2*ln(e*((b*x+a)/(d*x+c
))^n)^2*a^4*b^4*d^8*n-2*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*b^8*c^4*d^4*n-2*B^
2*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^4*d^8*n^2-2*B^2*ln(e*((b*x+a)/(d*x+c))^n
)*b^8*c^4*d^4*n^2-8*A^2*x*a^3*b^5*d^8*n+8*A^2*x*b^8*c^3*d^5*n+24*A^2*ln(e*
((b*x+a)/(d*x+c))^n)*a^2*b^6*c^2*d^6+60*B^2*x^4*ln(e*((b*x+a)/(d*x+c))^n)*
b^8*d^8*n^2+24*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*b^8*d^8+16*B^2*x^3*ln(e
*((b*x+a)/(d*x+c))^n)^3*a*b^7*d^8-96*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a^2*b
^6*c*d^7*n+96*A*B*x*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*c^2*d^6*n-48*A*B*x^3*l
n(e*((b*x+a)/(d*x+c))^n)*a*b^7*d^8*n+48*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)*
b^8*c*d^7*n+192*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*a*b^7*c*d^7*n^2+96*A*...
```

### 3.208.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 3062 vs. 2(720) = 1440.

Time = 0.39 (sec) , antiderivative size = 3062, normalized size of antiderivative = 4.18

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x
, algorithm="fricas")
```

output
```
-1/4*(2*A^2*b^4*c^4 - 16*A^2*a*b^3*c^3*d + 16*A^2*a^3*b*c*d^3 - 2*A^2*a^4*
d^4 - 12*(2*A^2*b^4*c*d^3 - 2*A^2*a*b^3*d^4 + 5*(B^2*b^4*c*d^3 - B^2*a*b^3
*d^4)*n^2)*x^3 - 8*(B^2*b^4*d^4*n^2*x^4 + B^2*a^2*b^2*c^2*d^2*n^2 + 2*(B^2
*b^4*c*d^3 + B^2*a*b^3*d^4)*n^2*x^3 + (B^2*b^4*c^2*d^2 + 4*B^2*a*b^3*c*d^3
 + B^2*a^2*b^2*d^4)*n^2*x^2 + 2*(B^2*a*b^3*c^2*d^2 + B^2*a^2*b^2*c*d^3)*n^
2*x)*log((b*x + a)/(d*x + c))^3 + (B^2*b^4*c^4 - 32*B^2*a*b^3*c^3*d + 32*B
^2*a^3*b*c*d^3 - B^2*a^4*d^4)*n^2 - 6*(6*A^2*b^4*c^2*d^2 - 6*A^2*a^2*b^2*d
^4 + 15*(B^2*b^4*c^2*d^2 - B^2*a^2*b^2*d^4)*n^2 + 4*(A*B*b^4*c^2*d^2 - 2*A
*B*a*b^3*c*d^3 + A*B*a^2*b^2*d^4)*n)*x^2 + 2*(B^2*b^4*c^4 - 8*B^2*a*b^3*c^
3*d + 8*B^2*a^3*b*c*d^3 - B^2*a^4*d^4 - 12*(B^2*b^4*c*d^3 - B^2*a*b^3*d^4)
*x^3 - 18*(B^2*b^4*c^2*d^2 - B^2*a^2*b^2*d^4)*x^2 - 4*(B^2*b^4*c^3*d + 6*B
^2*a*b^3*c^2*d^2 - 6*B^2*a^2*b^2*c*d^3 - B^2*a^3*b*d^4)*x - 12*(B^2*b^4*d^
4*x^4 + B^2*a^2*b^2*c^2*d^2 + 2*(B^2*b^4*c*d^3 + B^2*a*b^3*d^4)*x^3 + (B^2
*b^4*c^2*d^2 + 4*B^2*a*b^3*c*d^3 + B^2*a^2*b^2*d^4)*x^2 + 2*(B^2*a*b^3*c^2
*d^2 + B^2*a^2*b^2*c*d^3)*x)*log((b*x + a)/(d*x + c)))*log(e)^2 - 2*(12*A*
B*b^4*d^4*n*x^4 + 12*A*B*a^2*b^2*c^2*d^2*n + 12*((B^2*b^4*c*d^3 - B^2*a*b^
3*d^4)*n^2 + 2*(A*B*b^4*c*d^3 + A*B*a*b^3*d^4)*n)*x^3 - (B^2*b^4*c^4 - 8*B
^2*a*b^3*c^3*d + 8*B^2*a^3*b*c*d^3 - B^2*a^4*d^4)*n^2 + 6*(3*(B^2*b^4*c^2*
d^2 - B^2*a^2*b^2*d^4)*n^2 + 2*(A*B*b^4*c^2*d^2 + 4*A*B*a*b^3*c*d^3 + A*B*
a^2*b^2*d^4)*n)*x^2 + 4*((B^2*b^4*c^3*d + 6*B^2*a*b^3*c^2*d^2 - 6*B^2*a...
```

### 3.208.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**3/(d*i*x+c*i)**
3,x)
```

output
```
Timed out
```

### 3.208.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 5594 vs. 2(720) = 1440.

Time = 0.66 (sec) , antiderivative size = 5594, normalized size of antiderivative = 7.64

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x
, algorithm="maxima")
```

output
```
1/2*B^2*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c*d^2 - a^3*d
^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c*d^2 + a^2*b
*d^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 - 4*a^3*b^3*c
*d^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d^2 + 2*a^2*b
^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*g^3*i^3*x^3
+ (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^2*c^2*d^4 +
a^6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a^3*b^3*c^4*d^2
+ 2*a^4*b^2*c^3*d^3 - 3*a^5*b*c^2*d^4 + a^6*c*d^5)*g^3*i^3*x + (a^2*b^4*c^
6 - 4*a^3*b^3*c^5*d + 6*a^4*b^2*c^4*d^2 - 4*a^5*b*c^3*d^3 + a^6*c^2*d^4)*g
^3*i^3) + 12*b^2*d^2*log(b*x + a)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c
^3*d^2 - 10*a^3*b^2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3) - 12*b^2*d
^2*log(d*x + c)/((b^5*c^5 - 5*a*b^4*c^4*d + 10*a^2*b^3*c^3*d^2 - 10*a^3*b^
2*c^2*d^3 + 5*a^4*b*c*d^4 - a^5*d^5)*g^3*i^3))*log(e*(b*x/(d*x + c) + a/(d
*x + c))^n)^2 + A*B*((12*b^3*d^3*x^3 - b^3*c^3 + 7*a*b^2*c^2*d + 7*a^2*b*c
*d^2 - a^3*d^3 + 18*(b^3*c*d^2 + a*b^2*d^3)*x^2 + 4*(b^3*c^2*d + 7*a*b^2*c
*d^2 + a^2*b*d^3)*x)/((b^6*c^4*d^2 - 4*a*b^5*c^3*d^3 + 6*a^2*b^4*c^2*d^4 -
 4*a^3*b^3*c*d^5 + a^4*b^2*d^6)*g^3*i^3*x^4 + 2*(b^6*c^5*d - 3*a*b^5*c^4*d
^2 + 2*a^2*b^4*c^3*d^3 + 2*a^3*b^3*c^2*d^4 - 3*a^4*b^2*c*d^5 + a^5*b*d^6)*
g^3*i^3*x^3 + (b^6*c^6 - 9*a^2*b^4*c^4*d^2 + 16*a^3*b^3*c^3*d^3 - 9*a^4*b^
2*c^2*d^4 + a^6*d^6)*g^3*i^3*x^2 + 2*(a*b^5*c^6 - 3*a^2*b^4*c^5*d + 2*a...
```

### 3.208.8 Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx=\int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^3(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^3/(d*i*x+c*i)^3,x
, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/((b*g*x + a*g)^3*(d*i*x
 + c*i)^3), x)
```

### 3.208.9 Mupad [B] (verification not implemented)

Time = 8.43 (sec) , antiderivative size = 2419, normalized size of antiderivative = 3.30

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx=\text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^3*(c*i + d*i*x
)^3),x)
```

output
```
((3*x^2*(6*A^2*a*b^2*d^3 + 6*A^2*b^3*c*d^2 + 15*B^2*a*b^2*d^3*n^2 + 15*B^2
*b^3*c*d^2*n^2 - 4*A*B*a*b^2*d^3*n + 4*A*B*b^3*c*d^2*n))/(a*d - b*c) - (2*
A^2*a^3*d^3 + 2*A^2*b^3*c^3 + B^2*a^3*d^3*n^2 + B^2*b^3*c^3*n^2 - 14*A^2*a
*b^2*c^2*d - 14*A^2*a^2*b*c*d^2 - 2*A*B*a^3*d^3*n + 2*A*B*b^3*c^3*n - 31*B
^2*a*b^2*c^2*d*n^2 - 31*B^2*a^2*b*c*d^2*n^2 - 30*A*B*a*b^2*c^2*d*n + 30*A*
B*a^2*b*c*d^2*n)/(2*(a*d - b*c)) + (2*x*(2*A^2*a^2*b*d^3 + 2*A^2*b^3*c^2*d
 + 14*A^2*a*b^2*c*d^2 + 7*B^2*a^2*b*d^3*n^2 + 7*B^2*b^3*c^2*d*n^2 + 31*B^2
*a*b^2*c*d^2*n^2 - 6*A*B*a^2*b*d^3*n + 6*A*B*b^3*c^2*d*n))/(a*d - b*c) + (
6*x^3*(2*A^2*b^3*d^3 + 5*B^2*b^3*d^3*n^2))/(a*d - b*c))/(x^4*(2*a^3*b^2*d^
5*g^3*i^3 - 2*b^5*c^3*d^2*g^3*i^3 + 6*a*b^4*c^2*d^3*g^3*i^3 - 6*a^2*b^3*c*
d^4*g^3*i^3) - x*(4*a*b^4*c^5*g^3*i^3 - 4*a^5*c*d^4*g^3*i^3 - 8*a^2*b^3*c^
4*d*g^3*i^3 + 8*a^4*b*c^2*d^3*g^3*i^3) + x^3*(4*a^4*b*d^5*g^3*i^3 - 4*b^5*
c^4*d*g^3*i^3 + 8*a*b^4*c^3*d^2*g^3*i^3 - 8*a^3*b^2*c*d^4*g^3*i^3) + x^2*(
2*a^5*d^5*g^3*i^3 - 2*b^5*c^5*g^3*i^3 - 2*a*b^4*c^4*d*g^3*i^3 + 2*a^4*b*c*
d^4*g^3*i^3 + 16*a^2*b^3*c^3*d^2*g^3*i^3 - 16*a^3*b^2*c^2*d^3*g^3*i^3) - 2
*a^2*b^3*c^5*g^3*i^3 + 2*a^5*c^2*d^3*g^3*i^3 + 6*a^3*b^2*c^4*d*g^3*i^3 - 6
*a^4*b*c^3*d^2*g^3*i^3) + log(e*((a + b*x)/(c + d*x))^n)^2*((x*((3*B^2*b*d
*(a*d + b*c)^2)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^2 - (B^2*b*d)/(a^2*d^2 + b
^2*c^2 - 2*a*b*c*d) + (6*B^2*a*b^2*c*d^2)/(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)^
2) - (B^2*(a*d + b*c))/(2*(a^2*d^2 + b^2*c^2 - 2*a*b*c*d)) + (6*B^2*b^3...
```

3.208. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^3(ci+dix)^3}\,dx$

# 3.209 $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx$

### 3.209.1 Optimal result

Integrand size = 45, antiderivative size = 908

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = -\frac{B^2d^5n^2(a+bx)^2}{4(bc-ad)^6g^4i^3(c+dx)^2} - \frac{10AbBd^4n(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} + \frac{10bB^2d^4n^2(a+bx)}{(bc-ad)^6g^4i^3(c+dx)} - \frac{20b^3B^2d^2n^2(c+dx)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4B^2dn^2(c+dx)^2}{4(bc-ad)^6g^4i^3(a+bx)^2} - \frac{2b^5B^2n^2(c+dx)^3}{27(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10bB^2d^4n(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)^6g^4i^3(c+dx)} + \frac{Bd^5n(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(c+dx)^2} - \frac{20b^3Bd^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4Bdn(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{2b^5Bn(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{9(bc-ad)^6g^4i^3(a+bx)^3} - \frac{d^5(a+bx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^6g^4i^3(c+dx)^2} + \frac{5bd^4(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^6g^4i^3(c+dx)} - \frac{10b^3d^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)^6g^4i^3(a+bx)} + \frac{5b^4d(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{2(bc-ad)^6g^4i^3(a+bx)^2} - \frac{b^5(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{3(bc-ad)^6g^4i^3(a+bx)^3} - \frac{10b^2d^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{3B(bc-ad)^6g^4i^3n}$$

output
```
-1/4*B^2*d^5*n^2*(b*x+a)^2/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-10*A*b*B*d^4*n*(
b*x+a)/(-a*d+b*c)^6/g^4/i^3/(d*x+c)+10*b*B^2*d^4*n^2*(b*x+a)/(-a*d+b*c)^6/
g^4/i^3/(d*x+c)-20*b^3*B^2*d^2*n^2*(d*x+c)/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/
4*b^4*B^2*d*n^2*(d*x+c)^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-2/27*b^5*B^2*n^2*
(d*x+c)^3/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-10*b*B^2*d^4*n*(b*x+a)*ln(e*((b*x
+a)/(d*x+c))^n)/(-a*d+b*c)^6/g^4/i^3/(d*x+c)+1/2*B*d^5*n*(b*x+a)^2*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(d*x+c)^2-20*b^3*B*d^2*n*(d*
x+c)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*
B*d*n*(d*x+c)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^4/i^3/(b*x+
a)^2-2/9*b^5*B*n*(d*x+c)^3*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)^6/g^
4/i^3/(b*x+a)^3-1/2*d^5*(b*x+a)^2*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+
b*c)^6/g^4/i^3/(d*x+c)^2+5*b*d^4*(b*x+a)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2
/(-a*d+b*c)^6/g^4/i^3/(d*x+c)-10*b^3*d^2*(d*x+c)*(A+B*ln(e*((b*x+a)/(d*x+c
))^n))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)+5/2*b^4*d*(d*x+c)^2*(A+B*ln(e*((b*x+
a)/(d*x+c))^n))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^2-1/3*b^5*(d*x+c)^3*(A+B*ln
(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)^6/g^4/i^3/(b*x+a)^3-10/3*b^2*d^3*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))^3/B/(-a*d+b*c)^6/g^4/i^3/n
```

### 3.209.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 2138 vs. $2(908) = 1816$.

Time = 1.90 (sec) , antiderivative size = 2138, normalized size of antiderivative = 2.35

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Result too large to show}$$

input
```
Integrate[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i +
 d*i*x)^3),x]
```

output
```
-1/108*(360*b^2*B^2*d^3*n^2*(a + b*x)^3*(c + d*x)^2*Log[(a + b*x)/(c + d*x
)]^3 + 18*B*n*Log[(a + b*x)/(c + d*x)]^2*(60*a^3*A*b^2*c^2*d^3 + 2*b^5*B*c
^5*n - 15*a*b^4*B*c^4*d*n + 60*a^2*b^3*B*c^3*d^2*n - 30*a^4*b*B*c*d^4*n +
3*a^5*B*d^5*n + 180*a^2*A*b^3*c^2*d^3*x + 120*a^3*A*b^2*c*d^4*x - 5*b^5*B*
c^4*d*n*x + 60*a*b^4*B*c^3*d^2*n*x + 180*a^2*b^3*B*c^2*d^3*n*x - 120*a^3*b
^2*B*c*d^4*n*x - 15*a^4*b*B*d^5*n*x + 180*a*A*b^4*c^2*d^3*x^2 + 360*a^2*A*
b^3*c*d^4*x^2 + 60*a^3*A*b^2*d^5*x^2 + 20*b^5*B*c^3*d^2*n*x^2 + 270*a*b^4*
B*c^2*d^3*n*x^2 - 90*a^3*b^2*B*d^5*n*x^2 + 60*A*b^5*c^2*d^3*x^3 + 360*a*A*
b^4*c*d^4*x^3 + 180*a^2*A*b^3*d^5*x^3 + 110*b^5*B*c^2*d^3*n*x^3 + 180*a*b^
4*B*c*d^4*n*x^3 - 90*a^2*b^3*B*d^5*n*x^3 + 120*A*b^5*c*d^4*x^4 + 180*a*A*b
^4*d^5*x^4 + 100*b^5*B*c*d^4*n*x^4 + 60*A*b^5*d^5*x^5 + 20*b^5*B*d^5*n*x^5
 + 60*b^2*B*d^3*(a + b*x)^3*(c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n] - 6
0*b^2*B*d^3*n*(a + b*x)^3*(c + d*x)^2*Log[(a + b*x)/(c + d*x)]) + 6*b^2*d^
2*(b*c - a*d)*(a + b*x)^2*(c + d*x)^2*(108*A^2 + 282*A*B*n + 319*B^2*n^2 +
 108*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(36*A + 47*B*n)*Log[(a +
 b*x)/(c + d*x)] + 108*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*((a
+ b*x)/(c + d*x))^n]*(36*A + 47*B*n - 36*B*n*Log[(a + b*x)/(c + d*x)])) -
3*b^2*d*(b*c - a*d)^2*(a + b*x)*(c + d*x)^2*(54*A^2 + 66*A*B*n + 37*B^2*n^
2 + 54*B^2*Log[e*((a + b*x)/(c + d*x))^n]^2 - 6*B*n*(18*A + 11*B*n)*Log[(a
 + b*x)/(c + d*x)] + 54*B^2*n^2*Log[(a + b*x)/(c + d*x)]^2 + 6*B*Log[e*...
```

### 3.209.3 Rubi [A] (verified)

Time = 0.77 (sec) , antiderivative size = 653, normalized size of antiderivative = 0.72, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.067, Rules used = {2961, 2795, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{(ag+bgx)^4(ci+dix)^3}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{\int \frac{(c+dx)^4\left(b-\frac{d(a+bx)}{c+dx}\right)^5\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)^4}d\frac{a+bx}{c+dx}}{g^4i^3(bc-ad)^6}$$

$$\Big\downarrow \text{ 2795}$$

$$\frac{\int \left( \frac{(c+dx)^4\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^5}{(a+bx)^4} - \frac{5d(c+dx)^3\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^4}{(a+bx)^3} + \frac{10d^2(c+dx)^2\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 b^3}{(a+bx)^2} - \frac{10d^3(c+d}{}\right.}{g^4i^3(bc-ad)^6}$$

$$\Big\downarrow \text{2009}$$

$$-\frac{b^5(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{3(a+bx)^3} - \frac{2b^5Bn(c+dx)^3\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{9(a+bx)^3} + \frac{5b^4d(c+dx)^2\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2(a+bx)^2} + \frac{5b^4Bdn(c+dx}{}$$

input
```
Int[(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/((a*g + b*g*x)^4*(c*i + d*i*x
)^3),x]
```

output
```
(-1/4*(B^2*d^5*n^2*(a + b*x)^2)/(c + d*x)^2 - (10*A*b*B*d^4*n*(a + b*x))/(
c + d*x) + (10*b*B^2*d^4*n^2*(a + b*x))/(c + d*x) - (20*b^3*B^2*d^2*n^2*(c
 + d*x))/(a + b*x) + (5*b^4*B^2*d*n^2*(c + d*x)^2)/(4*(a + b*x)^2) - (2*b^
5*B^2*n^2*(c + d*x)^3)/(27*(a + b*x)^3) - (10*b*B^2*d^4*n*(a + b*x)*Log[e*
((a + b*x)/(c + d*x))^n])/(c + d*x) + (B*d^5*n*(a + b*x)^2*(A + B*Log[e*((
a + b*x)/(c + d*x))^n]))/(2*(c + d*x)^2) - (20*b^3*B*d^2*n*(c + d*x)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(a + b*x) + (5*b^4*B*d*n*(c + d*x)^2*(A
 + B*Log[e*((a + b*x)/(c + d*x))^n]))/(2*(a + b*x)^2) - (2*b^5*B*n*(c + d*
x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(9*(a + b*x)^3) - (d^5*(a + b
*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(c + d*x)^2) + (5*b*d^4
*(a + b*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(c + d*x) - (10*b^3*d
^2*(c + d*x)*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(a + b*x) + (5*b^4*
d*(c + d*x)^2*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(2*(a + b*x)^2) -
(b^5*(c + d*x)^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2)/(3*(a + b*x)^3)
 - (10*b^2*d^3*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^3)/(3*B*n))/((b*c -
a*d)^6*g^4*i^3)
```

3.209. $\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx$

### 3.209.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2795
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((f_.)*(x_))^(m_.)*((d_) +
(e_.)*(x_)^(r_.))^(q_.), x_Symbol] :> With[{u = ExpandIntegrand[(a + b*Log[
c*x^n])^p, (f*x)^m*(d + e*x^r)^q, x]}, Int[u, x] /; SumQ[u]] /; FreeQ[{a, b
, c, d, e, f, m, n, p, q, r}, x] && IntegerQ[q] && (GtQ[q, 0] || (IGtQ[p, 0
] && IntegerQ[m] && IntegerQ[r]))
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.209.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 4485 vs. $2(888) = 1776$.

Time = 205.16 (sec) , antiderivative size = 4486, normalized size of antiderivative = 4.94

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 4486 |

input
```
int((A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x,method
=_RETURNVERBOSE)
```

output
```
-1/108*(1080*A*B*x^3*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*c^2*d^7-720*A*B*x^3*
a^2*b^8*d^9*n^2+2160*A*B*x^3*b^10*c^2*d^7*n^2+2160*B^2*x^2*ln(e*((b*x+a)/(
d*x+c))^n)^3*a^2*b^8*c*d^8+1080*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^9*
c^2*d^7-1620*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^7*d^9*n+360*B^2*x^2
*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*c^3*d^6*n+3780*B^2*x^2*ln(e*((b*x+a)/(d*
x+c))^n)*a^3*b^7*d^9*n^2+1320*B^2*x^2*ln(e*((b*x+a)/(d*x+c))^n)*b^10*c^3*d
^6*n^2-5880*B^2*x^2*a^2*b^8*c*d^8*n^3+9210*B^2*x^2*a*b^9*c^2*d^7*n^3+6480*
A^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c*d^8+6600*B^2*x^4*ln(e*((b*x+a)/(
d*x+c))^n)*b^10*c*d^8*n^2+3240*A*B*x^4*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b^9*d
^9+16200*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)*a*b^9*c*d^8*n^2-540*B^2*ln(e*((
b*x+a)/(d*x+c))^n)^2*a^4*b^6*c*d^8*n+1080*B^2*ln(e*((b*x+a)/(d*x+c))^n)^2*
a^2*b^8*c^3*d^6*n-90*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*b^10*c^4*d^5*n+810*
B^2*x*ln(e*((b*x+a)/(d*x+c))^n)*a^4*b^6*d^9*n^2-150*B^2*x*ln(e*((b*x+a)/(d
*x+c))^n)*b^10*c^4*d^5*n^2-6280*B^2*x*a^3*b^7*c*d^8*n^3+1080*A^2*x^3*a*b^9
*c*d^8*n+1080*A*B*x^2*ln(e*((b*x+a)/(d*x+c))^n)^2*a^3*b^7*d^9+840*A*B*x^2*
a^3*b^7*d^9*n^2+1320*A*B*x^2*b^10*c^3*d^6*n^2+720*B^2*x*ln(e*((b*x+a)/(d*x
+c))^n)^3*a^3*b^7*c*d^8+1080*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^3*a^2*b^8*c^2
*d^7-270*B^2*x*ln(e*((b*x+a)/(d*x+c))^n)^2*a^4*b^6*d^9*n+2160*A*B*x^4*ln(e
*((b*x+a)/(d*x+c))^n)^2*b^10*c*d^8-720*A*B*x^4*a*b^9*d^9*n^2+720*A*B*x^4*b
^10*c*d^8*n^2+2160*B^2*x^3*ln(e*((b*x+a)/(d*x+c))^n)^3*a*b^9*c*d^8-1620...
```

### 3.209.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 4725 vs. 2(888) = 1776.

Time = 0.49 (sec) , antiderivative size = 4725, normalized size of antiderivative = 5.20

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x
, algorithm="fricas")
```

output
```
-1/108*(36*A^2*b^5*c^5 - 270*A^2*a*b^4*c^4*d + 1080*A^2*a^2*b^3*c^3*d^2 -
360*A^2*a^3*b^2*c^2*d^3 - 540*A^2*a^4*b*c*d^4 + 54*A^2*a^5*d^5 + 60*(18*A^
2*b^5*c*d^4 - 18*A^2*a*b^4*d^5 + 49*(B^2*b^5*c*d^4 - B^2*a*b^4*d^5)*n^2 +
12*(A*B*b^5*c*d^4 - A*B*a*b^4*d^5)*n)*x^4 + 30*(54*A^2*b^5*c^2*d^3 + 36*A^
2*a*b^4*c*d^4 - 90*A^2*a^2*b^3*d^5 + (159*B^2*b^5*c^2*d^3 + 74*B^2*a*b^4*c
*d^4 - 233*B^2*a^2*b^3*d^5)*n^2 + 24*(3*A*B*b^5*c^2*d^3 - 2*A*B*a*b^4*c*d^
4 - A*B*a^2*b^3*d^5)*n)*x^3 + 360*(B^2*b^5*d^5*n^2*x^5 + B^2*a^3*b^2*c^2*d
^3*n^2 + (2*B^2*b^5*c*d^4 + 3*B^2*a*b^4*d^5)*n^2*x^4 + (B^2*b^5*c^2*d^3 +
6*B^2*a*b^4*c*d^4 + 3*B^2*a^2*b^3*d^5)*n^2*x^3 + (3*B^2*a*b^4*c^2*d^3 + 6*
B^2*a^2*b^3*c*d^4 + B^2*a^3*b^2*d^5)*n^2*x^2 + (3*B^2*a^2*b^3*c^2*d^3 + 2*
B^2*a^3*b^2*c*d^4)*n^2*x)*log((b*x + a)/(d*x + c))^3 + (8*B^2*b^5*c^5 - 13
5*B^2*a*b^4*c^4*d + 2160*B^2*a^2*b^3*c^3*d^2 - 980*B^2*a^3*b^2*c^2*d^3 - 1
080*B^2*a^4*b*c*d^4 + 27*B^2*a^5*d^5)*n^2 + 10*(36*A^2*b^5*c^3*d^2 + 378*A
^2*a*b^4*c^2*d^3 - 216*A^2*a^2*b^3*c*d^4 - 198*A^2*a^3*b^2*d^5 + (170*B^2*
b^5*c^3*d^2 + 921*B^2*a*b^4*c^2*d^3 - 588*B^2*a^2*b^3*c*d^4 - 503*B^2*a^3*
b^2*d^5)*n^2 + 12*(11*A*B*b^5*c^3*d^2 + 21*A*B*a*b^4*c^2*d^3 - 39*A*B*a^2*
b^3*c*d^4 + 7*A*B*a^3*b^2*d^5)*n)*x^2 + 18*(2*B^2*b^5*c^5 - 15*B^2*a*b^4*c
^4*d + 60*B^2*a^2*b^3*c^3*d^2 - 20*B^2*a^3*b^2*c^2*d^3 - 30*B^2*a^4*b*c*d^
4 + 3*B^2*a^5*d^5 + 60*(B^2*b^5*c*d^4 - B^2*a*b^4*d^5)*x^4 + 30*(3*B^2*b^5
*c^2*d^3 + 2*B^2*a*b^4*c*d^4 - 5*B^2*a^2*b^3*d^5)*x^3 + 10*(2*B^2*b^5*c...
```

### 3.209.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*((b*x+a)/(d*x+c))**n))**2/(b*g*x+a*g)**4/(d*i*x+c*i)**
3,x)
```

output
```
Timed out
```

### 3.209.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 9293 vs. 2(888) = 1776.

Time = 1.04 (sec) , antiderivative size = 9293, normalized size of antiderivative = 10.23

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x
, algorithm="maxima")
```

output
```
-1/6*B^2*((60*b^4*d^4*x^4 + 2*b^4*c^4 - 13*a*b^3*c^3*d + 47*a^2*b^2*c^2*d^
2 + 27*a^3*b*c*d^3 - 3*a^4*d^4 + 30*(3*b^4*c*d^3 + 5*a*b^3*d^4)*x^3 + 10*(
2*b^4*c^2*d^2 + 23*a*b^3*c*d^3 + 11*a^2*b^2*d^4)*x^2 - 5*(b^4*c^3*d - 11*a
*b^3*c^2*d^2 - 35*a^2*b^2*c*d^3 - 3*a^3*b*d^4)*x)/((b^8*c^5*d^2 - 5*a*b^7*
c^4*d^3 + 10*a^2*b^6*c^3*d^4 - 10*a^3*b^5*c^2*d^5 + 5*a^4*b^4*c*d^6 - a^5*
b^3*d^7)*g^4*i^3*x^5 + (2*b^8*c^6*d - 7*a*b^7*c^5*d^2 + 5*a^2*b^6*c^4*d^3
+ 10*a^3*b^5*c^3*d^4 - 20*a^4*b^4*c^2*d^5 + 13*a^5*b^3*c*d^6 - 3*a^6*b^2*d
^7)*g^4*i^3*x^4 + (b^8*c^7 + a*b^7*c^6*d - 17*a^2*b^6*c^5*d^2 + 35*a^3*b^5
*c^4*d^3 - 25*a^4*b^4*c^3*d^4 - a^5*b^3*c^2*d^5 + 9*a^6*b^2*c*d^6 - 3*a^7*
b*d^7)*g^4*i^3*x^3 + (3*a*b^7*c^7 - 9*a^2*b^6*c^6*d + a^3*b^5*c^5*d^2 + 25
*a^4*b^4*c^4*d^3 - 35*a^5*b^3*c^3*d^4 + 17*a^6*b^2*c^2*d^5 - a^7*b*c*d^6 -
 a^8*d^7)*g^4*i^3*x^2 + (3*a^2*b^6*c^7 - 13*a^3*b^5*c^6*d + 20*a^4*b^4*c^5
*d^2 - 10*a^5*b^3*c^4*d^3 - 5*a^6*b^2*c^3*d^4 + 7*a^7*b*c^2*d^5 - 2*a^8*c*
d^6)*g^4*i^3*x + (a^3*b^5*c^7 - 5*a^4*b^4*c^6*d + 10*a^5*b^3*c^5*d^2 - 10*
a^6*b^2*c^4*d^3 + 5*a^7*b*c^3*d^4 - a^8*c^2*d^5)*g^4*i^3) + 60*b^2*d^3*log
(b*x + a)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c^3*
d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3) - 60*b^2*d^3*
log(d*x + c)/((b^6*c^6 - 6*a*b^5*c^5*d + 15*a^2*b^4*c^4*d^2 - 20*a^3*b^3*c
^3*d^3 + 15*a^4*b^2*c^2*d^4 - 6*a^5*b*c*d^5 + a^6*d^6)*g^4*i^3))*log(e*(b*
x/(d*x + c) + a/(d*x + c))^n)^2 - 1/3*A*B*((60*b^4*d^4*x^4 + 2*b^4*c^4 ...
```

### 3.209.8 Giac [F]

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \int \frac{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}{(bgx+ag)^4(dix+ci)^3}\,dx$$

input
```
integrate((A+B*log(e*((b*x+a)/(d*x+c))^n))^2/(b*g*x+a*g)^4/(d*i*x+c*i)^3,x
, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2/((b*g*x + a*g)^4*(d*i*x
 + c*i)^3), x)
```

### 3.209.9 Mupad [B] (verification not implemented)

Time = 10.11 (sec) , antiderivative size = 4649, normalized size of antiderivative = 5.12

$$\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(ag+bgx)^4(ci+dix)^3}\,dx = \text{Too large to display}$$

input
```
int((A + B*log(e*((a + b*x)/(c + d*x))^n))^2/((a*g + b*g*x)^4*(c*i + d*i*x
)^3),x)
```

output
```
log(e*((a + b*x)/(c + d*x))^n)*((x*((a*d + b*c)*(20*A*B*a*b*d^2 + 10*A*B*b
^2*c*d - (70*B^2*a*b*d^2*n)/3 + (10*B^2*b^2*c*d*n)/3) + a*c*(30*A*B*b^2*d^
2 - 20*B^2*b^2*d^2*n) + (5*B^2*a^2*b*d^3*n)/6 + (5*B^2*b^3*c^2*d*n)/6 - 5*
A*B*a^2*b*d^3 - 5*A*B*b^3*c^2*d + 10*A*B*a*b^2*c*d^2 - (5*B^2*a*b^2*c*d^2*
n)/3) + x^2*((a*d + b*c)*(30*A*B*b^2*d^2 - 20*B^2*b^2*d^2*n) + b*d*(20*A*B
*a*b*d^2 + 10*A*B*b^2*c*d - (70*B^2*a*b*d^2*n)/3 + (10*B^2*b^2*c*d*n)/3))
+ a*c*(20*A*B*a*b*d^2 + 10*A*B*b^2*c*d - (70*B^2*a*b*d^2*n)/3 + (10*B^2*b^
2*c*d*n)/3) - 3*A*B*a^3*d^3 - 2*A*B*b^3*c^3 + b*d*x^3*(30*A*B*b^2*d^2 - 20
*B^2*b^2*d^2*n) + (3*B^2*a^3*d^3*n)/2 - (2*B^2*b^3*c^3*n)/3 + A*B*a*b^2*c^
2*d + 4*A*B*a^2*b*c*d^2 + (17*B^2*a*b^2*c^2*d*n)/6 - (11*B^2*a^2*b*c*d^2*n
)/3)/(x^5*(3*a^4*b^3*d^6*g^4*i^3 + 3*b^7*c^4*d^2*g^4*i^3 - 12*a*b^6*c^3*d^
3*g^4*i^3 - 12*a^3*b^4*c*d^5*g^4*i^3 + 18*a^2*b^5*c^2*d^4*g^4*i^3) + x*(9*
a^2*b^5*c^6*g^4*i^3 + 6*a^7*c*d^5*g^4*i^3 - 30*a^3*b^4*c^5*d*g^4*i^3 - 15*
a^6*b*c^2*d^4*g^4*i^3 + 30*a^4*b^3*c^4*d^2*g^4*i^3) + x^2*(3*a^7*d^6*g^4*i
^3 + 9*a*b^6*c^6*g^4*i^3 + 6*a^6*b*c*d^5*g^4*i^3 - 18*a^2*b^5*c^5*d*g^4*i^
3 - 15*a^3*b^4*c^4*d^2*g^4*i^3 + 60*a^4*b^3*c^3*d^3*g^4*i^3 - 45*a^5*b^2*c
^2*d^4*g^4*i^3) + x^3*(3*b^7*c^6*g^4*i^3 + 9*a^6*b*d^6*g^4*i^3 + 6*a*b^6*c
^5*d*g^4*i^3 - 18*a^5*b^2*c*d^5*g^4*i^3 - 45*a^2*b^5*c^4*d^2*g^4*i^3 + 60*
a^3*b^4*c^3*d^3*g^4*i^3 - 15*a^4*b^3*c^2*d^4*g^4*i^3) + x^4*(9*a^5*b^2*d^6
*g^4*i^3 + 6*b^7*c^5*d*g^4*i^3 - 15*a*b^6*c^4*d^2*g^4*i^3 - 30*a^4*b^3*...
```

# 3.210 $\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p$



## 3.210.1 Optimal result

Integrand size = 49, antiderivative size = 189

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\frac{e^{-\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^m\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}}(i(c+dx))^{-m}\Gamma\left(1+p,-\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)(A+B}{(bc-ad)i^2(1+m)(c+dx)}$$

output
```
(b*x+a)*(g*(b*x+a))^m*GAMMA(p+1,-(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n
)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p/(-a*d+b*c)/exp(A*(1+m)/B/n)/i^2/(1+m)/
((e*((b*x+a)/(d*x+c))^n)^((1+m)/n))/(d*x+c)/((i*(d*x+c))^m)/((-(1+m)*(A+B*
ln(e*((b*x+a)/(d*x+c))^n))/B/n)^p)
```

## 3.210.2 Mathematica [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$

input
```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^p,x]
```

output
```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^p, x]
```

### 3.210.3 Rubi [A] (verified)

Time = 0.53 (sec) , antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2963, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^m(ci+dix)^{-m-2}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p\,dx$$

$$\Big\downarrow \text{ 2963}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int\left(\frac{a+bx}{c+dx}\right)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \text{ 2747}$$

$$\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\int\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{i^2n(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{ 2612}$$

$$\frac{(a+bx)e^{-\frac{A(m+1)}{Bn}}(g(a+bx))^m(i(c+dx))^{-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p\left(-\frac{(m+1)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{Bn}\right.}{i^2(m+1)(c+dx)(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^p,x]
```

output
```
((a + b*x)*(g*(a + b*x))^m*Gamma[1 + p, -(((1 + m)*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(B*n))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p)/((b*c -
 a*d)*E^((A*(1 + m))/(B*n))*i^2*(1 + m)*(e*((a + b*x)/(c + d*x))^n)^((1 +
m)/n)*(c + d*x)*(i*(c + d*x))^m*(-(((1 + m)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(B*n)))^p)
```

### 3.210.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

## 3.210.4 Maple [F]

$$\int \left(bgx+ag\right)^{m} \left(dix+ci\right)^{-2-m} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{p}dx$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p,x)
```

output
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p,x)
```

## 3.210.5 Fricas [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int (bgx+ag)^m(dix+ci)^{-m-2}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^p,x, algorithm="fricas")
```

output
```
integral((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log(e*((b*x + a)/(d*x +
 c))^n) + A)^p, x)
```

### 3.210.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**p,x)
```

output
```
Timed out
```

### 3.210.7 Maxima [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int (bgx+ag)^m(dix+ci)^{-m-2}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^p,x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^p, x)
```

### 3.210.8 Giac [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int (bgx+ag)^m(dix+ci)^{-m-2}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^p,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^p, x)
```

### 3.210.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int \frac{(a\,g+b\,g\,x)^m\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^p}{(c\,i+d\,i\,x)^{m+2}}\,dx$$

input
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^p)/(c*i + d*i*
x)^(m + 2),x)
```

output
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^p)/(c*i + d*i*
x)^(m + 2), x)
```

# 3.211 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p$



## 3.211.1 Optimal result

Integrand size = 49, antiderivative size = 190

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx =$$
$$-\frac{e^{\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\Gamma\left(1+p,\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)(A+}{(bc-ad)i^2(1+m)(c+dx)}$$

output
```
-exp(A*(1+m)/B/n)*(b*x+a)*(g*(b*x+a))^(-2-m)*(e*((b*x+a)/(d*x+c))^n)^((1+m
)/n)*(i*(d*x+c))^(2+m)*GAMMA(p+1,(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n
)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p/(-a*d+b*c)/i^2/(1+m)/(d*x+c)/(((1+m)*(
A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)^p)
```

## 3.211.2 Mathematica [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$

input
```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^p,x]
```

output
```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^p, x]
```

### 3.211.3 Rubi [A] (verified)

Time = 0.55 (sec) , antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2963, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^{-m-2}(ci+dix)^m \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p dx$$

$$\Big\downarrow \text{ 2963}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int \left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \text{ 2747}$$

$$\frac{(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\int \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{i^2n(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{ 2612}$$

$$-\frac{(a+bx)e^{\frac{A(m+1)}{Bn}}(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p\left(\frac{(m+1)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{Bn}\right)^{-p}\Gamma\left(p+1,\frac{(m+1)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{i^2(m+1)(c+dx)(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^p,x]
```

output
```
-((E^((A*(1 + m))/(B*n))*(a + b*x)*(g*(a + b*x))^(-2 - m)*(e*((a + b*x)/(c
 + d*x))^n)^((1 + m)/n)*(i*(c + d*x))^(2 + m)*Gamma[1 + p, ((1 + m)*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p)/((b*c - a*d)*i^2*(1 + m)*(c + d*x)*(((1 + m)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(B*n))^p))
```

### 3.211.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

## 3.211.4 Maple [F]

$$\int (bgx+ag)^{-2-m} (dix+ci)^{m} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{p}dx$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p,x)
```

output
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^p,x)
```

## 3.211.5 Fricas [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^{m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)^{p} dx = \int (bgx+ag)^{-m-2}(dix+ci)^{m}\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)+A\right)^{p} dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^p,x, algorithm="fricas")
```

output
```
integral((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log(e*((b*x + a)/(d*x +
 c))^n) + A)^p, x)
```

### 3.211.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**p,x)
```

output
```
Timed out
```

### 3.211.7 Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx$$
$$=\int (bgx+ag)^{-m-2}(dix+ci)^m\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^p,x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^p, x)
```

### 3.211.8 Giac [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx = \int (bgx+ag)^{-m-2}(dix+ci)^m\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^p\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^p,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^p, x)
```

### 3.211.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p\,dx = \int \frac{(c\,i+d\,i\,x)^m\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^p}{(a\,g+b\,g\,x)^{m+2}}\,dx$$

input
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^p)/(a*g + b*g*
x)^(m + 2),x)
```

output
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^p)/(a*g + b*g*
x)^(m + 2), x)
```

# 3.212 $\int (ag+bgx)^m(ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3$



## 3.212.1 Optimal result

Integrand size = 49, antiderivative size = 292

$$\begin{aligned}
&\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx \\
&= -\frac{6B^3n^3(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{(bc-ad)i^2(1+m)^4(c+dx)} \\
&\quad+\frac{6B^2n^2(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^3(c+dx)} \\
&\quad-\frac{3Bn(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)^2(c+dx)} \\
&\quad+\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)i^2(1+m)(c+dx)}
\end{aligned}$$

output
```
-6*B^3*n^3*(b*x+a)*(g*(b*x+a))^m/(-a*d+b*c)/i^2/(1+m)^4/(d*x+c)/((i*(d*x+c
))^m)+6*B^2*n^2*(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*
d+b*c)/i^2/(1+m)^3/(d*x+c)/((i*(d*x+c))^m)-3*B*n*(b*x+a)*(g*(b*x+a))^m*(A+
B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^2/(1+m)^2/(d*x+c)/((i*(d*x+c))
^m)+(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3/(-a*d+b*c)/i^2
/(1+m)/(d*x+c)/((i*(d*x+c))^m)
```

### 3.212.2 Mathematica [A] (verified)

Time = 4.71 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.71

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$$
$$=\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-1-m}\left(A^3(1+m)^3-3A^2B(1+m)^2n+6AB^2(1+m)n^2-6B^3n^3+3B(\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^3,x]
```

output
```
((a + b*x)*(g*(a + b*x))^m*(i*(c + d*x))^(-1 - m)*(A^3*(1 + m)^3 - 3*A^2*B
*(1 + m)^2*n + 6*A*B^2*(1 + m)*n^2 - 6*B^3*n^3 + 3*B*(1 + m)*(A^2*(1 + m)^
2 - 2*A*B*(1 + m)*n + 2*B^2*n^2)*Log[e*((a + b*x)/(c + d*x))^n] + 3*B^2*(1
 + m)^2*(A + A*m - B*n)*Log[e*((a + b*x)/(c + d*x))^n]^2 + B^3*(1 + m)^3*L
og[e*((a + b*x)/(c + d*x))^n]^3))/((b*c - a*d)*i*(1 + m)^4)
```

### 3.212.3 Rubi [A] (verified)

Time = 0.58 (sec) , antiderivative size = 237, normalized size of antiderivative = 0.81, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082, Rules used = {2963, 2742, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^m(ci+dix)^{-m-2}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3dx$$
$$\Big\downarrow \text{2963}$$
$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int\left(\frac{a+bx}{c+dx}\right)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$
$$\Big\downarrow \text{2742}$$
$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{m+1}-\frac{3Bn\int\left(\frac{a+bx}{c+dx}\right)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d\frac{a+bx}{c+dx}}{m+1}\right.}{i^2(bc-ad)}$$

$$\Big\downarrow \text{ 2742}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{m+1}-\frac{3Bn\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{m+1}-\frac{2Bn\int}{}\right.}{m+1}\right.}{i^2(bc-ad)}$$

$$\Big\downarrow \text{ 2741}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{m+1}-\frac{3Bn\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{m+1}-\frac{2Bn\left(\right.}{}\right.}{}\right.}{i^2(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^3,x]
```

output
```
((g*(a + b*x))^m*((((a + b*x)/(c + d*x))^(1 + m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^3)/(1 + m) - (3*B*n*((((a + b*x)/(c + d*x))^(1 + m)*(A + B*L
og[e*((a + b*x)/(c + d*x))^n])^2)/(1 + m) - (2*B*n*(-((B*n*((a + b*x)/(c +
 d*x))^(1 + m))/(1 + m)^2) + (((a + b*x)/(c + d*x))^(1 + m)*(A + B*Log[e*(
(a + b*x)/(c + d*x))^n]))/(1 + m)))/(1 + m)))/(1 + m)))/((b*c - a*d)*i^2*(
(a + b*x)/(c + d*x))^m*(i*(c + d*x))^m)
```

### 3.212.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

3.212. $\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

### 3.212.4 Maple [F]

$$\int (bgx+ag)^m (dix+ci)^{-2-m} \left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^3 dx$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

output
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

### 3.212.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2680 vs. $2(292) = 584$.

Time = 0.42 (sec) , antiderivative size = 2680, normalized size of antiderivative = 9.18

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="fricas")
```

output
```
(A^3*a*c*m^3 - 6*B^3*a*c*n^3 + 3*A^3*a*c*m^2 + 3*A^3*a*c*m + A^3*a*c + (B^
3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c + (B^3*b*d*m^3 + 3*B^3*b
*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*x^2 + (B^3*b*c + B^3*a*d + (B^3*b*c + B^3*
a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c + B^3*a*d)*m)*x)*log(e)^
3 + ((B^3*b*d*m^3 + 3*B^3*b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*n^3*x^2 + (B^3*
b*c + B^3*a*d + (B^3*b*c + B^3*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B
^3*b*c + B^3*a*d)*m)*n^3*x + (B^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m +
B^3*a*c)*n^3)*log((b*x + a)/(d*x + c))^3 + 6*(A*B^2*a*c*m + A*B^2*a*c)*n^2
 + (A^3*b*d*m^3 - 6*B^3*b*d*n^3 + 3*A^3*b*d*m^2 + 3*A^3*b*d*m + A^3*b*d +
6*(A*B^2*b*d*m + A*B^2*b*d)*n^2 - 3*(A^2*B*b*d*m^2 + 2*A^2*B*b*d*m + A^2*B
*b*d)*n)*x^2 + 3*(A*B^2*a*c*m^3 + 3*A*B^2*a*c*m^2 + 3*A*B^2*a*c*m + A*B^2*
a*c + (A*B^2*b*d*m^3 + 3*A*B^2*b*d*m^2 + 3*A*B^2*b*d*m + A*B^2*b*d - (B^3*
b*d*m^2 + 2*B^3*b*d*m + B^3*b*d)*n)*x^2 - (B^3*a*c*m^2 + 2*B^3*a*c*m + B^3
*a*c)*n + (A*B^2*b*c + A*B^2*a*d + (A*B^2*b*c + A*B^2*a*d)*m^3 + 3*(A*B^2*
b*c + A*B^2*a*d)*m^2 + 3*(A*B^2*b*c + A*B^2*a*d)*m - (B^3*b*c + B^3*a*d +
(B^3*b*c + B^3*a*d)*m^2 + 2*(B^3*b*c + B^3*a*d)*m)*n)*x + ((B^3*b*d*m^3 +
3*B^3*b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*n*x^2 + (B^3*b*c + B^3*a*d + (B^3*b
*c + B^3*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c + B^3*a*d)*m)*n
*x + (B^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c)*n)*log((b*x + a
)/(d*x + c)))*log(e)^2 - 3*((B^3*a*c*m^2 + 2*B^3*a*c*m + B^3*a*c)*n^3 -...
```

### 3.212.6 Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 \,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.212.7 Maxima [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3 (bgx+ag)^m (dix+ci)^{-m-2} dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="maxima")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^3*(b*g*x + a*g)^m*(d*i*x
+ c*i)^(-m - 2), x)
```

### 3.212.8 Giac [F]

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3 (bgx+ag)^m (dix+ci)^{-m-2} dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^3*(b*g*x + a*g)^m*(d*i*x
+ c*i)^(-m - 2), x)
```

### 3.212.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m (ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \int \frac{(a\,g+b\,g\,x)^m \left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^3}{(c\,i+d\,i\,x)^{m+2}} dx$$

input
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^3)/(c*i + d*i*
x)^(m + 2),x)
```

output
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^3)/(c*i + d*i*
x)^(m + 2), x)
```

3.212. $\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$

# 3.213 $\int (ag+bgx)^m(ci+dix)^{-2-m} \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2$



## 3.213.1 Optimal result

Integrand size = 49, antiderivative size = 210

$$\begin{aligned}\int (ag+bgx)^m(ci+dix)^{-2-m} \left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 \, dx &= \frac{2B^2n^2(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{(bc-ad)i^2(1+m)^3(c+dx)} \\ &\quad - \frac{2Bn(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^2(c+dx)} \\ &\quad + \frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)(c+dx)}\end{aligned}$$

output
```
2*B^2*n^2*(b*x+a)*(g*(b*x+a))^m/(-a*d+b*c)/i^2/(1+m)^3/(d*x+c)/((i*(d*x+c)
)^m)-2*B*n*(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c
)/i^2/(1+m)^2/(d*x+c)/((i*(d*x+c))^m)+(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*
x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^2/(1+m)/(d*x+c)/((i*(d*x+c))^m)
```

### 3.213.2 Mathematica [A] (verified)

Time = 1.91 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.64

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-1-m}\left(A^2(1+m)^2-2AB(1+m)n+2B^2n^2+2B(1+m)(A+Am-B\right.}{(bc-ad)i(1+m)^3}$$

input
```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^2,x]
```

output
```
((a + b*x)*(g*(a + b*x))^m*(i*(c + d*x))^(-1 - m)*(A^2*(1 + m)^2 - 2*A*B*(
1 + m)*n + 2*B^2*n^2 + 2*B*(1 + m)*(A + A*m - B*n)*Log[e*((a + b*x)/(c + d
*x))^n] + B^2*(1 + m)^2*Log[e*((a + b*x)/(c + d*x))^n]^2))/((b*c - a*d)*i*
(1 + m)^3)
```

### 3.213.3 Rubi [A] (verified)

Time = 0.48 (sec) , antiderivative size = 180, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2963, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^m(ci+dix)^{-m-2}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2\,dx$$
$$\Big\downarrow \text{ 2963}$$
$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int \left(\frac{a+bx}{c+dx}\right)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$
$$\Big\downarrow \text{ 2742}$$
$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{m+1}-\frac{2Bn\int \left(\frac{a+bx}{c+dx}\right)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}}{m+1}\right.}{i^2(bc-ad)}$$
$$\Big\downarrow \text{ 2741}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{m+1}-\frac{2Bn\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{m+1}-\frac{Bn\left(\frac{a\cdots}{c\cdots}\right.}{(m\cdots}\right.}{m+1}\right.}{i^2(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^2,x]
```

output
```
((g*(a + b*x))^m*((((a + b*x)/(c + d*x))^(1 + m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^2)/(1 + m) - (2*B*n*(-((B*n*((a + b*x)/(c + d*x))^(1 + m))/(
1 + m)^2) + (((a + b*x)/(c + d*x))^(1 + m)*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]))/(1 + m)))/(1 + m)))/((b*c - a*d)*i^2*((a + b*x)/(c + d*x))^m*(i*(
c + d*x))^m)
```

### 3.213.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

### 3.213.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2619 vs. 2(210) = 420.

Time = 134.76 (sec) , antiderivative size = 2620, normalized size of antiderivative = 12.48

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 2620 |

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x,m
ethod=_RETURNVERBOSE)
```

output
```
-(2*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c
*d*m^2*n+4*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n
)*a*b*d^2*m*n+4*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+
c))^n)*b^2*c*d*m*n+2*A*B*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d
*x+c))^n)*a*b*c*d*m^2*n+4*A*B*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+
a)/(d*x+c))^n)*a*b*c*d*m*n+B^2*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*(
(b*x+a)/(d*x+c))^n)^2*b^2*d^2*n-2*B^2*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)
*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*n^2+2*A^2*x^2*(g*(b*x+a))^m*(i*(d*x+c))
^(-2-m)*b^2*d^2*m*n+2*A*B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)
/(d*x+c))^n)*a*b*d^2*m^2*n+B^2*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*(
(b*x+a)/(d*x+c))^n)^2*b^2*d^2*m^2*n+2*B^2*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-
2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*b^2*d^2*m*n-2*B^2*x^2*(g*(b*x+a))^m*(i*(d
*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*m*n^2-2*A*B*x^2*(g*(b*x+a)
)^m*(i*(d*x+c))^(-2-m)*b^2*d^2*m*n^2+A^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m
)*a*b*d^2*m^2*n+A^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*c*d*m^2*n+2*A*B
*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*n+
B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*d^2
*n+B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*b^2*
c*d*n-2*B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a
*b*d^2*n^2-2*B^2*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+...
```

### 3.213.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 991 vs. $2(210) = 420$.

Time = 0.39 (sec) , antiderivative size = 991, normalized size of antiderivative = 4.72

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \frac{\left(A^2acm^2+2\,B^2acn^2+2\,A^2acm+A^2ac+\left(A^2bdm^2+2\,B^2bdn^2+2\,A^2bdm+A^2bd-2\left(ABbdm+ABb\right.\right.\right.}{}$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="fricas")
```

output
```
(A^2*a*c*m^2 + 2*B^2*a*c*n^2 + 2*A^2*a*c*m + A^2*a*c + (A^2*b*d*m^2 + 2*B^
2*b*d*n^2 + 2*A^2*b*d*m + A^2*b*d - 2*(A*B*b*d*m + A*B*b*d)*n)*x^2 + (B^2*
a*c*m^2 + 2*B^2*a*c*m + B^2*a*c + (B^2*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*x^
2 + (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m
)*x)*log(e)^2 + ((B^2*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*n^2*x^2 + (B^2*b*c
+ B^2*a*d + (B^2*b*c + B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m)*n^2*x + (B^
2*a*c*m^2 + 2*B^2*a*c*m + B^2*a*c)*n^2)*log((b*x + a)/(d*x + c))^2 - 2*(A*
B*a*c*m + A*B*a*c)*n + (A^2*b*c + A^2*a*d + (A^2*b*c + A^2*a*d)*m^2 + 2*(B
^2*b*c + B^2*a*d)*n^2 + 2*(A^2*b*c + A^2*a*d)*m - 2*(A*B*b*c + A*B*a*d + (
A*B*b*c + A*B*a*d)*m)*n)*x + 2*(A*B*a*c*m^2 + 2*A*B*a*c*m + A*B*a*c + (A*B
*b*d*m^2 + 2*A*B*b*d*m + A*B*b*d - (B^2*b*d*m + B^2*b*d)*n)*x^2 - (B^2*a*c
*m + B^2*a*c)*n + (A*B*b*c + A*B*a*d + (A*B*b*c + A*B*a*d)*m^2 + 2*(A*B*b*
c + A*B*a*d)*m - (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m)*n)*x + ((B^2*
b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*n*x^2 + (B^2*b*c + B^2*a*d + (B^2*b*c + B
^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m)*n*x + (B^2*a*c*m^2 + 2*B^2*a*c*m +
B^2*a*c)*n)*log((b*x + a)/(d*x + c)))*log(e) - 2*((B^2*a*c*m + B^2*a*c)*n^
2 + ((B^2*b*d*m + B^2*b*d)*n^2 - (A*B*b*d*m^2 + 2*A*B*b*d*m + A*B*b*d)*n)*
x^2 - (A*B*a*c*m^2 + 2*A*B*a*c*m + A*B*a*c)*n + ((B^2*b*c + B^2*a*d + (B^2
*b*c + B^2*a*d)*m)*n^2 - (A*B*b*c + A*B*a*d + (A*B*b*c + A*B*a*d)*m^2 + 2*
(A*B*b*c + A*B*a*d)*m)*n)*x)*log((b*x + a)/(d*x + c))*(b*g*x + a*g)^m*...
```

### 3.213.6 Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.213.7 Maxima [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2(bgx+ag)^m(dix+ci)^{-m-2}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="maxima")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2*(b*g*x + a*g)^m*(d*i*x
+ c*i)^(-m - 2), x)
```

### 3.213.8 Giac [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2(bgx+ag)^m(dix+ci)^{-m-2}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2*(b*g*x + a*g)^m*(d*i*x
+ c*i)^(-m - 2), x)
```

### 3.213.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int \frac{(a\,g+b\,g\,x)^m\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(c\,i+d\,i\,x)^{m+2}}\,dx$$

input
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^(m + 2),x)
```

output
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(c*i + d*i*
x)^(m + 2), x)
```

# 3.214 $\int (ag+bgx)^m(ci+dix)^{-2-m} \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) d$



## 3.214.1 Optimal result

Integrand size = 47, antiderivative size = 128

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = -\frac{Bn(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{(bc-ad)i^2(1+m)^2(c+dx)} + \frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)(c+dx)}$$

output
```
-B*n*(b*x+a)*(g*(b*x+a))^m/(-a*d+b*c)/i^2/(1+m)^2/(d*x+c)/((i*(d*x+c))^m)+
(b*x+a)*(g*(b*x+a))^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^2/(1+m)
/(d*x+c)/((i*(d*x+c))^m)
```

## 3.214.2 Mathematica [A] (verified)

Time = 0.63 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.61

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx = \frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-1-m}\left(A+Am-Bn+B(1+m)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i(1+m)^2}$$

input
```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]),x]
```

output
```
((a + b*x)*(g*(a + b*x))^m*(i*(c + d*x))^(-1 - m)*(A + A*m - B*n + B*(1 +
m)*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)*i*(1 + m)^2)
```

### 3.214.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 123, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.043$, Rules used = {2963, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^m(ci+dix)^{-m-2}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2963}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int\left(\frac{a+bx}{c+dx}\right)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{m+1}-\frac{Bn\left(\frac{a+bx}{c+dx}\right)^{m+1}}{(m+1)^2}\right)}{i^2(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]),x]
```

output
```
((g*(a + b*x))^m*(-((B*n*((a + b*x)/(c + d*x))^(1 + m))/(1 + m)^2) + (((a
+ b*x)/(c + d*x))^(1 + m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(1 + m))
)/((b*c - a*d)*i^2*((a + b*x)/(c + d*x))^m*(i*(c + d*x))^m)
```

### 3.214.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

## 3.214.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 816 vs. $2(128) = 256$.

Time = 32.12 (sec) , antiderivative size = 817, normalized size of antiderivative = 6.38

| method | result |
|---|---|
| parallelrisch | $-\frac{Ax(g(bx+a))^{m}(i(dx+c))^{-2-m}ab\,d^{2}mn+Ax(g(bx+a))^{m}(i(dx+c))^{-2-m}b^{2}cdmn+Bx(g(bx+a))^{m}(i(dx+c))^{-2-m}\ln\left(e\left(\frac{bx}{dx}\right.\right.}{}$ |

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,met
hod=_RETURNVERBOSE)
```

output
```
-(A*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*d^2*m*n+A*x*(g*(b*x+a))^m*(i*(d
*x+c))^(-2-m)*b^2*c*d*m*n+B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+
a)/(d*x+c))^n)*a*b*d^2*n+B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a
)/(d*x+c))^n)*b^2*c*d*n+A*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*c*d*m*n+B*(
g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*n+B*x^2*
(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*m*n+B*x
*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*m*n+B*
x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*m*n+B
*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*m*n+A*
x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*d^2*m*n-B*x^2*(g*(b*x+a))^m*(i*(d
*x+c))^(-2-m)*b^2*d^2*n^2+A*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*d^2*n
+B*x^2*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*
n-B*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*d^2*n^2-B*x*(g*(b*x+a))^m*(i*(d
*x+c))^(-2-m)*b^2*c*d*n^2+A*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*d^2*n+A
*x*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*b^2*c*d*n-B*(g*(b*x+a))^m*(i*(d*x+c))^
(-2-m)*a*b*c*d*n^2+A*(g*(b*x+a))^m*(i*(d*x+c))^(-2-m)*a*b*c*d*n)/d/b/(a*d-
b*c)/n/(1+m)^2
```

### 3.214.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 274 vs. $2(128) = 256$.

Time = 0.32 (sec) , antiderivative size = 274, normalized size of antiderivative = 2.14

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$

$$= \frac{(Aacm - Bacn + Aac + (Abdm - Bbdn + Abd)x^2 + (Abc + Aad + (Abc + Aad)m - (Bbc + Bad)n)x +}{}$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="fricas")
```

output
```
(A*a*c*m - B*a*c*n + A*a*c + (A*b*d*m - B*b*d*n + A*b*d)*x^2 + (A*b*c + A*
a*d + (A*b*c + A*a*d)*m - (B*b*c + B*a*d)*n)*x + (B*a*c*m + B*a*c + (B*b*d
*m + B*b*d)*x^2 + (B*b*c + B*a*d + (B*b*c + B*a*d)*m)*x)*log(e) + ((B*b*d*
m + B*b*d)*n*x^2 + (B*b*c + B*a*d + (B*b*c + B*a*d)*m)*n*x + (B*a*c*m + B*
a*c)*n)*log((b*x + a)/(d*x + c)))*(b*g*x + a*g)^m*e^(-(m + 2)*log(b*g*x +
a*g) + (m + 2)*log((b*x + a)/(d*x + c)) - (m + 2)*log(i/g))/((b*c - a*d)*m
^2 + b*c - a*d + 2*(b*c - a*d)*m)
```

### 3.214.6 Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*((b*x+a)/(d*x+c))**
n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.214.7 Maxima [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)(bgx+ag)^m(dix+ci)^{-m-2}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="maxima")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(b*g*x + a*g)^m*(d*i*x +
c*i)^(-m - 2), x)
```

### 3.214.8 Giac [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)(bgx+ag)^m(dix+ci)^{-m-2}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(b*g*x + a*g)^m*(d*i*x +
c*i)^(-m - 2), x)
```

### 3.214.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\int \frac{(a\,g+b\,g\,x)^m\,\left(A+B\,\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\right)}{(c\,i+d\,i\,x)^{m+2}}\,dx$$

input
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^(m + 2),x)
```

output
```
int(((a*g + b*g*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(c*i + d*i*x)
^(m + 2), x)
```

# 3.215 $\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$



## 3.215.1 Optimal result

Integrand size = 49, antiderivative size = 125

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{e^{-\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^m\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}}(i(c+dx))^{-m}\,\text{ExpIntegralEi}\left(\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)i^2n(c+dx)}$$

output
```
(b*x+a)*(g*(b*x+a))^m*Ei((1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a*
d+b*c)/exp(A*(1+m)/B/n)/i^2/n/((e*((b*x+a)/(d*x+c))^n)^((1+m)/n))/(d*x+c)/
((i*(d*x+c))^m)
```

## 3.215.2 Mathematica [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)
/(c + d*x))^n]),x]
```

output
```
Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)
/(c + d*x))^n]), x]
```

### 3.215.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2963, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-m-2}}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

$$\Big\downarrow \text{ 2963}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int \frac{\left(\frac{a+bx}{c+dx}\right)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \text{ 2747}$$

$$\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{i^2n(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{ 2609}$$

$$\frac{(a+bx)e^{-\frac{A(m+1)}{Bn}}(g(a+bx))^m(i(c+dx))^{-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\text{ExpIntegralEi}\left(\frac{(m+1)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bi^2n(c+dx)(bc-ad)}$$

input
```
Int[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/(c +
d*x))^n]),x]
```

output
```
((a + b*x)*(g*(a + b*x))^m*ExpIntegralEi[((1 + m)*(A + B*Log[e*((a + b*x)/
(c + d*x))^n]))/(B*n)])/(B*(b*c - a*d)*E^((A*(1 + m))/(B*n))*i^2*n*(e*((a
+ b*x)/(c + d*x))^n)^((1 + m)/n)*(c + d*x)*(i*(c + d*x))^m)
```

### 3.215.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

## 3.215.4 Maple [F]

$$\int \frac{(bgx+ag)^m (dix+ci)^{-2-m}}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

## 3.215.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 98, normalized size of antiderivative = 0.78

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\operatorname{Ei}\left(\frac{(Bm+B)n\log\left(\frac{bx+a}{dx+c}\right)+Am+(Bm+B)\log(e)+A}{Bn}\right)e^{\left(-\frac{(Bm+2\,B)n\log\left(\frac{i}{g}\right)+Am+(Bm+B)\log(e)+A}{Bn}\right)}}{(Bbc-Bad)g^2n}$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="fricas")
```

output
```
Ei(((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B*
n))*e^(-((B*m + 2*B)*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n))/((B*b
*c - B*a*d)*g^2*n)
```

### 3.215.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))**
n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.215.7 Maxima [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A), x)
```

### 3.215.8 Giac [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A), x)
```

### 3.215.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(a\,g+b\,g\,x)^m}{(c\,i+d\,i\,x)^{m+2}\,\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

input
```
int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))),x)
```

output
```
int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))), x)
```

# 3.216 $\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$



## 3.216.1 Optimal result

Integrand size = 49, antiderivative size = 206

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$
$$= \frac{e^{-\frac{A(1+m)}{Bn}}(1+m)(a+bx)(g(a+bx))^m\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}}(i(c+dx))^{-m}\operatorname{ExpIntegralEi}\left(\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{Bn}\right.}{B^2(bc-ad)i^2n^2(c+dx)}$$
$$-\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output
```
(1+m)*(b*x+a)*(g*(b*x+a))^m*Ei((1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/
B^2/(-a*d+b*c)/exp(A*(1+m)/B/n)/i^2/n^2/((e*((b*x+a)/(d*x+c))^n)^((1+m)/n)
)/(d*x+c)/((i*(d*x+c))^m)-(b*x+a)*(g*(b*x+a))^m/B/(-a*d+b*c)/i^2/n/(d*x+c)
/((i*(d*x+c))^m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

### 3.216.2 Mathematica [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2,x]
```

output
```
Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2, x]
```

### 3.216.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 201, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082, Rules used = {2963, 2743, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-m-2}}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2963}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int \frac{\left(\frac{a+bx}{c+dx}\right)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \text{2743}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)\int \frac{\left(\frac{a+bx}{c+dx}\right)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}}{Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}\right)}{i^2(bc-ad)}$$

$$\Big\downarrow \text{2747}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\int\frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn^2}-\frac{}{Bn\left(B\log\ldots\right.}\right.}{i^2(bc-ad)}$$

$\downarrow$ 2609

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)e^{-\frac{A(m+1)}{Bn}}\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\text{ExpIntegralEi}\left(\frac{(m+1)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\ldots\right.\right.}{Bn}\right.}{B^2n^2}\right.}{i^2(bc-ad)}$$

input
```
Int[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/(c +
d*x))^n])^2,x]
```

output
```
((g*(a + b*x))^m*(((1 + m)*((a + b*x)/(c + d*x))^(1 + m)*ExpIntegralEi[((1
 + m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/(B^2*E^((A*(1 + m))/
(B*n))*n^2*(e*((a + b*x)/(c + d*x))^n)^((1 + m)/n)) - ((a + b*x)/(c + d*x)
)^(1 + m)/(B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))))/((b*c - a*d)*i^2*
((a + b*x)/(c + d*x))^m*(i*(c + d*x))^m)
```

### 3.216.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

3.216. $\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

## 3.216.4 Maple [F]

$$\int \frac{(bgx + ag)^m (dix + ci)^{-2-m}}{\left(A + B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2} dx$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

## 3.216.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 293, normalized size of antiderivative = 1.42

$$\int \frac{(ag + bgx)^m (ci + dix)^{-2-m}}{\left(A + B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} dx =$$

$$-\frac{(Bbdg^2nx^2 + Bacg^2n + (Bbc + Bad)g^2nx)(bgx + ag)^m e^{\left(-(m+2)\log(bgx+ag)+(m+2)\log\left(\frac{bx+a}{dx+c}\right)-(m+2)\log\left(\frac{i}{g}\right)\right)}}{(B^3bc - B^3ad)g^2n^3\log\left(\frac{b}{d}\right.}$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="fricas")
```

output
```
-((B*b*d*g^2*n*x^2 + B*a*c*g^2*n + (B*b*c + B*a*d)*g^2*n*x)*(b*g*x + a*g)^
m*e^(-(m + 2)*log(b*g*x + a*g) + (m + 2)*log((b*x + a)/(d*x + c)) - (m + 2
)*log(i/g)) - ((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(
e) + A)*Ei(((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e)
+ A)/(B*n))*e^(-((B*m + 2*B)*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n
)))/((B^3*b*c - B^3*a*d)*g^2*n^3*log((b*x + a)/(d*x + c)) + (B^3*b*c - B^3
*a*d)*g^2*n^2*log(e) + (A*B^2*b*c - A*B^2*a*d)*g^2*n^2)
```

### 3.216.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.216.7 Maxima [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="maxima")
```

output
```
-g^m*(m + 1)*integrate(-(b*x + a)^m/((B^2*d^2*i^(m + 2)*n*x^2 + 2*B^2*c*d*
i^(m + 2)*n*x + B^2*c^2*i^(m + 2)*n)*(d*x + c)^m*log((b*x + a)^n) - (B^2*d
^2*i^(m + 2)*n*x^2 + 2*B^2*c*d*i^(m + 2)*n*x + B^2*c^2*i^(m + 2)*n)*(d*x +
 c)^m*log((d*x + c)^n) + (B^2*c^2*i^(m + 2)*n*log(e) + A*B*c^2*i^(m + 2)*n
 + (B^2*d^2*i^(m + 2)*n*log(e) + A*B*d^2*i^(m + 2)*n)*x^2 + 2*(B^2*c*d*i^(
m + 2)*n*log(e) + A*B*c*d*i^(m + 2)*n)*x)*(d*x + c)^m), x) - (b*g^m*x + a*
g^m)*(b*x + a)^m/(((b*c*d*i^(m + 2)*n - a*d^2*i^(m + 2)*n)*B^2*x + (b*c^2*
i^(m + 2)*n - a*c*d*i^(m + 2)*n)*B^2)*(d*x + c)^m*log((b*x + a)^n) - ((b*c
*d*i^(m + 2)*n - a*d^2*i^(m + 2)*n)*B^2*x + (b*c^2*i^(m + 2)*n - a*c*d*i^(
m + 2)*n)*B^2)*(d*x + c)^m*log((d*x + c)^n) + ((b*c^2*i^(m + 2)*n - a*c*d*
i^(m + 2)*n)*A*B + (b*c^2*i^(m + 2)*n*log(e) - a*c*d*i^(m + 2)*n*log(e))*B
^2 + ((b*c*d*i^(m + 2)*n - a*d^2*i^(m + 2)*n)*A*B + (b*c*d*i^(m + 2)*n*log
(e) - a*d^2*i^(m + 2)*n*log(e))*B^2)*x)*(d*x + c)^m)
```

### 3.216.8 Giac [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^2, x)
```

### 3.216.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(a\,g+b\,g\,x)^m}{(c\,i+d\,i\,x)^{m+2}\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^2),x)
```

output
```
int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^2), x)
```

# 3.217 $\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$



## 3.217.1 Optimal result

Integrand size = 49, antiderivative size = 295

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$$
$$= \frac{e^{-\frac{A(1+m)}{Bn}}(1+m)^2(a+bx)(g(a+bx))^m\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{1+m}{n}}(i(c+dx))^{-m}\,\text{ExpIntegralEi}\left(\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{Bn}\right.}{2B^3(bc-ad)i^2n^3(c+dx)}$$
$$-\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}$$
$$-\frac{(1+m)(a+bx)(g(a+bx))^m(i(c+dx))^{-m}}{2B^2(bc-ad)i^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output
```
1/2*(1+m)^2*(b*x+a)*(g*(b*x+a))^m*Ei((1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))
/B/n)/B^3/(-a*d+b*c)/exp(A*(1+m)/B/n)/i^2/n^3/((e*((b*x+a)/(d*x+c))^n)^((1
+m)/n))/(d*x+c)/((i*(d*x+c))^m)-1/2*(b*x+a)*(g*(b*x+a))^m/B/(-a*d+b*c)/i^2
/n/(d*x+c)/((i*(d*x+c))^m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2-1/2*(1+m)*(b*
x+a)*(g*(b*x+a))^m/B^2/(-a*d+b*c)/i^2/n^2/(d*x+c)/((i*(d*x+c))^m)/(A+B*ln(
e*((b*x+a)/(d*x+c))^n))
```

### 3.217.2 Mathematica [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$$

input
```
Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^3,x]
```

output
```
Integrate[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^3, x]
```

### 3.217.3 Rubi [A] (verified)

Time = 0.69 (sec) , antiderivative size = 266, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.102$, Rules used = {2963, 2743, 2743, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-m-2}}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}dx$$

$$\Big\downarrow \text{2963}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int \frac{\left(\frac{a+bx}{c+dx}\right)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \text{2743}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)\int \frac{\left(\frac{a+bx}{c+dx}\right)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{2Bn}-\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}}{2Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}\right)}{i^2(bc-ad)}$$

$$\Big\downarrow \text{2743}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)\left(\frac{(m+1)\int\frac{\left(\frac{a+bx}{c+dx}\right)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{\left(\frac{a+bx}{c+dx}\right)^{m+1}}{Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}\right)}{2Bn}-\frac{\left(\frac{a+b}{c+d}\right.}{2Bn\left(B\log\left(e\right.\right.}\right.}{i^2(bc-ad)}$$

$\downarrow$ 2747

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)\left(\frac{(m+1)\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\int\frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn^2}-\frac{}{Bn\left(B\right.}\right.}{2Bn}\right.}{i^2(bc-ad)}$$

$\downarrow$ 2609

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\left(\frac{(m+1)\left(\frac{(m+1)e^{-\frac{A(m+1)}{Bn}}\left(\frac{a+bx}{c+dx}\right)^{m+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\text{ExpIntegralEi}\left(\frac{(m+1)\left(A+B\log\left(e\left(\frac{a+}{c+}\right.\right.\right.}{Bn}\right.}{B^2n^2}\right.}{2Bn}\right.}{i^2(bc-ad)}$$

input
```
Int[((a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m))/(A + B*Log[e*((a + b*x)/(c +
d*x))^n])^3,x]
```

output
```
((g*(a + b*x))^m*(-1/2*((a + b*x)/(c + d*x))^(1 + m)/(B*n*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2) + ((1 + m)*(((1 + m)*((a + b*x)/(c + d*x))^(1 +
m)*ExpIntegralEi[((1 + m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)])/
(B^2*E^((A*(1 + m))/(B*n))*n^2*(e*((a + b*x)/(c + d*x))^n)^((1 + m)/n)) -
((a + b*x)/(c + d*x))^(1 + m)/(B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))
))/(2*B*n)))/((b*c - a*d)*i^2*((a + b*x)/(c + d*x))^m*(i*(c + d*x))^m)
```

3.217. $\int\frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$

### 3.217.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

## 3.217.4 Maple [F]

$$\int \frac{(bgx+ag)^{m} (dix+ci)^{-2-m}}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)^{3}}dx$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

output
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

### 3.217.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 818 vs. $2(289) = 578$.

Time = 0.34 (sec) , antiderivative size = 818, normalized size of antiderivative = 2.77

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx =$$

$$-\frac{\left(B^2acg^2n^2+(B^2bdg^2n^2+(ABbdg^2m+ABbdg^2)n)x^2+(ABacg^2m+ABacg^2)n+((B^2bc+B^2ad)g^2\right.}{}$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="fricas")
```

output
```
-1/2*((B^2*a*c*g^2*n^2 + (B^2*b*d*g^2*n^2 + (A*B*b*d*g^2*m + A*B*b*d*g^2)*
n)*x^2 + (A*B*a*c*g^2*m + A*B*a*c*g^2)*n + ((B^2*b*c + B^2*a*d)*g^2*n^2 +
((A*B*b*c + A*B*a*d)*g^2*m + (A*B*b*c + A*B*a*d)*g^2)*n)*x + ((B^2*b*d*g^2
*m + B^2*b*d*g^2)*n*x^2 + ((B^2*b*c + B^2*a*d)*g^2*m + (B^2*b*c + B^2*a*d)
*g^2)*n*x + (B^2*a*c*g^2*m + B^2*a*c*g^2)*n)*log(e) + ((B^2*b*d*g^2*m + B^
2*b*d*g^2)*n^2*x^2 + ((B^2*b*c + B^2*a*d)*g^2*m + (B^2*b*c + B^2*a*d)*g^2)
*n^2*x + (B^2*a*c*g^2*m + B^2*a*c*g^2)*n^2)*log((b*x + a)/(d*x + c)))*(b*g
*x + a*g)^m*e^(-(m + 2)*log(b*g*x + a*g) + (m + 2)*log((b*x + a)/(d*x + c)
) - (m + 2)*log(i/g)) - ((B^2*m^2 + 2*B^2*m + B^2)*n^2*log((b*x + a)/(d*x
+ c))^2 + A^2*m^2 + 2*A^2*m + (B^2*m^2 + 2*B^2*m + B^2)*log(e)^2 + 2*(A*B*
m^2 + 2*A*B*m + A*B)*n*log((b*x + a)/(d*x + c)) + A^2 + 2*(A*B*m^2 + 2*A*B
*m + (B^2*m^2 + 2*B^2*m + B^2)*n*log((b*x + a)/(d*x + c)) + A*B)*log(e))*E
i(((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B*n
))*e^(-((B*m + 2*B)*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n)))/((B^5
*b*c - B^5*a*d)*g^2*n^5*log((b*x + a)/(d*x + c))^2 + (B^5*b*c - B^5*a*d)*g
^2*n^3*log(e)^2 + 2*(A*B^4*b*c - A*B^4*a*d)*g^2*n^4*log((b*x + a)/(d*x + c
)) + (A^2*B^3*b*c - A^2*B^3*a*d)*g^2*n^3 + 2*((B^5*b*c - B^5*a*d)*g^2*n^4*
log((b*x + a)/(d*x + c)) + (A*B^4*b*c - A*B^4*a*d)*g^2*n^3)*log(e))
```

### 3.217.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)/(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**3,x)
```

output
```
Timed out
```

### 3.217.7 Maxima [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="maxima")
```

output
```
-(m^2 + 2*m + 1)*g^m*integrate(-1/2*(b*x + a)^m/((B^3*d^2*i^(m + 2)*n^2*x^
2 + 2*B^3*c*d*i^(m + 2)*n^2*x + B^3*c^2*i^(m + 2)*n^2)*(d*x + c)^m*log((b*
x + a)^n) - (B^3*d^2*i^(m + 2)*n^2*x^2 + 2*B^3*c*d*i^(m + 2)*n^2*x + B^3*c
^2*i^(m + 2)*n^2)*(d*x + c)^m*log((d*x + c)^n) + (B^3*c^2*i^(m + 2)*n^2*lo
g(e) + A*B^2*c^2*i^(m + 2)*n^2 + (B^3*d^2*i^(m + 2)*n^2*log(e) + A*B^2*d^2
*i^(m + 2)*n^2)*x^2 + 2*(B^3*c*d*i^(m + 2)*n^2*log(e) + A*B^2*c*d*i^(m + 2
)*n^2)*x)*(d*x + c)^m), x) - 1/2*((B*b*g^m*(m + 1)*x + B*a*g^m*(m + 1))*(b
*x + a)^m*log((b*x + a)^n) - (B*b*g^m*(m + 1)*x + B*a*g^m*(m + 1))*(b*x +
a)^m*log((d*x + c)^n) + (A*a*g^m*(m + 1) + (g^m*(m + 1)*log(e) + g^m*n)*B*
a + (A*b*g^m*(m + 1) + (g^m*(m + 1)*log(e) + g^m*n)*B*b)*x)*(b*x + a)^m)/(
((b*c*d*i^(m + 2)*n^2 - a*d^2*i^(m + 2)*n^2)*B^4*x + (b*c^2*i^(m + 2)*n^2
- a*c*d*i^(m + 2)*n^2)*B^4)*(d*x + c)^m*log((b*x + a)^n)^2 + ((b*c*d*i^(m
+ 2)*n^2 - a*d^2*i^(m + 2)*n^2)*B^4*x + (b*c^2*i^(m + 2)*n^2 - a*c*d*i^(m
+ 2)*n^2)*B^4)*(d*x + c)^m*log((d*x + c)^n)^2 + 2*((b*c^2*i^(m + 2)*n^2 -
a*c*d*i^(m + 2)*n^2)*A*B^3 + (b*c^2*i^(m + 2)*n^2*log(e) - a*c*d*i^(m + 2)
*n^2*log(e))*B^4 + ((b*c*d*i^(m + 2)*n^2 - a*d^2*i^(m + 2)*n^2)*A*B^3 + (b
*c*d*i^(m + 2)*n^2*log(e) - a*d^2*i^(m + 2)*n^2*log(e))*B^4)*x)*(d*x + c)^
m*log((b*x + a)^n) + ((b*c^2*i^(m + 2)*n^2 - a*c*d*i^(m + 2)*n^2)*A^2*B^2
+ 2*(b*c^2*i^(m + 2)*n^2*log(e) - a*c*d*i^(m + 2)*n^2*log(e))*A*B^3 + (b*c
^2*i^(m + 2)*n^2*log(e)^2 - a*c*d*i^(m + 2)*n^2*log(e)^2)*B^4 + ((b*c*d...
```

### 3.217.8 Giac [F]

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx=\int \frac{(bgx+ag)^m(dix+ci)^{-m-2}}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^3, x)
```

### 3.217.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^m(ci+dix)^{-2-m}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx=\int \frac{(a\,g+b\,g\,x)^m}{(c\,i+d\,i\,x)^{m+2}\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^3}\,dx$$

input
```
int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^3),x)
```

output
```
int((a*g + b*g*x)^m/((c*i + d*i*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^3), x)
```

# 3.218 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3$



## 3.218.1 Optimal result

Integrand size = 49, antiderivative size = 309

$$\begin{aligned}
&\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx \\
&= -\frac{6B^3n^3(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{(bc-ad)i^2(1+m)^4(c+dx)} \\
&\quad -\frac{6B^2n^2(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^3(c+dx)} \\
&\quad -\frac{3Bn(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)^2(c+dx)} \\
&\quad -\frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)i^2(1+m)(c+dx)}
\end{aligned}$$

output
```
-6*B^3*n^3*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)/(-a*d+b*c)/i^2/(1+
m)^4/(d*x+c)-6*B^2*n^2*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B*l
n(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^2/(1+m)^3/(d*x+c)-3*B*n*(b*x+a)*(g*
(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+
b*c)/i^2/(1+m)^2/(d*x+c)-(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B
*ln(e*((b*x+a)/(d*x+c))^n))^3/(-a*d+b*c)/i^2/(1+m)/(d*x+c)
```

### 3.218.2 Mathematica [A] (verified)

Time = 4.82 (sec) , antiderivative size = 206, normalized size of antiderivative = 0.67

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx =$$
$$-\frac{(g(a+bx))^{-1-m}(c+dx)(i(c+dx))^m \left(A^3(1+m)^3+3A^2B(1+m)^2n+6AB^2(1+m)n^2+6B^3n^3+3\dots\right.}{}$$

input
```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^3,x]
```

output
```
-(((g*(a + b*x))^(-1 - m)*(c + d*x)*(i*(c + d*x))^m*(A^3*(1 + m)^3 + 3*A^2
*B*(1 + m)^2*n + 6*A*B^2*(1 + m)*n^2 + 6*B^3*n^3 + 3*B*(1 + m)*(A^2*(1 + m
)^2 + 2*A*B*(1 + m)*n + 2*B^2*n^2)*Log[e*((a + b*x)/(c + d*x))^n] + 3*B^2*
(1 + m)^2*(A + A*m + B*n)*Log[e*((a + b*x)/(c + d*x))^n]^2 + B^3*(1 + m)^3
*Log[e*((a + b*x)/(c + d*x))^n]^3))/((b*c - a*d)*g*(1 + m)^4))
```

### 3.218.3 Rubi [A] (verified)

Time = 0.59 (sec) , antiderivative size = 252, normalized size of antiderivative = 0.82, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082, Rules used = {2963, 2742, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^{-m-2}(ci+dix)^m \left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3 dx$$

$$\Big\downarrow \text{2963}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int \left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \text{2742}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(\frac{3Bn\int \left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 d\frac{a+bx}{c+dx}}{m+1}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(B\log\left(e\left(\frac{a+\dots}{c+\dots}\right.\right.\right.}{m+1}\right.}{i^2(bc-ad)}$$

$$\Big\downarrow \text{2742}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(\frac{3Bn\left(\frac{2Bn\int\left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)\right)d\frac{a+bx}{c+dx}}{m+1}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)\cdots\right.\right.}{m+1}\right)}{m+1}\cdots\right.}{i^2(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(\frac{3Bn\left(\frac{2Bn\left(-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)+A\right)}{m+1}-\frac{Bn\left(\frac{a+bx}{c+dx}\right)^{-m-1}}{(m+1)^2}\right)}{m+1}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-\cdots}}{\cdots}\right)}{m+1}\cdots\right.}{i^2(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^3,x]
```

output
```
((g*(a + b*x))^(-2 - m)*((a + b*x)/(c + d*x))^(2 + m)*(i*(c + d*x))^(2 + m
)*(-((((a + b*x)/(c + d*x))^(-1 - m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^3)/(1 + m)) + (3*B*n*(-((((a + b*x)/(c + d*x))^(-1 - m)*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n])^2)/(1 + m)) + (2*B*n*(-((B*n*((a + b*x)/(c + d*x))^
(-1 - m))/(1 + m)^2) - (((a + b*x)/(c + d*x))^(-1 - m)*(A + B*Log[e*((a +
b*x)/(c + d*x))^n]))/(1 + m)))/(1 + m)))/(1 + m)))/((b*c - a*d)*i^2)
```

### 3.218.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

3.218. $\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

## 3.218.4 Maple [F]

$$\int (bgx + ag)^{-2-m} (dix + ci)^m \left(A + B\ln\left(e\left(\frac{bx + a}{dx + c}\right)^n\right)\right)^3 dx$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

output
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

## 3.218.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 2669 vs. $2(309) = 618$.

Time = 0.44 (sec) , antiderivative size = 2669, normalized size of antiderivative = 8.64

$$\int (ag + bgx)^{-2-m}(ci + dix)^m \left(A + B\log\left(e\left(\frac{a + bx}{c + dx}\right)^n\right)\right)^3 dx = \text{Too large to display}$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="fricas")
```

output
```
-(A^3*a*c*m^3 + 6*B^3*a*c*n^3 + 3*A^3*a*c*m^2 + 3*A^3*a*c*m + A^3*a*c + (B
^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c + (B^3*b*d*m^3 + 3*B^3*
b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*x^2 + (B^3*b*c + B^3*a*d + (B^3*b*c + B^3
*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c + B^3*a*d)*m)*x)*log(e)
^3 + ((B^3*b*d*m^3 + 3*B^3*b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*n^3*x^2 + (B^3
*b*c + B^3*a*d + (B^3*b*c + B^3*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(
B^3*b*c + B^3*a*d)*m)*n^3*x + (B^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m +
 B^3*a*c)*n^3)*log((b*x + a)/(d*x + c))^3 + 6*(A*B^2*a*c*m + A*B^2*a*c)*n^
2 + (A^3*b*d*m^3 + 6*B^3*b*d*n^3 + 3*A^3*b*d*m^2 + 3*A^3*b*d*m + A^3*b*d +
 6*(A*B^2*b*d*m + A*B^2*b*d)*n^2 + 3*(A^2*B*b*d*m^2 + 2*A^2*B*b*d*m + A^2*
B*b*d)*n)*x^2 + 3*(A*B^2*a*c*m^3 + 3*A*B^2*a*c*m^2 + 3*A*B^2*a*c*m + A*B^2
*a*c + (A*B^2*b*d*m^3 + 3*A*B^2*b*d*m^2 + 3*A*B^2*b*d*m + A*B^2*b*d + (B^3
*b*d*m^2 + 2*B^3*b*d*m + B^3*b*d)*n)*x^2 + (B^3*a*c*m^2 + 2*B^3*a*c*m + B^
3*a*c)*n + (A*B^2*b*c + A*B^2*a*d + (A*B^2*b*c + A*B^2*a*d)*m^3 + 3*(A*B^2
*b*c + A*B^2*a*d)*m^2 + 3*(A*B^2*b*c + A*B^2*a*d)*m + (B^3*b*c + B^3*a*d +
 (B^3*b*c + B^3*a*d)*m^2 + 2*(B^3*b*c + B^3*a*d)*m)*n)*x + ((B^3*b*d*m^3 +
 3*B^3*b*d*m^2 + 3*B^3*b*d*m + B^3*b*d)*n*x^2 + (B^3*b*c + B^3*a*d + (B^3*
b*c + B^3*a*d)*m^3 + 3*(B^3*b*c + B^3*a*d)*m^2 + 3*(B^3*b*c + B^3*a*d)*m)*
n*x + (B^3*a*c*m^3 + 3*B^3*a*c*m^2 + 3*B^3*a*c*m + B^3*a*c)*n)*log((b*x +
a)/(d*x + c)))*log(e)^2 + 3*((B^3*a*c*m^2 + 2*B^3*a*c*m + B^3*a*c)*n^3 ...
```

### 3.218.6 Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**3,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.218.7 Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$$
$$=\int\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3(bgx+ag)^{-m-2}(dix+ci)^m\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="maxima")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^3*(b*g*x + a*g)^(-m - 2)*
(d*i*x + c*i)^m, x)
```

### 3.218.8 Giac [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$$
$$=\int\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3(bgx+ag)^{-m-2}(dix+ci)^m\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^3*(b*g*x + a*g)^(-m - 2)*
(d*i*x + c*i)^m, x)
```

### 3.218.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\,dx$$
$$=\int\frac{(c\,i+d\,i\,x)^m\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^3}{(a\,g+b\,g\,x)^{m+2}}\,dx$$

input
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^3)/(a*g + b*g*
x)^(m + 2),x)
```

output
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^3)/(a*g + b*g*
x)^(m + 2), x)
```

3.218. $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3 \, dx$

# 3.219 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2$



## 3.219.1 Optimal result

Integrand size = 49, antiderivative size = 223

$$\begin{aligned}
&\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx \\
&= -\frac{2B^2n^2(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{(bc-ad)i^2(1+m)^3(c+dx)} \\
&\quad - \frac{2Bn(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)^2(c+dx)} \\
&\quad - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)i^2(1+m)(c+dx)}
\end{aligned}$$

output
```
-2*B^2*n^2*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)/(-a*d+b*c)/i^2/(1+
m)^3/(d*x+c)-2*B*n*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B*ln(e*
((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/i^2/(1+m)^2/(d*x+c)-(b*x+a)*(g*(b*x+a))^(
-2-m)*(i*(d*x+c))^(2+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2/(-a*d+b*c)/i^2/(
1+m)/(d*x+c)
```

### 3.219.2 Mathematica [A] (verified)

Time = 1.78 (sec) , antiderivative size = 134, normalized size of antiderivative = 0.60

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx = -\frac{(g(a+bx))^{-1-m}(c+dx)(i(c+dx))^m\left(A^2(1+m)^2+2AB(1+m)n+2B^2n^2+2B(1+m)(A+Am+\right.}{(bc-ad)g(1+m)^3}$$

input
```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(
c + d*x))^n])^2,x]
```

output
```
-(((g*(a + b*x))^(-1 - m)*(c + d*x)*(i*(c + d*x))^m*(A^2*(1 + m)^2 + 2*A*B
*(1 + m)*n + 2*B^2*n^2 + 2*B*(1 + m)*(A + A*m + B*n)*Log[e*((a + b*x)/(c +
 d*x))^n] + B^2*(1 + m)^2*Log[e*((a + b*x)/(c + d*x))^n]^2))/((b*c - a*d)*
g*(1 + m)^3))
```

### 3.219.3 Rubi [A] (verified)

Time = 0.50 (sec) , antiderivative size = 192, normalized size of antiderivative = 0.86, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.061, Rules used = {2963, 2742, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^{-m-2}(ci+dix)^m\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2dx$$

$$\Big\downarrow \ 2963$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int\left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \ 2742$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(\frac{2Bn\int\left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}}{m+1}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.\right.}{m+1}\right.}{i^2(bc-ad)}$$

$$\Big\downarrow \ 2741$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(\frac{2Bn\left(-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^{n}\right)+A\right)}{m+1}-\frac{Bn\left(\frac{a+bx}{c+dx}\right)^{-m-1}}{(m+1)^2}\right)}{m+1}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(\right.}{}\right.}{i^2(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c + d*
x))^n])^2,x]
```

output
```
((g*(a + b*x))^(-2 - m)*((a + b*x)/(c + d*x))^(2 + m)*(i*(c + d*x))^(2 + m
)*(-((((a + b*x)/(c + d*x))^(-1 - m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]
)^2)/(1 + m)) + (2*B*n*(-((B*n*((a + b*x)/(c + d*x))^(-1 - m))/(1 + m)^2)
- (((a + b*x)/(c + d*x))^(-1 - m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/
(1 + m)))/(1 + m)))/((b*c - a*d)*i^2)
```

### 3.219.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2742
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*((d_.)*(x_))^(m_.), x_Symbo
l] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^p/(d*(m + 1))), x] - Simp[b*n*
(p/(m + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p - 1), x], x] /; FreeQ[{a, b
, c, d, m, n}, x] && NeQ[m, -1] && GtQ[p, 0]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

### 3.219.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 2618 vs. 2(223) = 446.

Time = 131.30 (sec) , antiderivative size = 2619, normalized size of antiderivative = 11.74

| method | result | size |
|---|---|---|
| parallelrisch | Expression too large to display | 2619 |

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x,m
ethod=_RETURNVERBOSE)
```

output
```
(2*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^
2*n+2*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2
*c*d*n+2*A*B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*c*d*m*n^2+2*A*B*(i*(d*x+
c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*n+2*B^2*x*(i*(d
*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*m*n^2+2*B^2*
x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*m*n^2
+B^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*b*c*d*
m^2*n+2*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*m*n^2+4*A*B*x*(i*(d
*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*m*n+4*A*B*x*
(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*m*n+2*A
*B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*m^2*
n+4*A*B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d
*m*n+B^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2*a*
b*d^2*n+B^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)^2
*b^2*c*d*n+2*B^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))
^n)*a*b*d^2*n^2+2*B^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*
x+c))^n)*b^2*c*d*n^2+2*A^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*m*n+
2*A^2*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*m*n+A^2*(i*(d*x+c))^m*(g*
(b*x+a))^(-2-m)*a*b*c*d*m^2*n+2*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b
*d^2*n^2+2*A*B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*n^2+B^2*(i*(d...
```

### 3.219.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 983 vs. $2(223) = 446$.

Time = 0.34 (sec) , antiderivative size = 983, normalized size of antiderivative = 4.41

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx =$$

$$-\frac{\left(A^2acm^2+2B^2acn^2+2A^2acm+A^2ac+(A^2bdm^2+2B^2bdn^2+2A^2bdm+A^2bd+2(ABbdm+A\right.}{}$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="fricas")
```

output
```
-(A^2*a*c*m^2 + 2*B^2*a*c*n^2 + 2*A^2*a*c*m + A^2*a*c + (A^2*b*d*m^2 + 2*B
^2*b*d*n^2 + 2*A^2*b*d*m + A^2*b*d + 2*(A*B*b*d*m + A*B*b*d)*n)*x^2 + (B^2
*a*c*m^2 + 2*B^2*a*c*m + B^2*a*c + (B^2*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*x
^2 + (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*
m)*x)*log(e)^2 + ((B^2*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*n^2*x^2 + (B^2*b*c
 + B^2*a*d + (B^2*b*c + B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m)*n^2*x + (B
^2*a*c*m^2 + 2*B^2*a*c*m + B^2*a*c)*n^2)*log((b*x + a)/(d*x + c))^2 + 2*(A
*B*a*c*m + A*B*a*c)*n + (A^2*b*c + A^2*a*d + (A^2*b*c + A^2*a*d)*m^2 + 2*(
B^2*b*c + B^2*a*d)*n^2 + 2*(A^2*b*c + A^2*a*d)*m + 2*(A*B*b*c + A*B*a*d +
(A*B*b*c + A*B*a*d)*m)*n)*x + 2*(A*B*a*c*m^2 + 2*A*B*a*c*m + A*B*a*c + (A*
B*b*d*m^2 + 2*A*B*b*d*m + A*B*b*d + (B^2*b*d*m + B^2*b*d)*n)*x^2 + (B^2*a*
c*m + B^2*a*c)*n + (A*B*b*c + A*B*a*d + (A*B*b*c + A*B*a*d)*m^2 + 2*(A*B*b
*c + A*B*a*d)*m + (B^2*b*c + B^2*a*d + (B^2*b*c + B^2*a*d)*m)*n)*x + ((B^2
*b*d*m^2 + 2*B^2*b*d*m + B^2*b*d)*n*x^2 + (B^2*b*c + B^2*a*d + (B^2*b*c +
B^2*a*d)*m^2 + 2*(B^2*b*c + B^2*a*d)*m)*n*x + (B^2*a*c*m^2 + 2*B^2*a*c*m +
 B^2*a*c)*n)*log((b*x + a)/(d*x + c)))*log(e) + 2*((B^2*a*c*m + B^2*a*c)*n
^2 + ((B^2*b*d*m + B^2*b*d)*n^2 + (A*B*b*d*m^2 + 2*A*B*b*d*m + A*B*b*d)*n)
*x^2 + (A*B*a*c*m^2 + 2*A*B*a*c*m + A*B*a*c)*n + ((B^2*b*c + B^2*a*d + (B^
2*b*c + B^2*a*d)*m)*n^2 + (A*B*b*c + A*B*a*d + (A*B*b*c + A*B*a*d)*m^2 + 2
*(A*B*b*c + A*B*a*d)*m)*n)*x)*log((b*x + a)/(d*x + c))*(b*g*x + a*g)^(...
```

### 3.219.6 Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.219.7 Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 (bgx+ag)^{-m-2}(dix+ci)^m\, dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="maxima")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2*(b*g*x + a*g)^(-m - 2)*
(d*i*x + c*i)^m, x)
```

### 3.219.8 Giac [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2 dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2 (bgx+ag)^{-m-2}(dix+ci)^m\, dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)^2*(b*g*x + a*g)^(-m - 2)*
(d*i*x + c*i)^m, x)
```

### 3.219.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\,dx$$
$$=\int \frac{(c\,i+d\,i\,x)^m\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}{(a\,g+b\,g\,x)^{m+2}}\,dx$$

input
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^(m + 2),x)
```

output
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n))^2)/(a*g + b*g*
x)^(m + 2), x)
```

# 3.220 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) d$



## 3.220.1 Optimal result

Integrand size = 47, antiderivative size = 137

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = -\frac{Bn(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{(bc-ad)i^2(1+m)^2(c+dx)} - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)i^2(1+m)(c+dx)}$$

output
```
-B*n*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)/(-a*d+b*c)/i^2/(1+m)^2/(
d*x+c)-(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(A+B*ln(e*((b*x+a)/(d*
x+c))^n))/(-a*d+b*c)/i^2/(1+m)/(d*x+c)
```

## 3.220.2 Mathematica [A] (verified)

Time = 0.57 (sec) , antiderivative size = 78, normalized size of antiderivative = 0.57

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right) dx = -\frac{(g(a+bx))^{-1-m}(c+dx)(i(c+dx))^m\left(A+Am+Bn+B(1+m)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)g(1+m)^2}$$

input
```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(
c + d*x))^n]),x]
```

output
```
-(((g*(a + b*x))^(-1 - m)*(c + d*x)*(i*(c + d*x))^m*(A + A*m + B*n + B*(1
+ m)*Log[e*((a + b*x)/(c + d*x))^n]))/((b*c - a*d)*g*(1 + m)^2))
```

### 3.220.3 Rubi [A] (verified)

Time = 0.37 (sec) , antiderivative size = 132, normalized size of antiderivative = 0.96, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.043, Rules used = {2963, 2741}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^{-m-2}(ci+dix)^m\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)dx$$

$$\Big\downarrow \text{2963}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int\left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \text{2741}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{m+1}-\frac{Bn\left(\frac{a+bx}{c+dx}\right)^{-m-1}}{(m+1)^2}\right)}{i^2(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[e*((a + b*x)/(c + d*
x))^n]),x]
```

output
```
((g*(a + b*x))^(-2 - m)*((a + b*x)/(c + d*x))^(2 + m)*(i*(c + d*x))^(2 + m
)*(-((B*n*((a + b*x)/(c + d*x))^(-1 - m))/(1 + m)^2) - (((a + b*x)/(c + d*
x))^(-1 - m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(1 + m)))/((b*c - a*d
)*i^2)
```

### 3.220.3.1 Defintions of rubi rules used

rule 2741
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))*((d_.)*(x_))^(m_.), x_Symbol] :>
Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])/(d*(m + 1))), x] - Simp[b*n*((d*x)^(
m + 1)/(d*(m + 1)^2)), x] /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

## 3.220.4 Maple [B] (verified)

Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(137) = 274$.

Time = 31.01 (sec) , antiderivative size = 821, normalized size of antiderivative = 5.99

| method | result |
|---|---|
| parallelrisch | $\frac{Bx(i(dx+c))^m (g(bx+a))^{-2-m} \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) ab\, d^2 mn+Bx(i(dx+c))^m (g(bx+a))^{-2-m} \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right) b^2 cdmn+B(i(dx+c)}{}$ |

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x,met
hod=_RETURNVERBOSE)
```

output
```
(B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*m*
n+B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*m
*n+B*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*m*
n+B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*c*d*n
+A*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*c*d*m*n+B*(i*(d*x+c))^m*(g*(b*x+a)
)^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*c*d*n+A*x^2*(i*(d*x+c))^m*(g*(b*x+a
))^(-2-m)*b^2*d^2*n+B*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(
d*x+c))^n)*b^2*d^2*m*n+A*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*m*n+A*
x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*m*n+B*x*(i*(d*x+c))^m*(g*(b*x+a
))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*a*b*d^2*n+B*x^2*(i*(d*x+c))^m*(g*(b*x+
a))^(-2-m)*b^2*d^2*n^2+A*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*d^2*m*n+
B*x^2*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*ln(e*((b*x+a)/(d*x+c))^n)*b^2*d^2*n
+B*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*n^2+B*x*(i*(d*x+c))^m*(g*(b*
x+a))^(-2-m)*b^2*c*d*n^2+A*x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*d^2*n+A*
x*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*b^2*c*d*n+B*(i*(d*x+c))^m*(g*(b*x+a))^(
-2-m)*a*b*c*d*n^2+A*(i*(d*x+c))^m*(g*(b*x+a))^(-2-m)*a*b*c*d*n)/d/b/n/(a*d
*m-b*c*m+a*d-b*c)/(1+m)
```

### 3.220.5 Fricas [A] (verification not implemented)

Time = 0.37 (sec) , antiderivative size = 269, normalized size of antiderivative = 1.96

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx = -\frac{(Aacm+Bacn+Aac+(Abdm+Bbdn+Abd)x^2+(Abc+Aad+(Abc+Aad)m+(Bbc+Bad)n)x}{}$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="fricas")
```

output
```
-(A*a*c*m + B*a*c*n + A*a*c + (A*b*d*m + B*b*d*n + A*b*d)*x^2 + (A*b*c + A
*a*d + (A*b*c + A*a*d)*m + (B*b*c + B*a*d)*n)*x + (B*a*c*m + B*a*c + (B*b*
d*m + B*b*d)*x^2 + (B*b*c + B*a*d + (B*b*c + B*a*d)*m)*x)*log(e) + ((B*b*d
*m + B*b*d)*n*x^2 + (B*b*c + B*a*d + (B*b*c + B*a*d)*m)*n*x + (B*a*c*m + B
*a*c)*n)*log((b*x + a)/(d*x + c)))*(b*g*x + a*g)^(-m - 2)*e^(m*log(b*g*x +
 a*g) - m*log((b*x + a)/(d*x + c)) + m*log(i/g))/((b*c - a*d)*m^2 + b*c -
a*d + 2*(b*c - a*d)*m)
```

### 3.220.6 Sympy [F(-2)]

Exception generated.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*((b*x+a)/(d*x+c))**
n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.220.7 Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)(bgx+ag)^{-m-2}(dix+ci)^m\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="maxima")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(b*g*x + a*g)^(-m - 2)*(d
*i*x + c*i)^m, x)
```

### 3.220.8 Giac [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$= \int \left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)(bgx+ag)^{-m-2}(dix+ci)^m\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="giac")
```

output
```
integrate((B*log(e*((b*x + a)/(d*x + c))^n) + A)*(b*g*x + a*g)^(-m - 2)*(d
*i*x + c*i)^m, x)
```

### 3.220.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\,dx$$
$$=\int\frac{(c\,i+d\,i\,x)^m\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{(a\,g+b\,g\,x)^{m+2}}\,dx$$

input
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^(m + 2),x)
```

output
```
int(((c*i + d*i*x)^m*(A + B*log(e*((a + b*x)/(c + d*x))^n)))/(a*g + b*g*x)
^(m + 2), x)
```

# 3.221 $\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$



## 3.221.1 Optimal result

Integrand size = 49, antiderivative size = 128

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{e^{\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\,\text{ExpIntegralEi}\left(-\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{B(bc-ad)i^2n(c+dx)}$$

output
```
exp(A*(1+m)/B/n)*(b*x+a)*(g*(b*x+a))^(-2-m)*(e*((b*x+a)/(d*x+c))^n)^((1+m)
/n)*(i*(d*x+c))^(2+m)*Ei(-(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n)/B/(-a
*d+b*c)/i^2/n/(d*x+c)
```

## 3.221.2 Mathematica [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$

input
```
Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)
/(c + d*x))^n]),x]
```

output
```
Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)
/(c + d*x))^n]), x]
```

### 3.221.3 Rubi [A] (verified)

Time = 0.51 (sec) , antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.061$, Rules used = {2963, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^{-m-2}(ci+dix)^m}{B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A}dx$$

$$\Big\downarrow \ 2963$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int \frac{\left(\frac{a+bx}{c+dx}\right)^{-m-2}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \ 2747$$

$$\frac{(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{i^2n(c+dx)(bc-ad)}$$

$$\Big\downarrow \ 2609$$

$$\frac{(a+bx)e^{\frac{A(m+1)}{Bn}}(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\text{ExpIntegralEi}\left(-\frac{(m+1)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{Bi^2n(c+dx)(bc-ad)}$$

input
```
Int[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/(c +
d*x))^n]),x]
```

output
```
(E^((A*(1 + m))/(B*n))*(a + b*x)*(g*(a + b*x))^(-2 - m)*(e*((a + b*x)/(c +
 d*x))^n)^((1 + m)/n)*(i*(c + d*x))^(2 + m)*ExpIntegralEi[-(((1 + m)*(A +
B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n))])/(B*(b*c - a*d)*i^2*n*(c + d*x)
)
```

### 3.221.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

## 3.221.4 Maple [F]

$$\int \frac{(bgx+ag)^{-2-m}\,(dix+ci)^{m}}{A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)}dx$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

output
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x)
```

## 3.221.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 93, normalized size of antiderivative = 0.73

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\operatorname{Ei}\left(-\frac{(Bm+B)n\log\left(\frac{bx+a}{dx+c}\right)+Am+(Bm+B)\log(e)+A}{Bn}\right)e^{\left(\frac{Bmn\log\left(\frac{i}{g}\right)+Am+(Bm+B)\log(e)+A}{Bn}\right)}}{(Bbc-Bad)g^2n}$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="fricas")
```

output
```
Ei(-((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B
*n))*e^((B*m*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n))/((B*b*c - B*a
*d)*g^2*n)
```

### 3.221.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m/(A+B*ln(e*((b*x+a)/(d*x+c))**
n)),x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.221.7 Maxima [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A), x)
```

### 3.221.8 Giac [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A}\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
),x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A), x)
```

### 3.221.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{(c\,i+d\,i\,x)^m}{(a\,g+b\,g\,x)^{m+2}\,\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}\,dx$$

input
```
int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))),x)
```

output
```
int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))), x)
```

# 3.222 $\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$



## 3.222.1 Optimal result

Integrand size = 49, antiderivative size = 214

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = -\frac{e^{\frac{A(1+m)}{Bn}}(1+m)(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\,\text{ExpIntegralEi}\left(-\frac{(1+m)\left(A+B\log\left(\ldots\right.\right.}{Bn}\ldots\right.}{B^2(bc-ad)i^2n^2(c+dx)} - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output
```
-exp(A*(1+m)/B/n)*(1+m)*(b*x+a)*(g*(b*x+a))^(-2-m)*(e*((b*x+a)/(d*x+c))^n)
^((1+m)/n)*(i*(d*x+c))^(2+m)*Ei(-(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n))/B/n
)/B^2/(-a*d+b*c)/i^2/n^2/(d*x+c)-(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2
+m)/B/(-a*d+b*c)/i^2/n/(d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

### 3.222.2 Mathematica [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$$

input
```
Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2,x]
```

output
```
Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^2, x]
```

### 3.222.3 Rubi [A] (verified)

Time = 0.61 (sec) , antiderivative size = 209, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.082, Rules used = {2963, 2743, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^{-m-2}(ci+dix)^m}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}dx$$

$$\Big\downarrow \text{ 2963}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int \frac{\left(\frac{a+bx}{c+dx}\right)^{-m-2}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \text{ 2743}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)\int \frac{\left(\frac{a+bx}{c+dx}\right)^{-m-2}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}}{Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}\right)}{i^2(bc-ad)}$$

$$\Big\downarrow \text{ 2747}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\int\frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{Bn^2}-\right.}{i^2(bc-ad)}$$

$$\Big\downarrow \text{ 2609}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)e^{\frac{A(m+1)}{Bn}}\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\text{ExpIntegralEi}\left(-\frac{(m+1)\left(A+B\log(\right.}{Bn}\right.}{B^2n^2}\right.}{i^2(bc-ad)}$$

input
```
Int[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/(c +
d*x))^n])^2,x]
```

output
```
((g*(a + b*x))^(-2 - m)*((a + b*x)/(c + d*x))^(2 + m)*(i*(c + d*x))^(2 + m
)*(-((E^((A*(1 + m))/(B*n))*(1 + m)*(e*((a + b*x)/(c + d*x))^n)^((1 + m)/n
)*((a + b*x)/(c + d*x))^(-1 - m)*ExpIntegralEi[-(((1 + m)*(A + B*Log[e*((a
 + b*x)/(c + d*x))^n]))/(B*n))])/(B^2*n^2)) - ((a + b*x)/(c + d*x))^(-1 -
m)/(B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))))/((b*c - a*d)*i^2)
```

### 3.222.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

3.222. $\int\frac{(agi+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx$

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

### 3.222.4 Maple [F]

$$\int \frac{(bgx+ag)^{-2-m}(dix+ci)^m}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^2}dx$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

output
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x)
```

### 3.222.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 284, normalized size of antiderivative = 1.33

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx =$$

$$-\frac{(Bbdg^2nx^2+Bacg^2n+(Bbc+Bad)g^2nx)(bgx+ag)^{-m-2}e^{\left(m\log(bgx+ag)-m\log\left(\frac{bx+a}{dx+c}\right)+m\log\left(\frac{i}{g}\right)\right)}+((Bm}{(B^3bc-B^3ad)g^2n^3\log\left(\frac{bx+a}{dx+c}\right)+}$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="fricas")
```

output
```
-((B*b*d*g^2*n*x^2 + B*a*c*g^2*n + (B*b*c + B*a*d)*g^2*n*x)*(b*g*x + a*g)^
(-m - 2)*e^(m*log(b*g*x + a*g) - m*log((b*x + a)/(d*x + c)) + m*log(i/g))
+ ((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)*Ei(-
((B*m + B)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B*n))
*e^((B*m*n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n)))/((B^3*b*c - B^3*
a*d)*g^2*n^3*log((b*x + a)/(d*x + c)) + (B^3*b*c - B^3*a*d)*g^2*n^2*log(e)
 + (A*B^2*b*c - A*B^2*a*d)*g^2*n^2)
```

### 3.222.6 Sympy [F(-2)]

Exception generated.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \text{Exception raised: HeuristicGCDFailed}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m/(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**2,x)
```

output
```
Exception raised: HeuristicGCDFailed >> no luck
```

### 3.222.7 Maxima [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx = \int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="maxima")
```

output
```
i^m*(m + 1)*integrate(-(d*x + c)^m/((B^2*b^2*g^(m + 2)*n*x^2 + 2*B^2*a*b*g
^(m + 2)*n*x + B^2*a^2*g^(m + 2)*n)*(b*x + a)^m*log((b*x + a)^n) - (B^2*b^
2*g^(m + 2)*n*x^2 + 2*B^2*a*b*g^(m + 2)*n*x + B^2*a^2*g^(m + 2)*n)*(b*x +
a)^m*log((d*x + c)^n) + (B^2*a^2*g^(m + 2)*n*log(e) + A*B*a^2*g^(m + 2)*n
+ (B^2*b^2*g^(m + 2)*n*log(e) + A*B*b^2*g^(m + 2)*n)*x^2 + 2*(B^2*a*b*g^(m
 + 2)*n*log(e) + A*B*a*b*g^(m + 2)*n)*x)*(b*x + a)^m), x) - (d*i^m*x + c*i
^m)*(d*x + c)^m/(((b^2*c*g^(m + 2)*n - a*b*d*g^(m + 2)*n)*B^2*x + (a*b*c*g
^(m + 2)*n - a^2*d*g^(m + 2)*n)*B^2)*(b*x + a)^m*log((b*x + a)^n) - ((b^2*
c*g^(m + 2)*n - a*b*d*g^(m + 2)*n)*B^2*x + (a*b*c*g^(m + 2)*n - a^2*d*g^(m
 + 2)*n)*B^2)*(b*x + a)^m*log((d*x + c)^n) + ((a*b*c*g^(m + 2)*n - a^2*d*g
^(m + 2)*n)*A*B + (a*b*c*g^(m + 2)*n*log(e) - a^2*d*g^(m + 2)*n*log(e))*B^
2 + ((b^2*c*g^(m + 2)*n - a*b*d*g^(m + 2)*n)*A*B + (b^2*c*g^(m + 2)*n*log(
e) - a*b*d*g^(m + 2)*n*log(e))*B^2)*x)*(b*x + a)^m)
```

### 3.222.8 Giac [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^2}\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^2,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^2, x)
```

### 3.222.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}\,dx=\int \frac{(c\,i+d\,i\,x)^m}{(a\,g+b\,g\,x)^{m+2}\left(A+B\,\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^2}\,dx$$

input
```
int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^2),x)
```

output
```
int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^2), x)
```

# 3.223 $\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$



## 3.223.1 Optimal result

Integrand size = 49, antiderivative size = 306

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx = \frac{e^{\frac{A(1+m)}{Bn}}(1+m)^2(a+bx)(g(a+bx))^{-2-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1+m}{n}}(i(c+dx))^{2+m}\,\text{ExpIntegralEi}\left(-\frac{(1+m)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn}\right)}{2B^3(bc-ad)i^2n^3(c+dx)} - \frac{(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{2B(bc-ad)i^2n(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2} + \frac{(1+m)(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}}{2B^2(bc-ad)i^2n^2(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}$$

output
```
1/2*exp(A*(1+m)/B/n)*(1+m)^2*(b*x+a)*(g*(b*x+a))^(-2-m)*(e*((b*x+a)/(d*x+c
))^n)^((1+m)/n)*(i*(d*x+c))^(2+m)*Ei(-(1+m)*(A+B*ln(e*((b*x+a)/(d*x+c))^n)
)/B/n)/B^3/(-a*d+b*c)/i^2/n^3/(d*x+c)-1/2*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d
*x+c))^(2+m)/B/(-a*d+b*c)/i^2/n/(d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2+
1/2*(1+m)*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)/B^2/(-a*d+b*c)/i^2/
n^2/(d*x+c)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))
```

### 3.223.2 Mathematica [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx=\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$$

input
```
Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^3,x]
```

output
```
Integrate[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)
/(c + d*x))^n])^3, x]
```

### 3.223.3 Rubi [A] (verified)

Time = 0.71 (sec) , antiderivative size = 276, normalized size of antiderivative = 0.90, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.102, Rules used = {2963, 2743, 2743, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(ag+bgx)^{-m-2}(ci+dix)^m}{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}dx$$

$$\Big\downarrow \text{2963}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int \frac{\left(\frac{a+bx}{c+dx}\right)^{-m-2}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \text{2743}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)\int \frac{\left(\frac{a+bx}{c+dx}\right)^{-m-2}}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{2Bn}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}}{2Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}\right)}{i^2(bc-ad)}$$

$$\Big\downarrow \text{2743}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)\left(-\frac{(m+1)\int\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-2}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{Bn}-\frac{\left(\frac{a+bx}{c+dx}\right)^{-m-1}}{Bn\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}\right)}{2Bn}-\frac{}{2B}\right.}{i^2(bc-ad)}$$

$\downarrow$ 2747

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)\left(-\frac{(m+1)\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\int\frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}}{A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{Bn^2}\right.}{2Bn}\right.}{i^2(bc-ad)}$$

$\downarrow$ 2609

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\left(-\frac{(m+1)\left(-\frac{(m+1)e^{\frac{A(m+1)}{Bn}}\left(\frac{a+bx}{c+dx}\right)^{-m-1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\text{ExpIntegralEi}\left(-\frac{(m+1)(A+}{}\right.}{B^2n^2}\right.}{2Bn}\right.}{i^2(bc-ad)}$$

input

```
Int[((a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m)/(A + B*Log[e*((a + b*x)/(c +
d*x))^n])^3,x]
```

output

```
((g*(a + b*x))^(-2 - m)*((a + b*x)/(c + d*x))^(2 + m)*(i*(c + d*x))^(2 + m
)*(-1/2*((a + b*x)/(c + d*x))^(-1 - m)/(B*n*(A + B*Log[e*((a + b*x)/(c + d
*x))^n])^2) - ((1 + m)*(-((E^((A*(1 + m))/(B*n))*(1 + m)*(e*((a + b*x)/(c
+ d*x))^n)^((1 + m)/n)*((a + b*x)/(c + d*x))^(-1 - m)*ExpIntegralEi[-(((1
+ m)*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n))])/(B^2*n^2)) - ((a + b
*x)/(c + d*x))^(-1 - m)/(B*n*(A + B*Log[e*((a + b*x)/(c + d*x))^n]))))/(2*
B*n)))/((b*c - a*d)*i^2)
```

## 3.223. $\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx$

### 3.223.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2743
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)*((a + b*Log[c*x^n])^(p + 1)/(b*d*n*(p + 1))), x] -
Simp[(m + 1)/(b*n*(p + 1))   Int[(d*x)^m*(a + b*Log[c*x^n])^(p + 1), x], x]
 /; FreeQ[{a, b, c, d, m, n}, x] && NeQ[m, -1] && LtQ[p, -1]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

## 3.223.4 Maple [F]

$$\int \frac{(bgx+ag)^{-2-m}(dix+ci)^m}{\left(A+B\ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)\right)^3}dx$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

output
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^3,x)
```

### 3.223.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 815 vs. $2(299) = 598$.

Time = 0.39 (sec) , antiderivative size = 815, normalized size of antiderivative = 2.66

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx =$$

$$-\frac{\left(B^2acg^2n^2+(B^2bdg^2n^2-(ABbdg^2m+ABbdg^2)n)x^2-(ABacg^2m+ABacg^2)n+((B^2bc+B^2ad)g^2\right.}{}$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="fricas")
```

output
```
-1/2*((B^2*a*c*g^2*n^2 + (B^2*b*d*g^2*n^2 - (A*B*b*d*g^2*m + A*B*b*d*g^2)*
n)*x^2 - (A*B*a*c*g^2*m + A*B*a*c*g^2)*n + ((B^2*b*c + B^2*a*d)*g^2*n^2 -
((A*B*b*c + A*B*a*d)*g^2*m + (A*B*b*c + A*B*a*d)*g^2)*n)*x - ((B^2*b*d*g^2
*m + B^2*b*d*g^2)*n*x^2 + ((B^2*b*c + B^2*a*d)*g^2*m + (B^2*b*c + B^2*a*d)
*g^2)*n*x + (B^2*a*c*g^2*m + B^2*a*c*g^2)*n)*log(e) - ((B^2*b*d*g^2*m + B^
2*b*d*g^2)*n^2*x^2 + ((B^2*b*c + B^2*a*d)*g^2*m + (B^2*b*c + B^2*a*d)*g^2)
*n^2*x + (B^2*a*c*g^2*m + B^2*a*c*g^2)*n^2)*log((b*x + a)/(d*x + c)))*(b*g
*x + a*g)^(-m - 2)*e^(m*log(b*g*x + a*g) - m*log((b*x + a)/(d*x + c)) + m*
log(i/g)) - ((B^2*m^2 + 2*B^2*m + B^2)*n^2*log((b*x + a)/(d*x + c))^2 + A^
2*m^2 + 2*A^2*m + (B^2*m^2 + 2*B^2*m + B^2)*log(e)^2 + 2*(A*B*m^2 + 2*A*B*
m + A*B)*n*log((b*x + a)/(d*x + c)) + A^2 + 2*(A*B*m^2 + 2*A*B*m + (B^2*m^
2 + 2*B^2*m + B^2)*n*log((b*x + a)/(d*x + c)) + A*B)*log(e))*Ei(-((B*m + B
)*n*log((b*x + a)/(d*x + c)) + A*m + (B*m + B)*log(e) + A)/(B*n))*e^((B*m*
n*log(i/g) + A*m + (B*m + B)*log(e) + A)/(B*n)))/((B^5*b*c - B^5*a*d)*g^2*
n^5*log((b*x + a)/(d*x + c))^2 + (B^5*b*c - B^5*a*d)*g^2*n^3*log(e)^2 + 2*
(A*B^4*b*c - A*B^4*a*d)*g^2*n^4*log((b*x + a)/(d*x + c)) + (A^2*B^3*b*c -
A^2*B^3*a*d)*g^2*n^3 + 2*((B^5*b*c - B^5*a*d)*g^2*n^4*log((b*x + a)/(d*x +
 c)) + (A*B^4*b*c - A*B^4*a*d)*g^2*n^3)*log(e))
```

### 3.223.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m/(A+B*ln(e*((b*x+a)/(d*x+c))**
n))**3,x)
```

output
```
Timed out
```

### 3.223.7 Maxima [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx=\int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="maxima")
```

output
```
-(m^2 + 2*m + 1)*i^m*integrate(-1/2*(d*x + c)^m/((B^3*b^2*g^(m + 2)*n^2*x^
2 + 2*B^3*a*b*g^(m + 2)*n^2*x + B^3*a^2*g^(m + 2)*n^2)*(b*x + a)^m*log((b*
x + a)^n) - (B^3*b^2*g^(m + 2)*n^2*x^2 + 2*B^3*a*b*g^(m + 2)*n^2*x + B^3*a
^2*g^(m + 2)*n^2)*(b*x + a)^m*log((d*x + c)^n) + (B^3*a^2*g^(m + 2)*n^2*lo
g(e) + A*B^2*a^2*g^(m + 2)*n^2 + (B^3*b^2*g^(m + 2)*n^2*log(e) + A*B^2*b^2
*g^(m + 2)*n^2)*x^2 + 2*(B^3*a*b*g^(m + 2)*n^2*log(e) + A*B^2*a*b*g^(m + 2
)*n^2)*x)*(b*x + a)^m), x) + 1/2*((B*d*i^m*(m + 1)*x + B*c*i^m*(m + 1))*(d
*x + c)^m*log((b*x + a)^n) - (B*d*i^m*(m + 1)*x + B*c*i^m*(m + 1))*(d*x +
c)^m*log((d*x + c)^n) + (A*c*i^m*(m + 1) + (i^m*(m + 1)*log(e) - i^m*n)*B*
c + (A*d*i^m*(m + 1) + (i^m*(m + 1)*log(e) - i^m*n)*B*d)*x)*(d*x + c)^m)/(
((b^2*c*g^(m + 2)*n^2 - a*b*d*g^(m + 2)*n^2)*B^4*x + (a*b*c*g^(m + 2)*n^2
- a^2*d*g^(m + 2)*n^2)*B^4)*(b*x + a)^m*log((b*x + a)^n)^2 + ((b^2*c*g^(m
+ 2)*n^2 - a*b*d*g^(m + 2)*n^2)*B^4*x + (a*b*c*g^(m + 2)*n^2 - a^2*d*g^(m
+ 2)*n^2)*B^4)*(b*x + a)^m*log((d*x + c)^n)^2 + 2*((a*b*c*g^(m + 2)*n^2 -
a^2*d*g^(m + 2)*n^2)*A*B^3 + (a*b*c*g^(m + 2)*n^2*log(e) - a^2*d*g^(m + 2)
*n^2*log(e))*B^4 + ((b^2*c*g^(m + 2)*n^2 - a*b*d*g^(m + 2)*n^2)*A*B^3 + (b
^2*c*g^(m + 2)*n^2*log(e) - a*b*d*g^(m + 2)*n^2*log(e))*B^4)*x)*(b*x + a)^
m*log((b*x + a)^n) + ((a*b*c*g^(m + 2)*n^2 - a^2*d*g^(m + 2)*n^2)*A^2*B^2
+ 2*(a*b*c*g^(m + 2)*n^2*log(e) - a^2*d*g^(m + 2)*n^2*log(e))*A*B^3 + (a*b
*c*g^(m + 2)*n^2*log(e)^2 - a^2*d*g^(m + 2)*n^2*log(e)^2)*B^4 + ((b^2*c...
```

### 3.223.8 Giac [F]

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx=\int \frac{(bgx+ag)^{-m-2}(dix+ci)^m}{\left(B\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)+A\right)^3}\,dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m/(A+B*log(e*((b*x+a)/(d*x+c))^n)
)^3,x, algorithm="giac")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m/(B*log(e*((b*x + a)/(d*x
+ c))^n) + A)^3, x)
```

### 3.223.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(ag+bgx)^{-2-m}(ci+dix)^m}{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}\,dx=\int \frac{(c\,i+d\,i\,x)^m}{(a\,g+b\,g\,x)^{m+2}\,\left(A+B\,\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)^3}\,dx$$

input
```
int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^3),x)
```

output
```
int((c*i + d*i*x)^m/((a*g + b*g*x)^(m + 2)*(A + B*log(e*((a + b*x)/(c + d*
x))^n))^3), x)
```

# 3.224 $\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$



## 3.224.1 Optimal result

Integrand size = 35, antiderivative size = 41

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n(1+p)}$$

output
```
ln(e*((b*x+a)/(d*x+c))^n)^(p+1)/(-a*d+b*c)/n/(p+1)
```

## 3.224.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bcn-adn)(1+p)}$$

input
```
Integrate[Log[e*((a + b*x)/(c + d*x))^n]^p/((a + b*x)*(c + d*x)),x]
```

output
```
Log[e*((a + b*x)/(c + d*x))^n]^(1 + p)/((b*c*n - a*d*n)*(1 + p))
```

## 3.224.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2739}$$

$$\frac{\int \log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

$$\Big\downarrow \text{15}$$

$$\frac{\log^{p+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(p+1)(bc-ad)}$$

input
```
Int[Log[e*((a + b*x)/(c + d*x))^n]^p/((a + b*x)*(c + d*x)),x]
```

output
```
Log[e*((a + b*x)/(c + d*x))^n]^(1 + p)/((b*c - a*d)*n*(1 + p))
```

### 3.224.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.224.4 Maple [A] (verified)

Time = 7.37 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{p+1}}{n(ad-cb)(p+1)}$ | 43 |
| default | $-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{p+1}}{n(ad-cb)(p+1)}$ | 43 |
| parallelrisch | $-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{p}}{n(adp-bcp+ad-cb)}$ | 63 |

input
```
int(ln(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/n/(a*d-b*c)*ln(e*((b*x+a)/(d*x+c))^n)^(p+1)/(p+1)
```

## 3.224.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)^p}{(bc-ad)np+(bc-ad)n}$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x, algorithm="frica
s")
```

output
```
(n*log((b*x + a)/(d*x + c)) + log(e))*(n*log((b*x + a)/(d*x + c)) + log(e)
)^p/((b*c - a*d)*n*p + (b*c - a*d)*n)
```

### 3.224.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)**p/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

### 3.224.7 Maxima [F]

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^p}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x, algorithm="maxim
a")
```

output
```
integrate(log(e*((b*x + a)/(d*x + c))^n)^p/((b*x + a)*(d*x + c)), x)
```

### 3.224.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx = \frac{\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)^{p+1}}{(bcn-adn)(p+1)}$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(b*x+a)/(d*x+c),x, algorithm="giac"
)
```

output
```
(n*log((b*x + a)/(d*x + c)) + log(e))^(p + 1)/((b*c*n - a*d*n)*(p + 1))
```

### 3.224.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^p}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(log(e*((a + b*x)/(c + d*x))^n)^p/((a + b*x)*(c + d*x)),x)
```

output
```
int(log(e*((a + b*x)/(c + d*x))^n)^p/((a + b*x)*(c + d*x)), x)
```

# 3.225 $\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx$



## 3.225.1 Optimal result

Integrand size = 42, antiderivative size = 41

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bc-ad)n(1+p)}$$

output
```
ln(e*((b*x+a)/(d*x+c))^n)^(p+1)/(-a*d+b*c)/n/(p+1)
```

## 3.225.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx=\frac{\log^{1+p}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(bcn-adn)(1+p)}$$

input
```
Integrate[Log[e*((a + b*x)/(c + d*x))^n]^p/(a*c + (b*c + a*d)*x + b*d*x^2)
,x]
```

output
```
Log[e*((a + b*x)/(c + d*x))^n]^(1 + p)/((b*c*n - a*d*n)*(1 + p))
```

### 3.225.3 Rubi [A] (verified)

Time = 0.36 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {2974, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{x(ad+bc)+ac+bdx^2}dx$$

$$\Big\downarrow \ 2974$$

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \ 2961$$

$$\frac{\int \frac{(c+dx)\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{a+bx}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \ 2739$$

$$\frac{\int \log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

$$\Big\downarrow \ 15$$

$$\frac{\log^{p+1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(p+1)(bc-ad)}$$

input
```
Int[Log[e*((a + b*x)/(c + d*x))^n]^p/(a*c + (b*c + a*d)*x + b*d*x^2),x]
```

output
```
Log[e*((a + b*x)/(c + d*x))^n]^(1 + p)/((b*c - a*d)*n*(1 + p))
```

### 3.225.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2974
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_) + (h_.)*(x_)^2)^(m_.), x_Symbol] :> Simp[h^
m/(b^m*d^m)   Int[(a + b*x)^m*(c + d*x)^m*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, n, p}, x] && EqQ[b*
d*f - a*c*h, 0] && EqQ[b*d*g - h*(b*c + a*d), 0] && IntegerQ[m]
```

## 3.225.4 Maple [A] (verified)

Time = 10.28 (sec) , antiderivative size = 63, normalized size of antiderivative = 1.54

| method | result | size |
|---|---|---|
| parallelrisch | $-\frac{\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)^{p}}{n(adp-bcp+ad-cb)}$ | 63 |

input
```
int(ln(e*((b*x+a)/(d*x+c))^n)^p/(c*a+(a*d+b*c)*x+b*d*x^2),x,method=_RETURN
VERBOSE)
```

output
```
-ln(e*((b*x+a)/(d*x+c))^n)*ln(e*((b*x+a)/(d*x+c))^n)^p/n/(a*d*p-b*c*p+a*d-
b*c)
```

### 3.225.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 65, normalized size of antiderivative = 1.59

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)^p}{(bc-ad)np+(bc-ad)n}$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(a*c+(a*d+b*c)*x+b*d*x^2),x, algori
thm="fricas")
```

output
```
(n*log((b*x + a)/(d*x + c)) + log(e))*(n*log((b*x + a)/(d*x + c)) + log(e)
)^p/((b*c - a*d)*n*p + (b*c - a*d)*n)
```

### 3.225.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \text{Timed out}$$

input
```
integrate(ln(e*((b*x+a)/(d*x+c))**n)**p/(a*c+(a*d+b*c)*x+b*d*x**2),x)
```

output
```
Timed out
```

### 3.225.7 Maxima [F]

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \int \frac{\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)^p}{bdx^2+ac+(bc+ad)x}\,dx$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(a*c+(a*d+b*c)*x+b*d*x^2),x, algori
thm="maxima")
```

output
```
integrate(log(e*((b*x + a)/(d*x + c))^n)^p/(b*d*x^2 + a*c + (b*c + a*d)*x)
, x)
```

### 3.225.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \frac{\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)^{p+1}}{(bcn-adn)(p+1)}$$

input
```
integrate(log(e*((b*x+a)/(d*x+c))^n)^p/(a*c+(a*d+b*c)*x+b*d*x^2),x, algori
thm="giac")
```

output
```
(n*log((b*x + a)/(d*x + c)) + log(e))^(p + 1)/((b*c*n - a*d*n)*(p + 1))
```

### 3.225.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log^p\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{ac+(bc+ad)x+bdx^2}\,dx = \int \frac{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)^p}{b\,d\,x^2+(a\,d+b\,c)\,x+a\,c}\,dx$$

input
```
int(log(e*((a + b*x)/(c + d*x))^n)^p/(a*c + x*(a*d + b*c) + b*d*x^2),x)
```

output
```
int(log(e*((a + b*x)/(c + d*x))^n)^p/(a*c + x*(a*d + b*c) + b*d*x^2), x)
```

# 3.226 $\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c\right.\right.$



## 3.226.1 Optimal result

Integrand size = 50, antiderivative size = 193

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\frac{e^{-\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(e(a+bx)^n(c+dx)^{-n}\right)^{-\frac{1+m}{n}}\Gamma\left(1+p,-\frac{(1+m)(A+B\log(e(a+bx)^n(}{Bn}\right.}{(bc-ad)i^2(1+m)(c+}$$

output
```
(b*x+a)*(g*(b*x+a))^m*GAMMA(p+1,-(1+m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B
/n)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(-a*d+b*c)/exp(A*(1+m)/B/n)/i^2/(1
+m)/(d*x+c)/((i*(d*x+c))^m)/((e*(b*x+a)^n/((d*x+c)^n))^((1+m)/n))/((-(1+m)
*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/n)^p)
```

## 3.226.2 Mathematica [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$

input
```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^p,x]
```

output
```
Integrate[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^p, x]
```

## 3.226.3 Rubi [A] (warning: unable to verify)

Time = 0.74 (sec) , antiderivative size = 189, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2973, 2963, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^m(ci+dix)^{-m-2}\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^p\,dx$$

$$\Big\downarrow \text{2973}$$

$$\int (ag+bgx)^m(ci+dix)^{-m-2}\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^p\,dx$$

$$\Big\downarrow \text{2963}$$

$$\frac{(g(a+bx))^m(i(c+dx))^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int\left(\frac{a+bx}{c+dx}\right)^m\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \text{2747}$$

$$\frac{(a+bx)(g(a+bx))^m(i(c+dx))^{-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\int\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\log\left(e\left(\frac{a+bx}{c+dx}\right.\right.}{i^2n(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{2612}$$

$$\frac{(a+bx)e^{-\frac{A(m+1)}{Bn}}(g(a+bx))^m(i(c+dx))^{-m}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p\left(-\frac{(m+1)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)\right.\right.}{Bn}\right.}{i^2(m+1)(c+dx)(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^m*(c*i + d*i*x)^(-2 - m)*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])^p,x]
```

output
```
((a + b*x)*(g*(a + b*x))^m*Gamma[1 + p, -(((1 + m)*(A + B*Log[e*((a + b*x)
/(c + d*x))^n]))/(B*n))]*(A + B*Log[e*((a + b*x)/(c + d*x))^n])^p)/((b*c -
 a*d)*E^((A*(1 + m))/(B*n))*i^2*(1 + m)*(e*((a + b*x)/(c + d*x))^n)^((1 +
m)/n)*(c + d*x)*(i*(c + d*x))^m*(-(((1 + m)*(A + B*Log[e*((a + b*x)/(c + d
*x))^n]))/(B*n)))^p)
```

### 3.226.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

### 3.226.4 Maple [F]

$$\int (bgx+ag)^{m} (dix+ci)^{-2-m} \left(A+B\ln \left(e(bx+a)^{n} (dx+c)^{-n}\right)\right)^{p}dx$$

input
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p,x
)
```

output
```
int((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p,x
)
```

### 3.226.5 Fricas [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m} \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p \,dx$$
$$= \int (bgx+ag)^{m}(dix+ci)^{-m-2}\left(B\log \left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{p} dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n
)))^p,x, algorithm="fricas")
```

output
```
integral((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log((b*x + a)^n*e/(d*x
+ c)^n) + A)^p, x)
```

### 3.226.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m} \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p \,dx=\text{Timed out}$$

input
```
integrate((b*g*x+a*g)**m*(d*i*x+c*i)**(-2-m)*(A+B*ln(e*(b*x+a)**n/((d*x+c)
**n)))**p,x)
```

output
```
Timed out
```

### 3.226.7 Maxima [F]

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\int (bgx+ag)^m(dix+ci)^{-m-2}\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p\,dx$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n
)))^p,x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)^m*(d*i*x + c*i)^(-m - 2)*(B*log((b*x + a)^n*e/(d*x
 + c)^n) + A)^p, x)
```

### 3.226.8 Giac [F(-2)]

Exception generated.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\text{Exception raised: RuntimeError}$$

input
```
integrate((b*g*x+a*g)^m*(d*i*x+c*i)^(-2-m)*(A+B*log(e*(b*x+a)^n/((d*x+c)^n
)))^p,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,0,5,5,0,2,2,3,3,0,0,0,2]%%%}+%%%{-2,[0,0,5,4,1,3,1,3,3,0,0,0,2]%%%}+%%%{1
,[0,0,5,3,2,
```

### 3.226.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^m(ci+dix)^{-2-m}\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\int \frac{(a\,g+b\,g\,x)^m\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{(c\,i+d\,i\,x)^{m+2}}\,dx$$

input
```
int(((a*g + b*g*x)^m*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p)/(c*i + d*
i*x)^(m + 2),x)
```

output
```
int(((a*g + b*g*x)^m*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p)/(c*i + d*
i*x)^(m + 2), x)
```

## 3.226. $\int (ag + bgx)^m (ci + dix)^{-2-m} \left(A + B \log \left(e(a + bx)^n (c + dx)^{-n}\right)\right)^p \, dx$

# 3.227 $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c\right.\right.$



## 3.227.1 Optimal result

Integrand size = 50, antiderivative size = 194

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx =$$

$$-\frac{e^{\frac{A(1+m)}{Bn}}(a+bx)(g(a+bx))^{-2-m}(i(c+dx))^{2+m}\left(e(a+bx)^n(c+dx)^{-n}\right)^{\frac{1+m}{n}}\Gamma\Big(1+p,\frac{(1+m)(A+B\log(e(a+bx}{Bn}}{(bc-ad)i^2(1+m)(c+}$$

output
```
-exp(A*(1+m)/B/n)*(b*x+a)*(g*(b*x+a))^(-2-m)*(i*(d*x+c))^(2+m)*(e*(b*x+a)^
n/((d*x+c)^n))^((1+m)/n)*GAMMA(p+1,(1+m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
/B/n)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(-a*d+b*c)/i^2/(1+m)/(d*x+c)/(((
1+m)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/n)^p)
```

## 3.227.2 Mathematica [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$
$$=\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p\,dx$$

input
```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^p,x]
```

output
```
Integrate[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[(e*(a + b*x)^n
)/(c + d*x)^n])^p, x]
```

### 3.227.3 Rubi [A] (warning: unable to verify)

Time = 0.76 (sec) , antiderivative size = 190, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.080, Rules used = {2973, 2963, 2747, 2612}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int (ag+bgx)^{-m-2}(ci+dix)^m \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^p dx$$

$$\Big\downarrow \text{ 2973}$$

$$\int (ag+bgx)^{-m-2}(ci+dix)^m \left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^p dx$$

$$\Big\downarrow \text{ 2963}$$

$$\frac{(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(\frac{a+bx}{c+dx}\right)^{m+2}\int \left(\frac{a+bx}{c+dx}\right)^{-m-2}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\frac{a+bx}{c+dx}}{i^2(bc-ad)}$$

$$\Big\downarrow \text{ 2747}$$

$$\frac{(a+bx)(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\int \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\log\left(e\right.}{i^2n(c+dx)(bc-ad)}$$

$$\Big\downarrow \text{ 2612}$$

$$-\frac{(a+bx)e^{\frac{A(m+1)}{Bn}}(g(a+bx))^{-m-2}(i(c+dx))^{m+2}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^p\left(\frac{(m+1)\left(B\log\left(e\left(\frac{a+b}{c+d}\right.\right.\right.}{Bn}\right.}{i^2(m+1)(c+dx)(bc-ad)}$$

input
```
Int[(a*g + b*g*x)^(-2 - m)*(c*i + d*i*x)^m*(A + B*Log[(e*(a + b*x)^n)/(c +
 d*x)^n])^p,x]
```

output
```
-((E^((A*(1 + m))/(B*n))*(a + b*x)*(g*(a + b*x))^(-2 - m)*(e*((a + b*x)/(c
 + d*x))^n)^((1 + m)/n)*(i*(c + d*x))^(2 + m)*Gamma[1 + p, ((1 + m)*(A + B
*Log[e*((a + b*x)/(c + d*x))^n]))/(B*n)]*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p)/((b*c - a*d)*i^2*(1 + m)*(c + d*x)*(((1 + m)*(A + B*Log[e*((a + b
*x)/(c + d*x))^n]))/(B*n))^p))
```

### 3.227.3.1 Defintions of rubi rules used

rule 2612
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))*((c_.) + (d_.)*(x_))^(m_), x_Symbol]
:> Simp[(-F^(g*(e - c*(f/d))))*((c + d*x)^FracPart[m]/(d*((-f)*g*(Log[F]/d)
)^(IntPart[m] + 1)*((-f)*g*Log[F]*((c + d*x)/d))^FracPart[m]))*Gamma[m + 1,
 ((-f)*g*(Log[F]/d))*(c + d*x)], x] /; FreeQ[{F, c, d, e, f, g, m}, x] &&
!IntegerQ[m]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

### 3.227.4 Maple [F]

$$\int (bgx+ag)^{-2-m} (dix+ci)^{m} \left(A+B\ln \left(e(bx+a)^{n} (dx+c)^{-n}\right)\right)^{p}dx$$

input
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p,x
)
```

output
```
int((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p,x
)
```

### 3.227.5 Fricas [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p \, dx$$
$$= \int (bgx+ag)^{-m-2}(dix+ci)^{m}\left(B\log \left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{p} dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*(b*x+a)^n/((d*x+c)^n
)))^p,x, algorithm="fricas")
```

output
```
integral((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log((b*x + a)^n*e/(d*x
+ c)^n) + A)^p, x)
```

### 3.227.6 Sympy [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p \, dx = \text{Timed out}$$

input
```
integrate((b*g*x+a*g)**(-2-m)*(d*i*x+c*i)**m*(A+B*ln(e*(b*x+a)**n/((d*x+c)
**n)))**p,x)
```

output
```
Timed out
```

### 3.227.7 Maxima [F]

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p \, dx$$
$$= \int (bgx+ag)^{-m-2}(dix+ci)^m \left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p \, dx$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*(b*x+a)^n/((d*x+c)^n
)))^p,x, algorithm="maxima")
```

output
```
integrate((b*g*x + a*g)^(-m - 2)*(d*i*x + c*i)^m*(B*log((b*x + a)^n*e/(d*x
 + c)^n) + A)^p, x)
```

### 3.227.8 Giac [F(-2)]

Exception generated.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p \, dx$$
$$= \text{Exception raised: RuntimeError}$$

input
```
integrate((b*g*x+a*g)^(-2-m)*(d*i*x+c*i)^m*(A+B*log(e*(b*x+a)^n/((d*x+c)^n
)))^p,x, algorithm="giac")
```

output
```
Exception raised: RuntimeError >> an error occurred running a Giac command
:INPUT:sage2OUTPUT:Unable to divide, perhaps due to rounding error%%%{1,[0
,0,5,5,0,2,2,3,3,0,0,0,2]%%%}+%%%{-2,[0,0,5,4,1,3,1,3,3,0,0,0,2]%%%}+%%%{1
,[0,0,5,3,2,
```

### 3.227.9 Mupad [F(-1)]

Timed out.

$$\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p \, dx$$
$$= \int \frac{(c\,i+d\,i\,x)^m \left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{(a\,g+b\,g\,x)^{m+2}} \, dx$$

input
```
int(((c*i + d*i*x)^m*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p)/(a*g + b*
g*x)^(m + 2),x)
```

output
```
int(((c*i + d*i*x)^m*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p)/(a*g + b*
g*x)^(m + 2), x)
```

## 3.227. $\int (ag+bgx)^{-2-m}(ci+dix)^m \left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p \, dx$

# 3.228 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)(c+dx)}\,dx$



## 3.228.1 Optimal result

Integrand size = 40, antiderivative size = 45

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)(c+dx)}\,dx=\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{4B(bc-ad)n}$$

output
```
1/4*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4/B/(-a*d+b*c)/n
```

## 3.228.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(a+bx)(c+dx)}\,dx=\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{4(bBcn-aBdn)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((a + b*x)*(c + d*x))
,x]
```

output
```
(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4/(4*(b*B*c*n - a*B*d*n))
```

### 3.228.3 Rubi [A] (warning: unable to verify)

Time = 0.45 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2973, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \; 2973$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \; 2961$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{a+bx}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \; 2739$$

$$\frac{\int \frac{(a+bx)^3}{(c+dx)^3}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\Big\downarrow \; 15$$

$$\frac{(a+bx)^4}{4Bn(c+dx)^4(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((a + b*x)*(c + d*x)),x]
```

output
```
(a + b*x)^4/(4*B*(b*c - a*d)*n*(c + d*x)^4)
```

### 3.228.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

## 3.228.4 Maple [A] (verified)

Time = 33.16 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| derivativedivides | $-\frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{4}}{4n(ad-cb)B}$ |
| default | $-\frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{4}}{4n(ad-cb)B}$ |
| parallelrisch | $-\frac{B^{3}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{4}b^{2}d^{2}+4A\,B^{2}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{3}b^{2}d^{2}+6A^{2}B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{2}b^{2}d^{2}+4A}{4n\,b^{2}d^{2}(ad-cb)}$ |
| parts | $\frac{A^{3}\ln(dx+c)}{ad-cb}-\frac{A^{3}\ln(bx+a)}{ad-cb}-\frac{B^{3}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{4}}{4n(ad-cb)}-\frac{A\,B^{2}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{3}}{n(ad-cb)}-\frac{3A^{2}B\ln\left(e(bx\right.}{2n(}$ |
| risch | Expression too large to display |

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)/(d*x+c),x,method=_RETURNVE
RBOSE)
```

output
```
-1/4/n/(a*d-b*c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4/B
```

### 3.228.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 375 vs. $2(43) = 86$.

Time = 0.32 (sec) , antiderivative size = 375, normalized size of antiderivative = 8.33

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx = \frac{B^3n^3\log(bx+a)^4 + B^3n^3\log(dx+c)^4 + 4(B^3n^2\log(e) + AB^2n^2)\log(bx+a)^3 - 4(B^3n^3\log(bx+a) \cdots}{}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)/(d*x+c),x, algorith
m="fricas")
```

output
```
1/4*(B^3*n^3*log(b*x + a)^4 + B^3*n^3*log(d*x + c)^4 + 4*(B^3*n^2*log(e) +
 A*B^2*n^2)*log(b*x + a)^3 - 4*(B^3*n^3*log(b*x + a) + B^3*n^2*log(e) + A*
B^2*n^2)*log(d*x + c)^3 + 6*(B^3*n*log(e)^2 + 2*A*B^2*n*log(e) + A^2*B*n)*
log(b*x + a)^2 + 6*(B^3*n^3*log(b*x + a)^2 + B^3*n*log(e)^2 + 2*A*B^2*n*lo
g(e) + A^2*B*n + 2*(B^3*n^2*log(e) + A*B^2*n^2)*log(b*x + a))*log(d*x + c)
^2 + 4*(B^3*log(e)^3 + 3*A*B^2*log(e)^2 + 3*A^2*B*log(e) + A^3)*log(b*x +
a) - 4*(B^3*n^3*log(b*x + a)^3 + B^3*log(e)^3 + 3*A*B^2*log(e)^2 + 3*A^2*B
*log(e) + A^3 + 3*(B^3*n^2*log(e) + A*B^2*n^2)*log(b*x + a)^2 + 3*(B^3*n*l
og(e)^2 + 2*A*B^2*n*log(e) + A^2*B*n)*log(b*x + a))*log(d*x + c))/(b*c - a
*d)
```

### 3.228.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

### 3.228.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 766 vs. 2(43) = 86.

Time = 0.25 (sec) , antiderivative size = 766, normalized size of antiderivative = 17.02

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx$$
$$= B^3\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^3$$
$$+3\,AB^2\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2$$
$$+3\,A^2B\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)$$
$$-\frac{1}{4}B^3\left(\frac{6\left(en\log(bx+a)^2-2\,en\log(bx+a)\log(dx+c)+en\log(dx+c)^2\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2}{(bc-ad)e}-\frac{4\left(e^2n^2\log\right.}{}\right.$$
$$+A^3\left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)$$
$$-AB^2\left(\frac{3\left(en\log(bx+a)^2-2\,en\log(bx+a)\log(dx+c)+en\log(dx+c)^2\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{(bc-ad)e}-\frac{e^2n^2\log(}{}\right.$$
$$-\frac{3\left(en\log(bx+a)^2-2\,en\log(bx+a)\log(dx+c)+en\log(dx+c)^2\right)A^2B}{2\,(bc-ad)e}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)/(d*x+c),x, algorith
m="maxima")
```

output
```
B^3*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log((b*x + a)^n*
e/(d*x + c)^n)^3 + 3*A*B^2*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c -
 a*d))*log((b*x + a)^n*e/(d*x + c)^n)^2 + 3*A^2*B*(log(b*x + a)/(b*c - a*d
) - log(d*x + c)/(b*c - a*d))*log((b*x + a)^n*e/(d*x + c)^n) - 1/4*B^3*(6*
(e*n*log(b*x + a)^2 - 2*e*n*log(b*x + a)*log(d*x + c) + e*n*log(d*x + c)^2
)*log((b*x + a)^n*e/(d*x + c)^n)^2/((b*c - a*d)*e) - (4*(e^2*n^2*log(b*x +
 a)^3 - 3*e^2*n^2*log(b*x + a)^2*log(d*x + c) + 3*e^2*n^2*log(b*x + a)*log
(d*x + c)^2 - e^2*n^2*log(d*x + c)^3)*log((b*x + a)^n*e/(d*x + c)^n)/((b*c
 - a*d)*e) - (e^3*n^3*log(b*x + a)^4 - 4*e^3*n^3*log(b*x + a)^3*log(d*x +
c) + 6*e^3*n^3*log(b*x + a)^2*log(d*x + c)^2 - 4*e^3*n^3*log(b*x + a)*log(
d*x + c)^3 + e^3*n^3*log(d*x + c)^4)/((b*c - a*d)*e^2))/e) + A^3*(log(b*x
+ a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)) - A*B^2*(3*(e*n*log(b*x + a)^
2 - 2*e*n*log(b*x + a)*log(d*x + c) + e*n*log(d*x + c)^2)*log((b*x + a)^n*
e/(d*x + c)^n)/((b*c - a*d)*e) - (e^2*n^2*log(b*x + a)^3 - 3*e^2*n^2*log(b
*x + a)^2*log(d*x + c) + 3*e^2*n^2*log(b*x + a)*log(d*x + c)^2 - e^2*n^2*l
og(d*x + c)^3)/((b*c - a*d)*e^2)) - 3/2*(e*n*log(b*x + a)^2 - 2*e*n*log(b*
x + a)*log(d*x + c) + e*n*log(d*x + c)^2)*A^2*B/((b*c - a*d)*e)
```

### 3.228.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bx+a)(dx+c)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(b*x+a)/(d*x+c),x, algorith
m="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/((b*x + a)*(d*x + c)),
x)
```

### 3.228.9 Mupad [B] (verification not implemented)

Time = 2.61 (sec) , antiderivative size = 141, normalized size of antiderivative = 3.13

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(a+bx)(c+dx)}\,dx$$
$$= -\frac{\frac{3A^2B\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^2}{2}+AB^2\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^3+\frac{B^3\ln\left(\frac{e(a+bx)^n}{(c+dx)^n}\right)^4}{4}}{n\,(a\,d-b\,c)}$$
$$+\frac{A^3\,\mathrm{atan}\left(\frac{a\,d\,1\mathrm{i}+b\,c\,1\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}\right)\,2\mathrm{i}}{a\,d-b\,c}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/((a + b*x)*(c + d*x)),x)
```

output
```
(A^3*atan((a*d*1i + b*c*1i + b*d*x*2i)/(a*d - b*c))*2i)/(a*d - b*c) - ((B^
3*log((e*(a + b*x)^n)/(c + d*x)^n)^4)/4 + (3*A^2*B*log((e*(a + b*x)^n)/(c
+ d*x)^n)^2)/2 + A*B^2*log((e*(a + b*x)^n)/(c + d*x)^n)^3)/(n*(a*d - b*c))
```

# 3.229 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx$



## 3.229.1 Optimal result

Integrand size = 40, antiderivative size = 45

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx = \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{3B(bc-ad)n}$$

output
```
1/3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/B/(-a*d+b*c)/n
```

## 3.229.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx = \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{3(bBcn-aBdn)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/((a + b*x)*(c + d*x))
,x]
```

output
```
(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(3*(b*B*c*n - a*B*d*n))
```

### 3.229.3 Rubi [A] (warning: unable to verify)

Time = 0.44 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2973, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{a+bx}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2739}$$

$$\frac{\int \frac{(a+bx)^2}{(c+dx)^2}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\Big\downarrow \text{15}$$

$$\frac{(a+bx)^3}{3Bn(c+dx)^3(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/((a + b*x)*(c + d*x)),x]
```

output
```
(a + b*x)^3/(3*B*(b*c - a*d)*n*(c + d*x)^3)
```

#### 3.229.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

### 3.229.4 Maple [A] (verified)

Time = 5.82 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{3}}{3n(ad-cb)B}$ | 44 |
| default | $-\frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{3}}{3n(ad-cb)B}$ | 44 |
| parallelrisch | $-\frac{B^{2}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{3}b^{2}d^{2}+3AB\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{2}b^{2}d^{2}+3A^{2}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)b^{2}d^{2}}{3n\,b^{2}d^{2}(ad-cb)}$ | 115 |
| parts | $\frac{A^{2}\ln(dx+c)}{ad-cb}-\frac{A^{2}\ln(bx+a)}{ad-cb}-\frac{B^{2}\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{3}}{3n(ad-cb)}-\frac{BA\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)^{2}}{n(ad-cb)}$ | 120 |
| risch | Expression too large to display | 1106 |

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)/(d*x+c),x,method=_RETURNVE
RBOSE)
```

output
```
-1/3/n/(a*d-b*c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/B
```

### 3.229.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 188 vs. $2(43) = 86$.

Time = 0.33 (sec) , antiderivative size = 188, normalized size of antiderivative = 4.18

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx = \frac{B^2n^2\log(bx+a)^3 - B^2n^2\log(dx+c)^3 + 3(B^2n\log(e) + ABn)\log(bx+a)^2 + 3(B^2n^2\log(bx+a) + B$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)/(d*x+c),x, algorith
m="fricas")
```

output
```
1/3*(B^2*n^2*log(b*x + a)^3 - B^2*n^2*log(d*x + c)^3 + 3*(B^2*n*log(e) + A
*B*n)*log(b*x + a)^2 + 3*(B^2*n^2*log(b*x + a) + B^2*n*log(e) + A*B*n)*log
(d*x + c)^2 + 3*(B^2*log(e)^2 + 2*A*B*log(e) + A^2)*log(b*x + a) - 3*(B^2*
n^2*log(b*x + a)^2 + B^2*log(e)^2 + 2*A*B*log(e) + A^2 + 2*(B^2*n*log(e) +
 A*B*n)*log(b*x + a))*log(d*x + c))/(b*c - a*d)
```

### 3.229.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

### 3.229.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(43) = 86$.

Time = 0.22 (sec) , antiderivative size = 387, normalized size of antiderivative = 8.60

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx$$
$$= B^2\left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)^2$$
$$+ 2\,AB\left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)$$
$$+ A^2\left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right)$$
$$-\frac{1}{3}B^2\left(\frac{3\left(en\log(bx+a)^2 - 2\,en\log(bx+a)\log(dx+c) + en\log(dx+c)^2\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}{(bc-ad)e} - \frac{e^2n^2\log\right.$$
$$- \frac{\left(en\log(bx+a)^2 - 2\,en\log(bx+a)\log(dx+c) + en\log(dx+c)^2\right)AB}{(bc-ad)e}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)/(d*x+c),x, algorith
m="maxima")
```

output
```
B^2*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log((b*x + a)^n*
e/(d*x + c)^n)^2 + 2*A*B*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a
*d))*log((b*x + a)^n*e/(d*x + c)^n) + A^2*(log(b*x + a)/(b*c - a*d) - log(
d*x + c)/(b*c - a*d)) - 1/3*B^2*(3*(e*n*log(b*x + a)^2 - 2*e*n*log(b*x + a
)*log(d*x + c) + e*n*log(d*x + c)^2)*log((b*x + a)^n*e/(d*x + c)^n)/((b*c
- a*d)*e) - (e^2*n^2*log(b*x + a)^3 - 3*e^2*n^2*log(b*x + a)^2*log(d*x + c
) + 3*e^2*n^2*log(b*x + a)*log(d*x + c)^2 - e^2*n^2*log(d*x + c)^3)/((b*c
- a*d)*e^2)) - (e*n*log(b*x + a)^2 - 2*e*n*log(b*x + a)*log(d*x + c) + e*n
*log(d*x + c)^2)*A*B/((b*c - a*d)*e)
```

### 3.229.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(bx+a)(dx+c)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(b*x+a)/(d*x+c),x, algorith
m="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/((b*x + a)*(d*x + c)),
x)
```

### 3.229.9 Mupad [B] (verification not implemented)

Time = 1.49 (sec) , antiderivative size = 100, normalized size of antiderivative = 2.22

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(a+bx)(c+dx)}\,dx$$
$$=-\frac{-6\mathrm{i}\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,A^2+3\,A\,B\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2+B^2\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^3}{3\,n\,(a\,d-b\,c)}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/((a + b*x)*(c + d*x)),x)
```

output
```
-(B^2*log((e*(a + b*x)^n)/(c + d*x)^n)^3 + 3*A*B*log((e*(a + b*x)^n)/(c +
d*x)^n)^2 - A^2*n*atan((b*c*2i + b*d*x*2i)/(a*d - b*c) + 1i)*6i)/(3*n*(a*d
 - b*c))
```

# 3.230 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx$



## 3.230.1 Optimal result

Integrand size = 38, antiderivative size = 45

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx=\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2B(bc-ad)n}$$

output
```
1/2*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/B/(-a*d+b*c)/n
```

## 3.230.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx=\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{2(bBcn-aBdn)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((a + b*x)*(c + d*x)),x
]
```

output
```
(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(2*(b*B*c*n - a*B*d*n))
```

### 3.230.3 Rubi [A] (warning: unable to verify)

Time = 0.39 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.079, Rules used = {2973, 2961, 2738}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{a+bx}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2738}$$

$$\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{2Bn(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((a + b*x)*(c + d*x)),x]
```

output
```
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^2/(2*B*(b*c - a*d)*n)
```

#### 3.230.3.1 Defintions of rubi rules used

rule 2738
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))/(x_), x_Symbol] :> Simp[(a + b*Lo
g[c*x^n])^2/(2*b*n), x] /; FreeQ[{a, b, c, n}, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

### 3.230.4 Maple [A] (verified)

Time = 3.84 (sec) , antiderivative size = 62, normalized size of antiderivative = 1.38

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\frac{B \ln\left(e (b x+a)^{n} (d x+c)^{-n}\right)^{2}}{2}+\ln\left(e (b x+a)^{n} (d x+c)^{-n}\right) A}{n (a d-c b)}$ | 62 |
| default | $-\frac{\frac{B \ln\left(e (b x+a)^{n} (d x+c)^{-n}\right)^{2}}{2}+\ln\left(e (b x+a)^{n} (d x+c)^{-n}\right) A}{n (a d-c b)}$ | 62 |
| parts | $\frac{A \ln(d x+c)}{a d-c b}-\frac{\ln(b x+a) A}{a d-c b}-\frac{B \ln\left(e (b x+a)^{n} (d x+c)^{-n}\right)^{2}}{2 n (a d-c b)}$ | 76 |
| parallelrisch | $-\frac{B a^{2} c^{2} \ln\left(e (b x+a)^{n} (d x+c)^{-n}\right)^{2}+2 A \ln\left(e (b x+a)^{n} (d x+c)^{-n}\right) a^{2} c^{2}}{2 c^{2} a^{2} n (a d-c b)}$ | 80 |
| risch | Expression too large to display | 1152 |

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)/(d*x+c),x,method=_RETURNVERB
OSE)
```

output
```
-1/n/(a*d-b*c)*(1/2*B*ln(e*(b*x+a)^n/((d*x+c)^n))^2+ln(e*(b*x+a)^n/((d*x+c
)^n))*A)
```

### 3.230.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx$$
$$=\frac{Bn\log\left(bx+a\right)^2+Bn\log\left(dx+c\right)^2+2\left(B\log\left(e\right)+A\right)\log\left(bx+a\right)-2\left(Bn\log\left(bx+a\right)+B\log\left(e\right)+A\right.}{2\left(bc-ad\right)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)/(d*x+c),x, algorithm=
"fricas")
```

output
```
1/2*(B*n*log(b*x + a)^2 + B*n*log(d*x + c)^2 + 2*(B*log(e) + A)*log(b*x +
a) - 2*(B*n*log(b*x + a) + B*log(e) + A)*log(d*x + c))/(b*c - a*d)
```

### 3.230.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

### 3.230.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 151 vs. $2(43)=86$.

Time = 0.21 (sec) , antiderivative size = 151, normalized size of antiderivative = 3.36

$$\begin{aligned}\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx &= B\left(\frac{\log\left(bx+a\right)}{bc-ad}-\frac{\log\left(dx+c\right)}{bc-ad}\right)\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\left(\frac{\log\left(bx+a\right)}{bc-ad}-\frac{\log\left(dx+c\right)}{bc-ad}\right)\\&\quad-\frac{\left(en\log\left(bx+a\right)^2-2\,en\log\left(bx+a\right)\log\left(dx+c\right)+en\log\left(dx+c\right)^2\right)B}{2\,(bc-ad)e}\end{aligned}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)/(d*x+c),x, algorithm=
"maxima")
```

output
```
B*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log((b*x + a)^n*e/
(d*x + c)^n) + A*(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d)) - 1
/2*(e*n*log(b*x + a)^2 - 2*e*n*log(b*x + a)*log(d*x + c) + e*n*log(d*x + c
)^2)*B/((b*c - a*d)*e)
```

### 3.230.8 Giac [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{(bx+a)(dx+c)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(b*x+a)/(d*x+c),x, algorithm=
"giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/((b*x + a)*(d*x + c)), x)
```

### 3.230.9 Mupad [B] (verification not implemented)

Time = 1.36 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.58

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(a+bx)(c+dx)}\,dx = -\frac{B\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2 - A\,n\,\mathrm{atan}\left(\frac{b\,c\,2\mathrm{i}+b\,d\,x\,2\mathrm{i}}{a\,d-b\,c}+1\mathrm{i}\right)\,4\mathrm{i}}{2\,n\,\left(a\,d-b\,c\right)}$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/((a + b*x)*(c + d*x)),x)
```

output
```
-(B*log((e*(a + b*x)^n)/(c + d*x)^n)^2 - A*n*atan((b*c*2i + b*d*x*2i)/(a*d
 - b*c) + 1i)*4i)/(2*n*(a*d - b*c))
```

# 3.231 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$



## 3.231.1 Optimal result

Integrand size = 40, antiderivative size = 41

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)n}$$

output
```
ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/(-a*d+b*c)/n
```

## 3.231.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bBcn-aBdn}$$

input
```
Integrate[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))
,x]
```

output
```
Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(b*B*c*n - a*B*d*n)
```

### 3.231.3 Rubi [A] (warning: unable to verify)

Time = 0.49 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2973, 2961, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{c+dx}{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2739}$$

$$\frac{\int \frac{c+dx}{a+bx}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{Bn(bc-ad)}$$

input
```
Int[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

output
```
Log[A + B*Log[e*((a + b*x)/(c + d*x))^n]]/(B*(b*c - a*d)*n)
```

#### 3.231.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

## 3.231.4 Maple [A] (verified)

Time = 21.34 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

| method | result |
|---|---|
| derivativedivides | $-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{n(ad-cb)B}$ |
| default | $-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{n(ad-cb)B}$ |
| parallelrisch | $-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{n(ad-cb)B}$ |
| risch | $-\frac{\ln\left(\ln((dx+c)^{n})-\frac{-iB\pi\ \mathrm{csgn}(ie)\,\mathrm{csgn}\left(i(bx+a)^{n}(dx+c)^{-n}\right)\mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a)^{n}\right)+iB\pi\ \mathrm{csgn}(ie)\,\mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a\right.\right.}{}}{}$ |

input
```
int(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVE
RBOSE)
```

output
```
-1/n/(a*d-b*c)*ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B
```

### 3.231.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-Bn\log\left(bx+a\right)+Bn\log\left(dx+c\right)-B\log\left(e\right)-A\right)}{(Bbc-Bad)n}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorith
m="fricas")
```

output
```
log(-B*n*log(b*x + a) + B*n*log(d*x + c) - B*log(e) - A)/((B*b*c - B*a*d)*
n)
```

### 3.231.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Timed out
```

### 3.231.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-\frac{B\log((bx+a)^n)-B\log((dx+c)^n)+B\log(e)+A}{B}\right)}{(bcn-adn)B}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorith
m="maxima")
```

output
```
log(-(B*log((b*x + a)^n) - B*log((d*x + c)^n) + B*log(e) + A)/B)/((b*c*n -
 a*d*n)*B)
```

### 3.231.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(Bn\log\left(bx+a\right)-Bn\log\left(dx+c\right)+B\log\left(e\right)+A\right)}{Bbcn-Badn}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorith
m="giac")
```

output
```
log(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)/(B*b*c*n - B*a*d*n
)
```

### 3.231.9 Mupad [B] (verification not implemented)

Time = 1.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = -\frac{\ln\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}{B\,a\,d\,n-B\,b\,c\,n}$$

input
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)*(c + d*x)),x)
```

output
```
-log(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(B*a*d*n - B*b*c*n)
```

# 3.232 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}\,dx$



## 3.232.1 Optimal result

Integrand size = 40, antiderivative size = 43

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = -\frac{1}{B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}$$

output
```
-1/B/(-a*d+b*c)/n/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))
```

## 3.232.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 41, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = -\frac{1}{(bBcn-aBdn)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}$$

input
```
Integrate[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^
2),x]
```

output
```
-(1/((b*B*c*n - a*B*d*n)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])))
```

### 3.232.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 30, normalized size of antiderivative = 0.70, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2973, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{c+dx}{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2739}$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\Big\downarrow \text{15}$$

$$-\frac{c+dx}{Bn(a+bx)(bc-ad)}$$

input
```
Int[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2),x]
```

output
```
-((c + d*x)/(B*(b*c - a*d)*n*(a + b*x)))
```

#### 3.232.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

### 3.232.4 Maple [A] (verified)

Time = 65.20 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

| method | result |
|---|---|
| derivativedivides | $\frac{1}{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)Bn(ad-cb)}$ |
| default | $\frac{1}{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)Bn(ad-cb)}$ |
| parallelrisch | $\frac{1}{\left(A+B\ln\left(e(bx+a)^n(dx+c)^{-n}\right)\right)Bn(ad-cb)}$ |
| risch | $\frac{}{Bn(ad-cb)\left(2A+2B\ln(e)+2B\ln((bx+a)^n)-2B\ln((dx+c)^n)-iB\pi\operatorname{csgn}(i(bx+a)^n)\operatorname{csgn}\left(i(dx+c)^{-n}\right)\operatorname{csgn}\left(i(bx+a)^n\right.\right.}$ |

input
```
int(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x,method=_RETURN
VERBOSE)
```

output
```
1/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/n/(a*d-b*c)
```

### 3.232.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 86, normalized size of antiderivative = 2.00

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx =$$
$$-\frac{1}{(B^2bc-B^2ad)n^2\log(bx+a)-(B^2bc-B^2ad)n^2\log(dx+c)+(B^2bc-B^2ad)n\log(e)+(ABbc-A}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algori
thm="fricas")
```

output
```
-1/((B^2*b*c - B^2*a*d)*n^2*log(b*x + a) - (B^2*b*c - B^2*a*d)*n^2*log(d*x
 + c) + (B^2*b*c - B^2*a*d)*n*log(e) + (A*B*b*c - A*B*a*d)*n)
```

### 3.232.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Timed out
```

### 3.232.7 Maxima [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.88

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx =$$
$$-\frac{1}{(bcn-adn)B^2\log\left((bx+a)^n\right)-(bcn-adn)B^2\log\left((dx+c)^n\right)+(bcn-adn)AB+(bcn\log(e)-adn}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algori
thm="maxima")
```

output
```
-1/((b*c*n - a*d*n)*B^2*log((b*x + a)^n) - (b*c*n - a*d*n)*B^2*log((d*x +
c)^n) + (b*c*n - a*d*n)*A*B + (b*c*n*log(e) - a*d*n*log(e))*B^2)
```

### 3.232.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 99 vs. $2(43) = 86$.

Time = 0.29 (sec) , antiderivative size = 99, normalized size of antiderivative = 2.30

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx =$$
$$-\frac{1}{B^2bcn^2\log\left(bx+a\right)-B^2adn^2\log\left(bx+a\right)-B^2bcn^2\log\left(dx+c\right)+B^2adn^2\log\left(dx+c\right)+B^2bcn\log\left(e\right.}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, algori
thm="giac")
```

output
```
-1/(B^2*b*c*n^2*log(b*x + a) - B^2*a*d*n^2*log(b*x + a) - B^2*b*c*n^2*log(
d*x + c) + B^2*a*d*n^2*log(d*x + c) + B^2*b*c*n*log(e) - B^2*a*d*n*log(e)
+ A*B*b*c*n - A*B*a*d*n)
```

### 3.232.9 Mupad [B] (verification not implemented)

Time = 1.25 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \frac{1}{B\,n\,\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)\,\left(a\,d-b\,c\right)}$$

input
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(a + b*x)*(c + d*x)),x)
```

output
```
1/(B*n*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a*d - b*c))
```

# 3.233 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}\,dx$



## 3.233.1 Optimal result

Integrand size = 40, antiderivative size = 45

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx = -\frac{1}{2B(bc-ad)n\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}$$

output
```
-1/2/B/(-a*d+b*c)/n/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2
```

## 3.233.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.96

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx = -\frac{1}{2(bBcn-aBdn)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}$$

input
```
Integrate[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^
3),x]
```

output
```
-1/2*1/((b*B*c*n - a*B*d*n)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)
```

### 3.233.3 Rubi [A] (warning: unable to verify)

Time = 0.47 (sec) , antiderivative size = 34, normalized size of antiderivative = 0.76, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2973, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}dx$$

$$\Big\downarrow \text{ 2973}$$

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^3}dx$$

$$\Big\downarrow \text{ 2961}$$

$$\frac{\int \frac{c+dx}{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{ 2739}$$

$$\frac{\int \frac{(c+dx)^3}{(a+bx)^3}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\Big\downarrow \text{ 15}$$

$$-\frac{(c+dx)^2}{2Bn(a+bx)^2(bc-ad)}$$

input
```
Int[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3),x]
```

output
```
-1/2*(c + d*x)^2/(B*(b*c - a*d)*n*(a + b*x)^2)
```

### 3.233.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

## 3.233.4 Maple [A] (verified)

Time = 163.31 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.98

| method | result |
|---|---|
| derivativedivides | $\frac{1}{2\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{2}Bn(ad-cb)}$ |
| default | $\frac{1}{2\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{2}Bn(ad-cb)}$ |
| parallelrisch | $\frac{1}{2\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{2}Bn(ad-cb)}$ |
| risch | $\frac{}{Bn(ad-cb)\left(2A+2B\ln(e)+2B\ln((bx+a)^{n})-2B\ln((dx+c)^{n})-iB\pi\ \mathrm{csgn}(i(bx+a)^{n})\ \mathrm{csgn}\left(i(dx+c)^{-n}\right)\ \mathrm{csgn}\left(i(bx+a)^{n}\right.\right.}$ |

input
```
int(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3,x,method=_RETURN
VERBOSE)
```

output
```
1/2/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/B/n/(a*d-b*c)
```

### 3.233.5 Fricas [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 238 vs. $2(43) = 86$.

Time = 0.33 (sec) , antiderivative size = 238, normalized size of antiderivative = 5.29

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx =$$

$$-\frac{}{2\left((B^3bc-B^3ad)n^3\log\left(bx+a\right)^2+(B^3bc-B^3ad)n^3\log\left(dx+c\right)^2+(B^3bc-B^3ad)n\log\left(e\right)^2+2\left(AB\right.\right.}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algori
thm="fricas")
```

output
```
-1/2/((B^3*b*c - B^3*a*d)*n^3*log(b*x + a)^2 + (B^3*b*c - B^3*a*d)*n^3*log
(d*x + c)^2 + (B^3*b*c - B^3*a*d)*n*log(e)^2 + 2*(A*B^2*b*c - A*B^2*a*d)*n
*log(e) + (A^2*B*b*c - A^2*B*a*d)*n + 2*((B^3*b*c - B^3*a*d)*n^2*log(e) +
(A*B^2*b*c - A*B^2*a*d)*n^2)*log(b*x + a) - 2*((B^3*b*c - B^3*a*d)*n^3*log
(b*x + a) + (B^3*b*c - B^3*a*d)*n^2*log(e) + (A*B^2*b*c - A*B^2*a*d)*n^2)*
log(d*x + c))
```

### 3.233.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3,x)
```

output
```
Timed out
```

### 3.233.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 220 vs. 2(43) = 86.

Time = 0.36 (sec) , antiderivative size = 220, normalized size of antiderivative = 4.89

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx =$$

$$-\frac{}{2\left((bcn-adn)B^3\log\left((bx+a)^n\right)^2+(bcn-adn)B^3\log\left((dx+c)^n\right)^2+(bcn-adn)A^2B+2\left(bcn\log\left(e\right.\right.\right.}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algori
thm="maxima")
```

output
```
-1/2/((b*c*n - a*d*n)*B^3*log((b*x + a)^n)^2 + (b*c*n - a*d*n)*B^3*log((d*
x + c)^n)^2 + (b*c*n - a*d*n)*A^2*B + 2*(b*c*n*log(e) - a*d*n*log(e))*A*B^
2 + (b*c*n*log(e)^2 - a*d*n*log(e)^2)*B^3 + 2*((b*c*n - a*d*n)*A*B^2 + (b*
c*n*log(e) - a*d*n*log(e))*B^3)*log((b*x + a)^n) - 2*((b*c*n - a*d*n)*B^3*
log((b*x + a)^n) + (b*c*n - a*d*n)*A*B^2 + (b*c*n*log(e) - a*d*n*log(e))*B
^3)*log((d*x + c)^n))
```

### 3.233.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 321 vs. 2(43) = 86.

Time = 0.29 (sec) , antiderivative size = 321, normalized size of antiderivative = 7.13

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx =$$

$$-\frac{}{2\left(B^3bcn^3\log\left(bx+a\right)^2-B^3adn^3\log\left(bx+a\right)^2-2\,B^3bcn^3\log\left(bx+a\right)\log\left(dx+c\right)+2\,B^3adn^3\log\left(bx\right.\right.}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3,x, algori
thm="giac")
```

output
```
-1/2/(B^3*b*c*n^3*log(b*x + a)^2 - B^3*a*d*n^3*log(b*x + a)^2 - 2*B^3*b*c*
n^3*log(b*x + a)*log(d*x + c) + 2*B^3*a*d*n^3*log(b*x + a)*log(d*x + c) +
B^3*b*c*n^3*log(d*x + c)^2 - B^3*a*d*n^3*log(d*x + c)^2 + 2*B^3*b*c*n^2*lo
g(b*x + a)*log(e) - 2*B^3*a*d*n^2*log(b*x + a)*log(e) - 2*B^3*b*c*n^2*log(
d*x + c)*log(e) + 2*B^3*a*d*n^2*log(d*x + c)*log(e) + 2*A*B^2*b*c*n^2*log(
b*x + a) - 2*A*B^2*a*d*n^2*log(b*x + a) - 2*A*B^2*b*c*n^2*log(d*x + c) + 2
*A*B^2*a*d*n^2*log(d*x + c) + B^3*b*c*n*log(e)^2 - B^3*a*d*n*log(e)^2 + 2*
A*B^2*b*c*n*log(e) - 2*A*B^2*a*d*n*log(e) + A^2*B*b*c*n - A^2*B*a*d*n)
```

### 3.233.9 Mupad [B] (verification not implemented)

Time = 1.28 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.60

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}\,dx = \frac{1}{2\,B\,n\,(a\,d-b\,c)\,\left(A^2+2\,A\,B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)+B^2\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)^2\right)}$$

input
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3*(a + b*x)*(c + d*x)),x)
```

output
```
1/(2*B*n*(a*d - b*c)*(B^2*log((e*(a + b*x)^n)/(c + d*x)^n)^2 + A^2 + 2*A*B
*log((e*(a + b*x)^n)/(c + d*x)^n)))
```

# 3.234 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx$



## 3.234.1 Optimal result

Integrand size = 40, antiderivative size = 49

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx=\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^{1+p}}{B(bc-ad)n(1+p)}$$

output
```
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(-a*d+b*c)/n/(p+1)
```

## 3.234.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 47, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx=\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^{1+p}}{(bBcn-aBdn)(1+p)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/((a + b*x)*(c + d*x))
,x]
```

output
```
(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^(1 + p)/((b*B*c*n - a*B*d*n)*(1 +
 p))
```

### 3.234.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 48, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2973, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^p}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^p}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p}{a+bx}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2739}$$

$$\frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\Big\downarrow \text{15}$$

$$\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^{p+1}}{Bn(p+1)(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/((a + b*x)*(c + d*x)),x]
```

output
```
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(1 + p)/(B*(b*c - a*d)*n*(1 + p))
```

### 3.234.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

## 3.234.4 Maple [A] (verified)

Time = 234.62 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.04

| method | result | siz |
|---|---|---|
| derivativedivides | $-\frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{p+1}}{n(ad-cb)B(p+1)}$ | 51 |
| default | $-\frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{p+1}}{n(ad-cb)B(p+1)}$ | 51 |
| parallelrisch | $-\frac{B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{p}a^{2}c^{2}+A\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{p}a^{2}c^{2}}{(adp-bcp+ad-cb)B\,a^{2}c^{2}n}$ | 12 |

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x,method=_RETURNVE
RBOSE)
```

output
```
-1/n/(a*d-b*c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(p+1)
```

### 3.234.5 Fricas [A] (verification not implemented)

Time = 0.35 (sec) , antiderivative size = 81, normalized size of antiderivative = 1.65

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx = \frac{(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)^p}{(Bbc-Bad)np+(Bbc-Bad)n}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x, algorith
m="fricas")
```

output
```
(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)*(B*n*log(b*x + a) - B
*n*log(d*x + c) + B*log(e) + A)^p/((B*b*c - B*a*d)*n*p + (B*b*c - B*a*d)*n
)
```

### 3.234.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**p/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

### 3.234.7 Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p}{(bx+a)(dx+c)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x, algorith
m="maxima")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^p/((b*x + a)*(d*x + c)),
x)
```

### 3.234.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.00

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx = \frac{(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)^{p+1}}{(Bbcn-Badn)(p+1)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*x+a)/(d*x+c),x, algorith
m="giac")
```

output
```
(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)^(p + 1)/((B*b*c*n - B
*a*d*n)*(p + 1))
```

### 3.234.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/((a + b*x)*(c + d*x)),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/((a + b*x)*(c + d*x)), x)
```

# 3.235 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^p}{(af+bfx)(cg+dgx)}\,dx$



## 3.235.1 Optimal result

Integrand size = 46, antiderivative size = 55

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx=\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^{1+p}}{B(bc-ad)fgn(1+p)}$$

output
```
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(-a*d+b*c)/f/g/n/(p+1)
```

## 3.235.2 Mathematica [A] (verified)

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.93

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx=\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^{1+p}}{(bBcfgn-aBdfgn)(1+p)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/((a*f + b*f*x)*(c*g +
 d*g*x)),x]
```

output
```
(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^(1 + p)/((b*B*c*f*g*n - a*B*d*f*g
*n)*(1 + p))
```

### 3.235.3 Rubi [A] (warning: unable to verify)

Time = 0.60 (sec) , antiderivative size = 54, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2973, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^p}{(af+bfx)(cg+dgx)}dx$$

$$\Big\downarrow \ 2973$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^p}{(af+bfx)(cg+dgx)}dx$$

$$\Big\downarrow \ 2961$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p}{a+bx}d\frac{a+bx}{c+dx}}{fg(bc-ad)}$$

$$\Big\downarrow \ 2739$$

$$\frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bfgn(bc-ad)}$$

$$\Big\downarrow \ 15$$

$$\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^{p+1}}{Bfgn(p+1)(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/((a*f + b*f*x)*(c*g + d*g*x
)),x]
```

output
```
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(1 + p)/(B*(b*c - a*d)*f*g*n*(1 + p
))
```

#### 3.235.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

### 3.235.4 Maple [A] (verified)

Time = 0.02 (sec) , antiderivative size = 57, normalized size of antiderivative = 1.04

$$-\frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{p+1}}{fgn\left(ad-cb\right)B\left(p+1\right)}$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x)
```

output
```
-1/f/g/n/(a*d-b*c)*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(p+1)
```

### 3.235.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 85, normalized size of antiderivative = 1.55

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx = \frac{(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)^p}{(Bbc-Bad)fgnp+(Bbc-Bad)fgn}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x,
algorithm="fricas")
```

output
```
(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)*(B*n*log(b*x + a) - B
*n*log(d*x + c) + B*log(e) + A)^p/((B*b*c - B*a*d)*f*g*n*p + (B*b*c - B*a*
d)*f*g*n)
```

### 3.235.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**p/(b*f*x+a*f)/(d*g*x+c*g),x
)
```

output
```
Timed out
```

### 3.235.7 Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(af+bfx)(cg+dgx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p}{(bfx+af)(dgx+cg)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x,
algorithm="maxima")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^p/((b*f*x + a*f)*(d*g*x +
 c*g)), x)
```

### 3.235.8 Giac [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 0.96

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^p}{(af + bfx)(cg + dgx)} dx = \frac{(Bn\log(bx + a) - Bn\log(dx + c) + B\log(e) + A)^{p+1}}{(Bbcfgn - Badfgn)(p + 1)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(b*f*x+a*f)/(d*g*x+c*g),x,
algorithm="giac")
```

output
```
(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)^(p + 1)/((B*b*c*f*g*n
 - B*a*d*f*g*n)*(p + 1))
```

### 3.235.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^p}{(af + bfx)(cg + dgx)} dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{(a\,f + b\,f\,x)\,(c\,g + d\,g\,x)} dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/((a*f + b*f*x)*(c*g + d*g*x
)),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/((a*f + b*f*x)*(c*g + d*g*x
)), x)
```

# 3.236 $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx$



## 3.236.1 Optimal result

Integrand size = 50, antiderivative size = 52

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx=\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^{1+p}}{B(bc-ad)fn(1+p)}$$

output
```
(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^(p+1)/B/(-a*d+b*c)/f/n/(p+1)
```

## 3.236.2 Mathematica [A] (verified)

Time = 0.00 (sec) , antiderivative size = 50, normalized size of antiderivative = 0.96

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx=\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^{1+p}}{f(bBcn-aBdn)(1+p)}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/(a*c*f + (b*c + a*d)*
f*x + b*d*f*x^2),x]
```

output
```
(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^(1 + p)/(f*(b*B*c*n - a*B*d*n)*(1
 + p))
```

### 3.236.3 Rubi [A] (warning: unable to verify)

Time = 0.55 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2973, 2974, 2961, 2739, 15}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^p}{fx(ad+bc)+acf+bdfx^2}dx$$

$$\Big\downarrow \; 2973$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^p}{fx(ad+bc)+acf+bdfx^2}dx$$

$$\Big\downarrow \; 2974$$

$$\frac{\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{(a+bx)(c+dx)}dx}{f}$$

$$\Big\downarrow \; 2961$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p}{a+bx}d\frac{a+bx}{c+dx}}{f(bc-ad)}$$

$$\Big\downarrow \; 2739$$

$$\frac{\int \left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^p d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bfn(bc-ad)}$$

$$\Big\downarrow \; 15$$

$$\frac{\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^{p+1}}{Bfn(p+1)(bc-ad)}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^p/(a*c*f + (b*c + a*d)*f*x +
b*d*f*x^2),x]
```

output
```
(A + B*Log[e*((a + b*x)/(c + d*x))^n])^(1 + p)/(B*(b*c - a*d)*f*n*(1 + p))
```

### 3.236.3.1 Defintions of rubi rules used

rule 15
```
Int[(a_.)*(x_)^(m_.), x_Symbol] :> Simp[a*(x^(m + 1)/(m + 1)), x] /; FreeQ[
{a, m}, x] && NeQ[m, -1]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 2974
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_) + (h_.)*(x_)^2)^(m_.), x_Symbol] :> Simp[h^
m/(b^m*d^m)   Int[(a + b*x)^m*(c + d*x)^m*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, n, p}, x] && EqQ[b*
d*f - a*c*h, 0] && EqQ[b*d*g - h*(b*c + a*d), 0] && IntegerQ[m]
```

## 3.236.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{p}}{acf+\left(ad+cb\right)fx+bdf\,x^{2}}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2),x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2),x)
```

### 3.236.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 83, normalized size of antiderivative = 1.60

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx$$
$$=\frac{(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)^p}{(Bbc-Bad)fnp+(Bbc-Bad)fn}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*
x^2),x, algorithm="fricas")
```

output
```
(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)*(B*n*log(b*x + a) - B
*n*log(d*x + c) + B*log(e) + A)^p/((B*b*c - B*a*d)*f*n*p + (B*b*c - B*a*d)
*f*n)
```

### 3.236.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**p/(a*c*f+(a*d+b*c)*f*x+b*d*
f*x**2),x)
```

output
```
Timed out
```

### 3.236.7 Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^p}{bdfx^2+acf+(bc+ad)fx}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*
x^2),x, algorithm="maxima")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^p/(b*d*f*x^2 + a*c*f + (b
*c + a*d)*f*x), x)
```

### 3.236.8 Giac [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 51, normalized size of antiderivative = 0.98

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx = \frac{(Bn\log(bx+a)-Bn\log(dx+c)+B\log(e)+A)^{p+1}}{(Bbcfn-Badfn)(p+1)}$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^p/(a*c*f+(a*d+b*c)*f*x+b*d*f*
x^2),x, algorithm="giac")
```

output
```
(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)^(p + 1)/((B*b*c*f*n -
 B*a*d*f*n)*(p + 1))
```

### 3.236.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^p}{acf+(bc+ad)fx+bdfx^2}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^p}{b\,d\,f\,x^2+f\,\left(a\,d+b\,c\right)\,x+a\,c\,f}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/(a*c*f + f*x*(a*d + b*c) +
b*d*f*x^2),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^p/(a*c*f + f*x*(a*d + b*c) +
b*d*f*x^2), x)
```

# 3.237 $\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$



## 3.237.1 Optimal result

Integrand size = 40, antiderivative size = 41

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)n}$$

output
```
ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/(-a*d+b*c)/n
```

## 3.237.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 39, normalized size of antiderivative = 0.95

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bBcn-aBdn}$$

input
```
Integrate[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]))
,x]
```

output
```
Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(b*B*c*n - a*B*d*n)
```

### 3.237.3 Rubi [A] (warning: unable to verify)

Time = 0.48 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2973, 2961, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\Big\downarrow \; 2973$$

$$\int \frac{1}{(a+bx)(c+dx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\Big\downarrow \; 2961$$

$$\frac{\int \frac{c+dx}{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \; 2739$$

$$\frac{\int \frac{c+dx}{a+bx}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bn(bc-ad)}$$

$$\Big\downarrow \; 14$$

$$\frac{\log\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{Bn(bc-ad)}$$

input
```
Int[1/((a + b*x)*(c + d*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x]
```

output
```
Log[A + B*Log[e*((a + b*x)/(c + d*x))^n]]/(B*(b*c - a*d)*n)
```

#### 3.237.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

## 3.237.4 Maple [A] (verified)

Time = 20.47 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.05

| method | result |
|---|---|
| derivativedivides | $-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{n(ad-cb)B}$ |
| default | $-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{n(ad-cb)B}$ |
| parallelrisch | $-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{n(ad-cb)B}$ |
| risch | $-\frac{\ln\left(\ln((dx+c)^{n})-\frac{-iB\pi\ \mathrm{csgn}(ie)\ \mathrm{csgn}\left(i(bx+a)^{n}(dx+c)^{-n}\right)\ \mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a)^{n}\right)+iB\pi\ \mathrm{csgn}(ie)\ \mathrm{csgn}\left(ie(dx+c)^{-n}(bx+a\right.}{}\right.}{}$ |

input
```
int(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_RETURNVE
RBOSE)
```

output
```
-1/n/(a*d-b*c)*ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B
```

### 3.237.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.10

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-Bn\log\left(bx+a\right)+Bn\log\left(dx+c\right)-B\log\left(e\right)-A\right)}{(Bbc-Bad)n}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorith
m="fricas")
```

output
```
log(-B*n*log(b*x + a) + B*n*log(d*x + c) - B*log(e) - A)/((B*b*c - B*a*d)*
n)
```

### 3.237.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Timed out
```

### 3.237.7 Maxima [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.20

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-\frac{B\log\left((bx+a)^n\right)-B\log\left((dx+c)^n\right)+B\log(e)+A}{B}\right)}{(bcn-adn)B}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorith
m="maxima")
```

output
```
log(-(B*log((b*x + a)^n) - B*log((d*x + c)^n) + B*log(e) + A)/B)/((b*c*n -
 a*d*n)*B)
```

### 3.237.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 41, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(Bn\log\left(bx+a\right)-Bn\log\left(dx+c\right)+B\log\left(e\right)+A\right)}{Bbcn-Badn}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algorith
m="giac")
```

output
```
log(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)/(B*b*c*n - B*a*d*n
)
```

### 3.237.9 Mupad [B] (verification not implemented)

Time = 0.00 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.98

$$\int \frac{1}{(a+bx)(c+dx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = -\frac{\ln\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}{B\,a\,d\,n-B\,b\,c\,n}$$

input
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a + b*x)*(c + d*x)),x)
```

output
```
-log(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(B*a*d*n - B*b*c*n)
```

# 3.238 $\int \frac{1}{(af+bfx)(cg+dgx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$



## 3.238.1 Optimal result

Integrand size = 46, antiderivative size = 47

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{B(bc-ad)fgn}$$

output
```
ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/(-a*d+b*c)/f/g/n
```

## 3.238.2 Mathematica [A] (verified)

Time = 0.10 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.91

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}{bBcfgn-aBdfgn}$$

input
```
Integrate[1/((a*f + b*f*x)*(c*g + d*g*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d
*x)^n])),x]
```

output
```
Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(b*B*c*f*g*n - a*B*d*f*g*n)
```

### 3.238.3 Rubi [A] (warning: unable to verify)

Time = 0.52 (sec) , antiderivative size = 46, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {2973, 2961, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\Big\downarrow \; 2973$$

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\Big\downarrow \; 2961$$

$$\frac{\int \frac{c+dx}{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{fg(bc-ad)}$$

$$\Big\downarrow \; 2739$$

$$\frac{\int \frac{c+dx}{a+bx}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bfgn(bc-ad)}$$

$$\Big\downarrow \; 14$$

$$\frac{\log\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{Bfgn(bc-ad)}$$

input
```
Int[1/((a*f + b*f*x)*(c*g + d*g*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]
)),x]
```

output
```
Log[A + B*Log[e*((a + b*x)/(c + d*x))^n]]/(B*(b*c - a*d)*f*g*n)
```

### 3.238.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

## 3.238.4 Maple [A] (verified)

Time = 66.62 (sec) , antiderivative size = 49, normalized size of antiderivative = 1.04

| method | result |
|---|---|
| default | $-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{fgn(ad-cb)B}$ |
| parallelrisch | $-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{fgn(ad-cb)B}$ |
| risch | $-\frac{\ln\left(\ln((dx+c)^{n})-\frac{-iB\pi\ \text{csgn}(ie)\,\text{csgn}\left(i(bx+a)^{n}(dx+c)^{-n}\right)\text{csgn}\left(ie(dx+c)^{-n}(bx+a)^{n}\right)+iB\pi\ \text{csgn}(ie)\,\text{csgn}\left(ie(dx+c)^{-n}(bx+a)^{n}\right)^{2}-i}{}\right.}{}$ |

input
```
int(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,method=_
RETURNVERBOSE)
```

output
```
-1/f/g/n/(a*d-b*c)*ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B
```

### 3.238.5 Fricas [A] (verification not implemented)

Time = 0.34 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.09

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-Bn\log\left(bx+a\right)+Bn\log\left(dx+c\right)-B\log\left(e\right)-A\right)}{(Bbc-Bad)fgn}$$

input
```
integrate(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x,
algorithm="fricas")
```

output
```
log(-B*n*log(b*x + a) + B*n*log(d*x + c) - B*log(e) - A)/((B*b*c - B*a*d)*
f*g*n)
```

### 3.238.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Timed out
```

### 3.238.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.13

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(-\frac{B\log((bx+a)^n)-B\log((dx+c)^n)+B\log(e)+A}{B}\right)}{(bcfgn-adfgn)B}$$

input
```
integrate(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x,
algorithm="maxima")
```

output
```
log(-(B*log((b*x + a)^n) - B*log((d*x + c)^n) + B*log(e) + A)/B)/((b*c*f*g
*n - a*d*f*g*n)*B)
```

### 3.238.8 Giac [A] (verification not implemented)

Time = 0.27 (sec) , antiderivative size = 45, normalized size of antiderivative = 0.96

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \frac{\log\left(Bn\log\left(bx+a\right)-Bn\log\left(dx+c\right)+B\log\left(e\right)+A\right)}{Bbcfgn-Badfgn}$$

input
```
integrate(1/(b*f*x+a*f)/(d*g*x+c*g)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x,
algorithm="giac")
```

output
```
log(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)/(B*b*c*f*g*n - B*a
*d*f*g*n)
```

### 3.238.9 Mupad [B] (verification not implemented)

Time = 1.34 (sec) , antiderivative size = 44, normalized size of antiderivative = 0.94

$$\int \frac{1}{(af+bfx)(cg+dgx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = -\frac{\ln\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}{B\,a\,d\,f\,g\,n-B\,b\,c\,f\,g\,n}$$

input
```
int(1/((a*f + b*f*x)*(c*g + d*g*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n)
)),x)
```

output
```
-log(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(B*a*d*f*g*n - B*b*c*f*g*n)
```

# 3.239 $\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$



## 3.239.1 Optimal result

Integrand size = 50, antiderivative size = 44

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx = \frac{\log(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{B(bc-ad)fn}$$

output
```
ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/(-a*d+b*c)/f/n
```

## 3.239.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\,(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx = \frac{\log(A+B\log(e(a+bx)^n(c+dx)^{-n}))}{f(bBcn-aBdn)}$$

input
```
Integrate[1/((a*c*f + (b*c + a*d)*f*x + b*d*f*x^2)*(A + B*Log[(e*(a + b*x)
^n)/(c + d*x)^n])),x]
```

output
```
Log[A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n]]/(f*(b*B*c*n - a*B*d*n))
```

### 3.239.3 Rubi [A] (warning: unable to verify)

Time = 0.50 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2973, 2974, 2961, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(fx(ad+bc)+acf+bdfx^2)\,(B\log(e(a+bx)^n(c+dx)^{-n})+A)}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{1}{(fx(ad+bc)+acf+bdfx^2)\,(B\log(e(a+bx)^n(c+dx)^{-n})+A)}dx$$

$$\Big\downarrow \text{2974}$$

$$\frac{\int \frac{1}{(a+bx)(c+dx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}dx}{f}$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{c+dx}{(a+bx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}d\frac{a+bx}{c+dx}}{f(bc-ad)}$$

$$\Big\downarrow \text{2739}$$

$$\frac{\int \frac{c+dx}{a+bx}d\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{Bfn(bc-ad)}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{Bfn(bc-ad)}$$

input
```
Int[1/((a*c*f + (b*c + a*d)*f*x + b*d*f*x^2)*(A + B*Log[(e*(a + b*x)^n)/(c
 + d*x)^n])),x]
```

output
```
Log[A + B*Log[e*((a + b*x)/(c + d*x))^n]]/(B*(b*c - a*d)*f*n)
```

### 3.239.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 2974
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_) + (h_.)*(x_)^2)^(m_.), x_Symbol] :> Simp[h^
m/(b^m*d^m)   Int[(a + b*x)^m*(c + d*x)^m*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p, x], x] /; FreeQ[{a, b, c, d, e, f, g, h, A, B, n, p}, x] && EqQ[b*
d*f - a*c*h, 0] && EqQ[b*d*g - h*(b*c + a*d), 0] && IntegerQ[m]
```

## 3.239.4 Maple [A] (verified)

Time = 55.31 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.05

| method | result |
|---|---|
| parallelrisch | $-\frac{\ln\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)}{Bfn(ad-cb)}$ |
| risch | $-\frac{\ln\left(\ln((dx+c)^{n})-\frac{-iB\pi\ \operatorname{csgn}(ie)\operatorname{csgn}\left(i(bx+a)^{n}(dx+c)^{-n}\right)\operatorname{csgn}\left(ie(dx+c)^{-n}(bx+a)^{n}\right)+iB\pi\ \operatorname{csgn}(ie)\operatorname{csgn}\left(ie(dx+c)^{-n}(bx+a)^{n}\right)^{2}-i}{}\right.}{}$ |

input
```
int(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x,
method=_RETURNVERBOSE)
```

output
```
-ln(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/B/f/n/(a*d-b*c)
```

### 3.239.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 48, normalized size of antiderivative = 1.09

$$\int \frac{1}{(acf + (bc + ad)fx + bdfx^2)(A + B\log(e(a + bx)^n(c + dx)^{-n}))}\,dx = \frac{\log(-Bn\log(bx + a) + Bn\log(dx + c) - B\log(e) - A)}{(Bbc - Bad)fn}$$

input
```
integrate(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2)/(A+B*log(e*(b*x+a)^n/((d*x+c)^
n))),x, algorithm="fricas")
```

output
```
log(-B*n*log(b*x + a) + B*n*log(d*x + c) - B*log(e) - A)/((B*b*c - B*a*d)*
f*n)
```

### 3.239.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(acf + (bc + ad)fx + bdfx^2)(A + B\log(e(a + bx)^n(c + dx)^{-n}))}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x**2)/(A+B*ln(e*(b*x+a)**n/((d*x+c)
**n))),x)
```

output
```
Timed out
```

### 3.239.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.16

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\,(A+B\log{(e(a+bx)^n(c+dx)^{-n})})}\,dx = \frac{\log{\left(-\frac{B\log((bx+a)^n)-B\log((dx+c)^n)+B\log(e)+A}{B}\right)}}{(bcfn-adfn)B}$$

input
```
integrate(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2)/(A+B*log(e*(b*x+a)^n/((d*x+c)^
n))),x, algorithm="maxima")
```

output
```
log(-(B*log((b*x + a)^n) - B*log((d*x + c)^n) + B*log(e) + A)/B)/((b*c*f*n
 - a*d*f*n)*B)
```

### 3.239.8 Giac [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 43, normalized size of antiderivative = 0.98

$$\int \frac{1}{(acf+(bc+ad)fx+bdfx^2)\,(A+B\log{(e(a+bx)^n(c+dx)^{-n})})}\,dx = \frac{\log{(Bn\log{(bx+a)}-Bn\log{(dx+c)}+B\log{(e)}+A)}}{Bbcfn-Badfn}$$

input
```
integrate(1/(a*c*f+(a*d+b*c)*f*x+b*d*f*x^2)/(A+B*log(e*(b*x+a)^n/((d*x+c)^
n))),x, algorithm="giac")
```

output
```
log(B*n*log(b*x + a) - B*n*log(d*x + c) + B*log(e) + A)/(B*b*c*f*n - B*a*d
*f*n)
```

### 3.239.9 Mupad [B] (verification not implemented)

Time = 1.20 (sec) , antiderivative size = 42, normalized size of antiderivative = 0.95

$$\int \frac{1}{(acf + (bc + ad)fx + bdfx^2)\,(A + B\log{(e(a + bx)^n(c + dx)^{-n})})}\,dx = -\frac{\ln\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}{B\,a\,d\,f\,n - B\,b\,c\,f\,n}$$

input
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(a*c*f + f*x*(a*d + b*c) +
 b*d*f*x^2)),x)
```

output
```
-log(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(B*a*d*f*n - B*b*c*f*n)
```

# 3.240 $\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log(e(a+bx)^n(c+dx)^{-n})}\,dx$



## 3.240.1 Optimal result

Integrand size = 40, antiderivative size = 88

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx = \frac{(a+bx)^{1+m}(c+dx)^{-1-m}\left(e(a+bx)^n(c+dx)^{-n}\right)^{-\frac{1+m}{n}}\text{ExpIntegralEi}\left(\frac{(1+m)\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{n}\right)}{(bc-ad)n}$$

output
```
(b*x+a)^(1+m)*(d*x+c)^(-1-m)*Ei((1+m)*ln(e*(b*x+a)^n/((d*x+c)^n))/n)/(-a*d
+b*c)/n/((e*(b*x+a)^n/((d*x+c)^n))^((1+m)/n))
```

## 3.240.2 Mathematica [F]

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx = \int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx$$

input
```
Integrate[((a + b*x)^m*(c + d*x)^(-2 - m))/Log[(e*(a + b*x)^n)/(c + d*x)^n
],x]
```

output
```
Integrate[((a + b*x)^m*(c + d*x)^(-2 - m))/Log[(e*(a + b*x)^n)/(c + d*x)^n
], x]
```

### 3.240.3 Rubi [A] (warning: unable to verify)

Time = 0.54 (sec) , antiderivative size = 86, normalized size of antiderivative = 0.98, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {2973, 2963, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^m(c+dx)^{-m-2}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(a+bx)^m(c+dx)^{-m-2}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}dx$$

$$\Big\downarrow \text{2963}$$

$$\frac{(a+bx)^m(c+dx)^{-m}\left(\frac{a+bx}{c+dx}\right)^{-m}\int \frac{\left(\frac{a+bx}{c+dx}\right)^m}{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2747}$$

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{m+1}{n}}(c+dx)}{a+bx}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{(a+bx)^{m+1}(c+dx)^{-m-1}\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-\frac{m+1}{n}}\text{ExpIntegralEi}\left(\frac{(m+1)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(bc-ad)}$$

input
```
Int[((a + b*x)^m*(c + d*x)^(-2 - m))/Log[(e*(a + b*x)^n)/(c + d*x)^n],x]
```

output
```
((a + b*x)^(1 + m)*(c + d*x)^(-1 - m)*ExpIntegralEi[((1 + m)*Log[e*((a + b
*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(e*((a + b*x)/(c + d*x))^n)^((1 + m)
/n))
```

### 3.240.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2963
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[d^2*((g*((a + b*x)/b))^m/(i^2*(b*c - a*d)*(i*((c + d*x)/d))^m*((a
 + b*x)/(c + d*x))^m))   Subst[Int[x^m*(A + B*Log[e*x^n])^p, x], x, (a + b*
x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[b*f - a*g, 0] && EqQ[d*h - c*i, 0] && EqQ[m +
 q + 2, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

## 3.240.4 Maple [F]

$$\int \frac{(bx+a)^m (dx+c)^{-2-m}}{\ln\left(e\,(bx+a)^n\,(dx+c)^{-n}\right)}dx$$

input
```
int((b*x+a)^m*(d*x+c)^(-2-m)/ln(e*(b*x+a)^n/((d*x+c)^n)),x)
```

output
```
int((b*x+a)^m*(d*x+c)^(-2-m)/ln(e*(b*x+a)^n/((d*x+c)^n)),x)
```

### 3.240.5 Fricas [F]

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx=\int \frac{(bx+a)^m(dx+c)^{-m-2}}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-2-m)/log(e*(b*x+a)^n/((d*x+c)^n)),x, algorit
hm="fricas")
```

output
```
integral((b*x + a)^m*(d*x + c)^(-m - 2)/log((b*x + a)^n*e/(d*x + c)^n), x)
```

### 3.240.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx=\text{Timed out}$$

input
```
integrate((b*x+a)**m*(d*x+c)**(-2-m)/ln(e*(b*x+a)**n/((d*x+c)**n)),x)
```

output
```
Timed out
```

### 3.240.7 Maxima [F]

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log\left(e(a+bx)^n(c+dx)^{-n}\right)}\,dx=\int \frac{(bx+a)^m(dx+c)^{-m-2}}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-2-m)/log(e*(b*x+a)^n/((d*x+c)^n)),x, algorit
hm="maxima")
```

output
```
integrate((b*x + a)^m*(d*x + c)^(-m - 2)/log((b*x + a)^n*e/(d*x + c)^n), x
)
```

### 3.240.8 Giac [F]

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log(e(a+bx)^n(c+dx)^{-n})}\,dx = \int \frac{(bx+a)^m(dx+c)^{-m-2}}{\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)}\,dx$$

input
```
integrate((b*x+a)^m*(d*x+c)^(-2-m)/log(e*(b*x+a)^n/((d*x+c)^n)),x, algorit
hm="giac")
```

output
```
integrate((b*x + a)^m*(d*x + c)^(-m - 2)/log((b*x + a)^n*e/(d*x + c)^n), x
)
```

### 3.240.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^m(c+dx)^{-2-m}}{\log(e(a+bx)^n(c+dx)^{-n})}\,dx = \int \frac{(a+b\,x)^m}{\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\,(c+d\,x)^{m+2}}\,dx$$

input
```
int((a + b*x)^m/(log((e*(a + b*x)^n)/(c + d*x)^n)*(c + d*x)^(m + 2)),x)
```

output
```
int((a + b*x)^m/(log((e*(a + b*x)^n)/(c + d*x)^n)*(c + d*x)^(m + 2)), x)
```

# 3.241 $\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$



## 3.241.1 Optimal result

Integrand size = 35, antiderivative size = 75

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(a+bx)^4 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-4/n} \text{ExpIntegralEi}\left(\frac{4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^4}$$

output
```
(b*x+a)^4*Ei(4*ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a*d+b*c)/n/((e*((b*x+a)/(d*x
+c))^n)^(4/n))/(d*x+c)^4
```

## 3.241.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(a+bx)^4 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-4/n} \text{ExpIntegralEi}\left(\frac{4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^4}$$

input
```
Integrate[(a + b*x)^3/((c + d*x)^5*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)^4*ExpIntegralEi[(4*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c -
a*d)*n*(e*((a + b*x)/(c + d*x))^n)^(4/n)*(c + d*x)^4)
```

## 3.241.3 Rubi [A] (verified)

Time = 0.31 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2961, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx$$

$$\Big\downarrow \ 2961$$

$$\frac{\int \frac{(a+bx)^3}{(c+dx)^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \ 2747$$

$$\frac{(a+bx)^4 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-4/n} \int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{4/n}(c+dx)}{a+bx}d\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(c+dx)^4(bc-ad)}$$

$$\Big\downarrow \ 2609$$

$$\frac{(a+bx)^4 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-4/n} \text{ExpIntegralEi}\left(\frac{4\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(c+dx)^4(bc-ad)}$$

input
```
Int[(a + b*x)^3/((c + d*x)^5*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)^4*ExpIntegralEi[(4*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c -
a*d)*n*(e*((a + b*x)/(c + d*x))^n)^(4/n)*(c + d*x)^4)
```

### 3.241.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.241.4 Maple [F]

$$\int \frac{(bx+a)^3}{(dx+c)^5 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((b*x+a)^3/(d*x+c)^5/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int((b*x+a)^3/(d*x+c)^5/ln(e*((b*x+a)/(d*x+c))^n),x)
```

## 3.241.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 110, normalized size of antiderivative = 1.47

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\text{log\_integral}\left(\frac{(b^4x^4+4\,ab^3x^3+6\,a^2b^2x^2+4\,a^3bx+a^4)e^{\frac{4}{n}}}{d^4x^4+4\,cd^3x^3+6\,c^2d^2x^2+4\,c^3dx+c^4}\right)}{(bc-ad)e^{\frac{4}{n}}n}$$

input
```
integrate((b*x+a)^3/(d*x+c)^5/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fri
cas")
```

output
```
log_integral((b^4*x^4 + 4*a*b^3*x^3 + 6*a^2*b^2*x^2 + 4*a^3*b*x + a^4)*e^(
4/n)/(d^4*x^4 + 4*c*d^3*x^3 + 6*c^2*d^2*x^2 + 4*c^3*d*x + c^4))/((b*c - a*
d)*e^(4/n)*n)
```

### 3.241.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx = \text{Timed out}$$

input
```
integrate((b*x+a)**3/(d*x+c)**5/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

### 3.241.7 Maxima [F]

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx = \int \frac{(bx+a)^3}{(dx+c)^5 \log \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)} \, dx$$

input
```
integrate((b*x+a)^3/(d*x+c)^5/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="max
ima")
```

output
```
integrate((b*x + a)^3/((d*x + c)^5*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.241.8 Giac [F]

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx = \int \frac{(bx+a)^3}{(dx+c)^5 \log \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)} \, dx$$

input
```
integrate((b*x+a)^3/(d*x+c)^5/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="gia
c")
```

output
```
integrate((b*x + a)^3/((d*x + c)^5*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.241.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^3}{(c+dx)^5 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(a+b\,x)^3}{\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(c+d\,x)^5}\,dx$$

input
```
int((a + b*x)^3/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^5),x)
```

output
```
int((a + b*x)^3/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^5), x)
```

# 3.242 $\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$



## 3.242.1 Optimal result

Integrand size = 35, antiderivative size = 75

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(a+bx)^3 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n} \operatorname{ExpIntegralEi}\left(\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^3}$$

output
```
(b*x+a)^3*Ei(3*ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a*d+b*c)/n/((e*((b*x+a)/(d*x
+c))^n)^(3/n))/(d*x+c)^3
```

## 3.242.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{(a+bx)^3 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n} \operatorname{ExpIntegralEi}\left(\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^3}$$

input
```
Integrate[(a + b*x)^2/((c + d*x)^4*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)^3*ExpIntegralEi[(3*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c -
a*d)*n*(e*((a + b*x)/(c + d*x))^n)^(3/n)*(c + d*x)^3)
```

## 3.242.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2961, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(a+bx)^2}{(c+dx)^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2747}$$

$$\frac{(a+bx)^3 \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n} \int \frac{\left(e \left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n}(c+dx)}{a+bx}d\log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{n(c+dx)^3(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{(a+bx)^3 \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n} \text{ExpIntegralEi}\left(\frac{3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(c+dx)^3(bc-ad)}$$

input
```
Int[(a + b*x)^2/((c + d*x)^4*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)^3*ExpIntegralEi[(3*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c -
a*d)*n*(e*((a + b*x)/(c + d*x))^n)^(3/n)*(c + d*x)^3)
```

### 3.242.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.242.4 Maple [F]

$$\int \frac{(bx+a)^2}{(dx+c)^4 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((b*x+a)^2/(d*x+c)^4/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int((b*x+a)^2/(d*x+c)^4/ln(e*((b*x+a)/(d*x+c))^n),x)
```

### 3.242.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\text{log\_integral}\left(\frac{(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)e^{\frac{3}{n}}}{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}\right)}{(bc-ad)e^{\frac{3}{n}}n}$$

input
```
integrate((b*x+a)^2/(d*x+c)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fri
cas")
```

output
```
log_integral((b^3*x^3 + 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*e^(3/n)/(d^3*x^3 +
3*c*d^2*x^2 + 3*c^2*d*x + c^3))/((b*c - a*d)*e^(3/n)*n)
```

### 3.242.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)**2/(d*x+c)**4/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

### 3.242.7 Maxima [F]

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bx+a)^2}{(dx+c)^4 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((b*x+a)^2/(d*x+c)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="max
ima")
```

output
```
integrate((b*x + a)^2/((d*x + c)^4*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.242.8 Giac [F]

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(bx+a)^2}{(dx+c)^4 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((b*x+a)^2/(d*x+c)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="gia
c")
```

output
```
integrate((b*x + a)^2/((d*x + c)^4*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.242.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(a+bx)^2}{(c+dx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(a+b\,x)^2}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(c+d\,x)^4}\,dx$$

input
```
int((a + b*x)^2/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^4),x)
```

output
```
int((a + b*x)^2/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^4), x)
```

# 3.243 $\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$



## 3.243.1 Optimal result

Integrand size = 33, antiderivative size = 75

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^2}$$

output
```
(b*x+a)^2*Ei(2*ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a*d+b*c)/n/((e*((b*x+a)/(d*x
+c))^n)^(2/n))/(d*x+c)^2
```

## 3.243.2 Mathematica [A] (verified)

Time = 0.02 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{(a+bx)^2\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}\text{ExpIntegralEi}\left(\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)^2}$$

input
```
Integrate[(a + b*x)/((c + d*x)^3*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)^2*ExpIntegralEi[(2*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c -
a*d)*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2)
```

### 3.243.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.091, Rules used = {2961, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{a+bx}{(c+dx)^3 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{a+bx}{(c+dx) \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2747}$$

$$\frac{(a+bx)^2 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n} \int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n}(c+dx)}{a+bx}d\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(c+dx)^2(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{(a+bx)^2 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n} \text{ExpIntegralEi}\left(\frac{2\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(c+dx)^2(bc-ad)}$$

input
```
Int[(a + b*x)/((c + d*x)^3*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)^2*ExpIntegralEi[(2*Log[e*((a + b*x)/(c + d*x))^n])/n])/((b*c -
a*d)*n*(e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2)
```

#### 3.243.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

### 3.243.4 Maple [F]

$$\int \frac{bx+a}{(dx+c)^3 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((b*x+a)/(d*x+c)^3/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int((b*x+a)/(d*x+c)^3/ln(e*((b*x+a)/(d*x+c))^n),x)
```

### 3.243.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int \frac{a+bx}{(c+dx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\text{log\_integral}\left(\frac{(b^2x^2+2\,abx+a^2)e^{\frac{2}{n}}}{d^2x^2+2\,cdx+c^2}\right)}{(bc-ad)e^{\frac{2}{n}}n}$$

input
```
integrate((b*x+a)/(d*x+c)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="frica
s")
```

output
```
log_integral((b^2*x^2 + 2*a*b*x + a^2)*e^(2/n)/(d^2*x^2 + 2*c*d*x + c^2))/
((b*c - a*d)*e^(2/n)*n)
```

### 3.243.6 Sympy [F(-1)]

Timed out.

$$\int \frac{a+bx}{(c+dx)^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate((b*x+a)/(d*x+c)**3/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

### 3.243.7 Maxima [F]

$$\int \frac{a+bx}{(c+dx)^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{bx+a}{(dx+c)^3\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((b*x+a)/(d*x+c)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxim
a")
```

output
```
integrate((b*x + a)/((d*x + c)^3*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.243.8 Giac [F]

$$\int \frac{a+bx}{(c+dx)^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{bx+a}{(dx+c)^3\log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((b*x+a)/(d*x+c)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac"
)
```

output
```
integrate((b*x + a)/((d*x + c)^3*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.243.9 Mupad [F(-1)]

Timed out.

$$\int \frac{a+bx}{(c+dx)^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{a+b\,x}{\ln\left(e\,\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,{\left(c+d\,x\right)}^3}\,dx$$

input
```
int((a + b*x)/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^3),x)
```

output
```
int((a + b*x)/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^3), x)
```

# 3.244 $\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx$



## 3.244.1 Optimal result

Integrand size = 28, antiderivative size = 72

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx = \frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n} \text{ExpIntegralEi}\left(\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)}$$

output
```
(b*x+a)*Ei(ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a*d+b*c)/n/((e*((b*x+a)/(d*x+c))
^n)^(1/n))/(d*x+c)
```

## 3.244.2 Mathematica [A] (verified)

Time = 0.07 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00

$$\int \frac{1}{(c+dx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx = \frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n} \text{ExpIntegralEi}\left(\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(c+dx)}$$

input
```
Integrate[1/((c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)*ExpIntegralEi[Log[e*((a + b*x)/(c + d*x))^n]/n])/((b*c - a*d)*n
*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x))
```

## 3.244.3 Rubi [A] (verified)

Time = 0.24 (sec) , antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2951, 2737, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(c+dx)^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)} \, dx$$

$$\Big\downarrow \; 2951$$

$$\frac{\int \frac{1}{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \; 2737$$

$$\frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}(c+dx)}{a+bx}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(c+dx)(bc-ad)}$$

$$\Big\downarrow \; 2609$$

$$\frac{(a+bx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}\text{ExpIntegralEi}\left(\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(c+dx)(bc-ad)}$$

input
```
Int[1/((c + d*x)^2*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((a + b*x)*ExpIntegralEi[Log[e*((a + b*x)/(c + d*x))^n]/n])/((b*c - a*d)*n
*(e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x))
```

### 3.244.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2737
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_), x_Symbol] :> Simp[x/(n*(c*x
^n)^(1/n))   Subst[Int[E^(x/n)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ
[{a, b, c, n, p}, x]
```

rule 2951
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/d)^m   Subst[Int[(A + B*Log[e*x^n])^p/(b - d*x)^(m + 2), x], x, (a +
b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && NeQ[b*c
- a*d, 0] && IntegersQ[m, p] && EqQ[d*f - c*g, 0] && (GtQ[p, 0] || LtQ[m, -
1])
```

### 3.244.4 Maple [F]

$$\int \frac{1}{\left(dx+c\right)^{2} \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)}dx$$

input
```
int(1/(d*x+c)^2/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int(1/(d*x+c)^2/ln(e*((b*x+a)/(d*x+c))^n),x)
```

### 3.244.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{1}{(c+dx)^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\text{log\_integral}\left(\frac{(bx+a)e^{\left(\frac{1}{n}\right)}}{dx+c}\right)}{(bc-ad)e^{\left(\frac{1}{n}\right)}n}$$

input
```
integrate(1/(d*x+c)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

output
```
log_integral((b*x + a)*e^(1/n)/(d*x + c))/((b*c - a*d)*e^(1/n)*n)
```

### 3.244.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(d*x+c)**2/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

### 3.244.7 Maxima [F]

$$\int \frac{1}{(c+dx)^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{(dx+c)^2 \log \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate(1/(d*x+c)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

output
```
integrate(1/((d*x + c)^2*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.244.8 Giac [F]

$$\int \frac{1}{(c+dx)^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{(dx+c)^2 \log \left(e \left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate(1/(d*x+c)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
```

output
```
integrate(1/((d*x + c)^2*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.244.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(c+dx)^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{\ln \left(e \left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(c+d\,x\right)^2}\,dx$$

input
```
int(1/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^2),x)
```

output
```
int(1/(log(e*((a + b*x)/(c + d*x))^n)*(c + d*x)^2), x)
```

# 3.245 $\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$



## 3.245.1 Optimal result

Integrand size = 35, antiderivative size = 33

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\log\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)n}$$

output
```
ln(ln(e*((b*x+a)/(d*x+c))^n))/(-a*d+b*c)/n
```

## 3.245.2 Mathematica [A] (verified)

Time = 0.05 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = -\frac{\log\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(-bc+ad)n}$$

input
```
Integrate[1/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
-(Log[Log[e*((a + b*x)/(c + d*x))^n]]/((-(b*c) + a*d)*n))
```

## 3.245.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.086, Rules used = {2961, 2739, 14}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{c+dx}{(a+bx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2739}$$

$$\frac{\int \frac{c+dx}{a+bx}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(bc-ad)}$$

$$\Big\downarrow \text{14}$$

$$\frac{\log\left(\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{n(bc-ad)}$$

input
```
Int[1/((a + b*x)*(c + d*x)*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
Log[Log[e*((a + b*x)/(c + d*x))^n]]/((b*c - a*d)*n)
```

### 3.245.3.1 Defintions of rubi rules used

rule 14
```
Int[(a_.)/(x_), x_Symbol] :> Simp[a*Log[x], x] /; FreeQ[a, x]
```

rule 2739
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/(x_), x_Symbol] :> Simp[1/(
b*n)   Subst[Int[x^p, x], x, a + b*Log[c*x^n]], x] /; FreeQ[{a, b, c, n, p}
, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.245.4 Maple [A] (verified)

Time = 1.10 (sec) , antiderivative size = 35, normalized size of antiderivative = 1.06

| method | result | size |
|---|---|---|
| derivativedivides | $-\frac{\ln\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{n(ad-cb)}$ | 35 |
| default | $-\frac{\ln\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{n(ad-cb)}$ | 35 |
| parallelrisch | $-\frac{\ln\left(\ln\left(e\left(\frac{bx+a}{dx+c}\right)^{n}\right)\right)}{n(ad-cb)}$ | 35 |

input
```
int(1/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))^n),x,method=_RETURNVERBOSE)
```

output
```
-1/n/(a*d-b*c)*ln(ln(e*((b*x+a)/(d*x+c))^n))
```

## 3.245.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 34, normalized size of antiderivative = 1.03

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\log\left(n\log\left(\frac{bx+a}{dx+c}\right)+\log(e)\right)}{(bc-ad)n}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="frica
s")
```

output
```
log(n*log((b*x + a)/(d*x + c)) + log(e))/((b*c - a*d)*n)
```

### 3.245.6 Sympy [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 146 vs. 2(24) = 48.

Time = 56.71 (sec) , antiderivative size = 146, normalized size of antiderivative = 4.42

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \begin{cases} -\frac{1}{(bc+bdx)\log(e)} & \text{for } a=\frac{bc}{d} \wedge n=0 \\ -\frac{1}{bc\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)+bdx\log\left(e\left(\frac{bc}{cd+d^2x}+\frac{bx}{c+dx}\right)^n\right)} & \text{for } a=\frac{bc}{d} \\ \frac{-\frac{\log\left(\frac{a}{b}+x\right)}{ad-bc}+\frac{\log\left(\frac{c}{d}+x\right)}{ad-bc}}{\log(e)} & \text{for } n=0 \\ -\frac{\log\left(\log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)\right)}{adn-bcn} & \text{otherwise} \end{cases}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Piecewise((-1/((b*c + b*d*x)*log(e)), Eq(n, 0) & Eq(a, b*c/d)), (-1/(b*c*l
og(e*(b*c/(c*d + d**2*x) + b*x/(c + d*x))**n) + b*d*x*log(e*(b*c/(c*d + d*
*2*x) + b*x/(c + d*x))**n)), Eq(a, b*c/d)), ((-log(a/b + x)/(a*d - b*c) +
log(c/d + x)/(a*d - b*c))/log(e), Eq(n, 0)), (-log(log(e*(a/(c + d*x) + b*
x/(c + d*x))**n))/(a*d*n - b*c*n), True))
```

### 3.245.7 Maxima [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 37, normalized size of antiderivative = 1.12

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{\log\left(-\log\left((bx+a)^n\right)+\log\left((dx+c)^n\right)-\log(e)\right)}{bcn-adn}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxim
a")
```

output
```
log(-log((b*x + a)^n) + log((d*x + c)^n) - log(e))/(b*c*n - a*d*n)
```

### 3.245.8 Giac [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 88 vs. $2(33) = 66$.

Time = 0.49 (sec) , antiderivative size = 88, normalized size of antiderivative = 2.67

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$$
$$= \frac{\left(\frac{bc}{(bc-ad)^2} - \frac{ad}{(bc-ad)^2}\right)\log\left(\frac{1}{4}\left(\pi(\operatorname{sgn}(bx+a)\operatorname{sgn}(dx+c)-1)n+\pi(\operatorname{sgn}(e)-1)\right)^2+\left(n\log\left(\frac{|bx+a|}{|dx+c|}\right)+\log\left(\right.\right.\right.}{2n}$$

input
```
integrate(1/(b*x+a)/(d*x+c)/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac"
)
```

output
```
1/2*(b*c/(b*c - a*d)^2 - a*d/(b*c - a*d)^2)*log(1/4*(pi*(sgn(b*x + a)*sgn(
d*x + c) - 1)*n + pi*(sgn(e) - 1))^2 + (n*log(abs(b*x + a)/abs(d*x + c)) +
 log(abs(e)))^2)/n
```

### 3.245.9 Mupad [B] (verification not implemented)

Time = 1.17 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)(c+dx)\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = -\frac{\ln\left(\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\right)}{a\,d\,n-b\,c\,n}$$

input
```
int(1/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)*(c + d*x)),x)
```

output
```
-log(log(e*((a + b*x)/(c + d*x))^n))/(a*d*n - b*c*n)
```

# 3.246 $\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$



## 3.246.1 Optimal result

Integrand size = 28, antiderivative size = 71

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}} (c+dx) \operatorname{ExpIntegralEi}\left(-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)}$$

output
```
(e*((b*x+a)/(d*x+c))^n)^(1/n)*(d*x+c)*Ei(-ln(e*((b*x+a)/(d*x+c))^n)/n)/(-a
*d+b*c)/n/(b*x+a)
```

## 3.246.2 Mathematica [A] (verified)

Time = 0.06 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}} (c+dx) \operatorname{ExpIntegralEi}\left(-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)}$$

input
```
Integrate[1/((a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(Log[e*((a +
b*x)/(c + d*x))^n]/n)])/((b*c - a*d)*n*(a + b*x))
```

### 3.246.3 Rubi [A] (verified)

Time = 0.27 (sec) , antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.107, Rules used = {2949, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(a+bx)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}dx$$

$$\Big\downarrow \text{2949}$$

$$\frac{\int \frac{(c+dx)^2}{(a+bx)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2747}$$

$$\frac{(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-1/n}(c+dx)}{a+bx}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(a+bx)(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{(c+dx)\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{\frac{1}{n}}\text{ExpIntegralEi}\left(-\frac{\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(a+bx)(bc-ad)}$$

input
```
Int[1/((a + b*x)^2*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((e*((a + b*x)/(c + d*x))^n)^n^(-1)*(c + d*x)*ExpIntegralEi[-(Log[e*((a +
b*x)/(c + d*x))^n]/n)])/((b*c - a*d)*n*(a + b*x))
```

#### 3.246.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2949
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.), x_Symbol] :> Simp[(b*c - a*d)^(m +
1)*(g/b)^m   Subst[Int[x^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + 2)), x], x,
 (a + b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, A, B, n}, x] && Ne
Q[b*c - a*d, 0] && IntegersQ[m, p] && EqQ[b*f - a*g, 0] && (GtQ[p, 0] || Lt
Q[m, -1])
```

### 3.246.4 Maple [F]

$$\int \frac{1}{\left(bx+a\right)^{2} \ln \left(e \left(\frac{bx+a}{dx+c}\right)^{n}\right)}dx$$

input
```
int(1/(b*x+a)^2/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int(1/(b*x+a)^2/ln(e*((b*x+a)/(d*x+c))^n),x)
```

### 3.246.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 40, normalized size of antiderivative = 0.56

$$\int \frac{1}{(a+bx)^2 \log \left(e \left(\frac{a+bx}{c+dx}\right)^n\right)}\, dx = \frac{e^{\left(\frac{1}{n}\right)} \operatorname{log\_integral}\left(\frac{dx+c}{(bx+a)e^{\left(\frac{1}{n}\right)}}\right)}{(bc-ad)n}$$

input
```
integrate(1/(b*x+a)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fricas")
```

output
```
e^(1/n)*log_integral((d*x + c)/((b*x + a)*e^(1/n)))/((b*c - a*d)*n)
```

### 3.246.6 Sympy [F]

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a}{c+dx}+\frac{bx}{c+dx}\right)^n\right)}\,dx$$

input
```
integrate(1/(b*x+a)**2/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Integral(1/((a + b*x)**2*log(e*(a/(c + d*x) + b*x/(c + d*x))**n)), x)
```

### 3.246.7 Maxima [F]

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{(bx+a)^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate(1/(b*x+a)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxima")
```

output
```
integrate(1/((b*x + a)^2*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.246.8 Giac [F]

$$\int \frac{1}{(a+bx)^2 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{1}{(bx+a)^2 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate(1/(b*x+a)^2/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac")
```

output
```
integrate(1/((b*x + a)^2*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.246.9 Mupad [F(-1)]

Timed out.

$$\int \frac{1}{(a+bx)^2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{1}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,\left(a+b\,x\right)^2}\,dx$$

input
```
int(1/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^2),x)
```

output
```
int(1/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^2), x)
```

# 3.247 $\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx$



## 3.247.1 Optimal result

Integrand size = 33, antiderivative size = 75

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n} (c+dx)^2 \operatorname{ExpIntegralEi}\left(-\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^2}$$

output
```
(e*((b*x+a)/(d*x+c))^n)^(2/n)*(d*x+c)^2*Ei(-2*ln(e*((b*x+a)/(d*x+c))^n)/n)
/(-a*d+b*c)/n/(b*x+a)^2
```

## 3.247.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)} dx = \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n} (c+dx)^2 \operatorname{ExpIntegralEi}\left(-\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^2}$$

input
```
Integrate[(c + d*x)/((a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpIntegralEi[(-2*Log[e*((a
 + b*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(a + b*x)^2)
```

### 3.247.3 Rubi [A] (verified)

Time = 0.28 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2961, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}dx$$

$$\Big\downarrow \; 2961$$

$$\frac{\int \frac{(c+dx)^3}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \; 2747$$

$$\frac{(c+dx)^2 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n} \int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-2/n}(c+dx)}{a+bx}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(a+bx)^2(bc-ad)}$$

$$\Big\downarrow \; 2609$$

$$\frac{(c+dx)^2 \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{2/n} \text{ExpIntegralEi}\left(-\frac{2\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(a+bx)^2(bc-ad)}$$

input
```
Int[(c + d*x)/((a + b*x)^3*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((e*((a + b*x)/(c + d*x))^n)^(2/n)*(c + d*x)^2*ExpIntegralEi[(-2*Log[e*((a
 + b*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(a + b*x)^2)
```

#### 3.247.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] && !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.247.4 Maple [F]

$$\int \frac{dx+c}{(bx+a)^3 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((d*x+c)/(b*x+a)^3/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int((d*x+c)/(b*x+a)^3/ln(e*((b*x+a)/(d*x+c))^n),x)
```

## 3.247.5 Fricas [A] (verification not implemented)

Time = 0.33 (sec) , antiderivative size = 66, normalized size of antiderivative = 0.88

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{e^{\frac{2}{n}}\operatorname{log\_integral}\left(\frac{d^2x^2+2\,cdx+c^2}{(b^2x^2+2\,abx+a^2)e^{\frac{2}{n}}}\right)}{(bc-ad)n}$$

input
```
integrate((d*x+c)/(b*x+a)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="frica
s")
```

output
```
e^(2/n)*log_integral((d^2*x^2 + 2*c*d*x + c^2)/((b^2*x^2 + 2*a*b*x + a^2)*
e^(2/n)))/((b*c - a*d)*n)
```

### 3.247.6 Sympy [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)/(b*x+a)**3/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

### 3.247.7 Maxima [F]

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{dx+c}{(bx+a)^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((d*x+c)/(b*x+a)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="maxim
a")
```

output
```
integrate((d*x + c)/((b*x + a)^3*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.247.8 Giac [F]

$$\int \frac{c+dx}{(a+bx)^3 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{dx+c}{(bx+a)^3 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((d*x+c)/(b*x+a)^3/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="giac"
)
```

output
```
integrate((d*x + c)/((b*x + a)^3*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.247.9 Mupad [F(-1)]

Timed out.

$$\int \frac{c+dx}{(a+bx)^3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\int \frac{c+d\,x}{\ln\left(e\,{\left(\frac{a+b\,x}{c+d\,x}\right)}^n\right)\,{\left(a+b\,x\right)}^3}\,dx$$

input
```
int((c + d*x)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^3),x)
```

output
```
int((c + d*x)/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^3), x)
```

# 3.248 $\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx$



## 3.248.1 Optimal result

Integrand size = 35, antiderivative size = 75

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n}(c+dx)^3\,\text{ExpIntegralEi}\left(-\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^3}$$

output
```
(e*((b*x+a)/(d*x+c))^n)^(3/n)*(d*x+c)^3*Ei(-3*ln(e*((b*x+a)/(d*x+c))^n)/n)
/(-a*d+b*c)/n/(b*x+a)^3
```

## 3.248.2 Mathematica [A] (verified)

Time = 0.01 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx=\frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n}(c+dx)^3\,\text{ExpIntegralEi}\left(-\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{(bc-ad)n(a+bx)^3}$$

input
```
Integrate[(c + d*x)^2/((a + b*x)^4*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((e*((a + b*x)/(c + d*x))^n)^(3/n)*(c + d*x)^3*ExpIntegralEi[(-3*Log[e*((a
 + b*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(a + b*x)^3)
```

### 3.248.3 Rubi [A] (verified)

Time = 0.30 (sec) , antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {2961, 2747, 2609}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(c+dx)^2}{(a+bx)^4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}dx$$

$$\Big\downarrow \text{2961}$$

$$\frac{\int \frac{(c+dx)^4}{(a+bx)^4\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}d\frac{a+bx}{c+dx}}{bc-ad}$$

$$\Big\downarrow \text{2747}$$

$$\frac{(c+dx)^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n}\int \frac{\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{-3/n}(c+dx)}{a+bx}d\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n(a+bx)^3(bc-ad)}$$

$$\Big\downarrow \text{2609}$$

$$\frac{(c+dx)^3\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)^{3/n}\text{ExpIntegralEi}\left(-\frac{3\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{n}\right)}{n(a+bx)^3(bc-ad)}$$

input
```
Int[(c + d*x)^2/((a + b*x)^4*Log[e*((a + b*x)/(c + d*x))^n]),x]
```

output
```
((e*((a + b*x)/(c + d*x))^n)^(3/n)*(c + d*x)^3*ExpIntegralEi[(-3*Log[e*((a
 + b*x)/(c + d*x))^n])/n])/((b*c - a*d)*n*(a + b*x)^3)
```

#### 3.248.3.1 Defintions of rubi rules used

rule 2609
```
Int[(F_)^((g_.)*((e_.) + (f_.)*(x_)))/((c_.) + (d_.)*(x_)), x_Symbol] :> Si
mp[(F^(g*(e - c*(f/d)))/d)*ExpIntegralEi[f*g*(c + d*x)*(Log[F]/d)], x] /; F
reeQ[{F, c, d, e, f, g}, x] &&  !TrueQ[$UseGamma]
```

rule 2747
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_)*((d_.)*(x_))^(m_.), x_Symbol
] :> Simp[(d*x)^(m + 1)/(d*n*(c*x^n)^((m + 1)/n))   Subst[Int[E^(((m + 1)/n
)*x)*(a + b*x)^p, x], x, Log[c*x^n]], x] /; FreeQ[{a, b, c, d, m, n, p}, x]
```

rule 2961
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(m + q + 1)*(g/b)^m*(i/d)^q   Subst[Int[x^m*((A + B*L
og[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*x)], x] /; Fre
eQ[{a, b, c, d, e, f, g, h, i, A, B, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[
b*f - a*g, 0] && EqQ[d*h - c*i, 0] && IntegersQ[m, q]
```

## 3.248.4 Maple [F]

$$\int \frac{(dx+c)^2}{(bx+a)^4 \ln\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}dx$$

input
```
int((d*x+c)^2/(b*x+a)^4/ln(e*((b*x+a)/(d*x+c))^n),x)
```

output
```
int((d*x+c)^2/(b*x+a)^4/ln(e*((b*x+a)/(d*x+c))^n),x)
```

## 3.248.5 Fricas [A] (verification not implemented)

Time = 0.30 (sec) , antiderivative size = 88, normalized size of antiderivative = 1.17

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \frac{e^{\frac{3}{n}} \operatorname{log\_integral}\left(\frac{d^3x^3+3\,cd^2x^2+3\,c^2dx+c^3}{(b^3x^3+3\,ab^2x^2+3\,a^2bx+a^3)e^{\frac{3}{n}}}\right)}{(bc-ad)n}$$

input
```
integrate((d*x+c)^2/(b*x+a)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="fri
cas")
```

output
```
e^(3/n)*log_integral((d^3*x^3 + 3*c*d^2*x^2 + 3*c^2*d*x + c^3)/((b^3*x^3 +
 3*a*b^2*x^2 + 3*a^2*b*x + a^3)*e^(3/n)))/((b*c - a*d)*n)
```

### 3.248.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \text{Timed out}$$

input
```
integrate((d*x+c)**2/(b*x+a)**4/ln(e*((b*x+a)/(d*x+c))**n),x)
```

output
```
Timed out
```

### 3.248.7 Maxima [F]

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(dx+c)^2}{(bx+a)^4 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((d*x+c)^2/(b*x+a)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="max
ima")
```

output
```
integrate((d*x + c)^2/((b*x + a)^4*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.248.8 Giac [F]

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(dx+c)^2}{(bx+a)^4 \log\left(e\left(\frac{bx+a}{dx+c}\right)^n\right)}\,dx$$

input
```
integrate((d*x+c)^2/(b*x+a)^4/log(e*((b*x+a)/(d*x+c))^n),x, algorithm="gia
c")
```

output
```
integrate((d*x + c)^2/((b*x + a)^4*log(e*((b*x + a)/(d*x + c))^n)), x)
```

### 3.248.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(c+dx)^2}{(a+bx)^4 \log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}\,dx = \int \frac{(c+d\,x)^2}{\ln\left(e\left(\frac{a+b\,x}{c+d\,x}\right)^n\right)\,(a+b\,x)^4}\,dx$$

input
```
int((c + d*x)^2/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^4),x)
```

output
```
int((c + d*x)^2/(log(e*((a + b*x)/(c + d*x))^n)*(a + b*x)^4), x)
```

# 3.249 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{(f+gx)(ah+bhx)}\,dx$



## 3.249.1 Optimal result

Integrand size = 43, antiderivative size = 361

$$\begin{aligned}
&\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx \\
&= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{4Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{12B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\,\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{24B^3n^3(A+B\log(e(a+bx)^n(c+dx)^{-n}))\,\text{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{24B^4n^4\,\text{PolyLog}\left(5,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}
\end{aligned}$$

output
```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b
*x+a))/(-a*g+b*f)/h+4*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*polylog(2,(-
a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+12*B^2*n^2*(A+B*ln(e*(b*
x+a)^n/((d*x+c)^n)))^2*polylog(3,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-
a*g+b*f)/h+24*B^3*n^3*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(4,(-a*g+b*
f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+24*B^4*n^4*polylog(5,(-a*g+b*f
)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h
```

### 3.249.2 Mathematica [F]

$$\int \frac{(A+B\log (e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx=\int \frac{(A+B\log (e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4/((f + g*x)*(a*h + b*h
*x)),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4/((f + g*x)*(a*h + b*h
*x)), x]
```

### 3.249.3 Rubi [A] (warning: unable to verify)

Time = 1.14 (sec) , antiderivative size = 312, normalized size of antiderivative = 0.86, number of steps used = 9, number of rules used = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.186, Rules used = {2973, 2967, 27, 2779, 2821, 2830, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log (e(a+bx)^n(c+dx)^{-n})+A)^4}{(f+gx)(ah+bhx)}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log (e(a+bx)^n(c+dx)^{-n})+A)^4}{(f+gx)(ah+bhx)}dx$$

$$\Big\downarrow \text{2967}$$

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^4}{(bc-ad)h(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)\left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^4}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{h}$$

$$\Big\downarrow \text{2779}$$

$$\frac{\frac{4Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{bf-ag}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^4}{bf-ag}}{h}$$

$\Big\downarrow$ 2821

$$\frac{\frac{4Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3-3Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{bf-ag}-\frac{\log\left(1-\right.}{}}{h}$$

$\Big\downarrow$ 2830

$$\frac{\frac{4Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3-3Bn\left(2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\text{PolyL}\right.\right.}{bf-ag}}{h}$$

$\Big\downarrow$ 2830

$$\frac{\frac{4Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3-3Bn\left(2Bn\left(Bn\int \frac{(c+dx)\text{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\text{PolyLog}\left(4,\frac{(bf-ag)(c+}{(df-cg)(a+}\right.\right.\right.\right.}{bf-ag}}{h}$$

$\Big\downarrow$ 7143

$$\frac{\frac{4Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3-3Bn\left(2Bn\left(-\left(\text{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)-Bn\,\text{PolyLog}\right.\right.\right.}{bf-ag}}{h}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^4/((f + g*x)*(a*h + b*h*x)),x
]
```

output
```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^4*Log[1 - ((b*f - a*g)*(c + d*x
))/((d*f - c*g)*(a + b*x))])/(b*f - a*g)) + (4*B*n*((A + B*Log[e*((a + b*x
)/(c + d*x))^n])^3*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*
x))] - 3*B*n*(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*PolyLog[3, ((b*f
- a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]) + 2*B*n*(-((A + B*Log[e*((a +
b*x)/(c + d*x))^n])*PolyLog[4, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b
*x))]) - B*n*PolyLog[5, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]))
))/(b*f - a*g))/h
```

## 3.249. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^4}{(f+gx)(ah+bhx)}\,dx$

### 3.249.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2967
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)   Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*(b*h - a*i
- (d*h - c*i)*x)^q*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a
+ b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

---

3.249. $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx$

### 3.249.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{4}}{\left(gx+f\right)\left(bhx+ah\right)}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x)
```

### 3.249.5 Fricas [F]

$$\int \frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{4}}{(f+gx)(ah+bhx)}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{4}}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x, algo
rithm="fricas")
```

output
```
integral((B^4*log((b*x + a)^n*e/(d*x + c)^n)^4 + 4*A*B^3*log((b*x + a)^n*e
/(d*x + c)^n)^3 + 6*A^2*B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 4*A^3*B*log
((b*x + a)^n*e/(d*x + c)^n) + A^4)/(b*g*h*x^2 + a*f*h + (b*f + a*g)*h*x),
x)
```

### 3.249.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{4}}{(f+gx)(ah+bhx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**4/(g*x+f)/(b*h*x+a*h),x)
```

output
```
Timed out
```

### 3.249.7 Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^4}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x, algo
rithm="maxima")
```

output
```
A^4*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) + integr
ate((B^4*log((b*x + a)^n)^4 + B^4*log((d*x + c)^n)^4 + B^4*log(e)^4 + 4*A*
B^3*log(e)^3 + 6*A^2*B^2*log(e)^2 + 4*A^3*B*log(e) + 4*(B^4*log(e) + A*B^3
)*log((b*x + a)^n)^3 - 4*(B^4*log((b*x + a)^n) + B^4*log(e) + A*B^3)*log((
d*x + c)^n)^3 + 6*(B^4*log(e)^2 + 2*A*B^3*log(e) + A^2*B^2)*log((b*x + a)^
n)^2 + 6*(B^4*log((b*x + a)^n)^2 + B^4*log(e)^2 + 2*A*B^3*log(e) + A^2*B^2
 + 2*(B^4*log(e) + A*B^3)*log((b*x + a)^n))*log((d*x + c)^n)^2 + 4*(B^4*lo
g(e)^3 + 3*A*B^3*log(e)^2 + 3*A^2*B^2*log(e) + A^3*B)*log((b*x + a)^n) - 4
*(B^4*log((b*x + a)^n)^3 + B^4*log(e)^3 + 3*A*B^3*log(e)^2 + 3*A^2*B^2*log
(e) + A^3*B + 3*(B^4*log(e) + A*B^3)*log((b*x + a)^n)^2 + 3*(B^4*log(e)^2
+ 2*A*B^3*log(e) + A^2*B^2)*log((b*x + a)^n))*log((d*x + c)^n))/(b*g*h*x^2
 + a*f*h + (b*f*h + a*g*h)*x), x)
```

### 3.249.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^4}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^4/(g*x+f)/(b*h*x+a*h),x, algo
rithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^4/((b*h*x + a*h)*(g*x + f
)), x)
```

### 3.249.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^4}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^4}{(f+g\,x)\,(a\,h+b\,h\,x)}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^4/((f + g*x)*(a*h + b*h*x)),x
)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^4/((f + g*x)*(a*h + b*h*x)),
x)
```

# 3.250 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(f+gx)(ah+bhx)}\,dx$



## 3.250.1 Optimal result

Integrand size = 43, antiderivative size = 282

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx = -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

output
```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b
*x+a))/(-a*g+b*f)/h+3*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,(-
a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+6*B^2*n^2*(A+B*ln(e*(b*x
+a)^n/((d*x+c)^n)))*polylog(3,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g
+b*f)/h+6*B^3*n^3*polylog(4,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b
*f)/h
```

### 3.250.2 Mathematica [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx = \int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((f + g*x)*(a*h + b*h
*x)),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((f + g*x)*(a*h + b*h
*x)), x]
```

### 3.250.3 Rubi [A] (warning: unable to verify)

Time = 1.00 (sec) , antiderivative size = 248, normalized size of antiderivative = 0.88, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}} = 0.163$, Rules used = {2973, 2967, 27, 2779, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(f+gx)(ah+bhx)}\,dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{(f+gx)(ah+bhx)}dx$$

$$\Big\downarrow \text{2967}$$

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(bc-ad)h(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{h}$$

$$\Big\downarrow \text{2779}$$

$$\frac{\frac{3Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{bf-ag}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^3}{bf-ag}}{h}$$

$\Big\downarrow$ 2821

$$\frac{\frac{3Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{bf-ag}-\frac{\log\Big(1-}{}}{h}$$

$\Big\downarrow$ 2830

$$\frac{\frac{3Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\left(Bn\int \frac{(c+dx)\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}-\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\Big(\right.\right.}{bf-ag}}{h}$$

$\Big\downarrow$ 7143

$$\frac{\frac{3Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\left(-\left(\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)\right)-Bn\,\text{PolyLog}\left(4,\frac{(bf}{(df}\right.\right.\right.}{bf-ag}}{h}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/((f + g*x)*(a*h + b*h*x)),x
]
```

output
```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^3*Log[1 - ((b*f - a*g)*(c + d*x
))/((d*f - c*g)*(a + b*x))])/(b*f - a*g)) + (3*B*n*((A + B*Log[e*((a + b*x
)/(c + d*x))^n])^2*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*
x))] - 2*B*n*(-((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[3, ((b*f -
a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]) - B*n*PolyLog[4, ((b*f - a*g)*(c
 + d*x))/((d*f - c*g)*(a + b*x))])))/(b*f - a*g))/h
```

### 3.250.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2967
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)   Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*(b*h - a*i
- (d*h - c*i)*x)^q*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a
+ b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

## 3.250. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{(f+gx)(ah+bhx)}\,dx$

### 3.250.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{3}}{\left(gx+f\right)\left(bhx+ah\right)}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x)
```

### 3.250.5 Fricas [F]

$$\int \frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{3}}{(f+gx)(ah+bhx)}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{3}}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x, algo
rithm="fricas")
```

output
```
integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e
/(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(b*g*h*x^2
 + a*f*h + (b*f + a*g)*h*x), x)
```

### 3.250.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))^{3}}{(f+gx)(ah+bhx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(g*x+f)/(b*h*x+a*h),x)
```

output
```
Timed out
```

### 3.250.7 Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x, algo
rithm="maxima")
```

output
```
A^3*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) - integr
ate(-(B^3*log((b*x + a)^n)^3 - B^3*log((d*x + c)^n)^3 + B^3*log(e)^3 + 3*A
*B^2*log(e)^2 + 3*A^2*B*log(e) + 3*(B^3*log(e) + A*B^2)*log((b*x + a)^n)^2
 + 3*(B^3*log((b*x + a)^n) + B^3*log(e) + A*B^2)*log((d*x + c)^n)^2 + 3*(B
^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B)*log((b*x + a)^n) - 3*(B^3*log((b*x +
 a)^n)^2 + B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B + 2*(B^3*log(e) + A*B^2)*
log((b*x + a)^n))*log((d*x + c)^n))/(b*g*h*x^2 + a*f*h + (b*f*h + a*g*h)*x
), x)
```

### 3.250.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(g*x+f)/(b*h*x+a*h),x, algo
rithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/((b*h*x + a*h)*(g*x + f
)), x)
```

### 3.250.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{(f+gx)(ah+bhx)}\,dx = \int \frac{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^3}{(f+g\,x)\,\left(a\,h+b\,h\,x\right)}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/((f + g*x)*(a*h + b*h*x)),x
)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/((f + g*x)*(a*h + b*h*x)),
x)
```

# 3.251 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(f+gx)(ah+bhx)}\,dx$



## 3.251.1 Optimal result

Integrand size = 43, antiderivative size = 203

$$\begin{aligned}\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(f+gx)(ah+bhx)}\,dx &= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\\ &+\frac{2Bn\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\\ &+\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\end{aligned}$$

output

```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b
*x+a))/(-a*g+b*f)/h+2*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-a*
g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+2*B^2*n^2*polylog(3,(-a*g+
b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h
```

### 3.251.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1415 vs. 2(203) = 406.

Time = 0.78 (sec) , antiderivative size = 1415, normalized size of antiderivative = 6.97

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/((f + g*x)*(a*h + b*h
*x)),x]
```

output
```
(3*Log[a + b*x]*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b
*x)^n)/(c + d*x)^n]))^2 - 3*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + L
og[(e*(a + b*x)^n)/(c + d*x)^n]))^2*Log[f + g*x] + 3*B*n*(A + B*(-(n*Log[a
 + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]))*(Log[a + b*
x]^2 - 2*(Log[a + b*x]*Log[(b*(f + g*x))/(b*f - a*g)] + PolyLog[2, (g*(a +
 b*x))/(-(b*f) + a*g)])) - 6*A*B*n*(Log[c + d*x]*(Log[(d*(a + b*x))/(-(b*c
) + a*d)] - Log[(d*(f + g*x))/(d*f - c*g)]) + PolyLog[2, (b*(c + d*x))/(b*
c - a*d)] - PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)]) + 6*B^2*n*(n*Log[a +
 b*x] - n*Log[c + d*x] - Log[(e*(a + b*x)^n)/(c + d*x)^n])*(Log[c + d*x]*(
Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[(d*(f + g*x))/(d*f - c*g)]) + Poly
Log[2, (b*(c + d*x))/(b*c - a*d)] - PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g
)]) + B^2*n^2*(Log[a + b*x]^2*(Log[a + b*x] - 3*Log[(b*(f + g*x))/(b*f - a
*g)]) - 6*Log[a + b*x]*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)] + 6*PolyLo
g[3, (g*(a + b*x))/(-(b*f) + a*g)]) + 3*B^2*n^2*(Log[(d*(a + b*x))/(-(b*c)
 + a*d)]*Log[c + d*x]^2 - Log[c + d*x]^2*Log[(d*(f + g*x))/(d*f - c*g)] +
2*Log[c + d*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*Log[c + d*x]*Poly
Log[2, (g*(c + d*x))/(-(d*f) + c*g)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a
*d)] + 2*PolyLog[3, (g*(c + d*x))/(-(d*f) + c*g)]) - 6*B^2*n^2*((Log[a + b
*x]^2*(Log[c + d*x] - Log[(b*(c + d*x))/(b*c - a*d)]))/2 - Log[a + b*x]*Lo
g[c + d*x]*Log[(b*(f + g*x))/(b*f - a*g)] - (Log[(g*(c + d*x))/(-(d*f) ...
```

### 3.251.3 Rubi [A] (warning: unable to verify)

Time = 0.87 (sec) , antiderivative size = 183, normalized size of antiderivative = 0.90, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.140$, Rules used = {2973, 2967, 27, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

---

3.251. $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\,dx$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(f+gx)(ah+bhx)}dx$$

$$\Big\downarrow \text{ 2973}$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{(f+gx)(ah+bhx)}dx$$

$$\Big\downarrow \text{ 2967}$$

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(bc-ad)h(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{ 27}$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{h}$$

$$\Big\downarrow \text{ 2779}$$

$$\frac{\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{bf-ag}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2}{bf-ag}}{h}$$

$$\Big\downarrow \text{ 2821}$$

$$\frac{\frac{2Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int \frac{(c+dx)\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{bf-ag}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\right.\right.}{bf-ag}}{h}$$

$$\Big\downarrow \text{ 7143}$$

$$\frac{\frac{2Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+Bn\,\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\right)}{bf-ag}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bf-ag}}{h}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/((f + g*x)*(a*h + b*h*x)),x
]
```

3.251. $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}dx$

output
```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - ((b*f - a*g)*(c + d*x
))/((d*f - c*g)*(a + b*x))])/(b*f - a*g)) + (2*B*n*((A + B*Log[e*((a + b*x
)/(c + d*x))^n])*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x)
)] + B*n*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]))/(b*
f - a*g))/h
```

### 3.251.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2967
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)   Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*(b*h - a*i
- (d*h - c*i)*x)^q*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a
+ b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

## 3.251. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(f+gx)(ah+bhx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.251.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}(dx+c)^{-n}\right)\right)^{2}}{(gx+f)(bhx+ah)}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x)
```

### 3.251.5 Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x, algo
rithm="fricas")
```

output
```
integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(
d*x + c)^n) + A^2)/(b*g*h*x^2 + a*f*h + (b*f + a*g)*h*x), x)
```

### 3.251.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{(f+gx)(ah+bhx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(g*x+f)/(b*h*x+a*h),x)
```

output
```
Timed out
```

### 3.251.7 Maxima [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(f + gx)(ah + bhx)} dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bhx + ah)(gx + f)} dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x, algo
rithm="maxima")
```

output
```
A^2*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) + integr
ate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)^n)^2 + B^2*log(e)^2 + 2*A*
B*log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n) - 2*(B^2*log((b*x + a)^n)
 + B^2*log(e) + A*B)*log((d*x + c)^n))/(b*g*h*x^2 + a*f*h + (b*f*h + a*g*h
)*x), x)
```

### 3.251.8 Giac [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(f + gx)(ah + bhx)} dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{(bhx + ah)(gx + f)} dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(g*x+f)/(b*h*x+a*h),x, algo
rithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/((b*h*x + a*h)*(g*x + f
)), x)
```

### 3.251.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{(f + gx)(ah + bhx)} dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{(f + g\,x)\,(a\,h + b\,h\,x)} dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/((f + g*x)*(a*h + b*h*x)),x
)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/((f + g*x)*(a*h + b*h*x)),
x)
```

## 3.251. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{(f+gx)(ah+bhx)}\,dx$

# 3.252 $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx$



## 3.252.1 Optimal result

Integrand size = 41, antiderivative size = 123

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{Bn\,\mathrm{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

output
```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x
+a))/(-a*g+b*f)/h+B*n*polylog(2,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a
*g+b*f)/h
```

## 3.252.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 304 vs. 2(123) = 246.

Time = 0.26 (sec) , antiderivative size = 304, normalized size of antiderivative = 2.47

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = -\frac{-2A\log(a+bx)+Bn\log^2(a+bx)-2Bn\log(a+bx)\log(c+dx)+2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(c+dx)-2}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((f + g*x)*(a*h + b*h*x
)),x]
```

output
```
-1/2*(-2*A*Log[a + b*x] + B*n*Log[a + b*x]^2 - 2*B*n*Log[a + b*x]*Log[c +
d*x] + 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 2*B*Log[a +
b*x]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*A*Log[f + g*x] - 2*B*n*Log[a + b
*x]*Log[f + g*x] + 2*B*n*Log[c + d*x]*Log[f + g*x] + 2*B*Log[(e*(a + b*x)^
n)/(c + d*x)^n]*Log[f + g*x] + 2*B*n*Log[a + b*x]*Log[(b*(f + g*x))/(b*f -
 a*g)] - 2*B*n*Log[c + d*x]*Log[(d*(f + g*x))/(d*f - c*g)] + 2*B*n*PolyLog
[2, (g*(a + b*x))/(-(b*f) + a*g)] + 2*B*n*PolyLog[2, (b*(c + d*x))/(b*c -
a*d)] - 2*B*n*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)])/((b*f - a*g)*h)
```

### 3.252.3 Rubi [A] (warning: unable to verify)

Time = 0.57 (sec) , antiderivative size = 120, normalized size of antiderivative = 0.98, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}} = 0.122$, Rules used = {2973, 2967, 27, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(f+gx)(ah+bhx)}\,dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A}{(f+gx)(ah+bhx)}dx$$

$$\Big\downarrow \text{2967}$$

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(bc-ad)h(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{27}$$

$$\frac{\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left(bf-ag-\frac{(df-cg)(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}}{h}$$

$$\Big\downarrow \text{2779}$$

$$\frac{\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{bf-ag}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bf-ag}}{h}$$

$$\Big\downarrow \text{2838}$$

$$\frac{\frac{Bn\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{bf-ag}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{bf-ag}}{h}$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/((f + g*x)*(a*h + b*h*x)),x]
```

output
```
(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((b*f - a*g)*(c + d*x))
/((d*f - c*g)*(a + b*x))])/(b*f - a*g)) + (B*n*PolyLog[2, ((b*f - a*g)*(c
+ d*x))/((d*f - c*g)*(a + b*x))])/(b*f - a*g))/h
```

### 3.252.3.1 Defintions of rubi rules used

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2967
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)   Subst[Int[(b*f - a*g - (d*f - c*g)*x)^m*(b*h - a*i
- (d*h - c*i)*x)^q*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a
+ b*x)/(c + d*x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] &&
NeQ[b*c - a*d, 0] && IntegersQ[m, q] && IGtQ[p, 0]
```

## 3.252. $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx$

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

### 3.252.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 22.58 (sec) , antiderivative size = 1447, normalized size of antiderivative = 11.76

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1447 |

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(g*x+f)/(b*h*x+a*h),x,method=_RETURN
VERBOSE)
```

output
```
1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csg
n(I*(b*x+a)^n/((d*x+c)^n))-1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*e)*csgn
(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I/h/(a*g-b*f
)*ln(g*x+f)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((
d*x+c)^n))+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d
*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/h/(a*g-b*f)*ln(b*x+a)*A+1/h/(a
*g-b*f)*ln(g*x+f)*A+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*(b*x+a)^n/((d*
x+c)^n))^3+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^3-1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I
/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I/h/(a*g
-b*f)*ln(b*x+a)*B*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I/h/(
a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-
1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*
x+c)^n))^2+1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*
(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/h/(a*g-b*f)
*ln(g*x+f)*B*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/h/(a*g-b
*f)*ln(g*x+f)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I
/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^
n))^2-1/h*B*n/(a*g-b*f)*ln(g*x+f)*ln((b*(g*x+f)+a*g-b*f)/(a*g-b*f))+1/h...
```

### 3.252.5 Fricas [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(g*x+f)/(b*h*x+a*h),x, algori
thm="fricas")
```

output
```
integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*g*h*x^2 + a*f*h + (b*f
+ a*g)*h*x), x)
```

### 3.252.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(g*x+f)/(b*h*x+a*h),x)
```

output
```
Timed out
```

### 3.252.7 Maxima [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx = \int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(g*x+f)/(b*h*x+a*h),x, algori
thm="maxima")
```

output
```
A*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) - B*integr
ate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(b*g*h*x^2 + a*f*h + (
b*f*h + a*g*h)*x), x)
```

### 3.252.8 Giac [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx=\int \frac{B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{(bhx+ah)(gx+f)}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(g*x+f)/(b*h*x+a*h),x, algori
thm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/((b*h*x + a*h)*(g*x + f))
, x)
```

### 3.252.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{(f+gx)(ah+bhx)}\,dx=\int \frac{A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{(f+g\,x)\,(a\,h+b\,h\,x)}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/((f + g*x)*(a*h + b*h*x)),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/((f + g*x)*(a*h + b*h*x)), x)
```

# 3.253 $\int \frac{1}{(f+gx)(ah+bhx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$



## 3.253.1 Optimal result

Integrand size = 43, antiderivative size = 43

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Subst}\left(\text{Int}\left(\frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right),e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

output
```
_eval(Unintegrable(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x
),e*((b*x+a)/(d*x+c))^n = e*(b*x+a)^n/((d*x+c)^n))
```

## 3.253.2 Mathematica [N/A]

Not integrable

Time = 0.17 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx$$

input
```
Integrate[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])),x]
```

output
```
Integrate[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])), x]
```

### 3.253.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2973, 2969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)(ah+bhx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$
$$\Big\downarrow \text{2973}$$
$$\int \frac{1}{(f+gx)(ah+bhx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$
$$\Big\downarrow \text{2969}$$
$$\int \frac{1}{(f+gx)(ah+bhx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

input
```
Int[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])),x
]
```

output
```
$Aborted
```

#### 3.253.3.1 Defintions of rubi rules used

rule 2969
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Unintegrable[(f + g*x)^m*(h + i*x)^q*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

### 3.253.4 Maple [N/A]

Not integrable

Time = 0.23 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\,(bhx+ah)\,\left(A+B\ln\left(e\,(bx+a)^{n}\,(dx+c)^{-n}\right)\right)}dx$$

input
```
int(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

output
```
int(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),x)
```

### 3.253.5 Fricas [N/A]

Not integrable

Time = 0.30 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.77

$$\int \frac{1}{(f+gx)(ah+bhx)\,(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right))}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algo
rithm="fricas")
```

output
```
integral(1/(A*b*g*h*x^2 + A*a*f*h + (A*b*f + A*a*g)*h*x + (B*b*g*h*x^2 + B
*a*f*h + (B*b*f + B*a*g)*h*x)*log((b*x + a)^n*e/(d*x + c)^n)), x)
```

### 3.253.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx=\text{Timed out}$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n))),x)
```

output
```
Timed out
```

### 3.253.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algo
rithm="maxima")
```

output
```
integrate(1/((b*h*x + a*h)*(g*x + f)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A
)), x)
```

### 3.253.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n))),x, algo
rithm="giac")
```

output
```
integrate(1/((b*h*x + a*h)*(g*x + f)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A
)), x)
```

### 3.253.9 Mupad [N/A]

Not integrable

Time = 1.24 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(f+g\,x)\,(a\,h+b\,h\,x)\,\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)}\,dx$$

input
```
int(1/((f + g*x)*(a*h + b*h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),x
)
```

output
```
int(1/((f + g*x)*(a*h + b*h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))),
x)
```

# 3.254 $\int \frac{1}{(f+gx)(ah+bhx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}\,dx$



## 3.254.1 Optimal result

Integrand size = 43, antiderivative size = 43

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \text{Subst}\left(\text{Int}\left(\frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right),e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)$$

output
```
_eval(Unintegrable(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2
,x),e*((b*x+a)/(d*x+c))^n = e*(b*x+a)^n/((d*x+c)^n))
```

## 3.254.2 Mathematica [N/A]

Not integrable

Time = 0.33 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx$$

input
```
Integrate[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])^2),x]
```

output
```
Integrate[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^
n])^2), x]
```

### 3.254.3 Rubi [N/A]

Not integrable

Time = 0.43 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2973, 2969}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(f+gx)(ah+bhx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{2973}$$
$$\int \frac{1}{(f+gx)(ah+bhx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}dx$$
$$\Big\downarrow \text{2969}$$
$$\int \frac{1}{(f+gx)(ah+bhx)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}dx$$

input
```
Int[1/((f + g*x)*(a*h + b*h*x)*(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2)
,x]
```

output
```
$Aborted
```

#### 3.254.3.1 Defintions of rubi rules used

rule 2969
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Unintegrable[(f + g*x)^m*(h + i*x)^q*(A + B*Log[e*((a + b*x)/(c + d*x)
)^n])^p, x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, m, n, p, q}, x]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

### 3.254.4 Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 43, normalized size of antiderivative = 1.00

$$\int \frac{1}{(gx+f)\,(bhx+ah)\left(A+B\ln\left(e\,(bx+a)^{n}\,(dx+c)^{-n}\right)\right)^{2}}dx$$

input
```
int(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

output
```
int(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2,x)
```

### 3.254.5 Fricas [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 145, normalized size of antiderivative = 3.37

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^{n}(c+dx)^{-n}\right)\right)^{2}}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{2}}\,dx$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, al
gorithm="fricas")
```

output
```
integral(1/(A^2*b*g*h*x^2 + A^2*a*f*h + (A^2*b*f + A^2*a*g)*h*x + (B^2*b*g
*h*x^2 + B^2*a*f*h + (B^2*b*f + B^2*a*g)*h*x)*log((b*x + a)^n*e/(d*x + c)^
n)^2 + 2*(A*B*b*g*h*x^2 + A*B*a*f*h + (A*B*b*f + A*B*a*g)*h*x)*log((b*x +
a)^n*e/(d*x + c)^n)), x)
```

### 3.254.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2,x)
```

output
```
Timed out
```

### 3.254.7 Maxima [N/A]

Not integrable

Time = 0.38 (sec) , antiderivative size = 506, normalized size of antiderivative = 11.77

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, al
gorithm="maxima")
```

output
```
(d*f - c*g)*integrate(1/((b*c*f^2*h*n - a*d*f^2*h*n)*A*B + (b*c*f^2*h*n*lo
g(e) - a*d*f^2*h*n*log(e))*B^2 + ((b*c*g^2*h*n - a*d*g^2*h*n)*A*B + (b*c*g
^2*h*n*log(e) - a*d*g^2*h*n*log(e))*B^2)*x^2 + 2*((b*c*f*g*h*n - a*d*f*g*h
*n)*A*B + (b*c*f*g*h*n*log(e) - a*d*f*g*h*n*log(e))*B^2)*x + ((b*c*g^2*h*n
 - a*d*g^2*h*n)*B^2*x^2 + 2*(b*c*f*g*h*n - a*d*f*g*h*n)*B^2*x + (b*c*f^2*h
*n - a*d*f^2*h*n)*B^2)*log((b*x + a)^n) - ((b*c*g^2*h*n - a*d*g^2*h*n)*B^2
*x^2 + 2*(b*c*f*g*h*n - a*d*f*g*h*n)*B^2*x + (b*c*f^2*h*n - a*d*f^2*h*n)*B
^2)*log((d*x + c)^n)), x) - (d*x + c)/((b*c*f*h*n - a*d*f*h*n)*A*B + (b*c*
f*h*n*log(e) - a*d*f*h*n*log(e))*B^2 + ((b*c*g*h*n - a*d*g*h*n)*A*B + (b*c
*g*h*n*log(e) - a*d*g*h*n*log(e))*B^2)*x + ((b*c*g*h*n - a*d*g*h*n)*B^2*x
+ (b*c*f*h*n - a*d*f*h*n)*B^2)*log((b*x + a)^n) - ((b*c*g*h*n - a*d*g*h*n)
*B^2*x + (b*c*f*h*n - a*d*f*h*n)*B^2)*log((d*x + c)^n))
```

### 3.254.8 Giac [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(bhx+ah)(gx+f)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(g*x+f)/(b*h*x+a*h)/(A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2,x, al
gorithm="giac")
```

output
```
integrate(1/((b*h*x + a*h)*(g*x + f)*(B*log((b*x + a)^n*e/(d*x + c)^n) + A
)^2), x)
```

### 3.254.9 Mupad [N/A]

Not integrable

Time = 1.29 (sec) , antiderivative size = 45, normalized size of antiderivative = 1.05

$$\int \frac{1}{(f+gx)(ah+bhx)\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(f+g\,x)\,(a\,h+b\,h\,x)\,{\left(A+B\,\ln\left(\frac{e\,{(a+b\,x)}^n}{{(c+d\,x)}^n}\right)\right)}^2}\,dx$$

input
```
int(1/((f + g*x)*(a*h + b*h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2)
,x)
```

output
```
int(1/((f + g*x)*(a*h + b*h*x)*(A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2)
, x)
```

3.254. $\int \frac{1}{(f+gx)(ah+bhx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}\,dx$

# 3.255 $\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx$



## 3.255.1 Optimal result

Integrand size = 40, antiderivative size = 33

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = -\frac{\operatorname{PolyLog}\left(2,1-\frac{c+dx}{a+bx}\right)}{(bc-ad)h}$$

output
```
-polylog(2,1+(-d*x-c)/(b*x+a))/(-a*d+b*c)/h
```

## 3.255.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 298 vs. 2(33) = 66.

Time = 0.13 (sec) , antiderivative size = 298, normalized size of antiderivative = 9.03

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx$$
$$= \frac{-\log^2\left(\frac{-bc+ad}{d(a+bx)}\right)+2\log\left(\frac{(b-d)(a+bx)}{bc-ad}\right)\log(a-c+bx-dx)-2\log\left(\frac{-bc+ad}{d(a+bx)}\right)\log\left(\frac{b(c+dx)}{bc-ad}\right)-2\log(a-c+}{}$$

input
```
Integrate[Log[(c + d*x)/(a + b*x)]/((a + b*x)*((a - c)*h + (b - d)*h*x)),x
]
```

output
```
(-Log[(-(b*c) + a*d)/(d*(a + b*x))]^2 + 2*Log[((b - d)*(a + b*x))/(b*c - a
*d)]*Log[a - c + b*x - d*x] - 2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(b*(
c + d*x))/(b*c - a*d)] - 2*Log[a - c + b*x - d*x]*Log[((b - d)*(c + d*x))/
(b*c - a*d)] + 2*Log[(-(b*c) + a*d)/(d*(a + b*x))]*Log[(c + d*x)/(a + b*x)
] + 2*Log[a - c + b*x - d*x]*Log[(c + d*x)/(a + b*x)] + 2*PolyLog[2, (d*(a
 + b*x))/(-(b*c) + a*d)] + 2*PolyLog[2, -((b*(a - c + b*x - d*x))/(b*c - a
*d))] - 2*PolyLog[2, -((d*(-a + c - b*x + d*x))/(-(b*c) + a*d))])/((2*b*c
- 2*a*d)*h)
```

### 3.255.3 Rubi [A] (verified)

Time = 0.29 (sec) , antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, $\frac{\text{number of rules}}{\text{integrand size}} = 0.100$, Rules used = {2965, 25, 27, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)(h(a-c)+hx(b-d))}dx$$

$$\Big\downarrow \text{2965}$$

$$\int -\frac{\log\left(\frac{c+dx}{a+bx}\right)}{h\left(-\frac{(c+dx)(bc-ad)}{a+bx}-ad+bc\right)}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \text{25}$$

$$-\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{h\left(bc-ad-\frac{(bc-ad)(c+dx)}{a+bx}\right)}d\frac{c+dx}{a+bx}$$

$$\Big\downarrow \text{27}$$

$$-\frac{\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{bc-ad-\frac{(bc-ad)(c+dx)}{a+bx}}d\frac{c+dx}{a+bx}}{h}$$

$$\Big\downarrow \text{2752}$$

$$-\frac{\text{PolyLog}\left(2,1-\frac{c+dx}{a+bx}\right)}{h(bc-ad)}$$

input
```
Int[Log[(c + d*x)/(a + b*x)]/((a + b*x)*((a - c)*h + (b - d)*h*x)),x]
```

output
```
-(PolyLog[2, 1 - (c + d*x)/(a + b*x)]/((b*c - a*d)*h))
```

### 3.255.3.1 Defintions of rubi rules used

rule 25
```
Int[-(Fx_), x_Symbol] :> Simp[Identity[-1]   Int[Fx, x], x]
```

rule 27
```
Int[(a_)*(Fx_), x_Symbol] :> Simp[a   Int[Fx, x], x] /; FreeQ[a, x] &&  !Ma
tchQ[Fx, (b_)*(Gx_) /; FreeQ[b, x]]
```

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2965
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(q + 1)*(i/d)^q   Subst[Int[(b*f - a*g - (d*f - c*g)*
x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*
x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
 0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

## 3.255.4 Maple [A] (verified)

Time = 1.34 (sec) , antiderivative size = 42, normalized size of antiderivative = 1.27

| method | result |
|---|---|
| derivativedivides | $\frac{\operatorname{dilog}\left(-\frac{ad-cb}{b(bx+a)}+\frac{d}{b}\right)}{h(ad-cb)}$ |
| default | $\frac{\operatorname{dilog}\left(-\frac{ad-cb}{b(bx+a)}+\frac{d}{b}\right)}{h(ad-cb)}$ |
| risch | $\frac{\operatorname{dilog}\left(-\frac{ad-cb}{b(bx+a)}+\frac{d}{b}\right)}{h(ad-cb)}$ |
| parts | $-\frac{\ln\left(\frac{dx+c}{bx+a}\right)\ln(bx-dx+a-c)b}{h(ad-cb)(b-d)}+\frac{\ln\left(\frac{dx+c}{bx+a}\right)\ln(bx-dx+a-c)d}{h(ad-cb)(b-d)}+\frac{\ln\left(\frac{dx+c}{bx+a}\right)\ln(bx+a)}{h(ad-cb)}-\frac{-\frac{\ln(bx+a)^2}{2(ad-cb)}+\frac{\operatorname{dilog}\left(\frac{-ad+c}{-}\right.}{ad-}}{}$ |

input
```
int(ln((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x,method=_RETURNVERBOS
E)
```

output
```
1/h/(a*d-b*c)*dilog(-(a*d-b*c)/b/(b*x+a)+d/b)
```

### 3.255.5 Fricas [A] (verification not implemented)

Time = 0.32 (sec) , antiderivative size = 32, normalized size of antiderivative = 0.97

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = -\frac{\operatorname{Li}_2\left(-\frac{dx+c}{bx+a}+1\right)}{(bc-ad)h}$$

input
```
integrate(log((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x, algorithm="f
ricas")
```

output
```
-dilog(-(d*x + c)/(b*x + a) + 1)/((b*c - a*d)*h)
```

### 3.255.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = \text{Timed out}$$

input
```
integrate(ln((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x)
```

output
```
Timed out
```

### 3.255.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 357 vs. $2(32) = 64$.

Time = 0.20 (sec) , antiderivative size = 357, normalized size of antiderivative = 10.82

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx$$
$$= \left(\frac{\log\left(-(b-d)x-a+c\right)}{(bc-ad)h} - \frac{\log\left(bx+a\right)}{(bc-ad)h}\right)\log\left(\frac{dx+c}{bx+a}\right)$$
$$+ \frac{2\log\left(-(b-d)x-a+c\right)\log\left(bx+a\right) - \log\left(bx+a\right)^2}{2\left(bch-adh\right)}$$
$$+ \frac{\log\left(bx+a\right)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \text{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{bch-adh}$$
$$- \frac{\log\left(bx+a\right)\log\left(-\frac{a(b-d)+(b^2-bd)x}{bc-ad}+1\right) + \text{Li}_2\left(\frac{a(b-d)+(b^2-bd)x}{bc-ad}\right)}{bch-adh}$$
$$- \frac{\log\left(-(b-d)x-a+c\right)\log\left(\frac{ad-cd+(bd-d^2)x}{bc-ad}+1\right) + \text{Li}_2\left(-\frac{ad-cd+(bd-d^2)x}{bc-ad}\right)}{bch-adh}$$

input
```
integrate(log((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x, algorithm="m
axima")
```

output
```
(log(-(b - d)*x - a + c)/((b*c - a*d)*h) - log(b*x + a)/((b*c - a*d)*h))*l
og((d*x + c)/(b*x + a)) + 1/2*(2*log(-(b - d)*x - a + c)*log(b*x + a) - lo
g(b*x + a)^2)/(b*c*h - a*d*h) + (log(b*x + a)*log((b*d*x + a*d)/(b*c - a*d
) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))/(b*c*h - a*d*h) - (log(b*x + a
)*log(-(a*(b - d) + (b^2 - b*d)*x)/(b*c - a*d) + 1) + dilog((a*(b - d) + (
b^2 - b*d)*x)/(b*c - a*d)))/(b*c*h - a*d*h) - (log(-(b - d)*x - a + c)*log
((a*d - c*d + (b*d - d^2)*x)/(b*c - a*d) + 1) + dilog(-(a*d - c*d + (b*d -
 d^2)*x)/(b*c - a*d)))/(b*c*h - a*d*h)
```

### 3.255.8 Giac [F]

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx = \int \frac{\log\left(\frac{dx+c}{bx+a}\right)}{((b-d)hx+(a-c)h)(bx+a)}\,dx$$

input
```
integrate(log((d*x+c)/(b*x+a))/(b*x+a)/((a-c)*h+(b-d)*h*x),x, algorithm="g
iac")
```

output
```
integrate(log((d*x + c)/(b*x + a))/(((b - d)*h*x + (a - c)*h)*(b*x + a)),
x)
```

### 3.255.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{c+dx}{a+bx}\right)}{(a+bx)((a-c)h+(b-d)hx)}\,dx=\int \frac{\ln\left(\frac{c+d\,x}{a+b\,x}\right)}{\left(h\,\left(a-c\right)+h\,x\,\left(b-d\right)\right)\,\left(a+b\,x\right)}\,dx$$

input
```
int(log((c + d*x)/(a + b*x))/((h*(a - c) + h*x*(b - d))*(a + b*x)),x)
```

output
```
int(log((c + d*x)/(a + b*x))/((h*(a - c) + h*x*(b - d))*(a + b*x)), x)
```

# 3.256 $\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$



## 3.256.1 Optimal result

Integrand size = 38, antiderivative size = 27

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\frac{\operatorname{PolyLog}\left(2,\frac{g(c+dx)}{a+bx}\right)}{bc-ad}$$

output
```
polylog(2,g*(d*x+c)/(b*x+a))/(-a*d+b*c)
```

## 3.256.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 375 vs. 2(27) = 54.

Time = 0.22 (sec) , antiderivative size = 375, normalized size of antiderivative = 13.89

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$$
$$=\frac{-\log^2\left(\frac{a}{b}+x\right)+2\log\left(\frac{a}{b}+x\right)\log(a+bx)-2\log\left(\frac{a-cg}{b-dg}+x\right)\log(a+bx)+2\log\left(\frac{a-cg}{b-dg}+x\right)\log\left(\frac{(b-dg)(a}{(bc-ad}\right.}{}$$

input
```
Integrate[Log[(a - c*g + (b - d*g)*x)/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

output
```
(-Log[a/b + x]^2 + 2*Log[a/b + x]*Log[a + b*x] - 2*Log[(a - c*g)/(b - d*g)
 + x]*Log[a + b*x] + 2*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(a + b*
x))/((b*c - a*d)*g)] - 2*Log[a/b + x]*Log[c + d*x] + 2*Log[(a - c*g)/(b -
d*g) + x]*Log[c + d*x] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2
*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(c + d*x))/(b*c - a*d)] + 2*L
og[a + b*x]*Log[(a - c*g + b*x - d*g*x)/(a + b*x)] - 2*Log[c + d*x]*Log[(a
 - c*g + b*x - d*g*x)/(a + b*x)] + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)] + 2*PolyLog[2, -((b*(a - c*g + b*x - d*g*x))/((b*c - a*d)*g))] - 2*Pol
yLog[2, -((d*(-a + c*g - b*x + d*g*x))/(-(b*c) + a*d))])/(2*b*c - 2*a*d)
```

### 3.256.3 Rubi [A] (verified)

Time = 0.26 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41, number of steps used = 3, number of rules used = 2, $\frac{\text{number of rules}}{\text{integrand size}} = 0.053$, Rules used = {2965, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{a+x(b-dg)-cg}{a+bx}\right)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{2965}$$

$$\int \frac{\log\left(\frac{a+x(b-dg)-cg}{a+bx}\right)}{-\frac{(bc-ad)(a+x(b-dg)-cg)}{a+bx}-ad+bc}d\frac{a+x(b-dg)-cg}{a+bx}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\text{PolyLog}\left(2,1-\frac{a-cg+(b-dg)x}{a+bx}\right)}{bc-ad}$$

input
```
Int[Log[(a - c*g + (b - d*g)*x)/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

output
```
PolyLog[2, 1 - (a - c*g + (b - d*g)*x)/(a + b*x)]/(b*c - a*d)
```

### 3.256.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2965
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(q + 1)*(i/d)^q   Subst[Int[(b*f - a*g - (d*f - c*g)*
x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*
x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
 0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

## 3.256.4 Maple [A] (verified)

Time = 1.90 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

| method | result |
|---|---|
| derivativedivides | $-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$ |
| default | $-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$ |
| risch | $-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$ |
| parts | $\frac{\ln\left(\frac{a-cg+(-dg+b)x}{bx+a}\right)\ln(dx+c)}{ad-cb}-\frac{\ln\left(\frac{a-cg+(-dg+b)x}{bx+a}\right)\ln(bx+a)}{ad-cb}-\frac{g\left(\frac{b\ln(bx+a)^2}{2g(ad-cb)}-\frac{b(-dg+b)\left(\frac{\operatorname{dilog}\left(\frac{(-dg+b)(bx+a)+\ldots}{adg-bcg}\right)}{-dg+b}\ldots\right)}{\ldots}\right)}{\ldots}$ |

input
```
int(ln((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE
)
```

output
```
-1/(a*d-b*c)*dilog((a*d*g-b*c*g)/b/(b*x+a)+(-d*g+b)/b)
```

### 3.256.5 Fricas [A] (verification not implemented)

Time = 0.31 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\operatorname{Li}_2\left(\frac{cg+(dg-b)x-a}{bx+a}+1\right)}{bc-ad}$$

input
```
integrate(log((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="fr
icas")
```

output
```
dilog((c*g + (d*g - b)*x - a)/(b*x + a) + 1)/(b*c - a*d)
```

### 3.256.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(ln((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

### 3.256.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 344 vs. 2(26) = 52.

Time = 0.20 (sec) , antiderivative size = 344, normalized size of antiderivative = 12.74

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$$
$$= \left(\frac{\log(bx+a)}{bc-ad} - \frac{\log(dx+c)}{bc-ad}\right)\log\left(-\frac{cg+(dg-b)x-a}{bx+a}\right) + \frac{\log(bx+a)^2 - 2\log(bx+a)\log(dx+c)}{2(bc-ad)} - \frac{\log(bx+a)\log\left(\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}+1\right) + \mathrm{Li}_2\left(-\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}\right)}{bc-ad} + \frac{\log(dx+c)\log\left(\frac{cdg-bc+(d^2g-bd)x}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{cdg-bc+(d^2g-bd)x}{bc-ad}\right)}{bc-ad} + \frac{\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right) + \mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{bc-ad}$$

input
```
integrate(log((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="ma
xima")
```

output
```
(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log(-(c*g + (d*g - b
)*x - a)/(b*x + a)) + 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c))/(
b*c - a*d) - (log(b*x + a)*log(((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*
d*g) + 1) + dilog(-((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g)))/(b*c
- a*d) + (log(d*x + c)*log((c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d) + 1
) + dilog(-(c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d)))/(b*c - a*d) + (lo
g(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))/(b*c - a*d)
```

### 3.256.8 Giac [F]

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\log\left(-\frac{cg+(dg-b)x-a}{bx+a}\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log((a-c*g+(-d*g+b)*x)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="gi
ac")
```

output
```
integrate(log(-(c*g + (d*g - b)*x - a)/(b*x + a))/((b*x + a)*(d*x + c)), x
)
```

### 3.256.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a-cg+(b-dg)x}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\ln\left(\frac{a-c\,g+x\,(b-d\,g)}{a+b\,x}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(log((a - c*g + x*(b - d*g))/(a + b*x))/((a + b*x)*(c + d*x)),x)
```

output
```
int(log((a - c*g + x*(b - d*g))/(a + b*x))/((a + b*x)*(c + d*x)), x)
```

# 3.257 $\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$



## 3.257.1 Optimal result

Integrand size = 33, antiderivative size = 27

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\frac{\operatorname{PolyLog}\left(2,\frac{g(c+dx)}{a+bx}\right)}{bc-ad}$$

output
```
polylog(2,g*(d*x+c)/(b*x+a))/(-a*d+b*c)
```

## 3.257.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 375 vs. 2(27) = 54.

Time = 0.17 (sec) , antiderivative size = 375, normalized size of antiderivative = 13.89

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$$
$$=\frac{-\log^2\left(\frac{a}{b}+x\right)+2\log\left(\frac{a}{b}+x\right)\log(a+bx)-2\log\left(\frac{a-cg}{b-dg}+x\right)\log(a+bx)+2\log\left(\frac{a-cg}{b-dg}+x\right)\log\left(\frac{(b-dg)(a}{(bc-ad}\right.}{}$$

input
```
Integrate[Log[1 - (g*(c + d*x))/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

output
```
(-Log[a/b + x]^2 + 2*Log[a/b + x]*Log[a + b*x] - 2*Log[(a - c*g)/(b - d*g)
 + x]*Log[a + b*x] + 2*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(a + b*
x))/((b*c - a*d)*g)] - 2*Log[a/b + x]*Log[c + d*x] + 2*Log[(a - c*g)/(b -
d*g) + x]*Log[c + d*x] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2
*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(c + d*x))/(b*c - a*d)] + 2*L
og[a + b*x]*Log[(a - c*g + b*x - d*g*x)/(a + b*x)] - 2*Log[c + d*x]*Log[(a
 - c*g + b*x - d*g*x)/(a + b*x)] + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)] + 2*PolyLog[2, -((b*(a - c*g + b*x - d*g*x))/((b*c - a*d)*g))] - 2*Pol
yLog[2, -((d*(-a + c*g - b*x + d*g*x))/(-(b*c) + a*d))])/(2*b*c - 2*a*d)
```

## 3.257.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {2997, 2965, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log \left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)} \, dx$$

$$\Big\downarrow \; 2997$$

$$\int \frac{\log \left(\frac{a+x(b-dg)-cg}{a+bx}\right)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \; 2965$$

$$\int \frac{\log \left(\frac{a+x(b-dg)-cg}{a+bx}\right)}{-\frac{(bc-ad)(a+x(b-dg)-cg)}{a+bx}-ad+bc}d\frac{a+x(b-dg)-cg}{a+bx}$$

$$\Big\downarrow \; 2752$$

$$\frac{\text{PolyLog}\left(2,1-\frac{a-cg+(b-dg)x}{a+bx}\right)}{bc-ad}$$

input
```
Int[Log[1 - (g*(c + d*x))/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

output
```
PolyLog[2, 1 - (a - c*g + (b - d*g)*x)/(a + b*x)]/(b*c - a*d)
```

---

3.257. $\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)} \, dx$

### 3.257.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2965
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(q + 1)*(i/d)^q   Subst[Int[(b*f - a*g - (d*f - c*g)*
x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*
x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
 0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

rule 2997
```
Int[Log[(e_.)*((f_.)*((g_) + (v_.)/(w_)))^(r_.)]^(s_.)*(u_.), x_Symbol] :>
Int[u*Log[e*(f*(ExpandToSum[v + g*w, x]/ExpandToSum[w, x]))^r]^s, x] /; Fre
eQ[{e, f, g, r, s}, x] && LinearQ[w, x] && (FreeQ[v, x] || LinearQ[v, x]) &
& AlgebraicFunctionQ[u, x]
```

## 3.257.4 Maple [A] (verified)

Time = 1.82 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

| method | result |
|---|---|
| derivativedivides | $-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$ |
| default | $-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$ |
| risch | $-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$ |
| parts | $\frac{\ln\left(1-\frac{g(dx+c)}{bx+a}\right)\ln(dx+c)}{ad-cb}-\frac{\ln\left(1-\frac{g(dx+c)}{bx+a}\right)\ln(bx+a)}{ad-cb}-\frac{g\Big(\frac{b\ln(bx+a)^2}{2g(ad-cb)}-\frac{b(-dg+b)\Big(\frac{\operatorname{dilog}\left(\frac{(-dg+b)(bx+a)+adg-bcg}{adg-bcg}\right)}{-dg+b}\cdots}{g(ad-\cdots}}{b}$ |

input
```
int(ln(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE)
```

output
```
-1/(a*d-b*c)*dilog((a*d*g-b*c*g)/b/(b*x+a)+(-d*g+b)/b)
```

3.257. $\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$

### 3.257.5 Fricas [A] (verification not implemented)

Time = 0.28 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\mathrm{Li}_2\left(\frac{cg+(dg-b)x-a}{bx+a}+1\right)}{bc-ad}$$

input
```
integrate(log(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="fricas")
```

output
```
dilog((c*g + (d*g - b)*x - a)/(b*x + a) + 1)/(b*c - a*d)
```

### 3.257.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(ln(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

### 3.257.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(26) = 52.

Time = 0.20 (sec) , antiderivative size = 336, normalized size of antiderivative = 12.44

$$\begin{aligned}\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx &= \left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)\log\left(-\frac{(dx+c)g}{bx+a}+1\right)\\
&+\frac{\log(bx+a)^2-2\log(bx+a)\log(dx+c)}{2(bc-ad)}\\
&-\frac{\log(bx+a)\log\left(\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}+1\right)+\mathrm{Li}_2\left(-\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}\right)}{bc-ad}\\
&+\frac{\log(dx+c)\log\left(\frac{cdg-bc+(d^2g-bd)x}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{cdg-bc+(d^2g-bd)x}{bc-ad}\right)}{bc-ad}\\
&+\frac{\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{bc-ad}\end{aligned}$$

input
```
integrate(log(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="maxima")
```

output
```
(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log(-(d*x + c)*g/(b*
x + a) + 1) + 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c))/(b*c - a*
d) - (log(b*x + a)*log(((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g) + 1
) + dilog(-((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g)))/(b*c - a*d) +
 (log(d*x + c)*log((c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d) + 1) + dilo
g(-(c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d)))/(b*c - a*d) + (log(b*x +
a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c - a*d)))
/(b*c - a*d)
```

### 3.257.8 Giac [F]

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\log\left(-\frac{(dx+c)g}{bx+a}+1\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log(1-g*(d*x+c)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="giac")
```

output
```
integrate(log(-(d*x + c)*g/(b*x + a) + 1)/((b*x + a)*(d*x + c)), x)
```

### 3.257.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(1-\frac{g(c+dx)}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\int \frac{\ln\left(1-\frac{g\,(c+d\,x)}{a+b\,x}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(log(1 - (g*(c + d*x))/(a + b*x))/((a + b*x)*(c + d*x)),x)
```

output
```
int(log(1 - (g*(c + d*x))/(a + b*x))/((a + b*x)*(c + d*x)), x)
```

# 3.258 $\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$



## 3.258.1 Optimal result

Integrand size = 38, antiderivative size = 27

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx=\frac{\operatorname{PolyLog}\left(2,\frac{g(c+dx)}{a+bx}\right)}{bc-ad}$$

output
```
polylog(2,g*(d*x+c)/(b*x+a))/(-a*d+b*c)
```

## 3.258.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 375 vs. 2(27) = 54.

Time = 0.01 (sec) , antiderivative size = 375, normalized size of antiderivative = 13.89

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{-\log^2\left(\frac{a}{b}+x\right)+2\log\left(\frac{a}{b}+x\right)\log(a+bx)-2\log\left(\frac{a-cg}{b-dg}+x\right)\log(a+bx)+2\log\left(\frac{a-cg}{b-dg}+x\right)\log\left(\frac{(b-dg)(a}{(bc-ad}\right.}{}$$

input
```
Integrate[Log[(a - c*g + b*x - d*g*x)/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

output
```
(-Log[a/b + x]^2 + 2*Log[a/b + x]*Log[a + b*x] - 2*Log[(a - c*g)/(b - d*g)
 + x]*Log[a + b*x] + 2*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(a + b*
x))/((b*c - a*d)*g)] - 2*Log[a/b + x]*Log[c + d*x] + 2*Log[(a - c*g)/(b -
d*g) + x]*Log[c + d*x] + 2*Log[a/b + x]*Log[(b*(c + d*x))/(b*c - a*d)] - 2
*Log[(a - c*g)/(b - d*g) + x]*Log[((b - d*g)*(c + d*x))/(b*c - a*d)] + 2*L
og[a + b*x]*Log[(a - c*g + b*x - d*g*x)/(a + b*x)] - 2*Log[c + d*x]*Log[(a
 - c*g + b*x - d*g*x)/(a + b*x)] + 2*PolyLog[2, (d*(a + b*x))/(-(b*c) + a*
d)] + 2*PolyLog[2, -((b*(a - c*g + b*x - d*g*x))/((b*c - a*d)*g))] - 2*Pol
yLog[2, -((d*(-a + c*g - b*x + d*g*x))/(-(b*c) + a*d))])/(2*b*c - 2*a*d)
```

## 3.258.3 Rubi [A] (verified)

Time = 0.32 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41, number of steps used = 4, number of rules used = 3, $\frac{\text{number of rules}}{\text{integrand size}} = 0.079$, Rules used = {2971, 2965, 2752}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{\log\left(\frac{a+bx-cg-dgx}{a+bx}\right)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{2971}$$

$$\int \frac{\log\left(\frac{a+x(b-dg)-cg}{a+bx}\right)}{(a+bx)(c+dx)}dx$$

$$\Big\downarrow \text{2965}$$

$$\int \frac{\log\left(\frac{a+x(b-dg)-cg}{a+bx}\right)}{-\frac{(bc-ad)(a+x(b-dg)-cg)}{a+bx}-ad+bc}d\frac{a+x(b-dg)-cg}{a+bx}$$

$$\Big\downarrow \text{2752}$$

$$\frac{\text{PolyLog}\left(2,1-\frac{a-cg+(b-dg)x}{a+bx}\right)}{bc-ad}$$

input
```
Int[Log[(a - c*g + b*x - d*g*x)/(a + b*x)]/((a + b*x)*(c + d*x)),x]
```

output
```
PolyLog[2, 1 - (a - c*g + (b - d*g)*x)/(a + b*x)]/(b*c - a*d)
```

---

3.258. $\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}dx$

### 3.258.3.1 Defintions of rubi rules used

rule 2752
```
Int[Log[(c_.)*(x_)]/((d_) + (e_.)*(x_)), x_Symbol] :> Simp[(-e^(-1))*PolyLo
g[2, 1 - c*x], x] /; FreeQ[{c, d, e}, x] && EqQ[e + c*d, 0]
```

rule 2965
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*((f_.) + (g_.)*(x_))^(m_.)*((h_.) + (i_.)*(x_))^(q_.), x_Symbol
] :> Simp[(b*c - a*d)^(q + 1)*(i/d)^q   Subst[Int[(b*f - a*g - (d*f - c*g)*
x)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(m + q + 2)), x], x, (a + b*x)/(c + d*
x)], x] /; FreeQ[{a, b, c, d, e, f, g, h, i, A, B, n}, x] && NeQ[b*c - a*d,
 0] && IntegersQ[m, q] && IGtQ[p, 0] && EqQ[d*h - c*i, 0]
```

rule 2971
```
Int[((A_.) + Log[(e_.)*((u_)/(v_))^(n_.)]*(B_.))^(p_.)*(w_)^(m_.)*(y_)^(q_.
), x_Symbol] :> Int[ExpandToSum[w, x]^m*ExpandToSum[y, x]^q*(A + B*Log[e*(E
xpandToSum[u, x]/ExpandToSum[v, x])^n])^p, x] /; FreeQ[{e, A, B, m, n, p, q
}, x] && LinearQ[{u, v, w, y}, x] &&  !LinearMatchQ[{u, v, w, y}, x]
```

## 3.258.4 Maple [A] (verified)

Time = 1.86 (sec) , antiderivative size = 46, normalized size of antiderivative = 1.70

| method | result |
|---|---|
| derivativedivides | $-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$ |
| default | $-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$ |
| risch | $-\frac{\operatorname{dilog}\left(\frac{adg-bcg}{b(bx+a)}+\frac{-dg+b}{b}\right)}{ad-cb}$ |
| parts | $\frac{\ln\left(\frac{-dgx+bx-cg+a}{bx+a}\right)\ln(dx+c)}{ad-cb}-\frac{\ln\left(\frac{-dgx+bx-cg+a}{bx+a}\right)\ln(bx+a)}{ad-cb}-\frac{g\left(\frac{b\ln(bx+a)^2}{2g(ad-cb)}-\frac{b(-dg+b)\left(\frac{\operatorname{dilog}\left(\frac{(-dg+b)(bx+a)+a}{adg-bcg}\right.}{-dg+b}\right.}{}\right.}{}$ |

input
```
int(ln((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x,method=_RETURNVERBOSE
)
```

output
```
-1/(a*d-b*c)*dilog((a*d*g-b*c*g)/b/(b*x+a)+(-d*g+b)/b)
```

3.258. $\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx$

### 3.258.5 Fricas [A] (verification not implemented)

Time = 0.29 (sec) , antiderivative size = 38, normalized size of antiderivative = 1.41

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \frac{\mathrm{Li}_2\left(\frac{cg+(dg-b)x-a}{bx+a}+1\right)}{bc-ad}$$

input
```
integrate(log((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="fr
icas")
```

output
```
dilog((c*g + (d*g - b)*x - a)/(b*x + a) + 1)/(b*c - a*d)
```

### 3.258.6 Sympy [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \text{Timed out}$$

input
```
integrate(ln((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x)
```

output
```
Timed out
```

### 3.258.7 Maxima [B] (verification not implemented)

Leaf count of result is larger than twice the leaf count of optimal. 343 vs. 2(26) = 52.

Time = 0.20 (sec) , antiderivative size = 343, normalized size of antiderivative = 12.70

$$\begin{aligned}\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx &= \left(\frac{\log(bx+a)}{bc-ad}-\frac{\log(dx+c)}{bc-ad}\right)\log\left(-\frac{dgx+cg-bx-a}{bx+a}\right)\\ &\quad+\frac{\log(bx+a)^2-2\log(bx+a)\log(dx+c)}{2(bc-ad)}\\ &\quad-\frac{\log(bx+a)\log\left(\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}+1\right)+\mathrm{Li}_2\left(-\frac{(dg-b)a+(bdg-b^2)x}{bcg-adg}\right)}{bc-ad}\\ &\quad+\frac{\log(dx+c)\log\left(\frac{cdg-bc+(d^2g-bd)x}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{cdg-bc+(d^2g-bd)x}{bc-ad}\right)}{bc-ad}\\ &\quad+\frac{\log(bx+a)\log\left(\frac{bdx+ad}{bc-ad}+1\right)+\mathrm{Li}_2\left(-\frac{bdx+ad}{bc-ad}\right)}{bc-ad}\end{aligned}$$

input
```
integrate(log((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="ma
xima")
```

output
```
(log(b*x + a)/(b*c - a*d) - log(d*x + c)/(b*c - a*d))*log(-(d*g*x + c*g -
b*x - a)/(b*x + a)) + 1/2*(log(b*x + a)^2 - 2*log(b*x + a)*log(d*x + c))/(
b*c - a*d) - (log(b*x + a)*log(((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*
d*g) + 1) + dilog(-((d*g - b)*a + (b*d*g - b^2)*x)/(b*c*g - a*d*g)))/(b*c
- a*d) + (log(d*x + c)*log((c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d) + 1
) + dilog(-(c*d*g - b*c + (d^2*g - b*d)*x)/(b*c - a*d)))/(b*c - a*d) + (lo
g(b*x + a)*log((b*d*x + a*d)/(b*c - a*d) + 1) + dilog(-(b*d*x + a*d)/(b*c
- a*d)))/(b*c - a*d)
```

### 3.258.8 Giac [F]

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\log\left(-\frac{dgx+cg-bx-a}{bx+a}\right)}{(bx+a)(dx+c)}\,dx$$

input
```
integrate(log((-d*g*x+b*x-c*g+a)/(b*x+a))/(b*x+a)/(d*x+c),x, algorithm="gi
ac")
```

output
```
integrate(log(-(d*g*x + c*g - b*x - a)/(b*x + a))/((b*x + a)*(d*x + c)), x
)
```

### 3.258.9 Mupad [F(-1)]

Timed out.

$$\int \frac{\log\left(\frac{a-cg+bx-dgx}{a+bx}\right)}{(a+bx)(c+dx)}\,dx = \int \frac{\ln\left(\frac{a-c\,g+b\,x-d\,g\,x}{a+b\,x}\right)}{(a+b\,x)\,(c+d\,x)}\,dx$$

input
```
int(log((a - c*g + b*x - d*g*x)/(a + b*x))/((a + b*x)*(c + d*x)),x)
```

output
```
int(log((a - c*g + b*x - d*g*x)/(a + b*x))/((a + b*x)*(c + d*x)), x)
```

# 3.259 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{afh+bghx^2+h(bfx+agx)}\,dx$



## 3.259.1 Optimal result

Integrand size = 51, antiderivative size = 282

$$\begin{aligned}\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx &= -\frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\\ &+\frac{3Bn(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\\ &+\frac{6B^2n^2(A+B\log(e(a+bx)^n(c+dx)^{-n}))\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\\ &+\frac{6B^3n^3\operatorname{PolyLog}\left(4,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}\end{aligned}$$

output
```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b
*x+a))/(-a*g+b*f)/h+3*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*polylog(2,(-
a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+6*B^2*n^2*(A+B*ln(e*(b*x
+a)^n/((d*x+c)^n)))*polylog(3,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g
+b*f)/h+6*B^3*n^3*polylog(4,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b
*f)/h
```

### 3.259.2 Mathematica [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx=\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a*f*h + b*g*h*x^2 +
h*(b*f*x + a*g*x)),x]
```

output
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a*f*h + b*g*h*x^2 +
h*(b*f*x + a*g*x)), x]
```

### 3.259.3 Rubi [A] (warning: unable to verify)

Time = 1.08 (sec) , antiderivative size = 279, normalized size of antiderivative = 0.99, number of steps used = 8, number of rules used = 7, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.137, Rules used = {2973, 2976, 2026, 2779, 2821, 2830, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{h(agx+bfx)+afh+bghx^2}\,dx$$

$$\Big\downarrow \ 2973$$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^3}{h(agx+bfx)+afh+bghx^2}dx$$

$$\Big\downarrow \ 2976$$

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{\frac{(bc-ad)(bf-ag)h(a+bx)}{c+dx}-\frac{(bc-ad)(df-cg)h(a+bx)^2}{(c+dx)^2}}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2026$$

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^3}{(a+bx)\left((bc-ad)(bf-ag)h-\frac{(bc-ad)(df-cg)h(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \ 2779$$

$$(bc-ad)\left(\frac{3Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{h(bc-ad)(bf-ag)}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+\right.}{h(bc-ad)(bf-ag)}\right.$$

$\Big\downarrow$ 2821

$$(bc-ad)\left(\frac{3Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\text{PolyLog}\left(2,\frac{(bf-}{(df-}\right.}{a+bx}\right.}{h(bc-ad)(bf-ag)}\right.$$

$\Big\downarrow$ 2830

$$(bc-ad)\left(\frac{3Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\left(Bn\int \frac{(c+dx)\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+b}{c+d}\right.\right.}{h(bc-ad)(bf-ag)}\right.$$

$\Big\downarrow$ 7143

$$(bc-ad)\left(\frac{3Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)^2-2Bn\left(-\left(\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(\right.\right.\right.\right.\right.}{h(bc-ad)(bf-ag)}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^3/(a*f*h + b*g*h*x^2 + h*(b*f
*x + a*g*x)),x]
```

output
```
(b*c - a*d)*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^3*Log[1 - ((b*f - a
*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*c - a*d)*(b*f - a*g)*h)) + (3
*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*PolyLog[2, ((b*f - a*g)*(c
+ d*x))/((d*f - c*g)*(a + b*x))] - 2*B*n*(-((A + B*Log[e*((a + b*x)/(c + d
*x))^n])*PolyLog[3, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))]) - B*
n*PolyLog[4, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])))/((b*c - a
*d)*(b*f - a*g)*h))
```

3.259. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{afh+bghx^2+h(bfx+agx)}\,dx$

### 3.259.3.1 Defintions of rubi rules used

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2830
```
Int[(((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)*PolyLog[k_, (e_.)*(x_)^(q_
.)])/(x_), x_Symbol] :> Simp[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^p/q)
, x] - Simp[b*n*(p/q)   Int[PolyLog[k + 1, e*x^q]*((a + b*Log[c*x^n])^(p -
1)/x), x], x] /; FreeQ[{a, b, c, e, k, n, q}, x] && GtQ[p, 0]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 2976
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

---

3.259. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^3}{afh+bghx^2+h(bfx+agx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.259.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{3}}{afh+bgh\,x^{2}+h\left(agx+bfx\right)}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),
x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),
x)
```

### 3.259.5 Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)^3}{bghx^2+afh+(bfx+agx)h}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b
*f*x)),x, algorithm="fricas")
```

output
```
integral((B^3*log((b*x + a)^n*e/(d*x + c)^n)^3 + 3*A*B^2*log((b*x + a)^n*e
/(d*x + c)^n)^2 + 3*A^2*B*log((b*x + a)^n*e/(d*x + c)^n) + A^3)/(b*g*h*x^2
 + a*f*h + (b*f + a*g)*h*x), x)
```

### 3.259.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx = \text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**3/(a*f*h+b*g*h*x**2+h*(a*g*
x+b*f*x)),x)
```

output
```
Timed out
```

### 3.259.7 Maxima [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{bghx^2+afh+(bfx+agx)h}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b
*f*x)),x, algorithm="maxima")
```

output
```
A^3*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) - integr
ate(-(B^3*log((b*x + a)^n)^3 - B^3*log((d*x + c)^n)^3 + B^3*log(e)^3 + 3*A
*B^2*log(e)^2 + 3*A^2*B*log(e) + 3*(B^3*log(e) + A*B^2)*log((b*x + a)^n)^2
 + 3*(B^3*log((b*x + a)^n) + B^3*log(e) + A*B^2)*log((d*x + c)^n)^2 + 3*(B
^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B)*log((b*x + a)^n) - 3*(B^3*log((b*x +
 a)^n)^2 + B^3*log(e)^2 + 2*A*B^2*log(e) + A^2*B + 2*(B^3*log(e) + A*B^2)*
log((b*x + a)^n))*log((d*x + c)^n))/(b*g*h*x^2 + a*f*h + (b*f*h + a*g*h)*x
), x)
```

### 3.259.8 Giac [F]

$$\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^3}{afh+bghx^2+h(bfx+agx)}\,dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^3}{bghx^2+afh+(bfx+agx)h}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^3/(a*f*h+b*g*h*x^2+h*(a*g*x+b
*f*x)),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^3/(b*g*h*x^2 + a*f*h + (b
*f*x + a*g*x)*h), x)
```

### 3.259.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log{(e(a+bx)^n(c+dx)^{-n})})^3}{afh + bghx^2 + h(bfx + agx)}\,dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,{(a+b\,x)}^n}{{(c+d\,x)}^n}\right)\right)^3}{h\,\left(a\,g\,x + b\,f\,x\right) + a\,f\,h + b\,g\,h\,x^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(h*(a*g*x + b*f*x) + a*f*h
+ b*g*h*x^2),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^3/(h*(a*g*x + b*f*x) + a*f*h
+ b*g*h*x^2), x)
```

# 3.260 $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{afh+bghx^2+h(bfx+agx)}\,dx$



## 3.260.1 Optimal result

Integrand size = 51, antiderivative size = 203

$$\begin{aligned}
&\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{afh+bghx^2+h(bfx+agx)}\,dx \\
&= -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{2Bn\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\operatorname{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} \\
&\quad+\frac{2B^2n^2\operatorname{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}
\end{aligned}$$

output
```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b
*x+a))/(-a*g+b*f)/h+2*B*n*(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*polylog(2,(-a*
g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h+2*B^2*n^2*polylog(3,(-a*g+
b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a*g+b*f)/h
```

### 3.260.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 1415 vs. $2(203) = 406$.

Time = 0.57 (sec) , antiderivative size = 1415, normalized size of antiderivative = 6.97

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{afh + bghx^2 + h(bfx + agx)}\,dx = \text{Too large to display}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a*f*h + b*g*h*x^2 +
h*(b*f*x + a*g*x)),x]
```

output
```
(3*Log[a + b*x]*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + Log[(e*(a + b
*x)^n)/(c + d*x)^n]))^2 - 3*(A + B*(-(n*Log[a + b*x]) + n*Log[c + d*x] + L
og[(e*(a + b*x)^n)/(c + d*x)^n]))^2*Log[f + g*x] + 3*B*n*(A + B*(-(n*Log[a
 + b*x]) + n*Log[c + d*x] + Log[(e*(a + b*x)^n)/(c + d*x)^n]))*(Log[a + b*
x]^2 - 2*(Log[a + b*x]*Log[(b*(f + g*x))/(b*f - a*g)] + PolyLog[2, (g*(a +
 b*x))/(-(b*f) + a*g)])) - 6*A*B*n*(Log[c + d*x]*(Log[(d*(a + b*x))/(-(b*c
) + a*d)] - Log[(d*(f + g*x))/(d*f - c*g)]) + PolyLog[2, (b*(c + d*x))/(b*
c - a*d)] - PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)]) + 6*B^2*n*(n*Log[a +
 b*x] - n*Log[c + d*x] - Log[(e*(a + b*x)^n)/(c + d*x)^n])*(Log[c + d*x]*(
Log[(d*(a + b*x))/(-(b*c) + a*d)] - Log[(d*(f + g*x))/(d*f - c*g)]) + Poly
Log[2, (b*(c + d*x))/(b*c - a*d)] - PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g
)]) + B^2*n^2*(Log[a + b*x]^2*(Log[a + b*x] - 3*Log[(b*(f + g*x))/(b*f - a
*g)]) - 6*Log[a + b*x]*PolyLog[2, (g*(a + b*x))/(-(b*f) + a*g)] + 6*PolyLo
g[3, (g*(a + b*x))/(-(b*f) + a*g)]) + 3*B^2*n^2*(Log[(d*(a + b*x))/(-(b*c)
 + a*d)]*Log[c + d*x]^2 - Log[c + d*x]^2*Log[(d*(f + g*x))/(d*f - c*g)] +
2*Log[c + d*x]*PolyLog[2, (b*(c + d*x))/(b*c - a*d)] - 2*Log[c + d*x]*Poly
Log[2, (g*(c + d*x))/(-(d*f) + c*g)] - 2*PolyLog[3, (b*(c + d*x))/(b*c - a
*d)] + 2*PolyLog[3, (g*(c + d*x))/(-(d*f) + c*g)]) - 6*B^2*n^2*((Log[a + b
*x]^2*(Log[c + d*x] - Log[(b*(c + d*x))/(b*c - a*d)]))/2 - Log[a + b*x]*Lo
g[c + d*x]*Log[(b*(f + g*x))/(b*f - a*g)] - (Log[(g*(c + d*x))/(-(d*f) ...
```

### 3.260.3 Rubi [A] (warning: unable to verify)

Time = 0.92 (sec) , antiderivative size = 214, normalized size of antiderivative = 1.05, number of steps used = 7, number of rules used = 6, $\frac{\text{number of rules}}{\text{integrand size}} = 0.118$, Rules used = {2973, 2976, 2026, 2779, 2821, 7143}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

3.260. $\int \frac{(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh+bghx^2+h(bfx+agx)}\,dx$

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{h(agx+bfx)+afh+bghx^2}\,dx$$

$\Big\downarrow$ 2973

$$\int \frac{(B\log(e(a+bx)^n(c+dx)^{-n})+A)^2}{h(agx+bfx)+afh+bghx^2}dx$$

$\Big\downarrow$ 2976

$$(bc-ad)\int \frac{\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{\frac{(bc-ad)(bf-ag)h(a+bx)}{c+dx}-\frac{(bc-ad)(df-cg)h(a+bx)^2}{(c+dx)^2}}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2026

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2}{(a+bx)\left((bc-ad)(bf-ag)h-\frac{(bc-ad)(df-cg)h(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$\Big\downarrow$ 2779

$$(bc-ad)\left(\frac{2Bn\int \frac{(c+dx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{h(bc-ad)(bf-ag)}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right.}{h(bc-ad)(bf-ag)}\right.$$

$\Big\downarrow$ 2821

$$(bc-ad)\left(\frac{2Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)-Bn\int \frac{(c+dx)\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}\right)}{h(bc-ad)(bf-ag)}-\frac{\mathrm{l}}{}\right.$$

$\Big\downarrow$ 7143

$$(bc-ad)\left(\frac{2Bn\left(\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)+Bn\,\text{PolyLog}\left(3,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)\right)}{h(bc-ad)(bf-ag)}-\frac{\log\left(1-\frac{(c}{(a}\right.}{}\right.$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])^2/(a*f*h + b*g*h*x^2 + h*(b*f
*x + a*g*x)),x]
```

3.260. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{afh+bghx^2+h(bfx+agx)}\,dx$

output
```
(b*c - a*d)*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])^2*Log[1 - ((b*f - a
*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*c - a*d)*(b*f - a*g)*h)) + (2
*B*n*((A + B*Log[e*((a + b*x)/(c + d*x))^n])*PolyLog[2, ((b*f - a*g)*(c +
d*x))/((d*f - c*g)*(a + b*x))] + B*n*PolyLog[3, ((b*f - a*g)*(c + d*x))/((
d*f - c*g)*(a + b*x))]))/((b*c - a*d)*(b*f - a*g)*h))
```

### 3.260.3.1 Defintions of rubi rules used

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] && !MonomialQ[Px, x] && (ILtQ[p, 0] || !PolyQ[u, x])
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2821
```
Int[(Log[(d_.)*((e_) + (f_.)*(x_)^(m_.))]*((a_.) + Log[(c_.)*(x_)^(n_.)]*(b
_.))^(p_.))/(x_), x_Symbol] :> Simp[(-PolyLog[2, (-d)*f*x^m])*((a + b*Log[c
*x^n])^p/m), x] + Simp[b*n*(p/m)   Int[PolyLog[2, (-d)*f*x^m]*((a + b*Log[c
*x^n])^(p - 1)/x), x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] && IGtQ[p,
0] && EqQ[d*e, 1]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] && !Intege
rQ[n]
```

rule 2976
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

3.260. $\int \frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}{afh+bghx^2+h(bfx+agx)}\,dx$

rule 7143
```
Int[PolyLog[n_, (c_.)*((a_.) + (b_.)*(x_))^(p_.)]/((d_.) + (e_.)*(x_)), x_S
ymbol] :> Simp[PolyLog[n + 1, c*(a + b*x)^p]/(e*p), x] /; FreeQ[{a, b, c, d
, e, n, p}, x] && EqQ[b*d, a*e]
```

### 3.260.4 Maple [F]

$$\int \frac{\left(A+B\ln\left(e(bx+a)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}{afh+bgh\,x^{2}+h\left(agx+bfx\right)}dx$$

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),
x)
```

output
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),
x)
```

### 3.260.5 Fricas [F]

$$\int \frac{(A+B\log(e(a+bx)^{n}(c+dx)^{-n}))^{2}}{afh+bghx^{2}+h(bfx+agx)}\,dx=\int \frac{\left(B\log\left(\frac{(bx+a)^{n}e}{(dx+c)^{n}}\right)+A\right)^{2}}{bghx^{2}+afh+(bfx+agx)h}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b
*f*x)),x, algorithm="fricas")
```

output
```
integral((B^2*log((b*x + a)^n*e/(d*x + c)^n)^2 + 2*A*B*log((b*x + a)^n*e/(
d*x + c)^n) + A^2)/(b*g*h*x^2 + a*f*h + (b*f + a*g)*h*x), x)
```

### 3.260.6 Sympy [F(-1)]

Timed out.

$$\int \frac{(A+B\log(e(a+bx)^{n}(c+dx)^{-n}))^{2}}{afh+bghx^{2}+h(bfx+agx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))**2/(a*f*h+b*g*h*x**2+h*(a*g*
x+b*f*x)),x)
```

output
```
Timed out
```

### 3.260.7 Maxima [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{afh + bghx^2 + h(bfx + agx)} dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{bghx^2 + afh + (bfx + agx)h} dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b
*f*x)),x, algorithm="maxima")
```

output
```
A^2*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) + integr
ate((B^2*log((b*x + a)^n)^2 + B^2*log((d*x + c)^n)^2 + B^2*log(e)^2 + 2*A*
B*log(e) + 2*(B^2*log(e) + A*B)*log((b*x + a)^n) - 2*(B^2*log((b*x + a)^n)
 + B^2*log(e) + A*B)*log((d*x + c)^n))/(b*g*h*x^2 + a*f*h + (b*f*h + a*g*h
)*x), x)
```

### 3.260.8 Giac [F]

$$\int \frac{(A + B\log(e(a + bx)^n(c + dx)^{-n}))^2}{afh + bghx^2 + h(bfx + agx)} dx = \int \frac{\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right) + A\right)^2}{bghx^2 + afh + (bfx + agx)h} dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))^2/(a*f*h+b*g*h*x^2+h*(a*g*x+b
*f*x)),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)^2/(b*g*h*x^2 + a*f*h + (b
*f*x + a*g*x)*h), x)
```

### 3.260.9 Mupad [F(-1)]

Timed out.

$$\int \frac{(A + B\log(e(a+bx)^n(c+dx)^{-n}))^2}{afh + bghx^2 + h(bfx + agx)}\,dx = \int \frac{\left(A + B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2}{h\,\left(a\,g\,x + b\,f\,x\right) + a\,f\,h + b\,g\,h\,x^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(h*(a*g*x + b*f*x) + a*f*h
+ b*g*h*x^2),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2/(h*(a*g*x + b*f*x) + a*f*h
+ b*g*h*x^2), x)
```

# 3.261 $\int \frac{A+B\log(e(a+bx)^n(c+dx)^{-n})}{afh+bghx^2+h(bfx+agx)}\,dx$



## 3.261.1 Optimal result

Integrand size = 49, antiderivative size = 123

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx = -\frac{\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h} + \frac{Bn\,\mathrm{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{(bf-ag)h}$$

output
```
-(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))*ln(1-(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x
+a))/(-a*g+b*f)/h+B*n*polylog(2,(-a*g+b*f)*(d*x+c)/(-c*g+d*f)/(b*x+a))/(-a
*g+b*f)/h
```

## 3.261.2 Mathematica [B] (verified)

Leaf count is larger than twice the leaf count of optimal. 303 vs. 2(123) = 246.

Time = 0.18 (sec) , antiderivative size = 303, normalized size of antiderivative = 2.46

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx = -\frac{-2A\log(a+bx)+Bn\log^2(a+bx)-2Bn\log(a+bx)\log(c+dx)+2Bn\log\left(\frac{d(a+bx)}{-bc+ad}\right)\log(c+dx)-2}{}$$

input
```
Integrate[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a*f*h + b*g*h*x^2 + h*
(b*f*x + a*g*x)),x]
```

output
```
-((-2*A*Log[a + b*x] + B*n*Log[a + b*x]^2 - 2*B*n*Log[a + b*x]*Log[c + d*x
] + 2*B*n*Log[(d*(a + b*x))/(-(b*c) + a*d)]*Log[c + d*x] - 2*B*Log[a + b*x
]*Log[(e*(a + b*x)^n)/(c + d*x)^n] + 2*A*Log[f + g*x] - 2*B*n*Log[a + b*x]
*Log[f + g*x] + 2*B*n*Log[c + d*x]*Log[f + g*x] + 2*B*Log[(e*(a + b*x)^n)/
(c + d*x)^n]*Log[f + g*x] + 2*B*n*Log[a + b*x]*Log[(b*(f + g*x))/(b*f - a*
g)] - 2*B*n*Log[c + d*x]*Log[(d*(f + g*x))/(d*f - c*g)] + 2*B*n*PolyLog[2,
 (g*(a + b*x))/(-(b*f) + a*g)] + 2*B*n*PolyLog[2, (b*(c + d*x))/(b*c - a*d
)] - 2*B*n*PolyLog[2, (g*(c + d*x))/(-(d*f) + c*g)])/((2*b*f - 2*a*g)*h))
```

### 3.261.3 Rubi [A] (warning: unable to verify)

Time = 0.62 (sec) , antiderivative size = 151, normalized size of antiderivative = 1.23, number of steps used = 6, number of rules used = 5, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.102, Rules used = {2973, 2976, 2026, 2779, 2838}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A}{h(agx+bfx)+afh+bghx^2}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{B\log \left(e(a+bx)^n(c+dx)^{-n}\right)+A}{h(agx+bfx)+afh+bghx^2}dx$$

$$\Big\downarrow \text{2976}$$

$$(bc-ad)\int \frac{A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)}{\frac{(bc-ad)(bf-ag)h(a+bx)}{c+dx}-\frac{(bc-ad)(df-cg)h(a+bx)^2}{(c+dx)^2}}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2026}$$

$$(bc-ad)\int \frac{(c+dx)\left(A+B\log \left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)}{(a+bx)\left((bc-ad)(bf-ag)h-\frac{(bc-ad)(df-cg)h(a+bx)}{c+dx}\right)}d\frac{a+bx}{c+dx}$$

$$\Big\downarrow \text{2779}$$

$$(bc-ad)\left(\frac{Bn\int \frac{(c+dx)\log\left(1-\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{a+bx}d\frac{a+bx}{c+dx}}{h(bc-ad)(bf-ag)}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{h(bc-ad)(bf-ag)}\right)$$

$$\Big\downarrow \text{2838}$$

$$(bc-ad)\left(\frac{Bn\,\text{PolyLog}\left(2,\frac{(bf-ag)(c+dx)}{(df-cg)(a+bx)}\right)}{h(bc-ad)(bf-ag)}-\frac{\log\left(1-\frac{(c+dx)(bf-ag)}{(a+bx)(df-cg)}\right)\left(B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)+A\right)}{h(bc-ad)(bf-ag)}\right)$$

input
```
Int[(A + B*Log[(e*(a + b*x)^n)/(c + d*x)^n])/(a*f*h + b*g*h*x^2 + h*(b*f*x
 + a*g*x)),x]
```

output
```
(b*c - a*d)*(-(((A + B*Log[e*((a + b*x)/(c + d*x))^n])*Log[1 - ((b*f - a*g
)*(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*c - a*d)*(b*f - a*g)*h)) + (B*n
*PolyLog[2, ((b*f - a*g)*(c + d*x))/((d*f - c*g)*(a + b*x))])/((b*c - a*d)
*(b*f - a*g)*h))
```

### 3.261.3.1 Defintions of rubi rules used

rule 2026
```
Int[(Fx_.)*(Px_)^(p_.), x_Symbol] :> With[{r = Expon[Px, x, Min]}, Int[x^(p
*r)*ExpandToSum[Px/x^r, x]^p*Fx, x] /; IGtQ[r, 0]] /; PolyQ[Px, x] && Integ
erQ[p] &&  !MonomialQ[Px, x] && (ILtQ[p, 0] ||  !PolyQ[u, x])
```

rule 2779
```
Int[((a_.) + Log[(c_.)*(x_)^(n_.)]*(b_.))^(p_.)/((x_)*((d_) + (e_.)*(x_)^(r
_.))), x_Symbol] :> Simp[(-Log[1 + d/(e*x^r)])*((a + b*Log[c*x^n])^p/(d*r))
, x] + Simp[b*n*(p/(d*r))   Int[Log[1 + d/(e*x^r)]*((a + b*Log[c*x^n])^(p -
 1)/x), x], x] /; FreeQ[{a, b, c, d, e, n, r}, x] && IGtQ[p, 0]
```

rule 2838
```
Int[Log[(c_.)*((d_) + (e_.)*(x_)^(n_.))]/(x_), x_Symbol] :> Simp[-PolyLog[2
, (-c)*e*x^n]/n, x] /; FreeQ[{c, d, e, n}, x] && EqQ[c*d, 1]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

3.261. $\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx$

rule 2976
```
Int[((A_.) + Log[(e_.)*(((a_.) + (b_.)*(x_))/((c_.) + (d_.)*(x_)))^(n_.)]*(
B_.))^(p_.)*(P2x_)^(m_.), x_Symbol] :> With[{f = Coeff[P2x, x, 0], g = Coef
f[P2x, x, 1], h = Coeff[P2x, x, 2]}, Simp[(b*c - a*d)   Subst[Int[(b^2*f -
a*b*g + a^2*h - (2*b*d*f - b*c*g - a*d*g + 2*a*c*h)*x + (d^2*f - c*d*g + c^
2*h)*x^2)^m*((A + B*Log[e*x^n])^p/(b - d*x)^(2*(m + 1))), x], x, (a + b*x)/
(c + d*x)], x]] /; FreeQ[{a, b, c, d, e, A, B, n}, x] && PolyQ[P2x, x, 2] &
& NeQ[b*c - a*d, 0] && IntegerQ[m] && IGtQ[p, 0]
```

### 3.261.4 Maple [C] (warning: unable to verify)

Result contains higher order function than in optimal. Order 9 vs. order 4.

Time = 25.90 (sec) , antiderivative size = 1447, normalized size of antiderivative = 11.76

| method | result | size |
|---|---|---|
| risch | Expression too large to display | 1447 |

input
```
int((A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x)),x,
method=_RETURNVERBOSE)
```

output
```
1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csg
n(I*(b*x+a)^n/((d*x+c)^n))-1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*e)*csgn
(I*(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/2*I/h/(a*g-b*f
)*ln(g*x+f)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((
d*x+c)^n))+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*e)*csgn(I*(b*x+a)^n/((d
*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)-1/h/(a*g-b*f)*ln(b*x+a)*A+1/h/(a
*g-b*f)*ln(g*x+f)*A+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*(b*x+a)^n/((d*
x+c)^n))^3+1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n
)^3-1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))^3-1/2*I
/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^3-1/2*I/h/(a*g
-b*f)*ln(b*x+a)*B*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I/h/(
a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2-
1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*
x+c)^n))^2+1/2*I/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I*(b*x+a)^n/((d*x+c)^n))*
csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2-1/2*I/h/(a*g-b*f)*ln(b*x+a)*B*Pi*csgn(I*
(b*x+a)^n/((d*x+c)^n))*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/h/(a*g-b*f)
*ln(g*x+f)*B*Pi*csgn(I*e)*csgn(I*e/((d*x+c)^n)*(b*x+a)^n)^2+1/2*I/h/(a*g-b
*f)*ln(g*x+f)*B*Pi*csgn(I*(b*x+a)^n)*csgn(I*(b*x+a)^n/((d*x+c)^n))^2+1/2*I
/h/(a*g-b*f)*ln(g*x+f)*B*Pi*csgn(I/((d*x+c)^n))*csgn(I*(b*x+a)^n/((d*x+c)^
n))^2-1/h*B*n/(a*g-b*f)*ln(g*x+f)*ln((b*(g*x+f)+a*g-b*f)/(a*g-b*f))+1/h...
```

### 3.261.5 Fricas [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx=\int \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A}{bghx^2+afh+(bfx+agx)h}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f
*x)),x, algorithm="fricas")
```

output
```
integral((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*g*h*x^2 + a*f*h + (b*f
+ a*g)*h*x), x)
```

### 3.261.6 Sympy [F(-1)]

Timed out.

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx=\text{Timed out}$$

input
```
integrate((A+B*ln(e*(b*x+a)**n/((d*x+c)**n)))/(a*f*h+b*g*h*x**2+h*(a*g*x+b
*f*x)),x)
```

output
```
Timed out
```

### 3.261.7 Maxima [F]

$$\int \frac{A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx=\int \frac{B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A}{bghx^2+afh+(bfx+agx)h}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f
*x)),x, algorithm="maxima")
```

output
```
A*(log(b*x + a)/((b*f - a*g)*h) - log(g*x + f)/((b*f - a*g)*h)) - B*integr
ate(-(log((b*x + a)^n) - log((d*x + c)^n) + log(e))/(b*g*h*x^2 + a*f*h + (
b*f*h + a*g*h)*x), x)
```

### 3.261.8 Giac [F]

$$\int \frac{A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx=\int \frac{B\log \left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A}{bghx^2+afh+(bfx+agx)h}\,dx$$

input
```
integrate((A+B*log(e*(b*x+a)^n/((d*x+c)^n)))/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f
*x)),x, algorithm="giac")
```

output
```
integrate((B*log((b*x + a)^n*e/(d*x + c)^n) + A)/(b*g*h*x^2 + a*f*h + (b*f
*x + a*g*x)*h), x)
```

### 3.261.9 Mupad [F(-1)]

Timed out.

$$\int \frac{A+B\log \left(e(a+bx)^n(c+dx)^{-n}\right)}{afh+bghx^2+h(bfx+agx)}\,dx=\int \frac{A+B\,\ln \left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)}{h\,\left(a\,g\,x+b\,f\,x\right)+a\,f\,h+b\,g\,h\,x^2}\,dx$$

input
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(h*(a*g*x + b*f*x) + a*f*h +
b*g*h*x^2),x)
```

output
```
int((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))/(h*(a*g*x + b*f*x) + a*f*h +
b*g*h*x^2), x)
```

# 3.262 $\int \frac{1}{(afh+bghx^2+h(bfx+agx))(A+B\log(e(a+bx)^n(c+dx)^{-n}))}\,dx$



## 3.262.1 Optimal result

Integrand size = 51, antiderivative size = 51

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\,(A+B\log{(e(a+bx)^n(c+dx)^{-n})})}\,dx = \frac{\text{Subst}\left(\text{Int}\left(\frac{1}{(a+bx)(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)},x\right),e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)}{h}$$

output
```
_eval(Unintegrable(1/(b*x+a)/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n)),x),e*
((b*x+a)/(d*x+c))^n = e*(b*x+a)^n/((d*x+c)^n))/h
```

## 3.262.2 Mathematica [N/A]

Not integrable

Time = 0.09 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\,(A+B\log{(e(a+bx)^n(c+dx)^{-n})})}\,dx = \int \frac{1}{(afh+bghx^2+h(bfx+agx))\,(A+B\log{(e(a+bx)^n(c+dx)^{-n})})}\,dx$$

input
```
Integrate[1/((a*f*h + b*g*h*x^2 + h*(b*f*x + a*g*x))*(A + B*Log[(e*(a + b*
x)^n)/(c + d*x)^n])),x]
```

output
```
Integrate[1/((a*f*h + b*g*h*x^2 + h*(b*f*x + a*g*x))*(A + B*Log[(e*(a + b*
x)^n)/(c + d*x)^n])), x]
```

### 3.262.3 Rubi [N/A]

Not integrable

Time = 0.75 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2973, 7292, 7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{\left(h(agx+bfx)+afh+bghx^2\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 2973}$$

$$\int \frac{1}{\left(h(agx+bfx)+afh+bghx^2\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 7292}$$

$$\int \frac{1}{\left(hx(ag+bf)+afh+bghx^2\right)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}dx$$

$$\Big\downarrow \text{ 7279}$$

$$\int \left(\frac{b}{h(a+bx)(bf-ag)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)}-\frac{g}{h(f+gx)(bf-ag)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right.}$$

$$\Big\downarrow \text{ 2009}$$

$$\frac{b\int \frac{1}{(a+bx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}dx}{h(bf-ag)}-\frac{g\int \frac{1}{(f+gx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))}dx}{h(bf-ag)}$$

input
```
Int[1/((a*f*h + b*g*h*x^2 + h*(b*f*x + a*g*x))*(A + B*Log[(e*(a + b*x)^n)/
(c + d*x)^n])),x]
```

output
```
$Aborted
```

### 3.262.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

## 3.262.4 Maple [N/A]

Not integrable

Time = 0.39 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{(afh + bgh\,x^2 + h\,(agx + bfx))\,\left(A + B\ln\left(e\,(bx + a)^n\,(dx + c)^{-n}\right)\right)}dx$$

input
```
int(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),
x)
```

output
```
int(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n))),
x)
```

### 3.262.5 Fricas [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 76, normalized size of antiderivative = 1.49

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bghx^2+afh+(bfx+agx)h)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*log(e*(b*x+a)^n/((d*x+c
)^n))),x, algorithm="fricas")
```

output
```
integral(1/(A*b*g*h*x^2 + A*a*f*h + (A*b*f + A*a*g)*h*x + (B*b*g*h*x^2 + B
*a*f*h + (B*b*f + B*a*g)*h*x)*log((b*x + a)^n*e/(d*x + c)^n)), x)
```

### 3.262.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \text{Timed out}$$

input
```
integrate(1/(a*f*h+b*g*h*x**2+h*(a*g*x+b*f*x))/(A+B*ln(e*(b*x+a)**n/((d*x+
c)**n))),x)
```

output
```
Timed out
```

### 3.262.7 Maxima [N/A]

Not integrable

Time = 0.34 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bghx^2+afh+(bfx+agx)h)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*log(e*(b*x+a)^n/((d*x+c
)^n))),x, algorithm="maxima")
```

output
```
integrate(1/((b*g*h*x^2 + a*f*h + (b*f*x + a*g*x)*h)*(B*log((b*x + a)^n*e/
(d*x + c)^n) + A)), x)
```

### 3.262.8 Giac [N/A]

Not integrable

Time = 0.31 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{(bghx^2+afh+(bfx+agx)h)\left(B\log\left(\frac{(bx+a)^ne}{(dx+c)^n}\right)+A\right)}\,dx$$

input
```
integrate(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*log(e*(b*x+a)^n/((d*x+c
)^n))),x, algorithm="giac")
```

output
```
integrate(1/((b*g*h*x^2 + a*f*h + (b*f*x + a*g*x)*h)*(B*log((b*x + a)^n*e/
(d*x + c)^n) + A)), x)
```

### 3.262.9 Mupad [N/A]

Not integrable

Time = 1.13 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)}\,dx = \int \frac{1}{\left(A+B\,\ln\left(\frac{e\,{\left(a+b\,x\right)}^n}{{\left(c+d\,x\right)}^n}\right)\right)\,\left(h\,\left(a\,g\,x+b\,f\,x\right)+a\,f\,h+b\,g\,h\,x^2\right)}\,dx$$

input
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(h*(a*g*x + b*f*x) + a*f*h
 + b*g*h*x^2)),x)
```

output
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))*(h*(a*g*x + b*f*x) + a*f*h
 + b*g*h*x^2)), x)
```

# 3.263 $\int \frac{1}{(afh+bghx^2+h(bfx+agx))(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}\,dx$



## 3.263.1 Optimal result

Integrand size = 51, antiderivative size = 51

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \frac{\text{Subst}\left(\text{Int}\left(\frac{1}{(a+bx)(f+gx)\left(A+B\log\left(e\left(\frac{a+bx}{c+dx}\right)^n\right)\right)^2},x\right),e\left(\frac{a+bx}{c+dx}\right)^n,e(a+bx)^n(c+dx)^{-n}\right)}{h}$$

output
```
_eval(Unintegrable(1/(b*x+a)/(g*x+f)/(A+B*ln(e*((b*x+a)/(d*x+c))^n))^2,x),
e*((b*x+a)/(d*x+c))^n = e*(b*x+a)^n/((d*x+c)^n))/h
```

## 3.263.2 Mathematica [N/A]

Not integrable

Time = 0.14 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx = \int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx$$

input
```
Integrate[1/((a*f*h + b*g*h*x^2 + h*(b*f*x + a*g*x))*(A + B*Log[(e*(a + b*
x)^n)/(c + d*x)^n])^2),x]
```

output
```
Integrate[1/((a*f*h + b*g*h*x^2 + h*(b*f*x + a*g*x))*(A + B*Log[(e*(a + b*
x)^n)/(c + d*x)^n])^2), x]
```

### 3.263.3 Rubi [N/A]

Not integrable

Time = 0.73 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {2973, 7292, 7279, 2009}

Below are the steps used by Rubi to obtain the solution. The rule number used for the transformation is given above next to the arrow. The rules definitions used are listed below.

$$\int \frac{1}{(h(agx+bfx)+afh+bghx^2)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{2973}$$

$$\int \frac{1}{(h(agx+bfx)+afh+bghx^2)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{7292}$$

$$\int \frac{1}{(hx(ag+bf)+afh+bghx^2)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}dx$$

$$\Big\downarrow \text{7279}$$

$$\int \left(\frac{b}{h(a+bx)(bf-ag)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right)+A\right)^2}-\frac{g}{h(f+gx)(bf-ag)\left(B\log\left(e(a+bx)^n(c+dx)^{-n}\right.\right.}\right.$$

$$\Big\downarrow \text{2009}$$

$$\frac{b\int \frac{1}{(a+bx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}dx}{h(bf-ag)}-\frac{g\int \frac{1}{(f+gx)(A+B\log(e(a+bx)^n(c+dx)^{-n}))^2}dx}{h(bf-ag)}$$

input
```
Int[1/((a*f*h + b*g*h*x^2 + h*(b*f*x + a*g*x))*(A + B*Log[(e*(a + b*x)^n)/
(c + d*x)^n])^2),x]
```

output
```
$Aborted
```

#### 3.263.3.1 Defintions of rubi rules used

rule 2009
```
Int[u_, x_Symbol] :> Simp[IntSum[u, x], x] /; SumQ[u]
```

rule 2973
```
Int[((A_.) + Log[(e_.)*(u_)^(n_.)*(v_)^(mn_)]*(B_.))^(p_.)*(w_.), x_Symbol]
 :> Subst[Int[w*(A + B*Log[e*(u/v)^n])^p, x], e*(u/v)^n, e*(u^n/v^n)] /; Fr
eeQ[{e, A, B, n, p}, x] && EqQ[n + mn, 0] && LinearQ[{u, v}, x] &&  !Intege
rQ[n]
```

rule 7279
```
Int[(u_)/((a_.) + (b_.)*(x_)^(n_.) + (c_.)*(x_)^(n2_.)), x_Symbol] :> With[
{v = RationalFunctionExpand[u/(a + b*x^n + c*x^(2*n)), x]}, Int[v, x] /; Su
mQ[v]] /; FreeQ[{a, b, c}, x] && EqQ[n2, 2*n] && IGtQ[n, 0]
```

rule 7292
```
Int[u_, x_Symbol] :> With[{v = NormalizeIntegrand[u, x]}, Int[v, x] /; v =!
= u]
```

### 3.263.4 Maple [N/A]

Not integrable

Time = 0.03 (sec) , antiderivative size = 51, normalized size of antiderivative = 1.00

$$\int \frac{1}{\left(afh+bgh\,x^{2}+h\left(agx+bfx\right)\right)\left(A+B\ln\left(e\left(bx+a\right)^{n}\left(dx+c\right)^{-n}\right)\right)^{2}}dx$$

input
```
int(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^
2,x)
```

output
```
int(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*ln(e*(b*x+a)^n/((d*x+c)^n)))^
2,x)
```

### 3.263.5 Fricas [N/A]

Not integrable

Time = 0.29 (sec) , antiderivative size = 145, normalized size of antiderivative = 2.84

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx$$
$$=\int \frac{1}{(bghx^2+afh+(bfx+agx)h)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*log(e*(b*x+a)^n/((d*x+c
)^n)))^2,x, algorithm="fricas")
```

output
```
integral(1/(A^2*b*g*h*x^2 + A^2*a*f*h + (A^2*b*f + A^2*a*g)*h*x + (B^2*b*g
*h*x^2 + B^2*a*f*h + (B^2*b*f + B^2*a*g)*h*x)*log((b*x + a)^n*e/(d*x + c)^
n)^2 + 2*(A*B*b*g*h*x^2 + A*B*a*f*h + (A*B*b*f + A*B*a*g)*h*x)*log((b*x +
a)^n*e/(d*x + c)^n)), x)
```

### 3.263.6 Sympy [F(-1)]

Timed out.

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx=\text{Timed out}$$

input
```
integrate(1/(a*f*h+b*g*h*x**2+h*(a*g*x+b*f*x))/(A+B*ln(e*(b*x+a)**n/((d*x+
c)**n)))**2,x)
```

output
```
Timed out
```

### 3.263.7 Maxima [N/A]

Not integrable

Time = 0.41 (sec) , antiderivative size = 506, normalized size of antiderivative = 9.92

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx$$
$$=\int \frac{1}{(bghx^2+afh+(bfx+agx)h)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*log(e*(b*x+a)^n/((d*x+c
)^n)))^2,x, algorithm="maxima")
```

output
```
(d*f - c*g)*integrate(1/((b*c*f^2*h*n - a*d*f^2*h*n)*A*B + (b*c*f^2*h*n*lo
g(e) - a*d*f^2*h*n*log(e))*B^2 + ((b*c*g^2*h*n - a*d*g^2*h*n)*A*B + (b*c*g
^2*h*n*log(e) - a*d*g^2*h*n*log(e))*B^2)*x^2 + 2*((b*c*f*g*h*n - a*d*f*g*h
*n)*A*B + (b*c*f*g*h*n*log(e) - a*d*f*g*h*n*log(e))*B^2)*x + ((b*c*g^2*h*n
 - a*d*g^2*h*n)*B^2*x^2 + 2*(b*c*f*g*h*n - a*d*f*g*h*n)*B^2*x + (b*c*f^2*h
*n - a*d*f^2*h*n)*B^2)*log((b*x + a)^n) - ((b*c*g^2*h*n - a*d*g^2*h*n)*B^2
*x^2 + 2*(b*c*f*g*h*n - a*d*f*g*h*n)*B^2*x + (b*c*f^2*h*n - a*d*f^2*h*n)*B
^2)*log((d*x + c)^n)), x) - (d*x + c)/((b*c*f*h*n - a*d*f*h*n)*A*B + (b*c*
f*h*n*log(e) - a*d*f*h*n*log(e))*B^2 + ((b*c*g*h*n - a*d*g*h*n)*A*B + (b*c
*g*h*n*log(e) - a*d*g*h*n*log(e))*B^2)*x + ((b*c*g*h*n - a*d*g*h*n)*B^2*x
+ (b*c*f*h*n - a*d*f*h*n)*B^2)*log((b*x + a)^n) - ((b*c*g*h*n - a*d*g*h*n)
*B^2*x + (b*c*f*h*n - a*d*f*h*n)*B^2)*log((d*x + c)^n))
```

### 3.263.8 Giac [N/A]

Not integrable

Time = 0.35 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx$$
$$=\int \frac{1}{(bghx^2+afh+(bfx+agx)h)\left(B\log\left(\frac{(bx+a)^n e}{(dx+c)^n}\right)+A\right)^2}\,dx$$

input
```
integrate(1/(a*f*h+b*g*h*x^2+h*(a*g*x+b*f*x))/(A+B*log(e*(b*x+a)^n/((d*x+c
)^n)))^2,x, algorithm="giac")
```

output
```
integrate(1/((b*g*h*x^2 + a*f*h + (b*f*x + a*g*x)*h)*(B*log((b*x + a)^n*e/
(d*x + c)^n) + A)^2), x)
```

### 3.263.9 Mupad [N/A]

Not integrable

Time = 1.21 (sec) , antiderivative size = 53, normalized size of antiderivative = 1.04

$$\int \frac{1}{(afh+bghx^2+h(bfx+agx))\left(A+B\log\left(e(a+bx)^n(c+dx)^{-n}\right)\right)^2}\,dx$$
$$=\int \frac{1}{\left(A+B\,\ln\left(\frac{e\,(a+b\,x)^n}{(c+d\,x)^n}\right)\right)^2\,\left(h\,\left(a\,g\,x+b\,f\,x\right)+a\,f\,h+b\,g\,h\,x^2\right)}\,dx$$

input
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(h*(a*g*x + b*f*x) + a*f
*h + b*g*h*x^2)),x)
```

output
```
int(1/((A + B*log((e*(a + b*x)^n)/(c + d*x)^n))^2*(h*(a*g*x + b*f*x) + a*f
*h + b*g*h*x^2)), x)
```

CHAPTER 4

# Appendix



## 4.1 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

### 4.1.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
```

```
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A"," "}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A"," "}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]
```

```
(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]
```

```
ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

### 4.1.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
```

```
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);

        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;
```

```
if ExpnType_result<=ExpnType_optimal then
    if debug then
         print("ExpnType_result<=ExpnType_optimal");
    fi;
    if is_contains_complex(result) then
        if is_contains_complex(optimal) then
            if debug then
                    print("both result and optimal complex");
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                return "A"," ";
            else
               return "B",cat("Both result and optimal contain complex but leaf count of
                               convert(leaf_count_result,string)," vs. $2 (",
                               convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_
            end if
        else #result contains complex but optimal is not
            if debug then
                    print("result contains complex but optimal is not");
            fi;
            return "C","Result contains complex when optimal does not.";
        fi;
    else # result do not contain complex
         # this assumes optimal do not as well. No check is needed here.
        if debug then
               print("result do not contain complex, this assumes optimal do not as well"
        fi;
        if leaf_count_result<=2*leaf_count_optimal then
            if debug then
                print("leaf_count_result<=2*leaf_count_optimal");
            fi;
            return "A"," ";
        else
            if debug then
                print("leaf_count_result>2*leaf_count_optimal");
            fi;
            return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
        fi;
    fi;
```

```
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
```

```
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
```

```
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

### 4.1.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
```

```
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]

def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
          return True
        else:
          return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
```

```
        return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
```

```
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is large
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(le
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_


    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```

### 4.1.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
```

```
            return False
    else:
        return False

def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m
```

```
def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in sagemath

def is_atom(expn):

    #debug=False
    if debug:
        print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
```

```
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:   #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:   #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))   #max(2,expnType(expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinst
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):
```

```
if debug:
    print ("Enter grade_antiderivative for sagemath")
    print("Enter grade_antiderivative, result=",result)
    print("Enter grade_antiderivative, optimal=",optimal)
    print("type(anti)=",type(result))
    print("type(optimal)=",type(optimal))

leaf_count_result  = tree_size(result) #leaf_count(result)
leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

#if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


expnType_result  = expnType(result)
expnType_optimal = expnType(optimal)

if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation =" "
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation =" "
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)
```

```
    return grade, grade_annotation
```